Music and the Making of Modern Science

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(*Music and the Making of Modern Science*, MIT Press and Apple iBook, 2014)

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Peter Pesic

A knowledge of the historic and philosophic background gives that kind of independence from prejudices of his generation from which most scientists are suffering. This independence created by philosophical thought is—in my opinion—the mark of distinction between a mere artisan or specialist and a real seeker after truth.

Albert Einstein, Letter to Robert Thornton (December 7, 1944)

Music gave the first connection between mathematics and physical phenomena

Pythagorean temperament

Octave 2:1

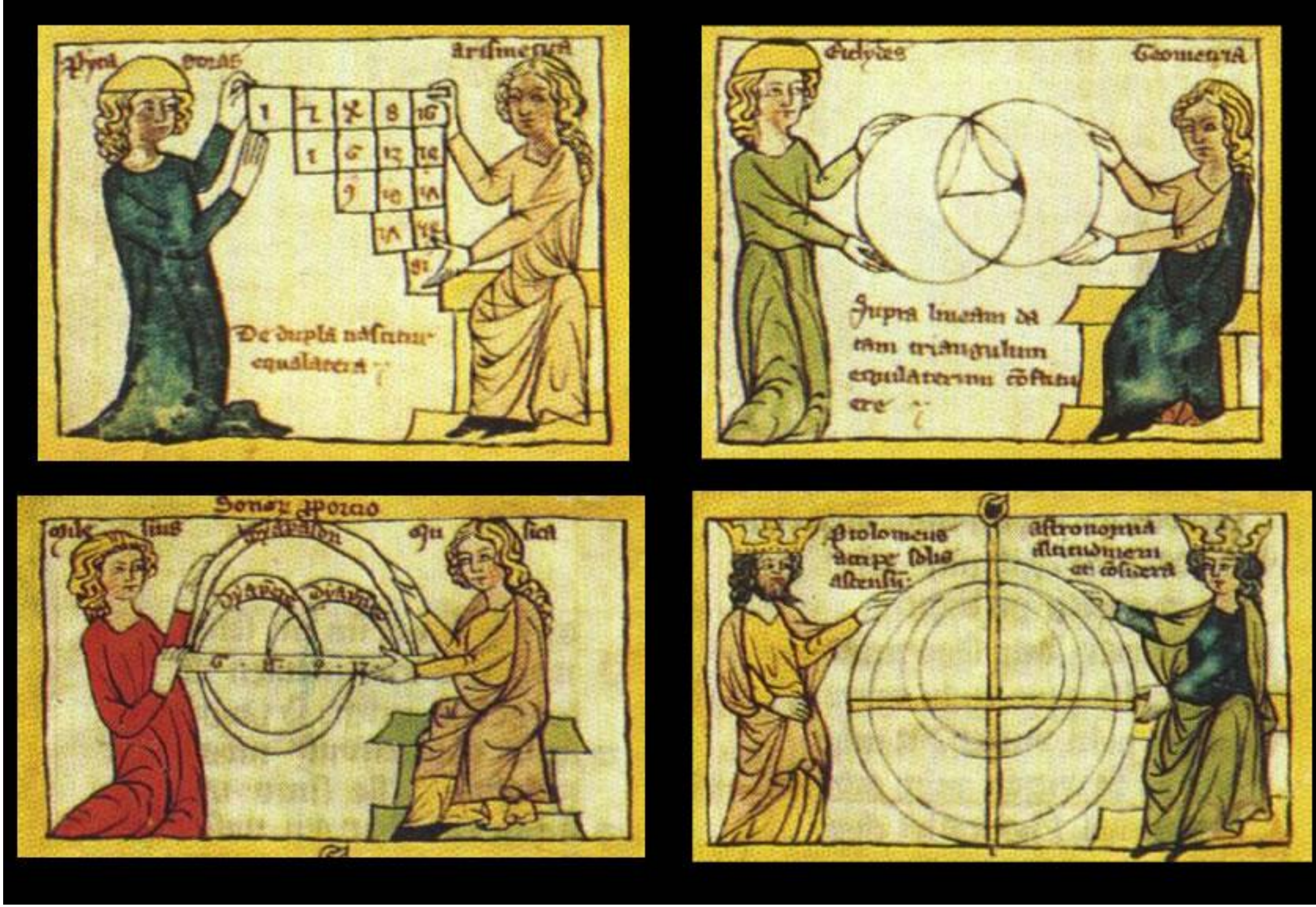
Fifth 3:2

Fourth 4:3

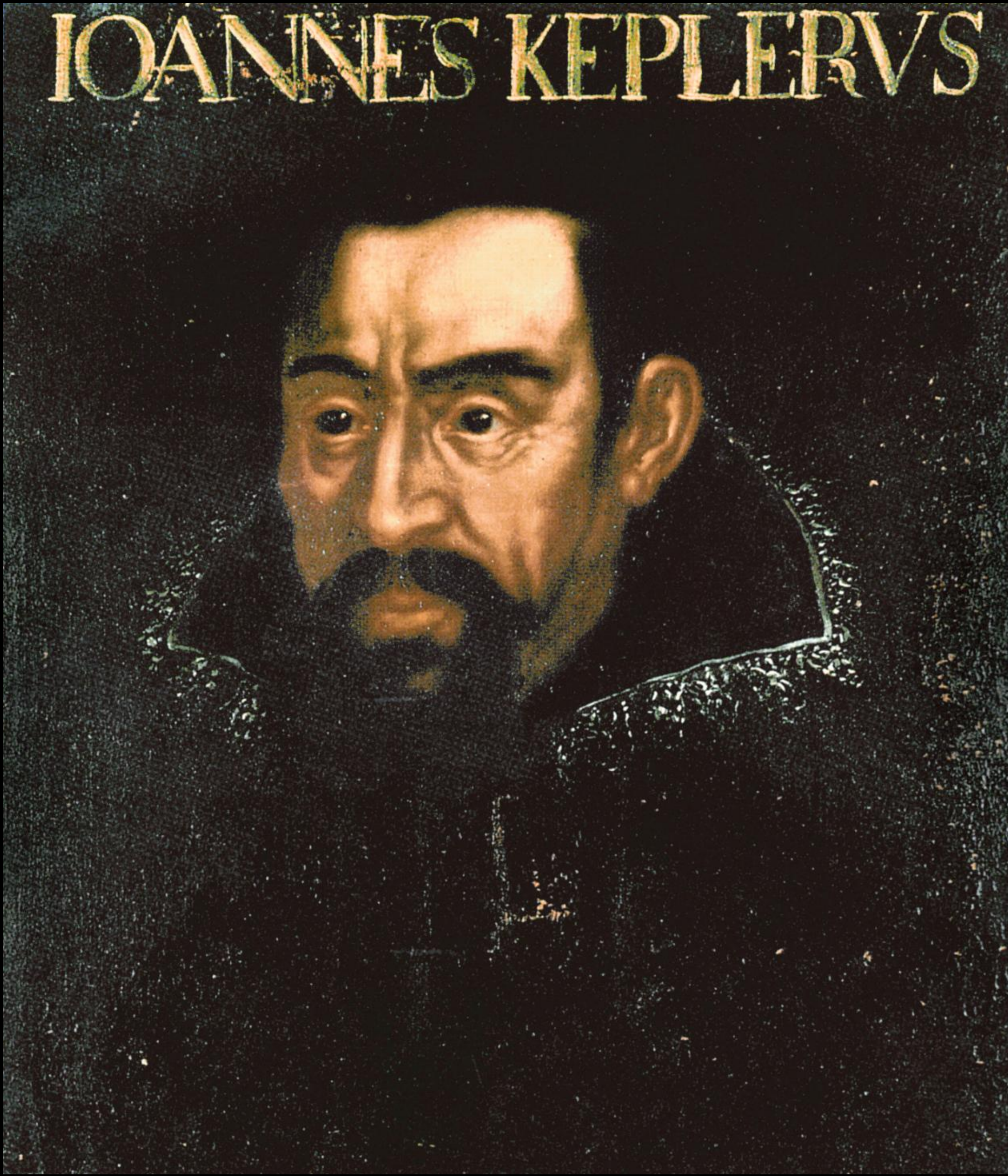
Tone 9:8 ($\times$ 4:3 = 3:2)

A tritone ($9^3:8^3$) is very close to $\sqrt{2}:1$

But seven tones overshoot an octave.



Based on Pythagorean ideas, Plato proposed a "four-fold path" (*quadrivium*) of sister "liberal arts" to form the basis of higher education: arithmetic, geometry, music, and astronomy.


