- Exotic Photonics

FDM Haldane *
KITP + Princeton Univ

- Photonics is interesting
  (not just "boring" "band structure")
  new effects
- Negative Index of Refraction
  - analogs of Quantum Hall
  - Maxwell on the lattice...
    Lattice QED → Band structure

- Sorry for messy transparencies!
  (last minute change of plan for talk)

* collab. with
S. Raghu (grad student + grad fellow here now)

"Photonics": could mean various things

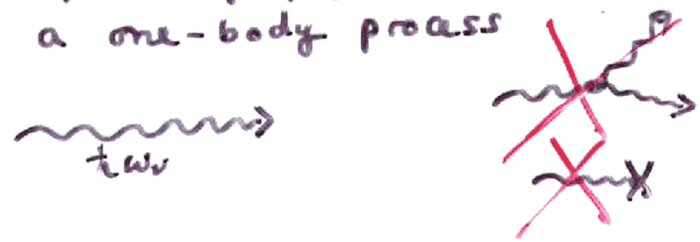
for my purposes:

= ballistic transport of electromagnetic energy in inhomogeneous media that are:

— Linear
— non-dissipative

(in some frequency range)

i.e. photon propagation is a one-body process

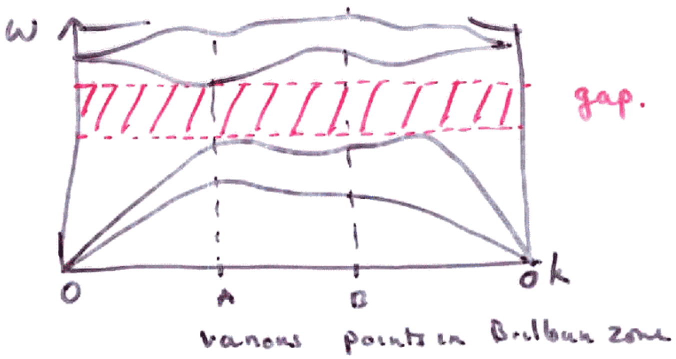


- — light ~ microns
  — microwaves ~ mm/cm

New possibilities are becoming realizable with "designer" artificial materials which are patterned on the wavelength scale

(a) initial development:

Photonic Band gap materials.



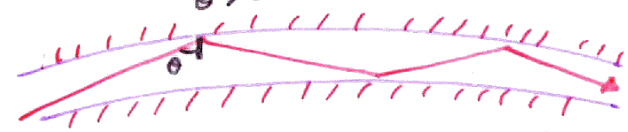
S. John, E. Yablonowitch, etc. J. Joannopoulos Soukoulis

- Goal: to create a periodic pattern of dielectric that has a full band gap in some frequency range
- easy in 2D, not easy in 3D.
  — achieved with "Lincoln Log" structure
- one goal:
  to understand what structures maximise such gaps.

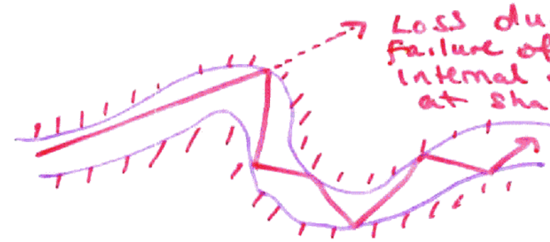
- to build / self-assemble them on an optical wavelength scale.

applications:

better waveguides / optical fibers



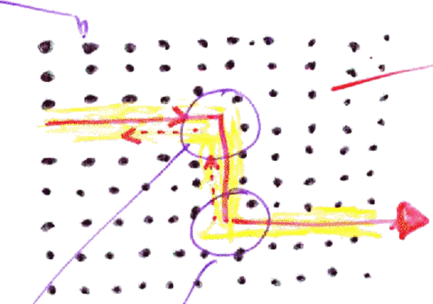
propagation by total internal reflection at fiber surface.



Loss due to failure of total internal reflection at sharp bends

array of silicon posts

ω in gap



photons at frequency ω in gap cannot propagate here!

propagate ⊥ to crystal

defect (2D localized mode)

need to fine-tune structure to avoid backscattering at a particular frequency ω^* in gap (move a few posts to tune phase shift to $2\pi n$)

review:
P. Vukusic
Physics world p35
Feb 2004

pbg occurence in nature:

minerals: opals, irridescence

biological: insect wing colors (butterflies)
fish scales, bird feathers
....

Photonic band gap effects are essentially straightforward to understand, but complicated to design/build so far

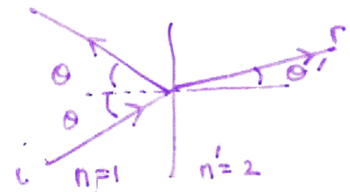
- "self-assembly" methods are being searched for.
- variants: PBG in quasicrystals? (expts by Chaikin, Steinhardt, group at Princeton).
- will now discuss more "exotic" effects.....

See FDMH cond-mat/0206420

"Negative Refractive index"

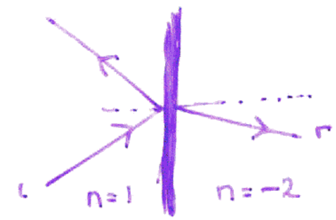
Veselago, (1964)

$n \sin\theta = n' \sin\theta'$
Snell's Law



$n = \sqrt{\varepsilon\mu}$ for $\varepsilon > 0$, $\mu > 0$

Veselago asked: what if $n < 0$?



