Two Coupled Flux Qubits with Controllable Interaction

- Introduction
- Experiments on a single qubit
- Controllable coupling of two flux qubits: theory
- Experiments on two flux qubits
  - Configuration and characterization
  - Coupled qubit spectroscopy
  - Coupled qubit manipulation
- Concluding remarks and future plans

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KITP
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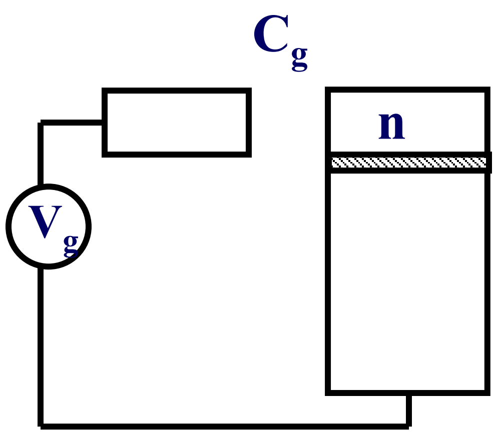
Types of Superconducting Qubits

Charge Qubit

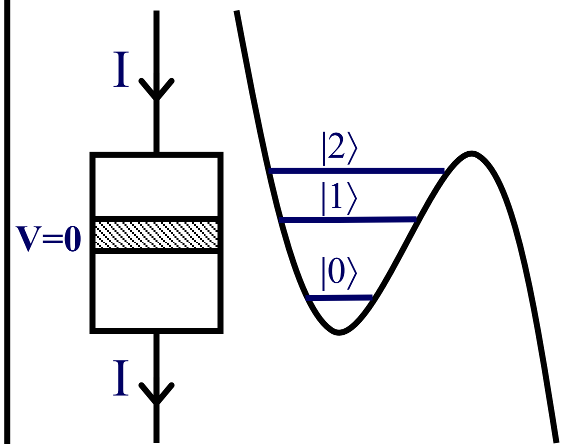


Pair charge on island

$|\mathbf{n}\rangle, \quad |\mathbf{n+1}\rangle$

Saclay
Yale
NEC
Chalmers
JPL

Phase Qubit

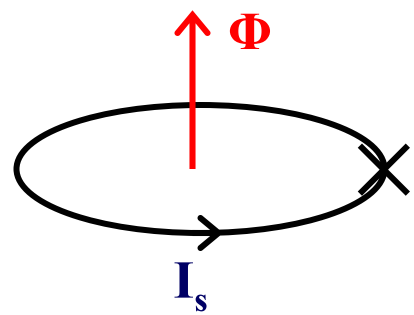


Phase across junction

$|\mathbf{0}\rangle, \quad |\mathbf{1}\rangle$

NIST
UC Santa Barbara
University of Maryland

Flux Qubit



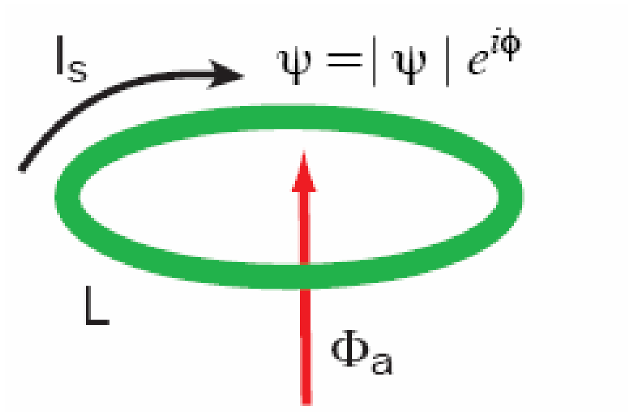
Current around loop

$|\circlearrowleft\rangle, \quad |\circlearrowright\rangle$

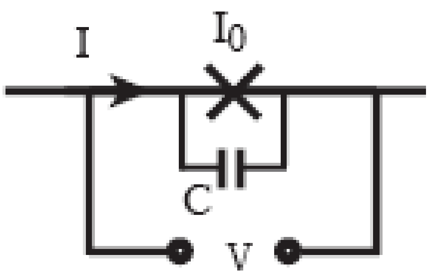
Stony Brook
TU Delft
NEC
NTT
MIT
IPHT Jena
UC Berkeley

Flux Quantization

- Pair condensate is described by a single quantum mechanical wave function
- Wave function ψ must be single valued
- Total flux $\Phi = \Phi_a - LI_s$
- $\Phi = n\Phi_0$, where $\Phi_0 \equiv h/2e \approx 2.07 \times 10^{-15}$ Tm^2

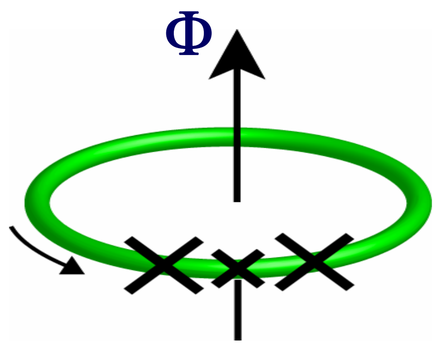


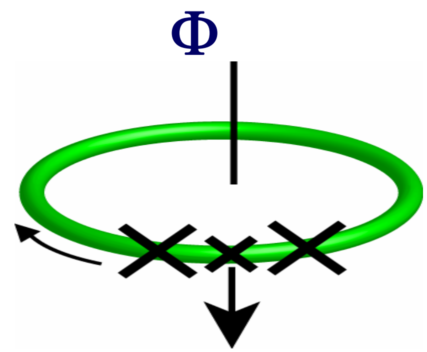
Josephson Tunneling



- $I = I_0 \sin \delta$
- $V = (\hbar/2e)d\delta/dt = (\Phi_0/2\pi)d\delta/dt$
- δ is phase difference across the barrier

Three-Junction Flux Qubit





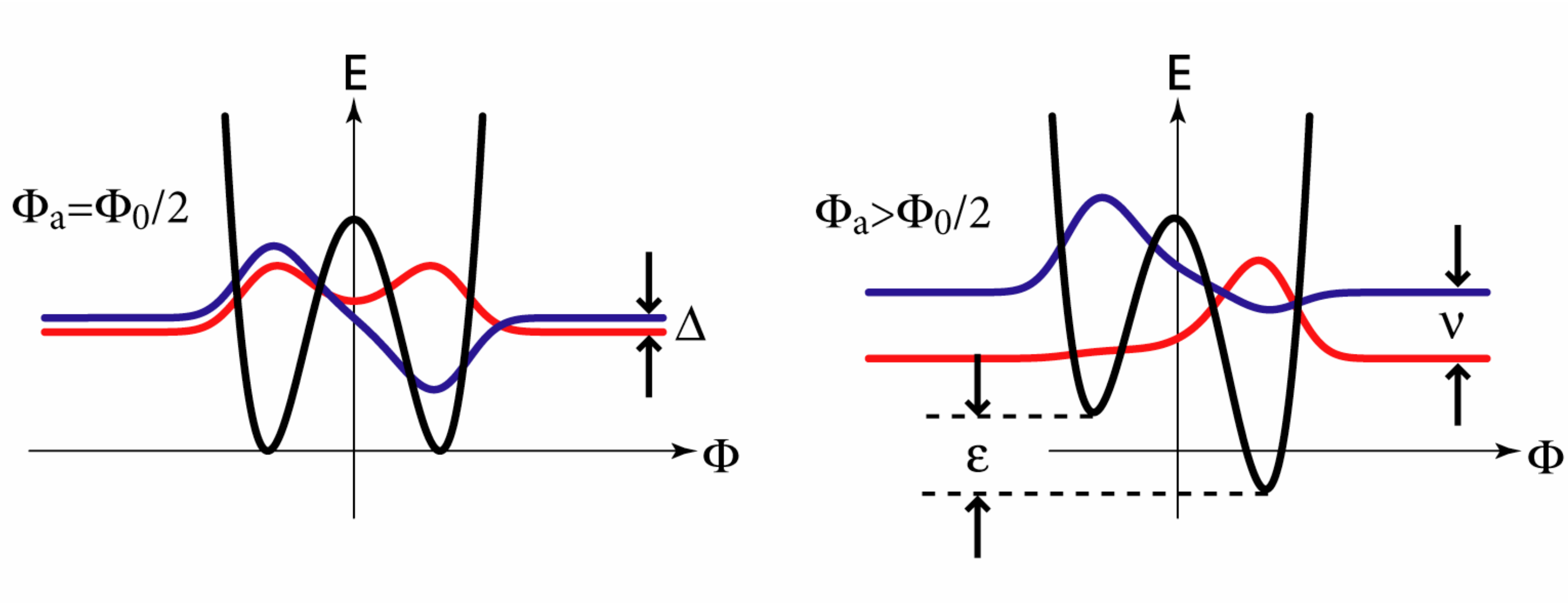
Degeneracy point $\Phi_a = \Phi_0/2$ $\uparrow$ $\Phi_a = \Phi_0/2$ $\uparrow$

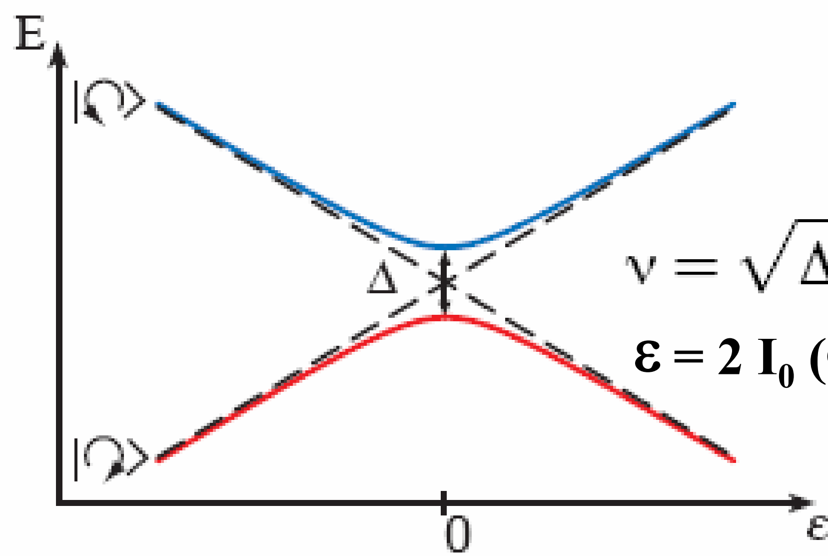
$$|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

J.E. Mooij *et al.*, *Science* **285**, 1036 (1999)
C.H. Van der Wal *et al.*, *Science* **290**, 773 (2000)

- Loop inductance << Josephson inductance $\Phi_0/2\pi I_0$

Energy of the Flux Qubit

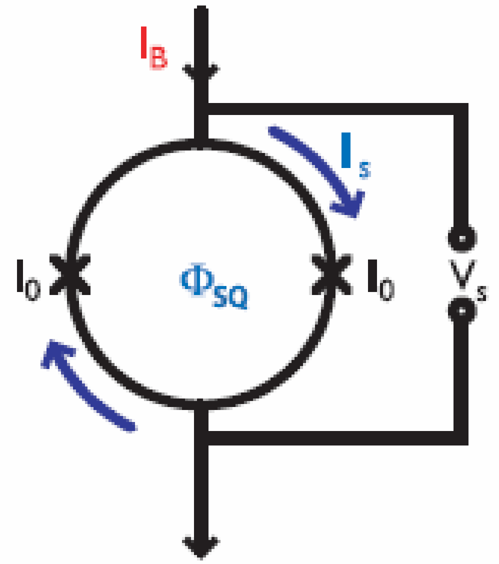




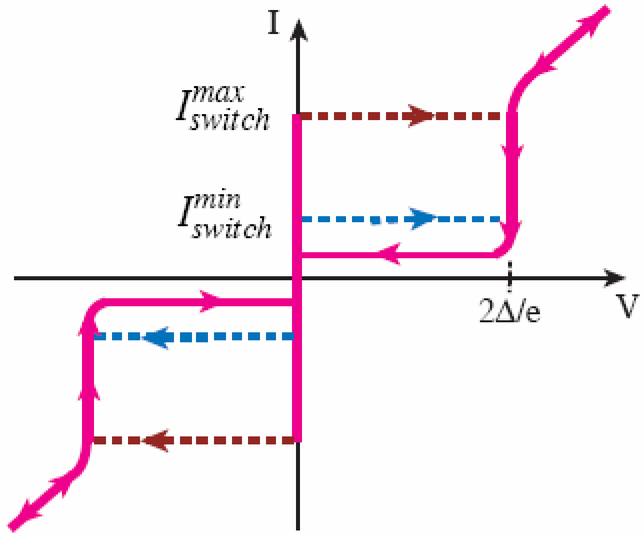
$$\nu = \sqrt{\Delta^2 + \varepsilon^2}$$

$$\boldsymbol{\varepsilon = 2\, I_0\, (\Phi_a - \Phi_0/2)}$$

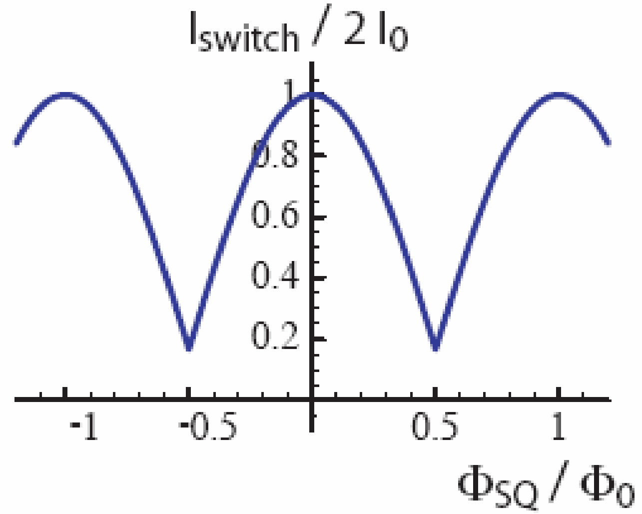
The dc SQUID



I_B
I_s
I_0
Φ_{SQ}
I_0
V_s



I
I_{switch}^{max}
I_{switch}^{min}
$2\Delta/e$
V

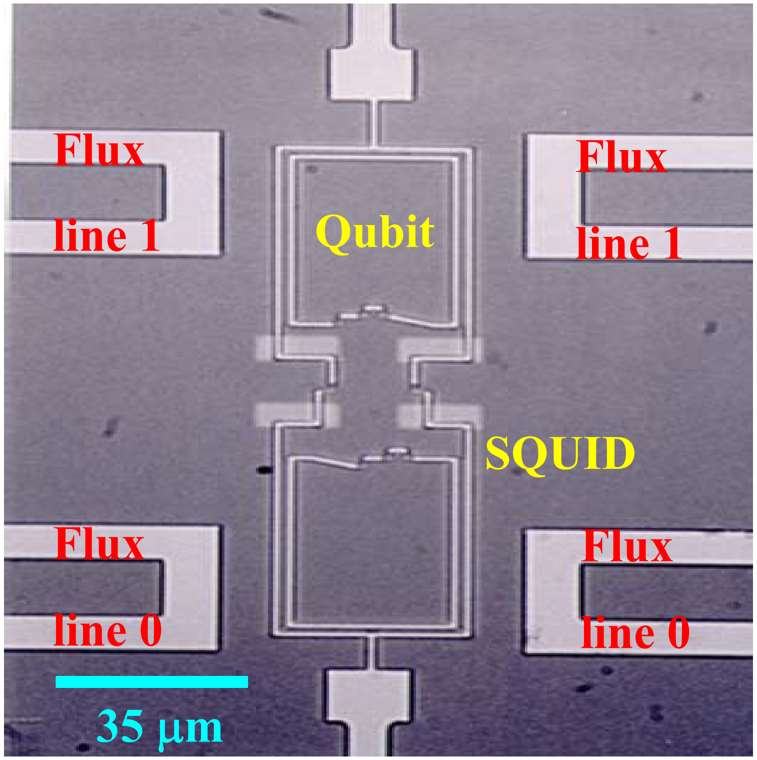


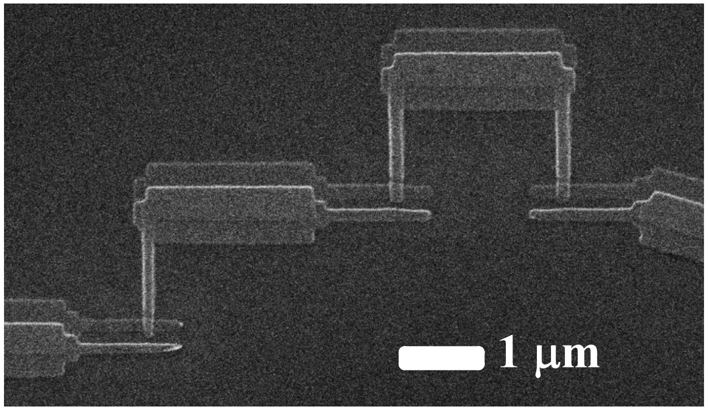
$I_{switch} / 2 I_0$
1
0.8
0.6
0.4
0.2
-1
-0.5
0.5
1
Φ_{SQ} / Φ_0

Experiments on a Single Flux Qubit

- Integrated design that can be scaled to many qubits with *controllable* interactions between them
- Multiple on-chip flux lines with independent current sources for separate bias of SQUID and qubit(s)
- Large geometrical inductance of qubit loop (~150 pH) to keep flux bias currents small
- Chip enclosed in a superconducting cavity to stabilize the magnetic field

