A mean-field model for the electron glass

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Motivated and influenced by experimental results of Zvi Ovadyahu et al, HU.

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Common features of glassy models

- Randomness, frustration.
- Many states close to the ground state.
- Aging and memory effects: relaxation is slower if perturbation lasts longer.

Physical examples

- Ordinary glasses.
- Various magnetic materials.
- Packing of hard spheres.
- **Electron glass! (long-ranged interactions, but not infinite!)**

OUR PROBLEM HERE!

General questions: structure of states, out-of-equilibrium dynamics?

Experimental motivation

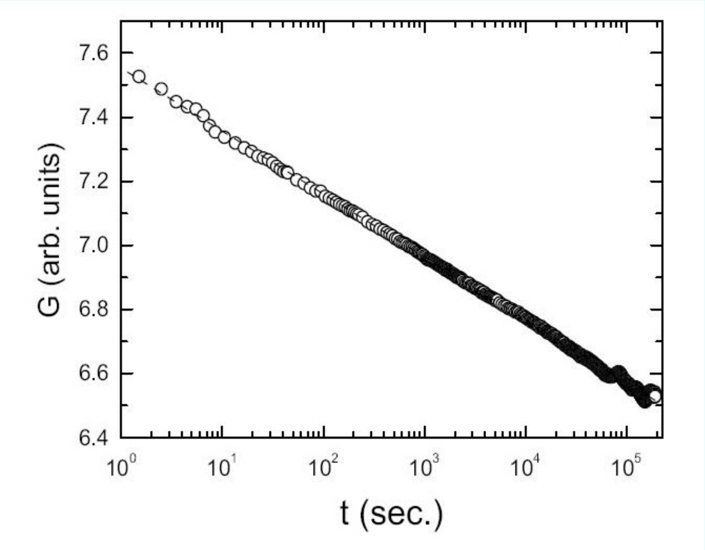
• Disordered samples,

excited out-of-equilibrium.

• Measured logarithmic decay of physical observable (conductance, capacitance) from times of a few seconds to a day!

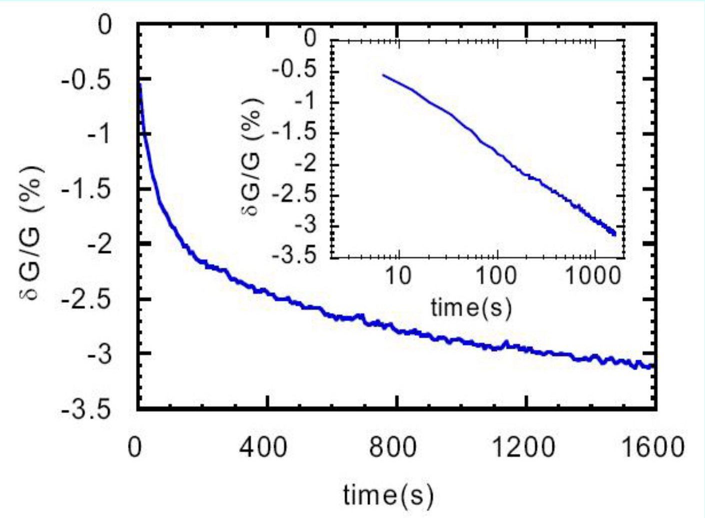
• Similar results for other systems, such as granular Al.

What are the ingredients leading to slow relaxations?



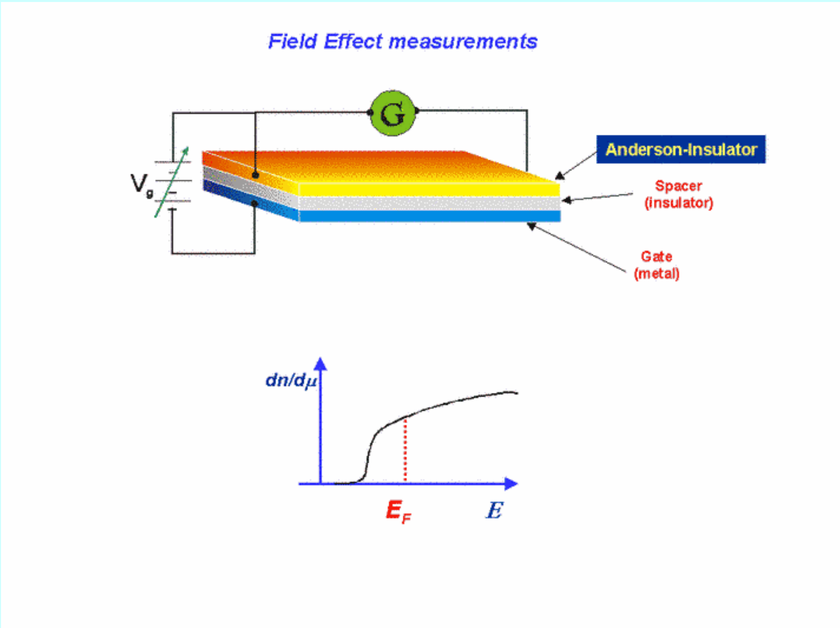
Z. Ovadyahu et al. (2003)

InO samples



T. Grenet (2004)
Granular Al samples

Electron Glass System



Courtesy of Z. Ovadyahu

- **Disordered InO samples**
- **High carrier densities~10[1920]/cc, Coulomb interactions may be important.**