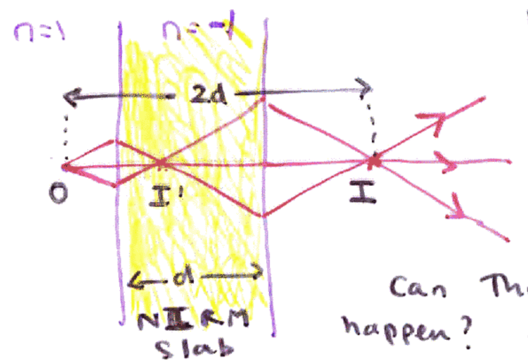
- showed this was the case if $\begin{cases} \varepsilon < 0 \\ \mu < 0 \\ \varepsilon\mu > 0 \end{cases}$!

$n = \sqrt{\varepsilon\mu}\,\mathrm{sgn}(\varepsilon)$.

Veselago flat lens focusses light when $n = -1$!

no lens axis, so no aberration!

$\varepsilon_1\mu_1 = \varepsilon_2\mu_2$



Can this really happen? — YES!

Pendry (2000) PRL $\underline{85}$, 3966 (2000)

If $\epsilon_1\mu_1 = \epsilon_2\mu_2$ $\quad |n_1| = |n_2|$

and $\frac{\mu_1}{\epsilon_1} = \frac{\mu_2}{\epsilon_2}$ impedence matched (no reflection)

So $\boxed{\begin{matrix}\mu_2 = -\mu_1 \\ \epsilon_2 = -\epsilon_1\end{matrix}}$

medium 1 | "Left-handed" medium 2

$\nabla\times E = -\frac{\partial B}{\partial t} = -\mu_0\mu_1\frac{\partial H}{\partial t}$ | $\nabla\times E = -\mu_0\mu_2\frac{\partial H}{\partial t}$

$\nabla\times H = \frac{\partial D}{\partial t} = \epsilon_0\epsilon_1\frac{\partial E}{\partial t}$ | $\nabla\times H = \epsilon_0\epsilon_2\frac{\partial E}{\partial t}$

- medium 2 works like medium 1 running backwards in time! (or $e^{ikz} \to -e^{ikz}$ in $\nabla\times$)

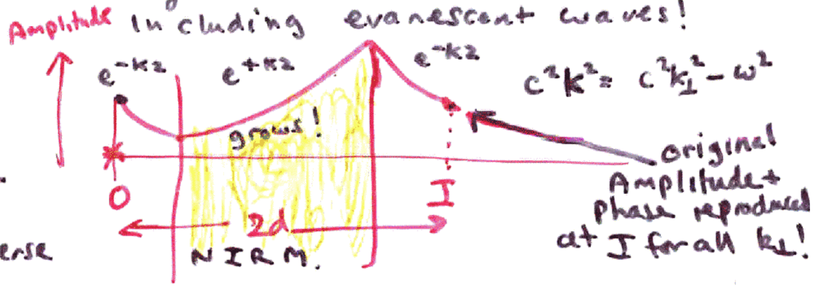
"PERFECT IMAGE"!

- Pendry looked at full maxwell eqn solution for veselago lens... Image formation not just in ray limit but at ALL lengthscales, including evanescent waves!

Steady State

$c^2k_\perp^2 > \omega^2$

Fine transverse detail



- Pendry's claim excited a lot of controversy, including claims that
  - "Violates causality"
  - "violates uncertainty principle"
  - "Violates energy conservation"
  - "not just perfect lens, but also veselago are wrong"

Pendry gave a mathematically apparently correct solution of steady-state Maxwell equations with $\epsilon_1 = -\epsilon_2$, $\mu_1 = -\mu_2$. using his "transfer matrix" formalism for transmission through a slab, without dissipation

but, taken literally, the image would be perfectly focussed at $\underline{\text{all}}$ lengthscales.

- "DISCOVER MAGAZINE" reports this; "Could burn 1000's of movies onto a single CD" with this technology (Subwavelength focussing) — Hollywood's nightmare when internet movie pirates buy this equipment!!!

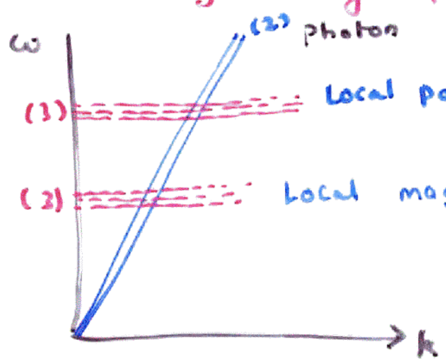
- All this sounds like "perpetual motion machine" etc. Is there a catch?

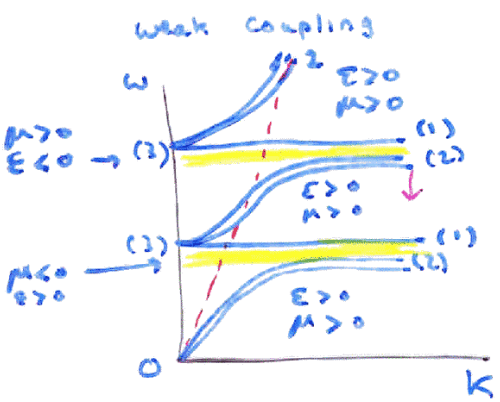
FDMH. cond-mat/0206420

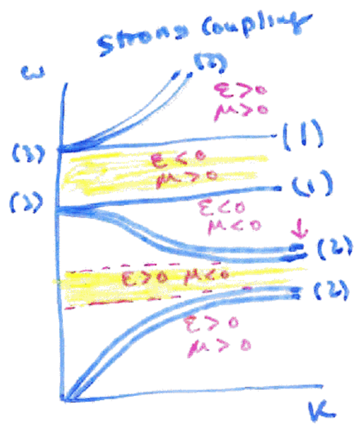
analyse this from a condensed-matter physics viewpoint.

- what is a "negative index of refraction material" ?