Fabrication of Flux Qubits and SQUID

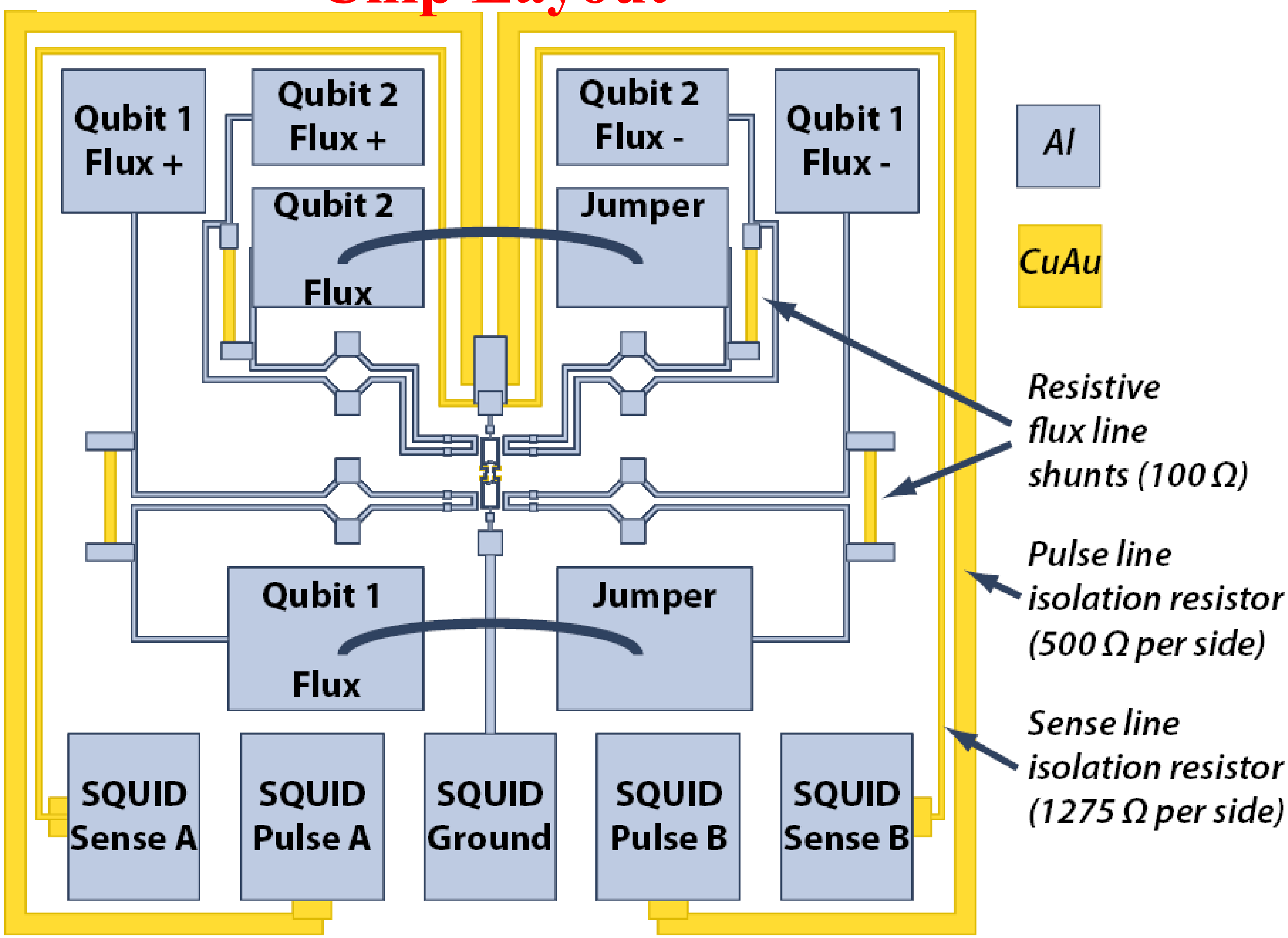




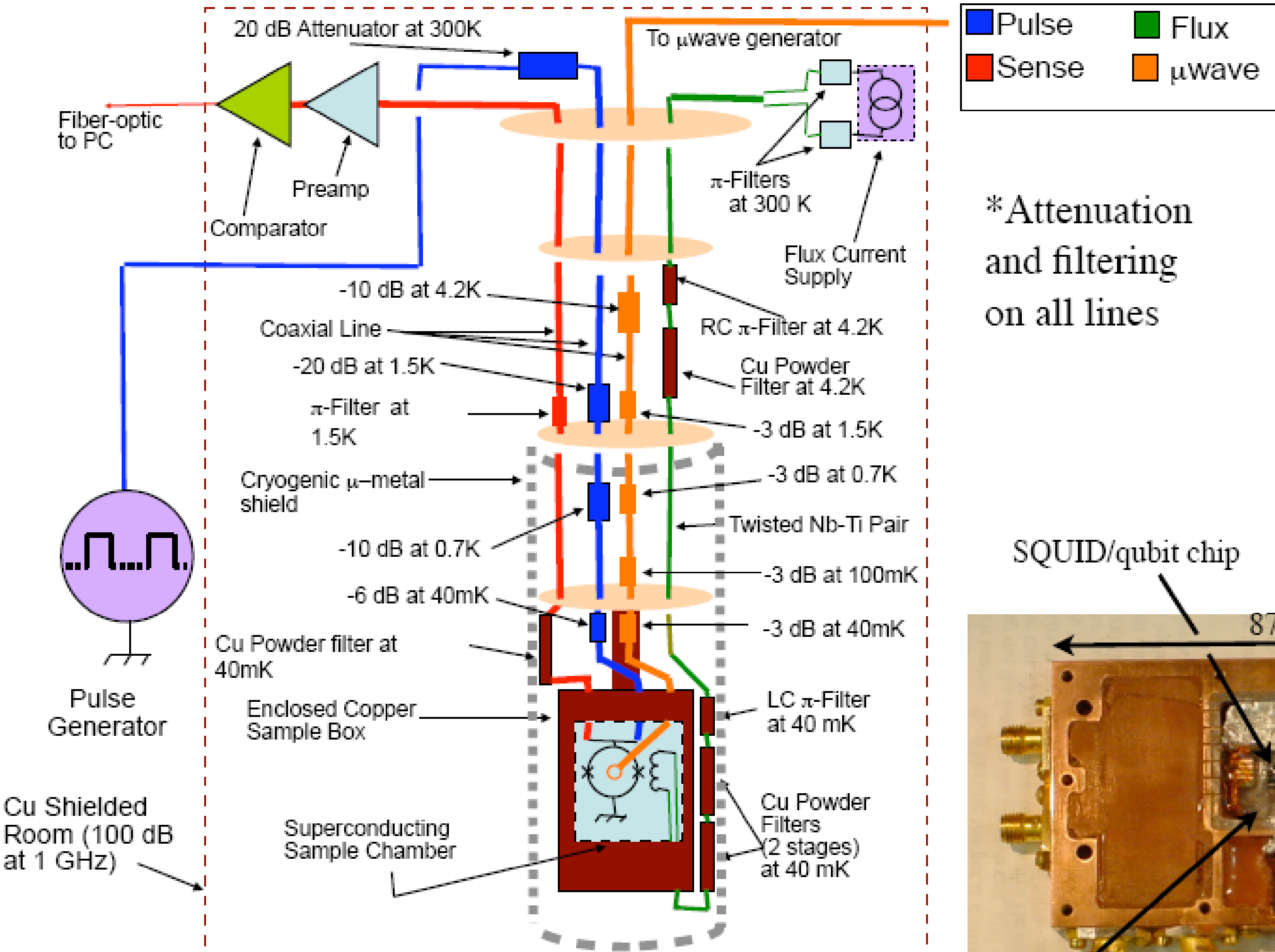
Qubit Junctions

- Electron-beam lithography
- Angled evaporation
- Al-AlOx-Al tunnel junctions

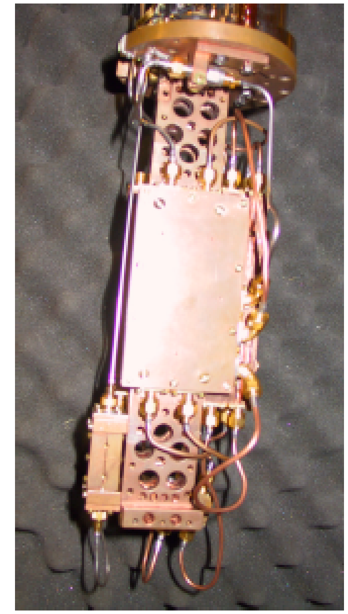
Chip Layout

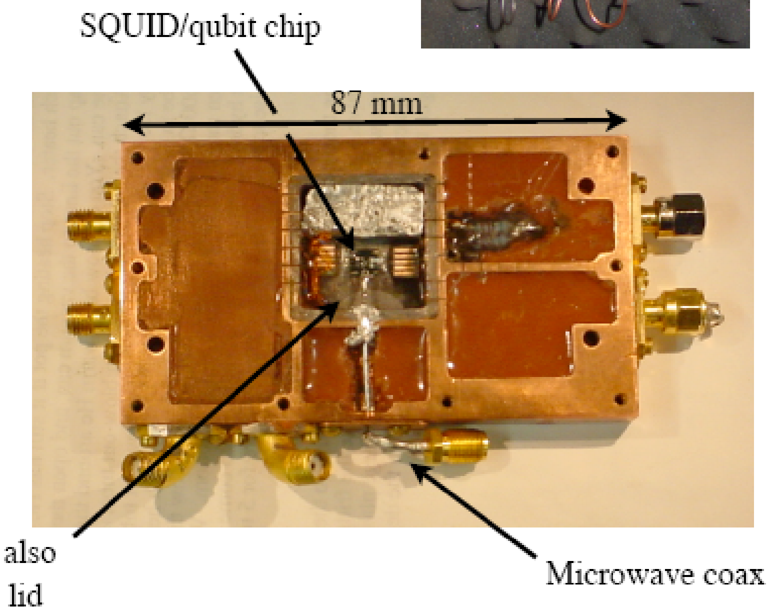

Qubit 1 Flux +
Qubit 2 Flux +
Qubit 2 Flux
Qubit 2 Flux -
Qubit 1 Flux -
Jumper
Qubit 1 Flux
Jumper
SQUID Sense A
SQUID Pulse A
SQUID Ground
SQUID Pulse B
SQUID Sense B
Al
CuAu
Resistive flux line shunts (100 Ω)
Pulse line isolation resistor (500 Ω per side)
Sense line isolation resistor (1275 Ω per side)

Measurement Configuration



*Attenuation and filtering on all lines

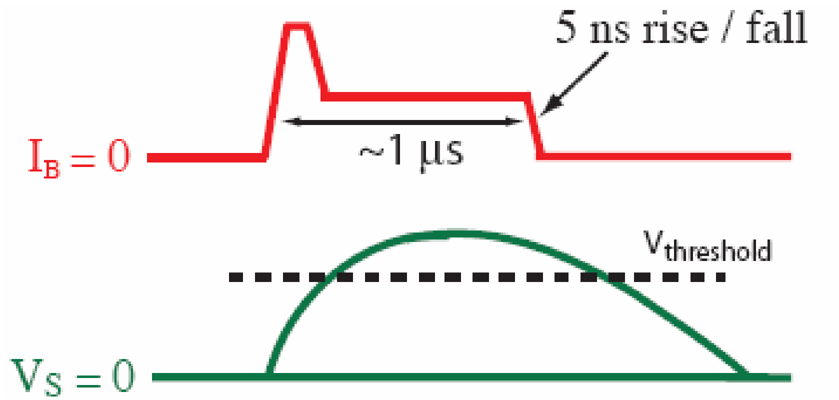




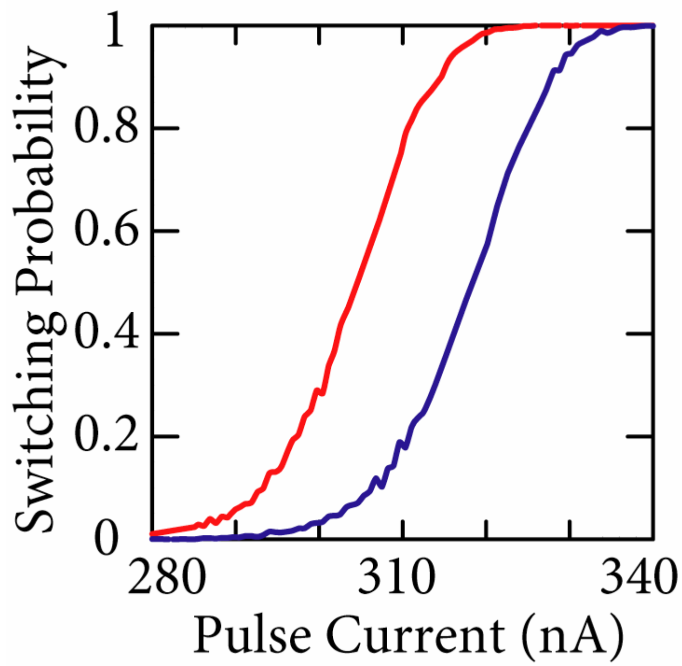
- Stable, low-noise, battery-operated current supply for setting flux bias
- Two-stage magnetic shielding

*Expect transverse cavity resonance around 6.7 GHz

SQUID Readout



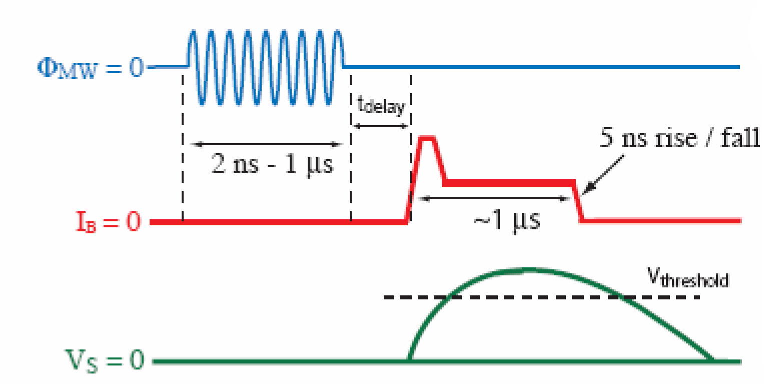
- Pulse bias current: detect switching events
- Repeat (say) 1000 times to determine probability
- Increment bias current and repeat



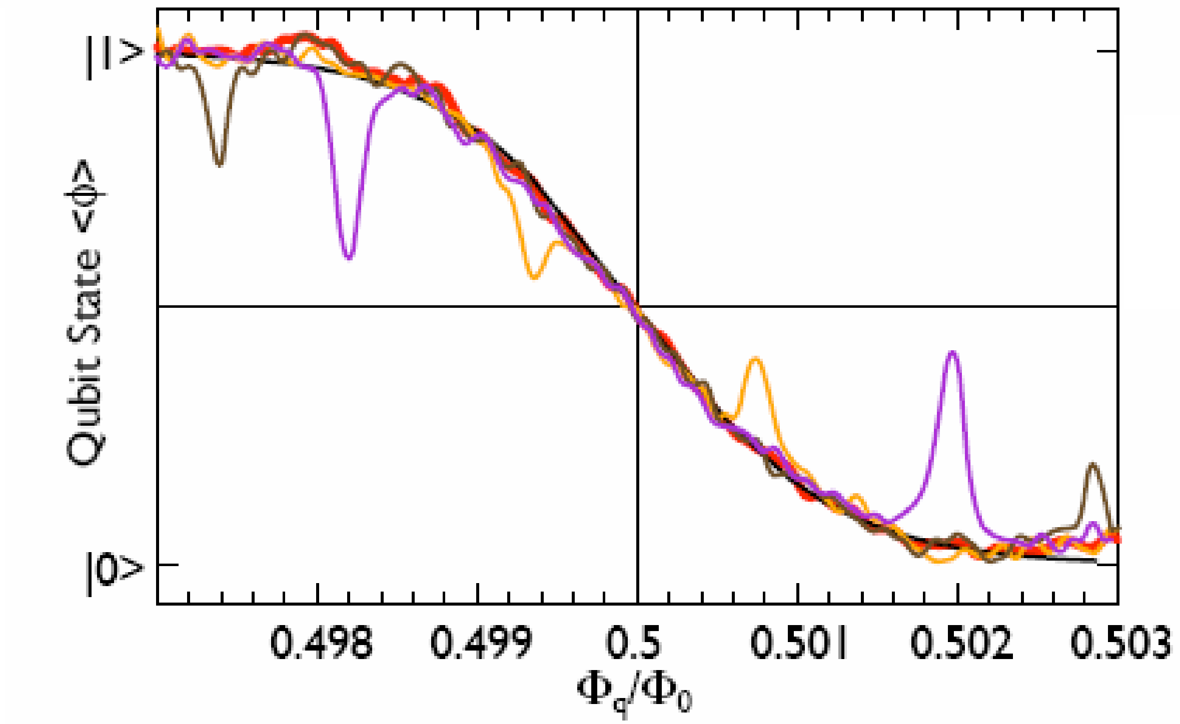
$\Phi_{QA} = 0.48\ \Phi_0$
$\Phi_{QA} = 0.52\ \Phi_0$
$\Phi_S = \text{constant}$

