General Relativity as a hybrid theory: The Genesis of Einstein’s work on the problem of motion

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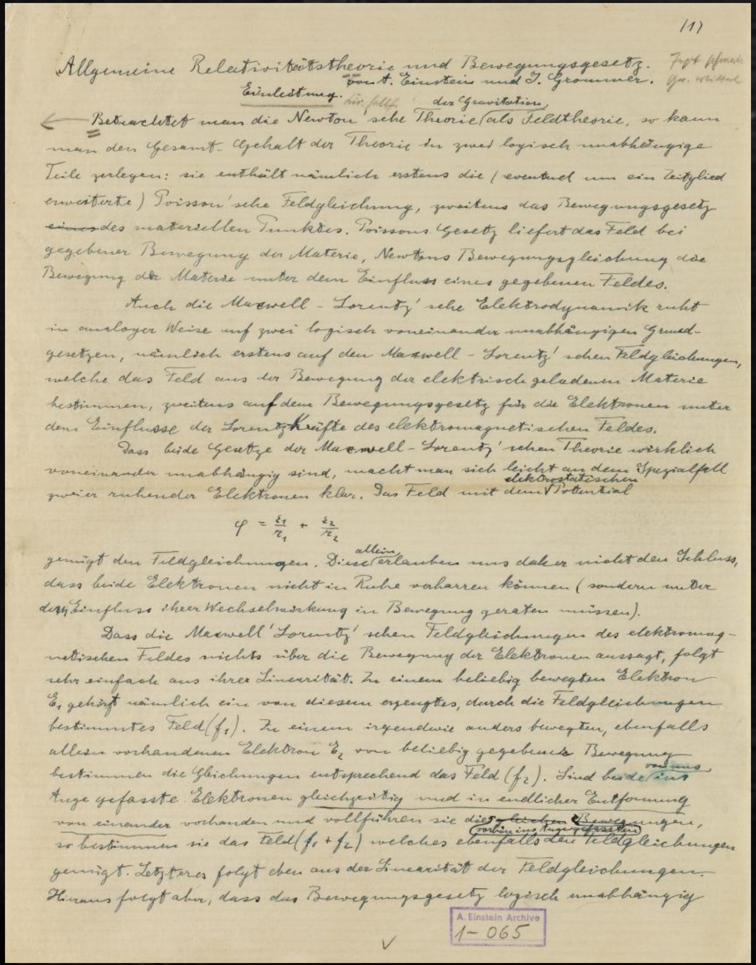
Allgemeine Relativitätstheorie und Bewegungsgesetz.
von A. Einstein und J. Grommer.
Einleitung.
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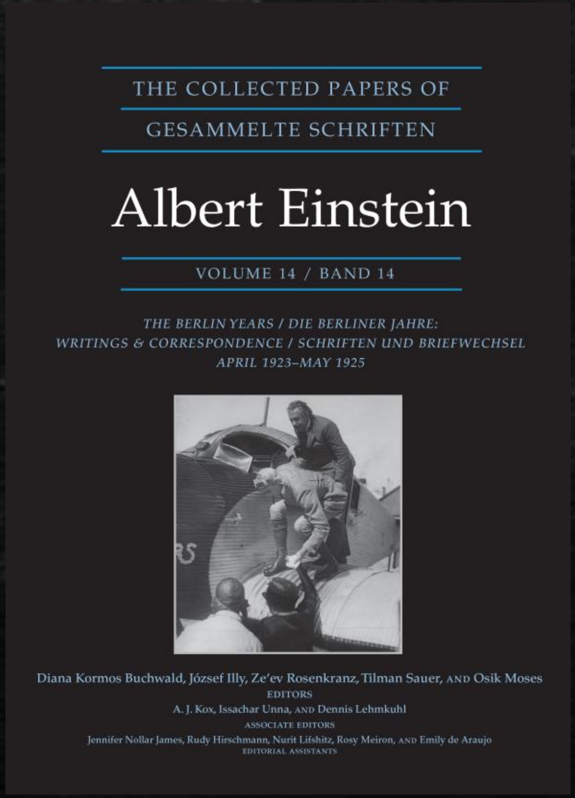
THE COLLECTED PAPERS OF
GESAMMELTE SCHRIFTEN
Albert Einstein
VOLUME 14 / BAND 14
THE BERLIN YEARS / DIE BERLINER JAHRE:
WRITINGS & CORRESPONDENCE / SCHRIFTEN UND BRIEFWECHSEL
APRIL 1923–MAY 1925
Diana Kormos Buchwald, József Illy, Ze'ev Rosenkranz, Tilman Sauer, AND Osik Moses
EDITORS
A. J. Kox, Issachar Unna, AND Dennis Lehmkuhl
ASSOCIATE EDITORS
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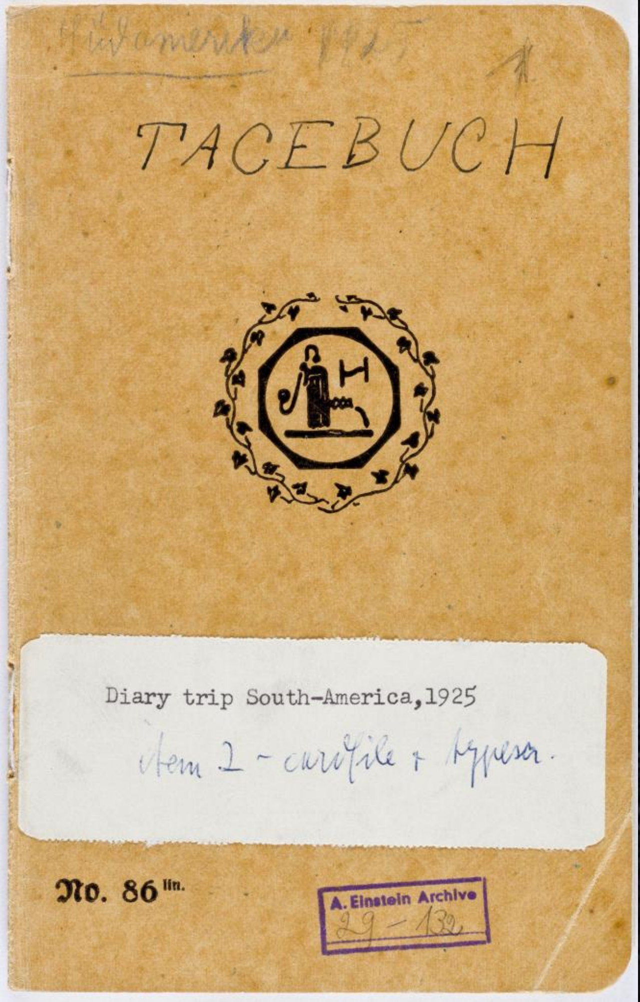
TAGEBUCH
Diary trip South-America,1925
No. 86
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Outline

1. Introduction
2. Einstein's interpretation of the Einstein field equations
   a. The left-hand side: geometry or gravitational field?
   b. The right-hand side: matter or place-holder for matter?
3. Einstein on the relationship between the Einstein field equations and the law of motion of matter subject to gravity
4. Conclusion

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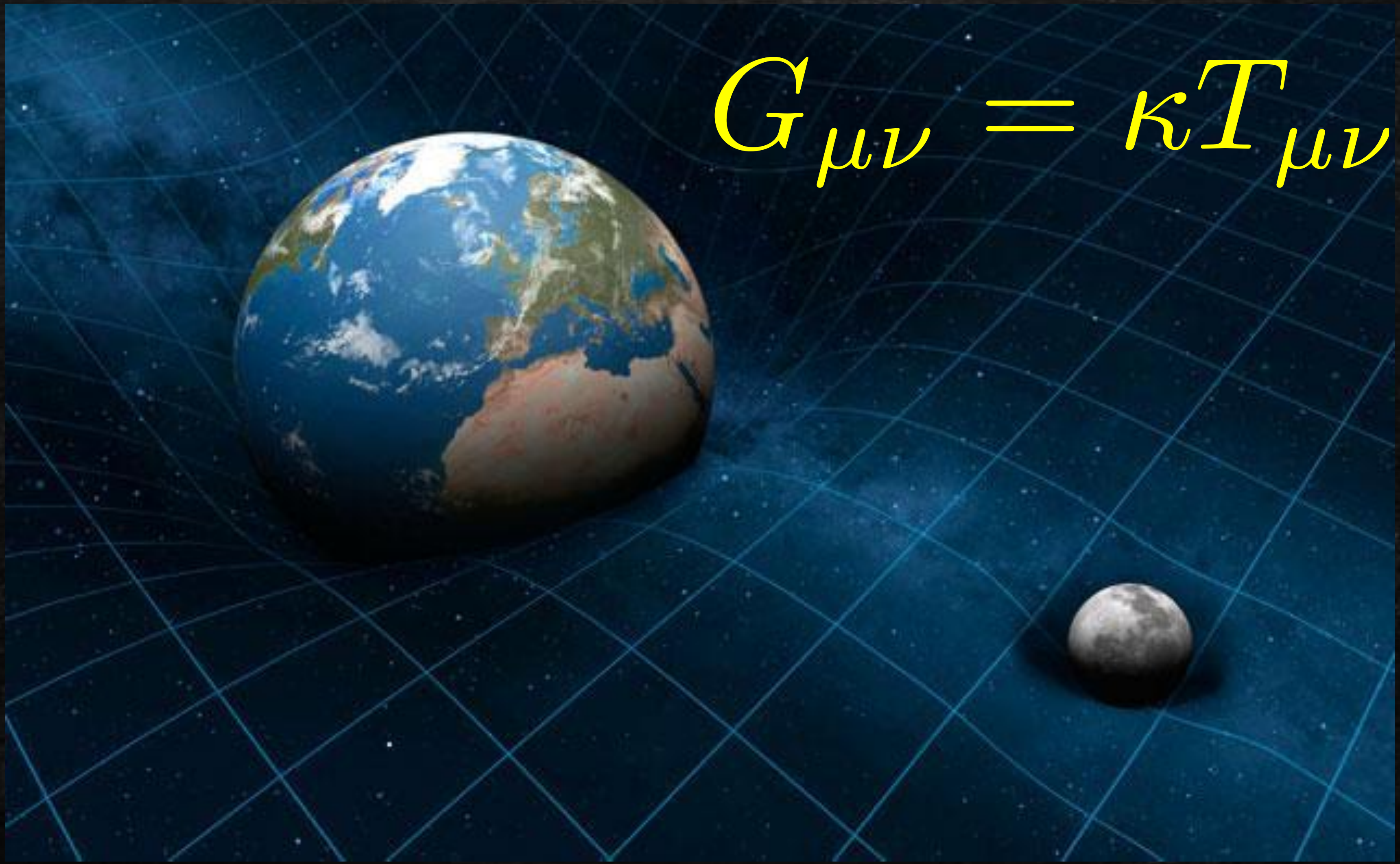
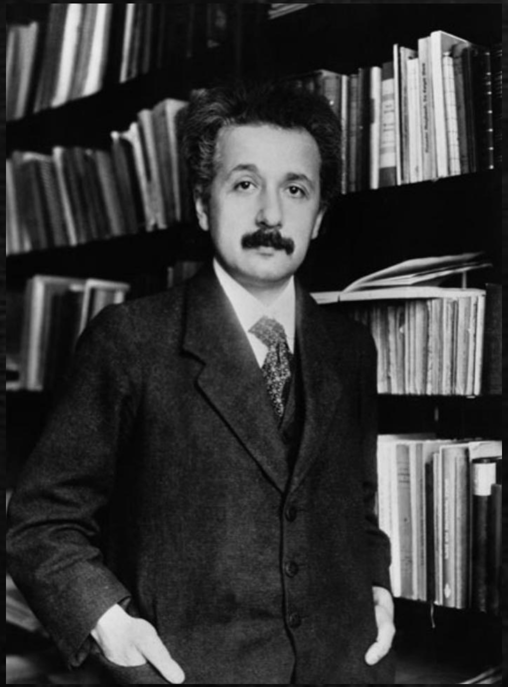
The geometric interpretation of GR