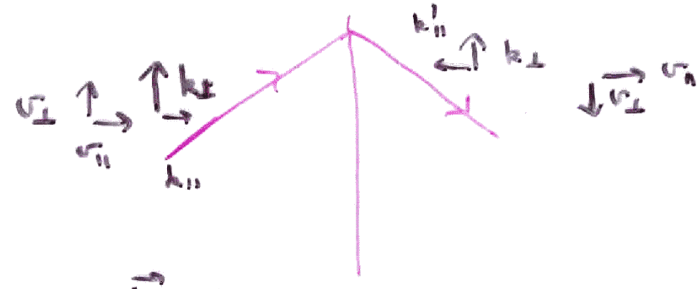
= negative group velocity







- In a NIRM, the group velocity is antiparallel to the wave vector

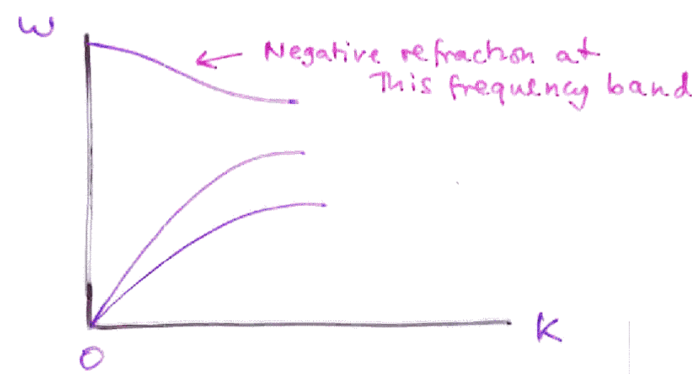


$\vec{k}_\perp$ (perpendicular to normal) is conserved at interface.

(microwaves)

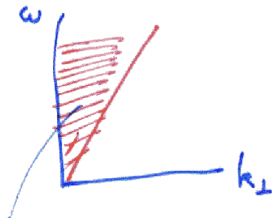
image formation

- Now experimentally confirmed. Also seen in higher bands of photonic crystals

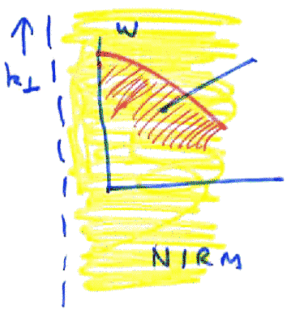


Universal Edge modes at a "NIRM" surface

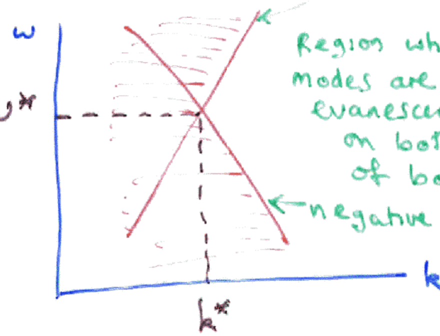
"surface polariton" noted by Ruppin, Phys. Lett. A 277 61 (2000).



Bulk modes
$\omega = c\sqrt{k_\parallel^2 + k_\perp^2}$



bulk single photon modes

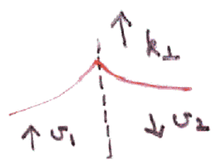


positive velocity v_1

Region where modes are evanescent on both sides of boundary

negative velocity v_2

Frequency at which $n_1 = -n_2$ (no change in wavelength on refraction)



Surface mode propagates at a velocity $v_2 < v < v_1$
negative positive

impedance matched limit: Surface mode is static!!!

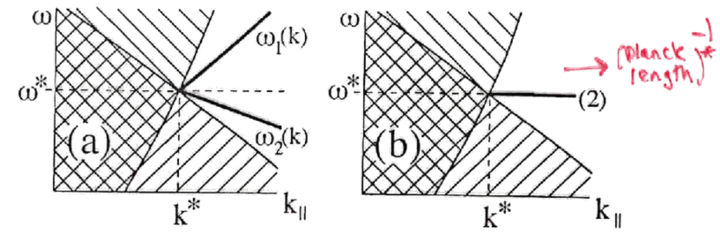


FIG. 2: (a): Generic spectrum of electromagnetic modes that propagate along a negative-refractive-index interface with surface wavenumber $k_\parallel > k^*$: there are two surface modes, respectively with positive and negative group velocity; the shaded regions of the $(\omega, k_\parallel)$ plane indicate where either one or both of the media supports propagating bulk modes, and ω^* is the special frequency at which waves have the same wavelength $\lambda^* = 2\pi/k^*$ in both media. (b): The degenerate spectrum predicted by the local Maxwell equations in the impedance-matched limit where "perfect lens" behavior has been predicted: the two surface modes become degenerate at the frequency ω^* for all $k_\parallel > k^*$. Such exactly-dispersionless degenerate surface modes are an artifact of the approximation which neglects wavenumber-dependence of the constitutive relations of the "left-handed" medium.

* if we take the theory too literally!

Still, some possibility for sub-wavelength engineering, need to include surface mode lifetime, etc.

ω^* gets exponentially closer to resonance as $k_\parallel$ increases.