- Determine current $I_S^{50\%}$ for 50% switching probability

Spectroscopy



Three microwave frequencies

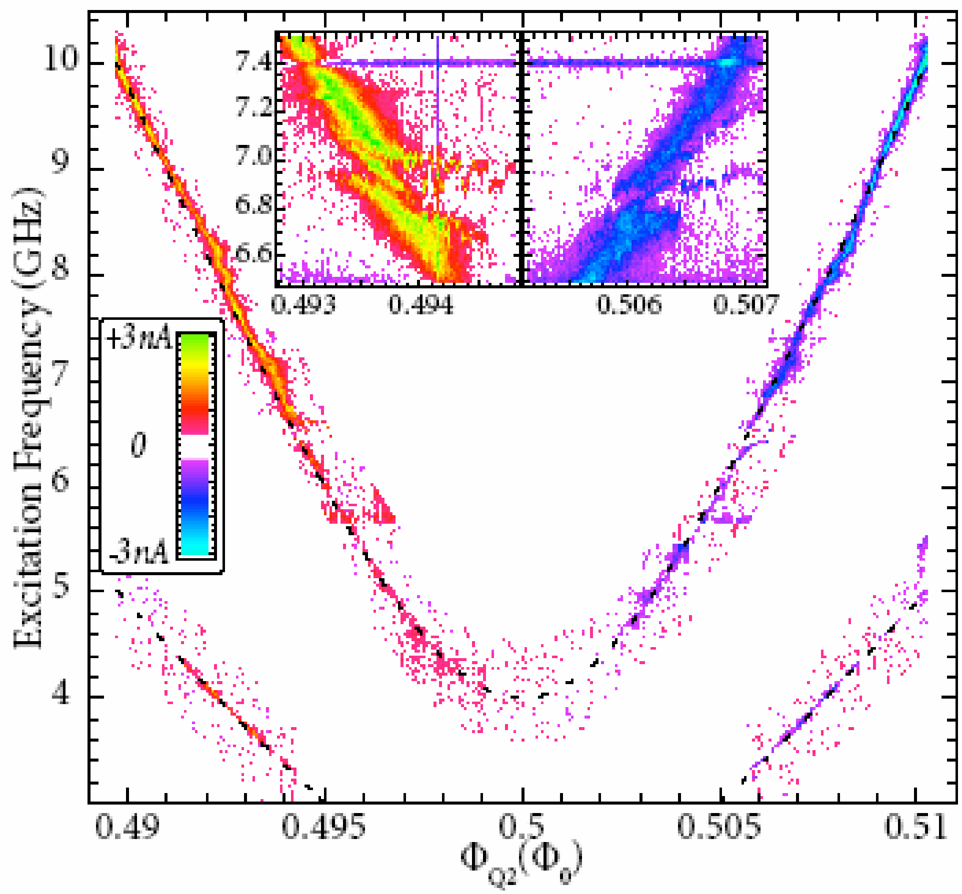


Qubit Spectroscopy

- Enhancement and suppression of $I_s^{50\%}$ relative to values without microwaves
- Dashed lines are fitted to hyperbolic dispersion for 1- and 2-photon qubit excitations
- Fitting values: $\Delta = 3.99$ GHz

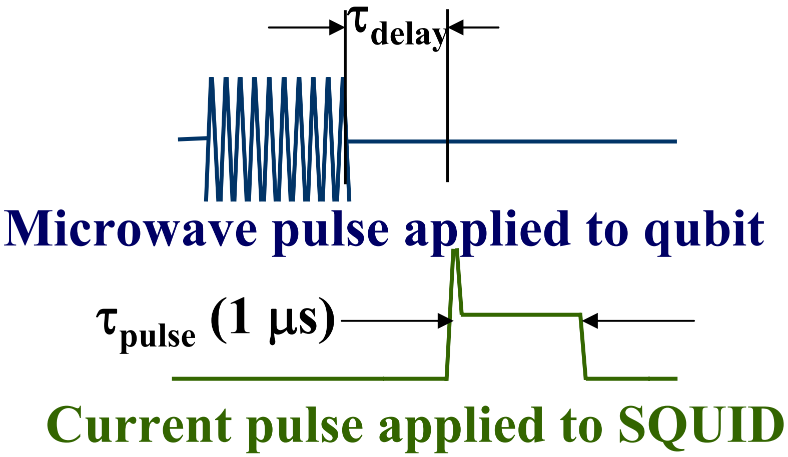
$$\frac{d\varepsilon}{d\Phi} = 896 \frac{\text{MHz}}{\text{m}\Phi_0}$$

- Number of points:
  75,000 (main panel)
  23,000 (inset)

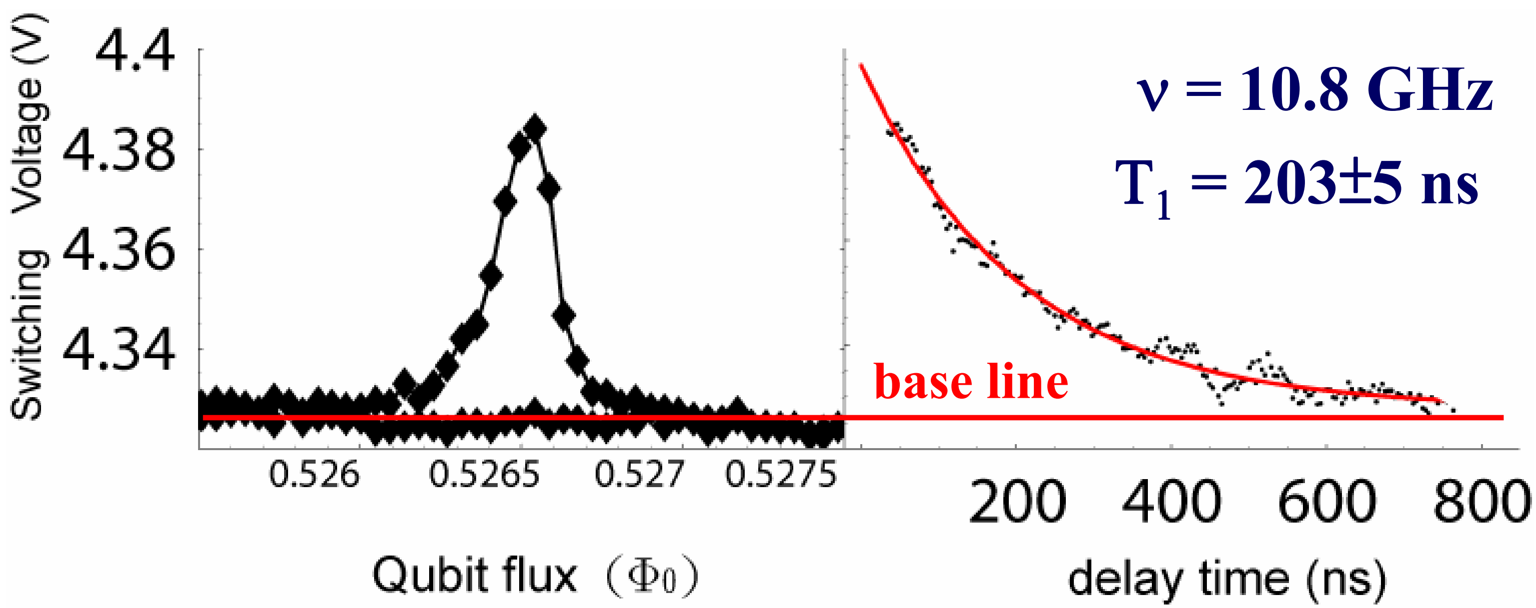


Time Domain Measurements:
Coherent Manipulation of Qubit Flux

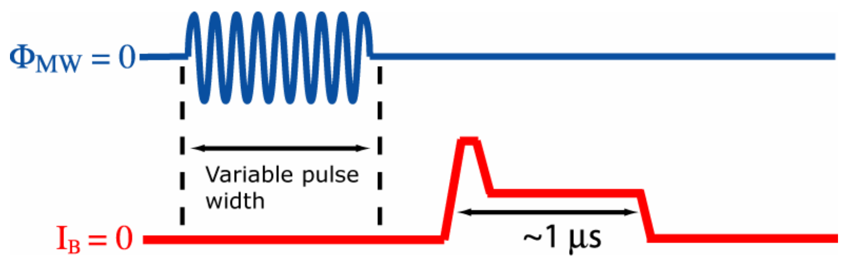
Measurement of Qubit Relaxation Time T_1



- Tune microwaves to the level splitting, vary τ_{delay}
- Peak height ~ $\exp(-\tau_{delay}/T_1)$
- Range of T_1 is 200 ~ 400 ns

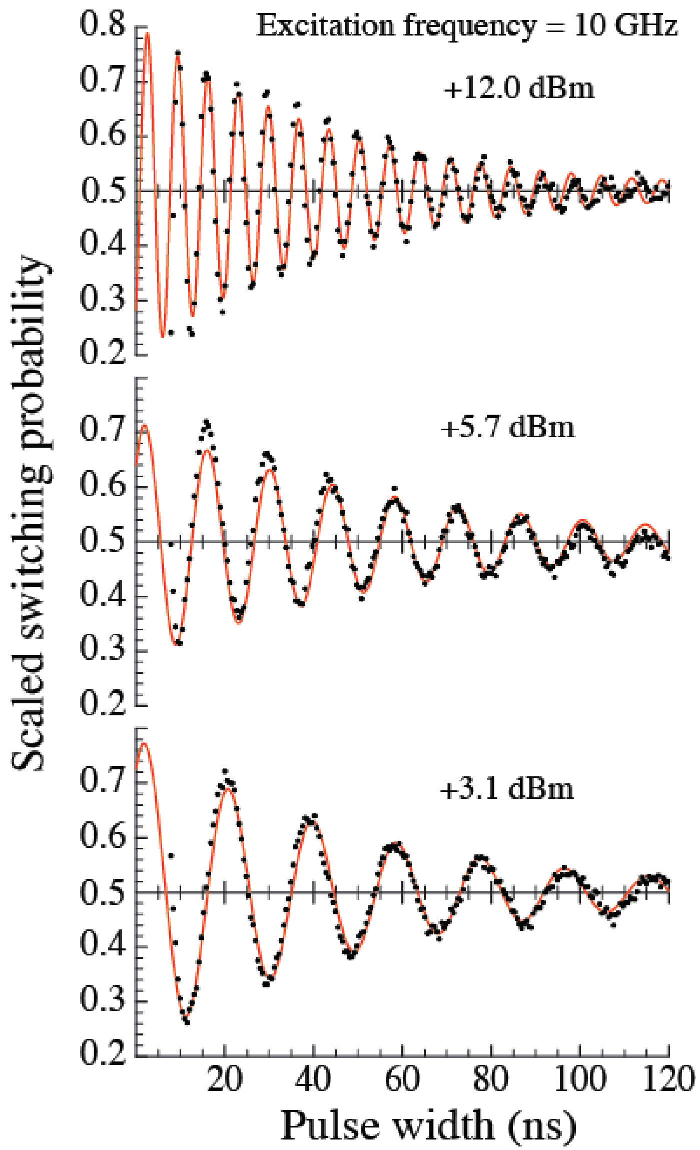


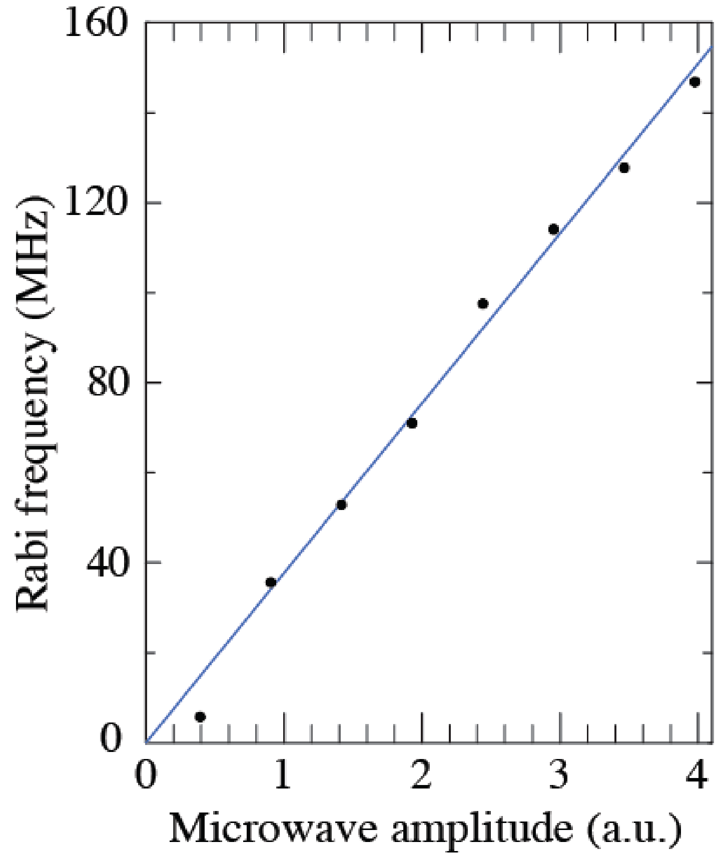
Measurement of Rabi Oscillations



- Apply on-resonance microwave pulses of variable width.
- Pulses drive qubit in coherent oscillations between its two states.

Rabi Oscillations





Dephasing times

T_2 **– dephasing time (homogeneous broadening)**

– measure with echo

T_2' **– due to inhomogeneous broadening**

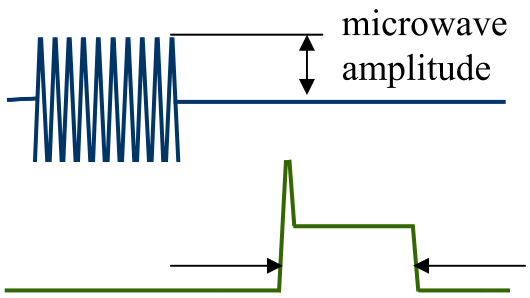
– measure from linewidth

T_2^* **– given by**

$$\frac{1}{T_2^*} \equiv \frac{1}{T_2'} + \frac{1}{T_2}$$

– measure with Ramsey fringes

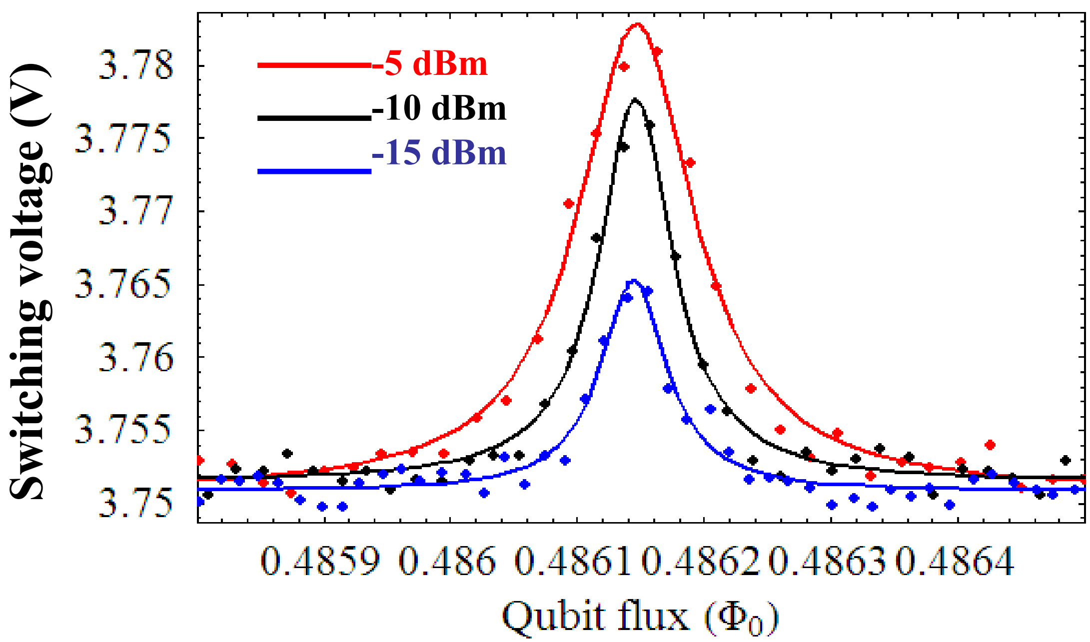
Measurement of T_2' from linewidth



τ_{pulse} **(1 μs)**

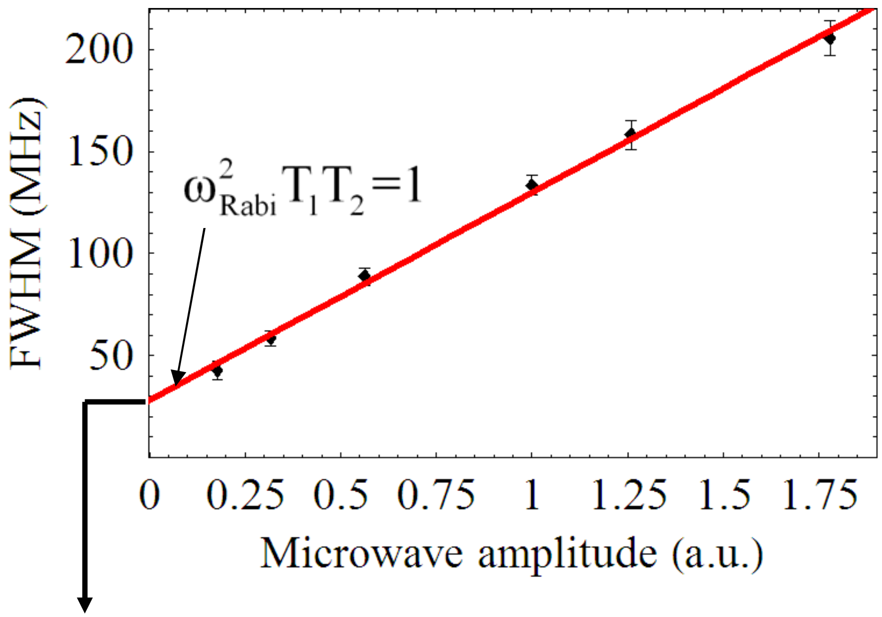
Make τ_{delay} as short as possible (~ 5ns)

Measure the absorption peak for different microwave amplitudes.