IOANNES KEPLERVS

Johannes Kepler's planetary songs (including the moon), from his *Harmonices mundi* (1609)

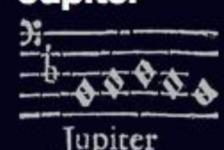
Mercury

Venus

Earth

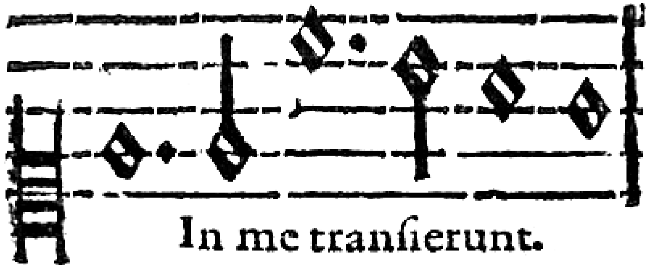
Mars

Jupiter

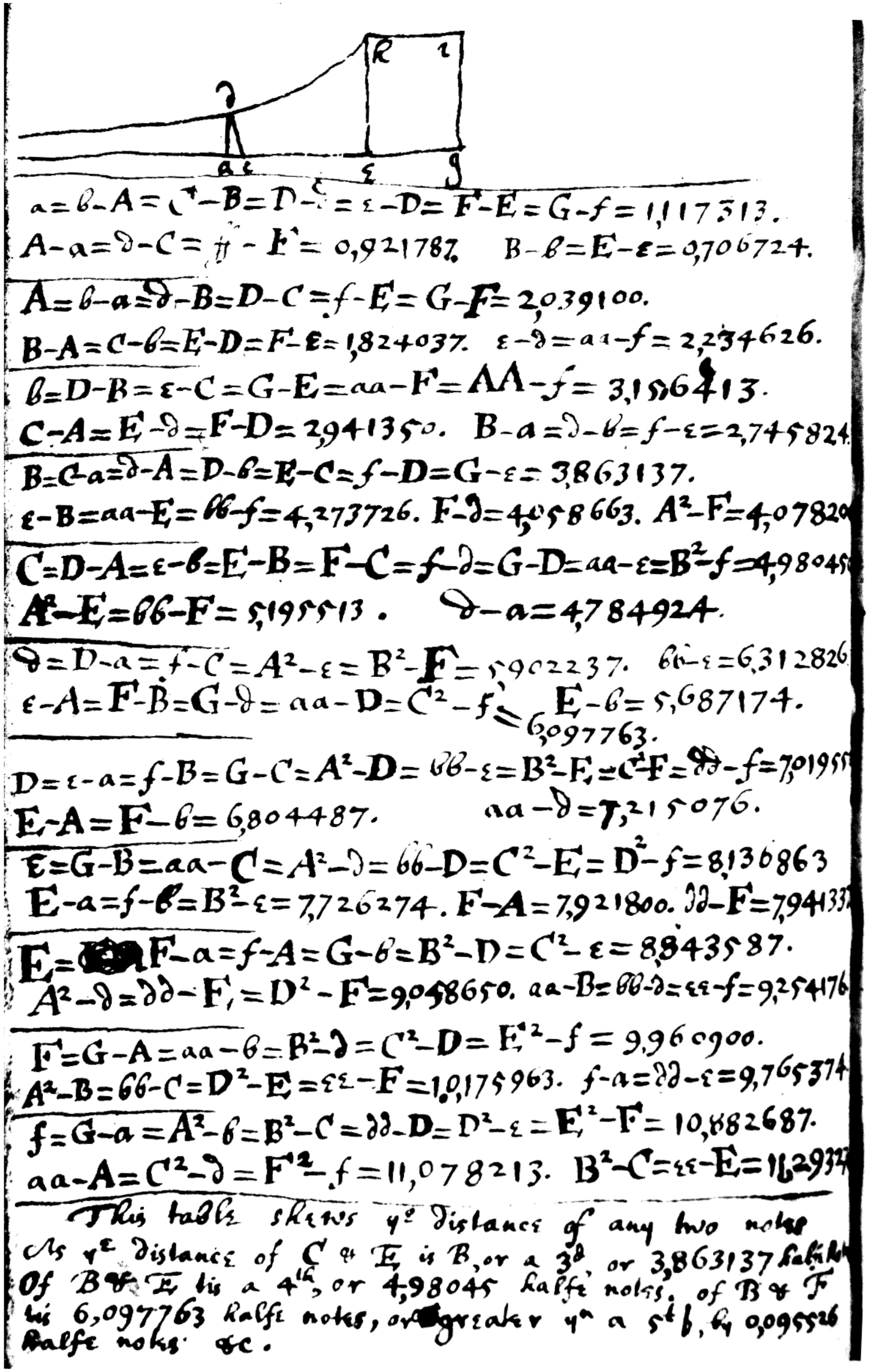


Saturn

Play All **Mute All**



Orlando di Lasso, “In me transierunt”



a = b − A = C − B = D − c = ε − D = F − E = G − f = 1,117313.
A − a = ∂ − C = # − F = 0,921787. B − b = E − ε = 0,706724.

A = b − a = ∂ − B = D − C = f − E = G − F = 2,039100.
B − A = C − b = E − D = F − ε = 1,824037. ε − ∂ = aa − f = 2,234626.
b = D − B = ε − C = G − E = aa − F = AA − f = 3,1[illegible]6413.
C − A = E − ∂ = F − D = 2,941350. B − a = ∂ − b = f − ε = 2,745824.

B = C − a = ∂ − A = D − b = E − C = f − D = G − ε = 3,863137.
ε − B = aa − E = bb − f = 4,273726. F − ∂ = 4,058663. A^2 − F = 4,0782[illegible]

C = D − A = ε − b = E − B = F − C = f − ∂ = G − D = aa − ε = B^2 − f = 4,98045[illegible]
A^2 − E = bb − F = 5,195513. ∂ − a = 4,784924.

∂ = D − a = f − C = A^2 − ε = B^2 − F = 5,902237. bb − ε = 6,312826
ε − A = F − B = G − ∂ = aa − D = C^2 − f = 6,097763. E − b = 5,687174.

D = ε − a = f − B = G − C = A^2 − D = bb − ε = B^2 − E = C^2 − F = ∂∂ − f = 7,01955[illegible]
E − A = F − b = 6,804487. aa − ∂ = 7,215076.

ε = G − B = aa − C = A^2 − ∂ = bb − D = C^2 − E = D^2 − f = 8,136863
E − a = f − b = B^2 − ε = 7,726274. F − A = 7,921800. ∂∂ − F = 7,94133[illegible]

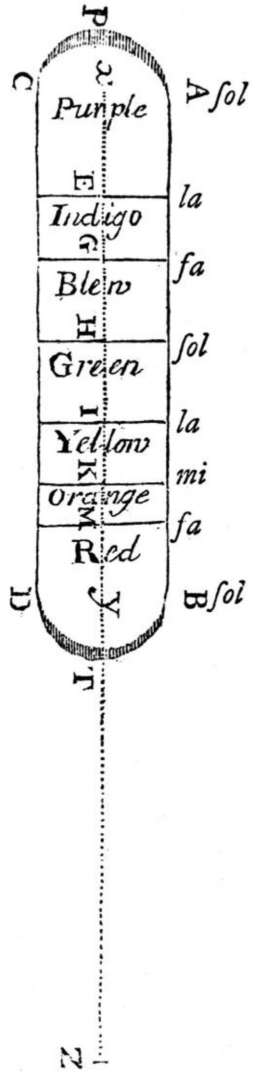
E = [illegible] F − a = f − A = G − b = B^2 − D = C^2 − ε = 8,843587.
A^2 − ∂ = ∂∂ − F = D^2 − F = 9,058650. aa − B = bb − ∂ = εε − f = 9,254176

F = G − A = aa − b = B^2 − ∂ = C^2 − D = E^2 − f = 9,960900.
A^2 − B = bb − C = D^2 − E = εε − F = 10,175963. f − a = ∂∂ − ε = 9,765374

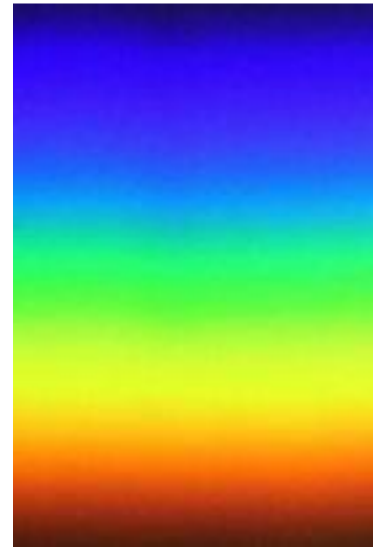
f = G − a = A^2 − b = B^2 − C = ∂∂ − D = D^2 − ε = E^2 − F = 10,882687.
aa − A = C^2 − ∂ = F^2 − f = 11,078213. B^2 − C = εε − E = 11,2933[illegible]

This table shews ye distance of any two notes
As ye distance of C & E is B, or a 3d, or 3,863137 halfe [illegible]
Of B & E is a 4th, or 4,98045 halfe notes, of B & F
is 6,097763 halfe notes, or greater yn a 5th, by 0,095526
halfe notes &c.