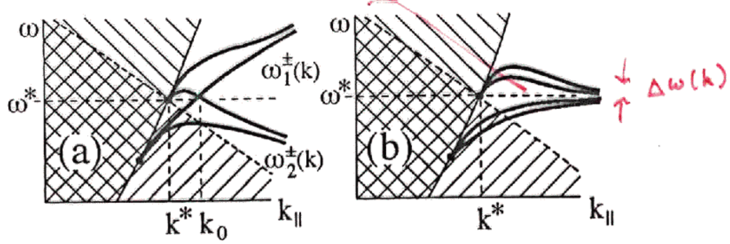


FIG. 3: (a): Generic spectrum of the coupled electromagnetic surface modes of a flat slab of "left-handed" medium, with finite thickness d, embedded in a standard "right-handed" medium, calculated assuming local (wavenumber-independent) constitutive relations. The splitting between the even and odd combinations of the surface modes becomes exponentially small for $k_\parallel$ large. An allowed band crossing occurs exactly at the frequency ω^*, at $k_\parallel = k_0$ (see text). (b): Predicted spectrum in the "perfect lens" limit (perfect impedance matching). The band-crossing point k_0 recedes to $k_\parallel = \infty$, and for large $k_\parallel$ the surface mode frequencies differ from the "perfect lens" frequency ω^* by exponentially small splittings proportional to $\exp(-k_\parallel d)$.

Illuminate at ω^*

Time taken to reach steady state after switching on illumination...

$\tau \sim \frac{1}{\Delta\omega(k)}$

$\sim e^{k_\parallel d}$!

This image is smeared by dispersion in a photonic crystal

but This image is experimentally seen (Sridhar, (2004)) (not "perfect", though....)

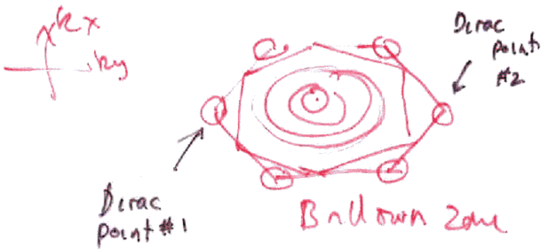
II • Quantum Hall analogs in photonics.

- quantum Hall effect
  - charged particles in a magnetic field
  - incompressible states
- non-interacting electrons exhibit Integer Quantum Hall effect, with incompressibility due to the Pauli Principle....
- Can any of This carry over to photons = neutral bosons ?
- First hint That This is possible

we do not need a uniform magnetic flux component, Landau levels, etc to get a quantum Hall effect:

"Simple" Bloch states suffice...

FDMH PRL 61, 2017 (1988)



half-filled system = semimetal

graphene

(now familiar from nanotubes)

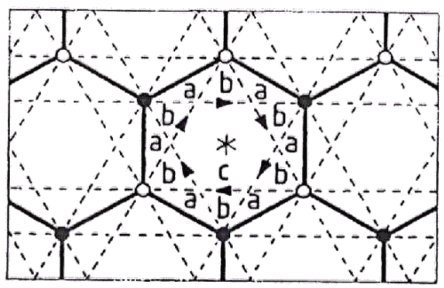


FIG. 1. The honeycomb-net model ("2D graphite") showing nearest-neighbor bonds (solid lines) and second-neighbor bonds (dashed lines). Open and solid points, respectively, mark the A and B sublattice sites. The Wigner-Seitz unit cell is conveniently centered on the point of sixfold rotation symmetry (marked "*") and is then bounded by the hexagon of nearest-neighbor bonds. Arrows on second-neighbor bonds mark the directions of positive phase hopping in the state with broken time-reversal invariance.

- Dirac point = degeneracy when 2 parameters, k_x, k_y are varied
- Requires both T (Time reversal sym) + I (inversion sym.)

(two $\vec{k} \to -\vec{k}$ symmetries)

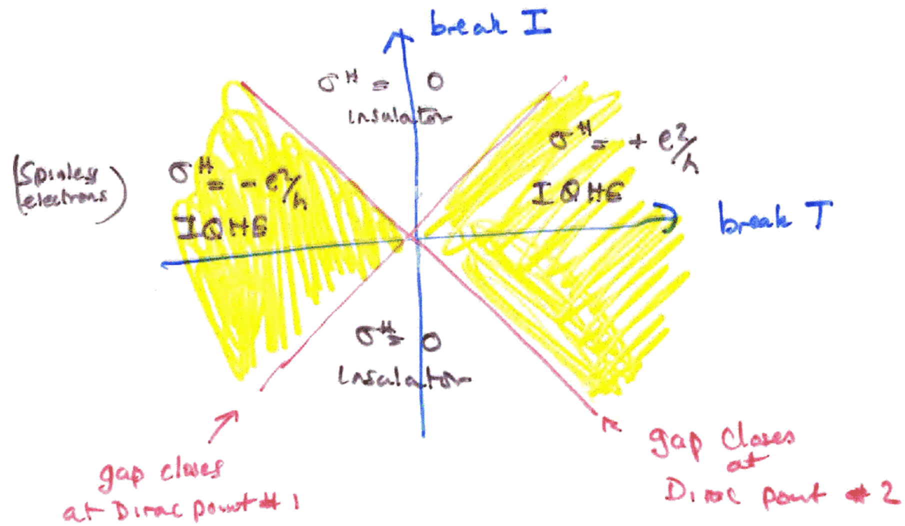
$\psi_{\vec{k}} = e^{ikx} u_k(x)$

$u_k = u_{-k}$ I

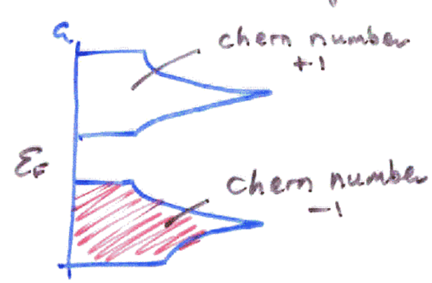
$u_k = u^*_{-k}$ T

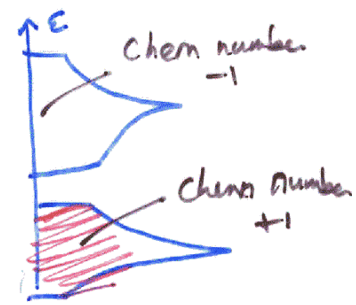
$u_k = u^*_k$ I + T $\xrightarrow{T}$ real