Measurement of T_2' from linewidth

A. Abragam, *The Principles of Nuclear Magnetism:*



T_2' **=11 ± 1ns**

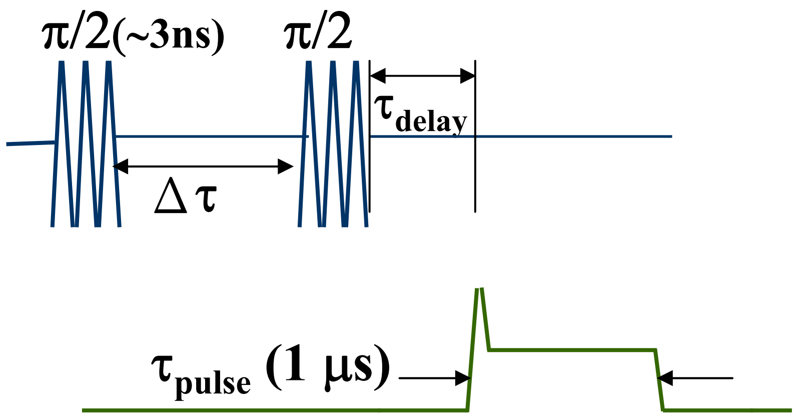
$$\mathrm{FWHM} = \frac{1}{\pi}\left(\frac{1}{T_2'} + \sqrt{\frac{1}{T_2^2} + \omega^2_{Rabi}\frac{T_1}{T_2}}\right)$$

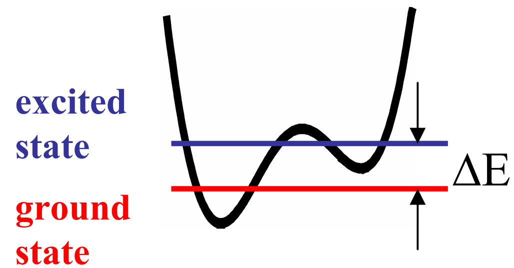
$$\approx \begin{cases} \dfrac{1}{\pi}\left(\dfrac{1}{T_2'} + \omega_{Rabi}\sqrt{\dfrac{T_1}{T_2}}\right) & \omega^2_{Rabi} T_1 T_2 \gg 1 \quad \text{(strong driving limit)} \\[2ex] \dfrac{1}{\pi}\left(\dfrac{1}{T_2'} + \dfrac{1}{T_2} + \dfrac{1}{2}\omega^2_{Rabi} T_1\right) & \omega^2_{Rabi} T_1 T_2 \ll 1 \quad \text{(weak driving limit)} \end{cases}$$

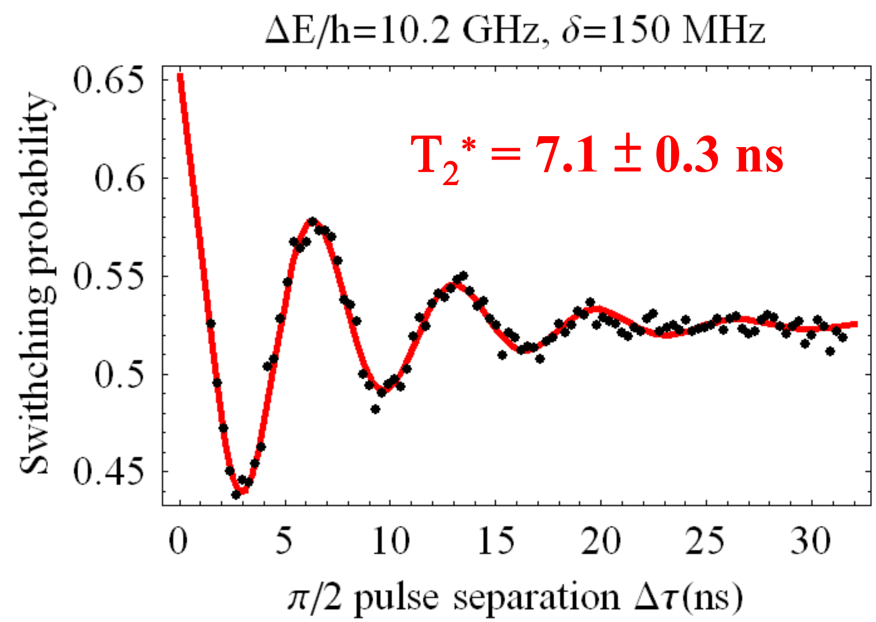
Measurements are in the strong driving regime

Measurement of T_2^* from Ramsey Oscillations

Microwave pulses







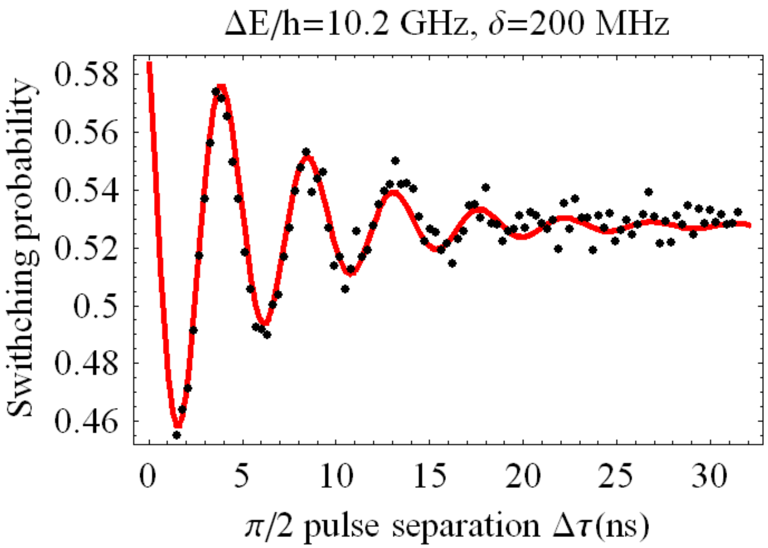
Choose qubit flux to fix ΔE

Make τ_{delay} as short as possible (~ 5ns)

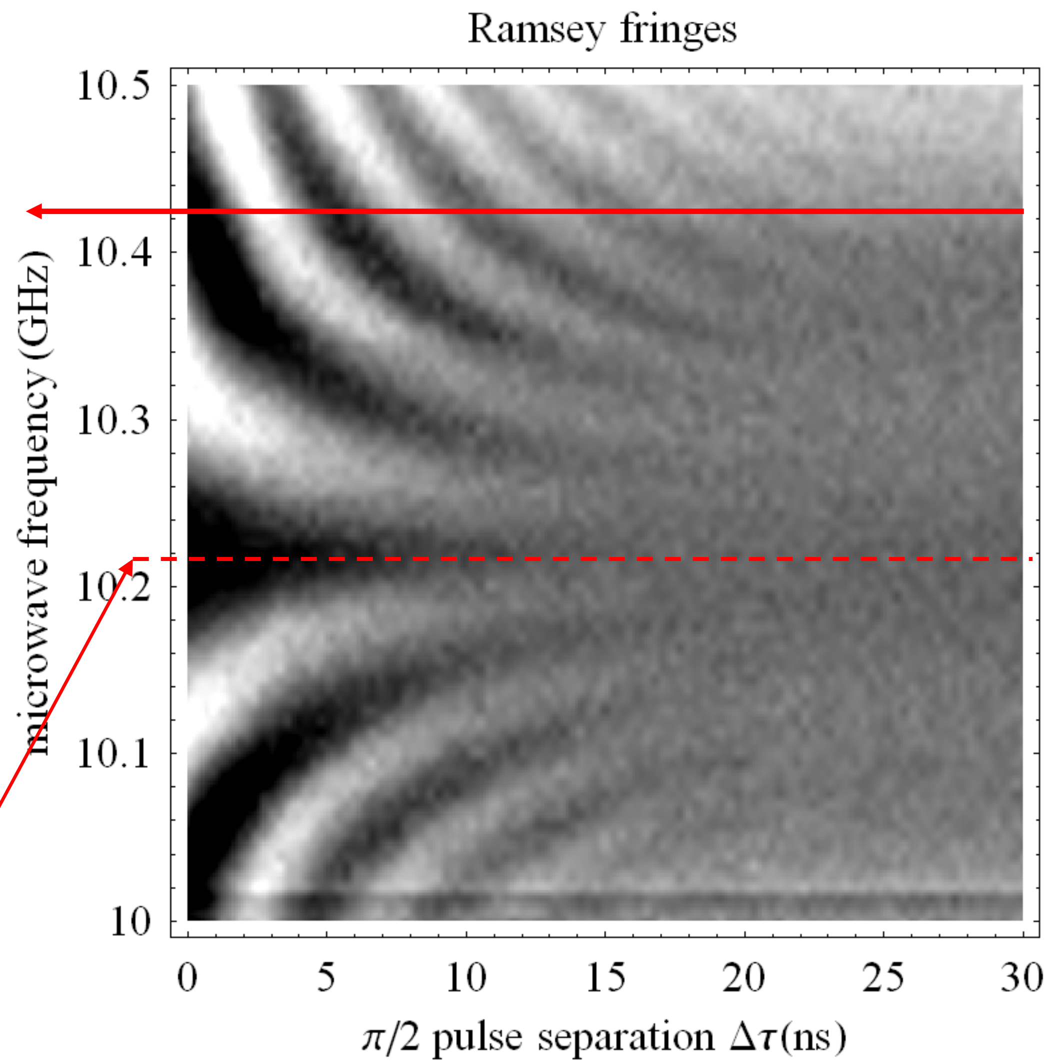
Tune microwave frequency slightly off-resonance: $\nu = \Delta E/h + \delta$

Measure the dependence of the switching probability on $\Delta\tau$

Measurement of T_2^* from Ramsey Oscillations

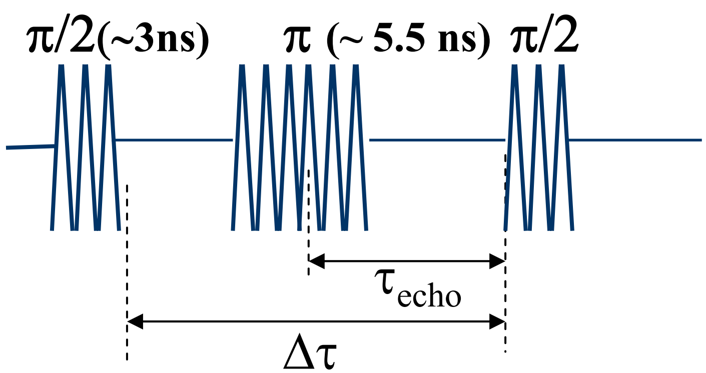


Measured 80 Sets of Ramsey oscillations with detuning δ from 300 MHz to - 200 MHz