Einstein and the geometric interpretation, 1925

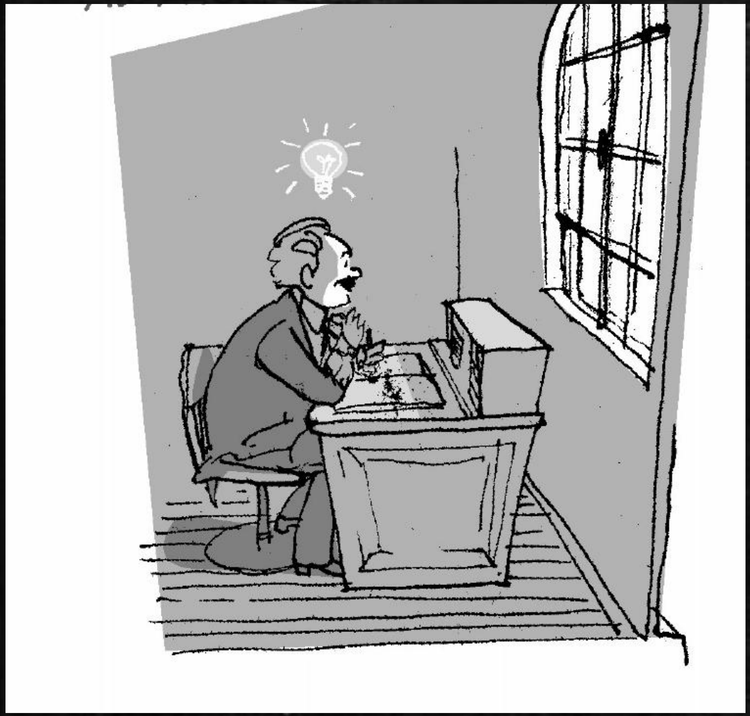


``I cannot, namely, admit that the assertion that the theory of relativity traces physics back to geometry has a clear meaning. [...] The fact that the metric tensor is denoted as ``geometrical'' is simply connected to the fact that this formal structure first appeared in the area of study denoted as ``geometry''. However, this is by no means a justification for denoting as ``geometry'' every area of study in which this formal structure plays a role, not even if for the sake of illustration one makes use of notions which one knows from geometry. Using a similar reasoning Maxwell and Hertz could have denoted the electromagnetic equations of the vaccuum as ``geometrical'' because the geometrical concept of a vector occurs in these equations.'' Einstein (1927), Review of Meyerson. (See DL [2014] for analysis and similar quotes from other decades.)

Geometrization is not Unification

``Thus, what is essential about Weyl's and Eddington's theories on the representation of the electromagnetic field is not that they have incorporated the theory of this field into geometry, but that they have shown a possible way to represent gravitation and electromagnetism from a unified point of view, whereas these fields entered the theory as logically independent structures beforehand.''
Einstein (1927)

What Einstein believed instead: Relativisation of the gravitational (force) field



``The existence of an electric field is a relative one, dependent on the state of motion of the coordinate system used, and only the electric and magnetic field together could be ascribed an objective reality. [...] Then I had the most fortunate thought of my life in the following form: the gravitational field only has a relative existence in a manner similar to the electric field generated by electro-magnetic induction. Because for an observer in free-fall from the roof of a house, there is during the fall - at least in his immediate vicinity - no gravitational field.''

Einstein (1920), unpublished; Vol. 7 Doc. 31 CPAE.

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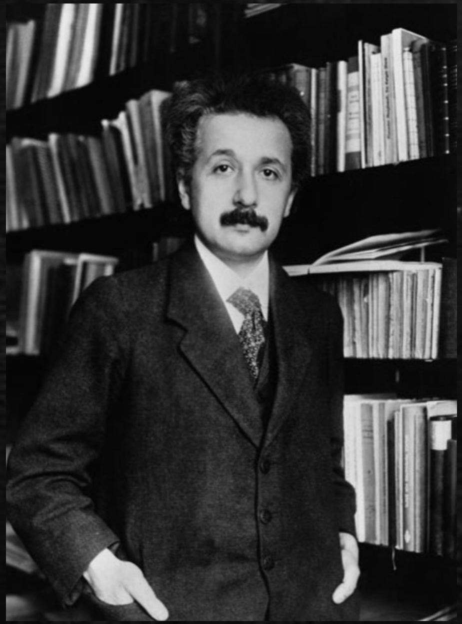
Einstein and the geometric interpretation



``[GR] is sufficient --- as far as we know --- for the representation of the observed facts of celestial mechanics. But it is similar to a building, one wing of which is made of fine marble (left part of the equation), but the other wing of which is built of low-grade wood (right side of equation). The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter." Einstein (1936)

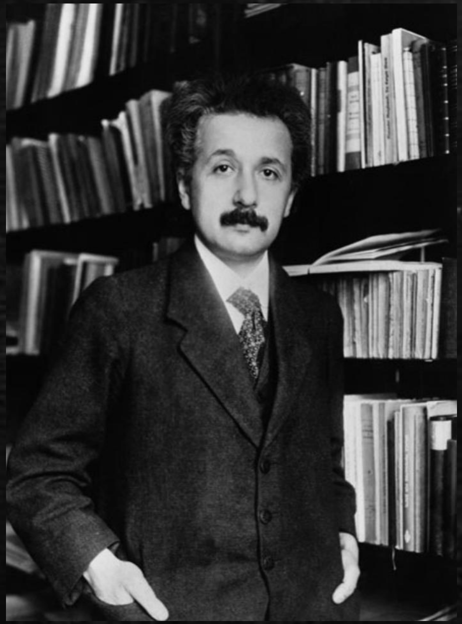
``In view of this geometrization, Einstein considered the role of the stress-energy tensor (the source-term of his field equations) a weak spot of the theory because it is a field devoid of any geometrical significance." Goenner (2004)

The energy tensor as phenomenological

"I do not care for Hilbert's presentation. It is unnecessarily specialised regarding ``matter'', unnecessarily complicated, and not honest (= Gaussian) in its structure (creating the impression of being superhuman by obfuscating one's methods)."   Einstein to Ehrenfest, 24 May 1916.