From Newton's student notebook "Of Musick" (1665)



Isaac Newton’s diagram of the solar spectrum, showing note names on the right (Dorian mode)

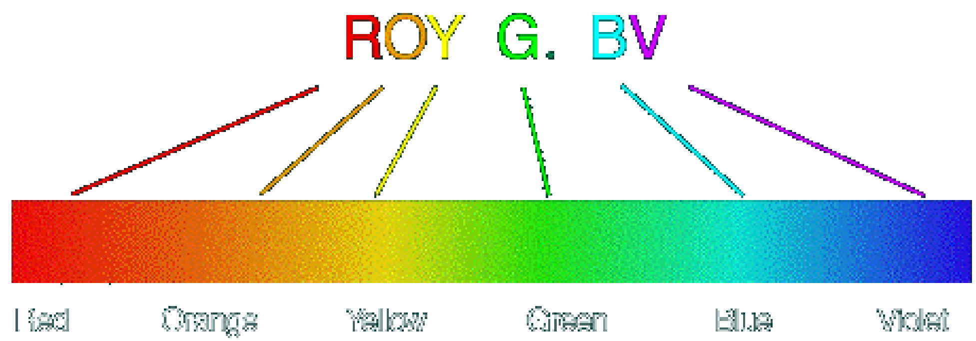


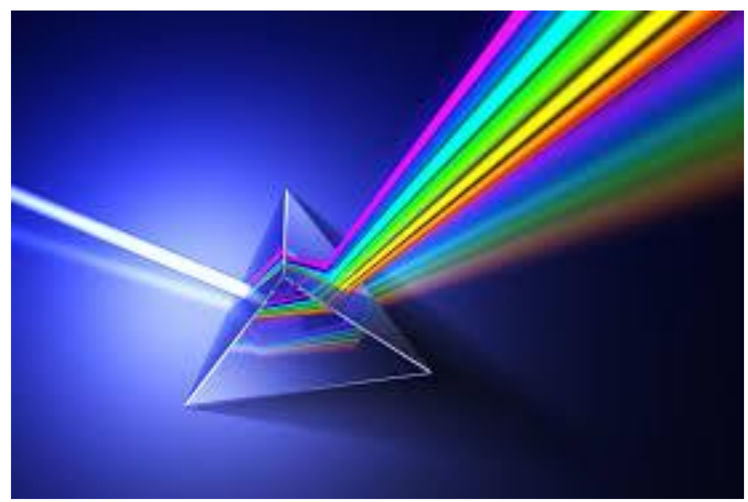
Solar spectrum

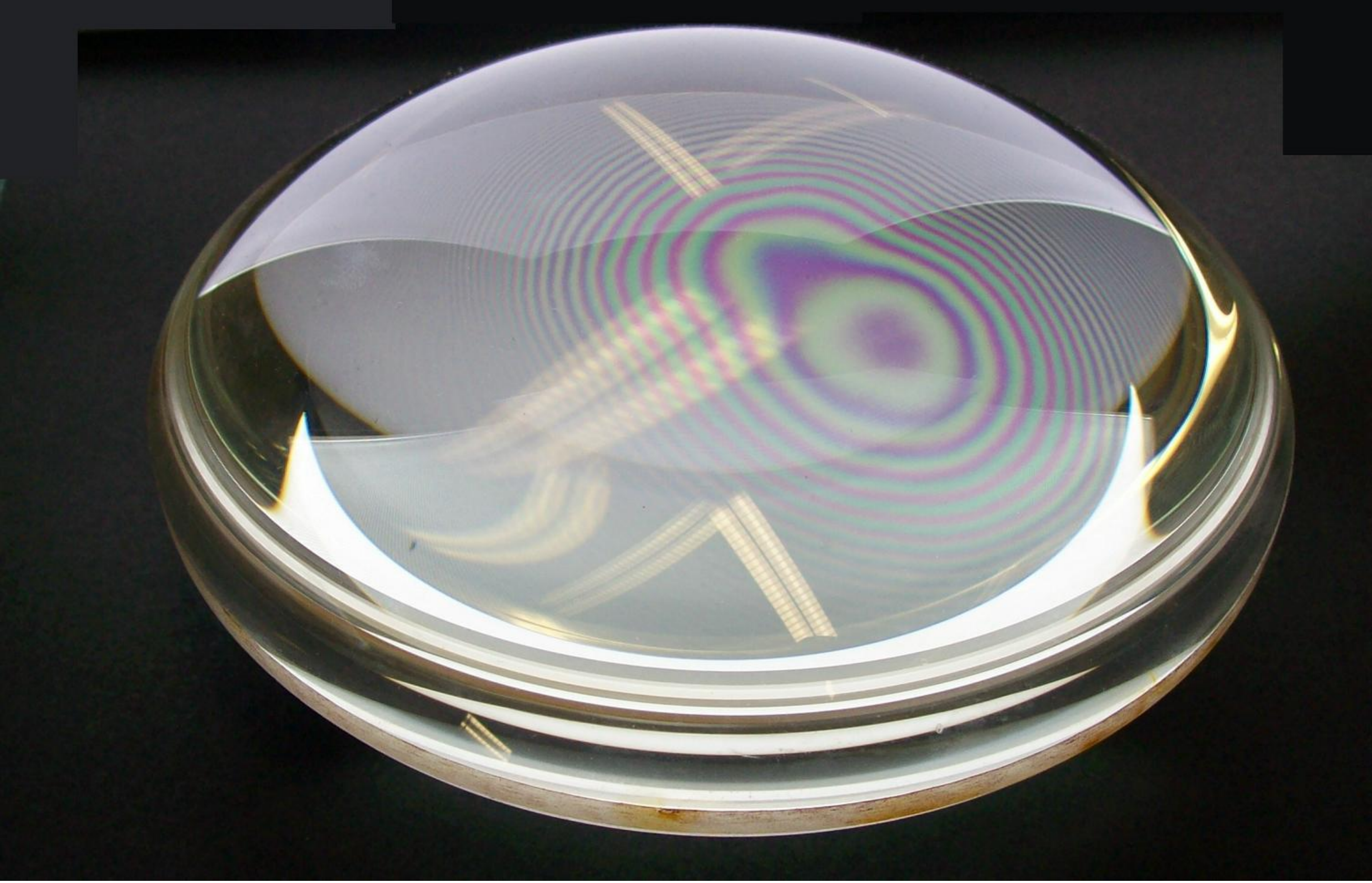
ROY G BIV

Roy in 1982

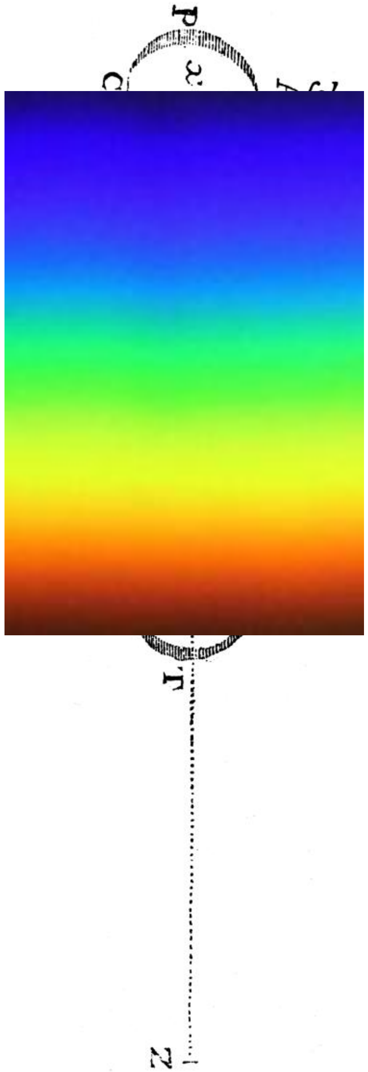
Roy G. Biv
Physics Club, Art Club, AV Club, Latin Club President, Glee Club, Debate Team, Student Council, Hall Monitor, Library Monitor, Color Guard, Soccer







Newton's rings, produced by two planoconvex lenses with their flat surfaces in contact

