

# Cosmic Singularities and String Theory

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# Plan

- What is a spacetime singularity?
- What is string theory?
- Two examples of stringy resolution of static singularities
- Cosmic singularities and string theory
- Lessons and implications

# What is a spacetime singularity

- A spacetime is **singular** if there exists **at least** one freely falling particle or photon which ends (or has begun) its existence within a **finite** “time”.
- Intuitively, **a spacetime singularity** is a “place” where some “**pathological behavior**” of the metric takes place, e.g. the curvature “**blows up**” or ...

## Some Examples

- **Big Bang/Big Crunch**

beginning or end of time?

- **Collapse of a star**

black holes

- **Infinitely thin cosmic strings**

Conical singularities

# Singularities in general relativity

- In the late sixties, Hawking and Penrose showed that “generic” classes of spacetimes in general relativity are **singular**.
- Standard notion of **spacetime breaks down** at the singularities.
- **General relativity** and other known physics laws such as **quantum field theory** also **break down**.

# Greatest crisis in physics of all time

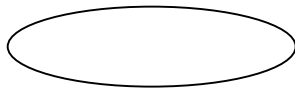
-- “Gravitation” by Misner, Thorne and Wheeler

To understand singularities, one must go **beyond** the general relativity, e.g.

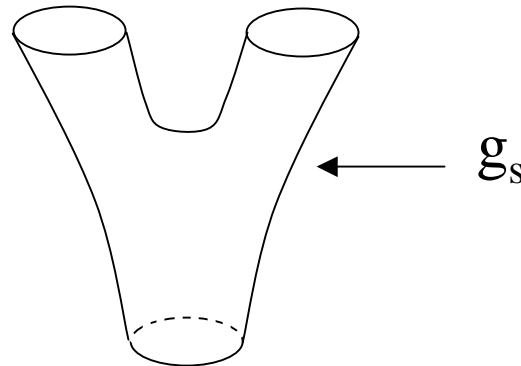
- Modify it at the classical level
- Quantum gravity

# String Theory I

- **Quantum field theory** : particles are **point-like** objects propagating in space time.
- **String theory**: gravitons, photons and all other elementary particles are **one-dimensional objects, strings**.
- Interactions are described by **splitting or joining of strings** in spacetime.
- Point-particle description arises only in a **low energy limit**.