``*Phenomenological description of the energy tensor of matter. Hydrodynamical equations*"
We know today that matter is built up of electrically charged elementary particles, but we do not know the field laws which ground the constitution of these particles. Thus, when investigating mechanical systems, we are forced to make use of an inexact description of matter, which corresponds to that of classical mechanics."   Einstein (1921), Princeton Lectures

Einstein to Besso



``But it is questionable whether the equation $G_{\mu\nu} = \kappa T_{\mu\nu}$ has any reality left within it in the face of quanta. I vigorously doubt it. In contrast, the left-hand side of the equation surely contains a deeper truth."
--- Einstein to Besso, 11 August 1926

``It has been attempted to remedy this lack of knowledge by considering the charged particles as proper singularities. But in my opinion this means giving up a real understanding of the structure of matter. It seems to me much better to admit our present inability rather than to be satisfied by a pseudo-solution." --- Einstein (1921), Princeton Lectures

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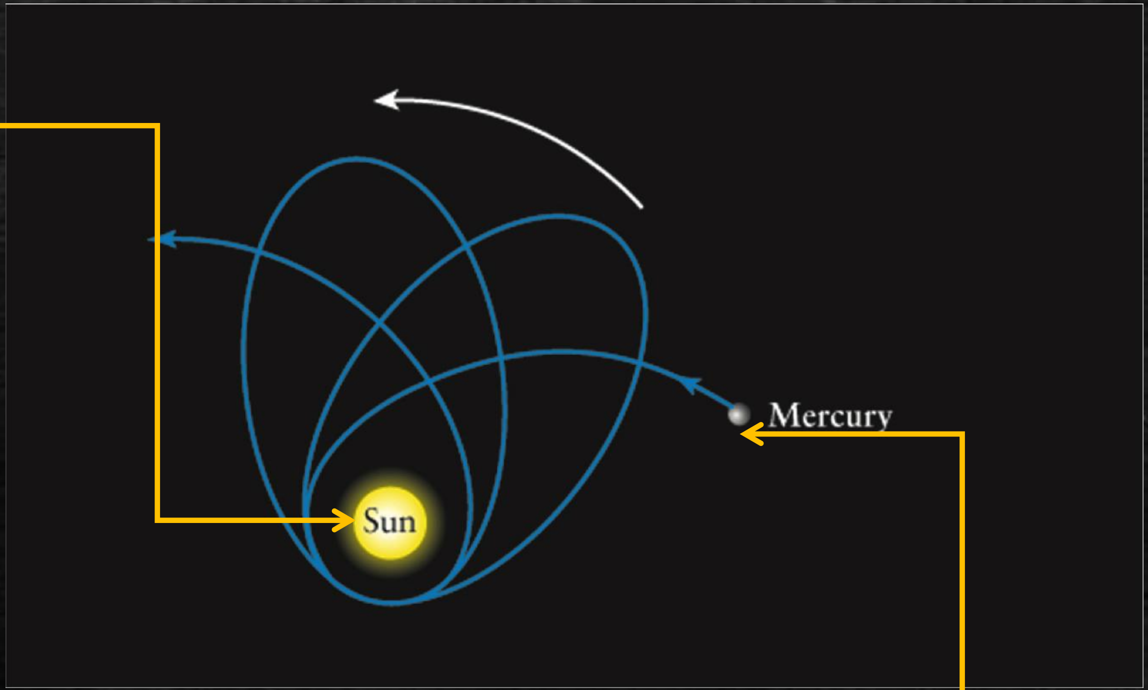
2 equations are at the heart of General Relativity (GR)

Gravitational field equations:

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

$$R_{\mu\nu} = 0$$

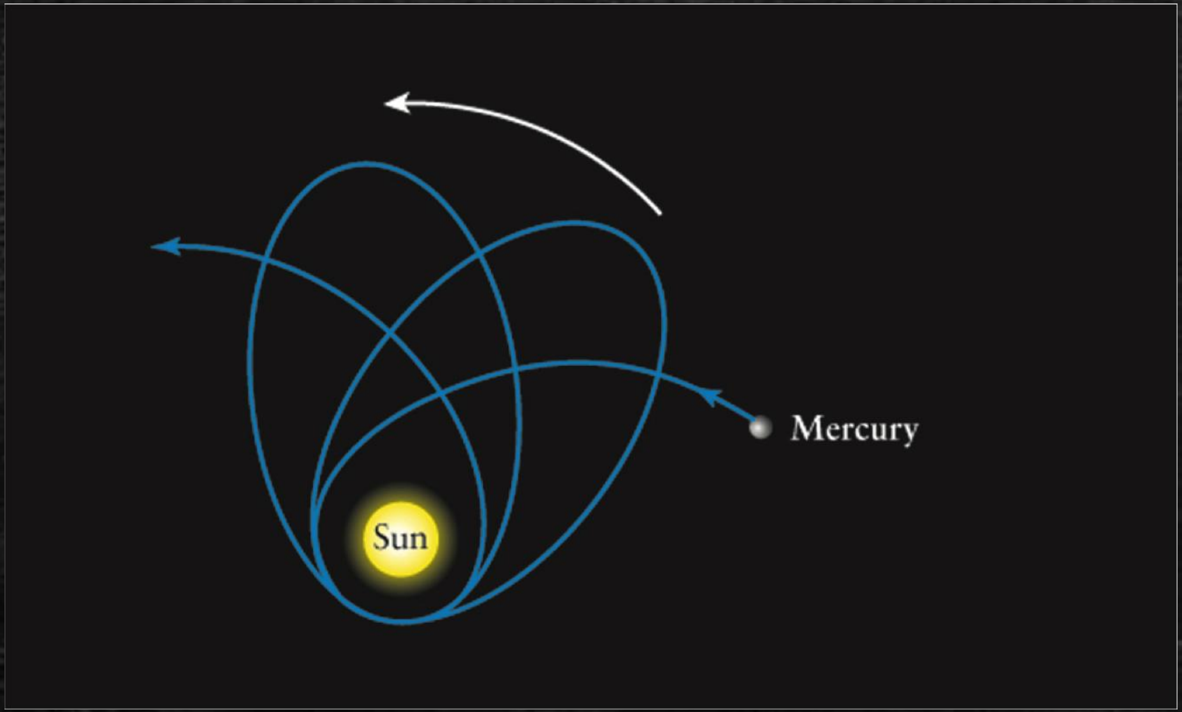


Equations of motion of (idealised) matter

$$\frac{d^2 x_\tau}{ds^2} + \Gamma^{\tau}{}_{\mu\nu} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds} = 0$$

- The problem of motion: How are the gravitational field equations (the Einstein field equations) and the equations of motion of matter subject to gravitational fields (the geodesic equation) related?

The problem of motion 1916 and 1927



Can the equations of motion of material bodies subject to gravitational fields be derived from the gravitational field equations?

Einstein 1915: No.
Einstein 1927: Yes.

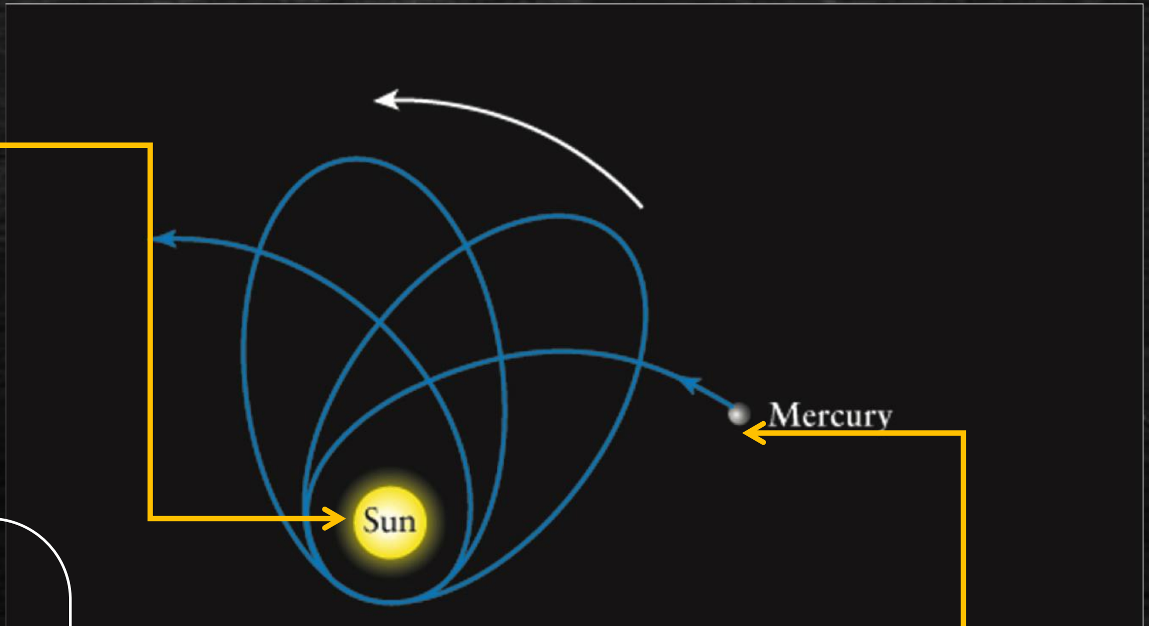
The T approach: deriving geodesic motion from energy-momentum conservation

The Einstein field equations:

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

Energy momentum conservation:

$$\nabla^\mu T_{\mu\nu} = 0$$



- SDEC: mass-energy of matter is always non-negative, and every observer will jude mass-energy to propagate along time-like curves only.
- BUT there is a price to pay: we have to assume the strengthened dominant energy condition

