The Equity Premium: Why is it a Puzzle?

by

Rajnish Mehra

University of California, Santa Barbara
and
National Bureau of Economic Research

Prepared for the

Kavli Institute for Theoretical Physics

May 3, 2006

Introduction

Introduction

Cosmology

What determines the ratio of the gravitational force to the electrostatic force?

Physics

Has the $\mu = \frac{\text{mass of proton}}{\text{mass of electron}}$ changed?

Engineering

Implication of $g = 9.81 m/sec^2$

Introduction

Cosmology

What determines the ratio of the gravitational force to the electrostatic force?

Physics

Has the $\mu = \frac{\text{mass of proton}}{\text{mass of electron}}$ changed?

Engineering

Implication of $g = 9.81 m/sec^2$

Macrofinance

What determines the equity premium?

Finance

What factors (book to market, size) affect equity returns?

Financial Engineering

Option pricing

- Physics postulates that consistent principles should govern all natural phenomena. By direct-analogy, economic science postulates that all economic phenomena, is the outcome of the interaction of the same individuals and firms, mediated by markets.

- Physics postulates that consistent principles should govern all natural phenomena. By direct-analogy, economic science postulates that all economic phenomena, is the outcome of the interaction of the same individuals and firms, mediated by markets.
- The equity premium puzzle is a glaring example of the inability of neoclassical theory to meet the challenge of cross model verification.

- Physics postulates that consistent principles should govern all natural phenomena. By direct-analogy, economic science postulates that all economic phenomena, is the outcome of the interaction of the same individuals and firms, mediated by markets.
- The equity premium puzzle is a glaring example of the inability of neoclassical theory to meet the challenge of cross model verification.
- Neoclassical theory does a good job of replicating macroeconomic phenomena but fails miserably when faced with financial data

- The equity premium is the return earned by a risky security such as a stock in excess of that earned by a risk free security such as a Treasury Bill.

- The equity premium is the return earned by a risky security such as a stock in excess of that earned by a risk free security such as a Treasury Bill.

- It is a crucial input for financial decision making such as portfolio allocation and corporate investment decisions.

Smithers and Wright "Valuing Wall Street" (2000)

- "The Equity Premium Puzzle is one of the most widely cited, and arguably one of the least understood pieces of economic research ever carried out".

Smithers and Wright "Valuing Wall Street" (2000)

- "The Equity Premium Puzzle is one of the most widely cited, and arguably one of the least understood pieces of economic research ever carried out".
- "Indeed, even many specialist economists struggle with the original Mehra and Prescott paper, which like many innovative papers is distinctly terse and at times almost impenetrable".

Historically this premium has been large.

	% real return on a market index	% real return on a relatively riskless security	% risk premium
Time period	mean	mean	mean
1802 - 2004	6.7	2.8	3.9
1889 - 2004	7.6	1.0	6.6
1889 - 1978	7.0	0.8	6.2
1926 - 2004	8.2	0.7	7.5
1947 - 2004	7.7	0.6	7.1

Source: 1802-1998 from Siegel (1998), 1889-2004 from Mehra & Prescott (1985). Data updated by the authors. The rest are the authors' estimates.

The equity premium in other capital markets

Country	Time period	%real return on a market index	%real return on a relatively riskless security	%risk premium
		mean	mean	mean
U.K.	1900 - 2005	5.50	0.64	6.14
Japan	1900 - 2005	4.51	5.33	9.84
Germany	1900 - 2005	3.09	5.47	9.07
France	1900 - 2005	3.60	5.67	9.27
Australia	1900 - 2005	7.70	0.79	8.49
Sweden	1900 - 2005	7.80	0.18	7.98
India	1991 - 2004	12.6	1.28	11.3

The equity premium in different sub-periods

Time period	%real return on a market index	%real return on a relatively riskless security	%risk premium
	mean	mean	mean
1889 - 1933	7.01	3.39	3.62
1934 - 2004	8.13	0.02	8.11
1946 - 2004	8.26	0.74	7.52

Source: Mehra and Prescott (1985). Updated by the authors.

The equity premium: 30 yr moving averages

Time Period	% real return on a market index	% real return on a relatively riskless security	% equity premium
	mean	mean	mean
1900-1950	7.45	2.95	4.50
1951-2004	8.51	1.07	7.44

Source: Mehra and Prescott (1985). Updated by the authors

- Although the premium has been increasing over time, this is largely due to the diminishing return on the riskless asset, rather than a dramatic increase in the return on equity.

- We find a dramatic change in the equity premium in the post 1933 period.

- We find a dramatic change in the equity premium in the post 1933 period.
- The premium rose from 3.62% to 8.11%, an increase of more than 125 percent.

- We find a dramatic change in the equity premium in the post 1933 period.
- The premium rose from 3.62% to 8.11%, an increase of more than 125 percent.
- Since 1933 marked the end of the period when the US was on the gold standard, this break can be seen as the change in the equity premium after the implementation of the new policy.

The dramatic investment implications of the differential rates of returns resulting from the equity premium.

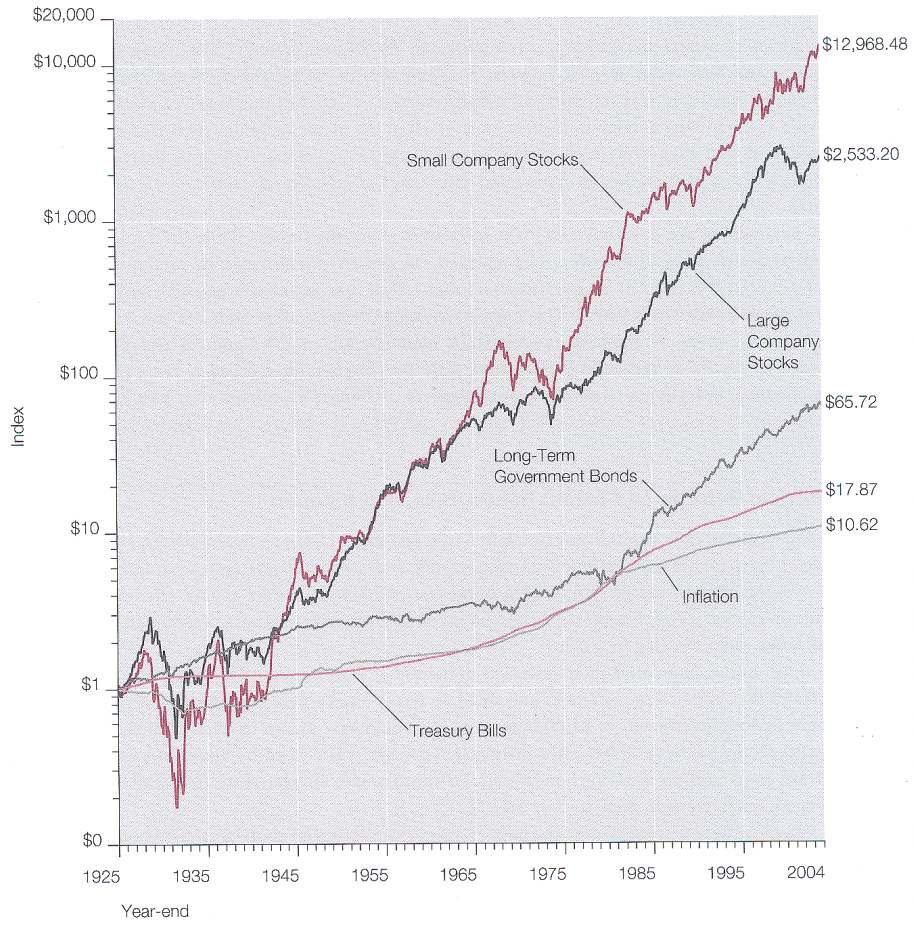
The dramatic investment implications of the differential rates of returns resulting from the equity premium.