$$\text{Tension} = \frac{1}{2\pi\alpha'}$$



# String theory II

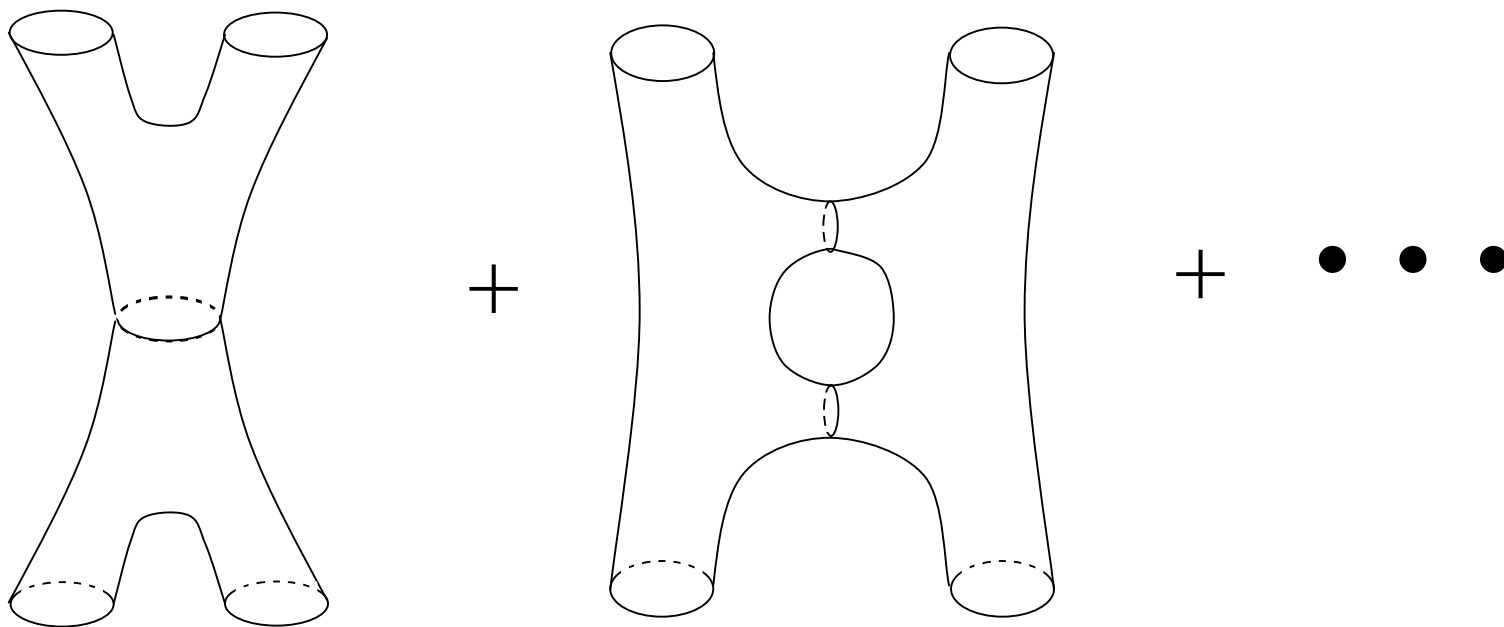
- Presently, string theory is mostly formulated in terms of **perturbation theory**.
  - **Perturbative degrees of freedom**: graviton, Yang-Mills fields, ...
  - Their masses have a well-defined limit as  $g_S \rightarrow 0$  .
- Much progress has been made toward a **non-perturbative** formulation.
  - **Non-perturbative degrees of freedom**: solitons, D-branes, black holes ...
  - Their masses depend on inverse powers of  $g_S$ , and thus are **not visible** in perturbation theory.
  - M(atric) theory, AdS/CFT .....



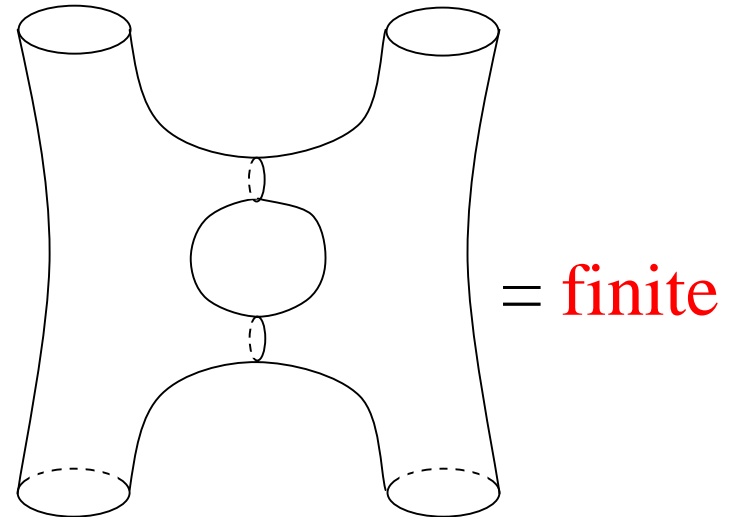
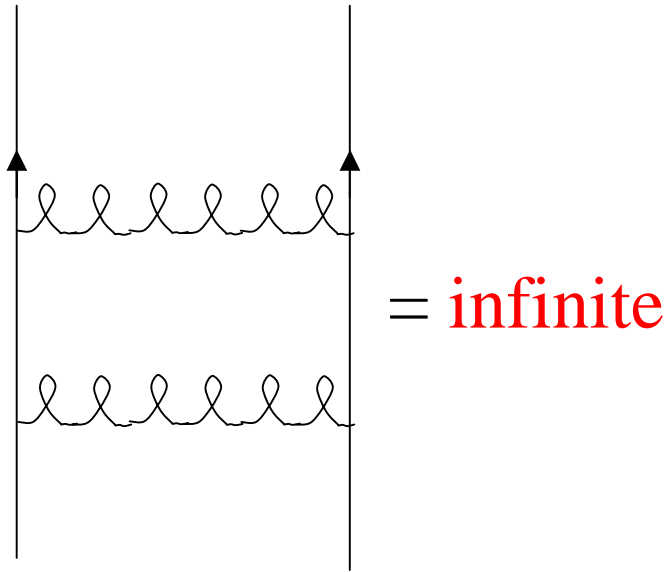
Example:

*Graviton + Graviton*  $\rightarrow$  *Graviton + Graviton*

$$A = \lambda_2 g_s^2 + \mathcal{O}(g_s^4) + \dots$$



# A consistent theory of gravity



In string theory, interactions no longer occur at points.

It **eliminates** the **UV** problem in general relativity.

Thus it is a promising candidate for a consistent theory of gravity.

## Low energy expansions

$$L_{eff} = \frac{1}{g_s^2 \alpha'} \left( R + \alpha' R^2 + \dots \right) + \dots$$

A **double** expansion in:

$$Energy^2 \alpha' \quad \text{and} \quad g_s^2$$

$$M_{pl}^2 = \frac{1}{g_s^2 \alpha'}$$

At **low energy** it reduces to Einstein relativity.

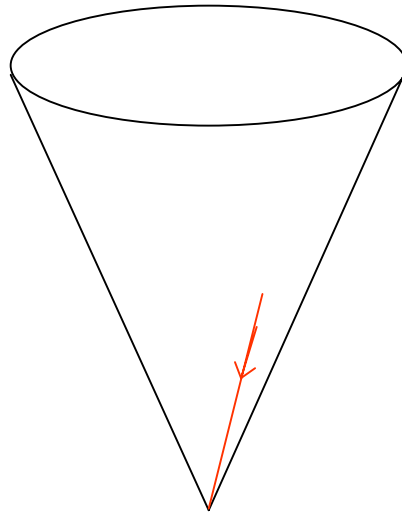
# String theory and singularities

Can singularities in general relativity be **resolved** in string theory?

- in perturbative string theory (due to extended nature of strings) ?
- Or does one need a non-perturbative formulation?

# Conical Singularities

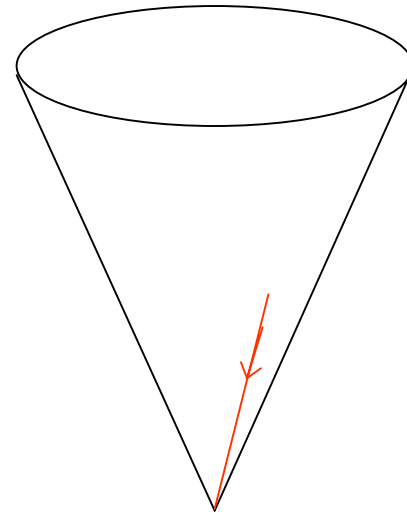
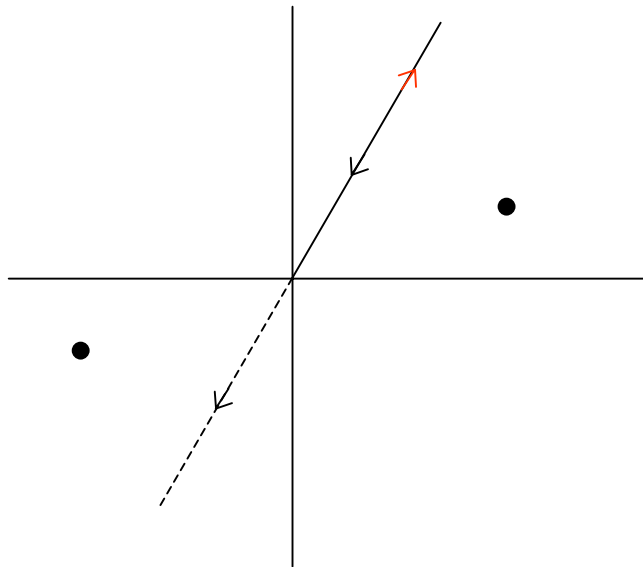
- Einstein gravity **breaks down** at a **generic** conical singularity. Those time- or null-like geodesics which hit the singularity cannot be continued beyond.