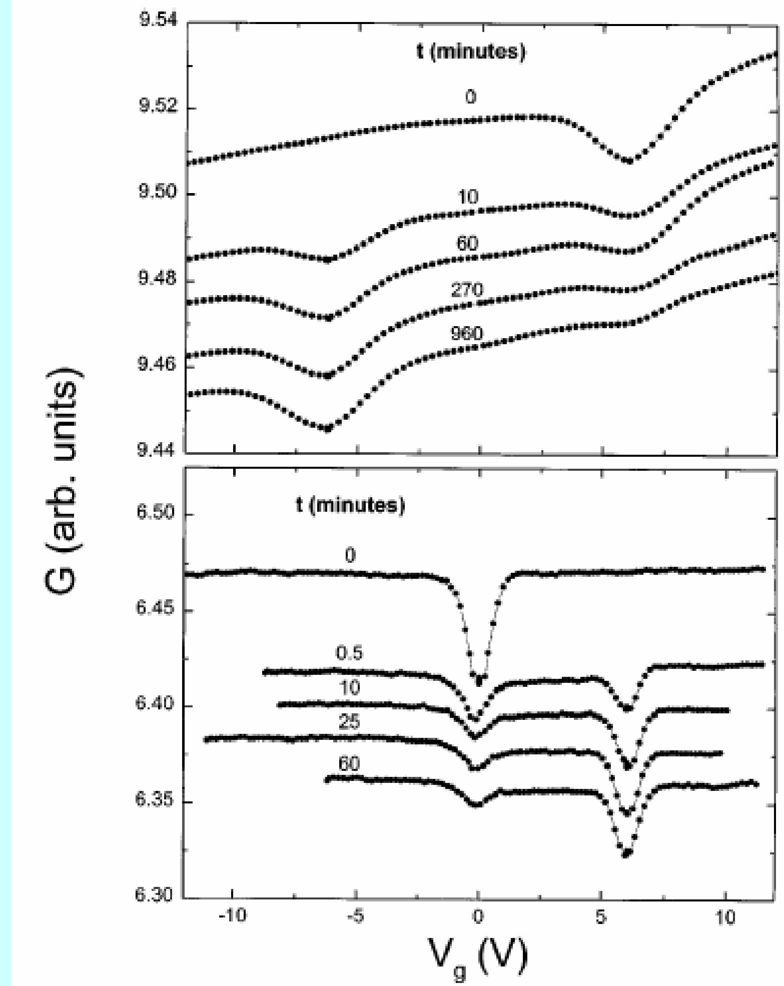


FIG. 1. Conductance versus gate voltage for a TDE of two samples with n of 4×10^{20} and 1×10^{20} cm^{-3} (upper and lower graphs, respectively). In each graph, the first trace was taken $\approx$12 hours after the initial cooldown with V_g^0 imposed (6 and 0 V for the upper and lower graphs, respectively). Then, V_g^n was applied and maintained between subsequent sweeps (-6 and 6 V, respectively). The other traces (shifted for clarity) are labeled by the time elapsed since V_g^n was first applied. Typical scan rate was 0.1–0.2 V/s.

Vaknin, Ovadyahu and Pollak, 1998

The model

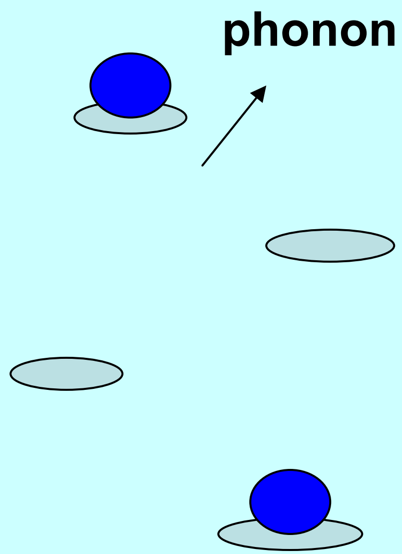
• Strong localization due to disorder

→ randomly positioned sites, on-site disorder.

• Coulomb interactions are important (e.g.: Efros-Shklovskii)

• Phonons induce transitions between configurations.

• Interference (quantum) effects neglected.

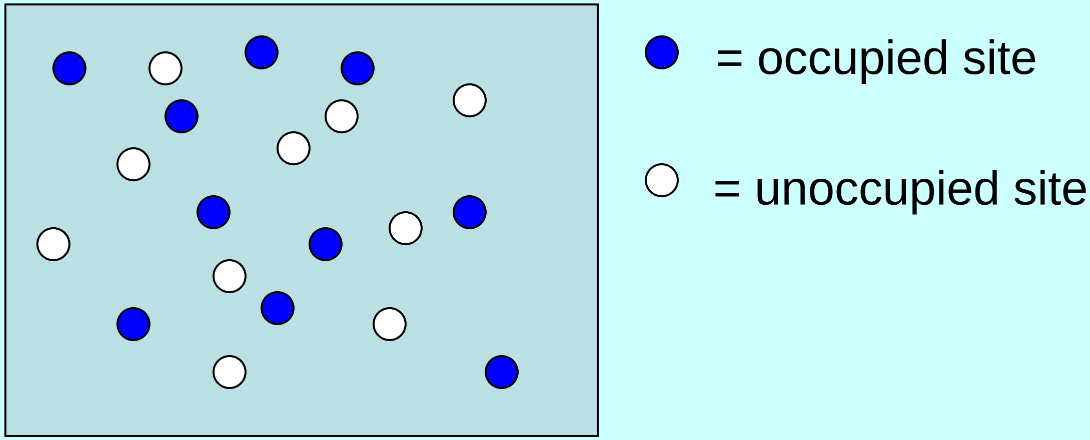


e.g:

Ovadyahu and Pollak (PRB, 2003)

M. Muller and S. Pankov (PRB, 2007)

Efros-Shklovskii argument , T=0



Cost of moving an electron: $\Delta E = E_i - E_j - \frac{e^2}{r_{ij}}$

For ground state: $\Delta E \geq 0$

Assume finite density of states at E_f → Contradiction.

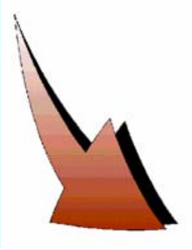
→ Upper bound is $g(E) \leq \alpha \, | E |^{d-1}$

Coulomb gap- estimation of scales

- For a 2D systems where the structural disorder is more important:

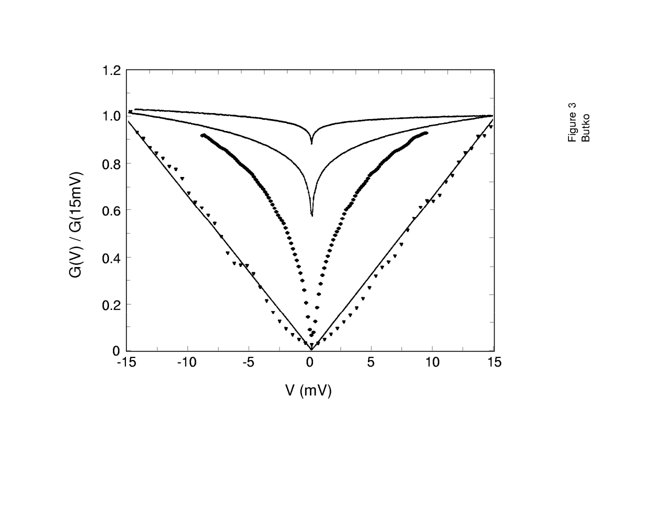
$$T_{washout} \sim \frac{e^2}{r_{nn}}$$

In a typical experiment W~ 10 nm, n ~10^{20}cm^{-3}, T~ 4K

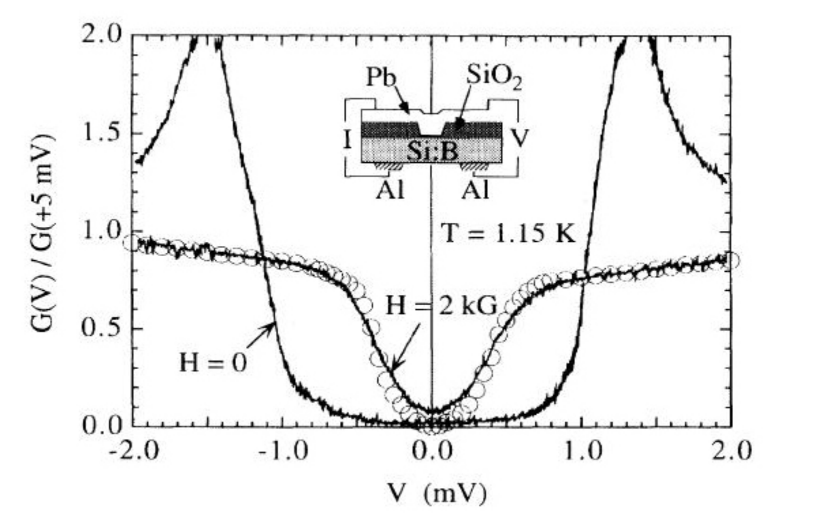


- The Coulomb gap should be observable.
- The system is close to 2D.

