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Quantum Gravity and its (dis-) contents

outline:

1. What is classical Gravity?
2. What is Quantum Gravity?
3. What is the Problem?
4. What is/are the Cure(s) ???

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1. What is it we (think we) want to quantize?

For this purpose, forget geometry etc..

GR is a special relativistic field theory, just like

$$L = \partial_\mu \varphi \partial^\mu \varphi$$

or $L_{Max} = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 = (\partial_\mu A_\nu)^2 + \dots$

but with more indices.

$$L_{(0)} \sim \partial_\mu h_{\alpha\beta} \partial^\mu h^{\alpha\beta} (+ \dots)$$

(free part)

$h_{\alpha\beta} = h_{\beta\alpha}$ is a tensor field.

Has to be tensor to couple to the special relativistic extension of mass density,

$\rho(\vec{r}, t)$: $T_{\mu\nu}^{MATT}(x^\mu)$: T_{00} = energy dens.

$\rightarrow \partial_\mu T^{\mu\nu} = 0 \Leftarrow$ closed system

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We will set coupling const.

$\sim \Box h_{\mu\nu} = G T_{\mu\nu}$

(like $\Box A_\mu = e J_\mu$)

Immediately "D" means ∃ degrees of freedom beyond Newtonian potential: GRAVITONS

they are both a challenge and a deep confrontation:

Just as the maxwell action must be $L \sim \ominus \frac{1}{4} F_{\mu\nu}^2 + J_\mu A^\mu$ to give ⊕ energy photons, i.e.

$L_{max} \sim \mp \partial_\mu A_\nu \partial^\mu A^\nu \sim +\dot{\vec{A}}^2 - (\vec{\partial}\vec{A})^2$

and then automatically gives repulsion,

$E_{int} = \int \rho \frac{1}{(-\nabla^2)} \rho \geq 0$

(for like sign charges)

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here $L = + (\dot{h}_{ij})^2 - (\partial h)^2 \ldots$

$\Rightarrow -\int T_{00} (-1/\nabla^2) T^{00} = E_{int} < 0$

all masses/energies attract.

The catch: when matter interacts with gravity, its $T_{\mu\nu}^{(M)}$ is no longer conserved – it's not "isolated". Photons are electrically neutral, "gravitons" weigh

So whereas

$(\Box h_{\mu\nu} + \ldots) \equiv G_{\mu\nu}^L(h) = G T_{\mu\nu}^M$

to lowest order, is OK:

$\partial^\mu G_{\mu\nu}^L \equiv 0$, $\partial^\mu T_{\mu\nu}^M = 0$

this breaks down when matter is dynamical

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So we must modify:

$$\Box h_{\mu\nu} = G\left[\underbrace{T_{\mu\nu}(\text{matt}) + T_{\mu\nu}(h)}_{\text{Total System}}\right]$$

Since the $T_{\mu\nu}(h)$ from $L_0 \sim (\partial h)^2$ is $\sim (\partial h \partial h)_{\mu\nu} \Rightarrow$ add $L_1 = h\partial h \partial h$; This gives the desired [illegible] BUT $L_1 = h\partial h\partial h \Rightarrow$ a new piece $(h\partial h\partial h)_{\mu\nu}$ and a new $L_2 \sim h^2 \partial h \partial h \ldots$ an ∞ series:

$$L = (\eta + Gh_{\mu\nu} + G^2 h_{\mu\nu}^2 + \cdots)\, \partial h \partial h$$

__Handwaving__ says this is GR. But that's never been proven this way: Instead, here's a 1-step way, where GR is __cubic__!

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One can write G.R. in 1st order form:

$$L_{Einst}(g, \Gamma) = g^{\mu\nu}(\partial_\alpha \Gamma^\alpha_{\mu\nu} + \Gamma\Gamma)_{\mu\nu}$$

(Like $L = p\dot{q} - \frac{p^2}{2}$).

Specifically, start with

$$L_0 = (\eta\Gamma\Gamma + h\partial\Gamma) \Rightarrow \partial\Gamma = 0,\ \Gamma = \partial h.$$

Then, to get $\partial\Gamma = T^{GRAV}_{\mu\nu}(h, \Gamma)$ add $L_1 = h\Gamma\Gamma$: this does __not__ yield any $\Delta T_{\mu\nu}$, so all ends there:

$$L_E = L_0 + L_1 = (\underbrace{\eta + h}_{g\ldots})(\partial\Gamma + \Gamma\Gamma)$$

__QED!__

This is what we must quantize, which forces back to perturbative:

$$L_{GR} = \sum_{n=0}^{\infty} (G^n h^n)\, \partial h \partial h$$

but never __derived__!

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2. What's (Q)GR ?

"Just" quantize the [nonlinear] theory with $[h, h'] = i\hbar\delta^3$ etc.

Non-linearity is not a priori an obstacle: all QM is nonlinear

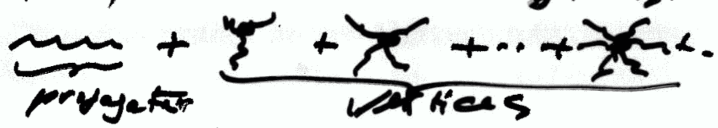
eg YM $\mathcal{L}_x = (\partial A - \partial A + g A \times A)^2$;

(self. coupling const) dimensionless

Here $\sum_n (G h)^n \partial h \partial h$

↳ Dimensionful: $G \sim 1/m^2$ (in Planck units)

First, let's write diagrams:



e.g.

grav-grav "compton" perfectly finite.

