Color Superconductivity and the Fermi Surface in Quark Matter

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Outline:

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(1) Quark matter and color superconductivity

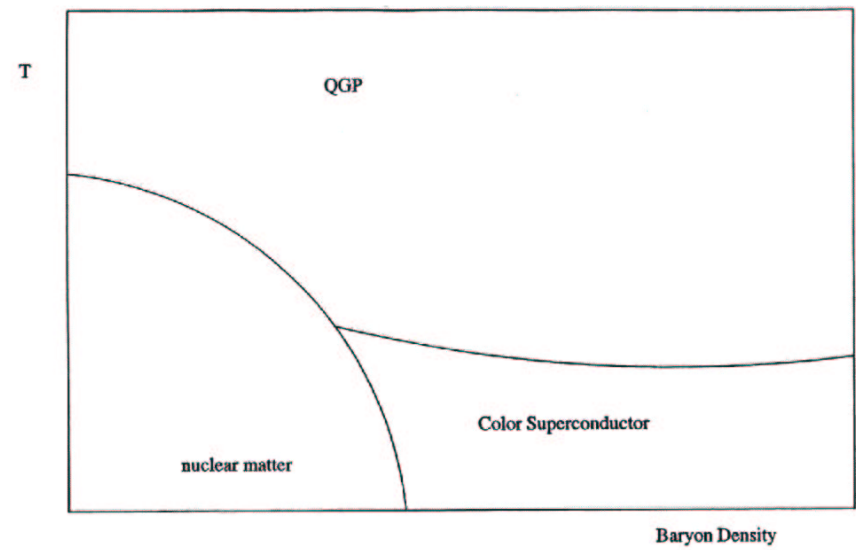
(2) Physics near a Fermi surface

(3) Results at intermediate and asymptotic densities

(4) Positivity of Fermi surface EFT

(5) Applications: Vafa-Witten theorem and lattice Monte Carlo simulation

(6) Future prospects: neutron stars?



QCD and its Phase Diagram

QCD: Non-Abelian gauge theory exhibiting asymptotic freedom.

$$\mathcal{L}_{\rm QCD} = \bar{\psi} i \not{D} \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \mu \bar{\psi} \gamma_0 \psi$$

Three qualitatively different density regimes:

Low Density: $\alpha_s(\mu) >> 1$; confinement, chiral symmetry breaking

Intermediate Density: $\alpha_s(\mu) \sim 1$; neutron star interior? color superconductivity?

High Density: $\alpha_s(\mu) << 1$; color superconductivity, color-flavor locking (CFL)

Quark Matter and Color Superconductivity

"Quark Matter" = matter in which effective degrees of freedom have quantum numbers of quarks (fundamental rep'n of SU(3) color) rather than baryons (color singlets). Must apply at sufficiently high density, due to asymptotic freedom.

Due to Pauli exclusion (quarks are fermions), must have filled Fermi sea and Fermi surface (FS).

Folk theorem: Near a FS, arbitrarily weak interactions can cause Cooper pairing.

In weak coupling (one gluon exchange), the antisymmetric $\bar{3}$ channel of quark-quark scattering is attractive. (No phonons required as in ordinary super-condutivity.)

Conjecture: (Frautschi, 1978) Quark matter is a color superconductor, with a diquark condensate $\langle \psi \Gamma \psi \rangle$ in the $\bar{3}$ channel.

$SU(3)_c \to SU(2)_c$, 5 massive and 3 massless gluons. 5 superconducting charges (chromo-Meissner effect, etc.)

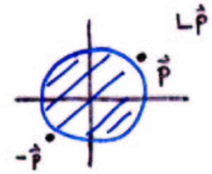
Some modern developments (1998-present):

1) Controlled calculations at asymptotic densities, accounting for long-range color-magnetic fluctuations.

2) Elaboration of flavor structure: CFL is the 3-flavor groundstate at high density.

3) Fermi surface effective theories and RG methods.

Physics near a Fermi surface :



Arbitrarily weak attraction can lead to instability near FS.

$\langle \psi(\vec{p})\, \psi(-\vec{p}) \rangle \neq 0$ Cooper pairing

• Dimensional reduction near FS !