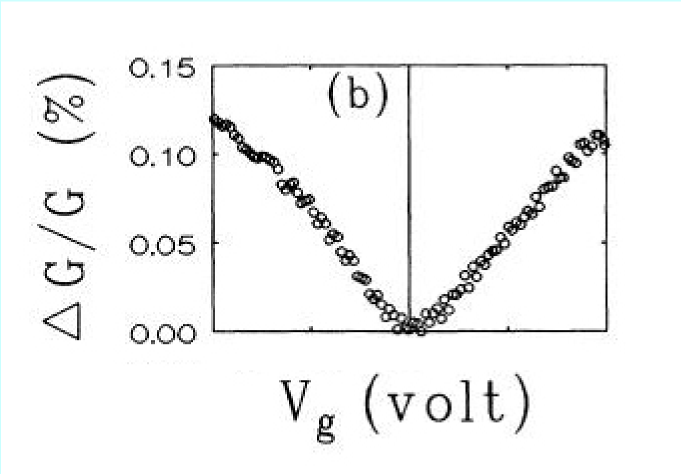
Experimental manifestation



2D: Butko et al. (PRL 84, 2000)



3D: Massey et al. (PRL, 1995)



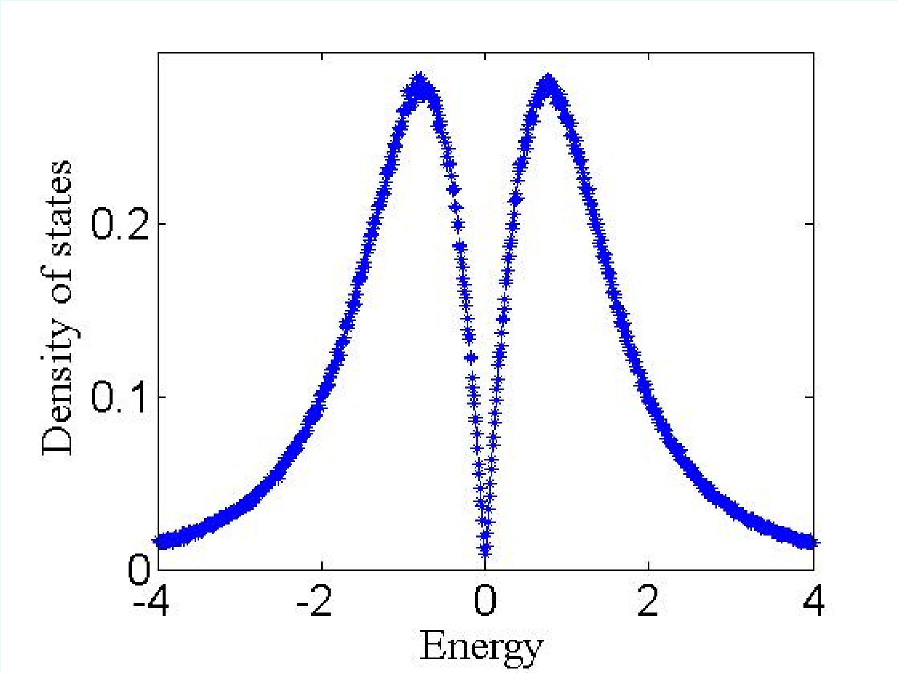
Ben Chorin et al. (PRB, 1992)

Mean-field approximation - Equilibrium

- Detailed balance leads to Fermi-Dirac statistics (f_j).
- Self-consistent set of equations for the energies:

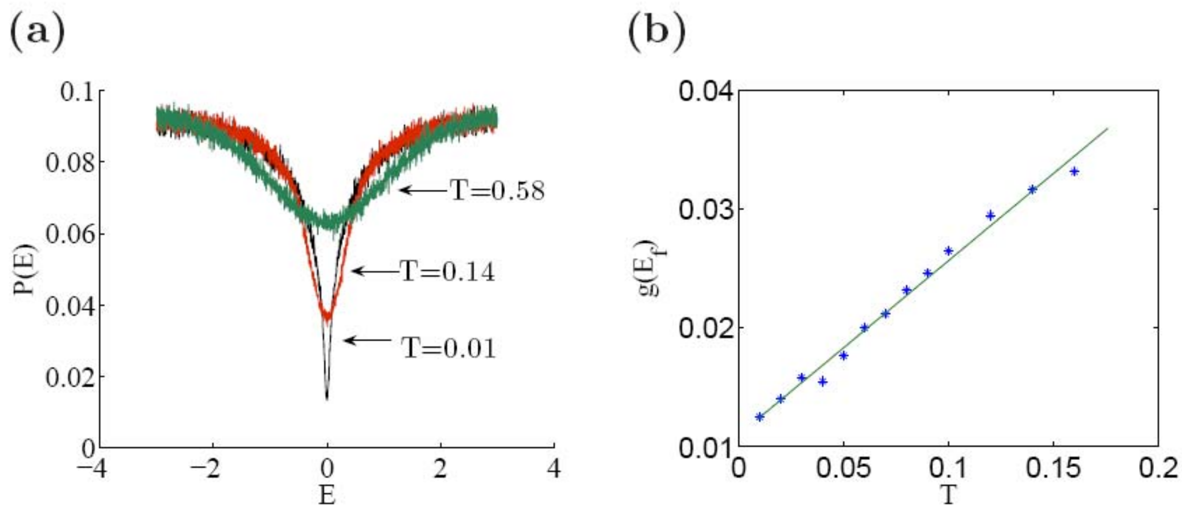
$$E_i = \varepsilon_i - \sum_{j \neq i} \frac{e^2 f_j}{r_{ij}}$$

Many solutions (valleys). Kogan, *PRB 57, 9736, 1998* – exp increase with N, in a numerical study.



Produces all features of Coulomb gap, incl temperature dependence (figure shown is for half filling).

Temp dependence of Coulomb gap, mean field



(a) (b)

FIG. 1: Fig.1(a) shows the DOS at different temperatures. Fig.1(b) shows the dependence of the finite DOS at the Fermi energy on temperature, with the zero temperature finite-size effect DOS subtracted. Each graph is an average over 2000 iterations. For each, $N = 1000$, and the ratio $\frac{e^2}{r_{nn}} = 1$, where r_{nn} is the nearest-neighbor distance. T, the temperature, was varied between 0.01 and 0.58, but only the linear regime is shown for Fig. 1(b). The on-site disorder was taken uniform in the interval $[-\frac{W}{2}, \frac{W}{2}]$, with $W = 10$.

Mean-field approximation - Dynamics

$$n_i \rightarrow \langle n_i \rangle, \frac{dn_i}{dt} = \sum_j (-\gamma_{i,j} + \gamma_{j,i})$$

$$\gamma_{i,j} \sim n_i (1 - n_j)[N(|\Delta E|) + \theta(\Delta E)]$$

ΔE contains all the interaction information

We saw: Mean-field works well for statics!