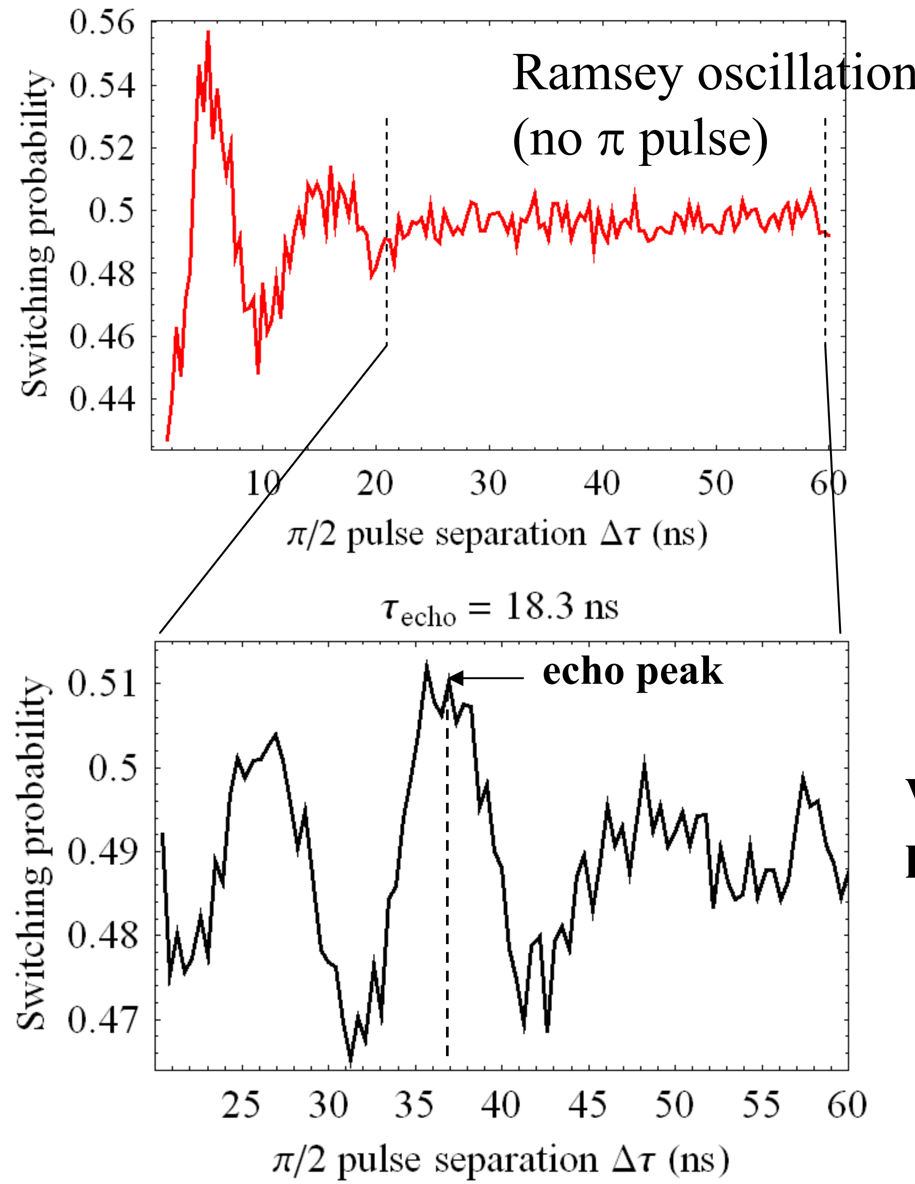


True resonance:
10.22 GHz

Measurement of T_2 from spin echo

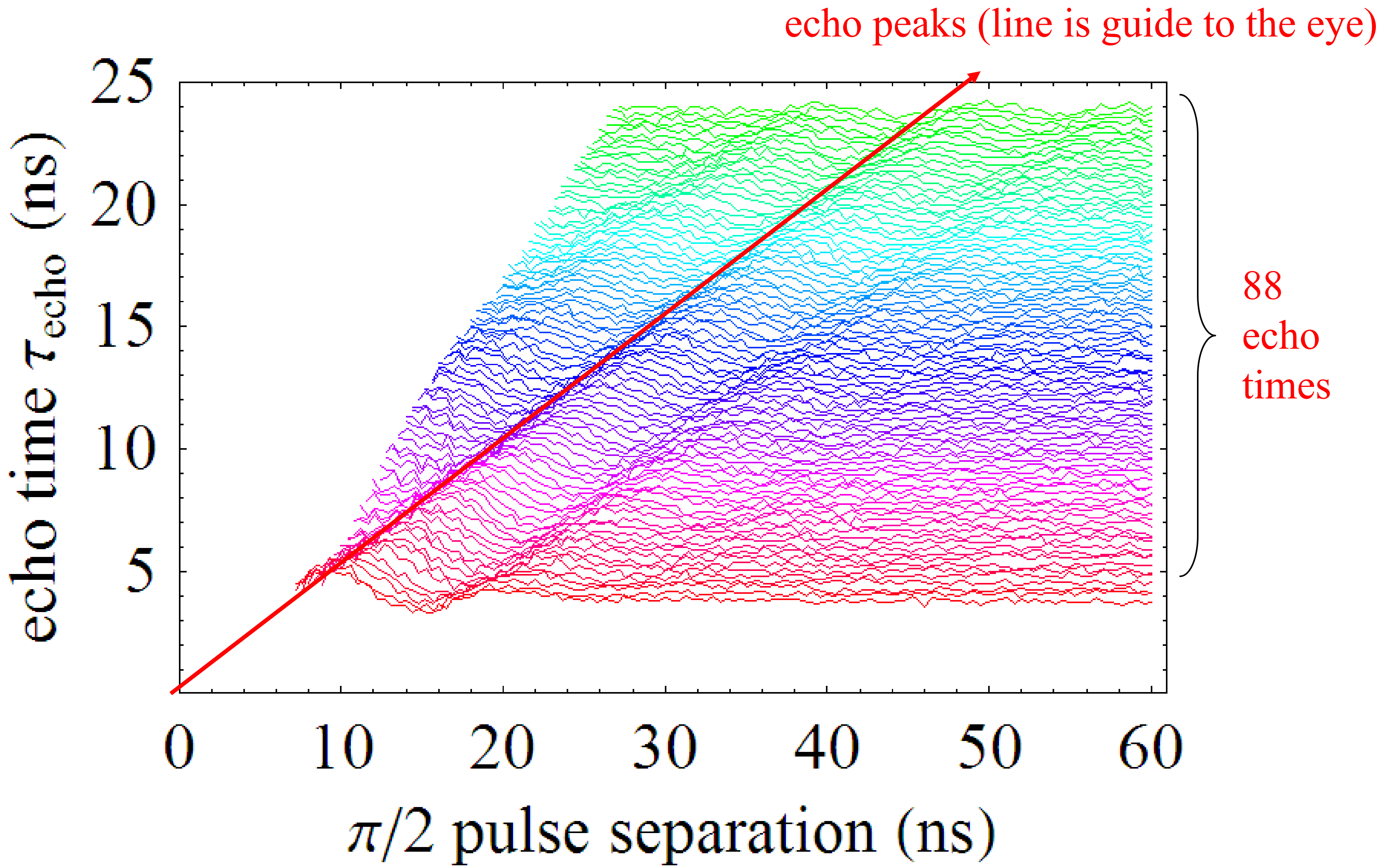


As a function of $\Delta\tau$, echo top is at time $2\tau_{echo}$

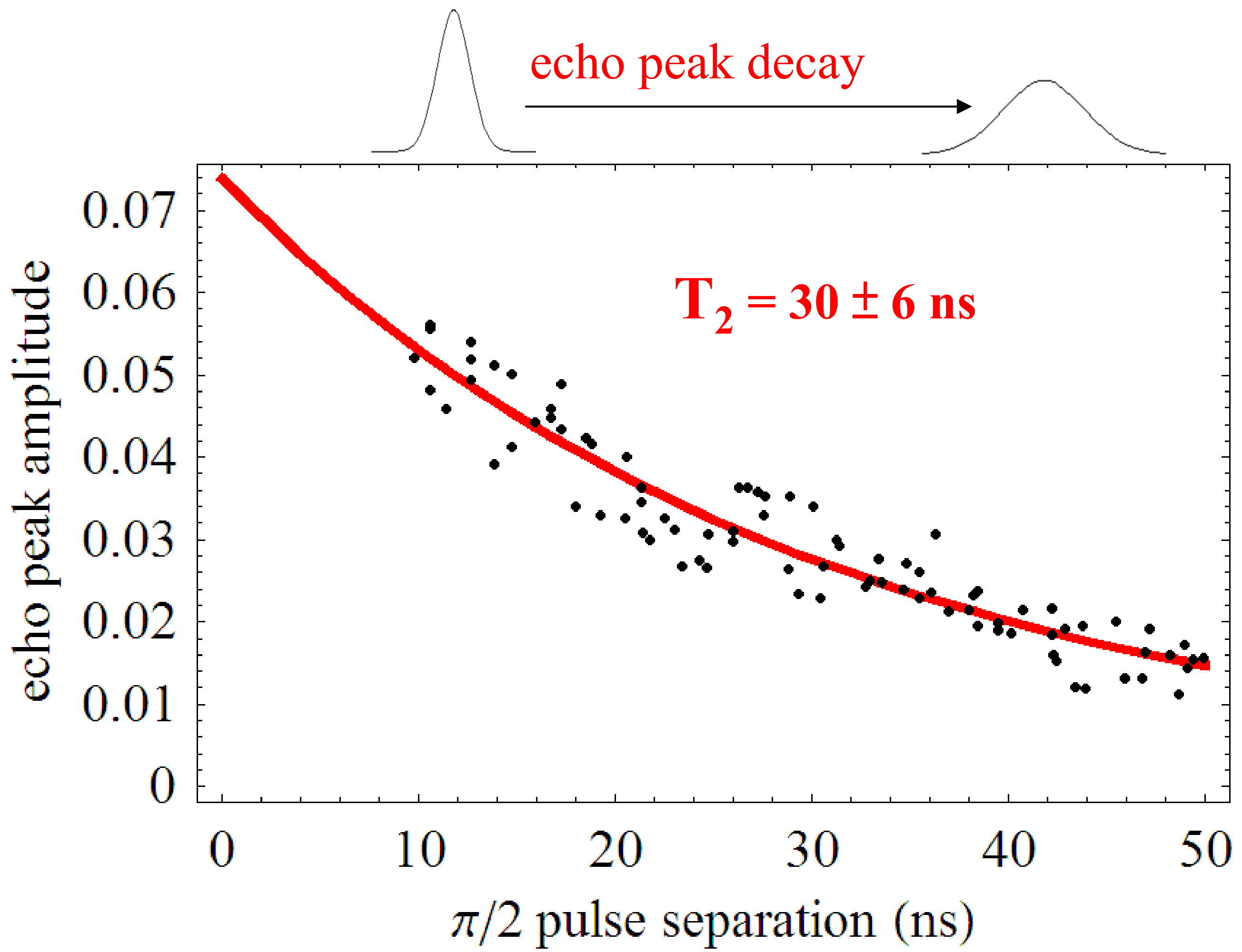


With π pulse on

Spin Echoes


echo peaks (line is guide to the eye)
25
20
15
10
5
echo time τ_{echo} (ns)
0
10
20
30
40
50
60
$\pi/2$ pulse separation (ns)
88
echo
times

T_2 from Spin Echo



Consistency of Dephasing Times

Measured: $T_2' = 11 \pm 1$ ns (linewidth)

$T_2 = 30 \pm 6$ ns (spin echo)

Inferred: $T_2^* = 8.0 \pm 1$ ns

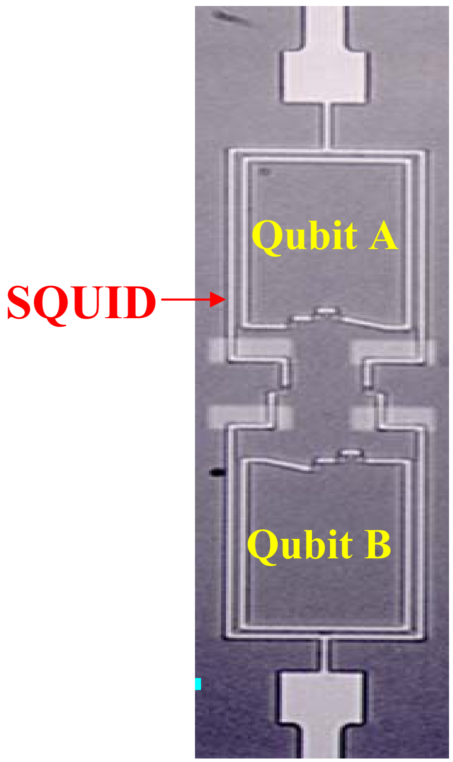
Measured: $T_2^* = 7.1 \pm 0.3$ ns (Ramsey)

- Away from the degeneracy point T_2 is not very different from the Delft values, despite the fact that the area of our qubit is ~ 500 greater.

- The fact that $T_2 << 2T_1$ implies that there is substantial low frequency noise.

Controllable Coupling of Two Qubits: Theory

Two Flux Qubits and a SQUID

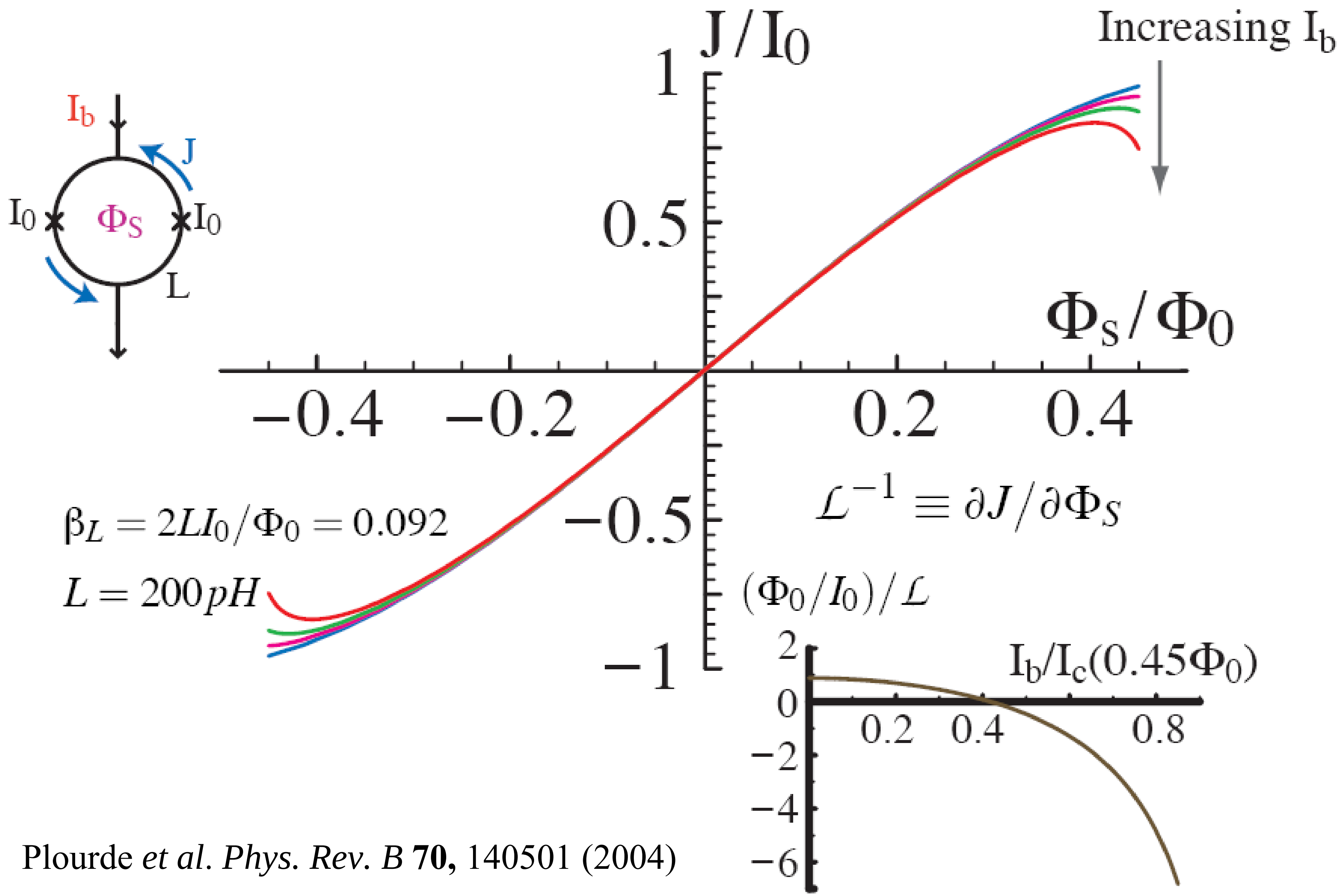


- Qubits have interaction of the form $\sigma_{AZ}\sigma_{BZ}$, where σ is a Pauli spin matrix
- Qubits coupled to each other via M_{qq}:

$$K_0 = -2M_{qq} \mid I_{qA} \mid \mid I_{qB} \mid$$

  where I_{qA} and I_{qB} are qubit circulating currents
- Qubits are also coupled via the SQUID: *this coupling depends on the SQUID current and flux biases*
- Thus, one can use the SQUID to control the total coupling between the qubits

Circulating Current in dc SQUID vs. Applied Flux



Plourde *et al*. *Phys. Rev. B* **70,** 140501 (2004)

Variable Qubit Coupling Using dc SQUID

- When *Qubit B* changes state, circulating current $I_q^{(B)}$ reverses direction, coupling flux $\Delta\Phi_S$ to the SQUID. The change in circulating current J is

$$\Delta J = -\Delta\Phi_S / \mathcal{L} = -2I_q M_{qs} / \mathcal{L}.$$

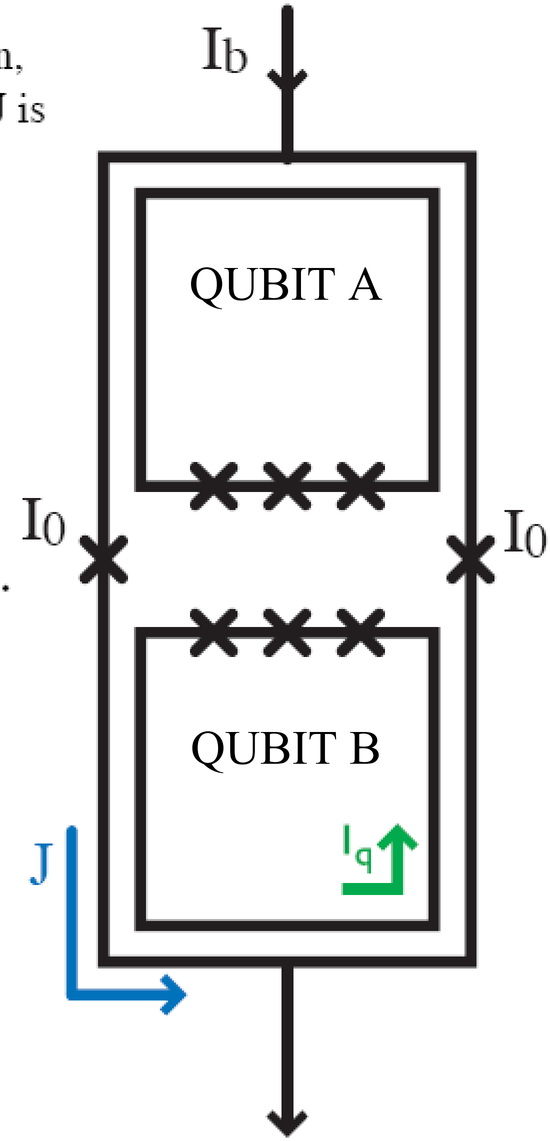
- In turn, ΔJ couples a flux to *Qubit A* in addition to the directly coupled flux:

$$\Delta\Phi_q^{(A)} = \Delta J M_{qs} - 2M_{qq} I_q = 2I_q(-M_{qs}^2 / \mathcal{L} - M_{qq}).$$

- The net coupling strength K is thus

$$K = I_q \Delta\Phi_q^{(A)} = 2I_q^2(-M_{qs}^2 / \mathcal{L} - M_{qq})$$
$$= K_s + K_0.$$

- Thus, one can use the same SQUID to vary K and to read out the flux state of the qubits.



Numerical Values

- Qubits

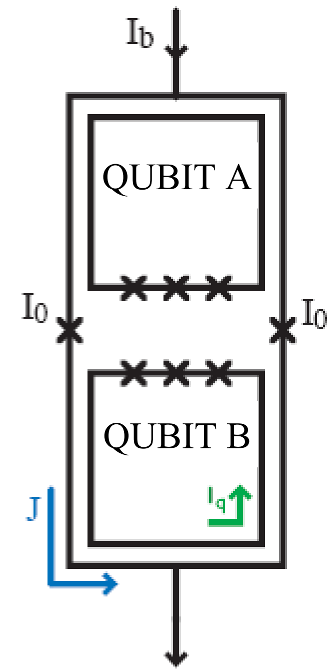
$$I_q = 0.46\,\mu\text{A},\ M_{qs} = 33\,\text{pH},\ M_{qq} = 0.25\,\text{pH}$$

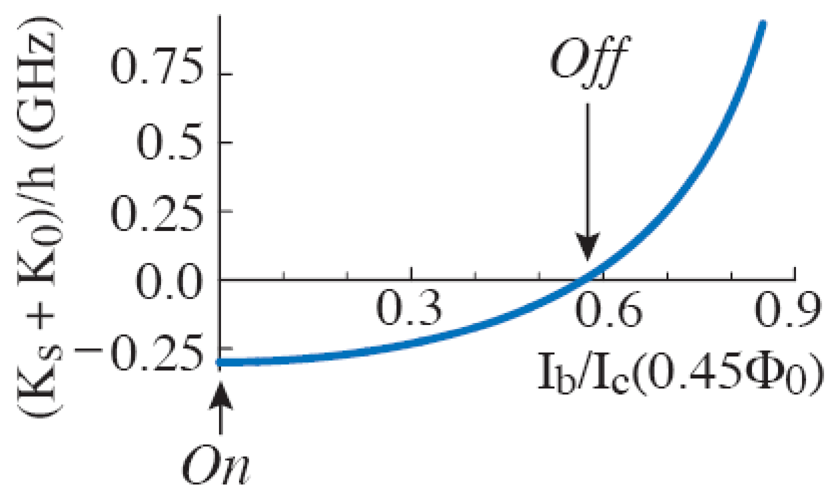
$$K_0/h = -2M_{qq}I_q^2/h = \underline{-0.16\,\text{GHz}}$$

- SQUID

$$L = 200\,\text{pH},\ I_0 = 0.48\,\mu\text{A},\ \beta_L = 0.092$$

$$\boxed{\Phi_S = 0.45\Phi_0 \qquad \begin{array}{l} I_b = 0:\ K_s/h = \underline{-0.14\,\text{GHz}} \\ I_b = 0.57 I_c(\Phi_S):\ K_s/h = \underline{0.16\,\text{GHz}} \end{array}}$$