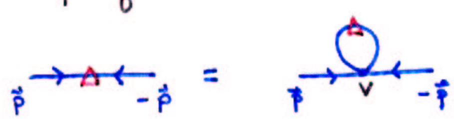
$\vec{p} = \vec{p}_F + \vec{\ell}$

$|\vec{p}_F| = \mu$

$\epsilon(\vec{p}) = p - \mu \simeq |\vec{\ell}|$

$\exists$ degeneracy, as $\epsilon(\vec{p})$ indep. of orientation of $\vec{p}_F$
$\to$ effectively 2D system

• Gap eq'n



$$\Delta = V \int \frac{d^4q}{(2\pi)^4} \frac{\Delta(q)}{q_0^2 + q_\ell^2 + \Delta^2} = V \underbrace{\int \frac{d\Omega_q\, \mu^2}{(2\pi)^3}}_{N/2\pi} \int dq_0\, dq_\ell \frac{\Delta(q)}{q_0^2 + q_\ell^2 + \Delta^2}$$

$$\Delta \simeq \mu \exp\left[-\frac{1}{VN}\right]$$

sol'n even for weak interaction

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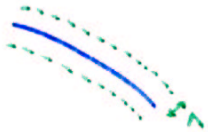
Renormalization Group Methods

Benfatto & Gallavotti
Shankar
Polchinski

$$S_{eff} = S_{kin} + S_{4\text{-}F} + \dots$$

Effective action for modes near FS:



$\epsilon(\vec{p}) \simeq |\vec{l}\,| < \Lambda$ low energy, but $|\vec{p}\,| \simeq |\vec{p}_F| \simeq \mu$

$\mu \gg \Lambda$

<u>RG scaling</u>:

$\Lambda \to 0$

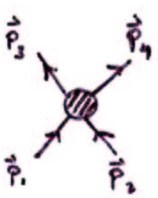
$\left.\begin{array}{l} E \to sE \\ l \to sl \\ \vec{p}_F \to \vec{p}_F \end{array}\right\}$ study s-scaling of interactions.

$\psi \sim s^{-1/2}$

All higher dimension operators are <u>irrelevant</u>

<u>Except</u> the 4-F operator with Cooper pairing kinematics.

$$S_{4\text{-}F} = \int dt\, d^3p_1\, d^3p_2\, d^3p_3\, d^3p_4 \quad V(p_i)\ \psi^{+}(p_3)\,\Gamma_1\,\psi(p_1) \cdot \psi^{+}(p_4)\,\Gamma_2\,\psi(p_2) \quad \delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3+\vec{p}_4)$$



this operator is <u>Marginal</u> if $\vec{p}_1 \approx -\vec{p}_2$

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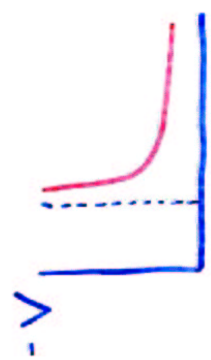
<u>Quantum</u> <u>Corrections</u>:

$$s\frac{\partial}{\partial s} V = N V^2$$

$$V(E) = \frac{V(E_i)}{1 + NV \ln(E_i/E)}$$



$V < 0$

Landau pole!

$E_L = E_i \exp\left[-1/|V|N\right]$

- RG invariant scale: related to gap Δ?
- Existence of Fermi liquid with Cooper pairing instability is <u>NATURAL</u> – generic result almost independent of dynamics at higher energy.

Asymptotic Density

Weak coupling, but main issue is how to handle long-range magnetic fluctuations. Can show that no magnetic mass is generated at any order of perturbation theory.

It turns out that Landau damping provides enough screening to obtain controlled results, but qualitatively changes the behavior relative to the naive expectation (Son, Hsu and Schwetz, Pisarski and Rischke, Schafer and Wilczek,...)

Landau damping is a mechanism by which (color-)magnetic fluctuations can lose energy to the Fermi sea. Vanishes when external magnetic frequency $\to 0$.

$$D_{\mu\nu} = \frac{P^T_{\mu\nu}}{q^2+G} + \frac{P^L_{\mu\nu}}{q^2+F}$$

- F is the Debye mass term $m_D^2 \sim g^2\mu^2$
- G the Landau damping term $\sim i m_D^2 \frac{|q_0|}{q}$.

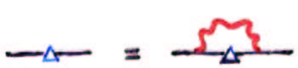
New momentum scale determined self-consistently :

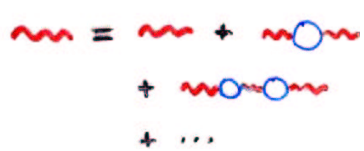
$$q_*^2 \sim G(q_*) \ \to \ q_* \sim m_D^{2/3}\Delta^{1/3}$$

(Δ = IR energy cutoff = gap)

Physics controlled by $g_s(q_*)$. Weak-coupling approximation self-consistent as long as $g_s(q_*) << 1$.

$$\Delta \sim \mu g^{-5} \exp\left[-\frac{3\pi^2}{\sqrt{2}g}\right] \quad (1+\mathcal{O}(g))$$





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Energetics: determining the QCD groundstate at high density

Evans, Hormuzdiar, Hsu + Schwetz '99
Schäfer '99

RG analysis : identifies FS instability
Schwinger-Dyson Eq'n : identifies extrema of energy

Additional analysis necessary to determine groundstate.