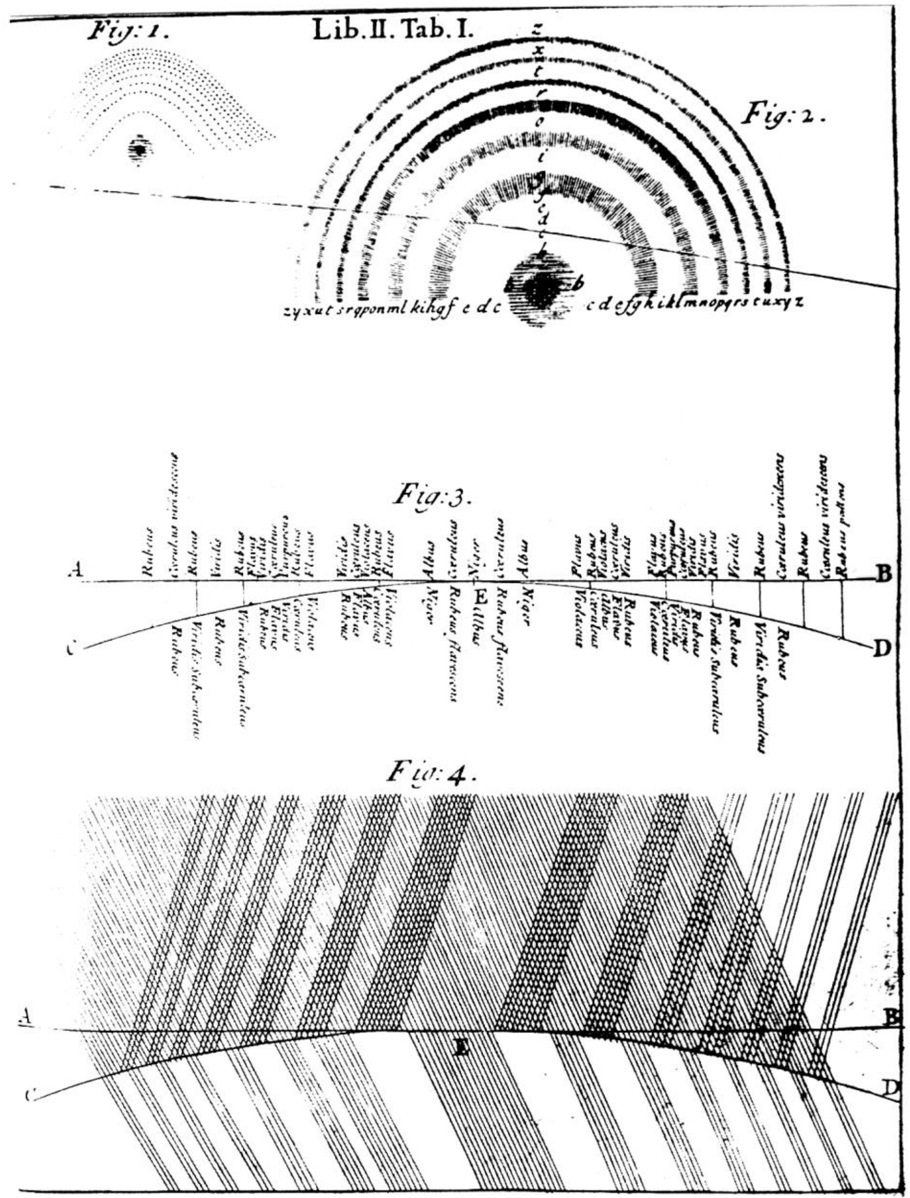


Isaac Newton's diagram of the solar spectrum, showing note names on the right (Dorian mode)

Newton ("An Hypothesis hinted at for explicating all the aforesaid properties of light," 1672) supposed that "the vibrations causing the deepest scarlet to be to those causing the deepest violet as two to one; for so there would be all that variety in colours which within the compass of an eight [octave] is found in sounds, & the reason why the extremes of colours Purple & scarlet resemble one another would be the same that causes Octaves (the extremes of sounds) to have in some measure the nature of unisons."

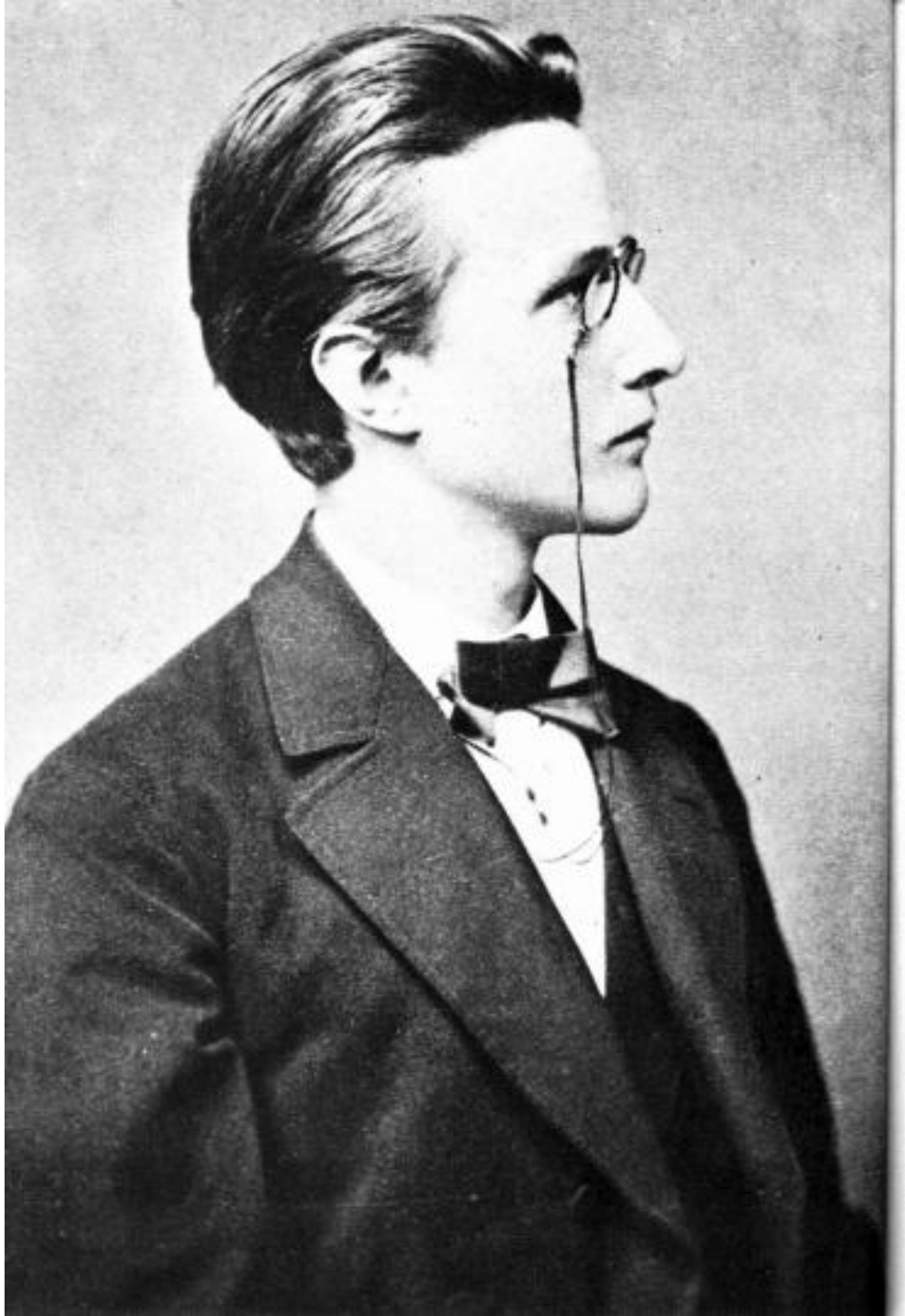
Newton ("An Hypothesis…," 1672): the ratio of the extreme colors in the rings was "greater than 3 to 2 & less than 5 to 3. By the most of my observations it was as 9 to 14."

Newton (*Opticks*, 1704): the rings "are to one another very nearly as the sixth lengths of a Chord which found the Notes in a sixth Major."



Newton's diagrams of the rings

Max Planck
in 1878



ein Kulturträger

a "bearer of culture"

“The outside world is something independent from man, something absolute, and the quest for the laws which apply to this absolute appeared to me as the most sublime scientific pursuit in life.”