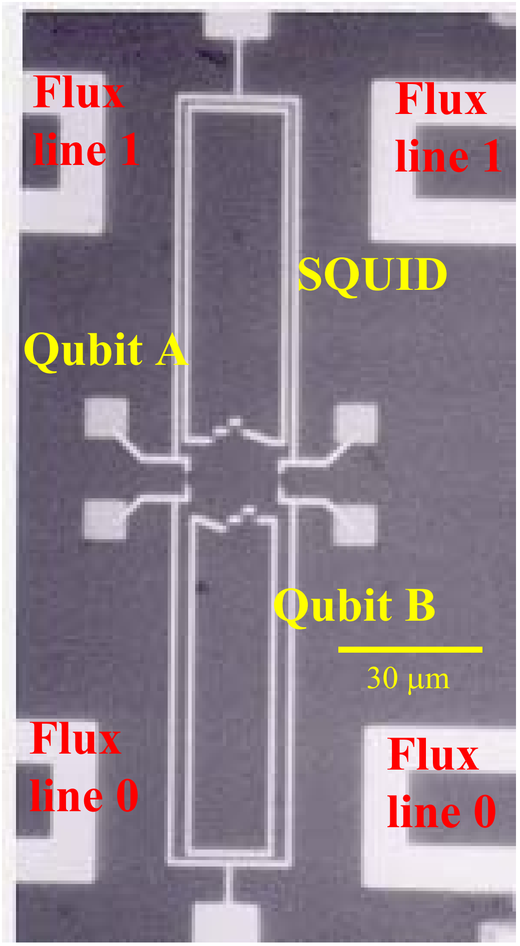




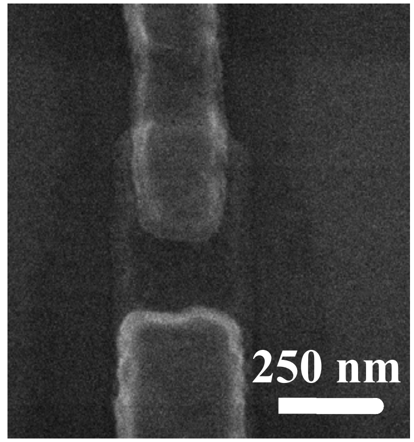
Experiments on Two Flux Qubits

Configuration and characterization

Two Flux Qubits, a SQUID and Flux Lines

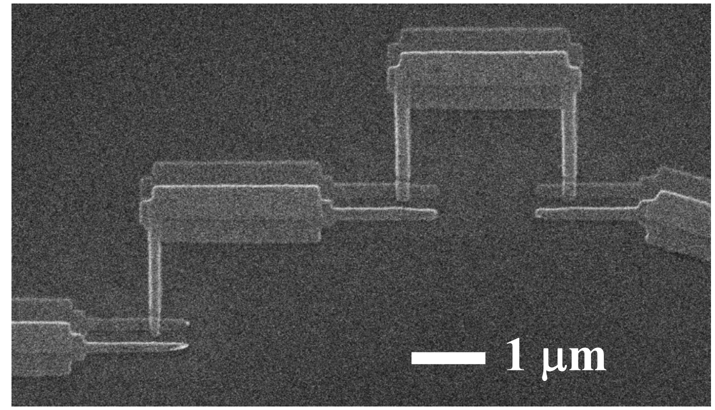


$L_q \sim 200$ pH
$L_J \sim 600$ pH
Loop inductance not negligible



SQUID junctions

215 x 250 nm^2 C ≈ 8.5 fF

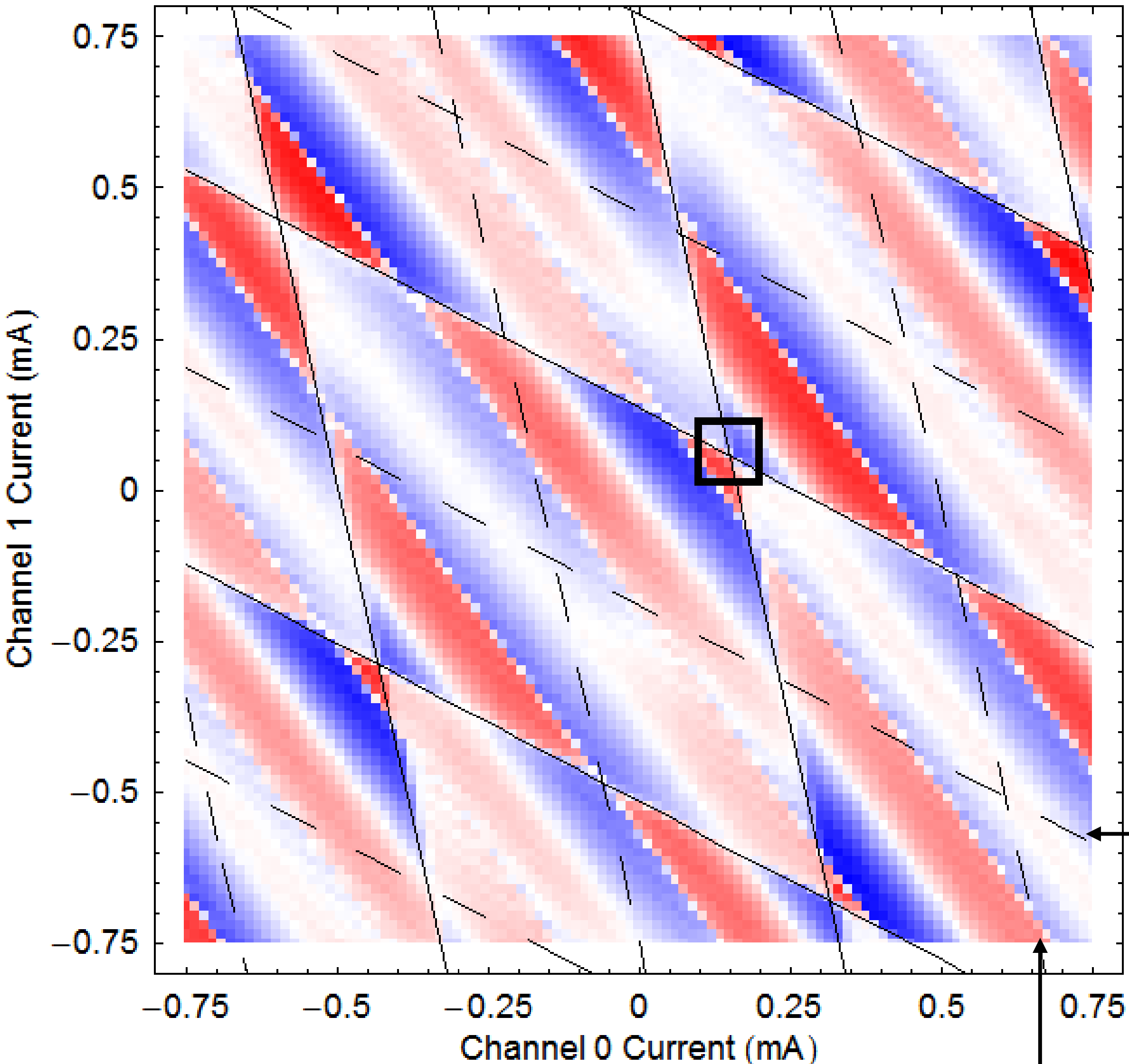


Qubit junctions

180x205 nm^2 C_0 ≈ 6.5 fF

- Two on-chip flux lines enable one to apply independent fluxes to any two of the three devices
- Large inductances to keep currents in flux lines small
- Need to measure the six mutual inductances: M_{foqA}, M_{fiqA}, M_{foqB}, M_{f1qB}, M_{f0s}, M_{f1s}
- Predictions require theory that includes loop inductance (Robertson *et al.,Phys. Rev. B* to be published)

Two-Qubit Flux Map



- $I_S^{50\%}$ vs flux 0 and flux 1
- SQUID contribution has been subtracted, leaving only the contributions of the two flux qubits
- Contains 10,000 flux values
- Yields values of: M_{foqA}, M_{fiqA}, M_{foqB}, M_{f1qB}, M_{f0s}, M_{f1s}

Lines of constant flux in qubit A

Lines of constant flux in qubit B

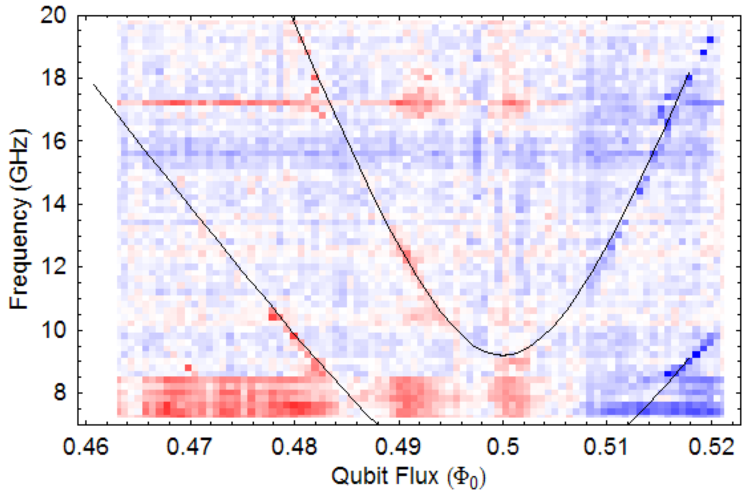
Typical double degeneracy point

Magnetic Flux Stability Over 1 Month

- From the flux map the double degeneracy point drifted by no more than the resolution of 0.1 mΦ_0.
- There were no evident flux jumps in the data.

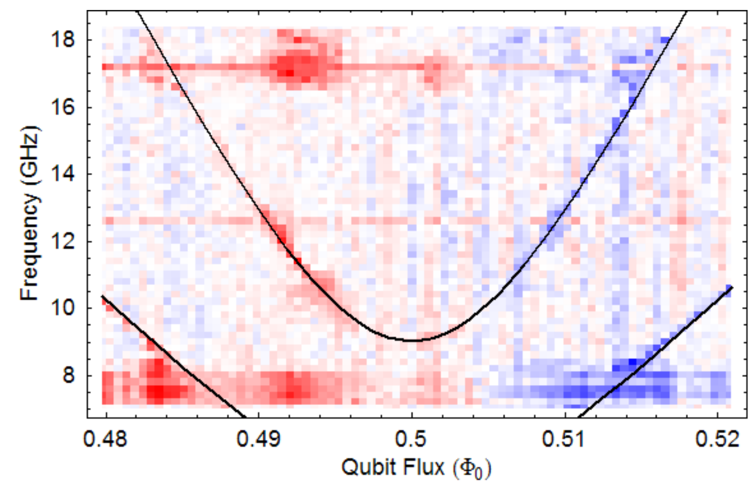
Microwave Spectroscopy of Qubits A and B

Qubit A

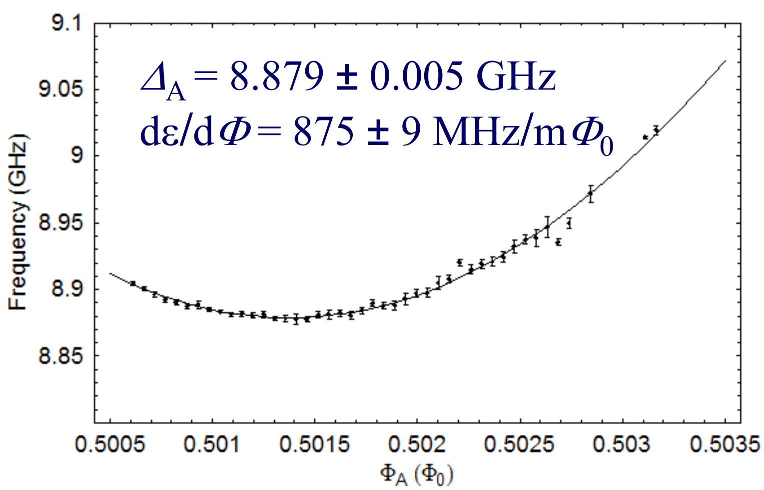


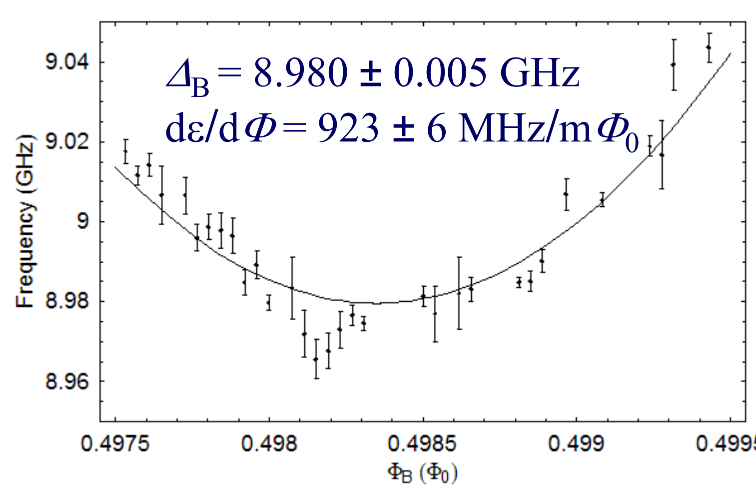
Qubit B

1 photon



2 photon





Fit data to $\nu = (\Delta^2 + \varepsilon^2)^{1/2}$, Δ is splitting at degeneracy point, $\varepsilon = 2I_q(\Phi - \Phi_0/2)$

Coupled Qubit Spectroscopy

- Examples of previous coupled qubits:

  Pashkin *et al*. (2003) Charge qubit

  Berkley *et al.* (2003) Phase qubits

  McDermott *et al*. (2005) Phase qubits

  Majer *et al*. (2005) Flux qubits

The Coupled-Qubit Hamiltonian

$$\boldsymbol{H}_{2\mathrm{qb}} = (-\tfrac{1}{2}\varepsilon_A\boldsymbol{\sigma}_{Az} - \tfrac{1}{2}\Delta_A\boldsymbol{\sigma}_{Ax}) + (-\tfrac{1}{2}\varepsilon_B\boldsymbol{\sigma}_{Bz} - \tfrac{1}{2}\Delta_B\boldsymbol{\sigma}_{Bx}) - \tfrac{1}{2}K\boldsymbol{\sigma}_{Az}\boldsymbol{\sigma}_{Bz}$$

Basis states:

Symmetric triplet $|11\rangle$, $|S\rangle \equiv (|01\rangle + |10\rangle)/2^{1/2}$, $|00\rangle$

Antisymmetric singlet $|A\rangle \equiv (|01\rangle - |10\rangle)/2^{1/2}$

$$H_{2qb} = -\frac{1}{2}\begin{array}{c} \begin{array}{cccc} (11) & (S) & (00) & (A) \end{array} \\ \left[\begin{array}{cccc} \varepsilon + K & \eta & 0 & -\Delta\eta \\ \eta & -K & \eta & \Delta\varepsilon \\ 0 & \eta & K-\varepsilon & \Delta\eta \\ -\Delta\eta & \Delta\varepsilon & \Delta\eta & -K \end{array}\right] \end{array} \begin{array}{c} \\ (11) \\ (S) \\ (00) \\ (A) \end{array}$$

Where:

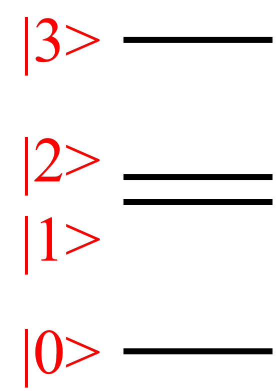
$$\varepsilon = \varepsilon_A + \varepsilon_B$$

$$\Delta\varepsilon = \varepsilon_A - \varepsilon_B$$

$$\eta = \tfrac{1}{\sqrt{2}}(\Delta_A + \Delta_B)$$

$$\Delta\eta = \tfrac{1}{\sqrt{2}}(\Delta_A - \Delta_B)$$

Eigenstates



Antiferromagnetic coupling: $E_1 < E_2$

Storcz and Wilhelm, Phys. Rev. A **67,** 042319 (2003)

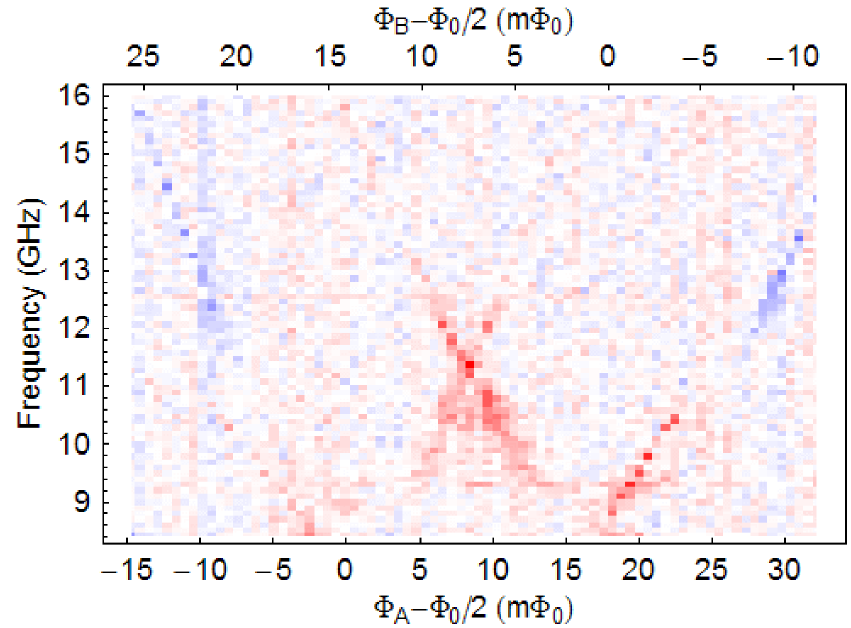
Coupled Qubit Spectroscopy

- Two regions of interest:

  "Intersecting" spectra ($\nu > \Delta_A, \Delta_B$)