Terminal Value of $1 Invested

Investment Period	Stocks	T-Bills	Ratio
1889-2004	$4,092.36	$3.14	1,303.30
1926-2004	$407.56	$1.67	244.05
1947–2004	$61.70	$1.33	46.39

Wealth Indices of Investments in the U.S. Capital Markets
Year-End 1925 = $1.00

from 1925 to 2004



- Adjusted for inflation, the average annual return on the US stock market in the past 115 years has been a robust 7.6%.

- Adjusted for inflation, the average annual return on the US stock market in the past 115 years has been a robust 7.6%.
- This premium exists even when one excludes the 1982- 2000 bull market.

- Adjusted for inflation, the average annual return on the US stock market in the past 115 years has been a robust 7.6%.
- This premium exists even when one excludes the 1982- 2000 bull market.
- Over the same period, the real return on comparatively safe securities like government T-bills was a paltry 1 per cent. The difference of 6.6% is the “equity premium”.

- This is puzzling because it defies easy explanation in standard theories of asset pricing.

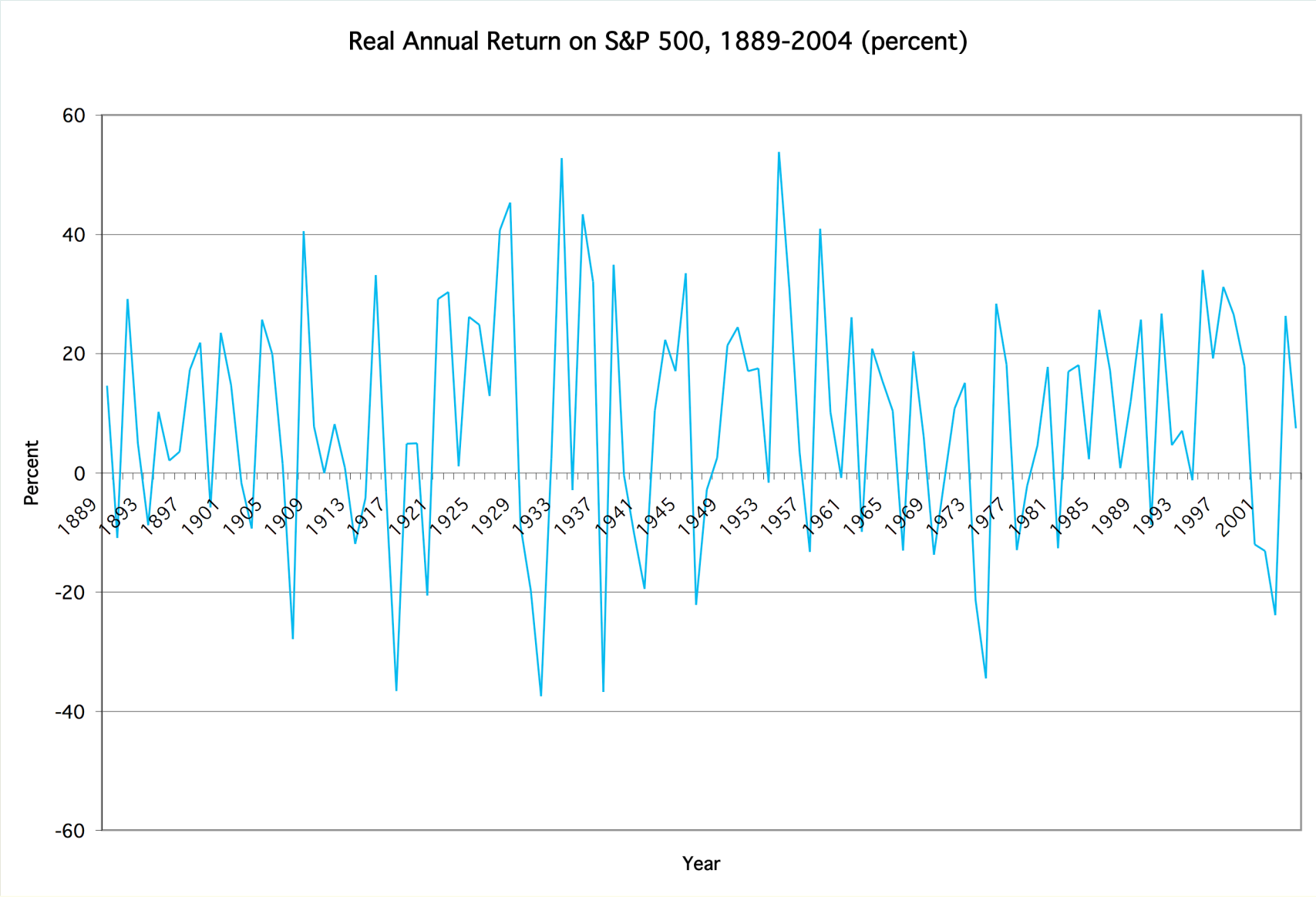
A Premium for Bearing Risk?

A Premium for Bearing Risk?