• The dynamics will make the system 'dig' its Coulomb gap, eventually.

• Many locally stable points ('glassy')

Solution near locally stable point

Close enough to the equilibrium (locally) stable point,

one can linearize the equations, leading to the equation:

$$\frac{d\delta\vec{n}}{dt} = A \cdot \delta\vec{n}$$

$$A_{i,j} = \frac{\gamma_{i,j}^{0}}{n_j^0(1-n_j^0)} - \frac{e^2}{T}\sum_{l \neq i,j} \gamma_{i,k}^{0} \left(\frac{1}{r_{i,j}} - \frac{1}{r_{i,k}}\right)$$

$$(i \neq j)$$

Negligible (checked numerically),

when T→0.

The sum of every column vanishes (particle number conservation)

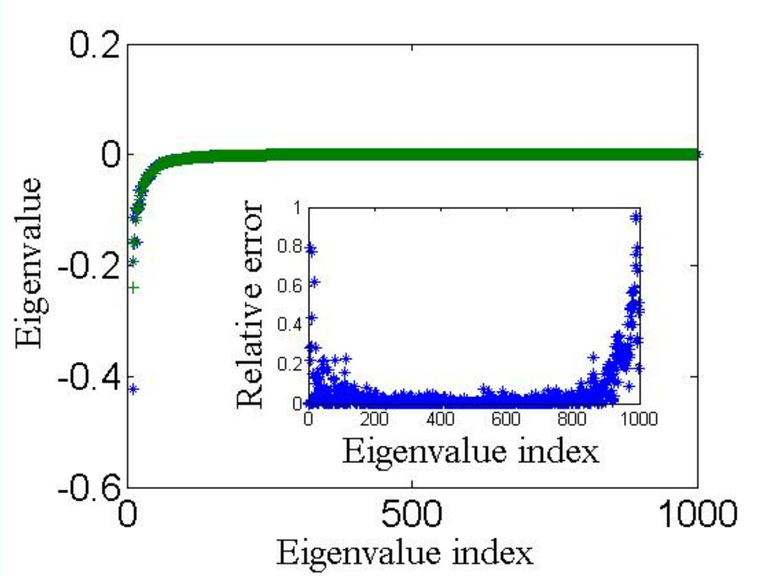
Off diagonals are positive

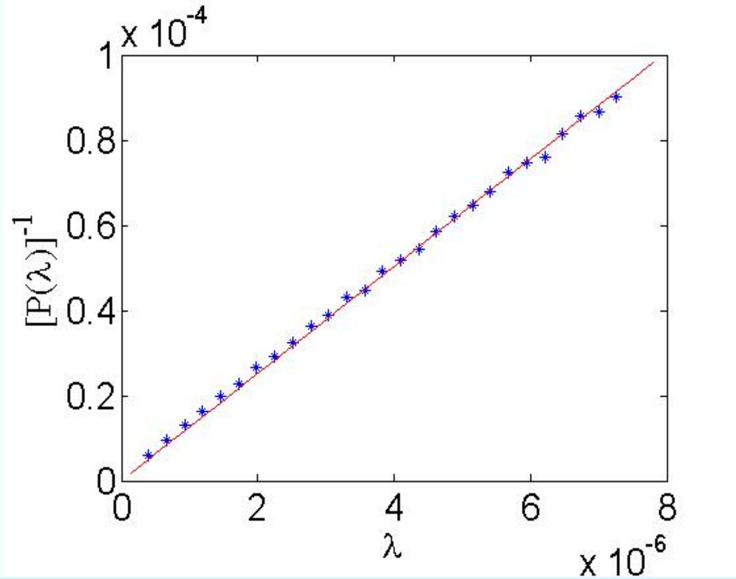
→ all Re(eigenvalues) are negative (Stability!)

Solution for dynamics

The eigenvalue distribution will determine the relaxation.

Solving numerically shows a distribution proportional to $1/\lambda$:

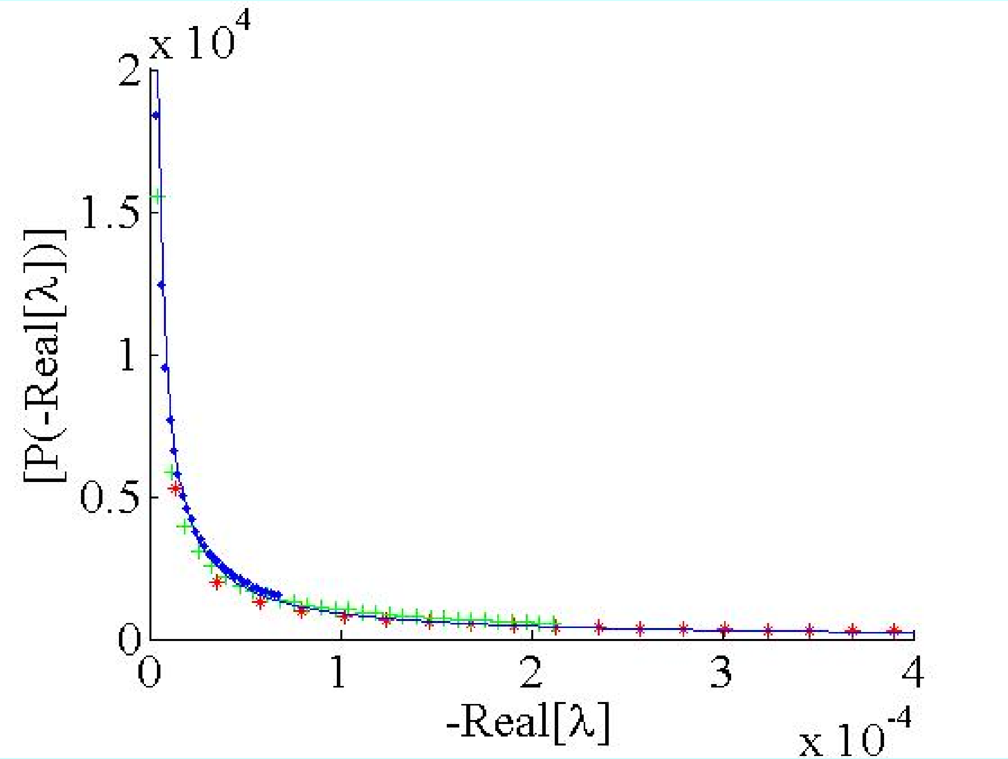




Implication on dynamics:

$$\delta n(t) = \int P(\lambda) e^{-\lambda t} d\lambda \sim -A \log(t)$$

Effect of Interactions



- Slow relaxations occur also without interactions.
- Interactions push the distribution to (s)lower values.