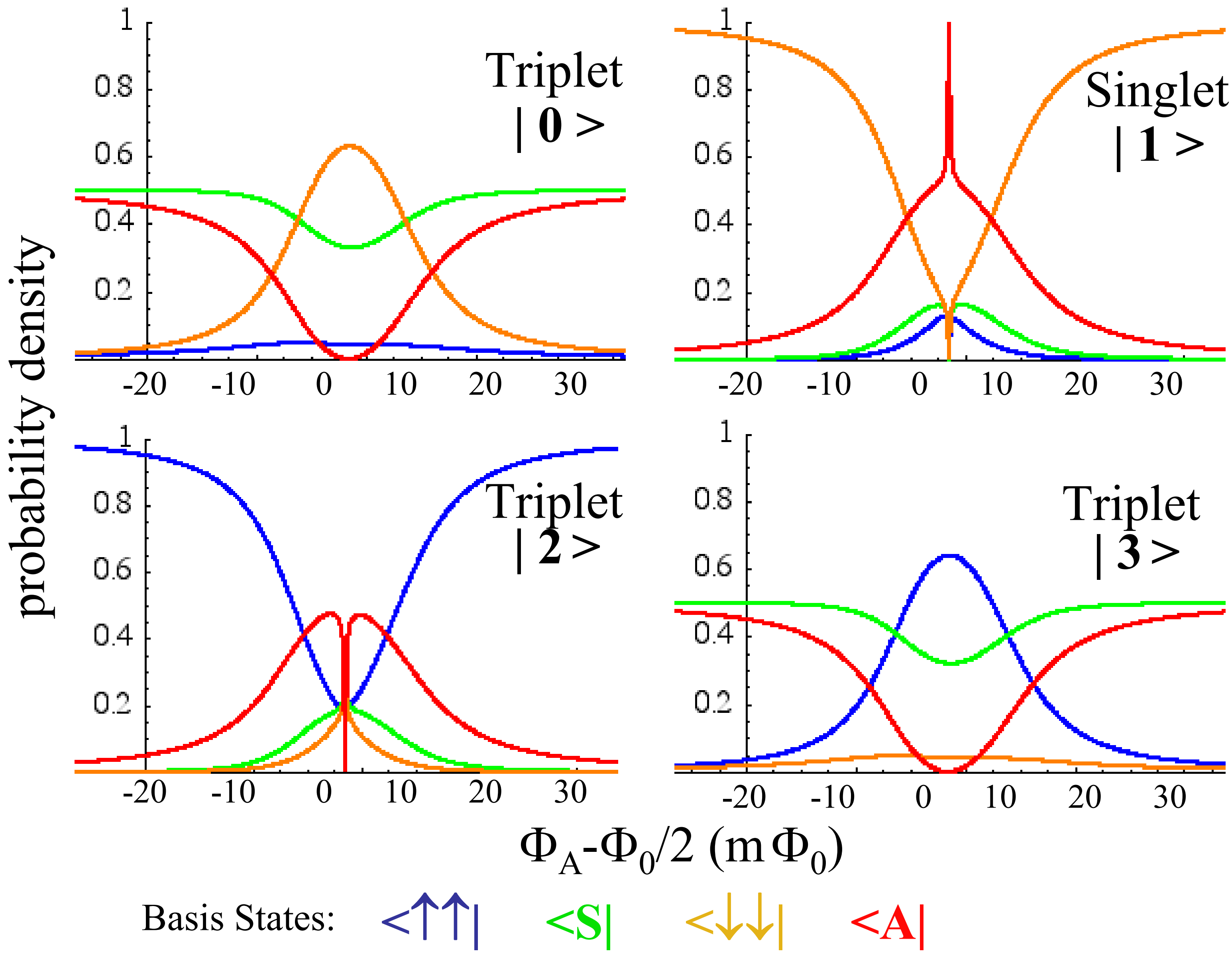
  Double degeneracy point ($\Phi_{\mathrm{A}} = \Phi_{\mathrm{B}} = \Phi_0/2$)

“Intersecting” Interaction

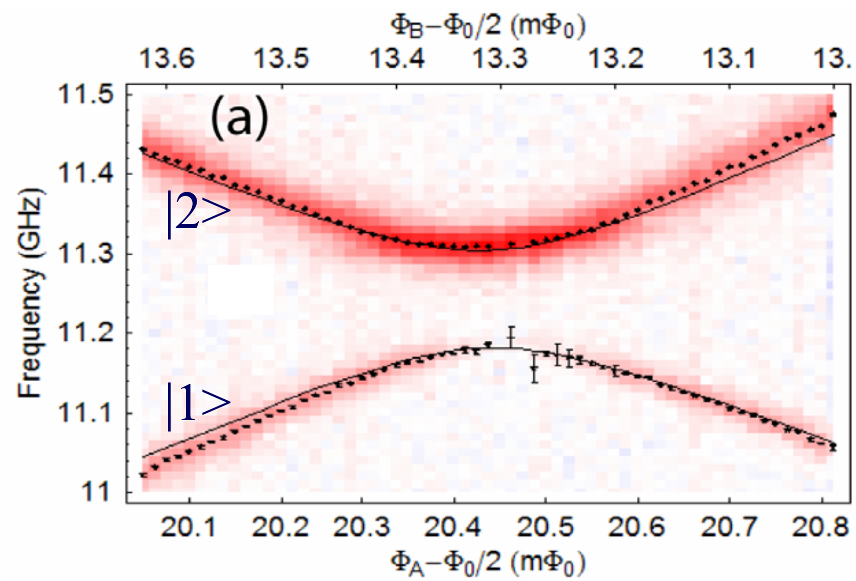


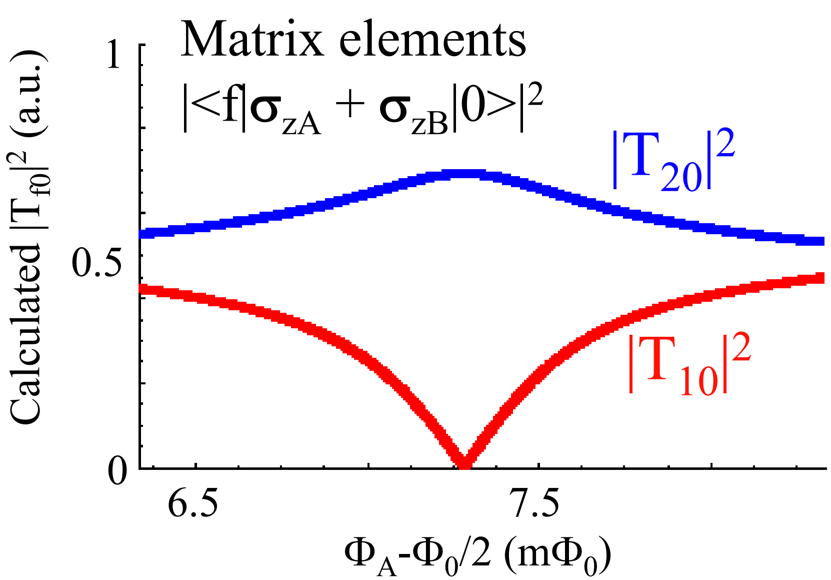
- Spectra measured at constant SQUID flux of 0.35 Φ_0: qubit-qubit coupling is constant

Wave Functions Near "Intersecting" Degeneracy



“Intersecting” Anticrossing of |1> and |2>





- Qubit intersection frequency 11.24 GHz
- Spectra measured at constant SQUID flux, 0.35 Φ_0, and hence at constant qubit-qubit coupling strength
- Zero SQUID bias current
- Minimum splitting 122.6 ± 0.8 MHz
- Note absence of data for |1> near anticrossing

Measured and Calculated Parameters

Mutual inductances

	Flux map (pH)*	Fast Henry (pH)*
M_{f0qA}	1.65	1.70
M_{f1qA}	-3.21	-3.20
M_{f0qB}	3.10	3.10
M_{f1qB}	-0.72	-0.70
M_{f0s}	7.52	7.27
M_{f1s}	-5.54	-5.54
M_{qq}	–	0.75
M_{qAs}	–	87.2
M_{qBs}	–	63.8

*Average error: 1.6 %

Self inductances

	Fast Henry (pH)
L_{qA}	194
L_{qB}	182
L_{s}	423

Qubit currents

$$d\varepsilon/d\Phi_q = 2I_{\mathrm{q}}$$

$$I_{\mathrm{qA}} = 140 \pm 0.9 \text{ nA}$$

$$I_{\mathrm{qB}} = 148 \pm 1.5 \text{ nA}$$

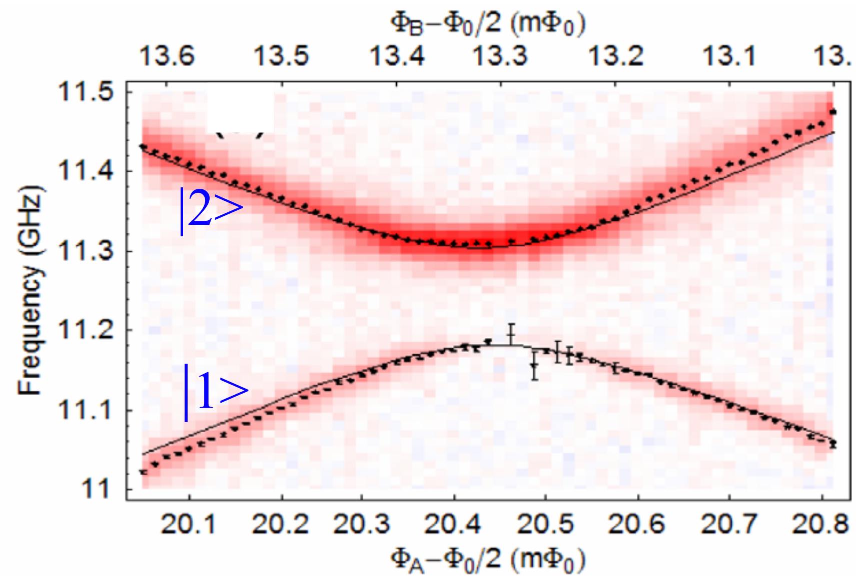
SQUID critical current

$$2I_0 = \pi\Delta/2R_{\mathrm{NN}}$$

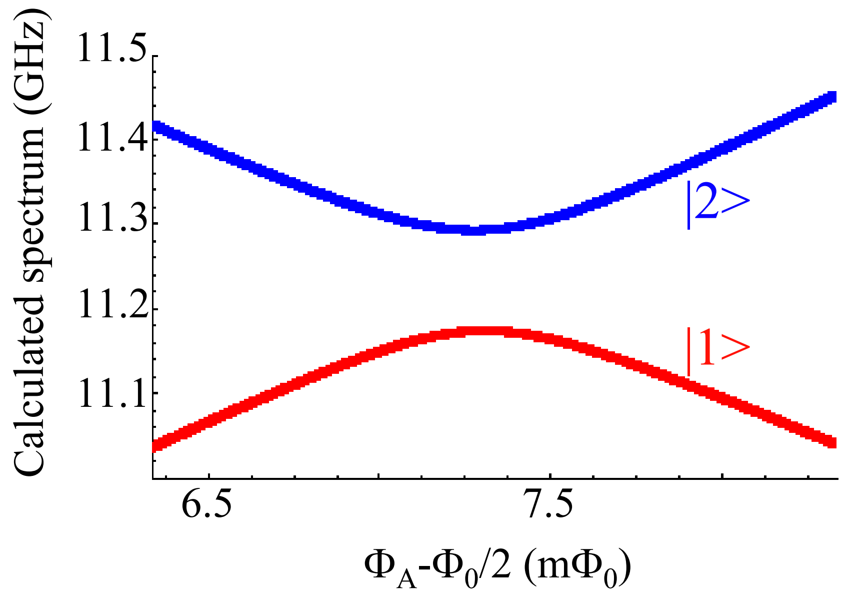
$$= 1.03\ \mu\text{A}$$

Anticrossing of |1> and |2>: Experiment and Theory

Experiment



Calculation

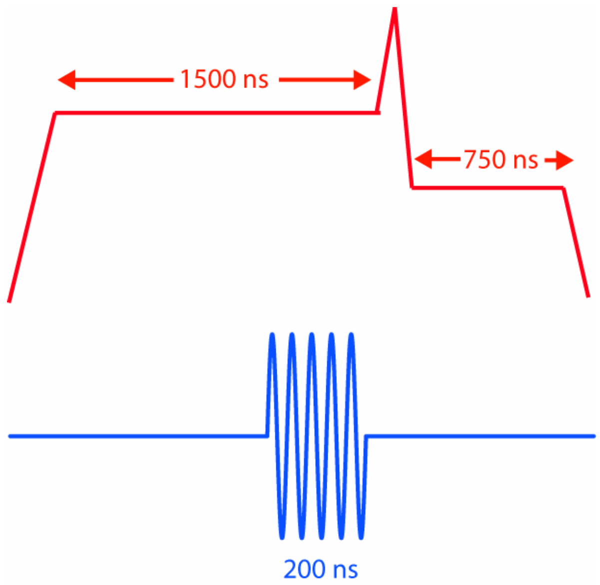


- Minimum splitting 122.6 ± 0.8 MHz

- For SQUID flux of 0.35 Φ_0:
  Minimum spitting 119 ± 2 MHz

Varying the Qubit-Qubit Coupling

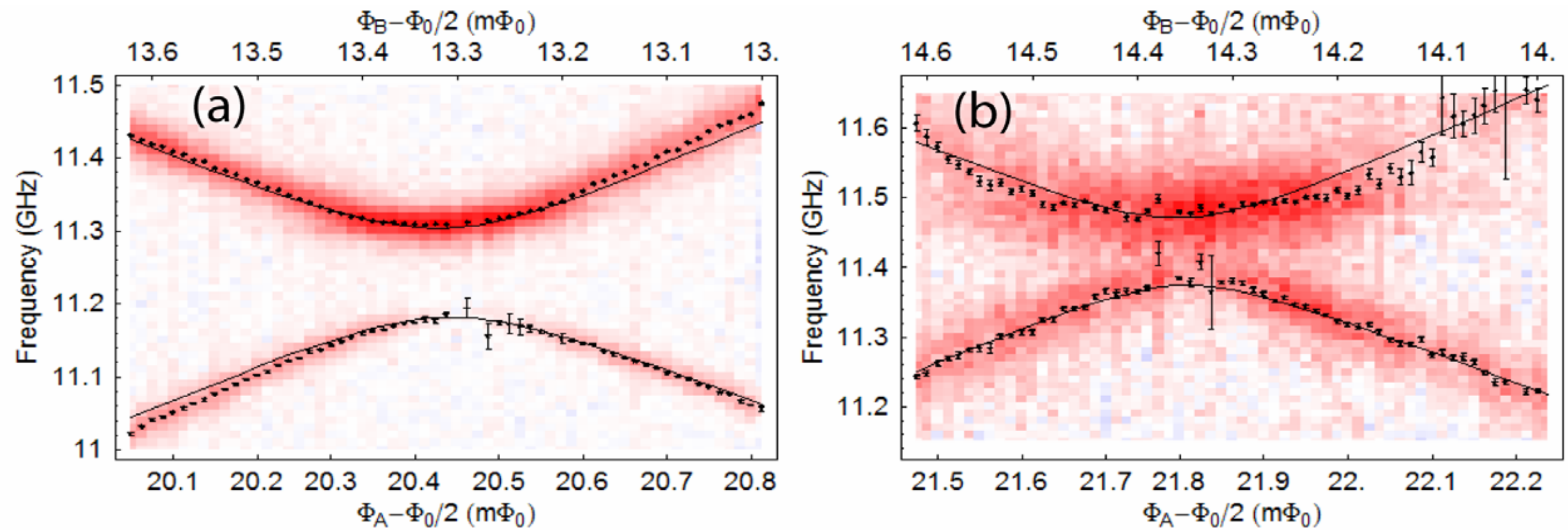
- Apply bias current pulse to SQUID to change its dynamic inductance in the zero vo;tage state