Planck, “A Scientific Autobiography”

Max Planck (1858–1947)

University study in Munich (1874–77)

Studies in Berlin with Hermann von Helmholtz (1877–8)

On the Sensations of Tone as a Physiological Basis for the Theory of Music (1863), fourth edition 1877

Work on thermodynamics in Munich and Kiel (1879–92)

Appointed professor of physics in Berlin (1892)

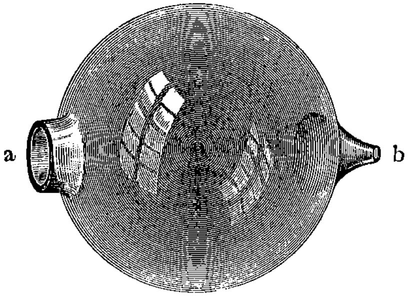
Work on Eitz harmonium (1893)

"On the Eitz harmonium" (1893)

"Tuning in modern vocal music" (1893)

Hermann von Helmholtz in 1881

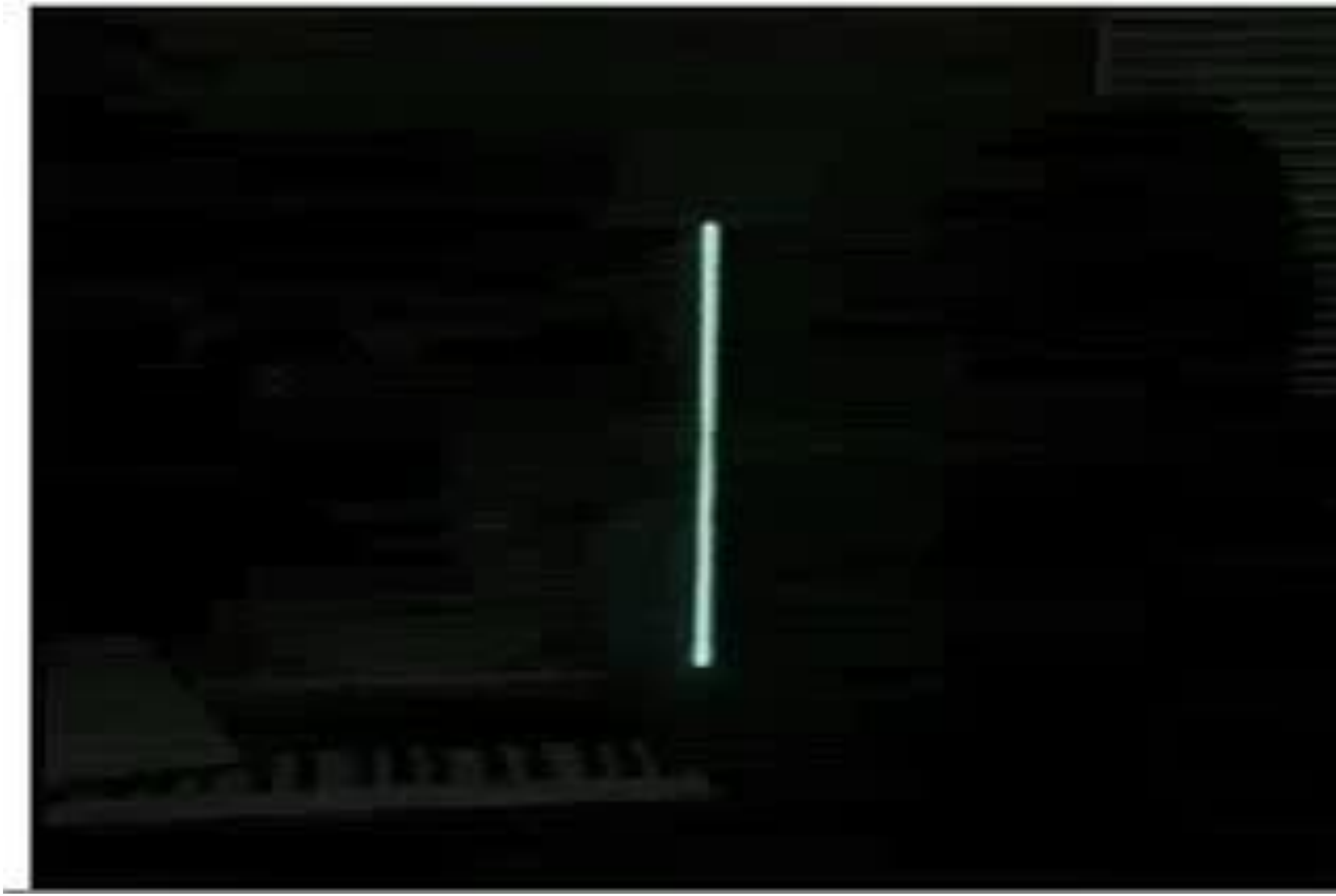
Detail of Helmholtz portrait



I think that no doubt can remain, if ever any doubt existed that *the [natural] intervals … are really natural for uncorrupted ears; that moreover the deviations of tempered intonation are really perceptible and unpleasant to uncorrupted ears; and lastly that … correct singing by natural intervals is much easier than singing in tempered intonation*.

Helmholtz, *On the Sensations of Tone* (fourth edition, 1877)

	Pythagorean Temperament	Just Intonation	Equal Temperament
octave	2:1	2:1	2:1
fifth	3:2	3:2	$\left(\sqrt[12]{2}\right)^7 : 1 = 1.4983....$
major third	81:64 = 1.265625	5:4 = 1.25	$\left(\sqrt[12]{2}\right)^4 : 1 = 1.25991....$
minor third	32:27 = 1.185185…	6:5 = 1.20	$\left(\sqrt[12]{2}\right)^3 : 1 = 1.1892....$



Just intonation ("natural tuning") vs. equal temperament ("tempered tuning")

Orlando di Lasso, Prologue (Carmen Chromatico) from *Prophetiae Sibyllarum* (ca. 1558)

Equal temperament

Adaptive just intonation

Planck's 1894 paper "Natural Tuning in Modern Vocal Music"

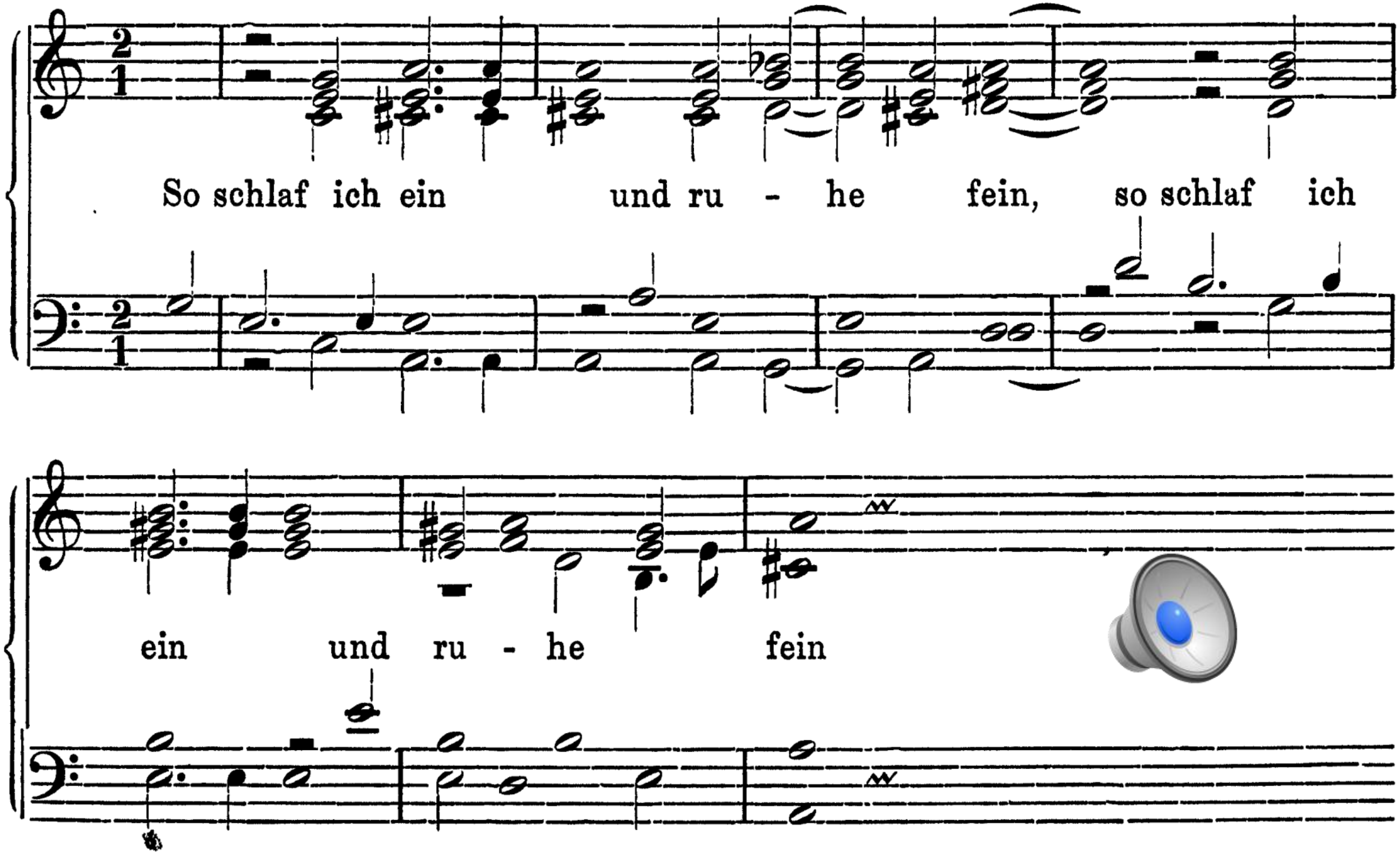
Die natürliche Stimmung in der modernen Vokalmusik.

