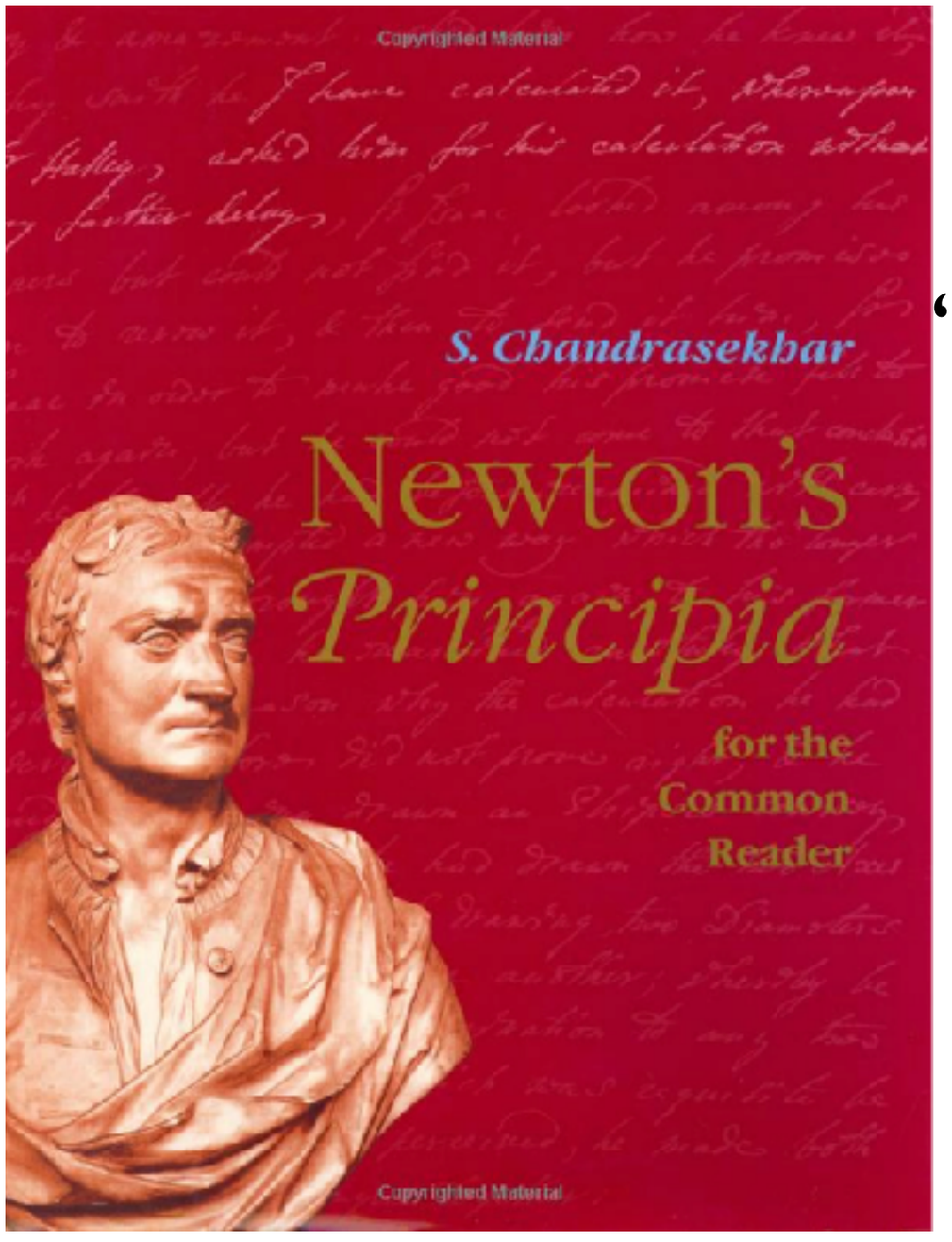
Visiting Newton's Atelier before the Principia
1679-1684

Michael Nauenberg
UC Santa Cruz

Oct 23, 2018

Pierre Simon Laplace
1729-1847

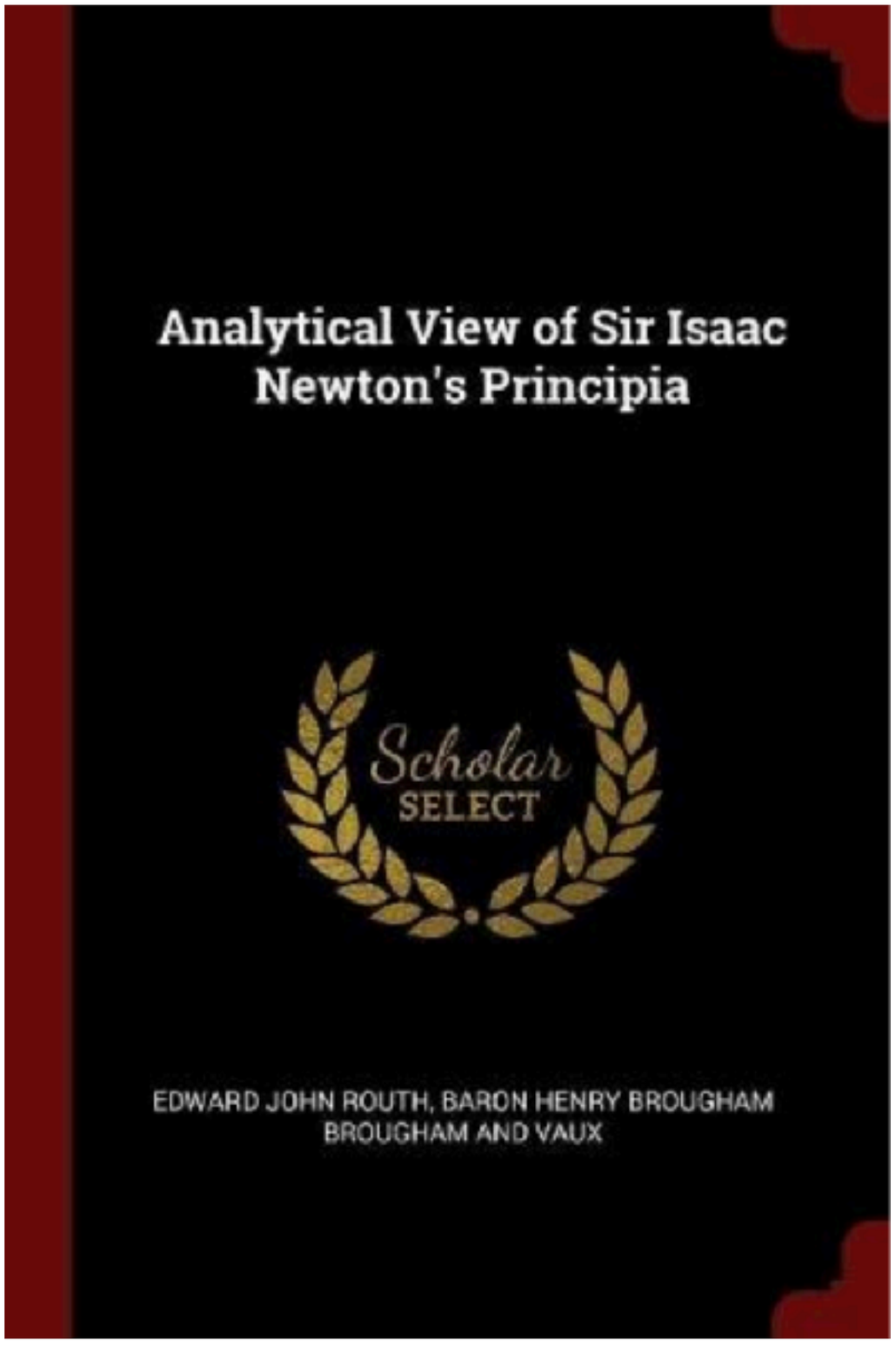
This admirable work contains the germs of all the great discoveries that have been made since, about the system of the world: the history of its development by the followers of that great geometer will be at the same time the most useful comment on his work, as well as the best guide to arrive at knew discoveries



Reading Newton became for Chandrasekhar a sustained epiphany:

"The view of science that he exhibit, the clarity with which he writes the number of new things that he finds, manifest a physical and mathematical insight of which there is no parallel in science at any time"

Scientific American,
March 1994



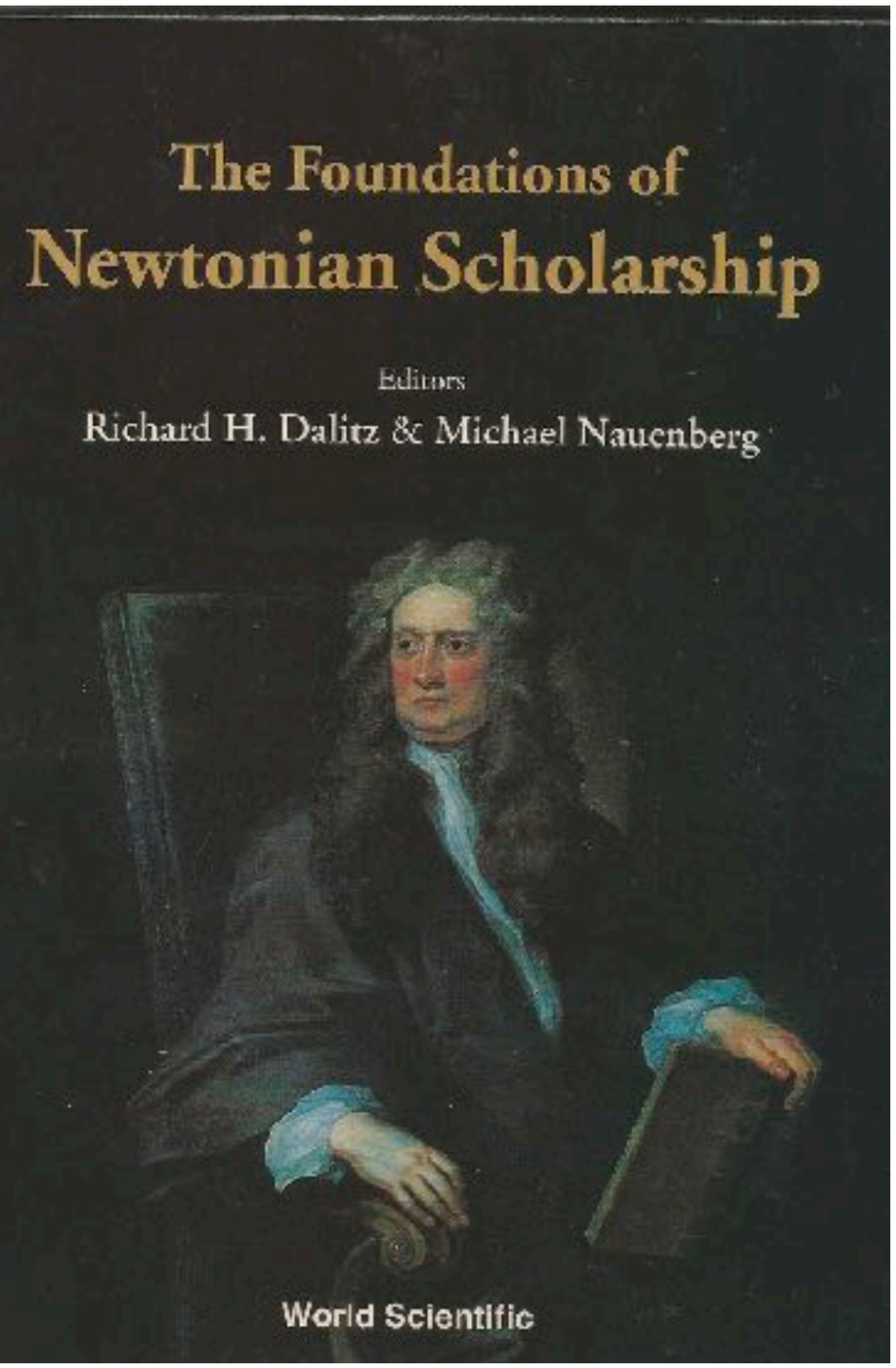
NEWTON'S PRINCIPIA.

ANALYTICAL VIEW.

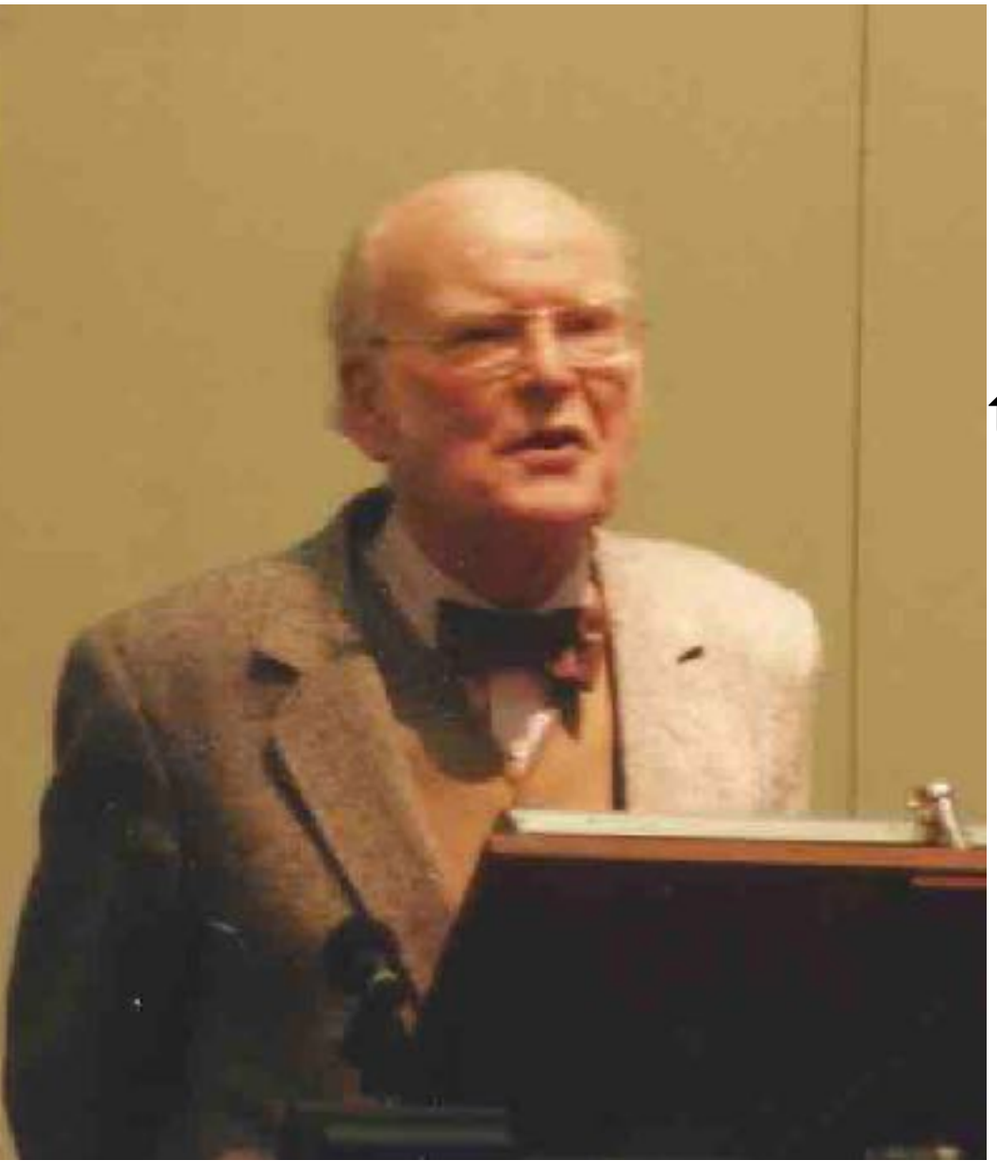
THIS work is justly considered by all men as the greatest of the monuments of human genius. It contains the exposition of the laws of motion in all its varieties, whether in free space or in resisting media, and of the action exerted by the masses or the particles of matter upon each other, those laws demonstrated by synthetic reasoning; and it unfolds the most magnificent discovery that was ever made by man — the Principle of Universal Gravitation, by which the system of the universe is governed under the superintendence of its Divine Maker.

E.J. Routh and H. Broughman 1898

Meeting on Newtonian Scholarship held at the London Royal Society in 1997



Back row (right to left): A.R. Hall, J.B. Brackenridge, A.E. Shapiro, J.G. Fauvel and M. Nauenberg.
Front row (right to left): D.T. Whiteside, I.B. Cohen, M. Boas Hall, A.H. Cook, P. Harman, R.H. Dalitz and G.I
(*Photo by David Fowler*).



"We now know that neither
Principia nor Opticks sprang
like Minerva from the head of Jove:
they are a palimpset of investigation
and tentative endeavors
we have been given glimpses
- more is hardly possible-
into the way Newton
created his sciences . . ."

Rupert Hall in "Review and Reminiscences",
The Foundations of Newtonian Scholarship, pg 201

In his introduction to Newton's Principia,
the eminent Newtonian scholar I. B. Cohen asked:

Whatever happened to the work-sheets
of the Principia? Do they still exist in some
obscure private or public collection?
Was this particular set of manuscripts - alone of all
the Newton papers - lost or mislaid,
either when the Portsmouth Collection was still in
Hurstbourne Castle or during the actual transfer
to the University Library in Cambridge?
Did such work-sheets still exist among
Newton's papers at the time of his death?
Or were they lost or destroyed - either
by chance or design
- during Newton's own lifetime?

We may possibly never be certain
of the answer to these questions.”