- weak Inversion sym. get gap at both points, (boring) Insulator
- Break Time reversal symmetry → get a gap at both Dirac Points!
- Surprise: system is a QHE system



- Chiral anomaly of 2+1-dim Dirac equation





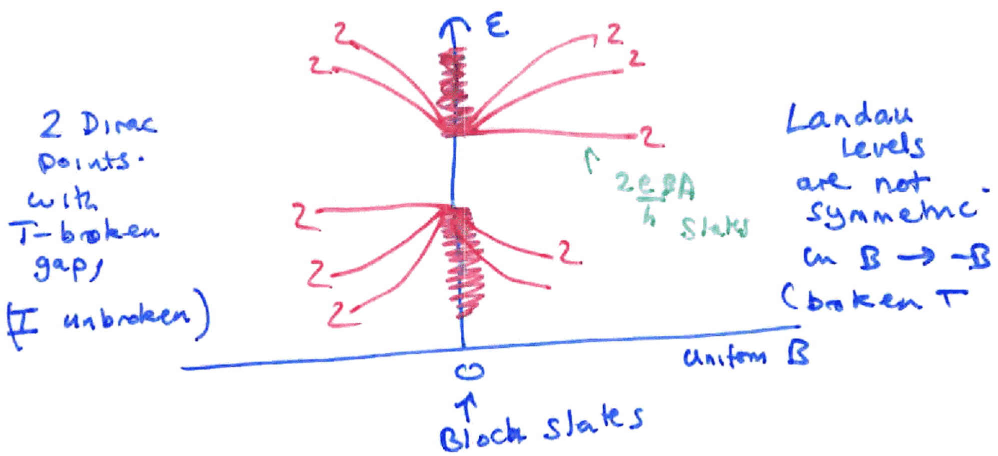
Quantum Hall effect

$$\left.\begin{aligned} J^x &= \sigma^H E^y \\ J^y &= -\sigma^H E^x \\ \rho &= \sigma^H B \end{aligned}\right\}$$

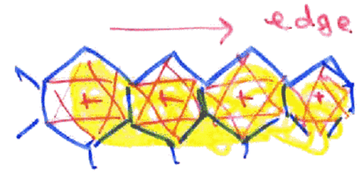
requires incompressibility, Pauli principle

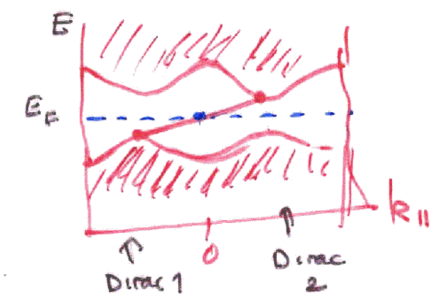
- No photonic Analog

Quantum Hall edge states

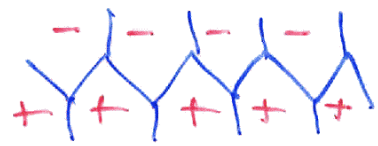


- Spectral flow occurs at edge of system!

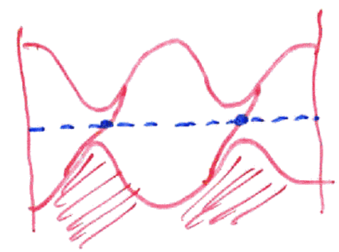




Chiral edge mode



Domain wall



2 chiral edge modes

$(\sigma^H = +e^2/h \rightarrow -e^2/h$

$\Delta\nu = 2 \;)$

- These are one body properties of Bloch states essential feature:

<u>Berry curvature</u>

$\psi_{k\nu} \quad e^{i\vec{k}\cdot\vec{x}} u_{k\nu}(x)$

as $\vec{k}$ changes

$$\boxed{F^{ab}_{\nu}(k) \quad \frac{\frac{1}{i}\int_{cell} d^3x \; \frac{\partial u^*(x)}{\partial k^a}\frac{\partial u(x)}{\partial k^b}}{\int_{cell} d^3x \quad u^*_k(x)\, u_k(x)} \quad - a \rightleftarrows b}$$

will occur for any Bloch-type state

$F^{ab}_{\nu}(k) = \quad F^{ab}(-k) \qquad$ Inversion sym

$F^{ab}_{\nu}(k) = \; -F^{ab}(-k) \qquad$ Time reversal sym

band index

$$\boxed{\text{• If } T + I \text{ are unbroken, } \quad F^{ab}_{\nu}(k) = 0!}$$

anomalous velocity correction to ray equation in slowly-varying photonic crystal:

~ like Sundaram + Niu (1999) in semiclassical electron dynamics (spin-Hall effect, etc).

<u>Ray optics</u> (eg Mirage

hot road

$\omega_{\nu}(\vec{k}, x)$

$\uparrow$ slowly varying medium.

- ω stays constant as photon propagates ballistically ..

$$\frac{d\vec{k}}{dt} = -\vec{\nabla}_x \,\omega_{\nu}(\vec{k}, x)$$

No Lorentz force in real space

$$\frac{d\vec{x}^a}{dt} = +\vec{\nabla}^a_k \,\omega_{\nu}(\vec{k}, x) \; + \underbrace{F^{ab}_{\nu}}_{\text{antisymmetric}}(\vec{k}, x)\,\frac{dk^b}{dt}$$

"Lorentz force" in k-space!

anomalous velocity.

2D. $\quad \boxed{\int_{BZ} d^2\vec{k} \quad F^{xy}_{\nu}(\vec{k}) \quad = \quad 2\pi N_{\nu}} \quad$ chern number

3D $\quad \int_{BZ} d^3\vec{k} \;\; F^{ab}_{\nu}(\vec{k}) \;\; = \;\; 2\pi\, \epsilon^{abc}\, G_{\nu c}$

$\uparrow$ a reciprocal vector.