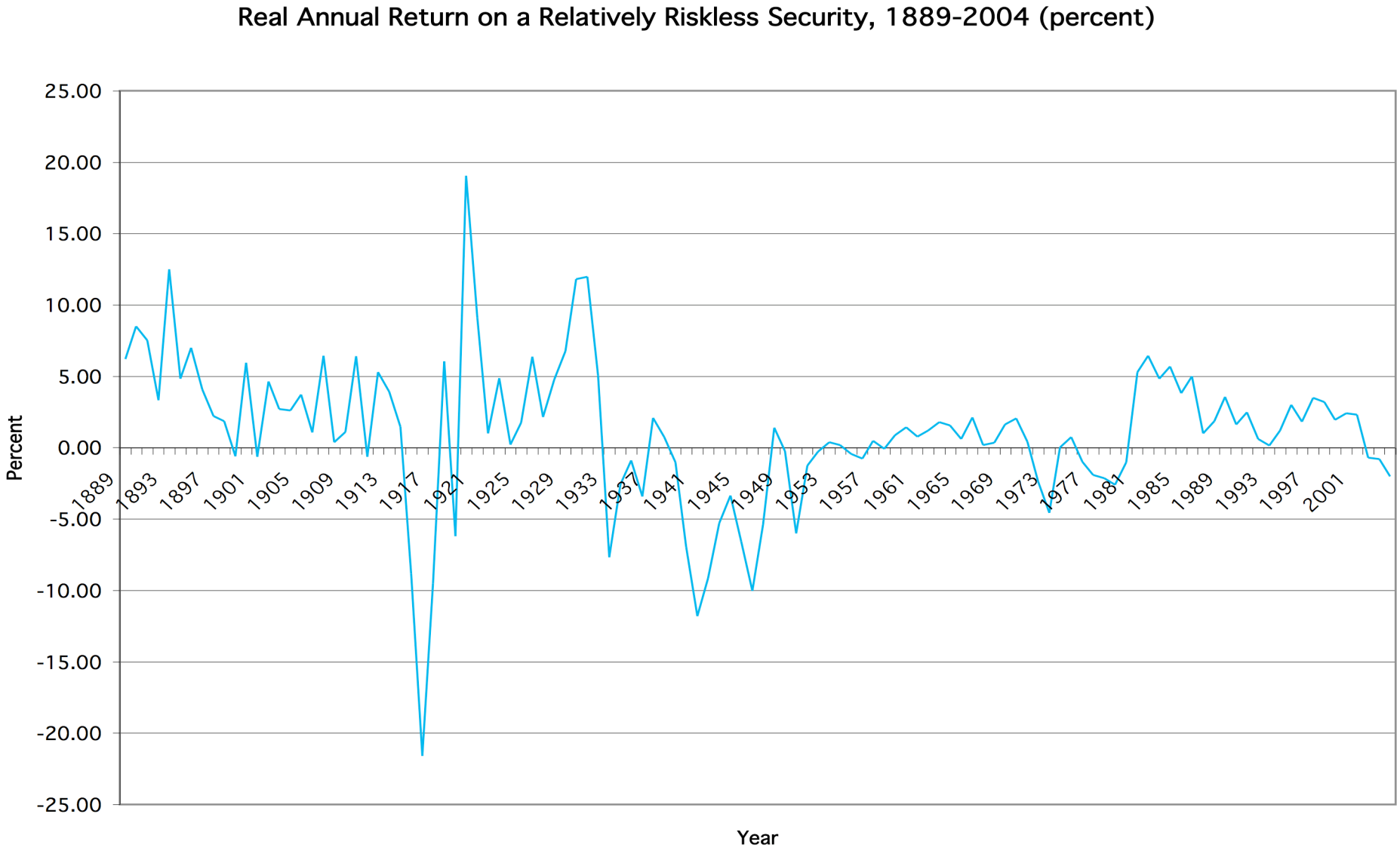
- Why have stocks been such an attractive investment relative to bonds?

- One intuitive answer is that since stocks are 'riskier' than bonds, investors require a larger premium for bearing this additional risk.

- One intuitive answer is that since stocks are 'riskier' than bonds, investors require a larger premium for bearing this additional risk.
- Indeed, the standard deviation of the returns to stocks (about 20% per annum historically) is larger than that of the returns to T-bills (about 4% per annum), so, obviously they are considerably more risky than bills!



Source: Mehra and Prescott (1985). Data updated by the author



Source: Mehra and Prescott (1985). Data updated by the author

- One intuitive answer is that since stocks are 'riskier' than bonds, investors require a larger premium for bearing this additional risk.
- Indeed, the standard deviation of the returns to stocks (about 20% per annum historically) is larger than that of the returns to T-bills (about 4% per annum), so, obviously they are considerably more risky than bills!
- But are they?

- Why do different assets yield different rates of return ?

- Why do different assets yield different rates of return ?
- Assets are priced such that, ex-ante, the loss in marginal utility incurred by sacrificing current consumption and buying an asset at a certain price is equal to the expected gain in marginal utility contingent on the anticipated increase in consumption when the asset pays off in the future.

- The *same* amount of consumption may result in *different* degrees of well-being at different times. (A five-course dinner after a heavy lunch yields considerably less satisfaction than a similar dinner when one is hungry!)

- From an investor's perspective, the desirability of an equity security depends on the relationship between future consumption and the future returns on the security.

- From an investor's perspective, the desirability of an equity security depends on the relationship between future consumption and the future returns on the security.
- If the security is likely to pay off handsomely when consumption is low, the investor will look more favorably on it.

- From an investor's perspective, the desirability of an equity security depends on the relationship between future consumption and the future returns on the security.
- If the security is likely to pay off handsomely when consumption is low, the investor will look more favorably on it.
- Why? The incremental improvement in wellbeing from a unit increase in consumption varies inversely with the level of consumption.

	State1 "Boom" (High consumption)	State 2 "Recession" (Low consumption)
Probability of state	0.5	0.5
Payoff of security A	$20,000	$0
Payoff of security B	$0	$20,000

Expected Pay off of security $A = \$10,000$
Expected Pay off of security $B = \$10,000$

Price of security $A = \$P$
Price of security $B = \$Q$

Expected Gross Rate of return of security $A = \$10,000/\$P$

Expected Gross Rate of return of security $B = \$10,000/\$Q$

- Assets that pay off when times are good and consumption levels are high, i.e.when the incremental value of additional consumption is low, are less desirable than those that pay off an equivalent amount when times are bad and additional consumption is both desirable and more highly valued.

- Let us illustrate this principle in the context of the standard, popular paradigm, the Capital Asset Pricing Model (CAPM).

- Let us illustrate this principle in the context of the standard, popular paradigm, the Capital Asset Pricing Model (CAPM).
- The model postulates a linear relationship between an asset's 'beta' and expected return. Thus, high beta stocks yield a high-expected rate of return.

$$\bar{R}_j = R_f + \beta_j(\bar{R}_m - R_f)$$

$$\beta_j = \frac{Cov(\tilde{R}_j, \tilde{R}_m)}{\sigma^2(\tilde{R}_m)} = \frac{\rho_{jm}\sigma_j}{\rho_{mm}\sigma_m}$$

- That is so because in the CAPM, good times and bad times are captured by the return on the market. The performance of the market as captured by a broad based index acts as a surrogate indicator for the relevant state of the economy.