I. B. Cohen, in “Introduction to Newton's Principia"
II. (Cambridge University Press, Cambridge, 1971) p. 81.

Certainly there can be no doubt that the peculiar geometrical form in which the exposition of the Principia is dressed up bears no resemblance at all to the mental processes by which Newton actually arrived at his conclusions”

J. Maynard Keynes in “Newton the Man” 1946

Copy of Newton bust at the London Royal Societ
by Michael Rysbrack

ONE POUND

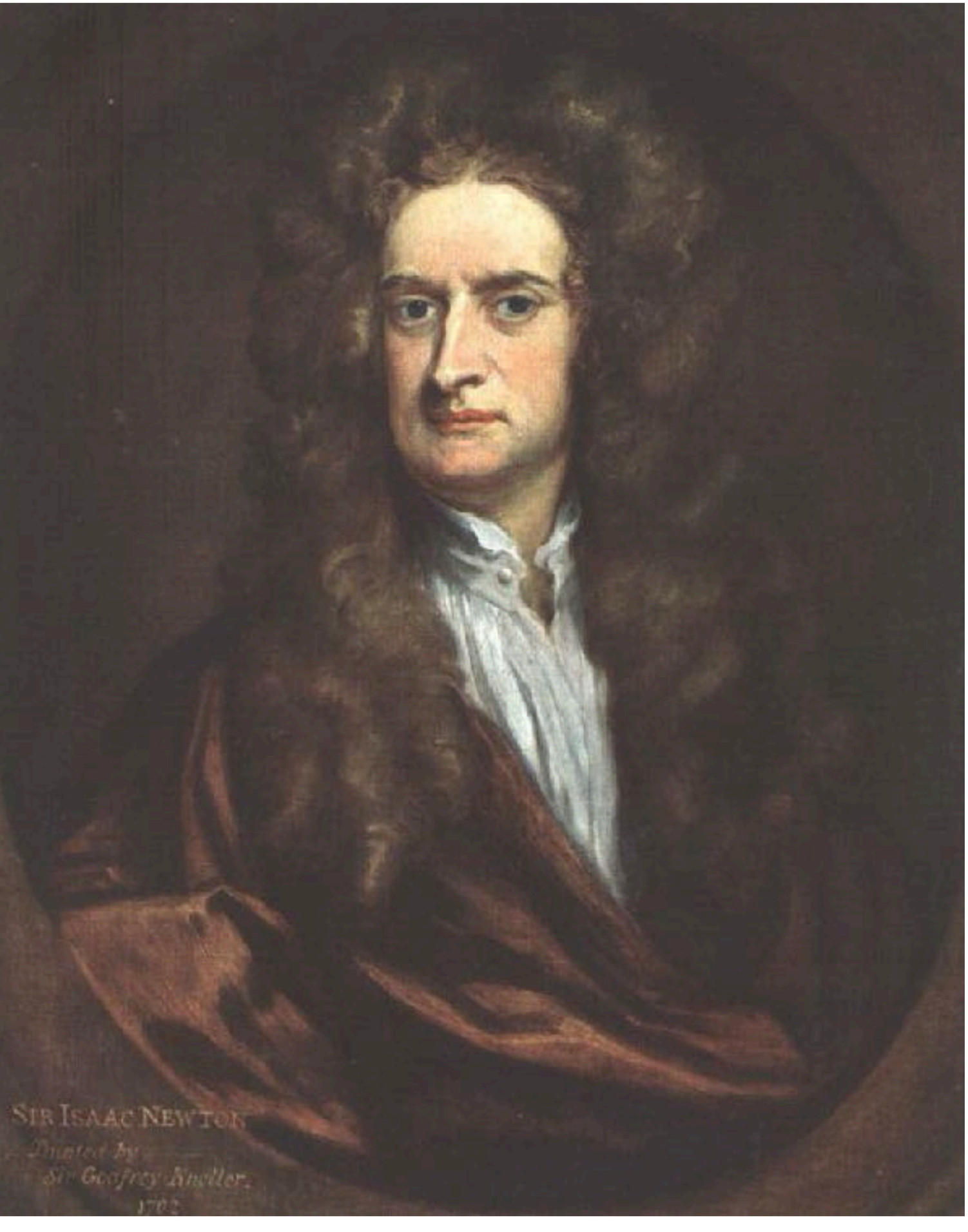
£1

Sir Isaac Newton
1642 · 1727

Diagram from Principia, Prop. 71
Newton's Superb Theorem

The apple story by John Conduitt

While he was musing in a garden it came into his thought that the power of gravity (which brought an apple from a tree to the ground) was not limited to a certain distance from the earth but that this power must extend to much farther than was usually thought. Why not as high as the moon said he to himself and if so that must influence her motion and perhaps retain her in her orbit



Newton
Portrait by Sir Godfrye Kneller
at the National Portrait Gallery, London

NATIONAL PORTRAIT GALLERY

Michael Nauenberg
Dept of Physics
University of Amsterdam
Valckemert Straat 65
Amsterdam
THE NETHERLANDS

22 April 1996

Dear Mr Nauenberg,

Thank you for filling out a visitor comment form on 24 March 1996.

The caption about Kneller's portrait of Sir Isaac Newton was written by my predecessor, who has now left the Gallery, and so I can only presume what his intentions were in writing it. However I am sure that he did not intend to imply that Newton did not develop his law of gravity through a scientific process. Indeed, it is not certain that the incident with the apple ever happened, and I think that in writing that the laws of gravity were '*traditionally said to have been* revealed to him' when he saw an apple falling from a tree, my predecessor was simply intending to record a tradition that grew up about Newton's discovery after his death. On the other hand, Newton was, of course, a religious man as well as a scientist, and it may well be that he believed that there was a degree of religious revelation involved in stimulating his discoveries.

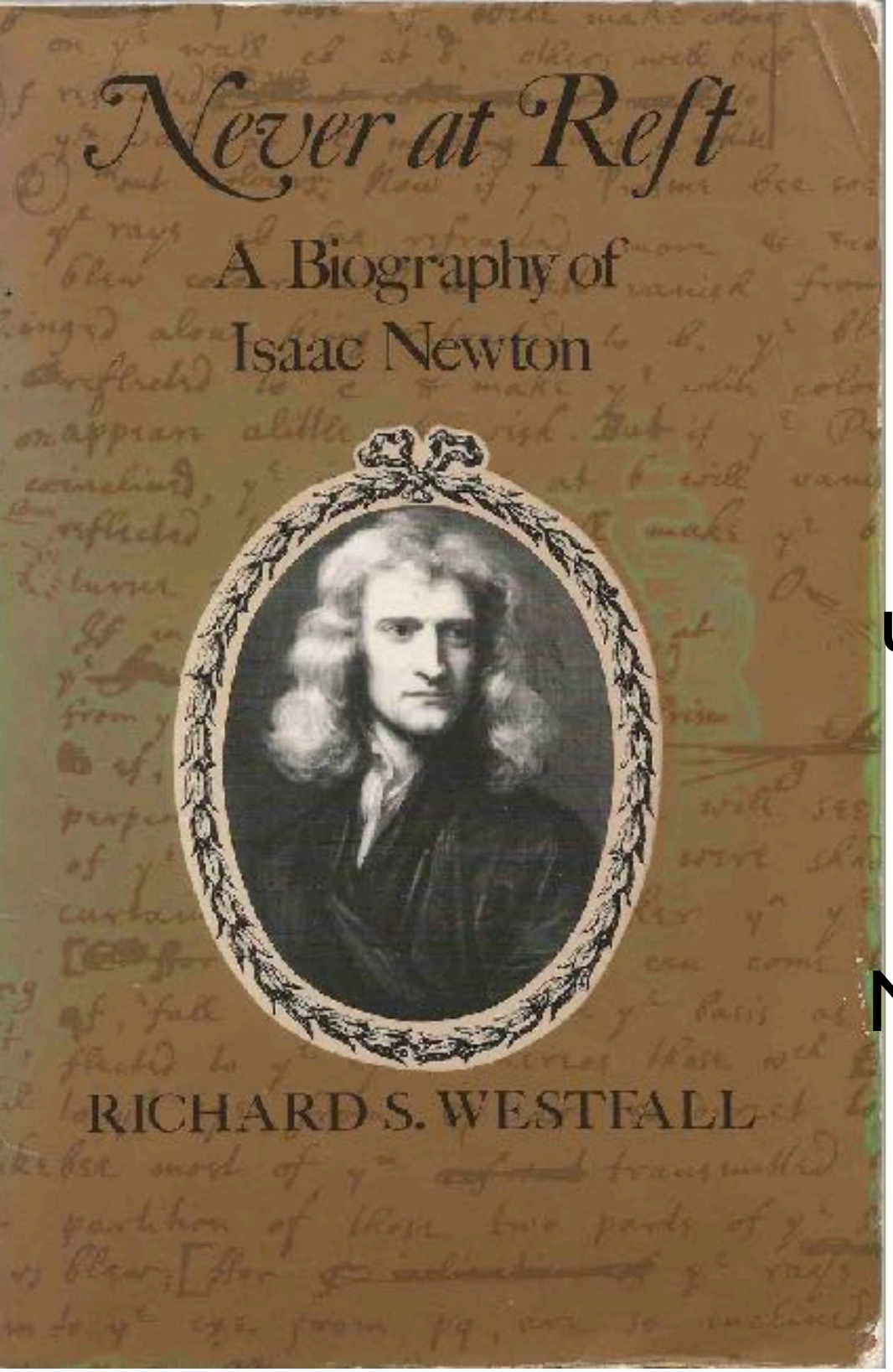
However, I agree that the caption is, perhaps, misleading in making Newton seem so passive in relation to his development of the law of gravity. I have amended the text, and I enclose a copy of the new caption. I hope that this meets with your approval.

Thank you for your interest in the Gallery. I hope that in spite of any annoyance caused by the Newton caption you enjoyed your visit.

Yours sincerely,

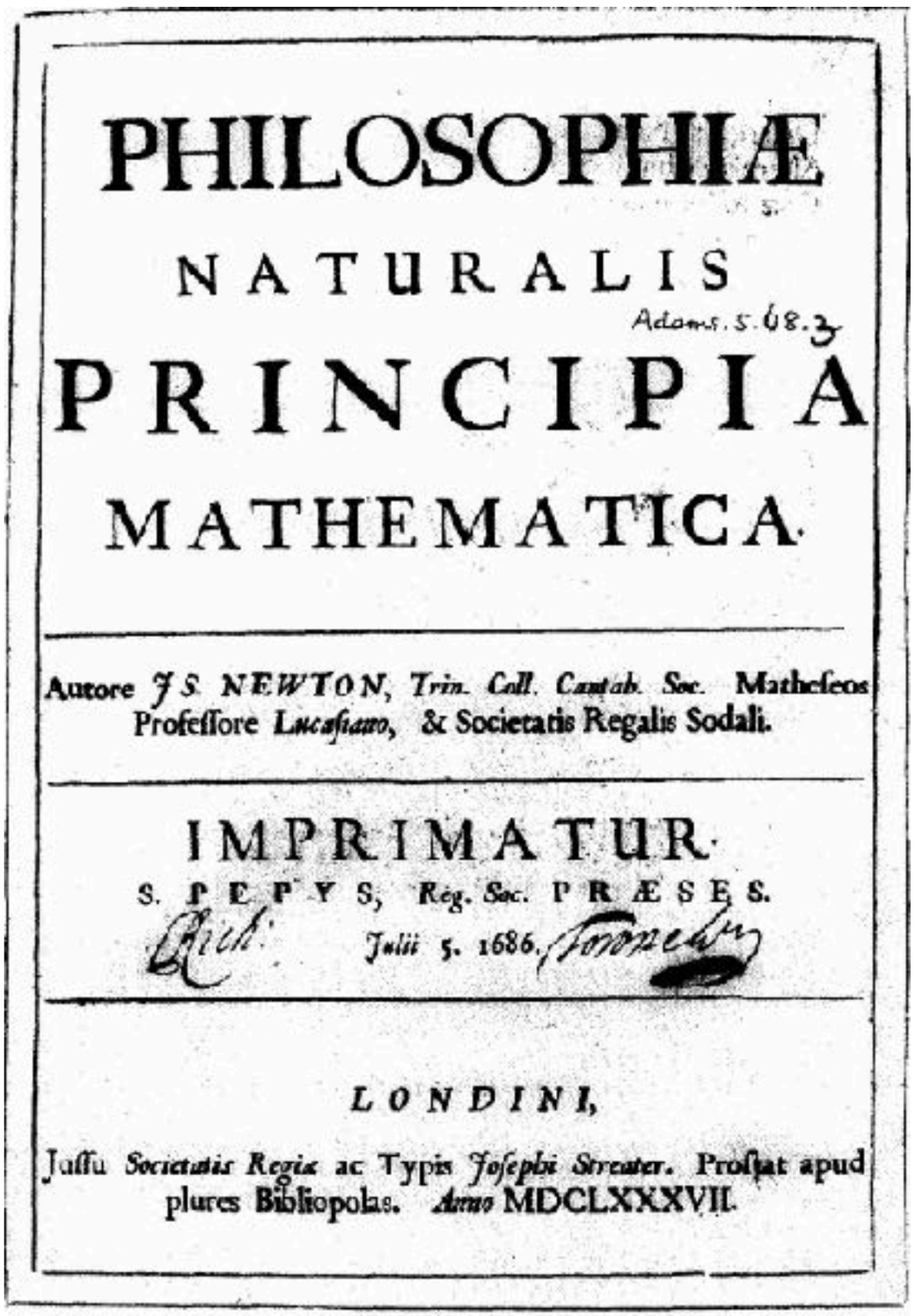
Catherine MacLeod

Catharine MacLeod



"…to avoid being baited by little Smatterers in Mathematics. . . he designedly made his Principia abstruse; but yet so as to be understood by able Mathematicians"

Newton, as told to William Derham
Keynes MS. 133, pg 10



PHILOSOPHIÆ
NATURALIS
Adams. 5. 68. 3
PRINCIPIA
MATHEMATICA.

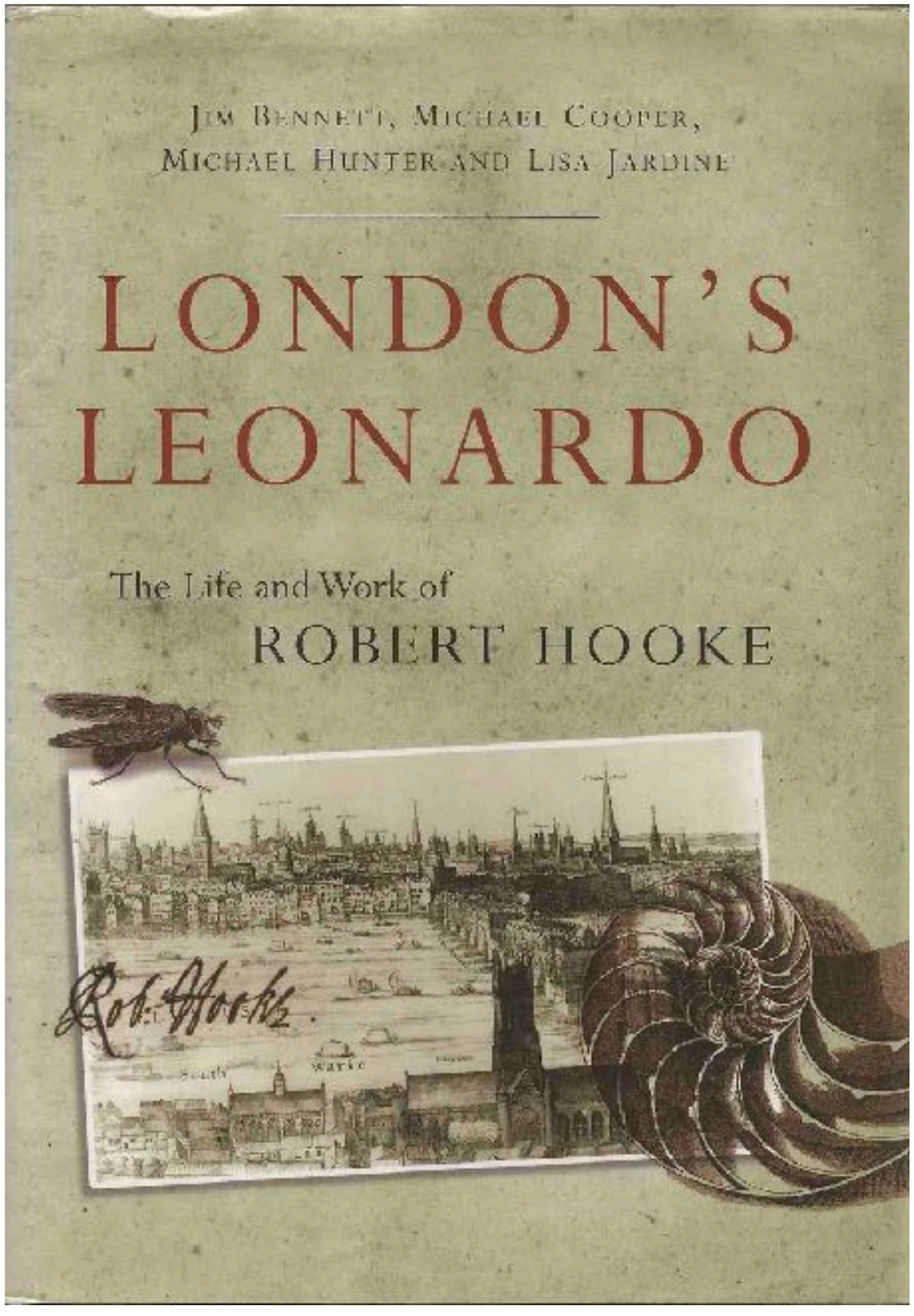
Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

"When Newton's Principia first appeared only the most advanced mathematicians were able to fathom its depths . . .the work acquiring a reputation as an impenetrable treatise presenting almost divine revelations about Nature."