- Non trivial bands characterized by non zero Chern number
- Vanishes if T-invariance is present.

Solving the maxwell normal mode problem (in a method inspired by Lattice QED!)

$$\boxed{\begin{aligned}\nabla\times E &= -\frac{\partial B}{\partial t}\\ \nabla\times H &= \frac{\partial D}{\partial t}\end{aligned}} \qquad \boxed{\begin{aligned}E &= \varepsilon^{-1}(x)\,D\\ H &= \mu^{-1}(x)\,B\end{aligned}}$$

$$\boxed{\begin{aligned}\nabla\cdot B &= 0\\ \nabla\cdot D &= 0\end{aligned}}$$

$$\underset{\text{real}}{E(x,t)} = E_\nu(x)\,e^{-i\omega t} + \underset{\text{complex}}{E_\nu^*(x)}\,e^{+i\omega t}$$

$$\begin{pmatrix} 0 & -i\nabla\times \\ i\nabla\times & 0\end{pmatrix}\begin{pmatrix}E_\nu\\ H_\nu\end{pmatrix} = \omega_\nu\begin{pmatrix}D_\nu\\ E_\nu\end{pmatrix}$$

$$\begin{pmatrix}\varepsilon^{-1} & 0\\ 0 & \mu^{-1}\end{pmatrix}\begin{pmatrix}D_\nu\\ B_\nu\end{pmatrix} = \begin{pmatrix}E_\nu\\ H_\nu\end{pmatrix}$$

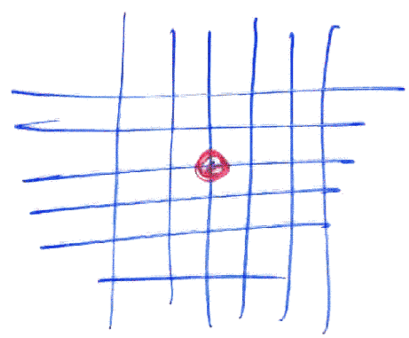
$$\left.\begin{aligned} -i\epsilon^{abc}\nabla_b E_{\nu c} &= \omega_\nu B^a\\ +i\epsilon^{abc}\nabla_b H_{\nu c} &= \omega_\nu D^a\\ \varepsilon^{-1}_{ab} D^b_\nu &= E_{\nu a}\\ \mu^{-1}_{ab} B^b_\nu &= H_{\nu a}\end{aligned}\right\}$$

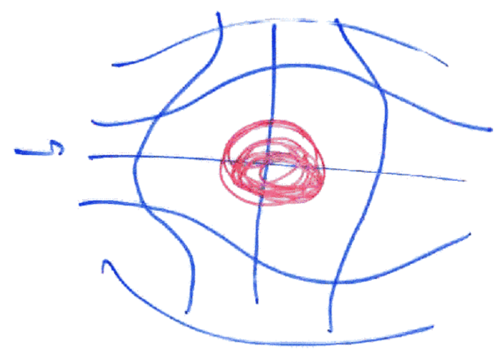
covariant.
No metric (in space)
(ε^{-1}_{ab} μ^{-1}_{ab}) instead

(• In vacuum only $\varepsilon^{-1}_{ab} = \varepsilon_0^{-1} g_{ab}$, $\mu^{-1}_{ab} = \mu_0^{-1} g_{ab}$)

• we can choose a curvilinear coordinate system in which we will perform a Euclidean metric Voronoi construction

$\rightarrow$ $\equiv$ Voronoi with arbitrary space metric, provided signature is Euclidean (+++)





magnify ~~the~~ critical regions before a discretization!

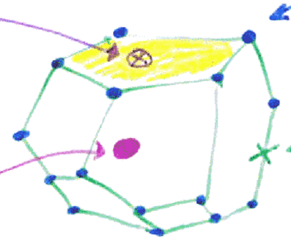
- VORONOI CONSTRUCTION
  (3D, Euclidean metric)

— Choose a set of points
("cell centers")

— Construct the convex polyhedron surrounding each center which encloses the region of 3D space closer to that point than to any other point.

centroid* of coplanar polygonal face (F)

vertex (V)



Edge (E)
(midpoint between two vertices)

Cell center (C) of convex Voronoi cell.

"anatomy" of the Voronoi cell.

* centroid (of coplanar convex polygon)

$\hat{n}$ ← normal

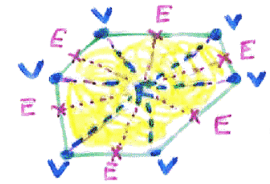
$\hat{n}\cdot(x_n - x_{n'}) = 0$ (coplanar)

centroid $\vec{x}_0$ = "center of gravity"

$$\vec{x}_0 = \frac{\sum_{nn'} \frac{1}{3}(\vec{x}_n + \vec{x}_{n+1} + \vec{x}_{n'})(\hat{n}\cdot(x_n - x_{n'}) \times (x_{n+1} - x_{n'}))}{\sum_{nn'} \hat{n}\cdot(x_n - x_{n'}) \times (x_{n+1} - x_{n'})}$$

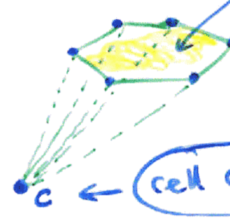
Disect space:

Cell Face:



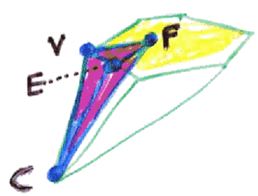
cell face

Voronoi cell can be disected into pyramids:
{ apex = center
{ base = cell face



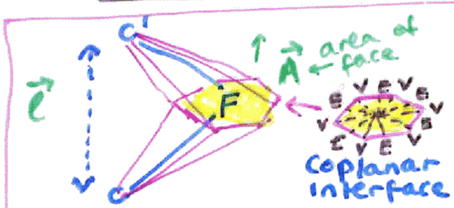
cell center

disect the pyramids into elementary tetrahedra

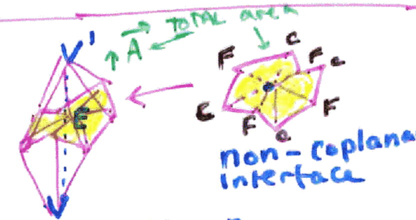


each tetrahedron has four vertices, one of each type C, V, F, E.