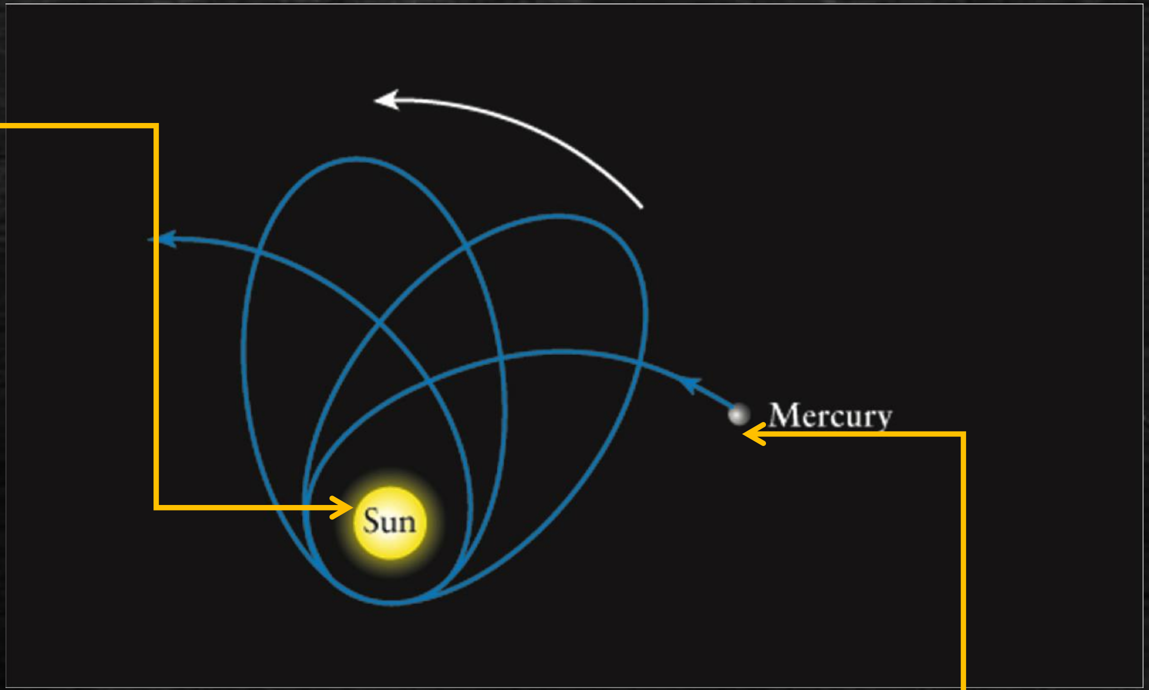
The geodesic equation:

$$\frac{d^2 x_\tau}{ds^2} + \Gamma^\tau{}_{\mu\nu}\frac{dx_\mu}{ds}\frac{dx_\nu}{ds} = 0$$

The V approach: geodesic motion via the vacuum field equations

The Einstein field equations:

$$R_{\mu\nu} = 0$$



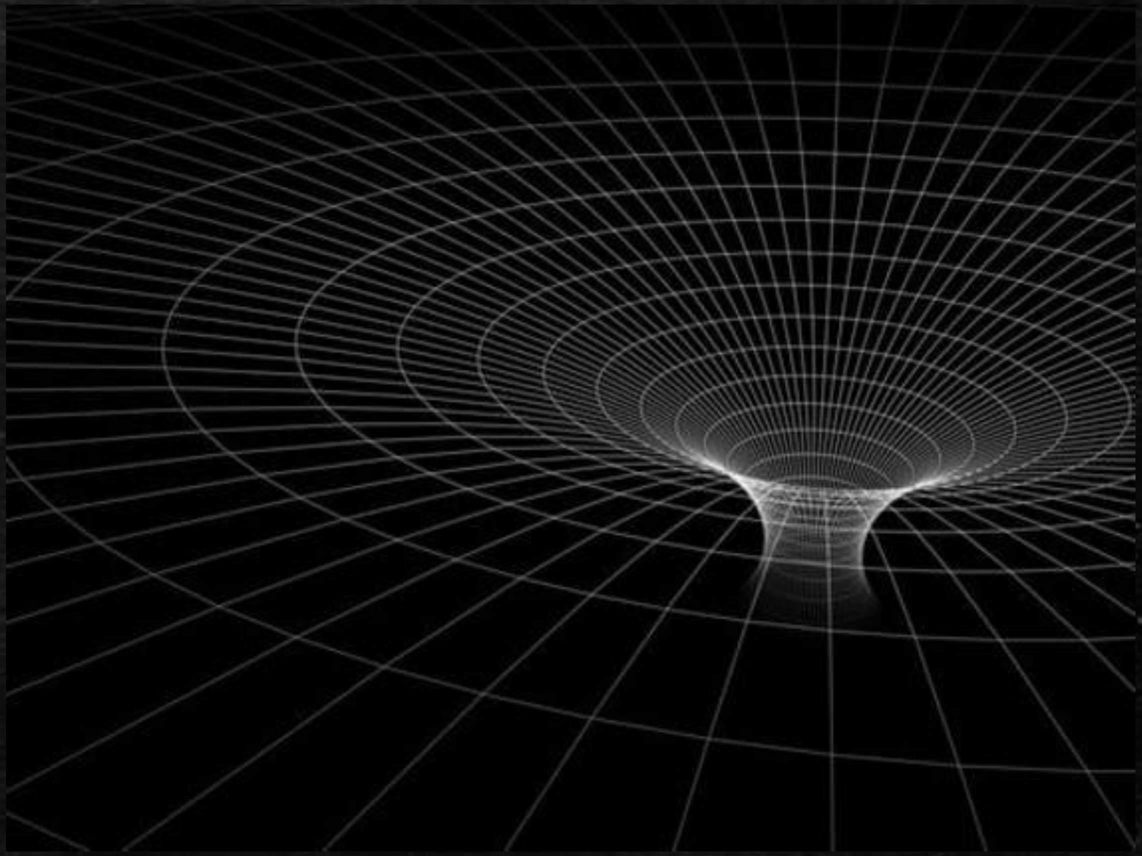
The geodesic equation:

$$\frac{d^2 x_\tau}{ds^2} + \Gamma^\tau{}_{\mu\nu} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds} = 0$$

- BUT there is a price to pay: it seems matter is represented by singularities.

What is a singularity?

- For Einstein, a singularity was a point in spacetime where the components of the gravitational field / the metric tensor become infinite.
- Now we know that there is more than one kind of singularity.



NOTE: field singularities in GR seem more problematic than in other classical field theories.

Criticism of the vacuum approach: ``Singularities moving through spacetime??''

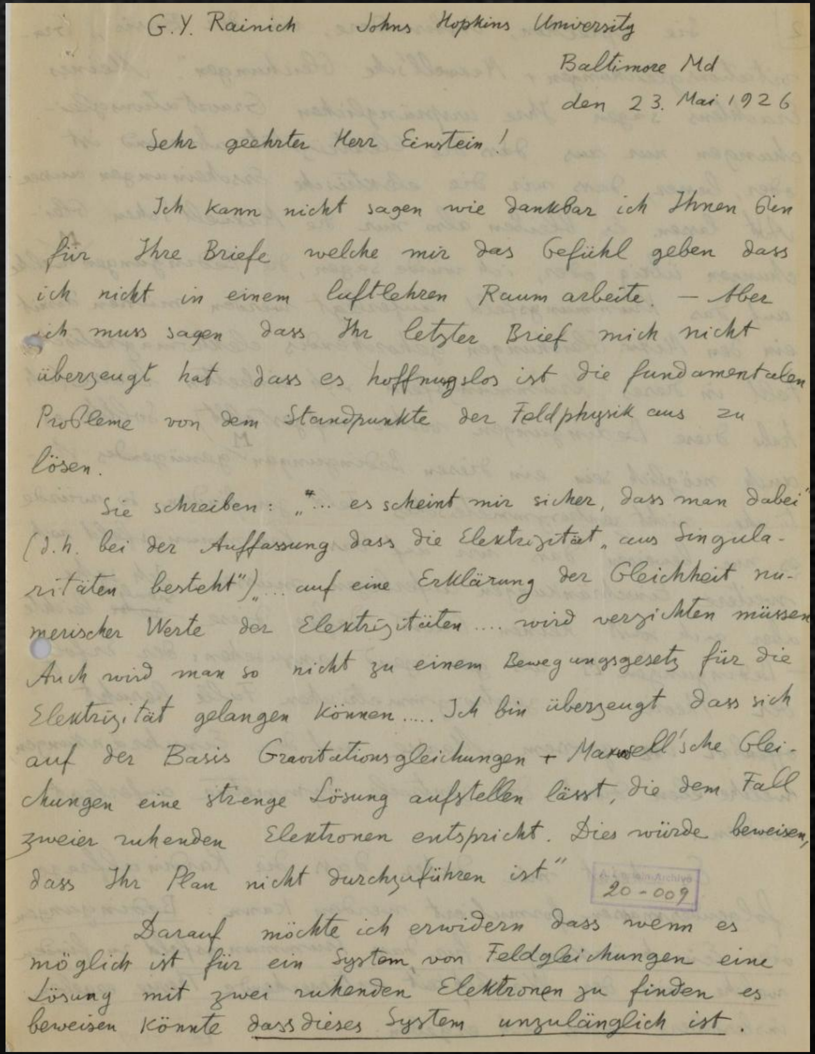
- ``[S]ingularities in the spacetime metric cannot be regarded as taking place at points of the spacetime manifold *M*. Thus, to speak of singularities in g_{ab} as geodesics of the spacetime is to speak in oxymorons.'' Earman (1995), p.12.

- ``A singularity is not even part of spacetime. How should it be possible to describe its motion *in* said spacetime? It does not make sense!'' Weatherall (2015), private discussion.

A careful investigation of the Einstein-Grommer argument

- I have argued elsewhere (DL 2017) that that despite appearances the V approach does not depend on representing matter by singularities, and that instead, we should interpret it similarly to Einstein's 1915 derivation of the perihelion of Mercury: material bodies are represented by their exterior gravitational field and/or identified by astronomical knowledge external to the mathematical model.
- In this paper I want to focus on how Einstein got to his approach to the problem of motion, and what he found attractive about it.

Einstein’s reinterpretation of Weyl’s two-body solution during his Correspondence with Yuri Rainich, 1925-1926



G. Y. Rainich Johns Hopkins University

Baltimore Md

den 23. Mai 1926

Sehr geehrter Herr Einstein!

Ich kann nicht sagen wie dankbar ich Ihnen bin für Ihre Briefe welche mir das Gefühl geben dass ich nicht in einem luftleeren Raum arbeite. — Aber ich muss sagen dass Ihr letzter Brief mich nicht überzeugt hat dass es hoffnungslos ist die fundamentalen Probleme von dem Standpunkte der Feldphysik aus zu lösen.