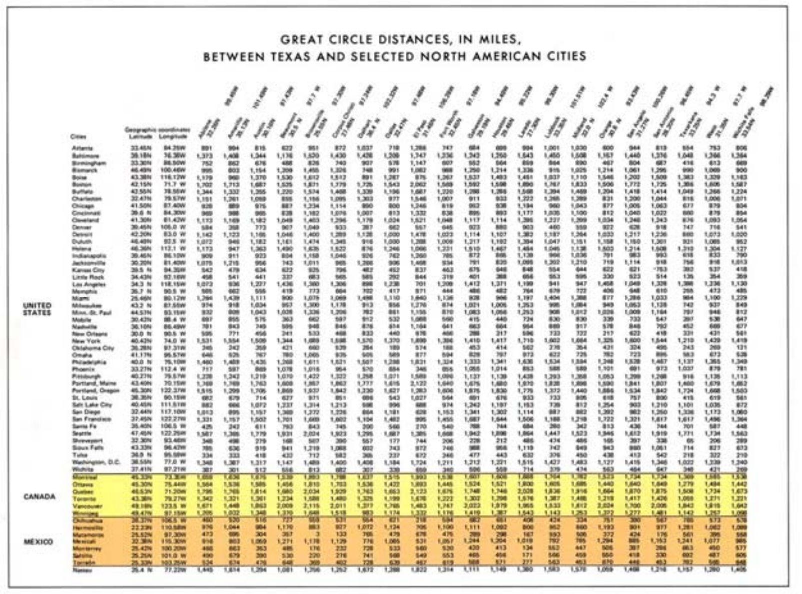
Distance matrices- definition

(cf. M. Mezard, G. Parisi, and A. Zee, Nucl. Phys. B **3**, 689, 1999).

- Off diagonals:

$$A_{i,j} = e^{-\frac{r_{ij}}{\xi}}$$

(Euclidean distances in D dimensions)



- Sum of every column vanishes

What is the eigenvalue distribution?

Distance (Euclidean) matrices- eigenvalue distribution

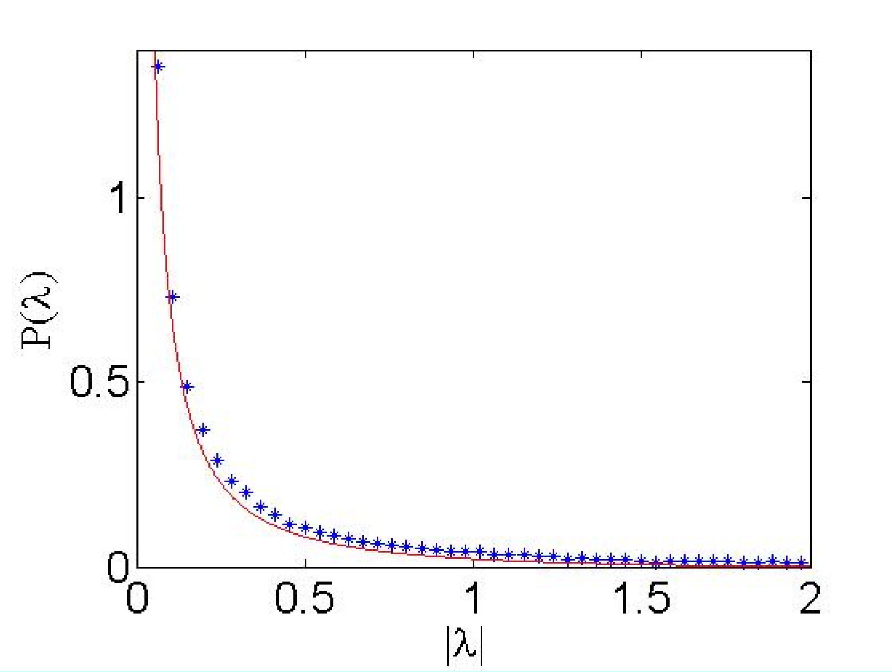
• Low density system ⟶ basically a set of independent dipoles.

• Calculation of the nearest-neighbor probability distribution will give approximately the eigenvalue distribution.

• Calculation gives, for **exponential** dependence on the distance:

$$P(\lambda) \approx \frac{N\xi}{L}\frac{1}{\lambda} \qquad \text{(1D)}$$

$$P(\lambda) \approx \frac{\pi N\xi^2}{L^2}\frac{\log[-\lambda/2]}{\lambda} \qquad \text{(2D)}$$



Heuristic calc—almost the same!

Is this more general?

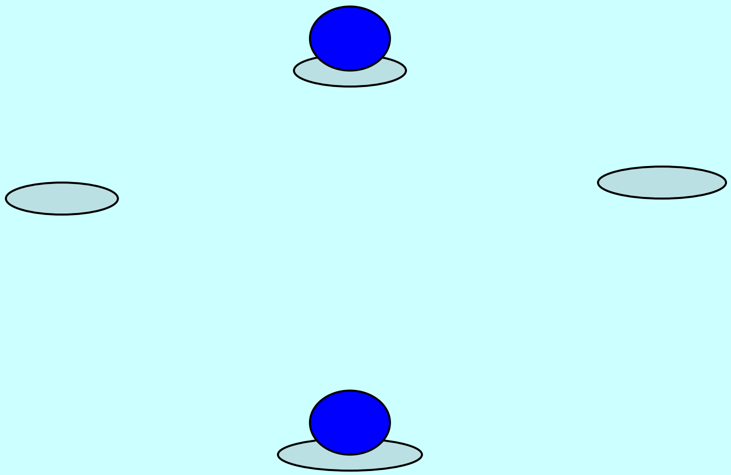
Benford's law: for many physical properties the *log* is uniformly distributed.

Examples: river lengths, phone bills, ***1/f*** noise etc.

Implies: distribution of first digit $\sim \log\left(\frac{1}{d}+1\right)$

More concretely:

What happens when we take multi-particle relaxations into account?



Connection to conductance relaxation?

If we assume:

$$\delta\sigma \sim g(\mu)$$

The energies of sites are changed, we get a finite DOS at the chemical potential.

$$\delta E = \sum \frac{e^2 \delta n_i}{r_{ij}}$$

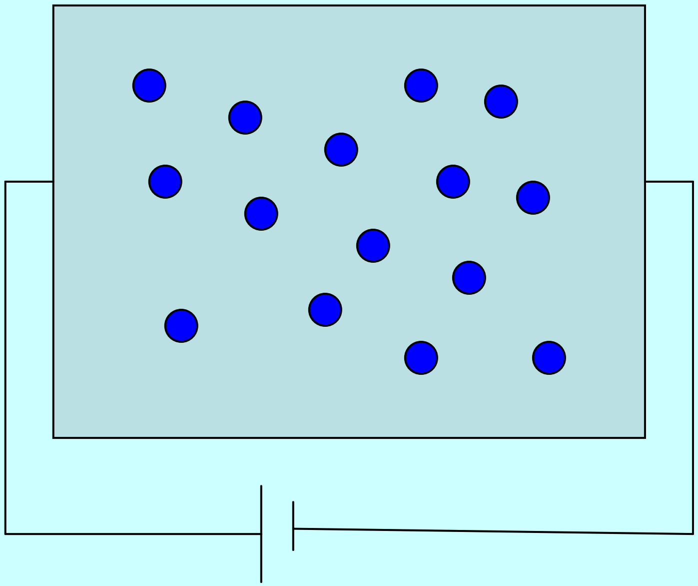
$g(\mu) \sim \delta n \rightarrow$ logarithmic decay of the conductance?

Other mechanisms might be involved.