→ reassemble to make dual cells centered on "V"



non-straight path C → F → C'

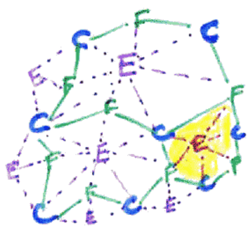


straight path V → E → V'

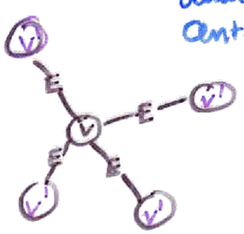
dual lattice cells (not Voronoi)

- centered at V
- non-coplanar faces centered at E
- perimeter of faces are paths
  C → F → C → F ..... → C
- each face has an outward-pointing area $\vec{A}_\alpha$, and displacement $\vec{\ell}_\alpha$ to neighboring cell center.

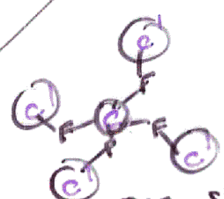
True for both types of cell → Volume of cell $= \frac{1}{3}\sum_\alpha \vec{\ell}_\alpha \cdot \vec{A}_\alpha$



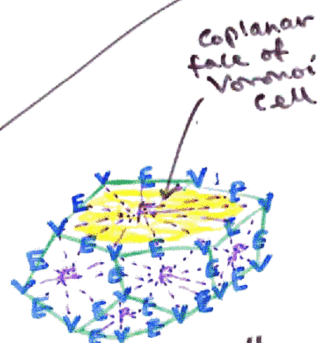
dual cell centered on V



Straight flux paths



non-straight flux paths



Original cell centered on C

- get two dual interpenetrating decompositions of 3D space into polyhedral cells
- Each type of cell has faces of definite area $\vec{A}_\alpha$ connecting neighbor cells. Electric or Magnetic flux flows across these faces.
- Assign an arbitrary direction to each flux: α

"Z_2 gauge choice"

$$g_\alpha(x) = \begin{cases} +1 & \text{if flux is away from cell center } x \\ -1 & \text{if flux is towards cell center } x \\ 0 & \text{if flux is not connected to cell center } x. \end{cases}$$

$\vec{\ell}_\alpha$: displacement between neighbor cell centers in direction of flux

$\vec{A}_\alpha$: total (vector sum of) cell face area in direction of flux (right-hand rule)

- $$\boxed{\mathcal{V}(x) = \sum_\alpha \frac{1}{3} g_\alpha(x)\, \vec{\ell}_\alpha \cdot \vec{A}_\alpha}$$

Volume of cell centered at $\underline{x}$ (Voronoi or dual)

Dynamics of Electromagnetic field:

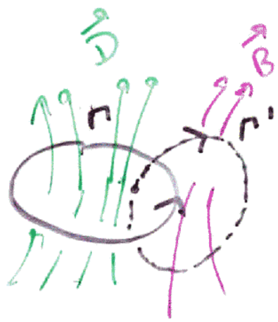
$$\{D^a(x), B^b(x')\}_{PB} = \epsilon^{abc} \nabla_c \delta^3(x-x')$$

Poisson Bracket (non-canonical)

$\nabla_a D^a = \nabla_a B^a = 0$ source free condition

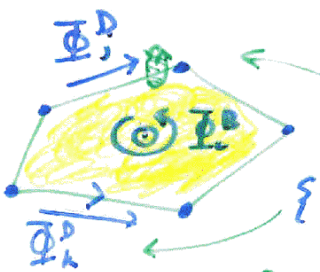
$$\rightarrow \{\Phi^D(r), \Phi^B(r')\}_{PB} = \mathcal{L}_G(r, r')$$

GAUSS LINKING NUMBER



$\mathcal{L}_G = +1$ (right-hand rule)

completely topological Poisson bracket!



$$\{\Phi^D_i, \Phi^B_j\}_{PB} = -1$$

$$\{\Phi^D_k, \Phi^B_j\} = +1$$

Poisson bracket vanishes unless Φ^D is along edge of Φ^B plaquette (face)

$\hbar \to 0$ Limit of: lattice QED:

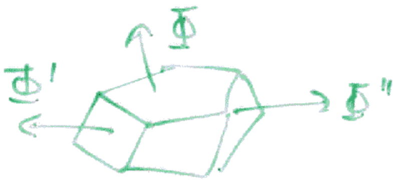
- electric charges "live" at Voronoi cell vertices V
- electric currents flow along Voronoi cell edges E
- magnetic monopoles "live" at Voronoi cell centers C
- magnetic monopoles tunnel across Voronoi cell faces F.
- electric fluxes along cell edges
- magnetic fluxes across cell faces.
- An "action-angle" pair of variables N_α, $e^{i\phi_\alpha}$ associated with each edge α

$\Phi^D_\alpha = e N_\alpha$ electric flux along edge

$e^{ie/\hbar \Phi^B_\Gamma} = \prod_{\alpha \,(\text{plaquette})} e^{i\phi_\alpha}$ magnetic flux through face