S.D. Snobelen, "On reading Isaac Newton's Principia in the 18th century"
Endeavour Vol. 22(4) 1998



On November 24, 1679, Robert Hooke wrote to Newton :

"For my own part I shall take it as a great favour if you please to communicate by Letter your objections against any hypothesis or opinion of mine,

And particularly if you will let me know your thoughts of that of compounding the celestiall motion of the planetts of a direct motion by the tangents and an attractive motion toward the central body"

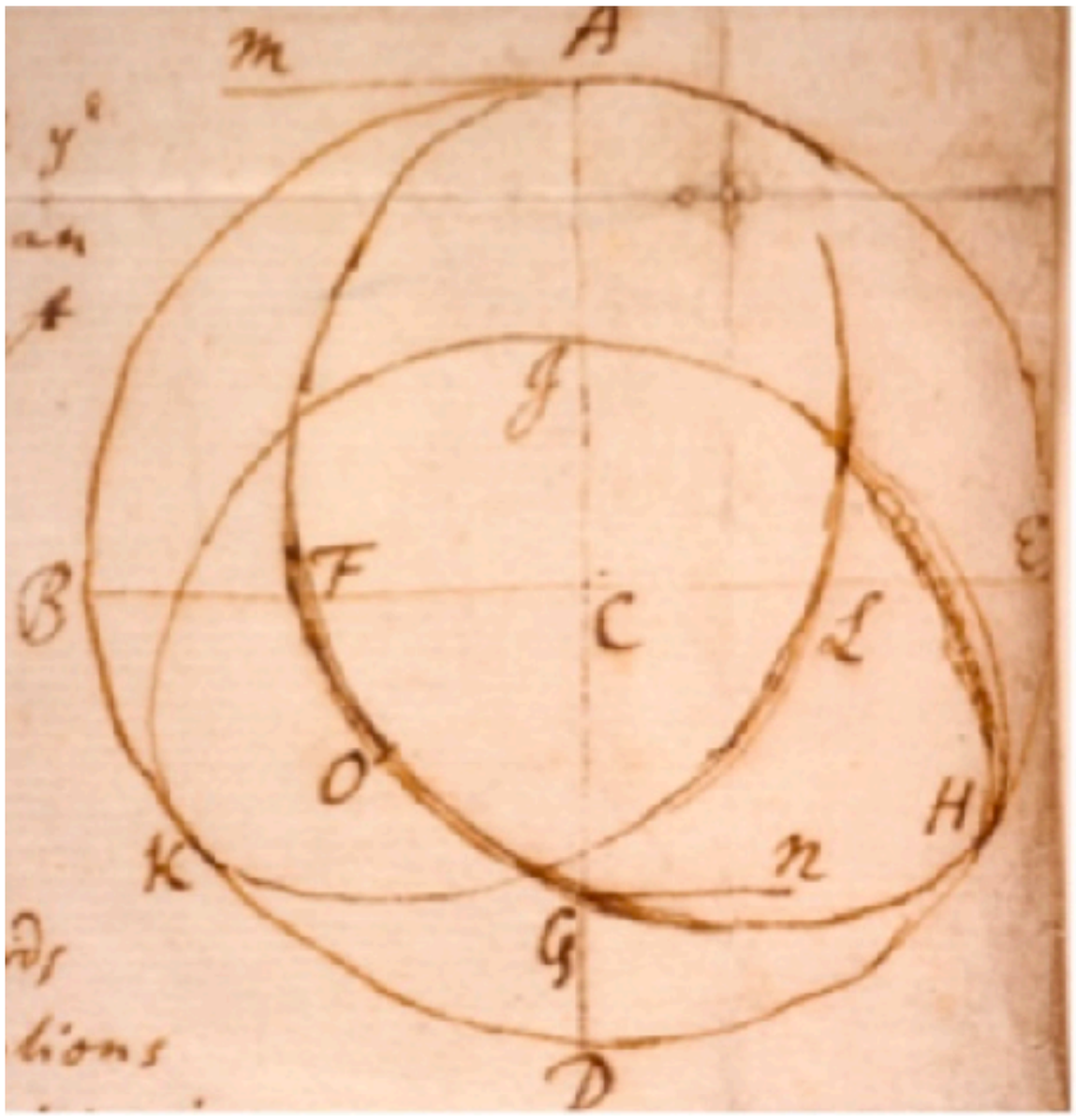
Hooke had elaborated his ideas in a short tract, published in 1674, entitled:

“An attempt to prove the motion of the Earth
by observations".

Hooke argued that attractive gravitational forces were universal. About terrestrial gravitation he wrote:

This propagated Pulse I take it to be the Cause of the descent of bodies towards the Earth

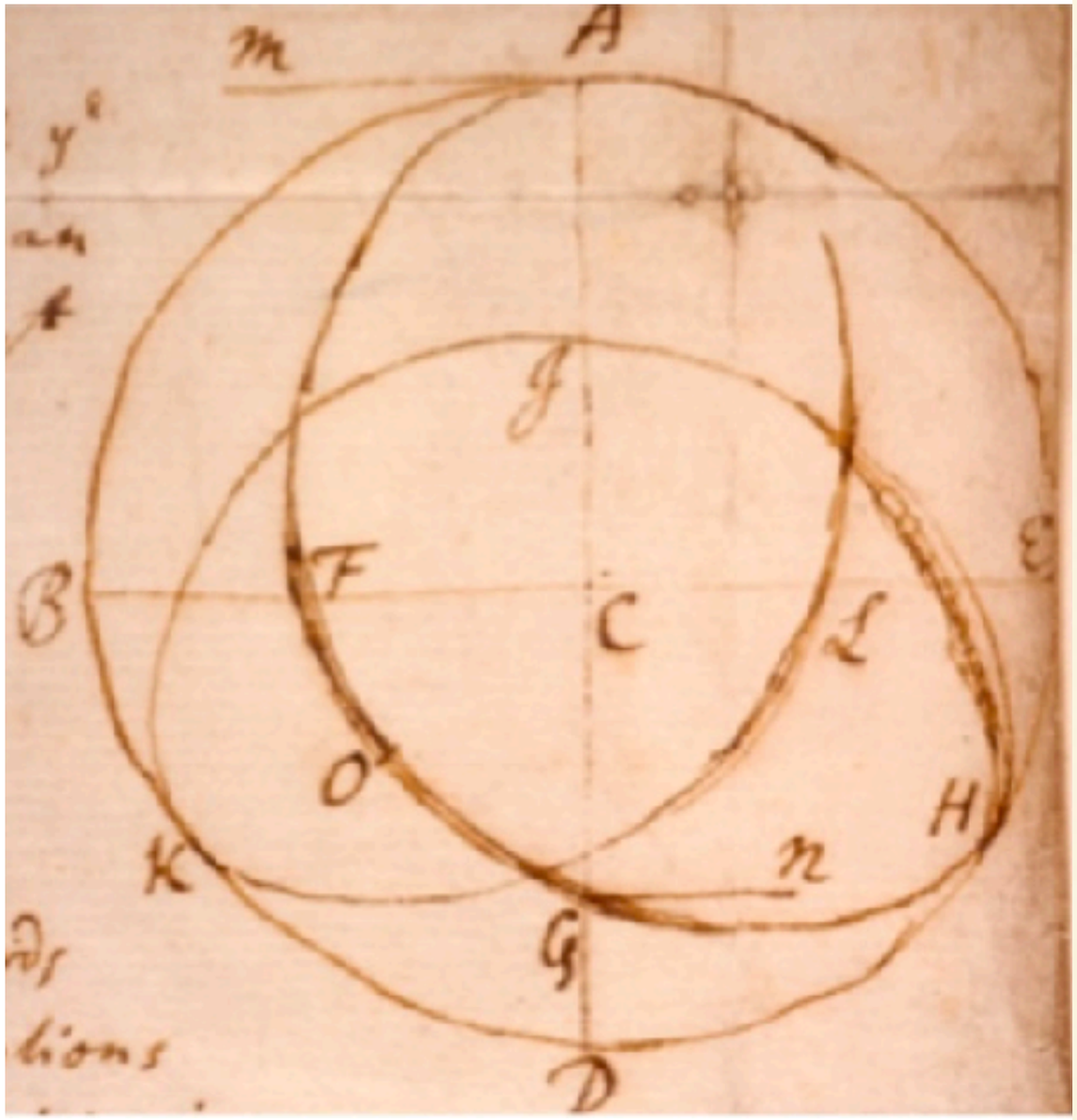
. . . Suppose for Instance there should be 1000 of these Pulses in a second of Time, then must the Grave body receive all those thousand impressions within the space of time of that Second, and a thousand more the next . . .".



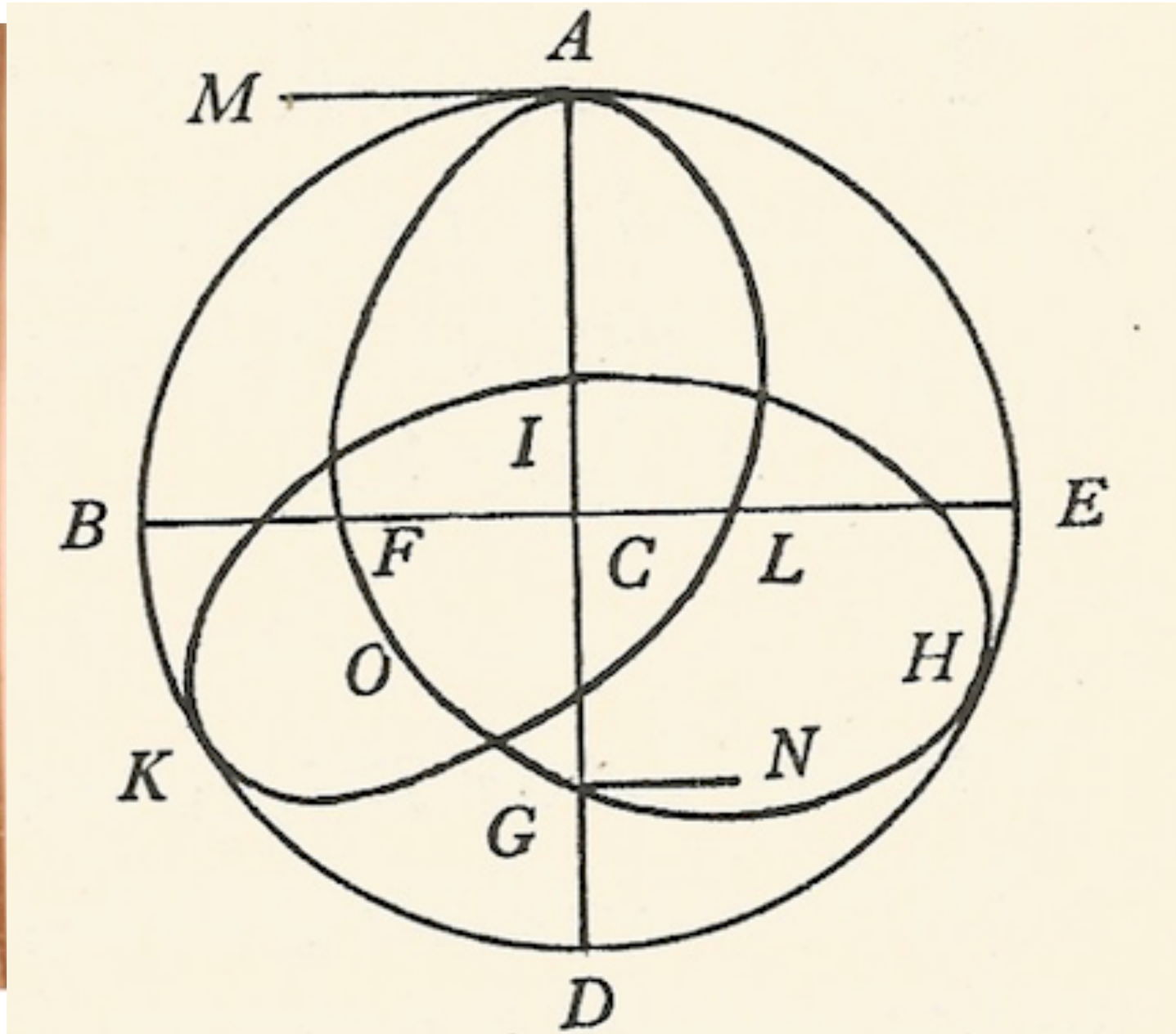
Newton's response to Hooke
on Dec 24, 1679:
"Your accute letter having put
me upon considering thus
far the species of this curve,
I might add something about
its description by points
quam proxime . . .

Newton's letter to Halley
on May 27, 1686
"I then took the simplest case
for computation, which was that
of Gravity uniform in a medium
not Resisting"

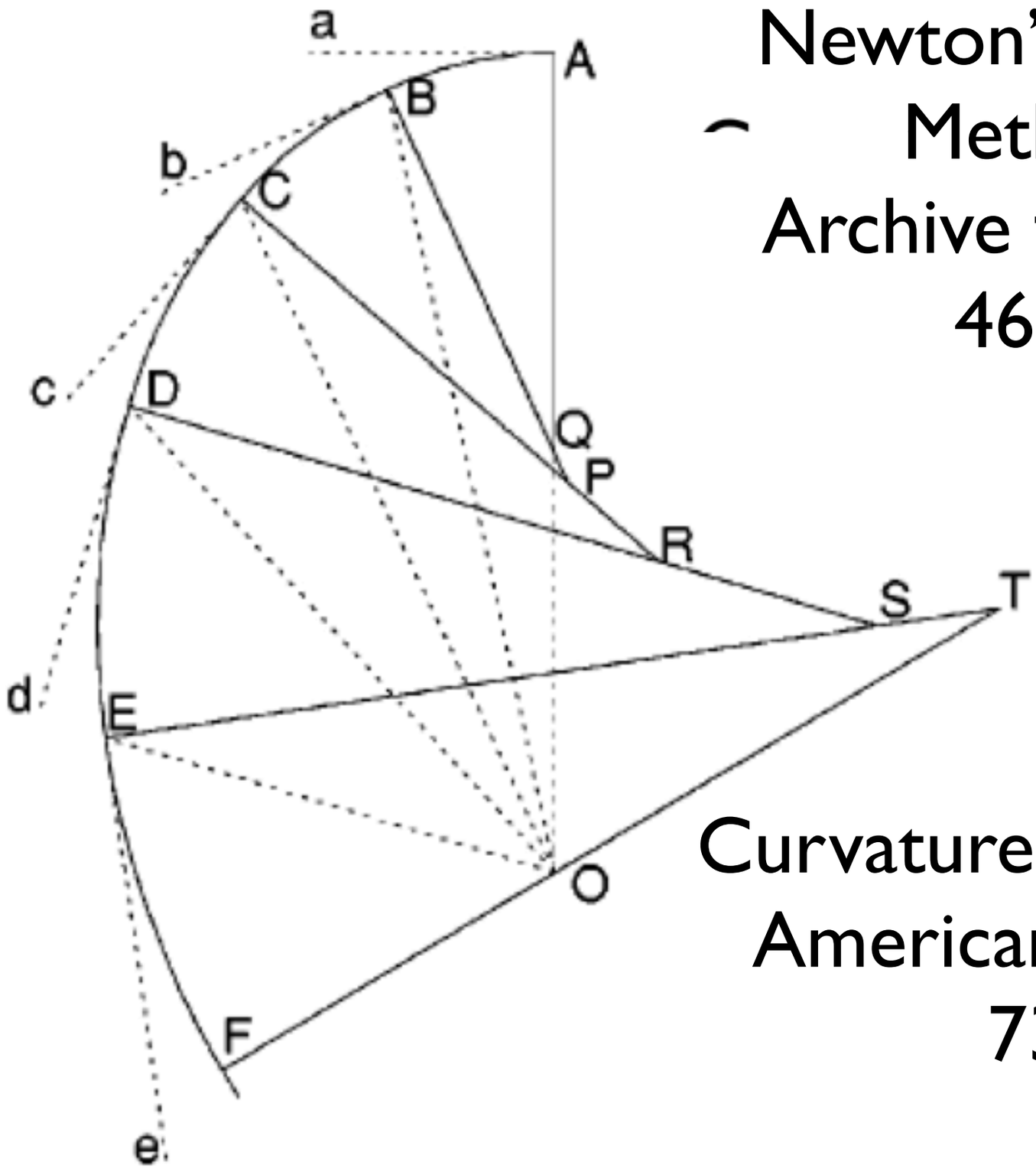
Newton's drawing



Historian's version



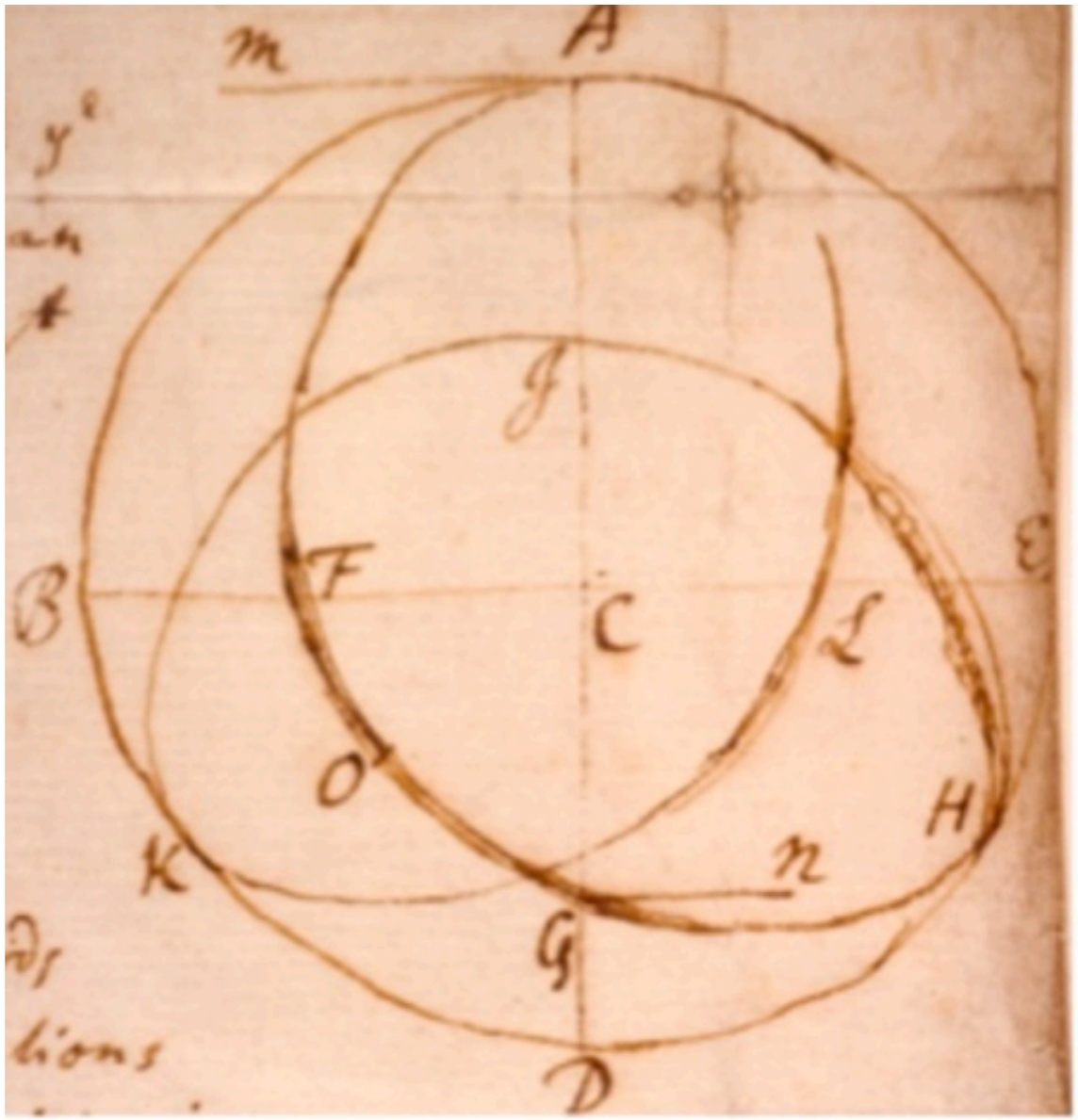
Newton's Graphical Method based on local curvature