- That is so because in the CAPM, good times and bad times are captured by the return on the market. The performance of the market as captured by a broad based index acts as a surrogate indicator for the relevant state of the economy.
- A high beta security tends to pay off more when the market return is high, that is, when times are good and consumption is plentiful; such a security provides less incremental utility than a security that pays off when consumption is low, is less valuable and consequently sells for less.

- Thus assets that pay off in states of low marginal utility will sell for a lower price than similar assets that pay off in states of high marginal utility. Since rates of return are inversely proportional to asset prices the latter class of assets will, on average, give a lower rate of return than the former.

- Another perspective on asset pricing emphasizes that economic agents prefer to smooth patterns of consumption over time.

- Another perspective on asset pricing emphasizes that economic agents prefer to smooth patterns of consumption over time.
- Assets that pay off a relatively larger amount at times when consumption is already high, "destabilize" these patterns of consumption, whereas assets that pay off when consumption levels are low "smooth" out consumption.

- Another perspective on asset pricing emphasizes that economic agents prefer to smooth patterns of consumption over time.
- Assets that pay off a relatively larger amount at times when consumption is already high, "destabilize" these patterns of consumption, whereas assets that pay off when consumption levels are low "smooth" out consumption.
- Insurance policies are a classic example of assets that smooth consumption. Individuals willingly purchase and hold them, in spite of their very low rates of return.

- To return to the original question: are stocks that much more risky than bills so as to justify a 7% differential in their rates of return?

- To return to the original question: are stocks that much more risky than bills so as to justify a 7% differential in their rates of return?
- What came as a surprise to many economists and researchers in finance was the conclusion of a research paper that Ed Prescott and I wrote in 1979.

- To return to the original question: are stocks that much more risky than bills so as to justify a 7% differential in their rates of return?
- What came as a surprise to many economists and researchers in finance was the conclusion of a research paper that Ed Prescott and I wrote in 1979.
- Stocks and bonds pay off in approximately the same states of nature or economic scenarios and hence, as argued earlier, they should command approximately the same rate of return.

- In fact, using standard theory to estimate risk adjusted returns, we found that stocks on average should command, at most, a 1% return premium over bills.

- In fact, using standard theory to estimate risk adjusted returns, we found that stocks on average should command, at most, a 1% return premium over bills.
- Since, for as long as we had reliable data, (about a hundred years), the mean premium on stocks over bills was considerably and consistently higher, we realized that we had a puzzle on our hands.

- It took us six more years to convince a skeptical profession and for our paper "The Equity Premium: A Puzzle" to be published. (Mehra and Prescott (1985)).

The central planning problem

$$w(k_0, \lambda_0) = \max \; E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

subject to

$$c_t + k_{t+1} \leq \lambda_t f(k_t, l_t), \quad \lambda_0, k_0 \text{ given}, \; l_t = 1 \; \forall t$$

The decentralized version

Household problem:

$$v(k_0, \underline{k}_0, \lambda_0) = \max E \left[\sum_{t=0}^{\infty} \beta^t \ln c^d(k_t, \underline{k}_t, \lambda_t) \right]$$

subject to

$$p_c\, c^d + p_i\, i^d \leq p_k\, k^s + p_l\, l^s$$

$$k_{t=1} \equiv k^s = i^d, \quad l^s \leq 1$$

and $\quad \underline{k}_{t+1} = \Psi(k_t, \lambda_t)$

The firm's problem:

$$\max \left[p_c \, c^s + p_i \, i^s - p_k \, k^d - p_l \, l^d \right]$$

subject to $$c_t^s + i_t^s \leq \lambda_t (k_t^d)^{\alpha} (l_t^d)^{1-\alpha}$$

The recursive representation:

$$v(k_t, \underline{k}_t, \lambda_t) = \max_{c^d, i^d, l^s, k^d} \left[\ln c^d + \beta \int v(i^d, \Psi, \lambda_{t+1}) dF(\lambda_{t+1}|\lambda_t) \right]$$

$$\text{subject to} \quad p_c\, c^d + p_i\, i^d \leq p_k\, k^s + p_l\, l^s$$

$$k_{t+1} \equiv k^s = i^d, \quad l^s \leq 1$$

$$\text{and} \quad \underline{k}_{t+1} = \Psi(k_t, \lambda_t)$$

- Hence the viability of using this class of models for any quantitative assessment, say, for instance, to gauge the welfare implications of alternative stabilization policies, is thrown open to question.

- For this reason, over the last 20 years or so, attempts to resolve the puzzle have become a major research impetus in finance and economics.

- For this reason, over the last 20 years or so, attempts to resolve the puzzle have become a major research impetus in finance and economics.
- Several generalizations of key features of the Mehra and Prescott (1985) model have been proposed to better reconcile observations with theory.

These include:

- Alternative assumptions on preferences

These include:

- Alternative assumptions on preferences
- Modified probability distributions to admit rare events

These include:

- Alternative assumptions on preferences
- Modified probability distributions to admit rare events
- Survival bias

These include:

- Alternative assumptions on preferences
- Modified probability distributions to admit rare events
- Survival bias
- Incomplete markets

These include:

- Alternative assumptions on preferences
- Modified probability distributions to admit rare events
- Survival bias
- Incomplete markets
- Market imperfections

These include:

- Alternative assumptions on preferences
- Modified probability distributions to admit rare events
- Survival bias
- Incomplete markets
- Market imperfections
- Limited participation of consumers in the stock market

These include:

- Alternative assumptions on preferences
- Modified probability distributions to admit rare events
- Survival bias
- Incomplete markets
- Market imperfections
- Limited participation of consumers in the stock market
- Problems of temporal aggregation

Incorporating Life Cycle Effects in an OLG Model

Constantinides, Donaldson and Mehra (2002)

Summary of the Model

Summary of the Model

* Three distinct Stages

Summary of the Model

* Three distinct Stages
* Two decisions in each stage: (1) consumption (2) portfolio

Summary of the Model

* Three distinct Stages
* Two decisions in each stage: (1) consumption (2) portfolio

Item	*Stage in Life*		
	Young	*Middle-age*	*Old*

Summary of the Model

* Three distinct Stages
* Two decisions in each stage: (1) consumption (2) portfolio

Item	*Stage in Life*		
	Young	*Middle-age*	*Old*
Wages	Low wages	High wages	Zero
Consumption Saving	Consume as much as possible	Save for old age	Consume everything

Summary of the Model

* Three distinct Stages
* Two decisions in each stage: (1) consumption (2) portfolio

Item	*Stage in Life*		
	Young	*Middle-age*	*Old*
Wages	Low wages	High wages	Zero
Consumption Saving	Consume as much as possible	Save for old age	Consume everything
Future Wage Uncertainty	High	Zero	-