Mean-field approximation – steady state

Miller-Abrahams resistance network: essentially mean-field

Find n_i *and* E_i such that the systems is in steady state.



$$I = \sum_j \frac{U_i - U_j}{R_{ij}}, \quad R_{ij} = \frac{T}{e^2 \gamma^0_{ij}}$$

equilibrium rates

$$U_i = -\vec{E} \cdot \vec{r} + e^2 \sum_{k \neq i} \frac{\delta n_k}{r_{ik}} + \delta\mu_i$$

Leads to variable-range-hopping: $\sigma \sim e^{CT^{-\frac{1}{d+1}}}$

A. Miller and E. Abrahams, (Phys. Rev. 1960)

Variable Range Hopping – back of the envelope derivation

Einstein formula:

$$\sigma = e^2 \frac{\partial n}{\partial \mu} D$$

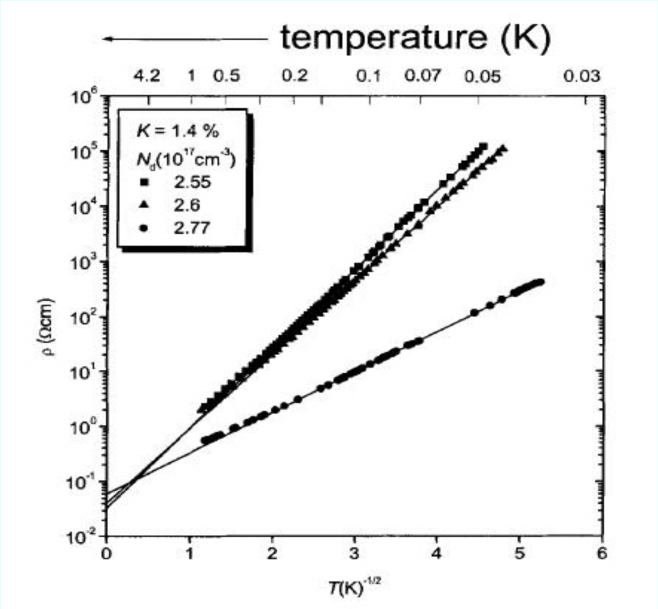
The typical diffusion coefficient:

$$r(T) \sim T^{-\frac{1}{d+1}}, \;\; E(T) \sim T^{\frac{d}{d+1}}$$

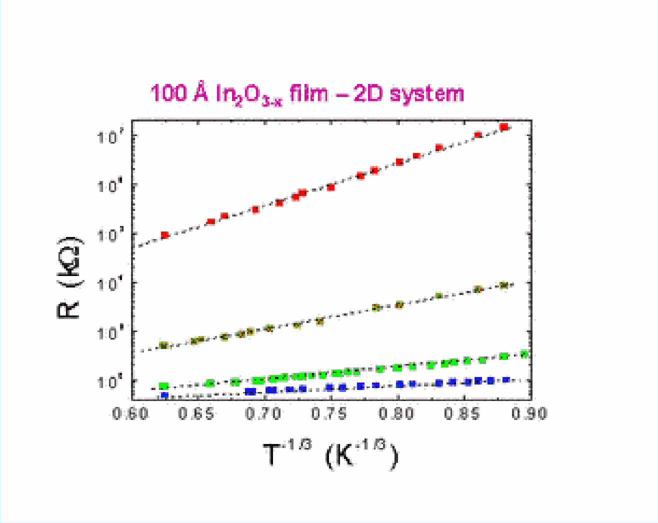
$$\sigma \sim e^{CT^{-\frac{1}{d+1}}}$$

Repeating the optimization with a Coulomb gap:

$$\sigma \sim e^{CT^{-\frac{1}{2}}}$$

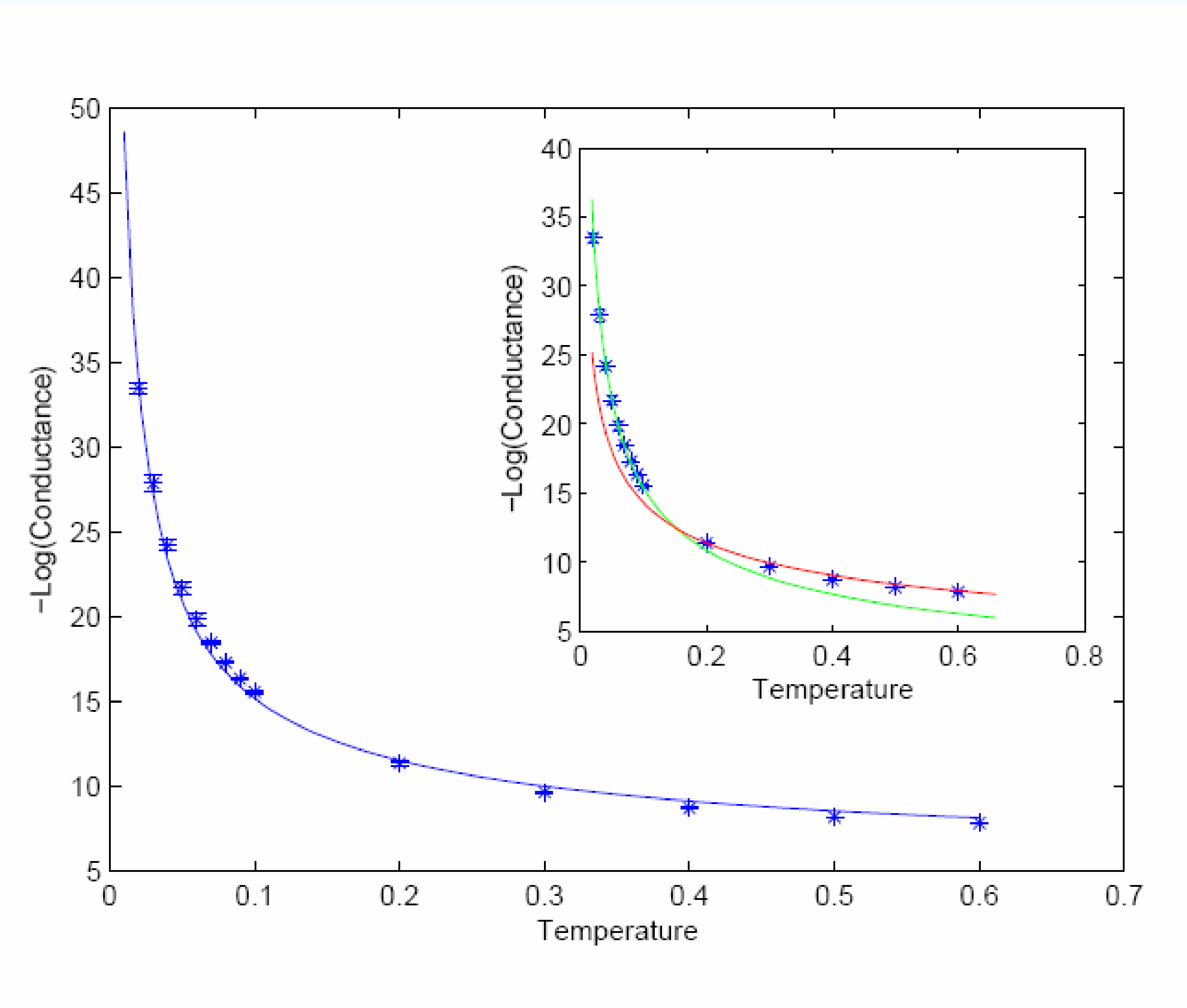


Rentzsch et al. (2001)