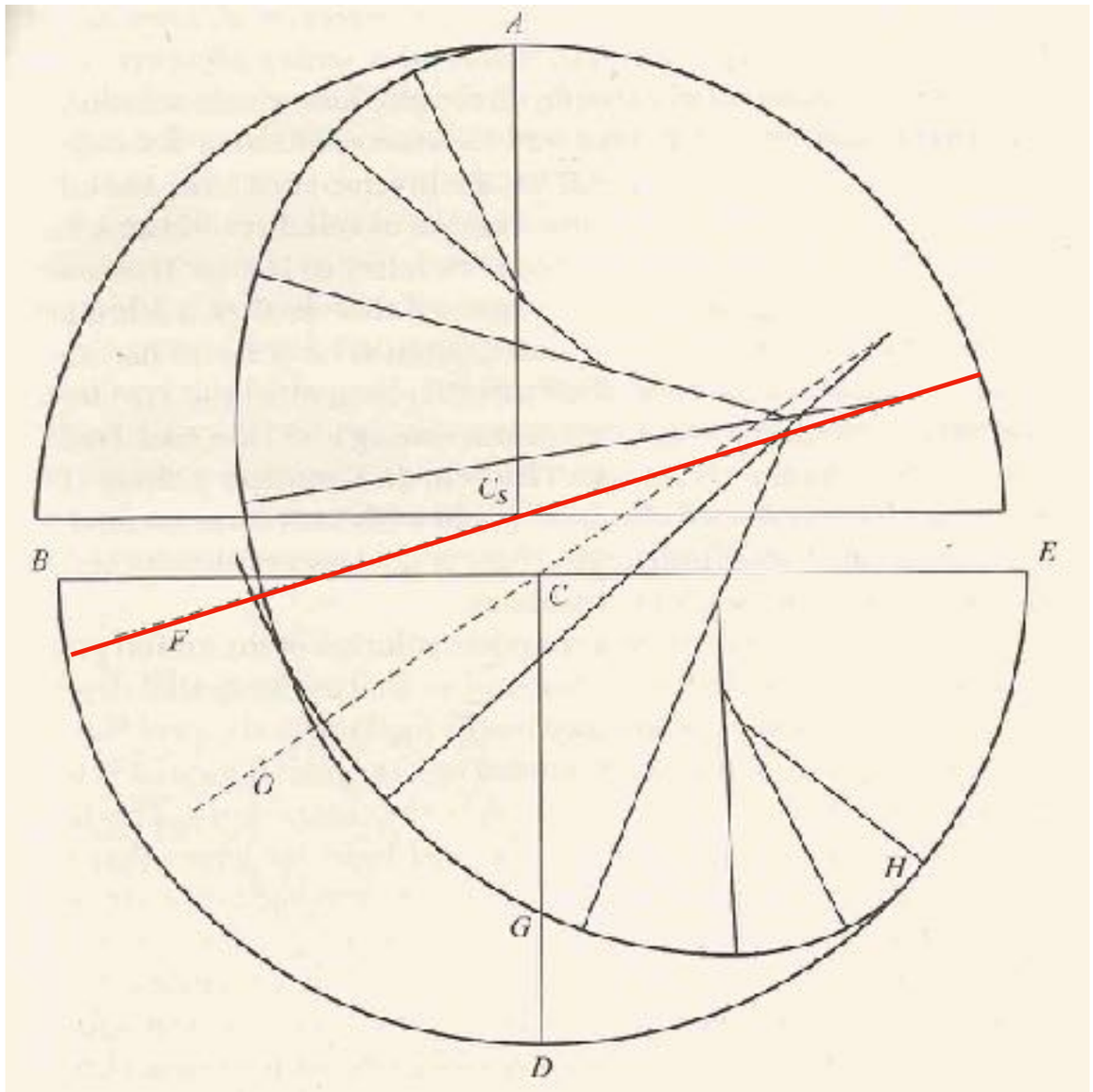
Newton's Early Computational Method for Dynamics
Archive for History of Science
46 (1994) 212-221

Curvature in Orbital Dynamics
American Journal of Physics
73(2005) 340

Newton's 1679 diagram



Section of Newton's diagram based on local curvature



This letter contains among other mistakes
an impossible picture of an orbit …
V. I. Arnold in "Huygens & Barrow, Newton & Hooke

According to several accounts originating with Newton, on August 1684 Halley visited him and asked:

" what he thought the curve would be that would be described by planets supposing the force of attraction towards the Sun to be reciprocal to the square of the distance from it".

"Sir Isaac replied immediately that it would be an ellipses . . .” but when asked for his calculation he claimed that he couldn't find it.

Actually Newton could only answer the converse to Halleys question, if the curve is an ellipse, the central force is an inverse square force. It is unlikely that he could have answered Halley's original question

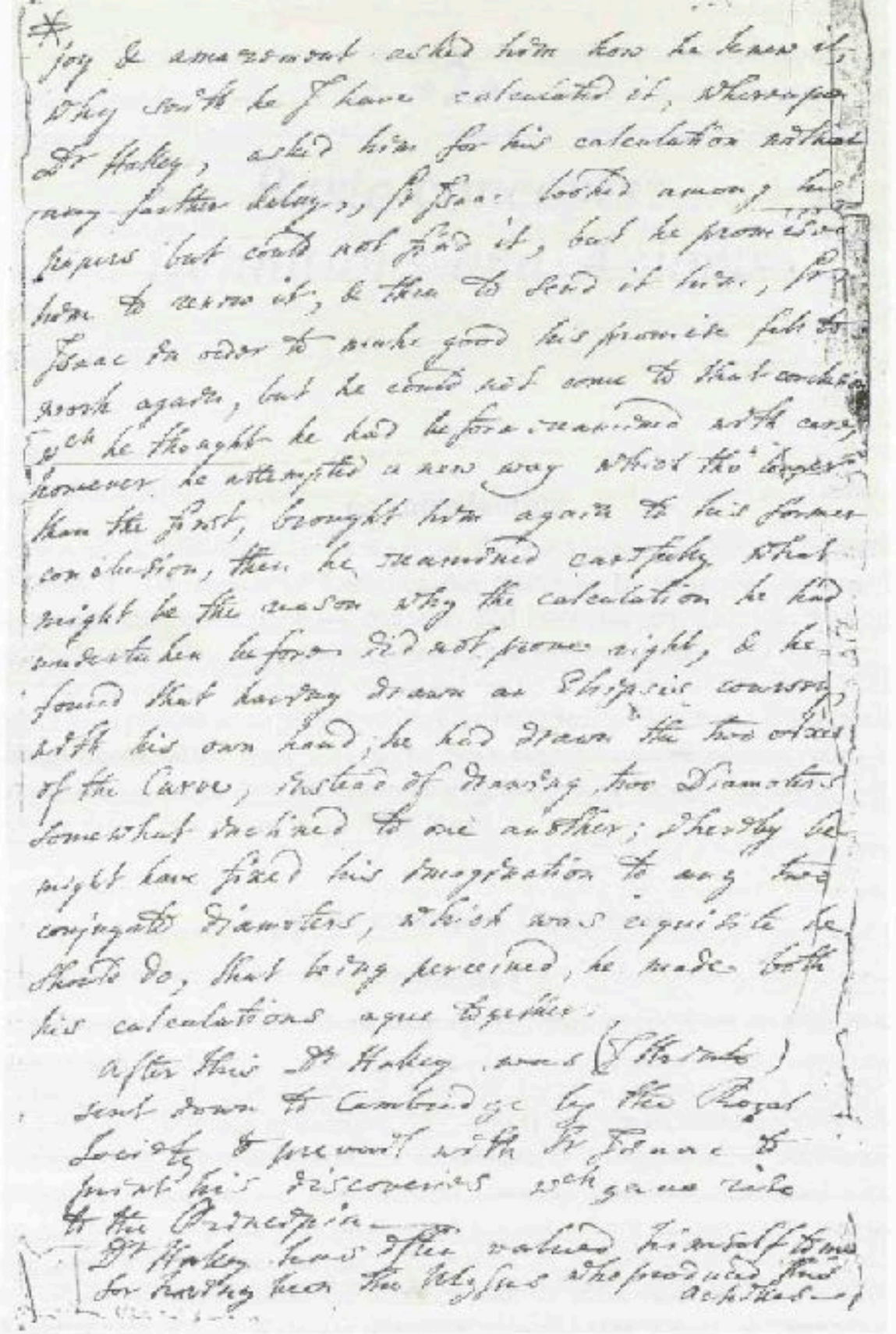
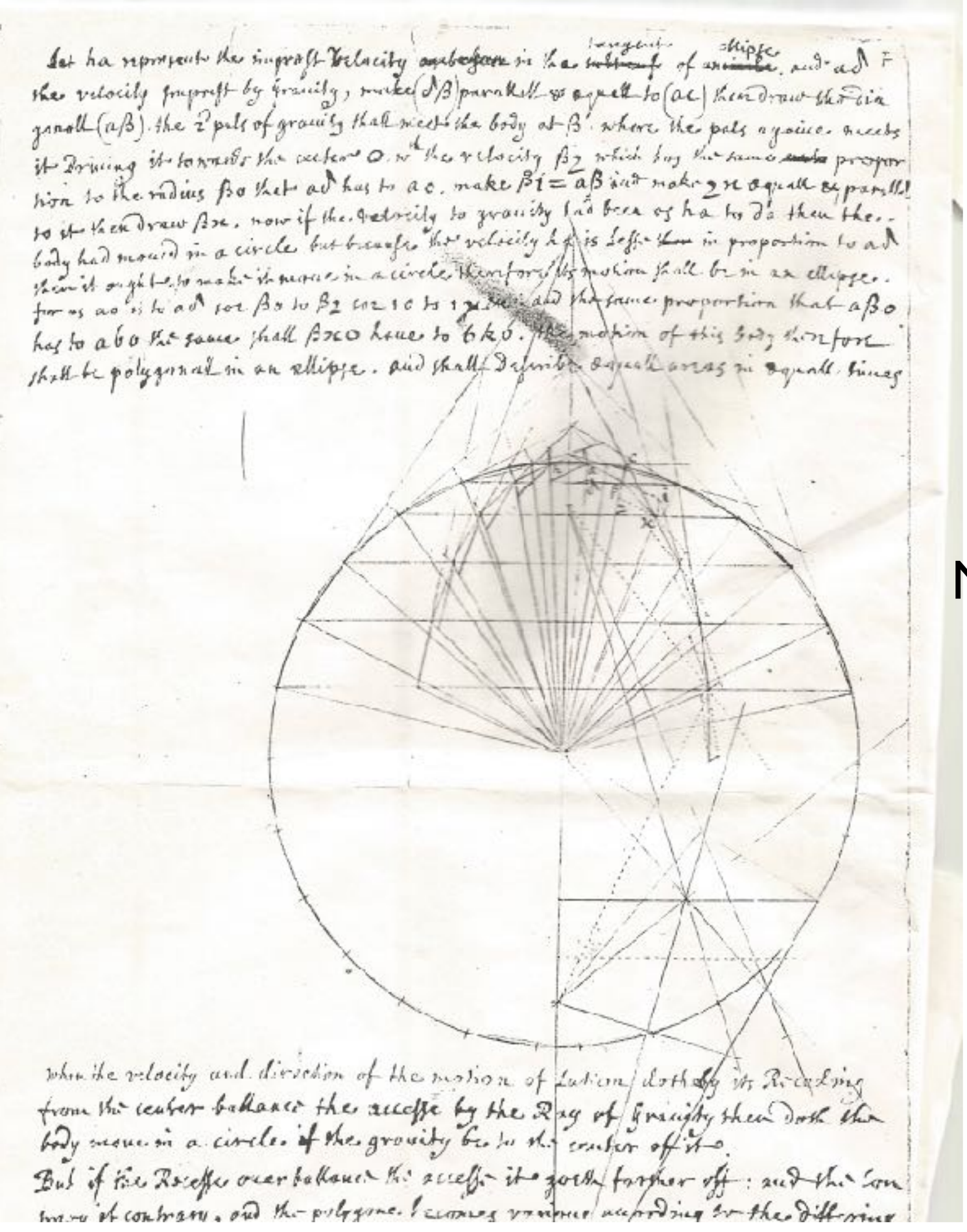


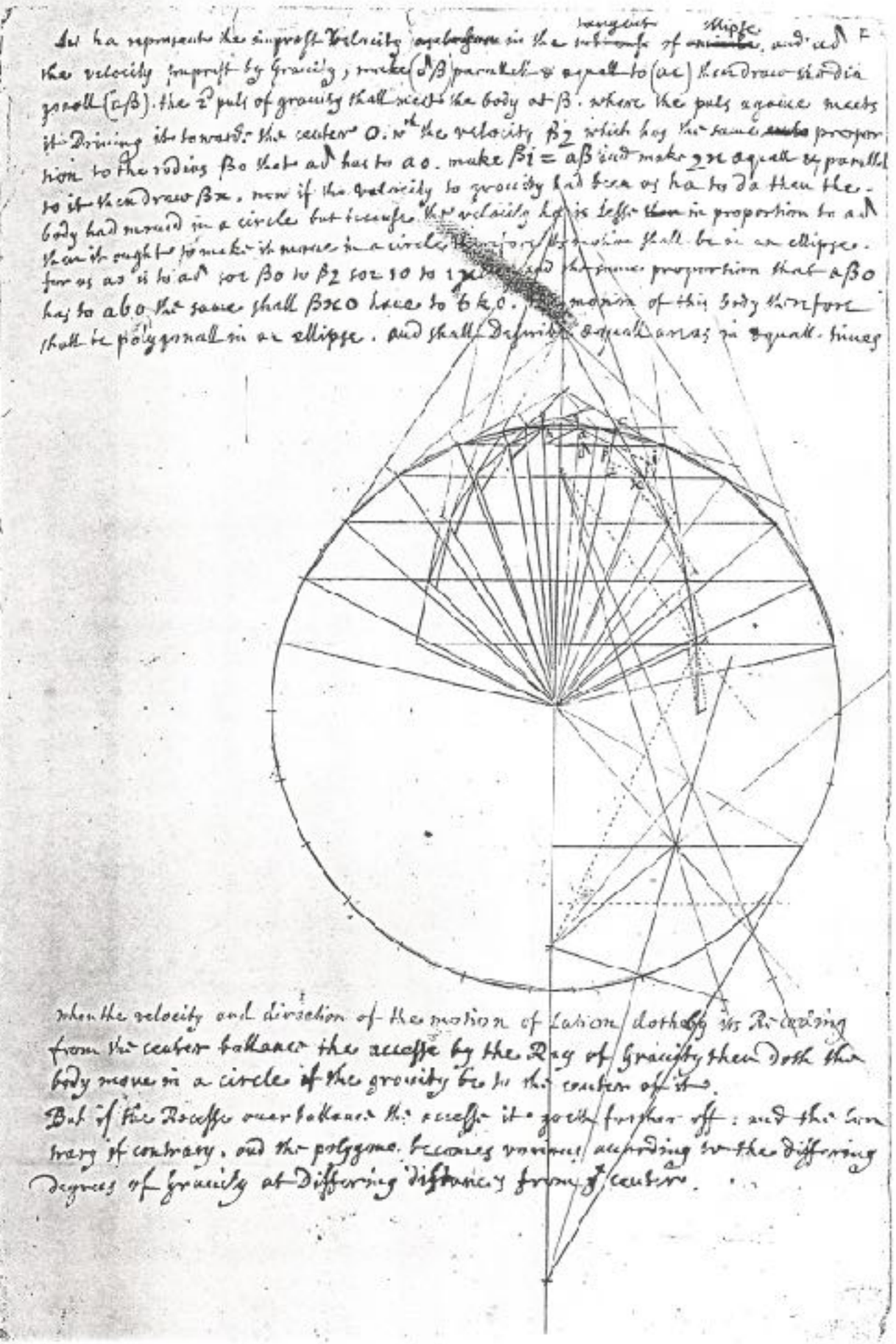
Plate 1 Copy of the original De Moivre memorandum, in the Joseph Halle Schaffner Collection of Scientific Manuscripts, Department of Special Collections of the University of Chicago Library; and published here with the permission of the Library.



Hooke graphical calculation of the orbit for a force that depends linearly on the distance from the center

Newton claimed that "Dr. Hook could not perform that which he pretented to: let him give Demonstrations of it: I know he hath not Geometry enough to do it."