E ~ 

$E_B \sim \mu^2\Delta^2$

Dominant channels for diquark condensation: candidates for extrema.

$\left.\begin{array}{l}\bar{3} \text{ of color (AS)} \\ J=L=S=0 \text{ (AS)}\end{array}\right\} \xrightarrow{\text{Pauli}}$ flavor AS channel

$N_f = 2$: (ud − du) isospin singlet

$N_f = 3$: 3 × $\bar{3}$'s of $SU(3)_f$

$N_f = 3$: Color-Flavor Locking and all that...

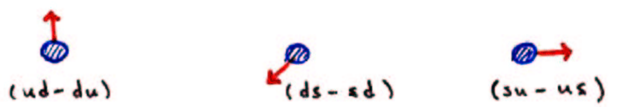
Alford, Rajagopal + Wilczek

$$\Delta^{ab\ L,R}_{ij} = A^{c\ L,R}_{k}\ \underset{\text{color}}{\varepsilon^{abc}}\ \underset{\text{flavor}}{\varepsilon_{ijk}}$$

using global $SU(3)_c$, $SU(3)_{L,R}$:

$$A = \begin{bmatrix} a & & \\ & b & \\ & & c \end{bmatrix}$$

CFL : $a = b = c$



(111)

Each flavor channel orients in a different, orthogonal direction in $SU(3)_c$ color space.

But, other values of a, b, c lead to sol'ns of the S-D eqns:

$\{a=1;\ b=c=0\}$ (100)

$\{a=b=1;\ c=0\}$ (110)

while $\Delta_{100} > \Delta_{110} > \Delta_{111}$, we find

$$E_{111} < E_{110} < E_{100}$$

Evans, Hormuzdiar, Hsu + Schwetz
Hong et al.

Intermediate Density

Quark matter, but strongly coupled. But note two expansion parameters: α_s , Δ/μ (width of gapped region likely much less than μ).

RG approach: Evans, Hsu and Schwetz; Schafer and Wilczek

Conjecture: magnetic fluctuations screened by non-perturbative effects. (Chromo-magnetic monopoles?)

$\rightarrow$ effectively local interactions

$$\mathcal{O}_{ij} \sim \frac{c_{ij}}{\Lambda^2}\, \bar{\psi}\Gamma_i\psi\ \bar{\psi}\Gamma_j\psi$$

(e.g., could use instanton interactions as a model.)

Study *most general* set of marginal operators near FS. Classify possible set of RG flows of these operators, given arbitrary matching conditions (initial conditions).

Generically find color superconductivity. Only small subset of initial conditions can avoid Landau pole in $\bar{3}$ channel. Such initial conditions generally require wrong (repulsive) signs of couplings that are attractive in perturbation theory or in instanton models.

Conclude that color superconductivity is likely even at intermediate density (neutron star interior). Gap size Δ given by position of Landau pole, typically $50 - 100$ MeV.

Positivity of Fermi surface effective theory

D.K. Hong and S. Hsu, hep-ph/0202236, to appear in PRD.

• A system of degenerate quarks with a fixed baryon number is described by the QCD Lagrangian density with a chemical potential μ,

$$\mathcal{L}_{\rm QCD} = \bar\psi i \not{D} \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \mu \bar\psi \gamma_0 \psi, \tag{1}$$

where the covariant derivative $D_\mu = \partial_\mu + i A_\mu$ and we neglect the mass of quarks for simplicity.

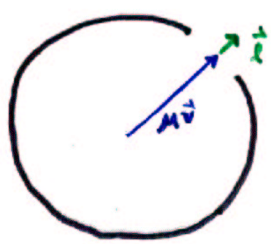
• In Euclidean space, the Dirac operator $i\not{D} + \mu\gamma_0$ is neither Hermitian nor anti-Hermitian, so it has complex eigenvalues $\to$ complex fermion determinant. No Monte Carlo simulation!

• Effective Fermi Surface theory

Decompose the momentum of a quark into a Fermi momentum and a residual momentum as

$$p_\mu = \mu v_\mu + l_\mu, \tag{2}$$

Region near the Fermi surface: $l_0, |\vec{l}| << \mu$.