Ovadyahu (2003)

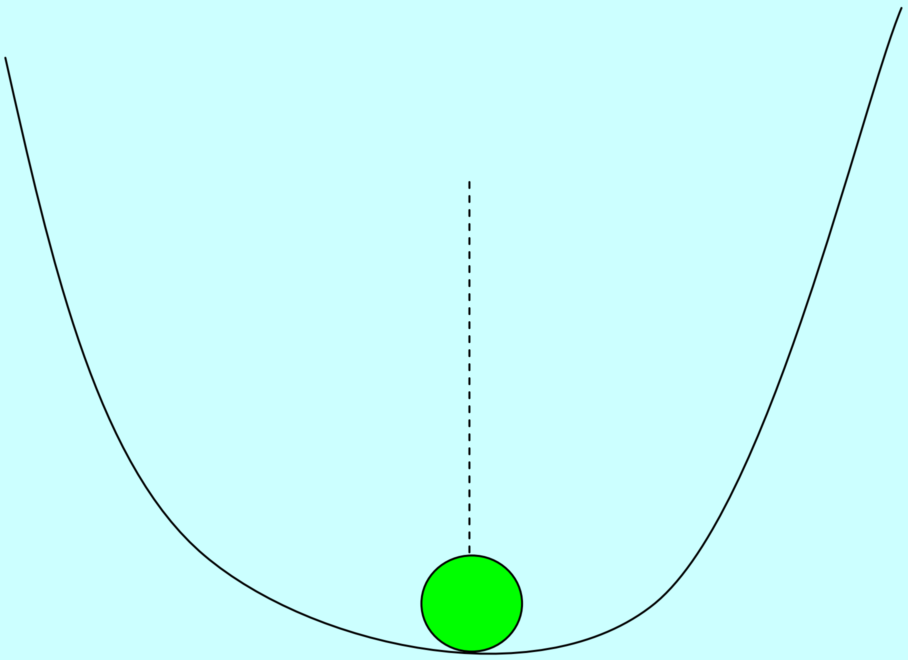
VRH (Mott) to E-S Crossover, from mean-field



Aging

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?

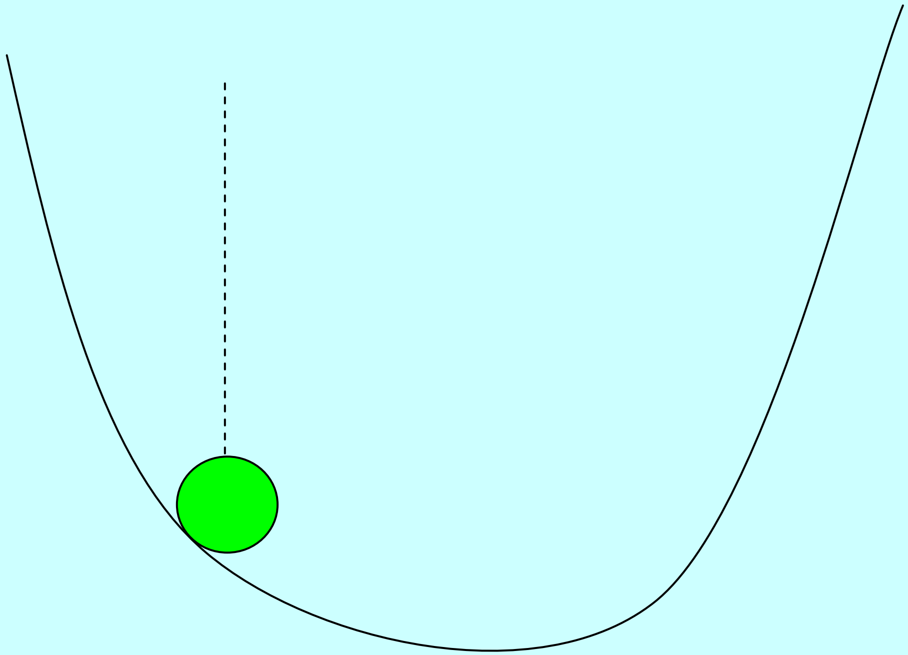


Initially, system is at some local minimum

Aging

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?



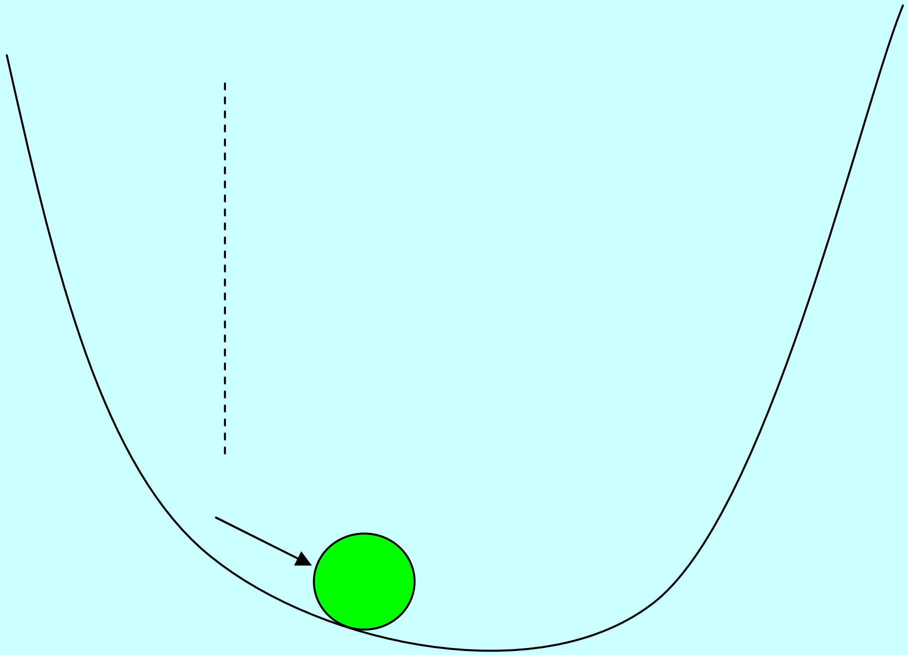
At time t=0 the potential changes,

and the system begins to roll towards the new minimum

Aging

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?