Letter of William Derham to Conduitt Esquire, July 18, 1733



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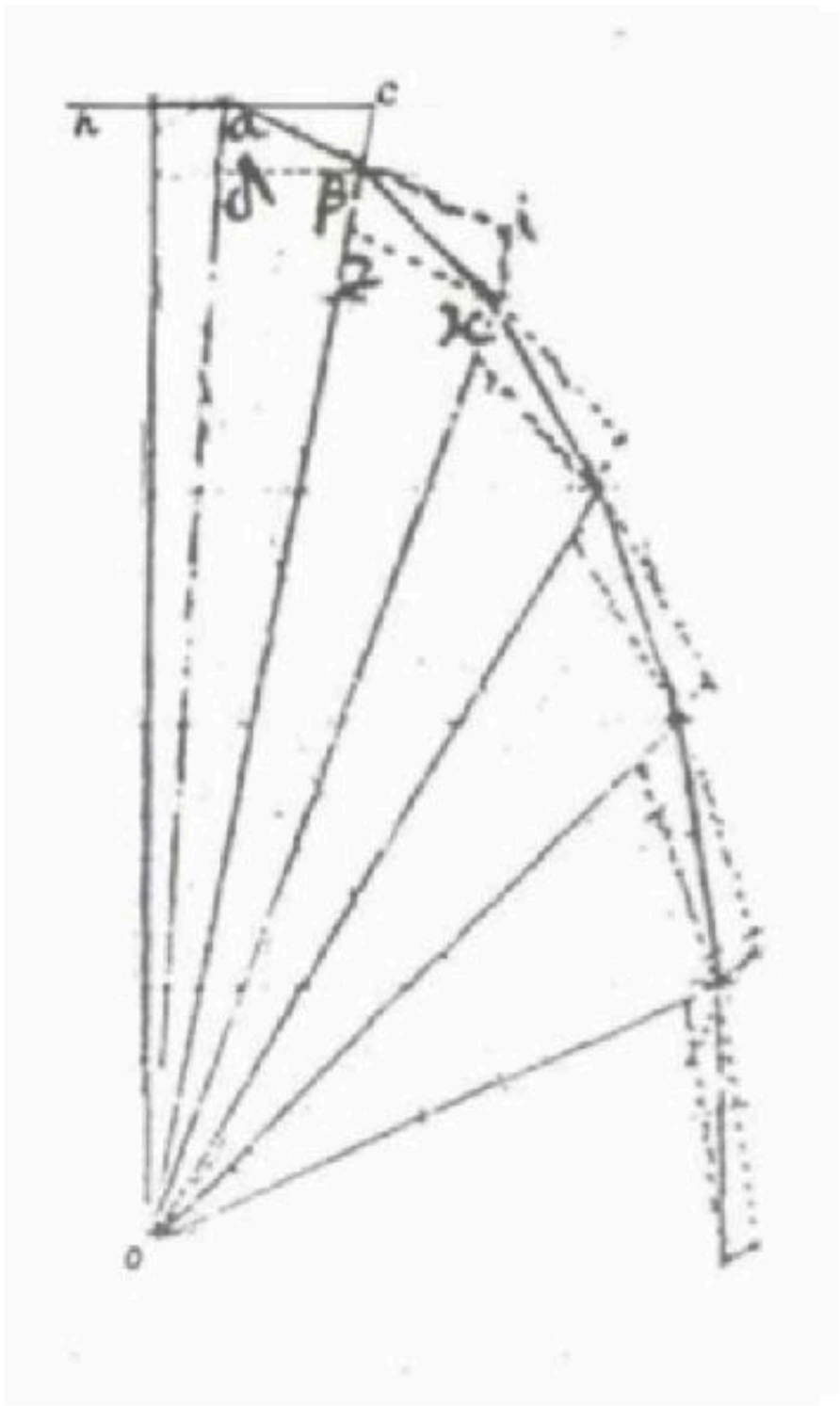
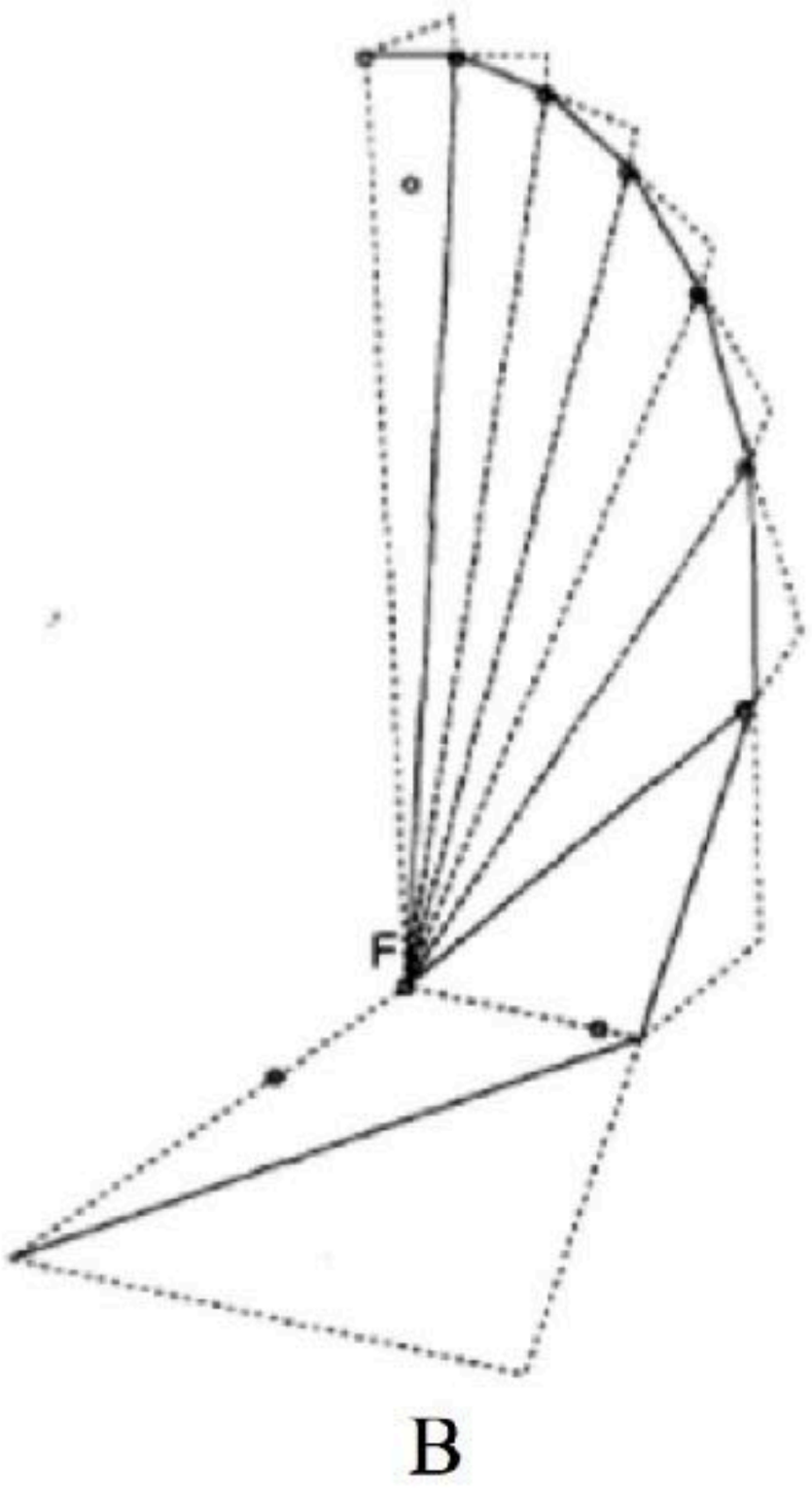


Hooke's graphical construction of an elliptic orbit for central force depending linearly on the distance from the center.

Drawn on Sept. 1685.



Graphical construction with inverse square force and Hooke's initial condition

In July 14, 1686, in a letter to Halley Newton admitted is indebtedness to Hooke. He wrote:

"This is true, that his Letters occasioned my finding the method of determining Figures which when I tried it in the Ellipsis "

But then Newton equivocated claiming that:

"I threw the calculation by being upon other studies & so it rested for about 5 years till upon your request I sought for yt paper, & not finding it did it again and reduced it into ye Proposition shown you for Mr. Paget . . ."

In November, 1684, Newton sent to the Royal Society a treatise of 9 pages "On the Motion of Bodies in an Orbit", that constituted the first draft of the Principia

During his conflict with Leibniz on the development of the Calculus, Newton wrote

"In the end of the year 1679 in answer to a letter from Dr.Hook then secretary of the R.S. . . I wrote that Whereupon I computed what would be the Orb described by the Planets, for I had found before by the sesquialterate proportion of the tempora periodica of the Planets with respect to their distances from the Sun, that the forces which kept them in their Orbs about the Sun were as the squares of their mean distances from the Sun reciprocally, & I found now that whatsoever was the law of the forces which kept the planets in their Orbs the areas described by a Radius drawn from them to the Sun would be proportional to the times in which they were described. And by the help of these Propositions I found that their Orbs would be such Ellipses as Kepler had described.

MS. Add 3958 b fol. 101

Graphical Construction of orbit for constant central impulses, based on Hooke's physical concept and Newton's mathematical implementation.

Let S be the center of force
and A the initial position of a body

Draw the initial displacement AB

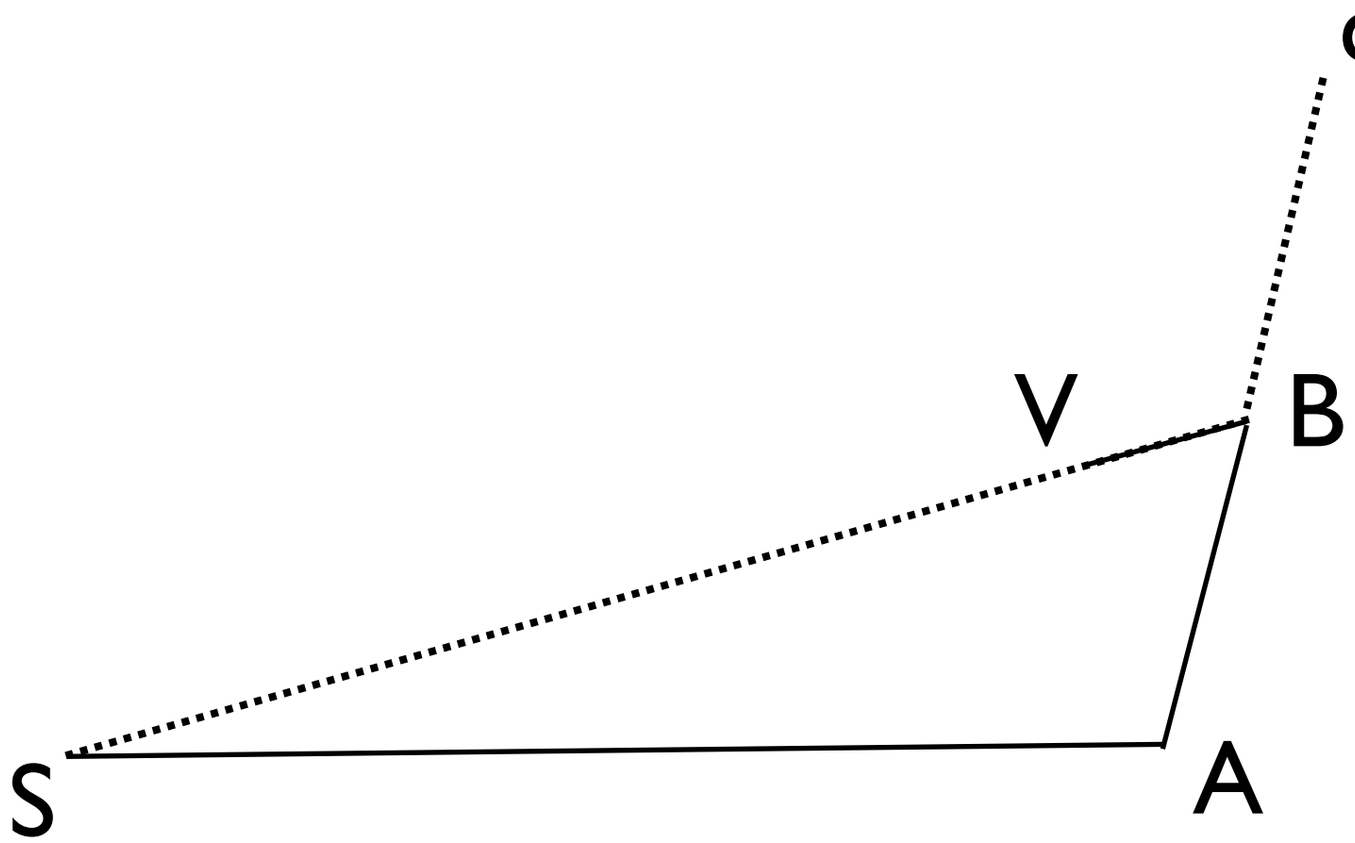
were $AB=v\delta t$

v= initial velocity

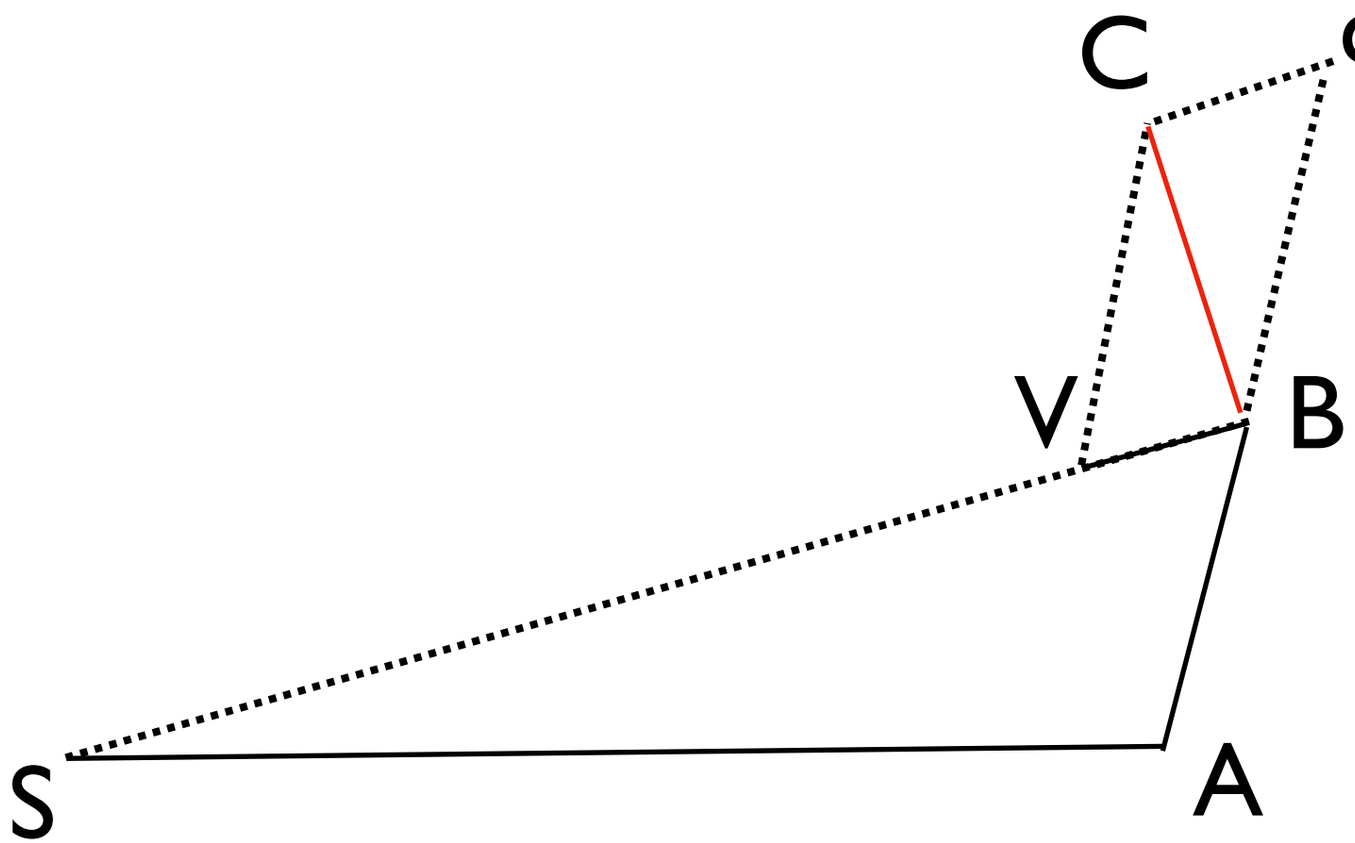
δt= time interval between

periodic impulses

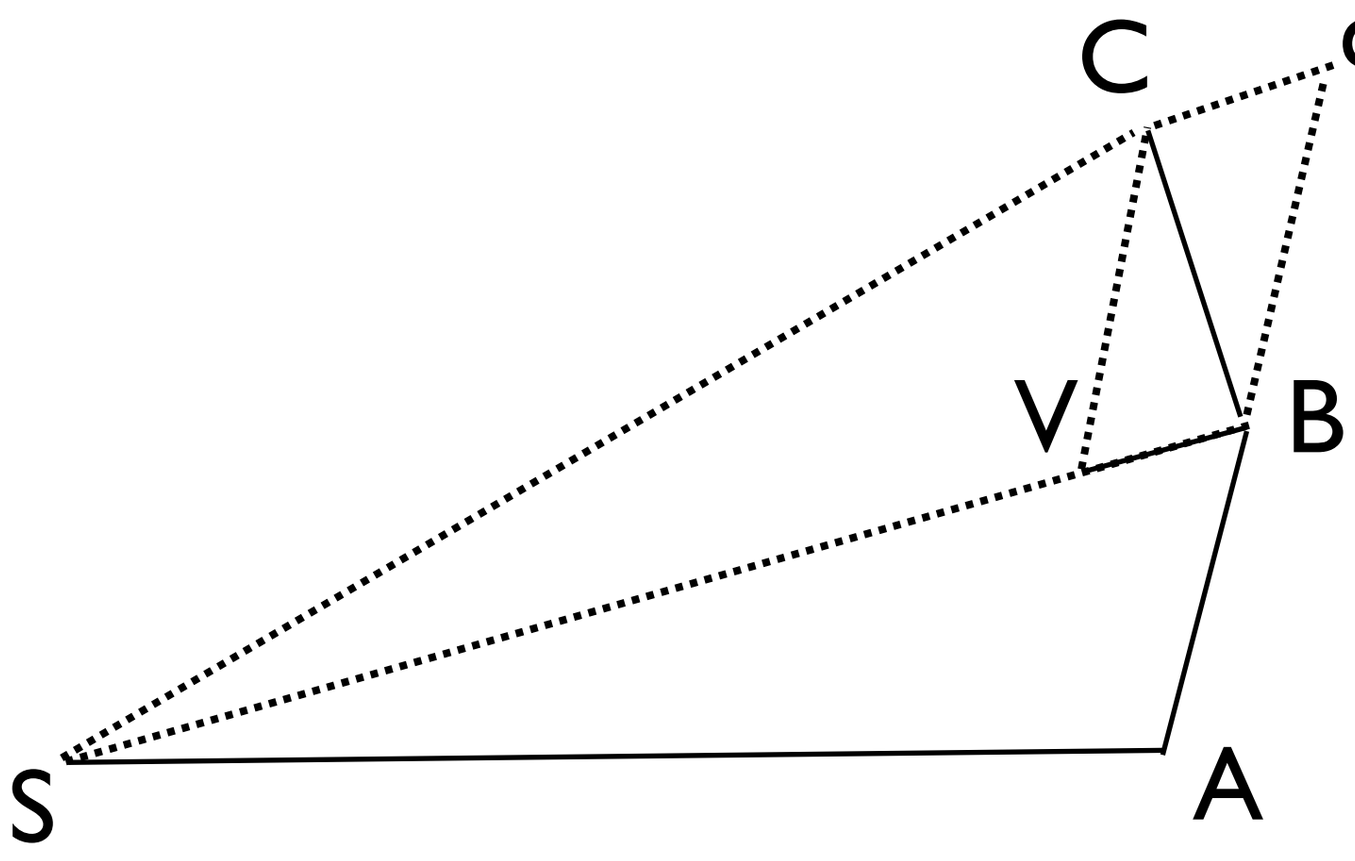
To obtain the next point
draw the extension Bc=AB,
and the impulse BV at B
directed along SB