Von

Max Planck.

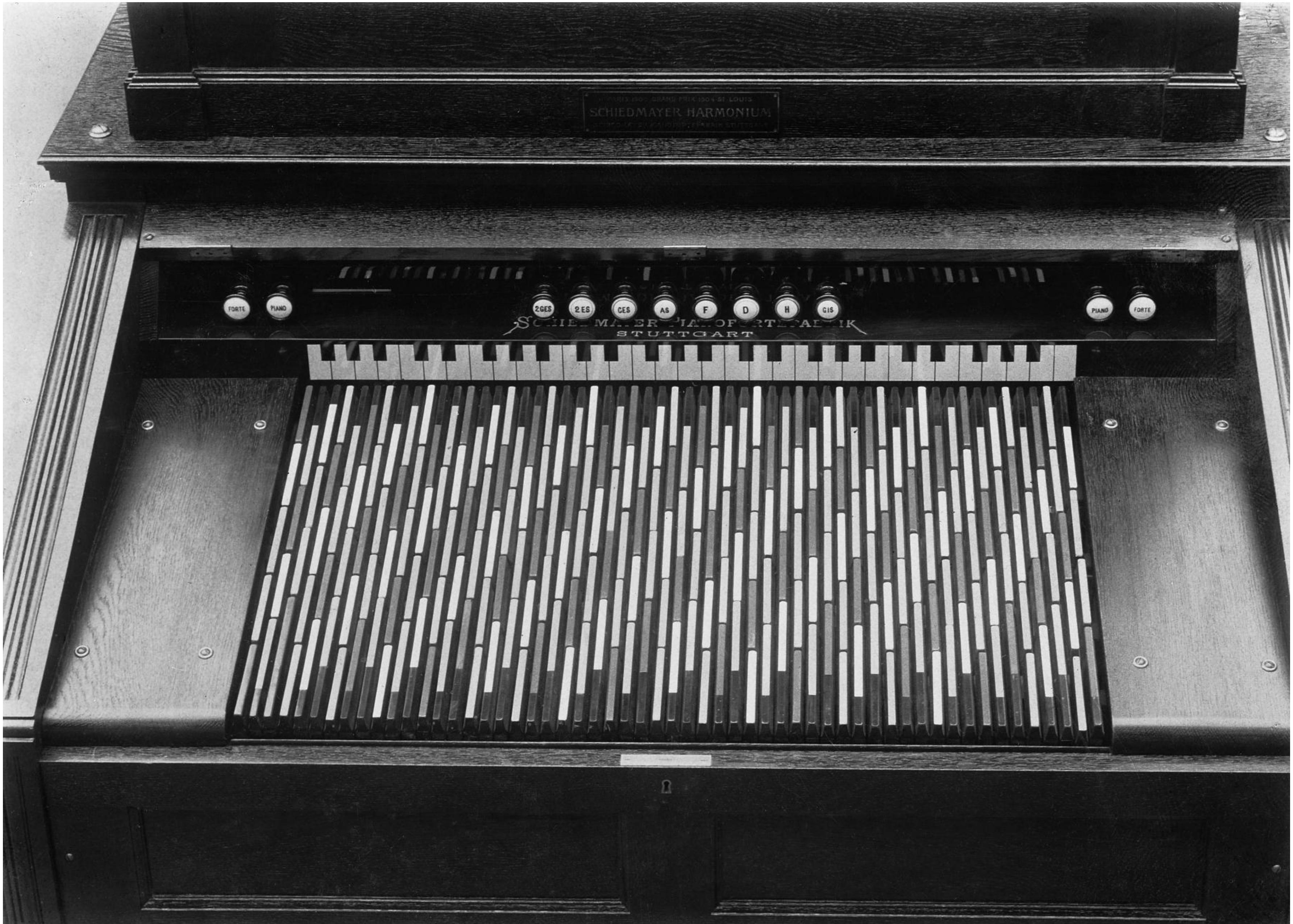
I.

Die Frage, ob der natürlichen Stimmung in der modernen Musik irgend eine praktische Bedeutung zukommt, trifft gegenwärtig in Musikerkreisen, soweit sich dieselben überhaupt mit ihr beschäftigen, noch auf sehr verschiedenartige Beurtheilung. Wohl die überwiegende Mehrzahl hält daran fest, daß es sich hier um geringfügige Unterschiede von mehr theoretischem Interesse handelt und daß in der praktischen Ausübung der Kunst weder heute noch auch in Zukunft sich irgend eine Veranlassung ergeben wird, eine andere als die nun schon durch mehr als zwei Jahrhunderte mit so glänzendem Erfolge bewährte temperirte Stimmung zu berücksichtigen. Daneben zeigen sich allerdings einzelne widersprechende Erscheinungen: so wird von aufmerksamen Geigenspielern ziemlich allgemein zugegeben, daß es gewisse Fälle giebt, wo ein Doppelgriff besser klingt, wenn er nicht genau im temperirten Intervall, sondern etwas abweichend davon genommen wird, und erfahrene Chordirigenten wissen, daß beim *a cappella*-Gesang in einem *piano* ausgehaltenen Durdreiklang die Terz leicht etwas zu tief genommen wird. Andere betrachten dagegen diese Erscheinungen nicht als sekundär, sondern gerade als die normalen: ich habe öfters von sachkundiger Seite die Ansicht aussprechen und sogar als ziemlich selbstverständlich hinstellen hören, daß das Streichquartett und der mehrstimmige Gesang sich immer nach der natürlichen Stimmung richte. Einige Theoretiker gehen sogar so weit, der temperirten Stimmung überhaupt jede Berechtigung abzusprechen, da sie sich von den natürlichen Verhältnissen entferne und sozusagen dem Ohr etwas vorlüge — ein Standpunkt, der sich allerdings Angesichts der thatsächlichen Leistungen der temperirten Stimmung von selber richtet.

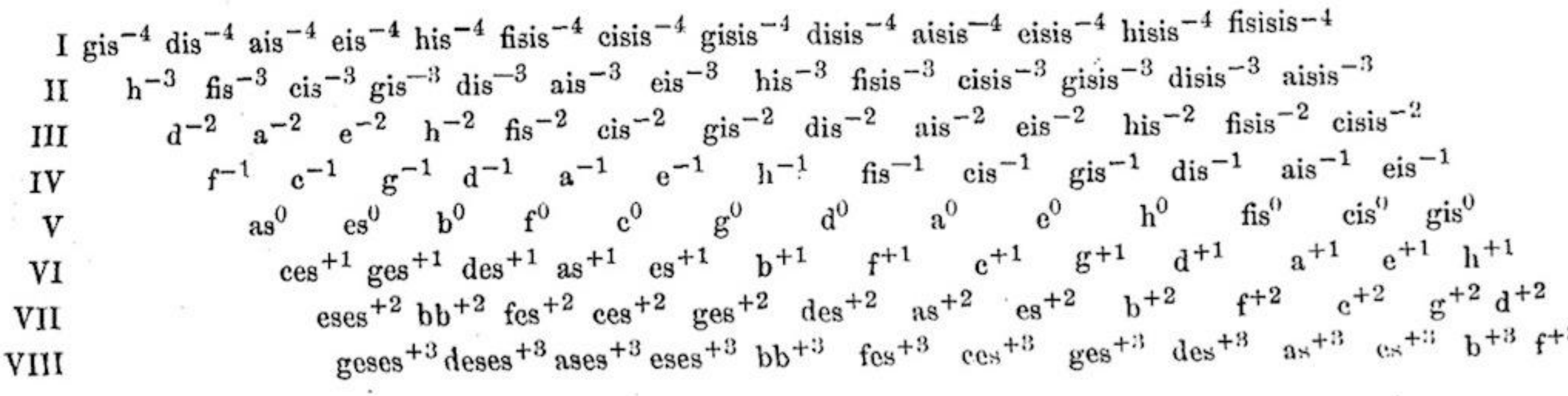


Planck's example from Heinrich Schütz's motet, "So fahr ich hin zu Jesu Christ" (SWV 379, *Geistliche Chorwerke* 1648). Text: "Thus I fall asleep and rest soundly."