All trees

etc OK.

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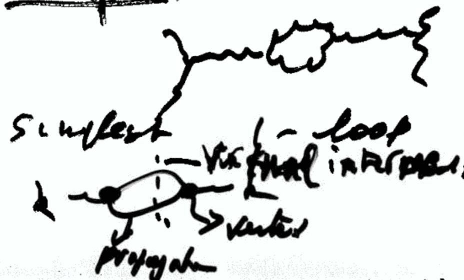
But Loops !!!



$$\sim \int \frac{d^4 p}{p^2 (p+k)^2} V(p+k)\, V(p+k)$$

But $V(p) \sim h\, \partial h \partial h \sim p^2$

So $G^2 \int^{\infty} \frac{d^4 p}{p^4} p^2 p^2 \sim \infty^4$.

"∞" we can deal with in QED too: need counterterms like $(\delta m)\bar{\psi}\psi + (\delta e) j^\mu A_\mu$.

But those were logarithmic ∞'s

Here ∞^4! the dimensions force counterterm $\sim (\partial\partial h)^4 + \cdots$

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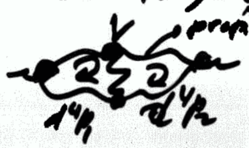
Such a term is necessarily

$\Delta L = \#(\infty^4)\, R^2_{\mu\nu\alpha\beta}$.

At 2 loops, e.g.



$\sim \int \frac{d^4q\, d^4k}{(p^2)^5} (p^2)^4$ ← 4V

↳ 5 propagators

$\Rightarrow \sim \frac{p^8 p^8}{p^{10}} \sim p^6 \Rightarrow (\infty^6) R^3 \ldots$

etc. ; R^4 3-loop …

Need an ∞ # of unknown counter terms, as if

$L = \frac{R}{\kappa_{\text{const.}}} + \infty R^2 + \infty R^3 + \ldots$

All predictive power lost!

The root is dimension of $G = [L^{+2}]$

So $(G^2 R)$ at each extra loop.

[Old disease, eg Fermi Theory 1934]

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Scary — but what "really" happens?

At 1-loop, pure Gravity, no sources, is accidentally OK:

$\infty R^2 \ldots$ can be "field-redefined" away, for special reasons in $D=4$.

But @ 2 loop, with horrible # of diagrams there are 2 independent calculations, both agree $\Rightarrow (\neq 0) R^3 \ldots$!

And even at 1 loop,

graviton loop → matter $\Rightarrow \infty(\neq 0)$ and cannot be removed

for scalars, spinors, photons, YM. All calculated,

$\Rightarrow$ no miracles.

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4. Solutions?

Not (yet), but, there are (too?) many paths open:

1. Denial: Don't quantize GR, let it stay classical, with

$$G_{\mu\nu}(g_{class}) = G\langle T^{matt}_{\mu\nu}(\psi)\rangle$$

This is inconsistent and violates QM, since g_{cl} is an external source.

So is: "Stay at tree level", forget loops. This violates unitarity.

2. Acceptance: "So what," the ∞s only kick in at Planck mass $\sim 10^{-5}$ gm or $L \sim 10^{-35}$ m.

No: ∞'s affect all processes; besides, at early universe QG matters.

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3. Homeopathic – small changes within GR may cure: different formalism (LQG..) or asymptotic safety (still alive)

4 Deeper: GR is an effective theory (like phonons) that is not valid at UV energy, where the "correct" theory takes over. For example, string theory: a non-local quantization that incorporates GR, but stays finite. Maybe – but jury is very much still out.

Of course, GR certainly isn't the be-all end-all model...

"Spacetime is an emergent property", a currently popular idea

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5. miracles?

SUGRA: Gravity + s=3/2 fermion

is safe, both at 1 loop (despite "matter") and at 2 loop by supersymmetry cancellations.

BUT! at 3 loops, unshielded.

and even the "ultimate" D=11, SUGRA is ∞ at .. 2 loops.

BUT BUT .. wait! There is a new hope, at D=4, even:

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"Maximal" N=8 SUGRA:

The largest that supports super-symmetry

N=8: 1 graviton, 8 spin 3/2, 28 spin 1, 56 spin 1/2, 70 spin 0 (all m=0)

out of this world, but has — so far — jumped through all hoops!

[*Like SYM$_4$ ⊗ SYM$_4$] (14b)

for technical reasons*, despite/because of the many new particles, it's much easier to calculate than with [illegible]. So far all calculated, and understood amplitudes up to ~5 loops and some series of diagrams to all loops are $< \infty$, and none (yet) $= \infty$! Warning flags keep being pushed back: currently, 9 loops is "dangerous".

BUT: "what if we win?" Even if finite, what does $N=8$ SUGRA say about the real, non-SUSY world??
only a few $m=0$ seen, lots of non-susy!

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(one sci-fi, for fun)

A last frontier, D>4 GR. Here, nothing is ~~known~~, but one a priori fact: the ∞s – if present, are less & less "[illegible]" – the counterterms $\sim R^{D}$ and only affect D-particle (& up) amplitudes / ultra Planck energies.

What does this mean, if anything?

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Coda

There is no consensus as to the right answer/theory. It's likely, as usual to come out of nowhere and finesse the question. But meanwhile keeps a lot of people off the street, and at many chowing-bowls.