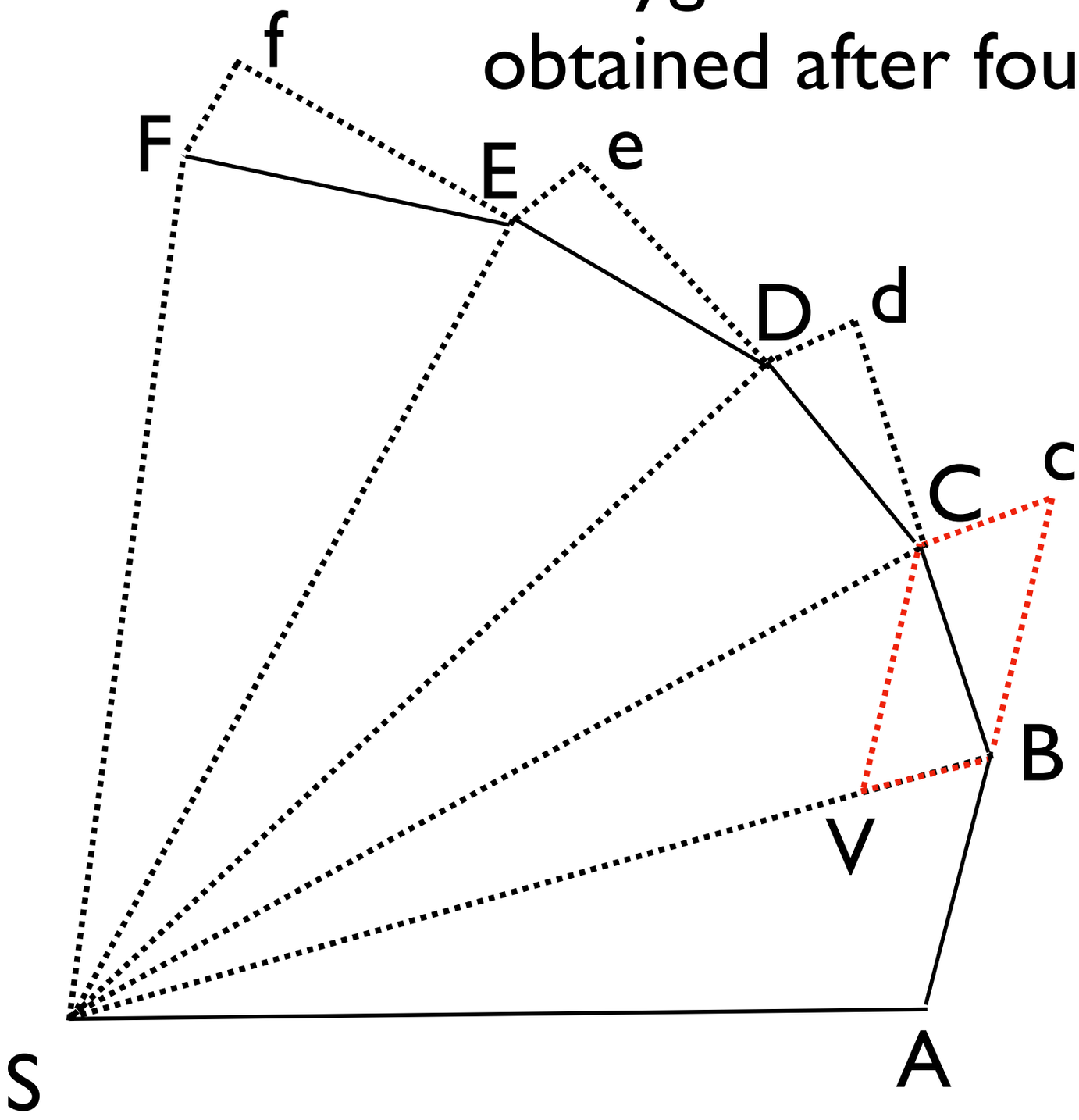
Obtain the next point C by Newton's parallelogram construction to add velocities vectorially:
draw VC parallel and equal to extension Bc
or draw Cc parallel and equal to impulse VB.
Then BC is the displacement after the impulse at B



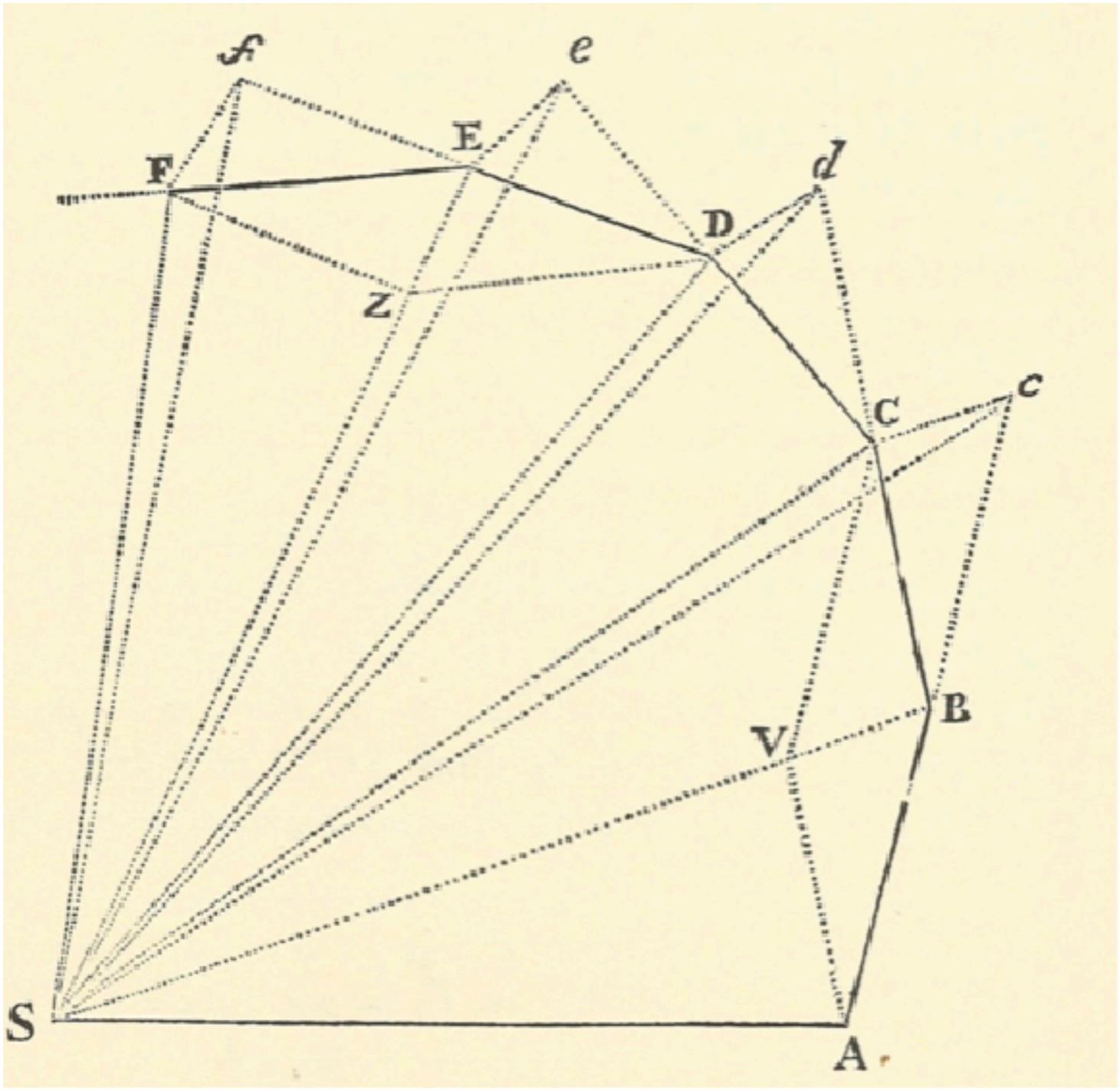
Join S to C and repeat this graphical construction for successive impulses at periodic intervals δt



Polygonal orbit ABCDEF
obtained after four impulses



Newton's diagram for Proposition 1 in Principia, Book 1



Analytic form of Newton's graphical construction

The velocity $\vec{v}(i)$ before the ith impulse is

$$\vec{v}(i) = \frac{\vec{r}(i) - \vec{r}(i-1)}{\delta t}, \tag{1}$$

and after the ith impulse

$$\vec{v}(i+1) = \vec{v}(i) + \vec{f}(i)\delta t, \tag{2}$$

where δt is the periodic time interval between impulses,

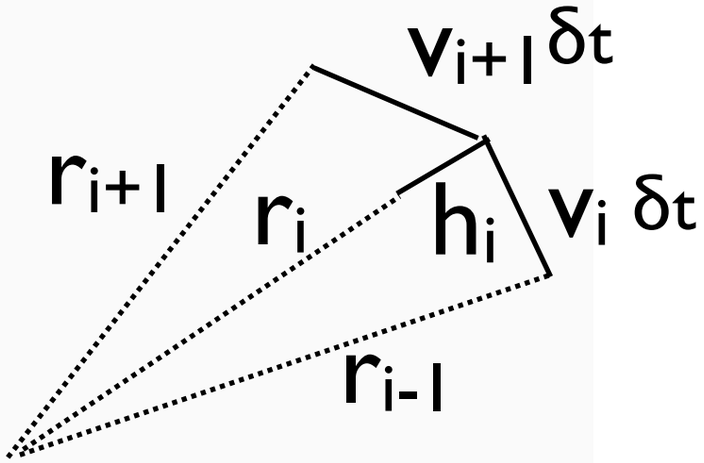
$$\vec{f}(i) = \frac{\vec{h}(i)}{\delta^2 t}, \tag{3}$$

$\vec{h}(i)$ is the magnitude of the impulse, and $\vec{f}(i)$ is the corresponding force.

According to Eq. 1,

$$\vec{r}(i+1) = \vec{r}(i) + \vec{v}(i+1)\delta t, \tag{4}$$

and Eqs. 2 and 4 are the analytic form of Newton's graphical equations of motion.



These two equations are symplectic (area preserving), and have been rediscovered several times in the past.

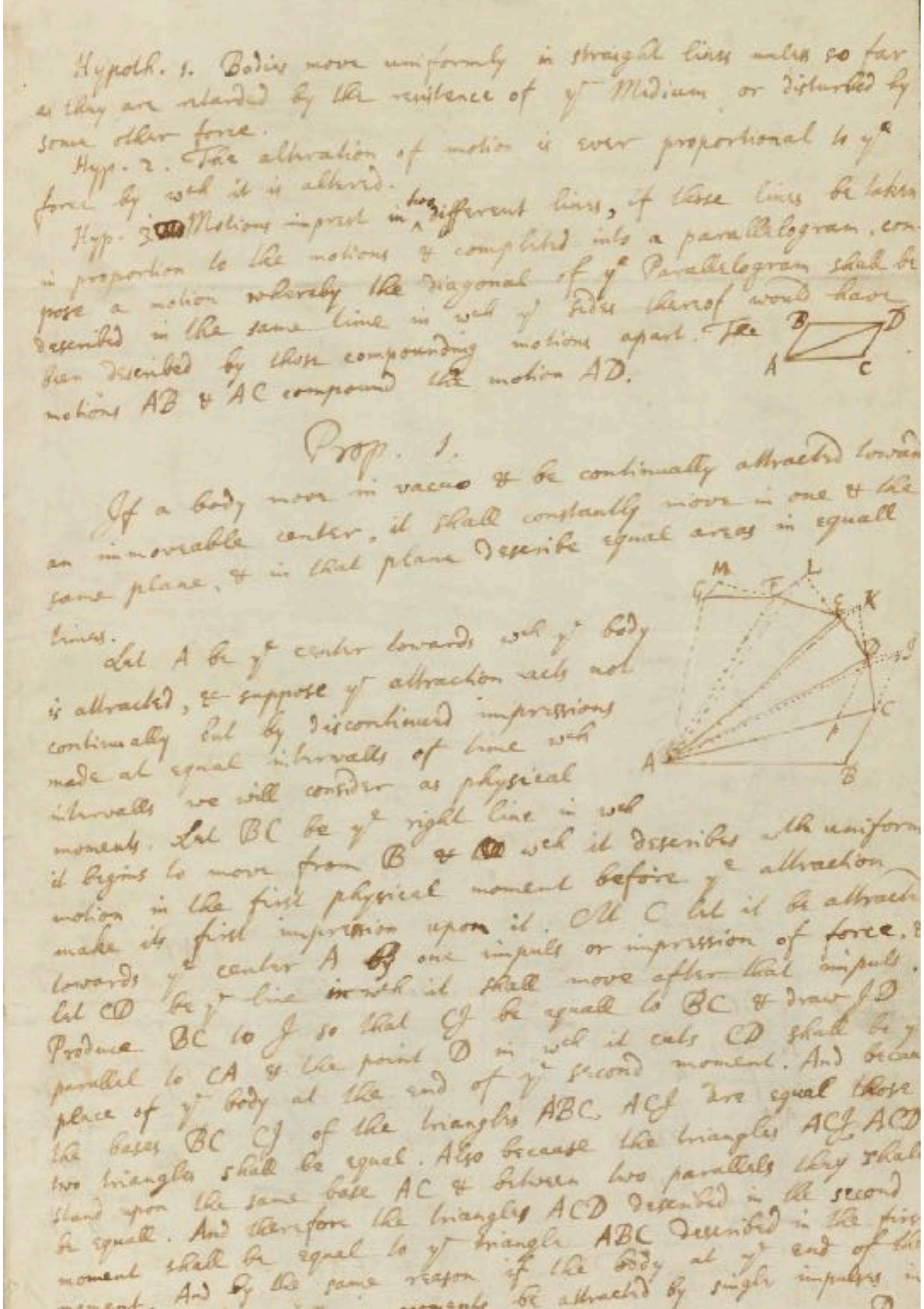
In the limit $\delta t \to 0$, Eq, 2 is the well known equation of classical mechanics

$$\vec{f}(t) = m\vec{a}(t), \tag{5}$$

where $\vec{a}(t) = \delta\vec{v}/\delta t$ and $m = 1$.

Early manuscript of Newton's Principia sent to the Royal Society in 1684

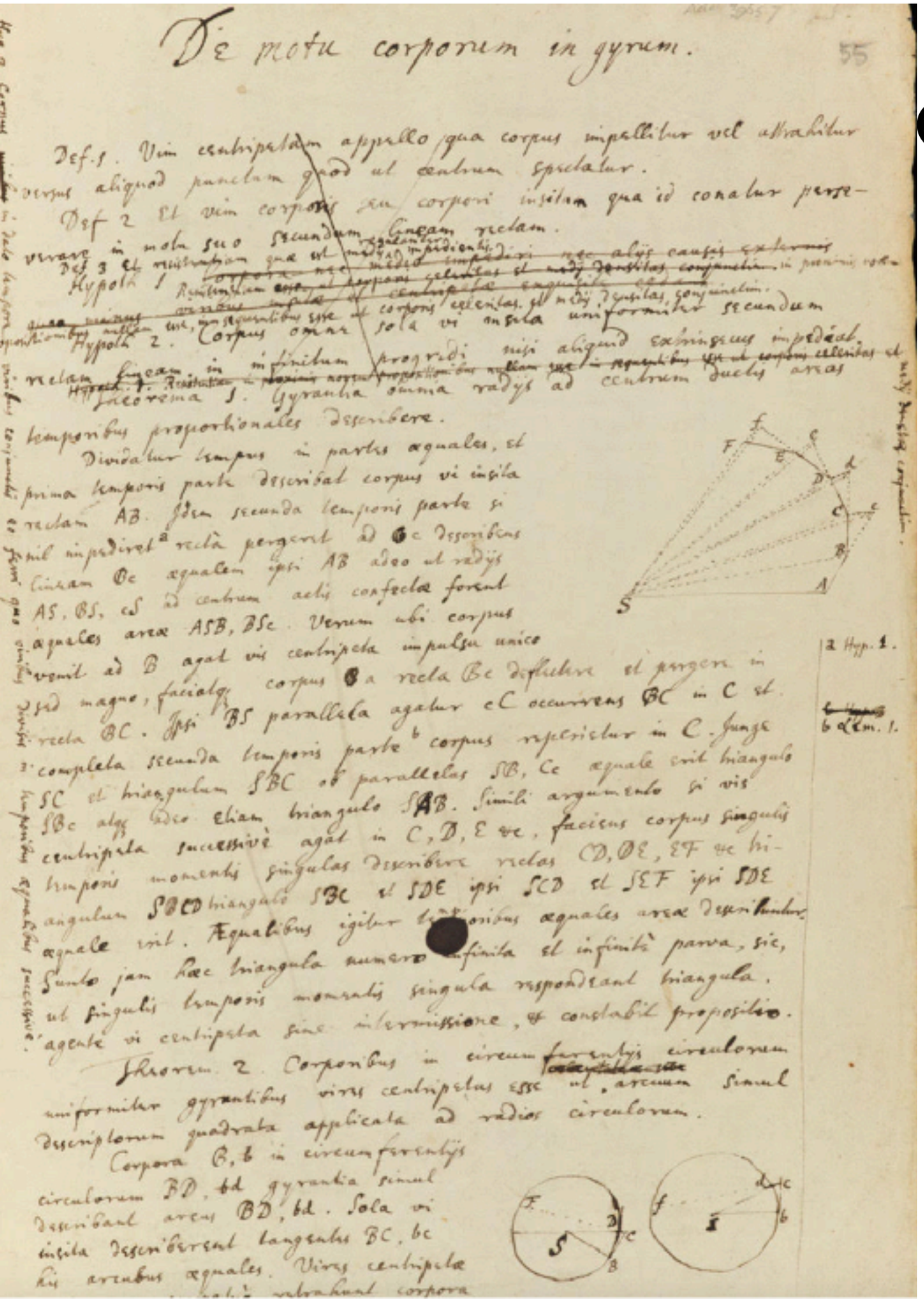
Proposition 1 diagram Principia Book1



Newton's Papers
Cambridge University Library

De Motu

Newton's preliminary manuscript for the Principia sent to the Royal Society in 1684


De motu corporum in gyrum.