Anticrossing of |1> and |2>: Changing the Coupling

SQUID bias current = 0

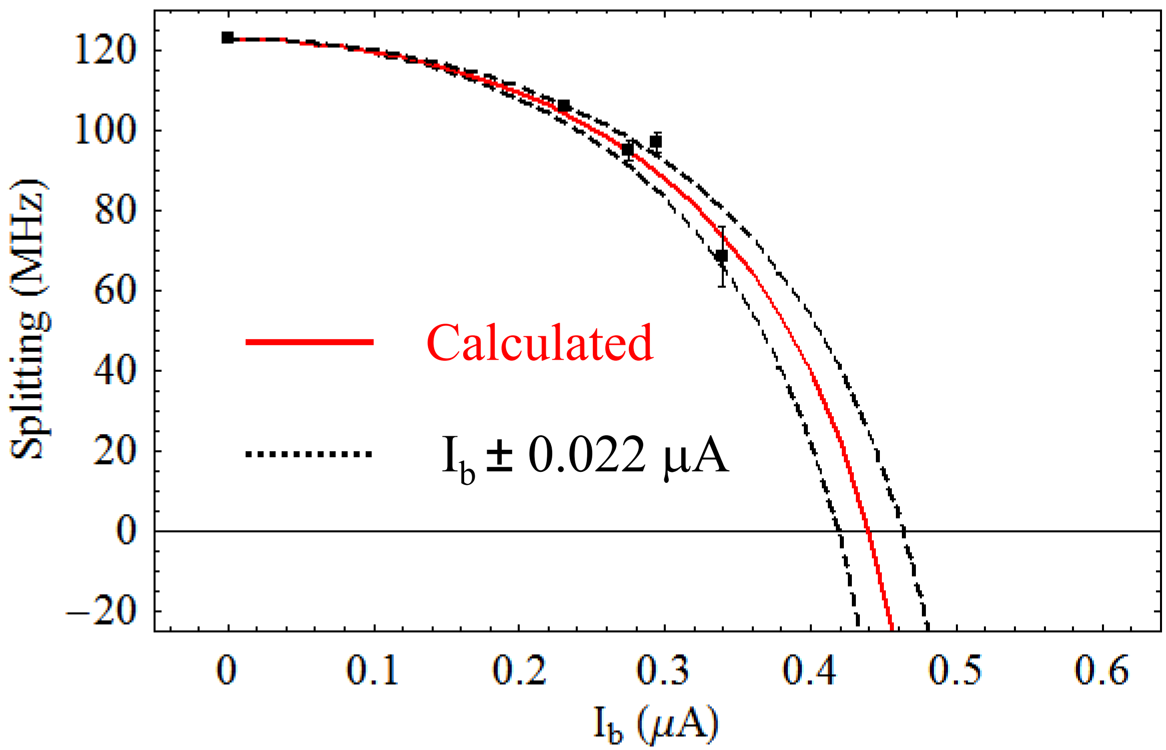
SQUID bias current = 0.46 μA



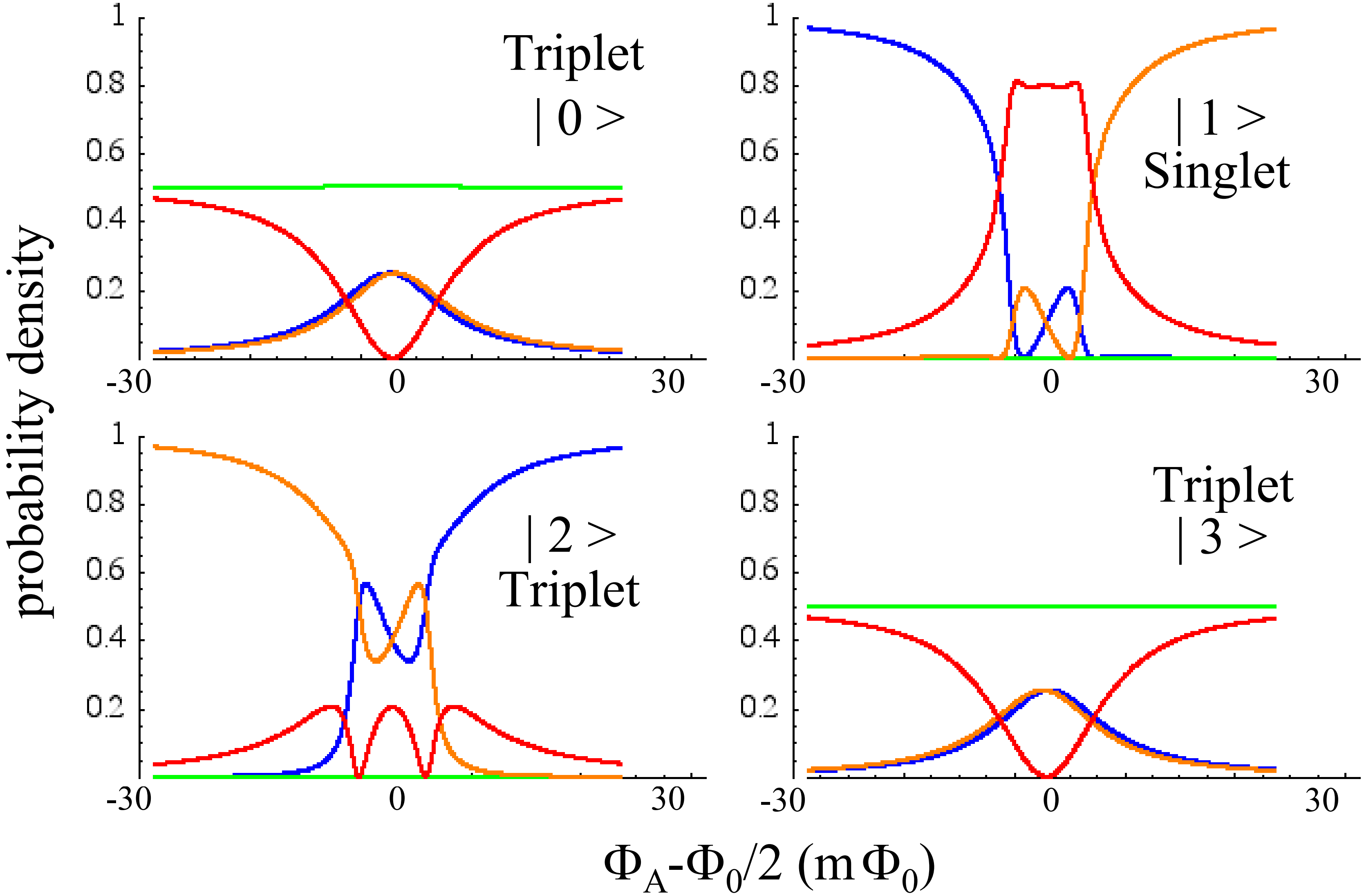
Minimum splitting 122.6 ± 0.8 MHz

Minimum splitting 96.7 ± 2.6 MHz

Splitting Frequency vs. SQUID Bias Current

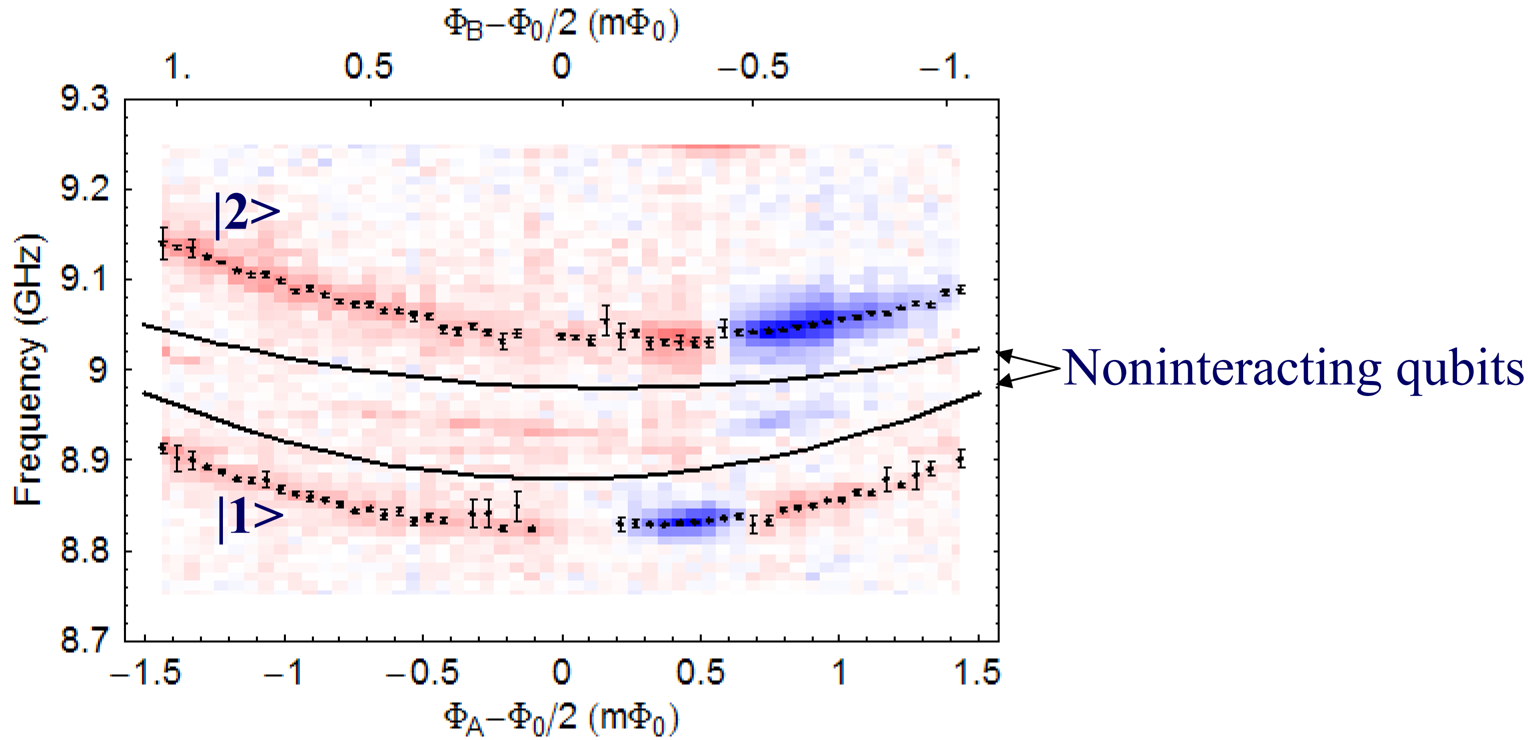


Wave Functions Near Double Degeneracy



Basis States: $\langle\uparrow\uparrow|$ $\langle S|$ $\langle\downarrow\downarrow|$ $\langle A|$

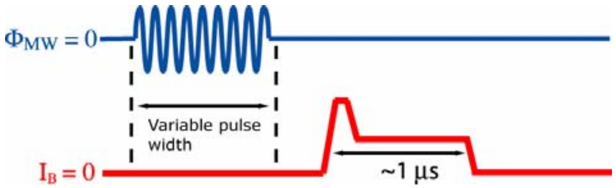
Anticrossing of |1> and |2> at Double Degeneracy Point

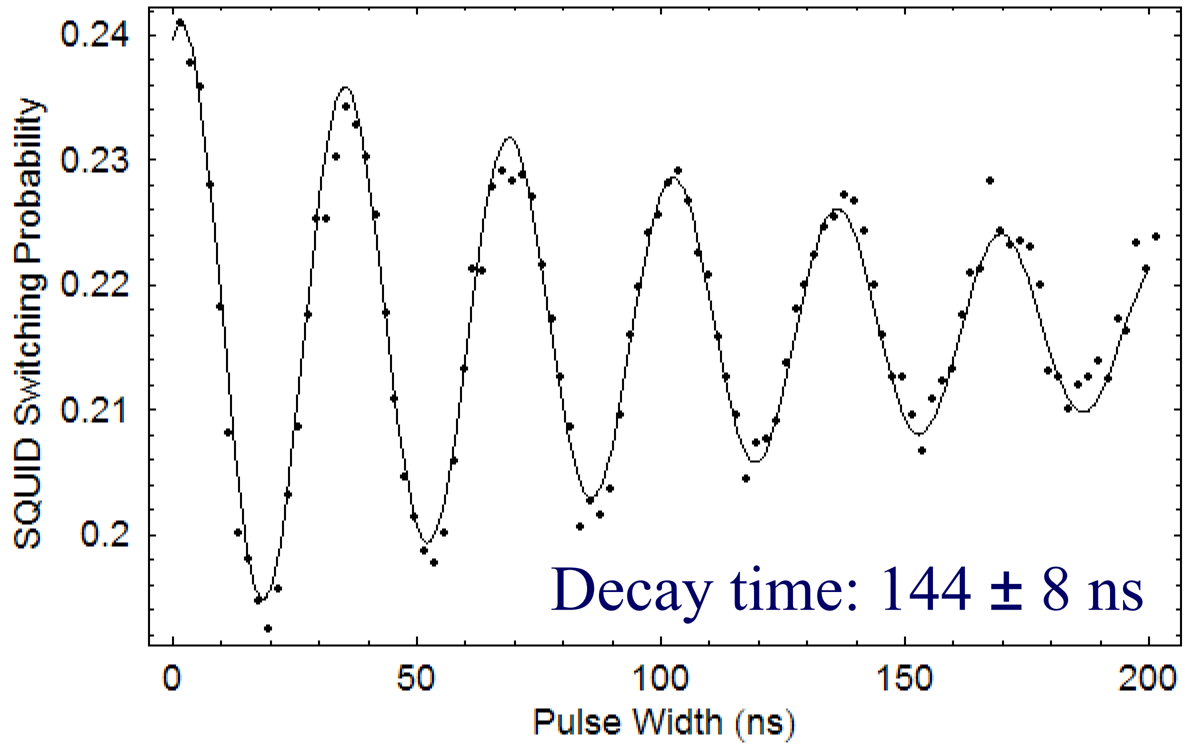


- SQUID flux = 0.35 Φ_0
- Energy of |2> increases by 55 ± 7 MHz
- Energy of |1> decreases by 53 ± 7 MHz
- Total energy repulsion 108 ± 10 MHz
- Predicted energy repulsion 118 ± 2 MHz

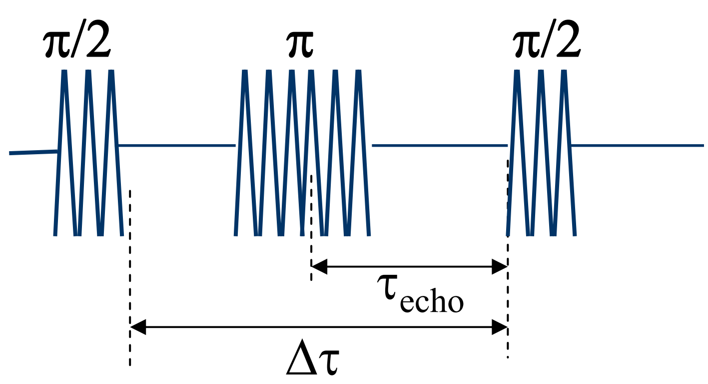
Coupled Qubit Manipulation

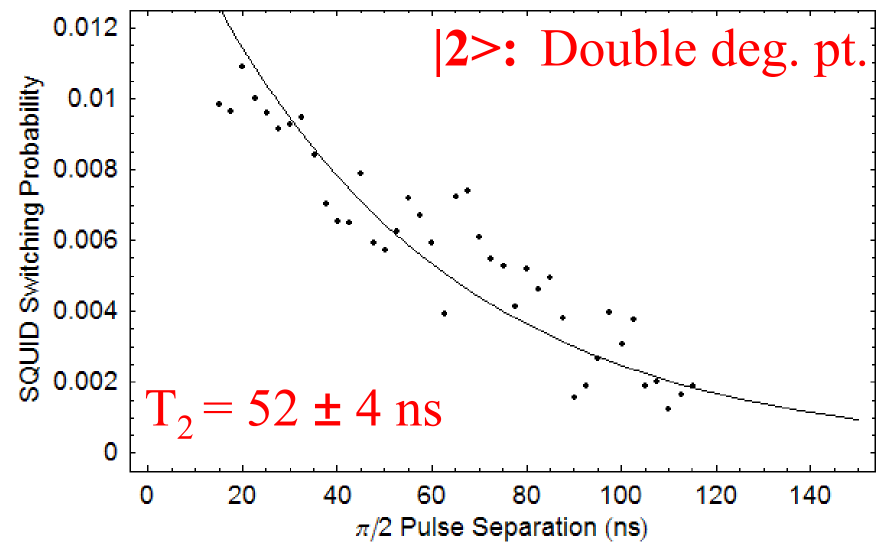
Rabi Oscillation on |2> at Double Degeneracy Point

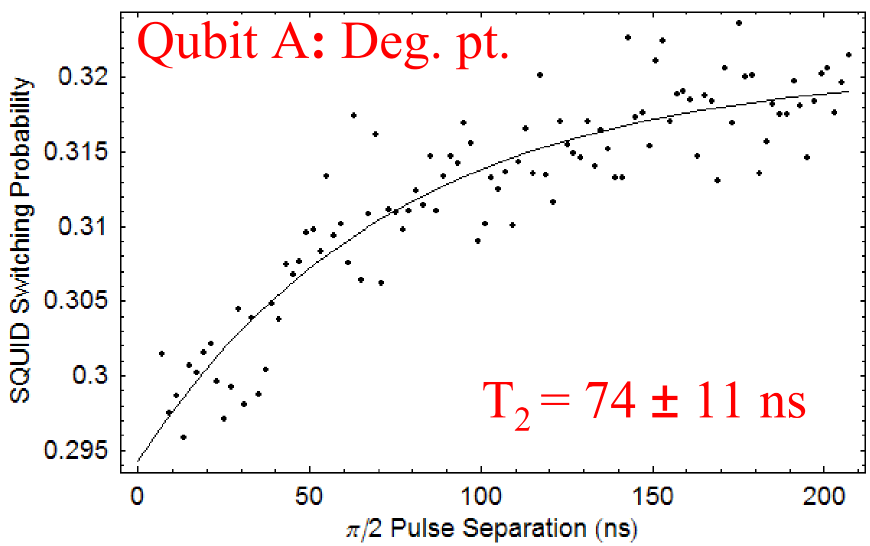


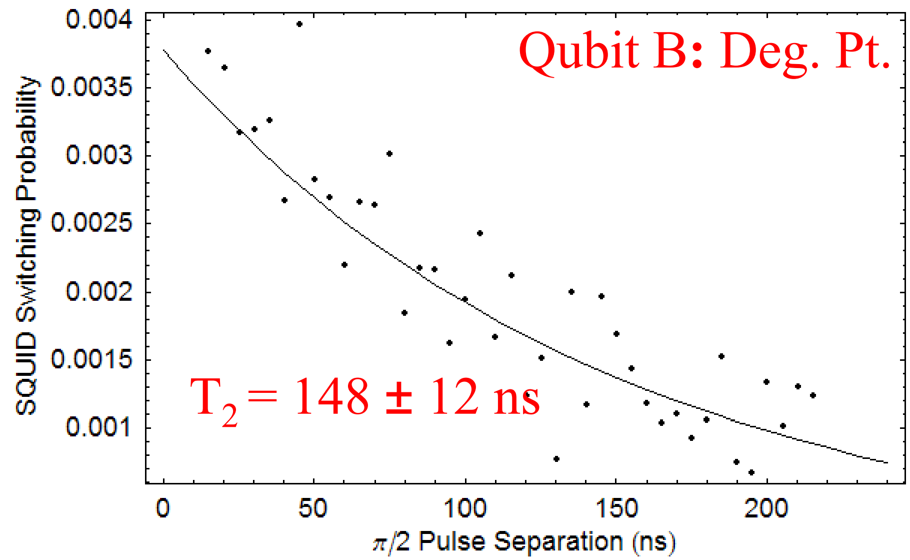


Flux Echoes









$$(1/T_{2A} + 1/T_{2B})^{-1} = 49 \text{ ns}$$

Concluding Remarks

- Fabricated two flux qubits with splittings within 1%
- Spectroscopy of coupled qubits near "intersecting" degeneracy point:
  - Splitting of |1> and |2> energies is within 3% of predicted value
  - Absence of transitions to |1> agrees qualitatively with calculated matrix elements
  - Splitting reduced by bias current in SQUID in good agreement with predictions
- Spectroscopy of coupled qubits near double degeneracy point:
  - Repulsion of |1> and |2> energies agrees with predictions within error bars
- Time domain measurements on |2> at double degeneracy point:
  - Rabi oscillations: decay time = 144 ± 8 ns
  - Flux echo: $T_2 = 52 \pm 4$ ns

    $1/T_{2|2>} = 1/T_{2A} + 1/T_{2B}$ to within experimental error

Future Plans

- Attempt to reduce splitting to zero at intersecting degeneracy point:
  - This would enable one to manipulate states of the two qubits independently
  - In turn, this would enable one to make a CNOT gate

- Coherent oscillations between |1> and |2>
- Replace dissipative readout scheme with dispersive readout scheme to reduce decoherence of readout process and increase readout speed