$\boxed{* \quad \bullet \text{ What about field energy?}}$

Energetic of electro-magnetic field



$$U(x) = \frac{1}{2}\sum_{ij} \underbrace{u_{ij}(x)\,\Phi_i\,\Phi_j}$$

electric or magnetic energy of cell centered at $\underline{x}$

quadratic form (gaussian) in fluxes attached to faces of cell $\underline{x}$

- Should be

magnetic energy

$$U(x) = \frac{1}{2}\, v^{\text{cell}}(x) \sum_{ab} B^a \mu^{-1}_{ab}(x) B^b$$

IF ~~[illegible]~~ flux density is uniform:

Then $\Phi_\alpha = B^a A_{a\alpha}$

area of face α

- Only three independent components of uniform flux density
- At least 4 fluxes (4 faces)
- Overcomplete!

- "Physical" (low frequency) modes correspond to (locally) uniform magnetic flux density (on lattice scale).

Projection operator:

1 or 0 if α is or is not attached to $\underline{x}$

areas of cell faces

Key idea

$$S_{ab}(x) = \sum_\alpha g_\alpha(x)^2 A_{a\alpha} A_{\alpha b}$$

$$\tilde{S}^{ab}(x)\, S_{bc}(x) = \delta^a_c \qquad \text{inverse}$$

$$\boxed{B^a(x) = \tilde{S}^{ab}(x) \sum_\alpha g_\alpha(x)^2 A_{\alpha b}\, \Phi^B_\alpha}$$

expresses flux density at center of cell in terms of fluxes through faces. gives correct answer for uniform flux density

$$U(x) = \frac{1}{2}\sum_\alpha \lambda_\alpha^2\, g_\alpha^2(x)\, \Phi_\alpha^2$$

$$+\frac{1}{2}\Big(\underbrace{v(x)\,\mu^{-1}_{ab}(x) - \sum_\alpha \lambda_\alpha^2 g_\alpha^2(x) A_{a\alpha} A_{\alpha b}}\Big) B^a(x) B^b(x)$$

choose λ_α^2 to make this vanish as much as possible

needed to give "unphysical" modes high frequency

- similar expression for electric field energy

Final Structure

$$\boxed{\begin{aligned} i\{\Phi_i, \Phi_j\}_{PB} &= A_{ij} \\ H &= \tfrac{1}{2}\sum_{ij} B_{ij}\Phi_i\Phi_j . \end{aligned}}$$

has 33% zero modes!

- lattice flux non-divergence
- lattice local energy conservation are exact

Topological • A_{ij} is imaginary antisymmetric (only couples different kinds of flux)

Continuous variable • B_{ij} is real symmetric positive (definite) (only couples same kind of flux)

• eigenvalue equation (generalized Hermitian)

$$\boxed{\underline{A}\,\underline{B}\,\underline{x}_\nu = \omega_\nu \underline{x}_\nu}$$

real because B is positive

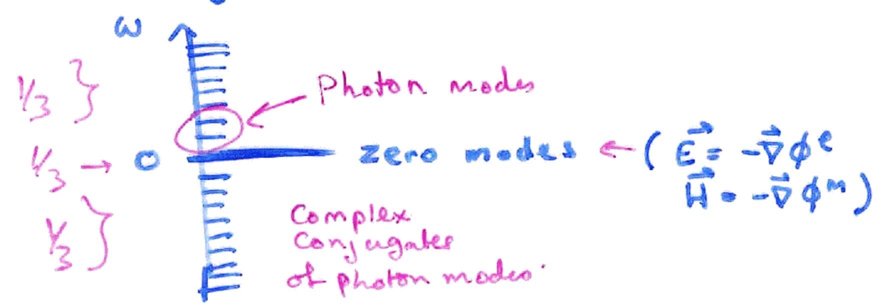
Only connects fluxes on ~~site~~ same edge/face

only connects fluxes from same cells

Very sparse matrices! (not like fourier space methods!)

→ LANCZOS TECHNIQUES

eigenvalue structure (symmetric in $\omega \to -\omega$)



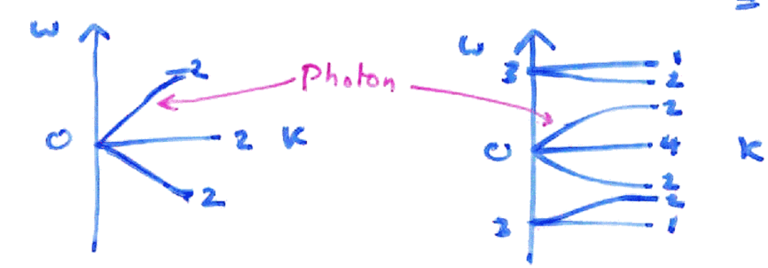
• want modes with small positive ω:

Solve $(ABA - 2\omega_0 A)B\,\underline{x}^\nu = (\omega_\nu^2 - 2\omega_0\omega_\nu)\,\underline{x}^\nu$

$\uparrow$ $(\omega_\nu - \omega_0)^2 - \omega_0^2$

for lowest eigenvalues (Lanczos)

• empty lattice: use a Bravais lattice of points, $\underline{B} \propto \underline{I}$



Cell = Cube (simple cubic)

$3+3=6$ fluxes/cell

self dual.

cell = rhombic dodecahedron (face-centered cubic)

$6+8$ fluxes/cell

Can easily include

- Local polarization/magnetization degrees of freedom (frequency-dependent constitutive relations)
- metallic regions (perfect conducting walls, $\varepsilon^{-1} = 0$)
- test implementations show This scheme works!

(simple cubic lattice scheme is related to earlier scheme of Pendry + McKinnon (1992)) but differs in detail.