Eitz harmonium (Deutsches Museum, Munich)



Eitz harmonium keyboard



Planck’s diagram of the Eitz harmonium keyboard

Planck's first composition

I	gis^{-4}	dis^{-4}	ais^{-4}	eis^{-4}	his^{-4}	fisis^{-4}	cisis^{-4}	gisis^{-4}	disis^{-4}	aisis^{-4}	eisis^{-4}	hisis^{-4}	fisisis^{-4}
II	h^{-3}	fis^{-3}	cis^{-3}	gis^{-3}	dis^{-3}	ais^{-3}	eis^{-3}	his^{-3}	fisis^{-3}	cisis^{-3}	gisis^{-3}	disis^{-3}	aisis^{-3}
III	d^{-2}	a^{-2}	e^{-2}	h^{-2}	fis^{-2}	cis^{-2}	gis^{-2}	dis^{-2}	ais^{-2}	eis^{-2}	his^{-2}	fisis^{-2}	cisis^{-2}
IV	f^{-1}	c^{-1}	g^{-1}	d^{-1}	a^{-1}	e^{-1}	h^{-1}	fis^{-1}	cis^{-1}	gis^{-1}	dis^{-1}	ais^{-1}	eis^{-1}
V	as^{0}	es^{0}	b^{0}	f^{0}	c^{0}	g^{0}	d^{0}	a^{0}	e^{0}	h^{0}	fis^{0}	cis^{0}	gis^{0}
VI	ces^{+1}	ges^{+1}	des^{+1}	as^{+1}	es^{+1}	b^{+1}	f^{+1}	c^{+1}	g^{+1}	d^{+1}	a^{+1}	e^{+1}	h^{+1}
VII	eses^{+2}	bb^{+2}	fes^{+2}	ces^{+2}	ges^{+2}	des^{+2}	as^{+2}	es^{+2}	b^{+2}	f^{+2}	c^{+2}	g^{+2}	d^{+2}
VIII	geses^{+3}	deses^{+3}	ases^{+3}	eses^{+3}	bb^{+3}	fes^{+3}	ces^{+3}	ges^{+3}	des^{+3}	as^{+3}	es^{+3}	b^{+3}	f^{+3}

Planck's diagram of the Eitz harmonium keyboard, showing *Tonnetz* for his test compositions

Planck’s second composition

Max Planck (1858–1947)

University study in Munich (1874–77)

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"The black year of German physics" (1894)
Deaths of August Kundt, Heinrich Hertz, and Hermann von Helmholtz

Return to studies of black-body radiation (1894–1914)

Planck's black-body radiation law (1900)

Figure from Otto Lummer and Ernst Pringsheim, “On the radiation of black bodies for long wavelengths” (1900). Their observations are the solid line; dotted lines show earlier theoretical predictions.

Eine bessere Uebersicht gewinnt man aus dem tabellarisch zusammengestellten Beobachtungsmaterial. Wir begnügen uns hier mit der Wiedergabe der Resultate für einige

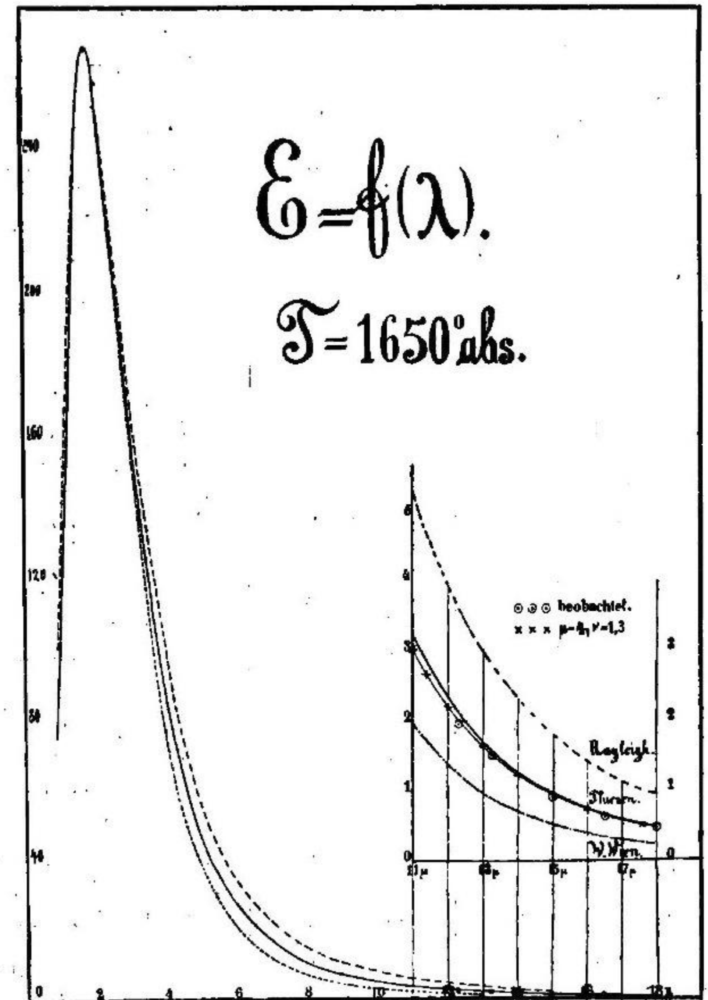
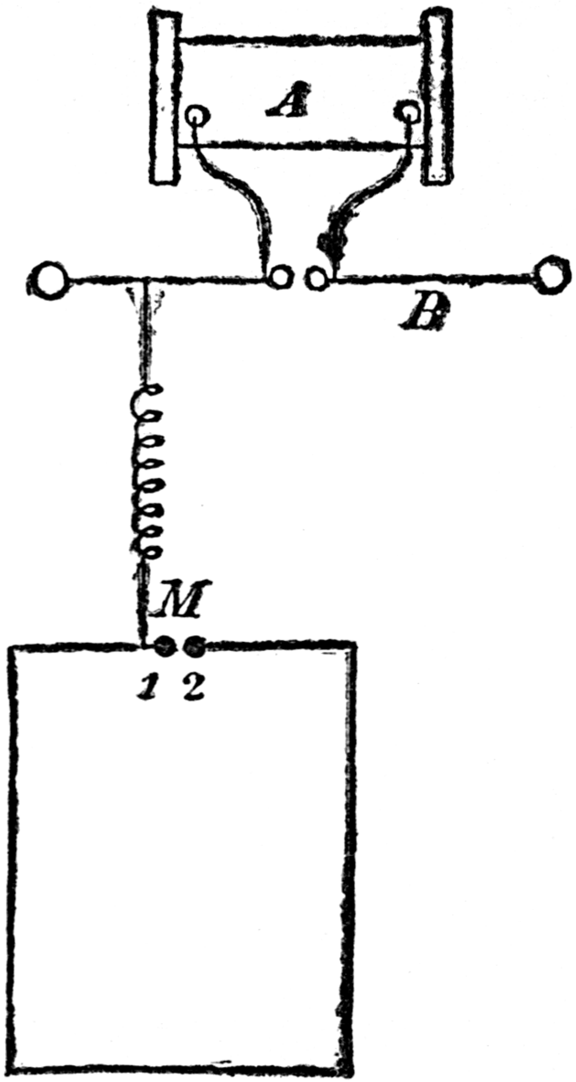


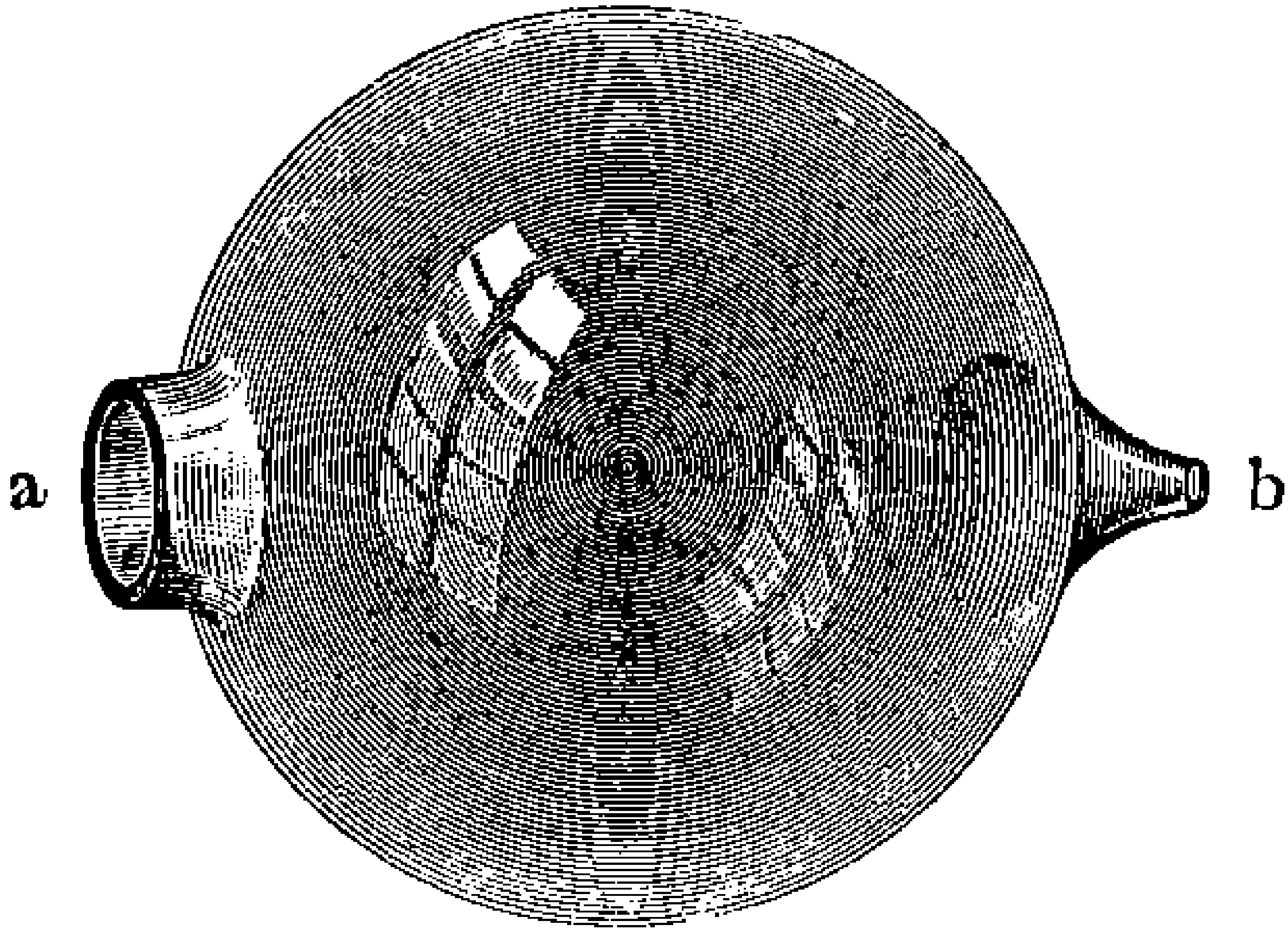
Fig. 3.