• Recall why the measure of dense QCD is complex in Euclidean space. Euclidean gamma matrices satisfy

$$\gamma_E^{\mu\dagger} = \gamma_E^\mu \quad , \quad \{\gamma_E^\mu, \gamma_E^\nu\} = 2\delta^{\mu\nu}. \tag{3}$$

• The Dirac-conjugated field, $\bar\psi = \psi^\dagger \gamma^0$, is mapped into a field, still denoted as $\bar\psi$, which is independent of ψ and transforms as $\psi^\dagger$ under $SO(4)$. Then, the grand canonical partition function for QCD is

$$Z(\mu) = \int {\rm d}A_\mu \det(M)\, e^{-S(A_\mu)}, \tag{4}$$

where $S(A_\mu)$ is the positive semi-definite gauge action, and the Dirac operator

$$M = \gamma_E^\mu D_E^\mu + \mu \gamma_E^4, \tag{5}$$

where $D_E = \partial_E + i A_E$ is the analytic continuation of the covariant derivative.

• The Hermitian conjugate of the Dirac operator is

$$M^\dagger = -\gamma_E^\mu D_E^\mu + \mu \gamma_E^4 \quad . \tag{6}$$

• The first term in (6) is anti-Hermitian, while the second is Hermitian, hence the generally complex eigenvalues. When $\mu = 0$, the eigenvalues are purely imaginary, but come in conjugate pairs (λ, λ^*), so the resulting determinant is real and positive.

• To demonstrate this, note that γ_5 anti-commutes with M, so if $M\phi = \lambda\phi$, then $M\gamma_5\phi = -\gamma_5 M\phi = -\lambda\gamma_5\phi$.

Effective Field Theory

Decompose the quark fields as (like Heavy Quark EFT!):

$$\psi(x) = \sum_{\vec{v}_F} \left[e^{i\mu\vec{v}_F\cdot\vec{x}}\psi_+(\vec{v}_F, x) + e^{i\mu\vec{v}_F\cdot\vec{x}}\psi_-(\vec{v}_F, x) \right], \tag{7}$$

where $\vec{\alpha}\cdot\vec{v}_F\psi_\pm(\vec{v}_F, x) = \pm\psi_\pm(\vec{v}_F, x)$ is a projector on (anti)particle states.

The quark Lagrangian in Eq. (1) then becomes

$$\begin{aligned}\bar{\psi}\left(i\not{D} + \mu\gamma^0\right)\psi \;&=\; \sum_{\vec{v}_F}\left[\bar{\psi}_+(\vec{v}_F, x)i\gamma_\parallel^\mu D_\mu\psi_+(\vec{v}_F, x) + \bar{\psi}_-(\vec{v}_F, x)\gamma^0\left(2\mu + iD_\parallel\right)\psi_-(\vec{v}_F, x)\right] \\ &+\; \sum_{\vec{v}_F}\left[\bar{\psi}_-(\vec{v}_F, x)i\not{D}_\perp\psi_+(\vec{v}_F, x) + \text{h.c.}\right]\end{aligned} \tag{8}$$

where $\gamma_\parallel^\mu \equiv (\gamma^0, \vec{v}_F\vec{v}_F\cdot\vec{\gamma})$, $\gamma_\perp^\mu = \gamma^\mu - \gamma_\parallel^\mu$, $D_\parallel = \bar{V}^\mu D_\mu$ with $\bar{V}^\mu = (1, -\vec{v}_F)$, and $\not{D}_\perp = \gamma_\perp^\mu D_\mu$.

• Eliminating massive ψ_- modes, we obtain the tree-level Lagrangian for ψ_+

$$\mathcal{L}_{\text{eff}}^0 = \bar{\psi}_+ i\gamma_\parallel^\mu D_\mu\psi_+ - \frac{1}{2\mu}\bar{\psi}_+\gamma^0(\not{D}_\perp)^2\psi_+ \;+\; \cdots, \tag{9}$$

where the ellipsis denotes terms with higher derivatives.

• The leading (Euclidean) Dirac operator is of the form

$$\mathcal{L}_+ = \bar{\psi}_+ e^{-iX}\left(i\not{\partial} - \not{A} + \mu\gamma_0\right)e^{+iX}\psi_+ \quad , \tag{10}$$

where X is a Hermitian operator which removes the "large" component of momentum $\mu\vec{v}_F$. We can rewrite this as:

$$\mathcal{L}_+ = \bar{\psi}_+\gamma_\parallel^\mu\left(\partial^\mu + iA_+^\mu\right)\psi_+ \quad , \tag{11}$$

where

$$A_+^\mu = e^{-iX}\;A^\mu\;e^{+iX} \quad . \tag{12}$$

• The operator in (11) is anti-Hermitian and leads to a positive, semidefinite determinant since it anti-commutes with γ_5.