Summary of the Model

* Three distinct Stages
* Two decisions in each stage: (1) consumption (2) portfolio

Item	*Stage in Life*		
	Young	*Middle-age*	*Old*
Wages	Low wages	High wages	Zero
Consumption Saving	Consume as much as possible	Save for old age	Consume everything

Future Wage Uncertainty	High	Zero	-

Portfolio holdings -with borrowing constraints	ZERO bonds **and** equity	May hold equity and bonds	ZERO: Sell all bonds and stocks

Summary of the Model

* Three distinct Stages
* Two decisions in each stage: (1) consumption (2) portfolio

Item	*Stage in Life*		
	Young	*Middle-age*	*Old*
Wages	Low wages	High wages	Zero
Consumption Saving	Consume as much as possible	Save for old age	Consume everything

Future Wage Uncertainty	High	Zero	-

Portfolio holdings -with borrowing constraints	ZERO bonds **and** equity	May hold equity and bonds	ZERO: Sell all bonds and stocks

Portfolio holdings -unconstrained	Borrow (Sell Bonds Short)	May hold equity and bonds	ZERO: Sell all bonds and stocks

In an infinitely-lived, representative-agent model,

$$\text{consumption}_{t+1} = \text{div}_{t+1} + \text{coupon}_{t+1} + \text{wages}$$

Since wages are a large part of consumption,

$COV(\text{consumption}_{t+1}, \text{equity}_{t+1} + \text{div}_{t+1})$ is *low*.

In an OLG model,

the elderly agents consumption is

$$\text{consumption}_{t+1} = \text{equity}_{t+1} + \text{div}_{t+1} + \text{bond}_{t+1} + \text{coupon}_{t+1}$$

and

$COV(\text{consumption}_{t+1}, \text{equity}_{t+1} + \text{div}_{t+1})$ is *high*.

Is the Equity Premium likely to persist?

Is the Equity Premium likely to persist?

- There is point of view, held by a group of academicians and professionals who claim that *at present there is no equity premium* and by implication no equity premium puzzle.

Is the Equity Premium likely to persist?

- There is point of view, held by a group of academicians and professionals who claim that *at present there is no equity premium* and by implication no equity premium puzzle.
- To address these claims we need to differentiate between two different interpretations of the term "equity premium".

- One is the *ex-post* or realized equity premium over long periods of time. This is the actual, historically observed difference between the return on the market, as captured by a stock index, and the risk free rate, as proxied by the return on government bills.

- This is what Edward Prescott and I addressed in our 1985 paper.

- This is what Edward Prescott and I addressed in our 1985 paper.
- However, there is a related concept – the ex ante equity premium. This is a forward-looking measure of the premium, that is, the equity premium that is *expected* to prevail in the future or the conditional equity premium given the current state of the economy. This must be positive!

- To elaborate, after a bull market, when stock valuations are high relative to fundamentals the ex ante equity premium is likely to be low.

- To elaborate, after a bull market, when stock valuations are high relative to fundamentals the ex ante equity premium is likely to be low.
- However, it is precisely in these times, when the market has risen sharply, that the ex-post, or the realized premium is high.

- To elaborate, after a bull market, when stock valuations are high relative to fundamentals the ex ante equity premium is likely to be low.
- However, it is precisely in these times, when the market has risen sharply, that the ex-post, or the realized premium is high.
- Conversely, after a major downward correction, the ex-ante (expected) premium is likely to be high while the realized premium will be low. This should not come as a surprise since returns to stock have been documented to be mean reverting.

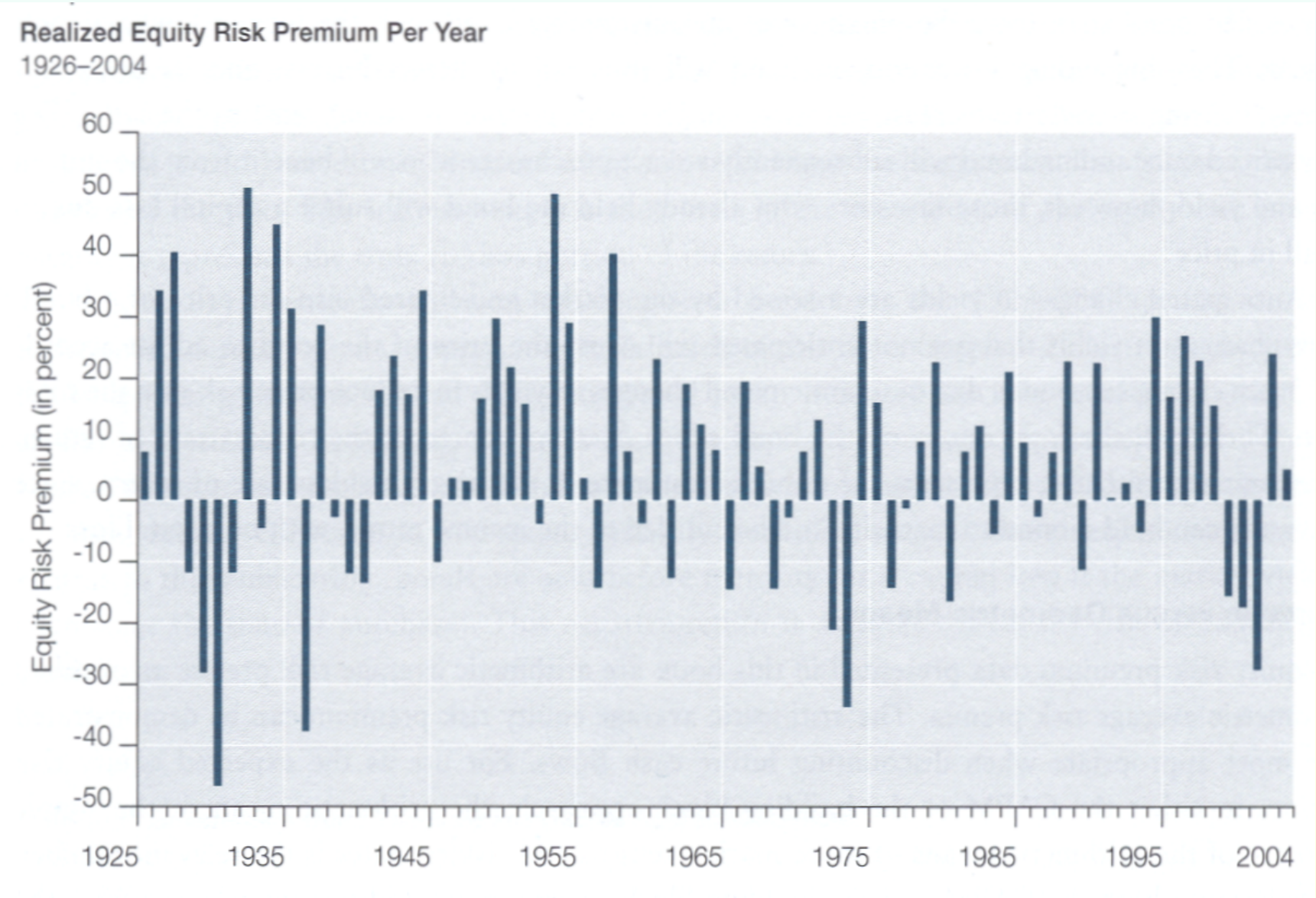
- Which of these interpretations of the equity premium is relevant for an investment advisor?