# Orbifolds

- For those obtainable from 2-dimensional flat space by **discrete identifications**, one can still make sense the classical theory, e.g.

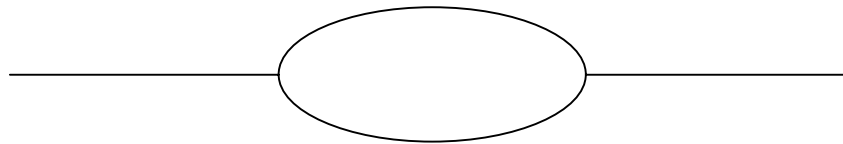
$$(x_1, x_2) \sim (-x_1, -x_2)$$



$\mathbb{R}^2/\mathbb{Z}_2$

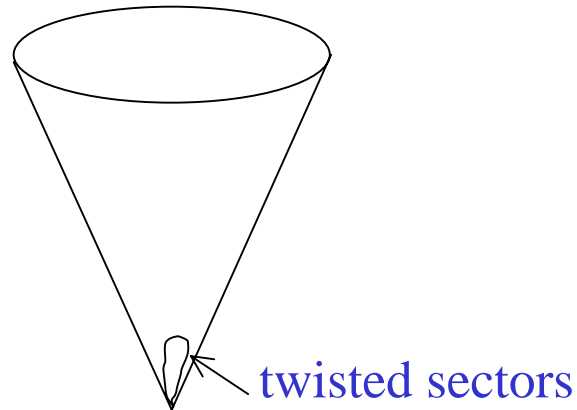
## Orbifolds II

- Similarly, one can try to define a quantum field theory in the cone by projecting onto the subspace of the Hilbert space in  $\mathcal{R}^2$  which are **invariant** under the identifications.
- However, a field theory defined this way is **not unitary** at the **quantum** level.



# String theory on orbifolds

- The **extended nature** of string theory introduces additional degrees of freedom **localized** at the tip of the cone: **twisted sectors**.

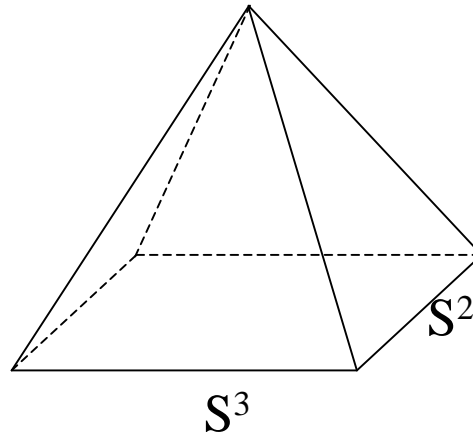


- Including the twisted sectors, **string amplitudes are unitary and physics is completely smooth.**
- This is an example where string theory **resolves** the singularity at the **perturbative** level.



# Conifold

- General relativity is **singular**.
- Classical string theory is **singular**.
- By including the **non-perturbative** degrees of freedom at the tip of the cone, the physics is again **smooth**.



# Lessons

- String theory introduces **new degrees of freedom**. By including them the physics at those singularities becomes completely **smooth**.
- Those singularities arise in general relativity simply because the **relevant** degrees of freedom are **not visible**.

# Cosmological singularities

- **Beginning of time**
  - Need initial conditions, wave functions of the Universe etc.
- **Time has no beginning or end**
  - Need to understand how to pass through the singularity. ....