Sie schreiben: „*... es scheint mir sicher, dass man dabei" (d.h. bei der Auffassung dass die Elektrizität „aus Singularitäten besteht") ... auf eine Erklärung der Gleichheit numerischer Werte der Elektrizitäten .... wird verzichten müssen Auch wird man so nicht zu einem Bewegungsgesetz für die Elektrizität gelangen können ..... Ich bin überzeugt dass sich auf der Basis Gravitationsgleichungen + Maxwell'sche Gleichungen eine strenge Lösung aufstellen lässt, die dem Fall zweier ruhenden Elektronen entspricht. Dies würde beweisen, dass Ihr Plan nicht durchzuführen ist"

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Darauf möchte ich erwidern dass wenn es möglich ist für ein System von Feldgleichungen eine Lösung mit zwei ruhenden Elektronen zu finden es beweisen könnte dass dieses System unzulänglich ist.

Rainich on linear vs non-linear field equations

- Rainich pointed out to Einstein that in a *linear* theory the existence of a solution representing a static single-body solution would imply a static two-body solution.
- However, in a *non-linear* theory like GR, the existence of a two-body solution is not implied. In a letter to Einstein from 5 April 1926, Rainich adds that in contrast to a linear theory, in a non-linear theory the field of one particle may heavily constrain the properties the second particle can have.
- Rainich connects these remarks with his own research project: represent and investigate the behaviour of material bodies only in terms of their exterior gravitational fields.

Einstein on two-body solutions

``I am convinced that one could find an exact solution on the basis of the gravitational equations + Maxwell equations, which would represent the case of two electrons at rest (as singularities). For the case in which the particles in question have no electric charge this has already been shown by Weyl and Levi-Civita (special case of axial symmetry). This would show that your plan cannot be carried out."

Einstein to Rainich, 18 April 1926.

Rainich insists

``I cannot tell you how grateful I am for your letters, which give me the feeling that I am not working in a vaccuum. - But I have to say that your last letter did not convince me… . […] '' Rainich to Einstein, 23 May 1926.

- In what follows, Rainich insists on the points of his previous letter: it is not clear that GR admits a solution that should be interpreted as representing two particles (represented as singularities) at rest with respect to one another.

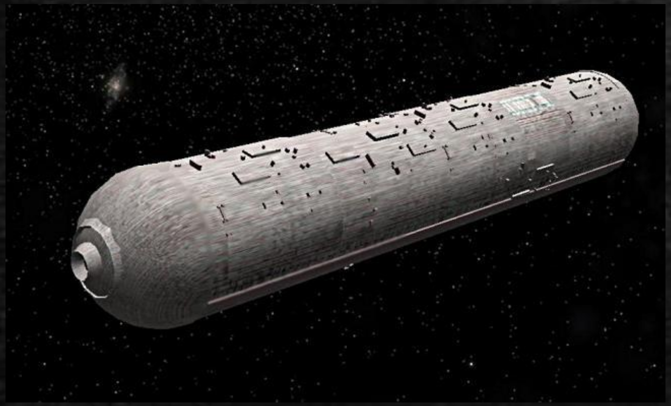
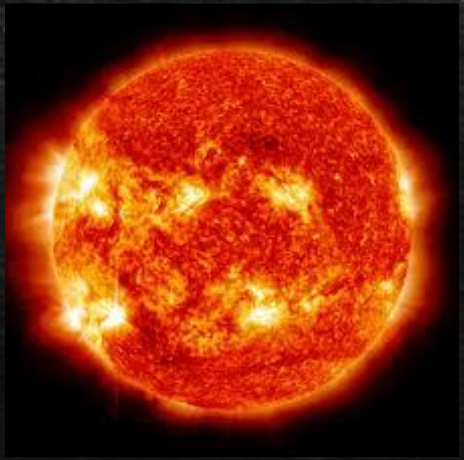
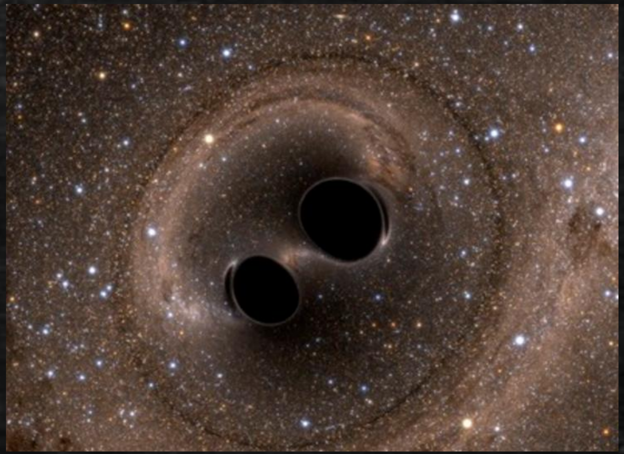
Einstein to Rainich: the U turn

``I completely agree with your main point. If a theory has a solution which represents two electrons *at rest*, then it is inadequate. This was indeed the reason why I thought that I had to reject a theory which regards electrons *as singularities*. For I had thought to have seen that any such theory would have solutions with electrons at rest. But it now seems that I was wrong about this." Einstein to Rainich, 6 June 1926 (emphasis in original).

Between 23 May and 6 June 1926

- On 18 April 1926, Einstein had pointed to Weyl and Levi-Civta's solutions as representing a static two-body solution. On 6 June 1926 he agrees with Rainich that a static two-body solution does not exist. What happened?
- I conjecture that between Rainich's letter of 23 May and Einstein's answer of 6 June, Einstein must have gone back to the papers by Levi-Civita and Weyl (and Bach) that he had referred to in his previous letter.
- He found reason to judge Weyl's two-body solution as unsatisfactory, as a *non-physical* two-body solution.

1917: The Weyl class of solutions: axially symmetric, static

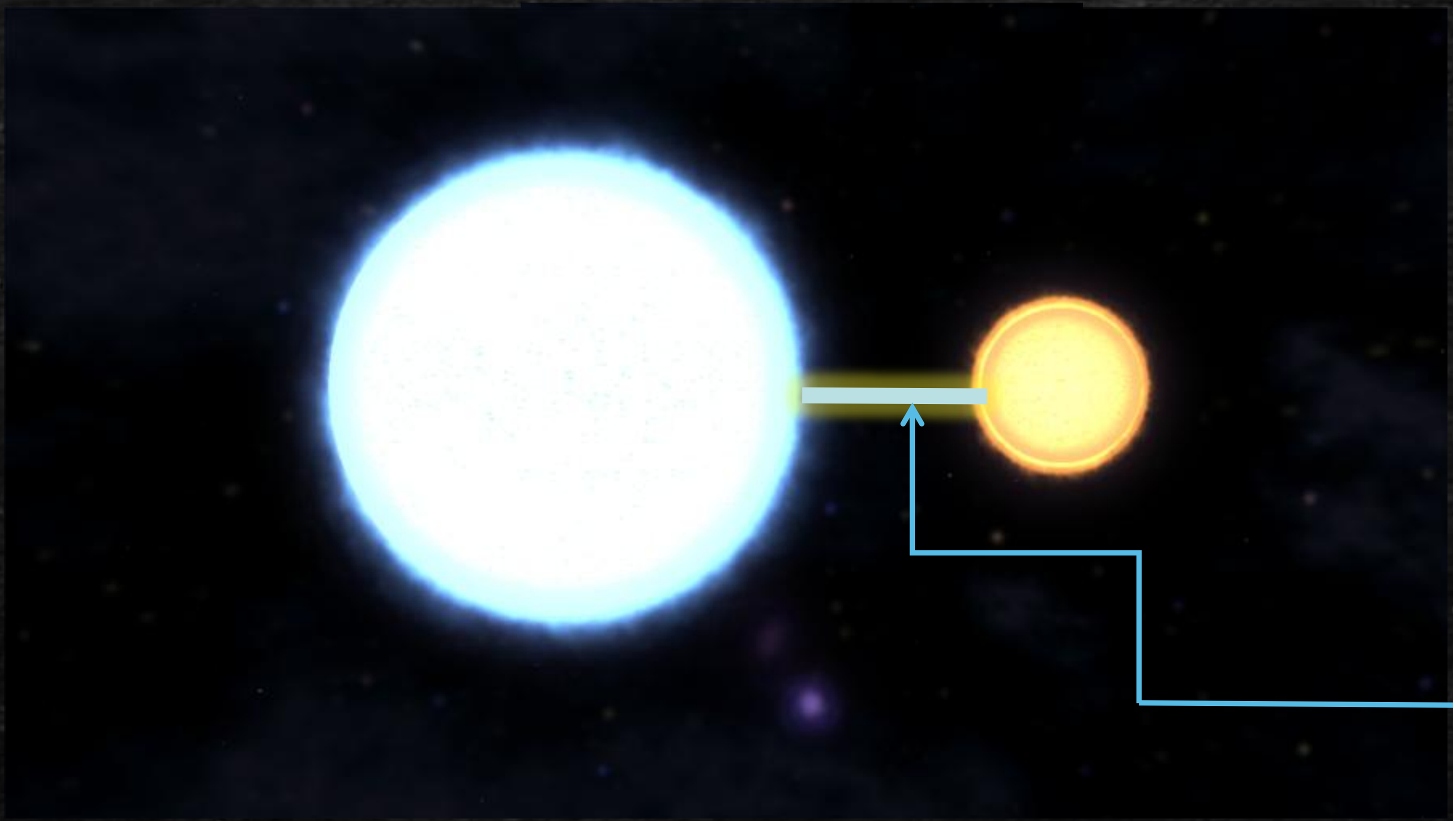
$$ds^2 = e^{2\psi} dt^2 - e^{-2\psi}[r^2 d\theta^2 + e^{2\gamma}(dr^2 + dz^2)]$$

where

$$\Delta\psi = \frac{1}{r}[\frac{\partial}{\partial z}(r\frac{\partial\psi}{\partial z}) + \frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r})] = 0$$

$$d\gamma = 2r\frac{\partial\psi}{\partial z}\frac{\partial\psi}{\partial r}dz + r(\frac{\partial^2\psi}{\partial r^2} - \frac{\partial^2\psi}{\partial z^2})dr$$