55

Def. 1. Vim centripetam appello qua corpus impellitur vel attrahitur versus aliquod punctum quod ut centrum spectatur.

Def 2 Et vim corporis seu corpori insitam qua id conatur perseverare in motu suo secundum lineam rectam.

Def 3 Et resistantiam qua ut modo impedirii nec aliis causis externis corpora nec medio impediri nec aliis causis externis ...

Hypoth 1. Resistantiam esse ut velocitatem et spatium celeritas et radij densitas conjunctim.

Hypoth 2. Corpus omne sola vi insita uniformiter secundum lineam rectam in infinitum progredi nisi aliquid extrinsecus impedeat.

Theorema 1. Gyranta omnia radijs ad centrum ductis areas temporibus proportionales describere.

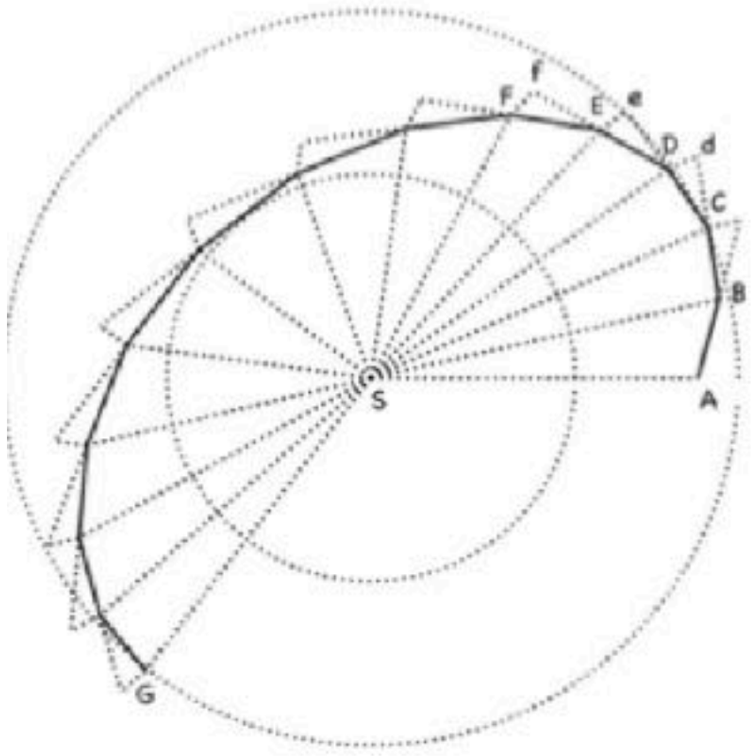
Dividatur tempus in partes aequales, et prima temporis parte describat corpus vi insita rectam AB. Idem secunda temporis parte si nihil impediret recta pergeret ad c describens lineam Bc aequalem ipsi AB adeo ut radijs AS, BS, cS ad centrum actis confecta forent aequales areae ASB, BSc. Verum ubi corpus venit ad B agat vis centripeta impulsu unico sed magno, faciatq; corpus a recta Bc deflectere et pergere in recta BC. Ipsi BS parallela agatur cC occurrens BC in C et 2 Hyp. 1. 2 completa secunda temporis parte corpus reperietur in C. Junge SC et triangulum SBC ob parallelas SB, Cc aequale erit triangulo SBc atq; adeo etiam triangulo SAB. Simili argumento si vis centripeta successivè agat in C, D, E &c. faciens corpus singulis temporis momentis singulas describere rectas CD, DE, EF &c. hi- 6 Lem. 1. angulum SBCD triangulo SBC et SDE ipsi SCD et SEF ipsi SDE aequale erit. Aequalibus igitur temporibus aequales areae describuntur. Sunto jam haec triangula numero infinita et infinitè parva, sic, ut singulis temporis momentis singula respondeant triangula, agente vi centripeta sine intermissione, & constabit propositio.

Theorem. 2. Corporibus in circumferentijs circulorum uniformiter gyrantibus vires centripetas esse ut arcuum simul descriptorum quadrata applicata ad radios circulorum.

Corpora B, b in circumferentijs circulorum BD, bd gyrantia simul describant arcus BD, bd. Sola vi insita describerent tangentes BC, bc his arcubus aequales. Vires centripetae

Proposition 1, Principia, Book 1

"The area which bodies made to move in orbits described by radii drawn to an unmoving center of forces lie in unmoving planes and are proportional to the times"



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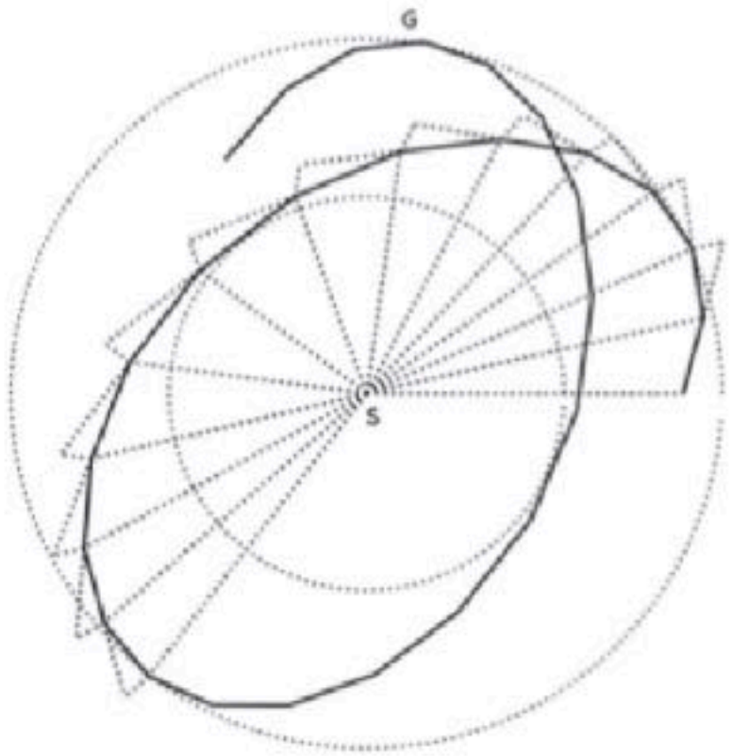

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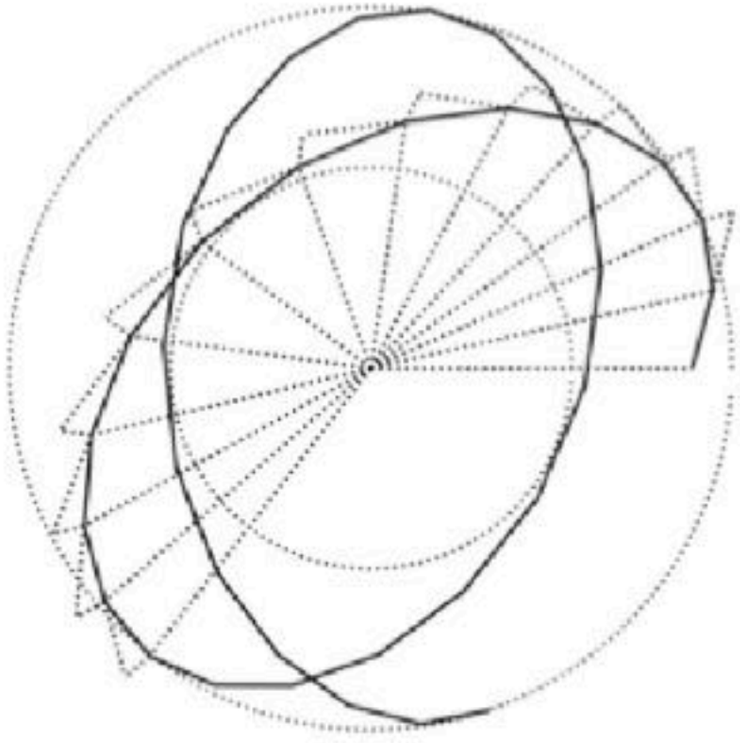

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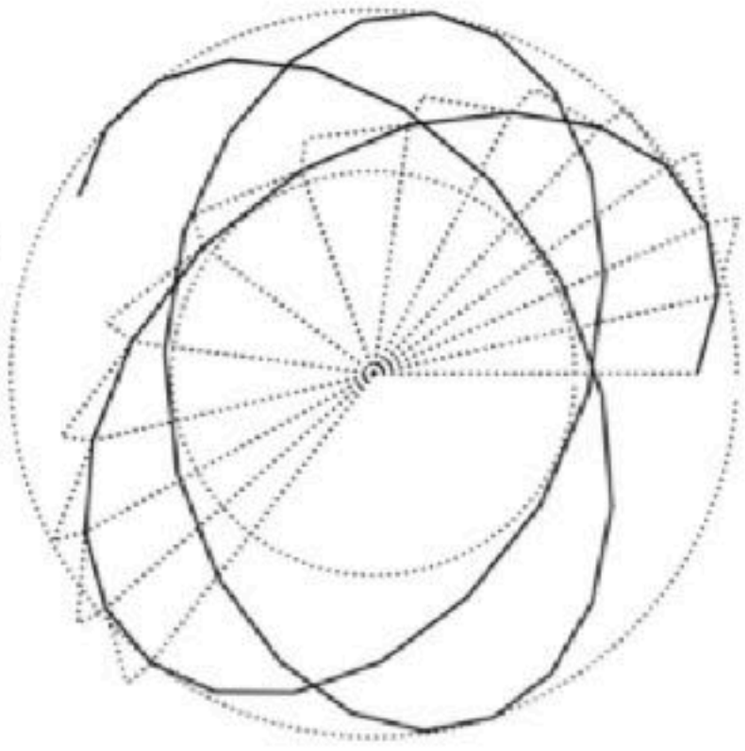

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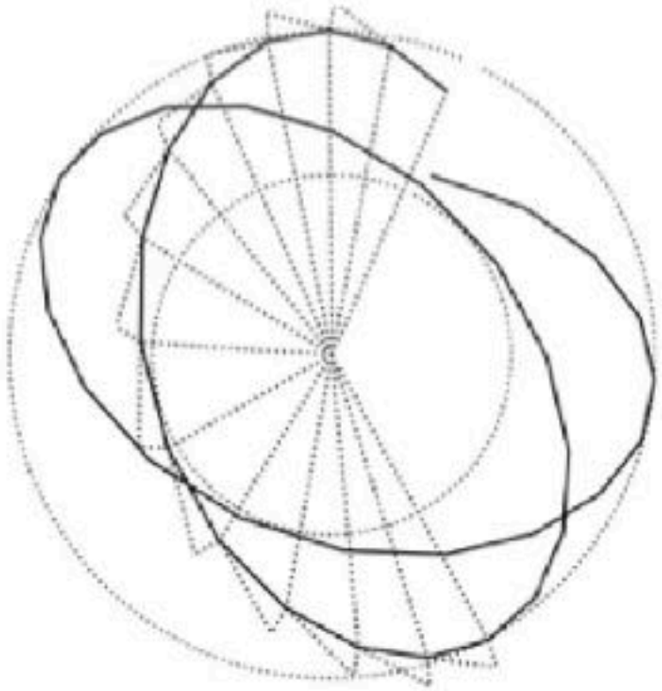

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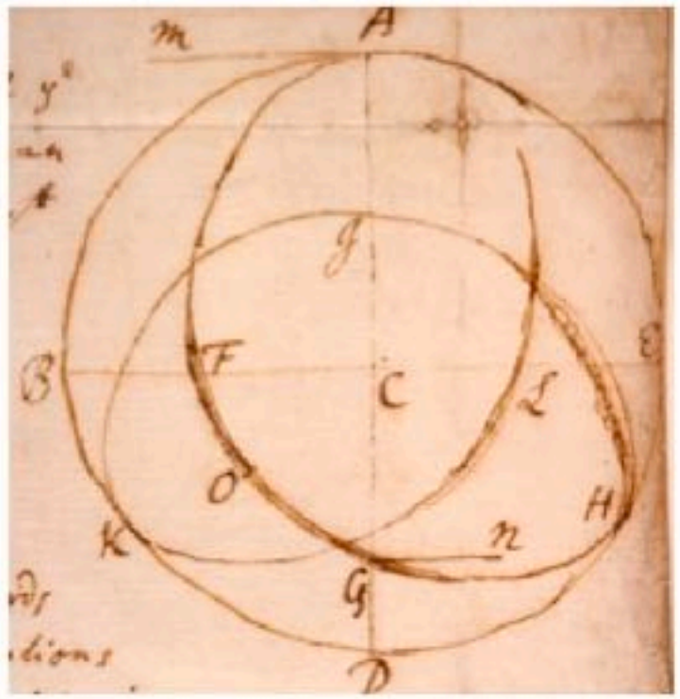

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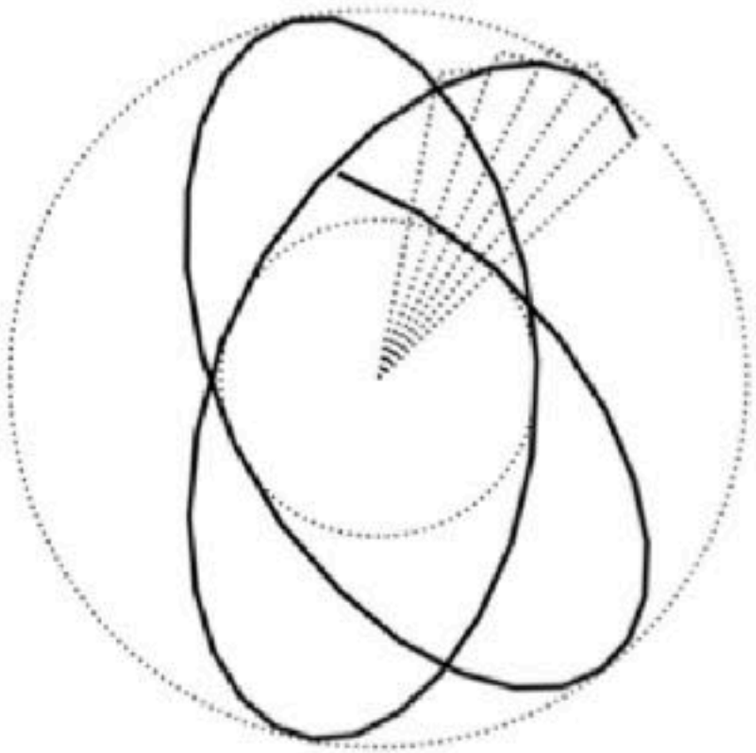

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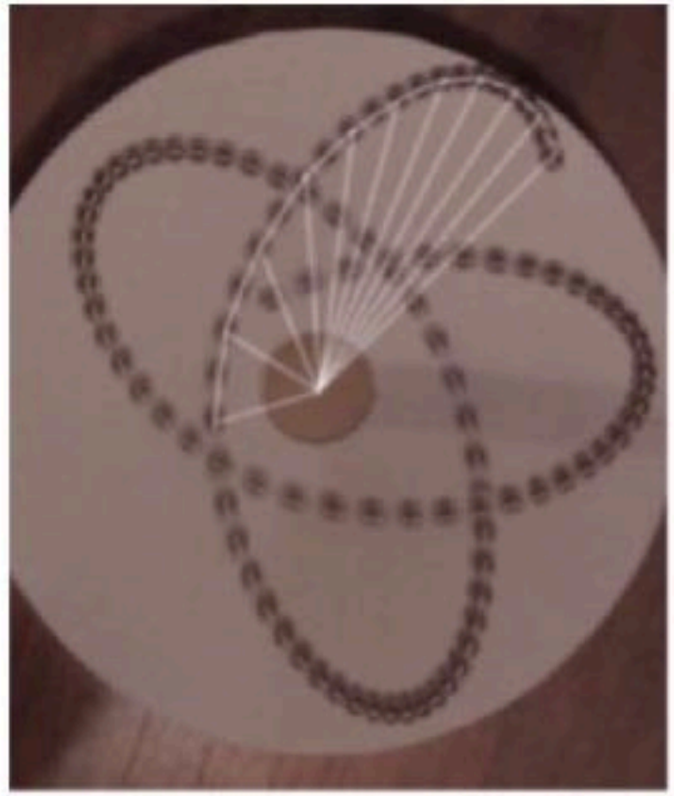

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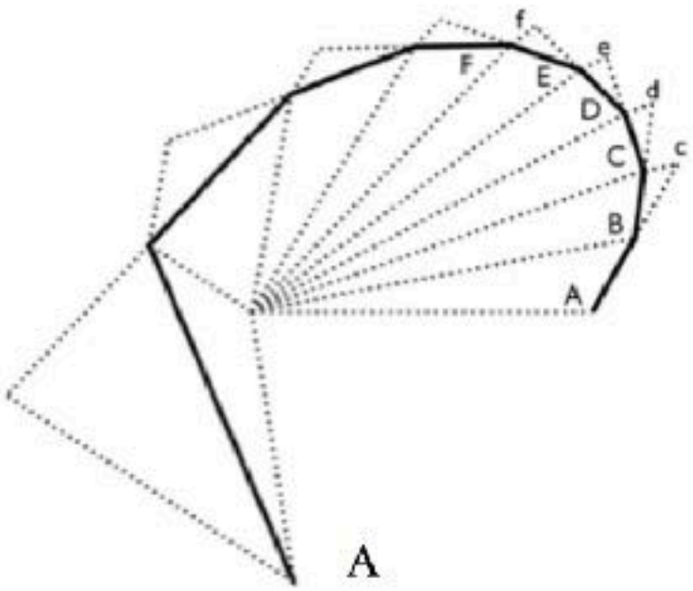

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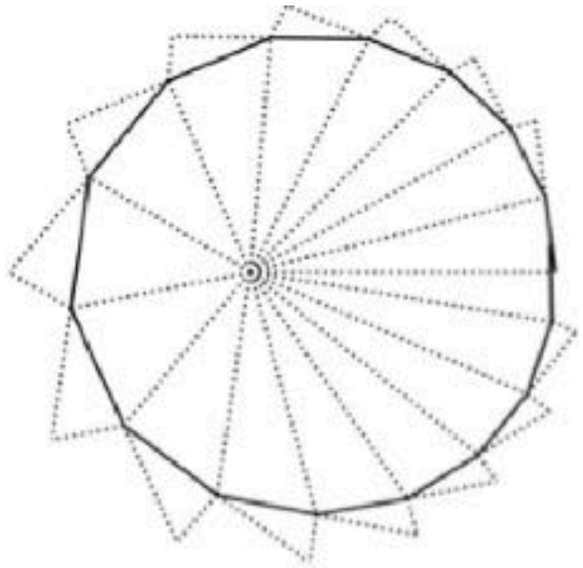