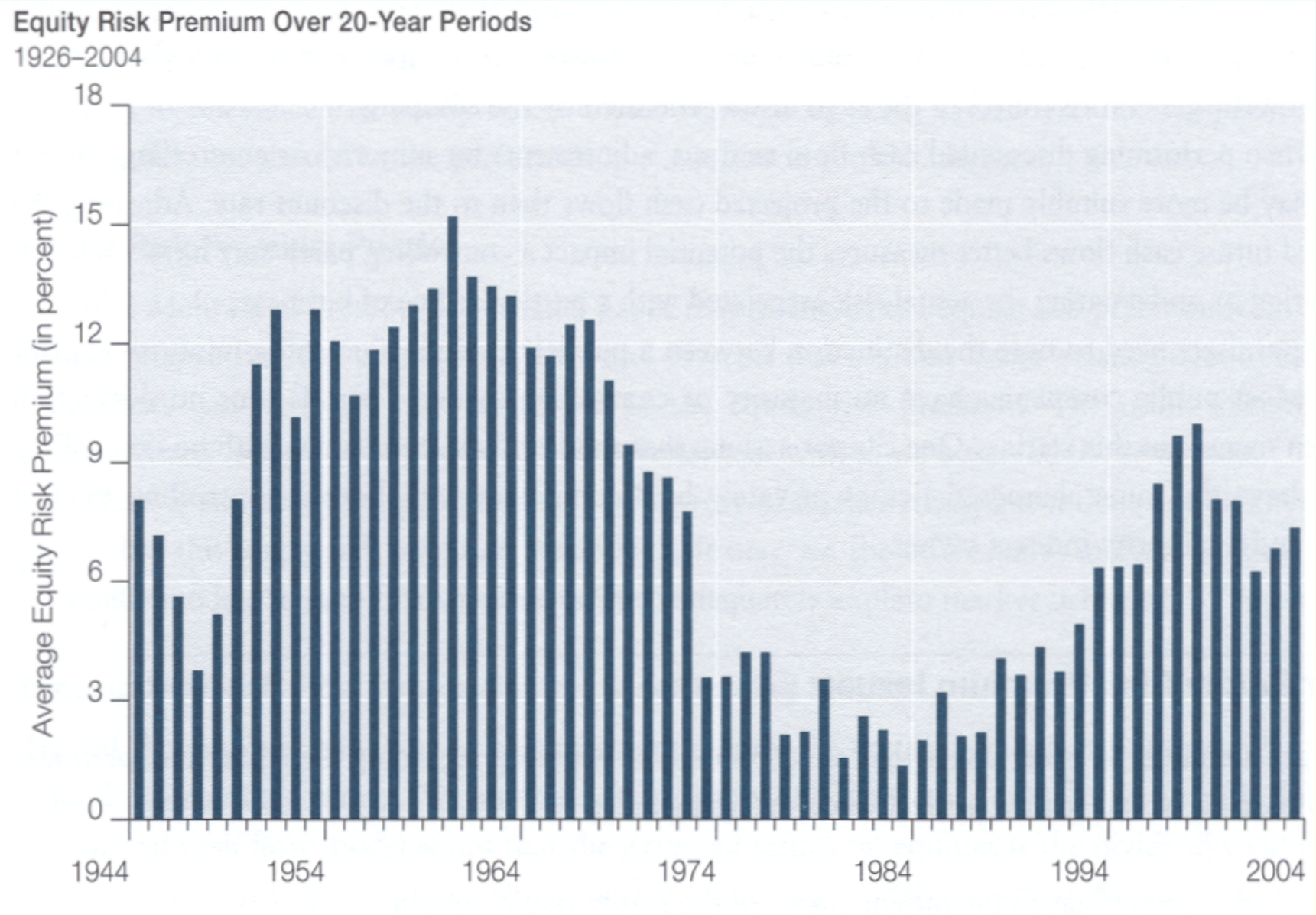
- Which of these interpretations of the equity premium is relevant for an investment advisor?
- Clearly this depends on the planning horizon.

- The equity premium that we documented in our 1985 paper is for long investment horizons.