Special Case: Weyl's static two-body solution

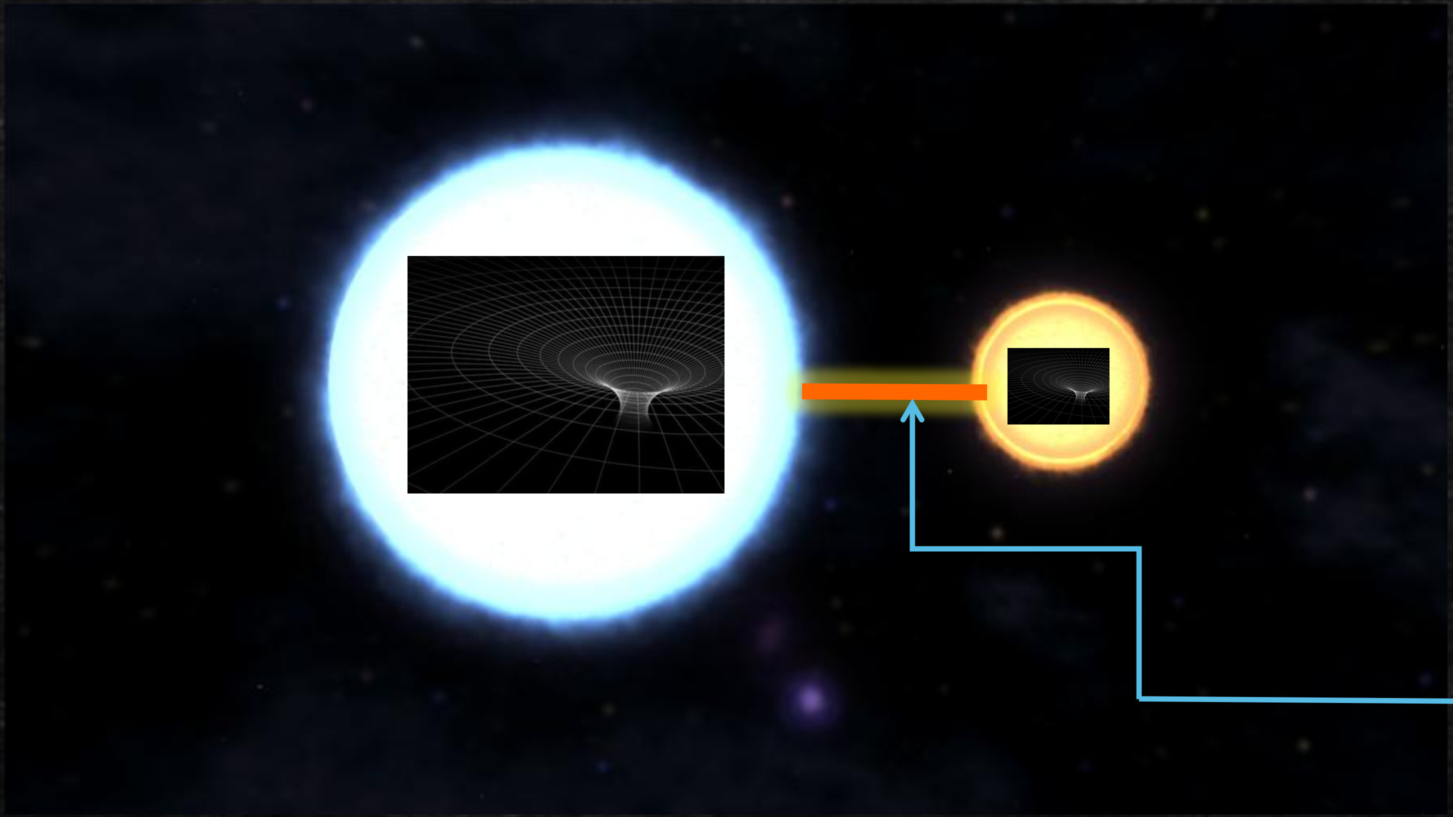


"Weyl strut"

$$T^1_1 + T^2_2 = 0$$

Weyl makes clear that the introduction of a "Weyl strut" is the only way to avoid a singularity along the rotation axis, for it ensures that $\gamma = 0$ along the axis.

Weyl's static two-body solution without "Weyl strut"



Line singularity along the z-axis.

Einstein was perfectly fine to have the two material bodies correspond to singularities, but he took the appearance of a line singularity *between* the two bodies to be sufficient to dismiss the solution as unphysical. BUT WHY?

General Relativity as a hybrid theory → Good and bad singularities

- Einstein regarded general relativity as what I would call a hybrid theory:
  - fundamentally correct with regard to spacetime regions containing only gravitational fields, and
  - only phenomenologically correct with regard to spacetime regions in which matter is present. The energy-momentum tensor in GR was only a place-holder for an adequate (quantum) theory of matter not yet found.
- Thus, he was fine with introducing singularities to stand in for matter: it just meant switching one placeholder for another.
- But in spacetime regions free of matter no singularities where to be allowed.
- This implied a selection principle for physical vs. non-physical solutions.

In search for an acceptable solution

- Einstein now made two moves:
  1. He turned Weyl's two-body problem into the problem of finding an axially symmetric solution capable of representing one body subject to an external gravitational field.
  2. He chose a simpler ansatz: while Weyl aimed to find a solution capable of representing extended material bodies, Einstein wanted an axially symmetric solution capable of representing a point mass subject to an external field.

From Newtonian point particle to the Curzon solution

Einstein and Grommer's Ansatz:

$$\psi_1 = -\frac{m}{r^2 + z^2}$$

$$\Delta\psi = \frac{1}{r}[\frac{\partial}{\partial z}(r\frac{\partial\psi}{\partial z}) + \frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r})] = 0$$

$$ds^2 = e^{2\psi}dt^2 - e^{-2\psi}[r^2 d\theta^2 + e^{2\gamma}(dr^2 + dz^2)]$$

From Curzon solution to a point particle subject to an external field

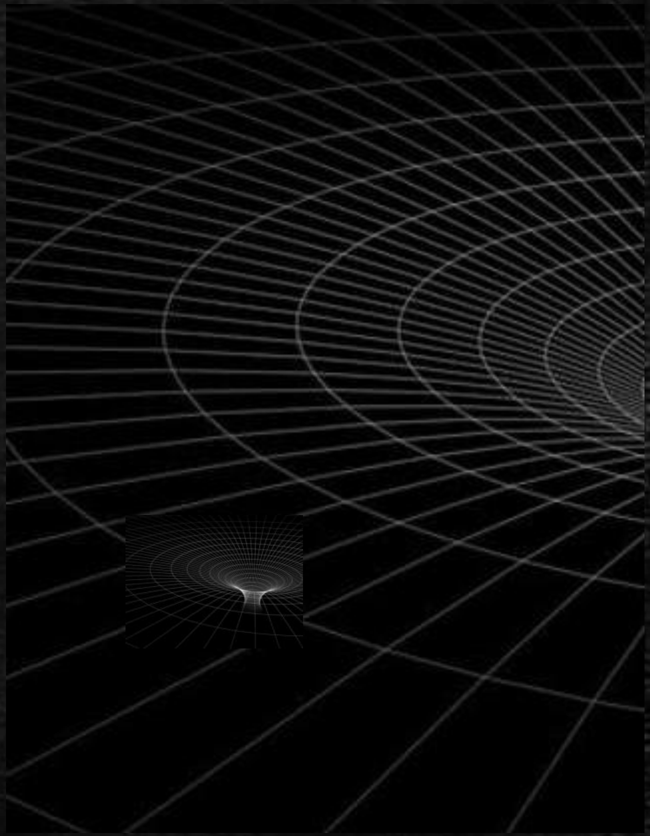
Einstein and Grommer's Ansatz: $$\psi_{total} = \psi_1 + \hat{\psi}$$

with $$\psi_1 = -\frac{m}{r^2 + z^2}$$

- Like Weyl, Einstein and Grommer had argued that the only way to avoid a singularity along the rotation axis is to ensure that $\gamma = 0$ along the axis.
- They find that the only way to do this without introducing stresses is for the external field $\hat{\psi}$ to vanish.