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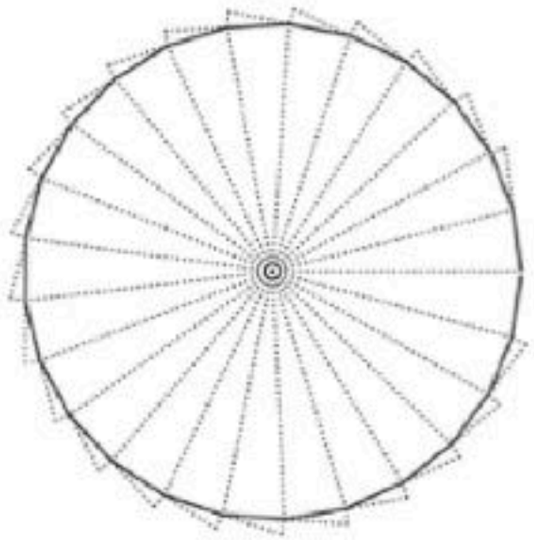

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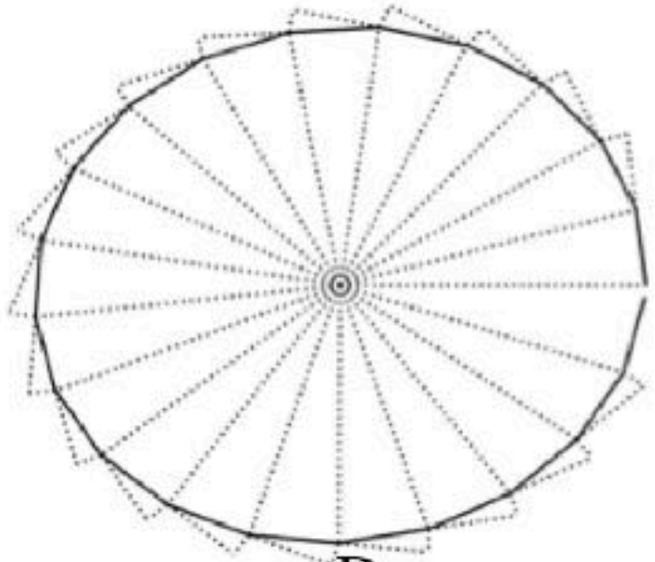

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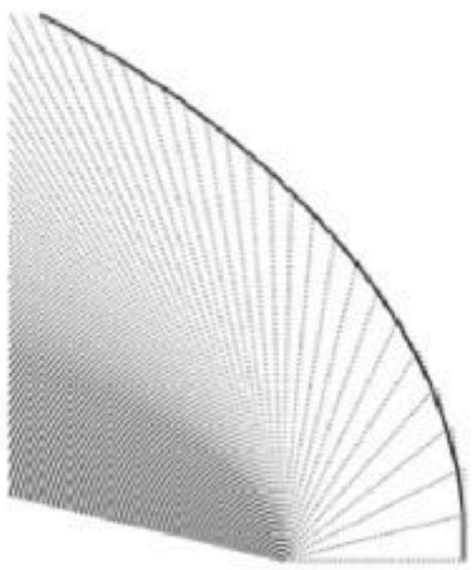

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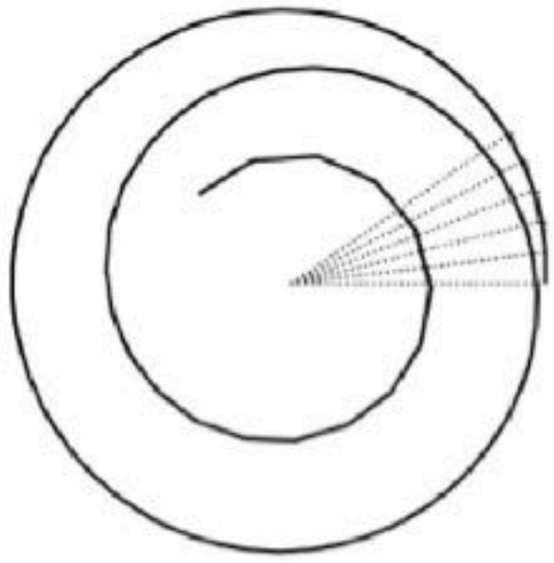

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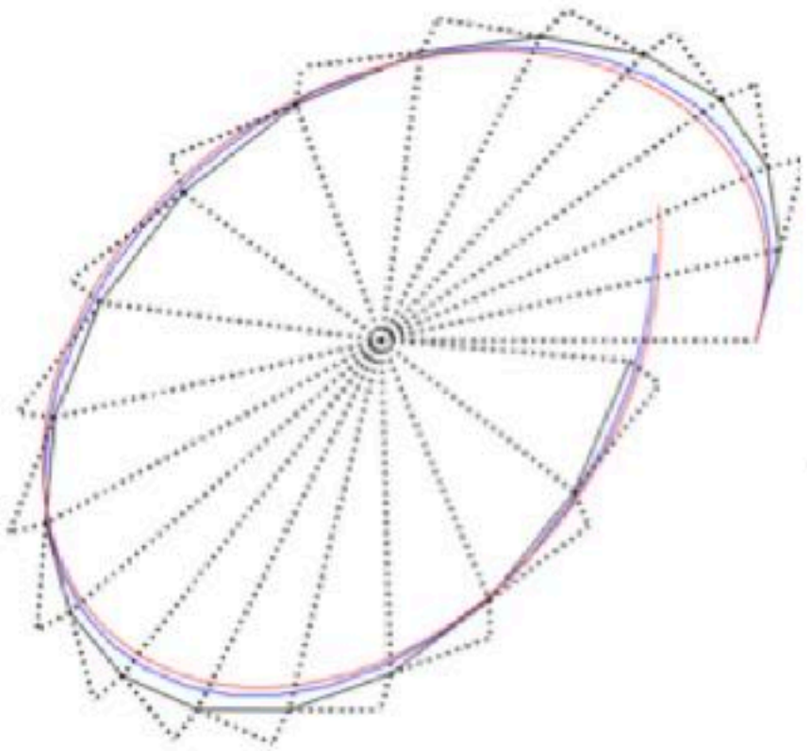

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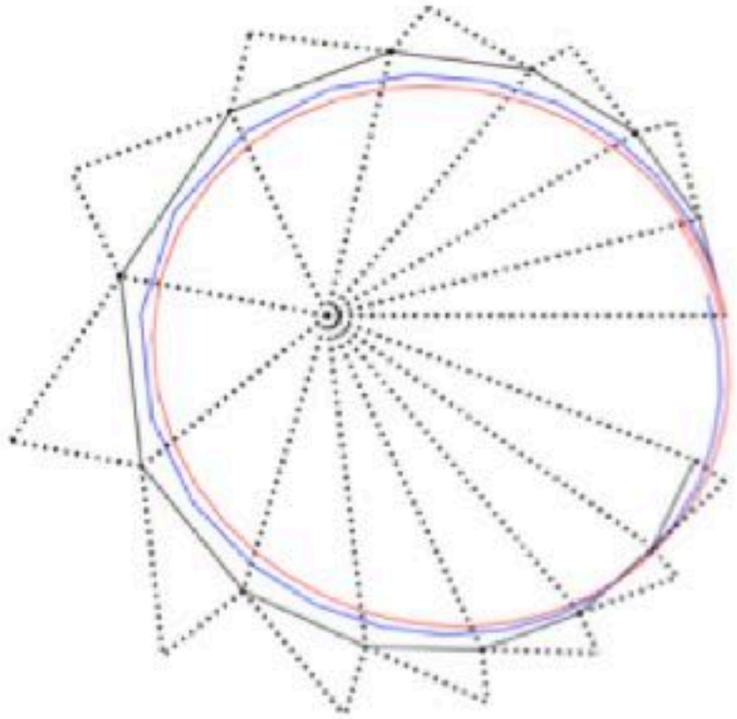

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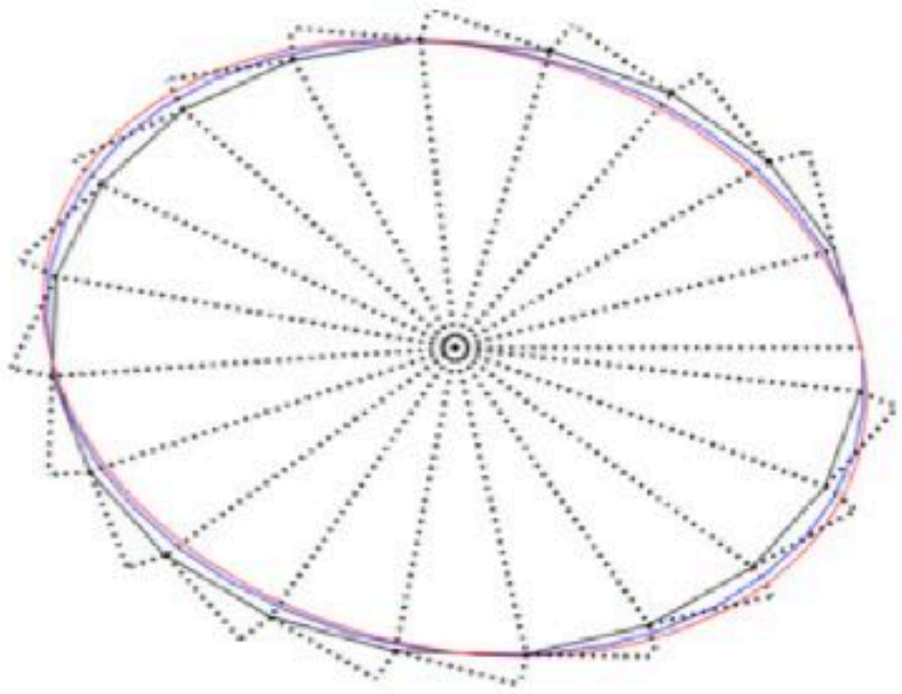

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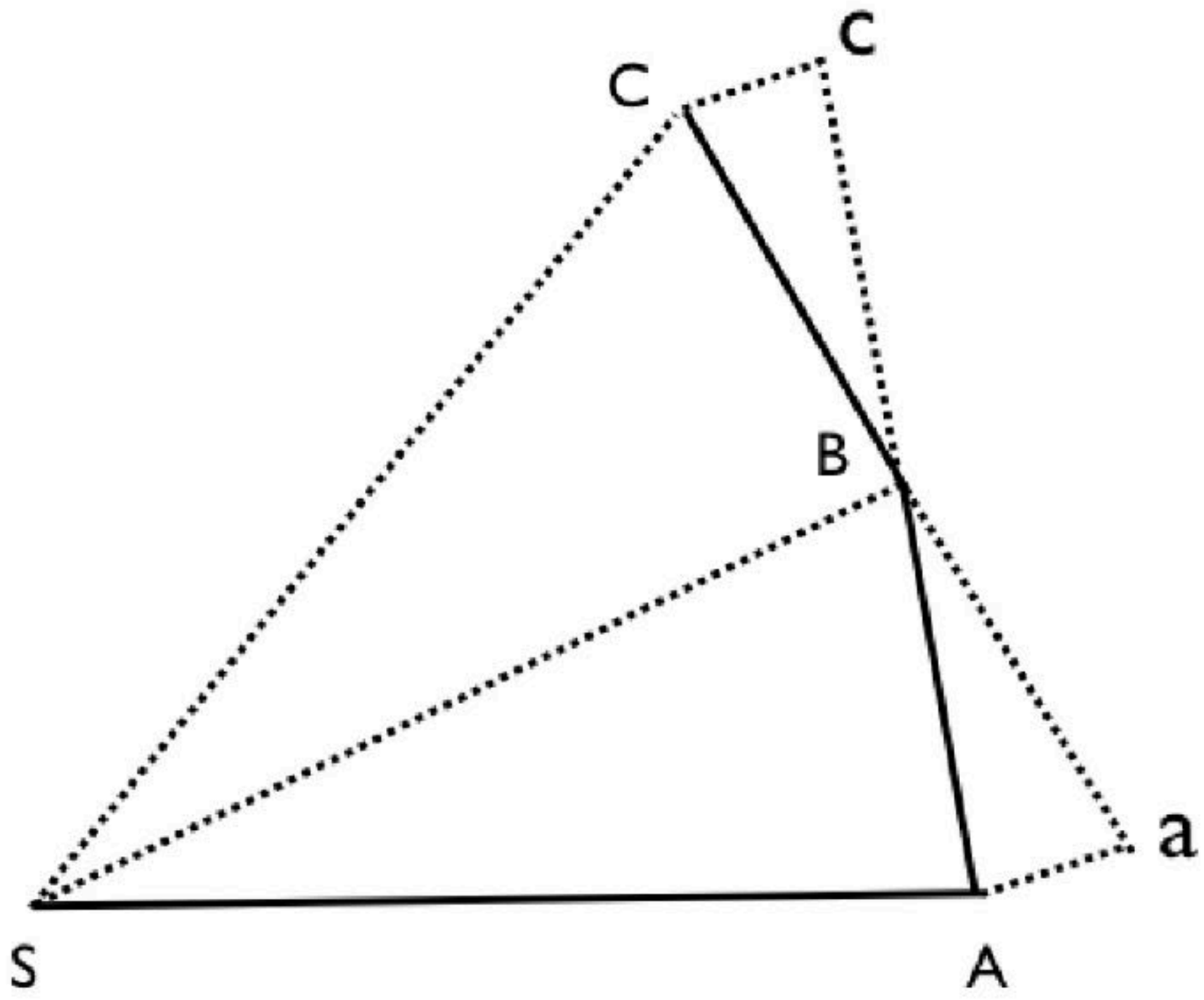
Time Reversal

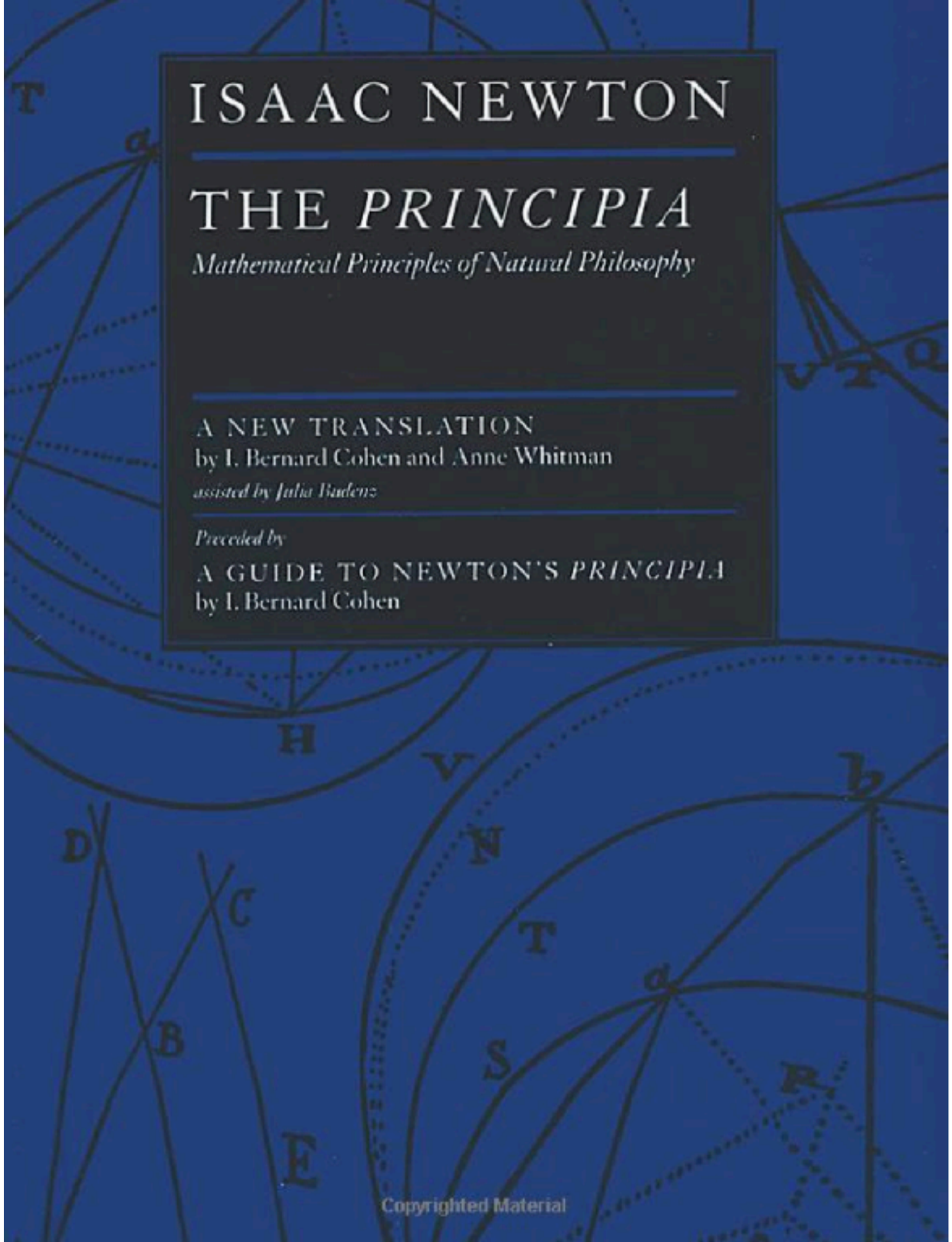


Conversation between Laplace and Napoleon at a reception in 1802

Napoleon: Newton spoke of God in his book. I perused yours but failed to find his name even once

Laplace: I had no need for that hypothesis

Knowledge of the method that has guided a man of genius is no less useful to the progress of science and to his glory than his discoveries; the method is often the most interesting part".


ISAAC NEWTON
THE PRINCIPIA
Mathematical Principles of Natural Philosophy
A NEW TRANSLATION
by I. Bernard Cohen and Anne Whitman
assisted by Julia Budenz
Preceded by
A GUIDE TO NEWTON'S PRINCIPIA
by I. Bernard Cohen
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