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Application: lattice simulation

Back to the partition function:

$$Z(\mu) = \int \mathrm{d}A_\mu \det(M)\, e^{-S(A_\mu)} \; .$$

It is easy to show that at $\mathbf{A} = 0$ (zero background gauge field), the Dirac determinant is real even at finite density.

Now, consider background gauge fields $\mathbf{A}$ whose magnitude and derivatives $\partial\mathbf{A}$ are small relative to μ. (e.g. $\mu >> \Lambda_{\text{QCD}} \sim \mathbf{A}\ ,\ \partial\mathbf{A}$.)

Expanding about the FS, we obtain

$$\det(M) \;=\; [\text{real, positive}]\left(1 \;+\; \mathcal{O}(\frac{\mathbf{A}}{\mu}, \frac{\partial\mathbf{A}}{\mu^2})\right)$$

Use two lattices with different spacings a_{det}, a_{gauge}. Compute determinant on lattice with spacing $a_{\text{det}} \sim \mu^{-1} << a_{\text{gauge}}$.

• Nontrivial check on analytic results at asymptotic density. Extrapolate to intermediate density?

• Applications to condensed matter systems? High-T_c superconductors?

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Application: Vafa-Witten inequalities at finite density

- Vafa-Witten theorem applies to HDET: CFL is exact at high density

- Vector current correlators fall off exponentially, if all quarks are gapped.

$$\left\langle J_\mu^a(\vec{v}_F, x) J_\nu^b(\vec{v}_F, y)\right\rangle^A = -\mathrm{Tr}\,\gamma_\mu T^a S^A(x,y;\Delta)\gamma_\nu T^b S^A(y,x;\Delta),$$

with $J_\mu^a(\vec{v}_F, x) = \bar{\psi}_+(\vec{v}_F, x)\gamma_\mu T^a \psi_+(\vec{v}_F, x)$. The propagator is

$$\langle x|\,\frac{1}{M}\,|y\rangle = \int_0^\infty \mathrm{d}\tau\, \langle x|\, e^{-i\tau(-iM)}\,|y\rangle$$

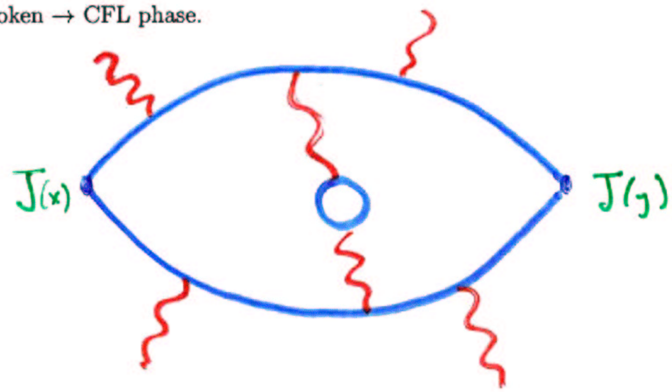
where with $D = \partial + iA$

$$M = \gamma_0 \begin{pmatrix} D\cdot V & \Delta \\ \Delta & D\cdot\bar{V} \end{pmatrix},$$

- Eigenvalues of M bounded from below by Δ, yielding the inequality:

$$\left|\langle x|\,\frac{1}{M}\,|y\rangle\right| \;\leq\; \int_R^\infty \mathrm{d}\tau\, e^{-\Delta\tau}\sqrt{\langle x|x\rangle}\sqrt{\langle y|y\rangle} \;=\; \frac{e^{-\Delta R}}{\Delta}\sqrt{\langle x|x\rangle}\sqrt{\langle y|y\rangle}.$$

No Goldstone mode in the vector channel. For three light flavors $SU(3)_V$ is unbroken $\rightarrow$ CFL phase.



Conclusion and Future Prospects

- Recent developments have vastly improved our understanding of the QCD phase diagram. New phases of matter: color superconductor, CFL, 2SC, etc.

- Analytic results and rigorous theorems at asymptotic densities. Positivity may allow lattice simulations. Special properties of FS play an important role.

- Strong hints of behavior at intermediate densities: color superconductivity likely to persist even in strongly coupled quark matter.

- Phenomenological impact on neutron star physics still an open question.