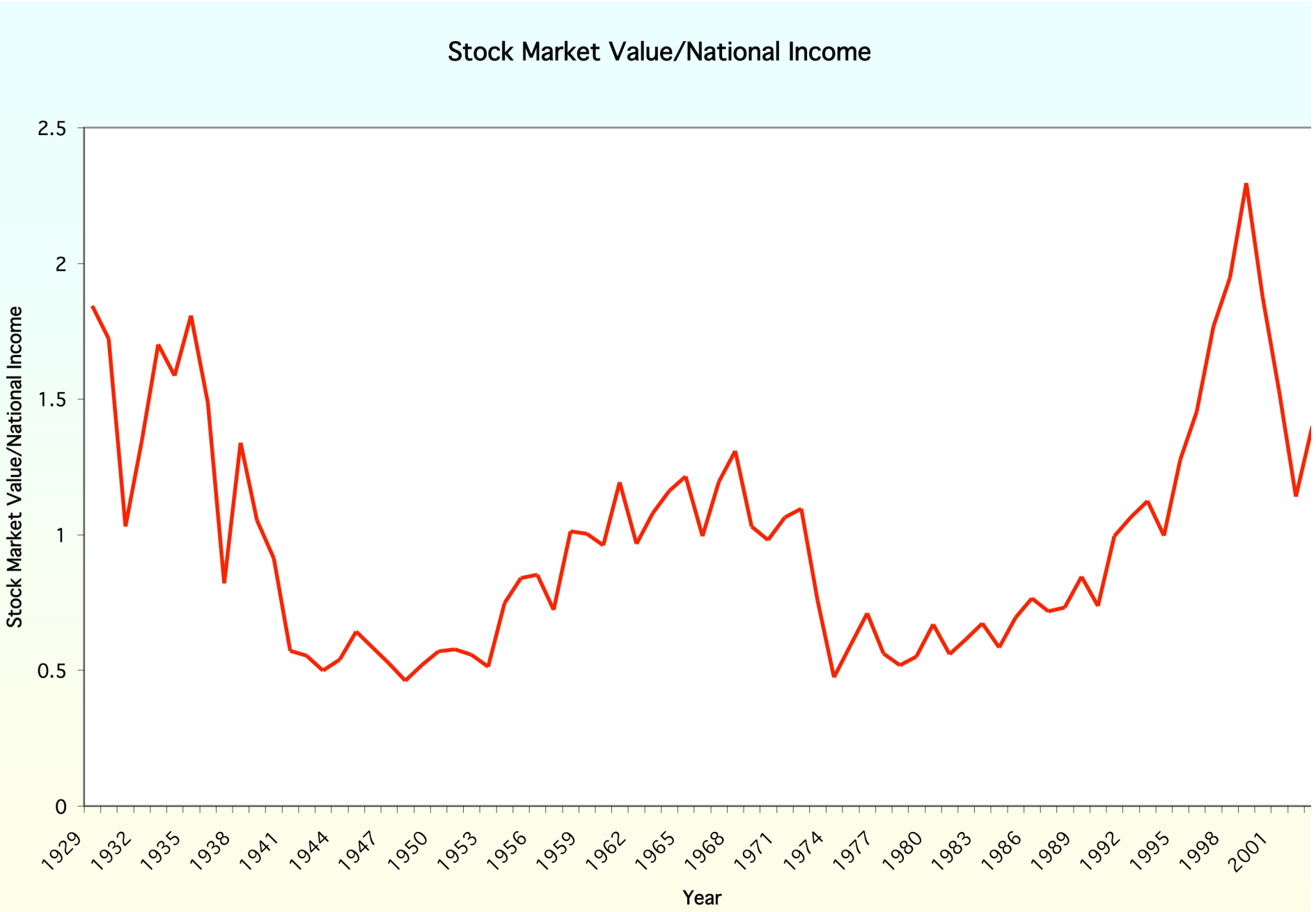
- The equity premium that we documented in our 1985 paper is for long investment horizons.
- It has little to do with what the premium is going to be over the next year.

- The equity premium that we documented in our 1985 paper is for long investment horizons.
- It has little to do with what the premium is going to be over the next year.
- The ex-post equity premium is the realization of a stochastic process over a certain period and it has varied considerably over time. Furthermore, the variation depends on the time horizon over which it is measured.

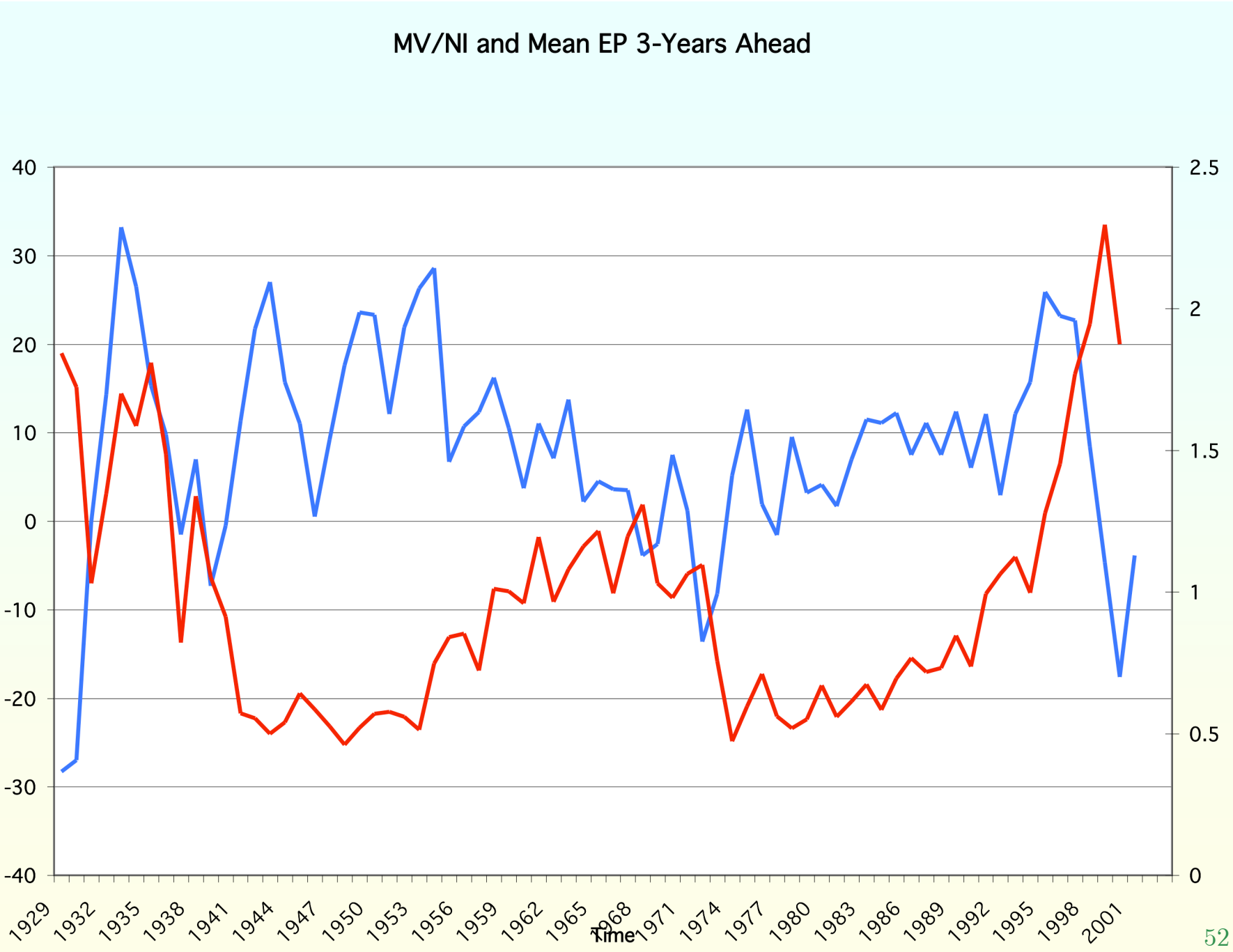

Realized Equity Risk Premium Per Year
1926–2004
Equity Risk Premium (in percent)
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
1925
1935
1945
1955
1965
1975
1985
1995
2004


Equity Risk Premium Over 20-Year Periods
1926–2004
Average Equity Risk Premium (in percent)
18
15
12
9
6
3
0
1944
1954
1964
1974
1984
1994
2004

- The low frequency variation has been *counter cyclical*.