Temperaturen. In Tab. III sind die auf den Maassstab der Flussspatbeobachtungen reducirten Energien unter „beobachtet“ eingetragen und mit den Resultaten der Lummer-Jahnke'schen Spectralgleichung (8) für die Wertepaare $\mu = 5$; $\nu = 1$ (W. Wien), $\mu = 4,5$; $\nu = 1$ (Thiesen) und $\mu = 4$; $\nu = 1,3$ zusammengestellt.

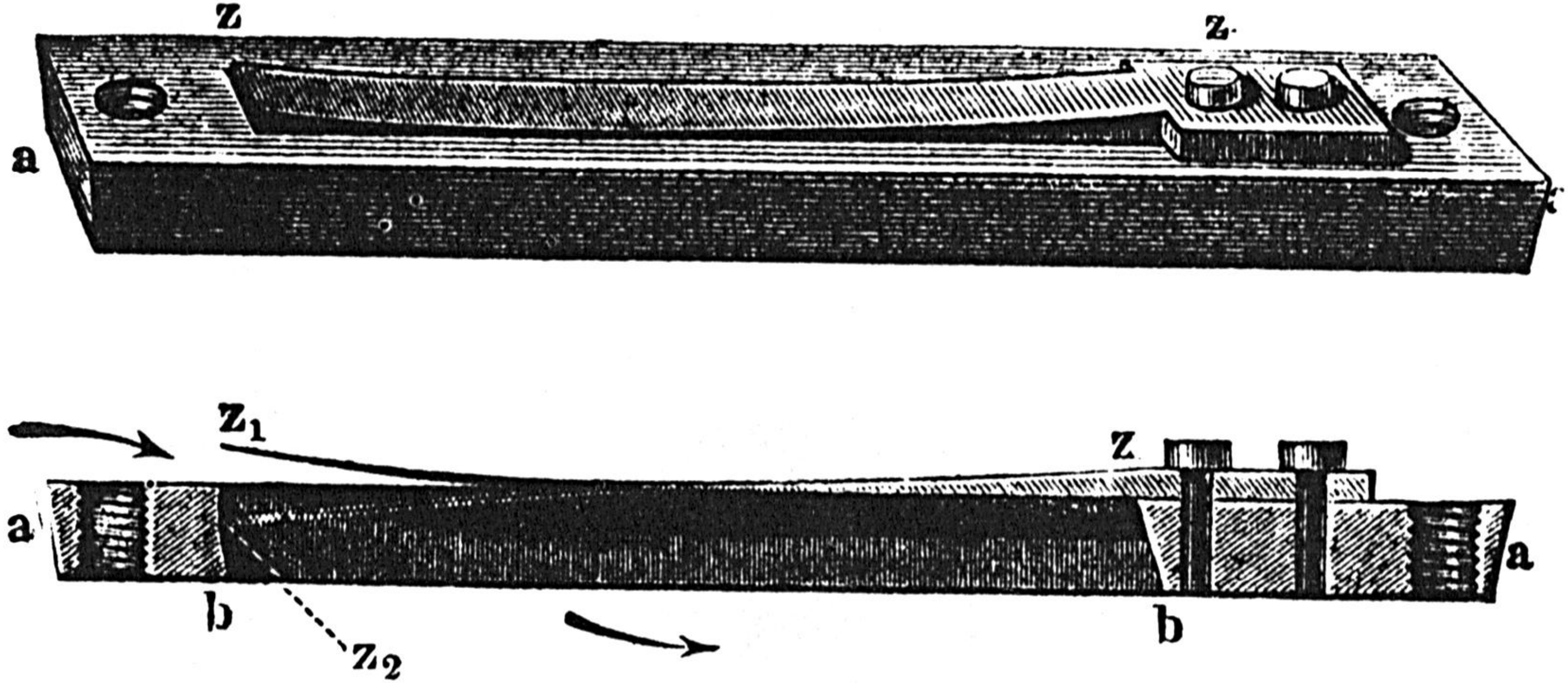


Heinrich Hertz, oscillator to produce electromagnetic waves (18)

Herz oscillator in operation



Resonator to isolate overtone, from Helmholtz, *On the Sensations of Tone* (1863)



Harmonium reed resonator, from Helmholtz, *On the Sensations of Tone* (1863)

§ 10. Wenden wir das Wien'sche Verschiebungsgesetz in der letzten Fassung auf den Ausdruck (6) der Entropie S an, so erkennen wir, dass das Energieelement ε proportional der Schwingungszahl ν sein muss, also:

$$\varepsilon = h \, . \, \nu$$

und somit:

$$S = k \left\{ \left(1 + \frac{U}{h\nu}\right) \log \left(1 + \frac{U}{h\nu}\right) - \frac{U}{h\nu} \log \frac{U}{h\nu} \right\}.$$

Hierbei sind h und k universelle Constante.

Planck's original statement of the quantization of the "energy element," $\varepsilon = h\nu$ (October, 1900) aussermenschliche Culturen nothwendig behalten und welche daher als »natürliche Maasseinheiten« bezeichnet werden können.

Die Mittel zur Festsetzung der vier Einheiten für Länge, Masse, Zeit und Temperatur werden gegeben durch die beiden erwähnten Constanten a und b, ferner durch die Grösse der Lichtfortpflanzungsgeschwindigkeit c im Vacuum und durch die der Gravitationsconstante f. Bezogen auf Centimeter, Gramm, Secunde und Celsiusgrad sind die Zahlenwerthe dieser vier Constanten die folgenden:

$$a = 0.4818 \cdot 10^{-10} [\mathrm{sec} \times \mathrm{Celsiusgrad}]$$

$$b = 6.885 \cdot 10^{-27} \left[\frac{\mathrm{cm}^2 \mathrm{gr}}{\mathrm{sec}}\right]$$

$$c = 3.00 \cdot 10^{10} \left[\frac{\mathrm{cm}}{\mathrm{sec}}\right]$$

$$f = 6.685 \cdot 10^{-8} \left[\frac{\mathrm{cm}^3}{\mathrm{gr.\,sec}^2}\right]$$ [1].

Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 an-

Planck’s original statement of his natural units (May, 1900)

These necessarily maintain their meaning for all times and for all cultures, even extraterrestrial and non-human ones, and can therefore be designated as "natural units."

Planck's comment (May, 1899)

Planck time: $\sqrt{\frac{\hbar G}{c^5}} = 5.391 \times 10^{-44}$ sec

"Planck frequency": $\sqrt{\frac{c^5}{\hbar G}} = 1.855 \times 10^{43}\, Hz$

Assuming standard A440, this corresponds to the pitch
426 Hz 135 octaves above middle C:
in terms of present standard pitch A440, a quite low A

Alternatively, any energy E can be converted into its *absolute pitch* $n = \frac{E}{h}$.

A knowledge of the historic and philosophic background gives that kind of independence from prejudices of his generation from which most scientists are suffering. This independence created by philosophical thought is—in my opinion—the mark of distinction between a mere artisan or specialist and a real seeker after truth.

Albert Einstein, Letter to Robert Thornton (December 7, 1944)