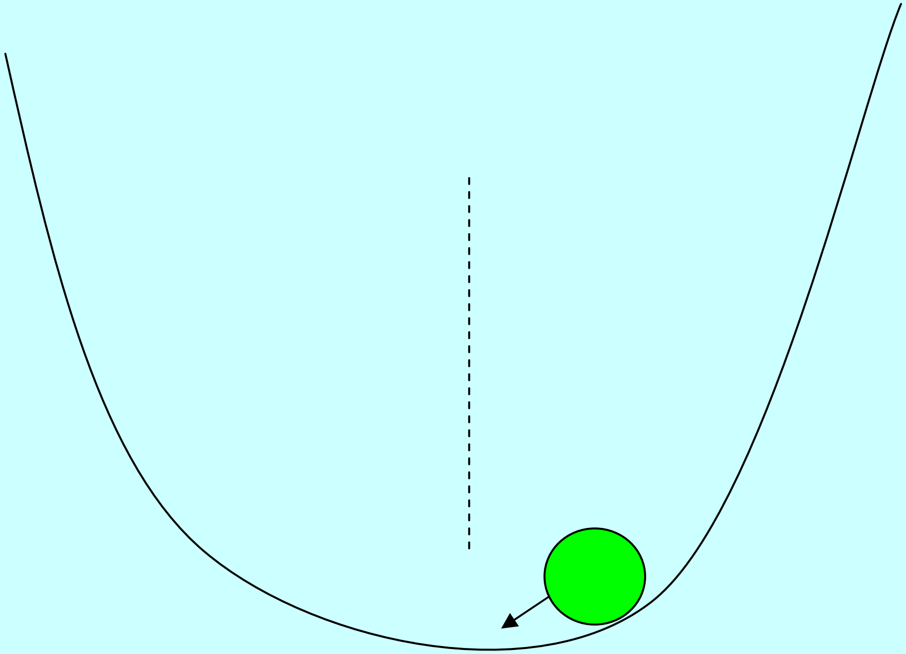


At time t_w the system reached some new configuration

Aging

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?



Now the potential is changed back to the initial form-

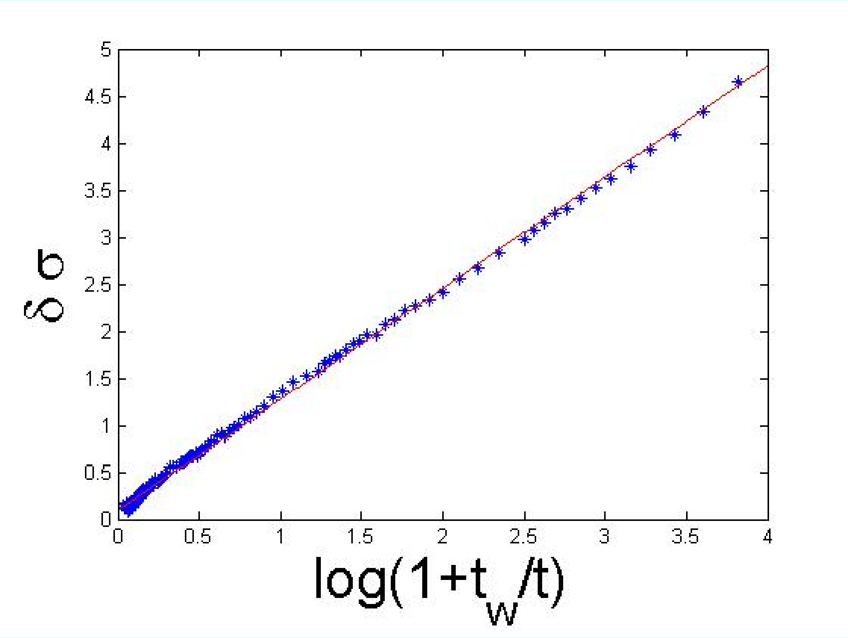
the particle is not in the minima!

The longer t_w, the further it got away from it.

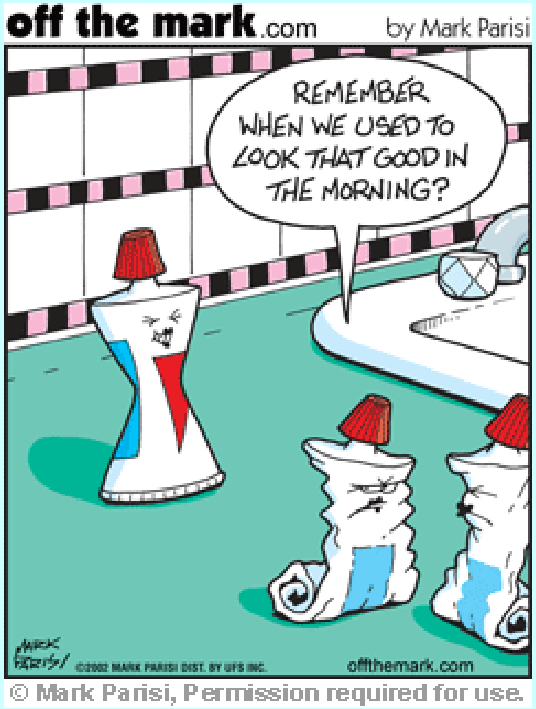
It will begin to roll down the hill.

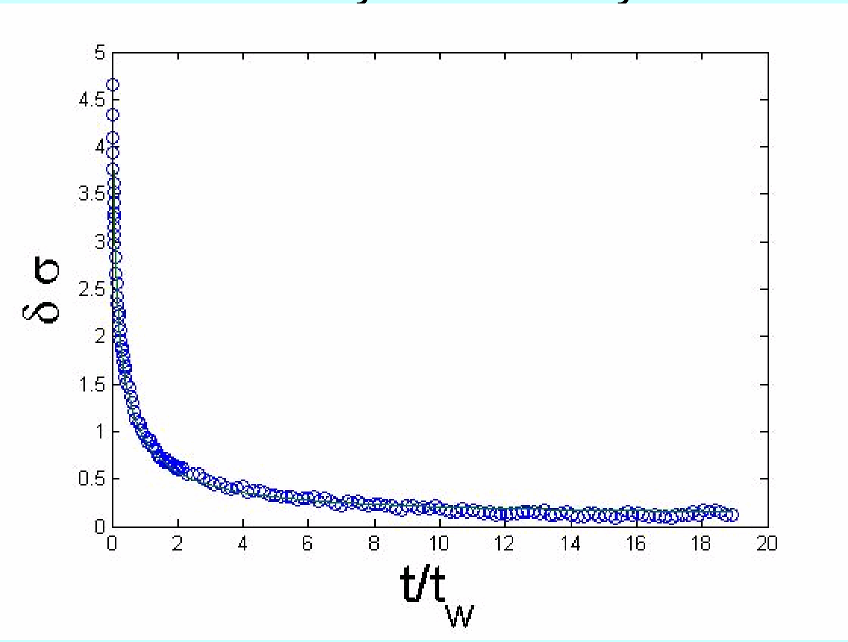
Results

- Simple aging (function of t/t_w)
- *log(t)* turns to a power-law at large times
- Not stretched exponential!
- Fits experimental data!



Data courtesy of Z. Ovadyhau





Conclusions and future work

• Dynamics near locally stable point: slow, log t, relaxations. How universal is the $1/\lambda$ distribution? **a very relevant RMT class.**

•Slow dynamics may arise without transitions between *different* metastable states. (work in progress). How will the inter-state transitions connect with intra-state ones? **We believe: They'll push the distribution to lower λ!**

•It is interesting to see if the mean-field model can predict the 'two-dip' experiment, where the system shows memory effects.

More details: arXiv 0712.0895, Phys. Rev. B 77, 1, 2008