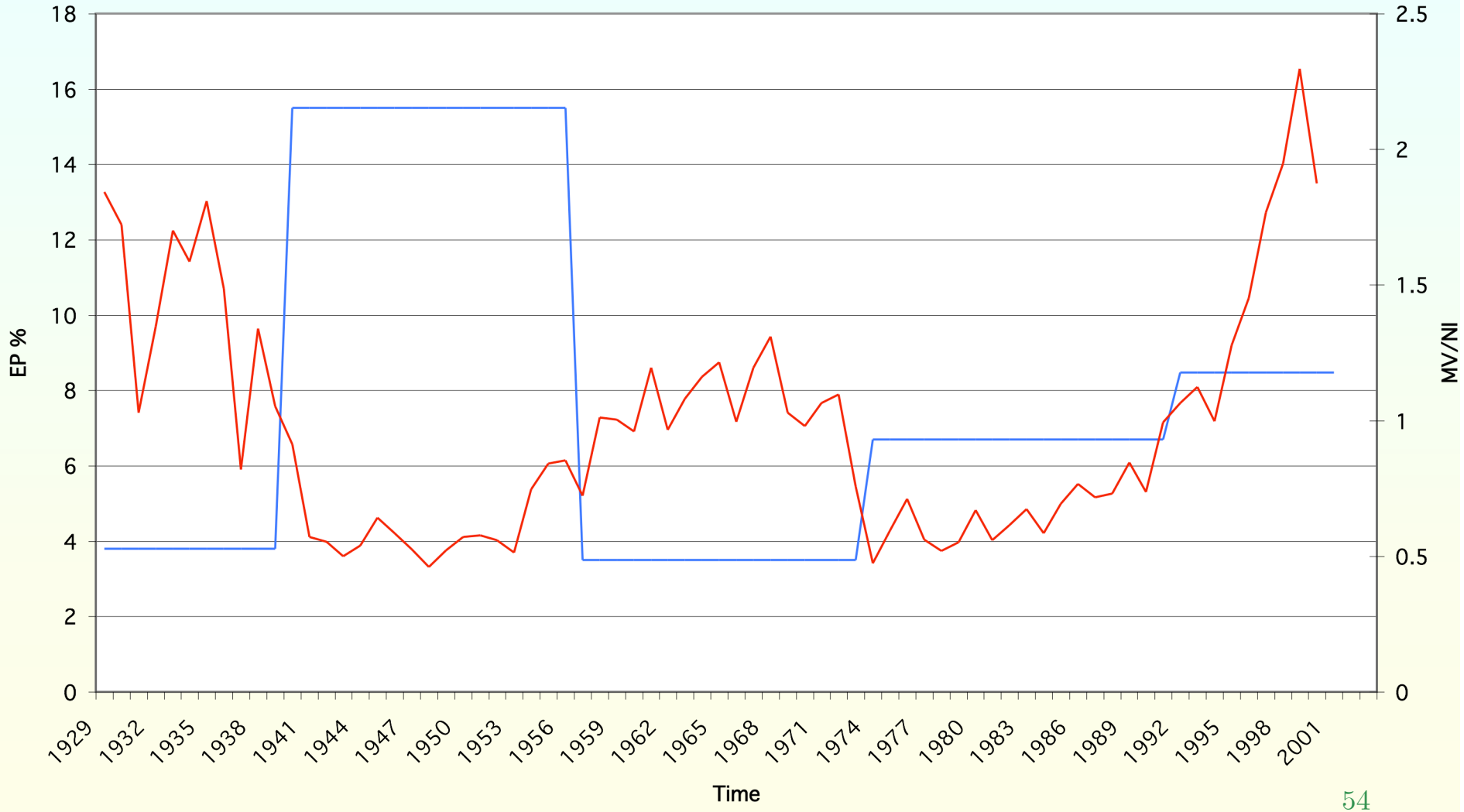
Source: Updated from R. Mehra "On the Volatility of Stock Prices: An Exercise in Quantitative Theory" International Journal of Systems Science (1998) Volume 29 No 11 pg 1203 -11


MV/NI and Mean EP 3-Years Ahead
40
30
20
10
0
-10
-20
-30
-40
2.5
2
1.5
1
0.5
0
1929
1932
1935
1938
1941
1944
1947
1950
1953
1956
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
2001
Time

- We have divided the time period from 1929 to 2000 into sub- periods where the ratio market value of equity to national income was greater than 1 and when it was less than 1.

- We have divided the time period from 1929 to 2000 into sub- periods where the ratio market value of equity to national income was greater than 1 and when it was less than 1.
- Historically, as the figure illustrates, subsequent to periods when this ratio was high the realized equity premium was low.

MV/NI and Mean EP 3 Year Ahead (Averaged over time periods when MV/NI>1 and MV/NI<1)



- Estimating the conditional equity premium is by no means a simple task. It is isomorphic to forecasting equity returns.

- Estimating the conditional equity premium is by no means a simple task. It is isomorphic to forecasting equity returns.
- Since returns have a standard deviation of 20% the "noise" dominates the drift.

- Estimating the conditional equity premium is by no means a simple task. It is isomorphic to forecasting equity returns.
- Since returns have a standard deviation of 20% the "noise" dominates the drift.
- Operationally how much information is there in knowing that the mean is 2% rather than 6% when the σ is 20%?

- Even if the conditional equity premium given current market conditions is small, and there appears to be general consensus that it is, this in itself does not imply that it was obvious that either the historical premium was too high or that the unconditional equity premium has diminished

How did the world look to an investor at the end of 1928 - before the Great Crash?

How did the world look to an investor at the end of 1928 - before the Great Crash?

- The mean real return on the S&P 500 for the period 1889 - 1928 was 8.52%

How did the world look to an investor at the end of 1928 - before the Great Crash?

- The mean real return on the S&P 500 for the period 1889 - 1928 was 8.52%
- The mean real return on risk free assets for the period 1889 - 1928 was 2.77%

How did the world look to an investor at the end of 1928 - before the Great Crash?

- The mean real return on the S&P 500 for the period 1889 - 1928 was 8.52%
- The mean real return on risk free assets for the period 1889 - 1928 was 2.77%
- The mean equity premium for this period was was 5.75%

How did the world look to an investor at the end of 1928 - before the Great Crash?

- The mean real return on the S&P 500 for the period 1889 - 1928 was 8.52%
- The mean real return on risk free assets for the period 1889 - 1928 was 2.77%
- The mean equity premium for this period was was 5.75%
- An analysis similar to Mehra and Prescott (1985) would have yielded an equity premium of 2.02%

- The data used to document the equity premium over the past hundred years is as good an economic data set as we have and a hundred years is long series when it comes to economic data.

- The data used to document the equity premium over the past hundred years is as good an economic data set as we have and a hundred years is long series when it comes to economic data.
- Before we dismiss the premium, not only do we need to understand the observed phenomena but we also need a plausible explanation as to why the future is likely to be any different from the past.