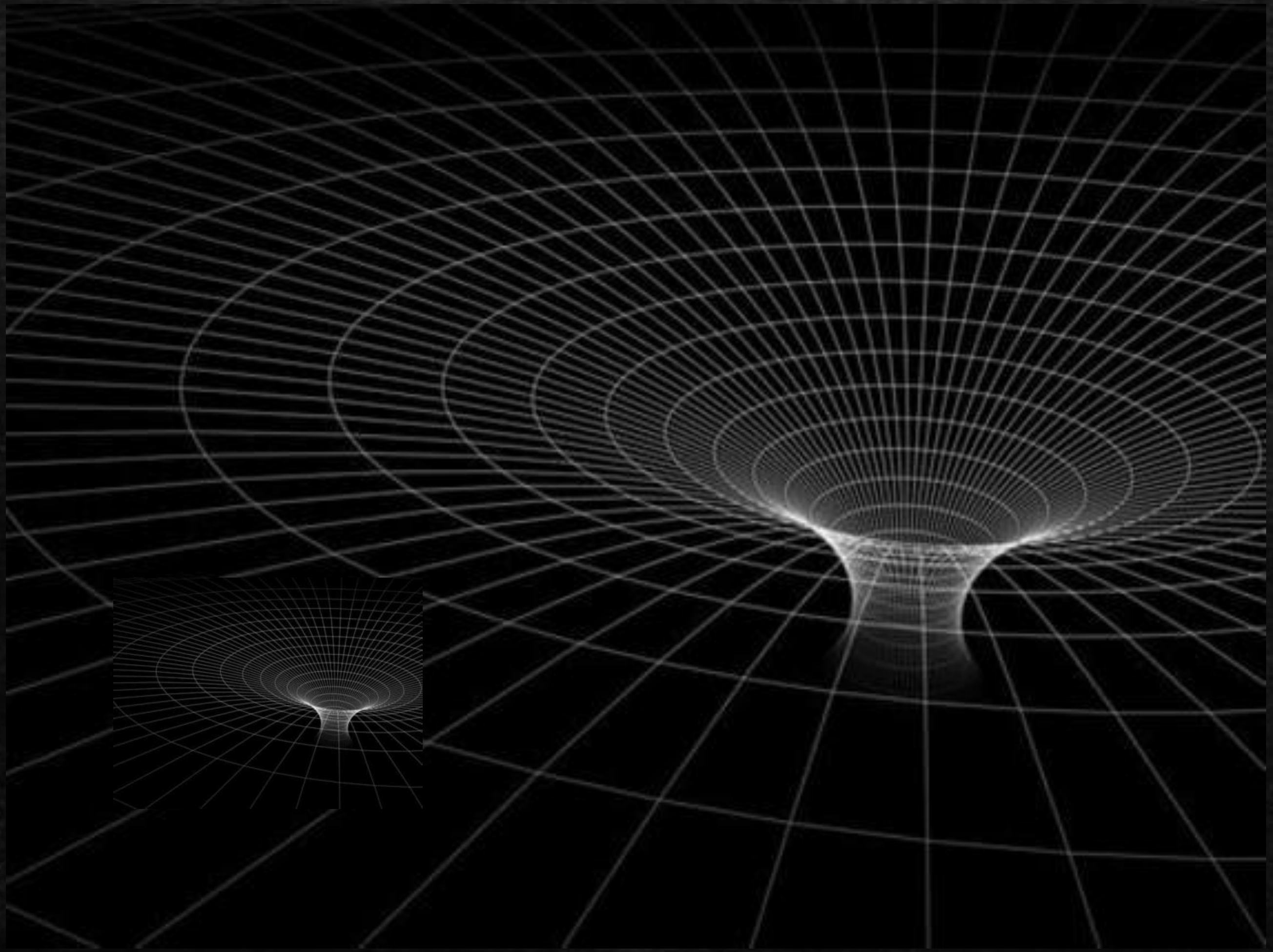
From two-body vaccuum solution to problem of motion

- Einstein and Grommer conclude that in the full, non-linear theory, there is no physical solution of a particle at rest but subject to an external gravitational field.
- Thus, they say, in GR it follows from the field equations that a particle cannot be at rest when subject to a gravitational field. (Big difference to Newtonian theory of gravity and Maxwellian theory of electrodynamics.)
- So the field equations predict whether a particle moves; they predict *that* it will move.
- From here it is only a small step to expect the field equations to determine *how* the particle will move.

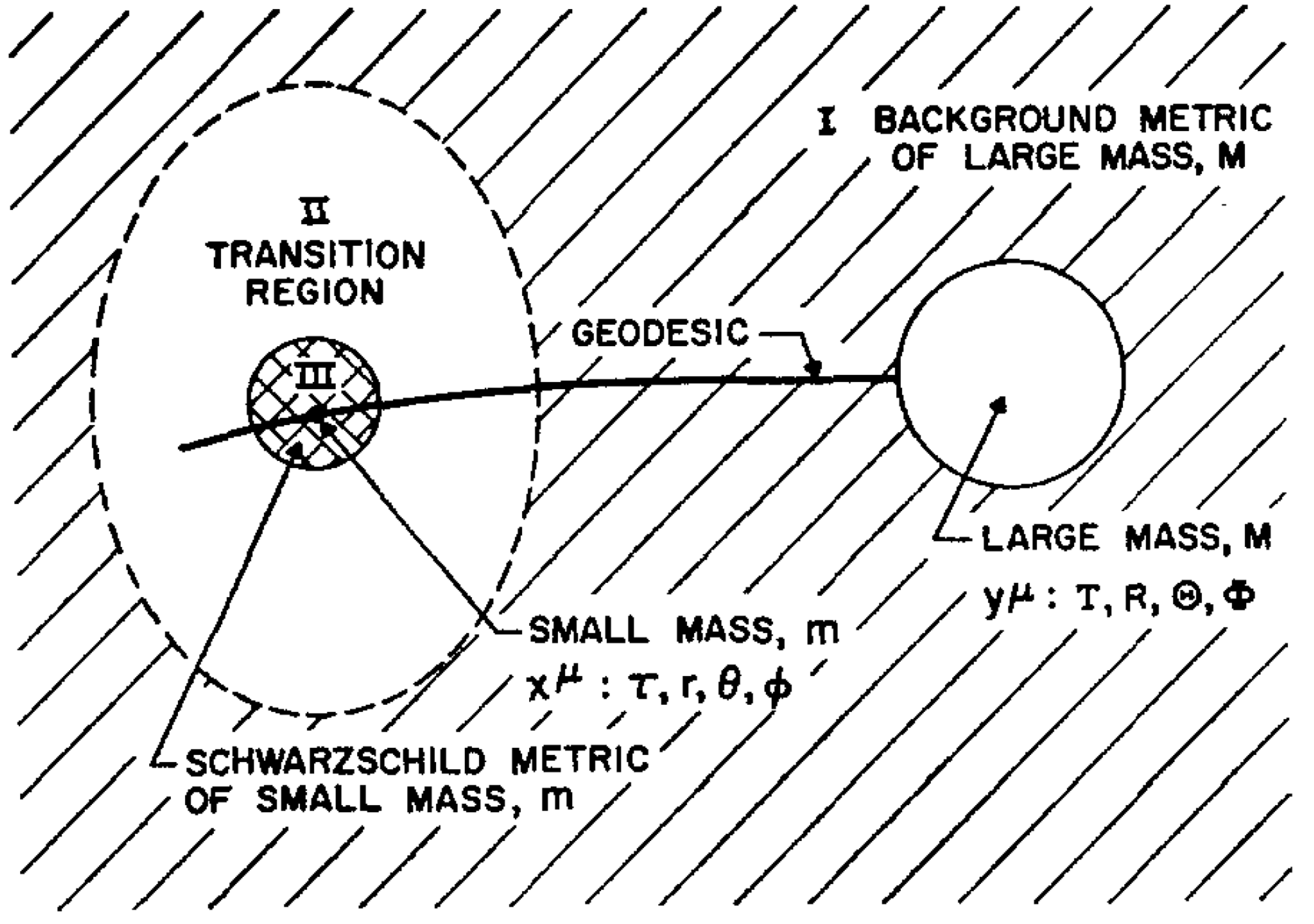


➢ The problem of motion.

From two-body solution to problem of motion



Using black hole solutions in the problem of motion



From:
Mannasse
(1963)

Generalisations of Mannasse's approach: deriving the motion of black holes in spacetime

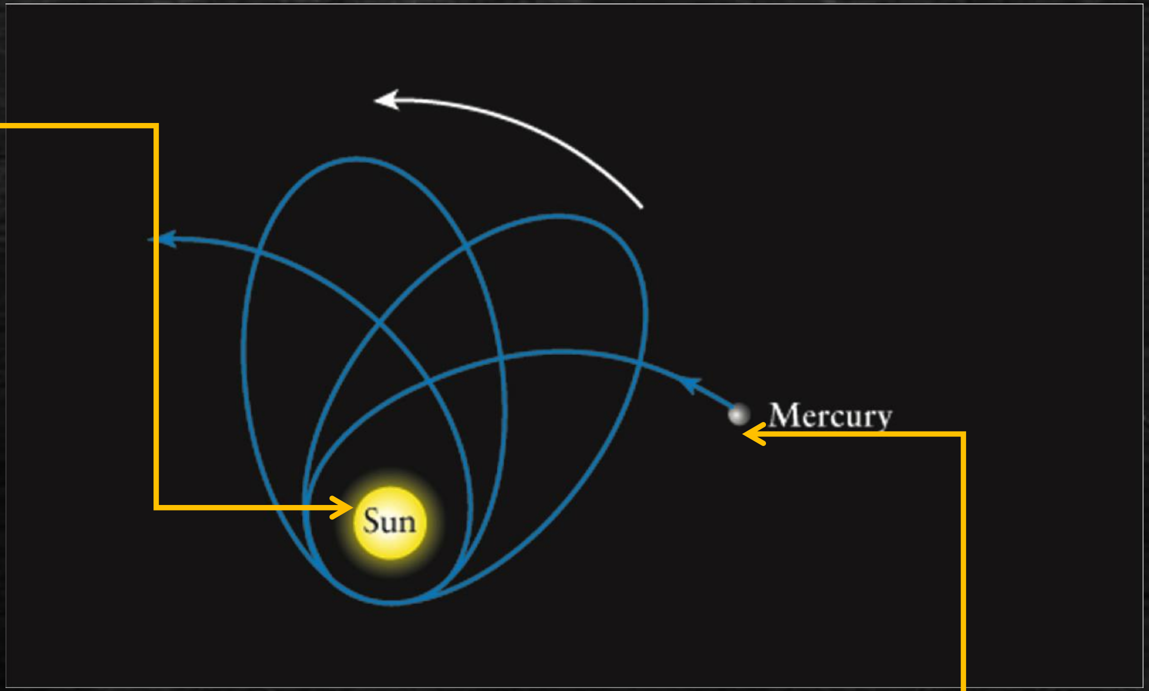
- Manasse (1963) focused on deriving the pertubations of the large Schwarzschild black hole on the small Schwarzschild black hole and vice versa, but essentially assumed that the small black hole would move on a geodesic towards the big one.
- D'Eath (1974) now made a similar move as Einstein and Grommer (1927) had: he abstracted from the two-body problem to the problem of the motion of one black hole subject to an external gravitational field. (He used a Kerr black hole.)
- He could show that the Kerr black hole moves approximately along a timelike geodesic of the background spacetime. (Further generalisations: D'Eath 1975, Thorne and Hartle 1985.)