# Challenges for string theory

- **Hope**

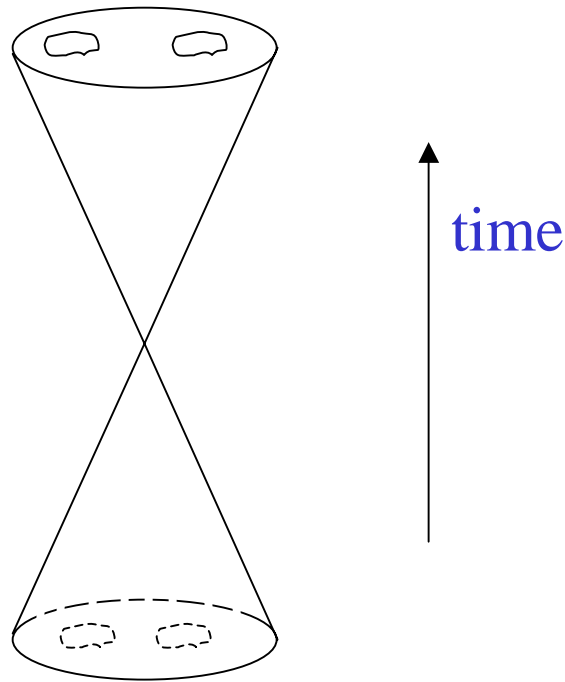
- String theory will lead to a detailed theory of the Big Bang.
- Experimental tests of string theory.

- **Question**

Is the cosmological singularity smoothed by

*classical string theory* or *quantum string theory* ?

# From Big Crunch to Big Bang: is it possible? (A Toy Model)



Time-dependent orbifolds: obtainable from **discrete** identifications of a flat **Lorentzian** spacetime. (Horowitz, Steif)

# Motivations

- Basis for some recently proposed cosmological scenarios:  
**Ekpyrotic/Cyclic Model.** Khoury, Ovrut, Seiberg, Steinhardt, Turok
- **Simplicity:** it can be subjected to an **exact perturbative string analysis.**
- **Universality:** the structure of singularity is the same as in certain black holes and certain more complicated cosmological backgrounds.

# Results from string perturbation theory

Liu, Moore, Seiberg

- One can compute the **S-matrix** from one cone to the other.
- For **generic** kinematics the amplitudes in classical string theory are **finite** (while they diverge in GR).  
This may be attributed to the **softness** of strings at **high energies**.
- For special kinematics (**near forward scattering**) the string amplitudes **diverge**.

# Origin of the divergence

Liu, Moore, Seiberg  
Horowitz, Polchinski  
Lawrence  
Martinec, McElgin

- Since the background depends on time, **energy is not conserved**.
- The energy of an incoming particle is blue shifted to **infinity** by the contraction at the singularity.
- The infinite energy generates infinitely large gravitational field and **distorts the geometry**.
- String perturbative expansion **breaks down** as a result of large backreaction.



# Lessons

- Classical string theory is **singular** in time dependent singular orbifolds.
- The extended nature of strings is **not sufficient** to resolve the singularity.
- Need to understand the full (**non-perturbative**) quantum theory to explain the physics at the singularity.

# Implication for a non-singular bounce?

- The idea of going from a big crunch to a big bang through a non-singular bounce has a long history:
  - 30's Einstein, Tolman .....
- The singularity theorems of Hawking and Penrose ruled out this possibility in general relativity.
- The recent suggestions that the universe passes through the singularity is motivated by the orbifold construction of string theory.
- We now see that **classical** string theory is also **singular** and cannot be trusted.

# Summary

- String theory is a promising candidate for a consistent theory of quantum gravity.
- Certain singularities in GR are resolved in perturbative string theory, while others are resolved by invoking non-perturbative degrees of freedom.
- Understanding the cosmic singularities is a big challenge for string theory. String theory has the potential to make important progress in cosmology by addressing this question.
- Our investigation indicates one needs to develop new non-perturbative tools to solve this problem.