➢ Indeed, using just solutions of the vacuum field equations, we can derive the equations of motion of a material body, represented by a asymptotically matched Kerr black hole solution to the vacuum field equations. These later works are arguably the fulfillment of Einstein's and Grommer's vision.

The vacuum approach: geodesic motion via the vacuum field equations

The Einstein field equations:

$$R_{\mu\nu} = 0$$



The geodesic equation:

$$\frac{d^2 x_\tau}{ds^2} + \Gamma^{\tau}{}_{\mu\nu} \frac{dx_\mu}{ds} \frac{dx_\nu}{ds} = 0$$

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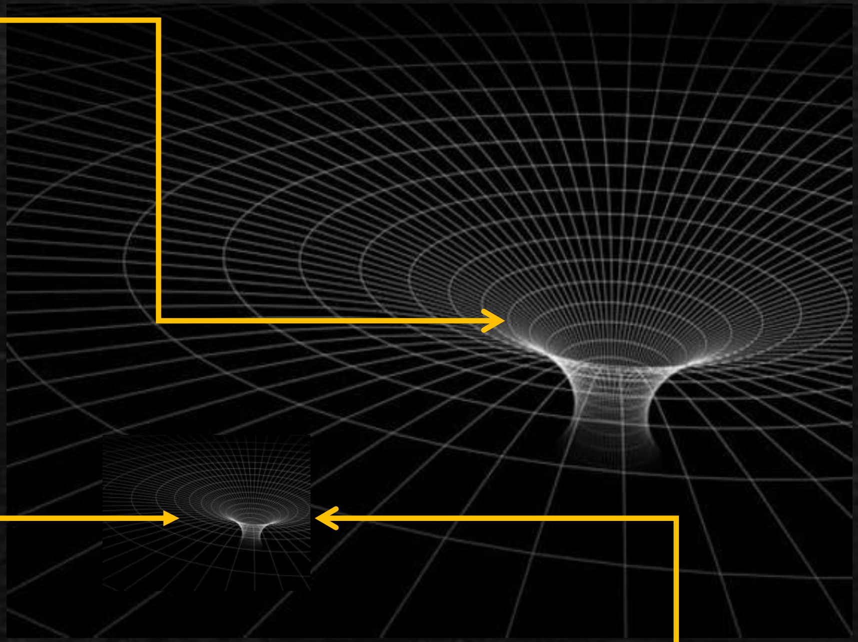

The geodesic equation:

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Conclusion

- Einstein opposed the idea that GR showed that the gravitational field is “nothing but spacetime geometry”.
- He also argued that the energy-momentum tensor of matter in the Einstein equations is nothing but a place-holder for a “proper” theory of matter, a quantum theory of matter.
- According to Einstein, GR is a a hybrid theory: fundamentally correct with regard to spacetime regions containing only gravitational fields, and only phenomenologically correct with regard to spacetime regions in which matter is present.
- Thus, exchanging the representation of matter by energy-momentum tensors to singularities was ok: it just meant switching one place-holder for the other.
- The work by Einstein and Grommer of 1927 foreshadowed some of the later works on equations of motion of black holes by Mannasse, D’Eath, Thorne and Hartle.

Thank you!

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Interpreting the Bach-Weyl solution by looking at its Newtonian counterpart.

- As we saw, the Weyl class of solution includes a Poisson-like equation:

$$ds^2 = e^{2\psi}dt^2 - e^{-2\psi}[r^2 d\theta^2 + e^{2\gamma}(dr^2 + dz^2)]$$

where

$$\Delta\psi = \frac{1}{r}[\frac{\partial}{\partial z}(r\frac{\partial\psi}{\partial z}) + \frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r})] = 0$$

$$d\gamma = 2r\frac{\partial\psi}{\partial z}\frac{\partial\psi}{\partial r}dz + r(\frac{\partial^2\psi}{\partial r^2} - \frac{\partial^2\psi}{\partial z^2})dr$$

- This suggest a solution-generating technique: start with the exact Newtonian potential ψ for some classical axially symmetric system in a flat space expressed in terms of standard cylindrical coordinates. Then...

Interpreting the Bach-Weyl solution by looking at its Newtonian counterpart.

- This suggest a solution-generating technique: start with the exact Newtonian potential ψ for some classical axially symmetric system in a flat space expressed in terms of standard cylindrical coordinates.
- Plug ψ into the Laplace-like equation of the Weyl metric, and determine γ
- Together ψ and γ suffice to determine a particular axially symmetric solution, a specific member of the Weyl class of solutions.
- Interpret the solution as the gravitational field of the analogous Newtonian source.
- (This is what Einstein and Grommer did, as we will see in the following. Note, however, that the last step can be treacherous, as we will also see.)

From Newtonian point particle to the Curzon solution

Einstein and Grommer's Ansatz: $$\psi_1 = -\frac{m}{r^2 + z^2}$$

$$\Delta\psi = \frac{1}{r}[\frac{\partial}{\partial z}(r\frac{\partial\psi}{\partial z}) + \frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r})] = 0$$

$$ds^2 = e^{2\psi}dt^2 - e^{-2\psi}[r^2 d\theta^2 + e^{2\gamma}(dr^2 + dz^2)]$$

From Curzon solution to a point particle subject to an external field

Einstein and Grommer's Ansatz: $$\psi_{total} = \psi_1 + \hat{\psi}$$

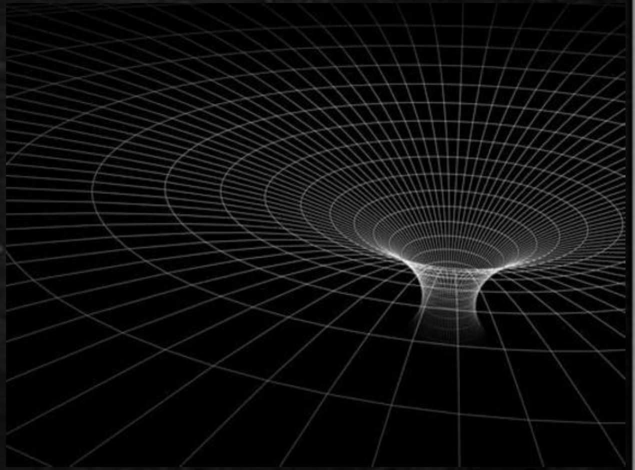
with $$\psi_1 = -\frac{m}{r^2 + z^2}$$

- Like Weyl, Einstein and Grommer had argued that the only way to avoid a singularity along the rotation axis is to ensure that $\gamma = 0$ along the axis.
- They find that the only way to do this without introducing stresses is:

$$\text{No line singularity along z-axis} \iff \gamma = 0 \quad \text{when} \quad r \to 0 \iff \oint_{r \to 0} d\gamma = 0 \iff \hat{\psi} = 0$$

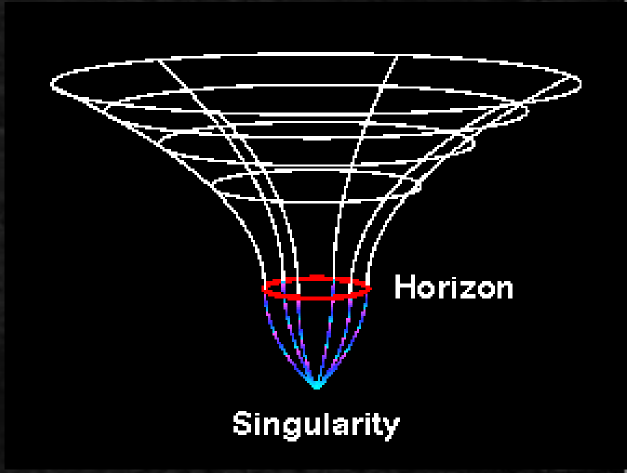
Step 1: Representing isolated bodies by vacuum spacetimes

- Ehlers (1979) suggested that for something to be "a model of an isolated system" in spacetime, the spacetime has to be asymptotically flat.
- This allows for vacuum spacetimes in which, as Thorne and Hartle (1985) put it, "one can separate spacetime into a part that represents the body and a part which represents the spacetime of the external universe".



Step 2: Make sure that if your vacuum solution has a singularity, it's not a naked one

- Naked singularities threaten a breakdown of determinism but a non-naked singularity is "hidden" behind a black-hole event horizon: it is causally isolated from the exterior.
- If a singularity is non-naked, then for astrophysical purposes it does not really matter if it's there; a black hole is then just a very massive body.
- The Schwarzschild metric has a non-naked singularity at its center.



➢ In virtue of it being asymptotically flat and involving only non-naked singularities, we are able to represent an astronomical body like the Sun by the exterior Schwarzschild metric.