Triangle anomalies in Landau's Fermi liquid theory

Dam Thanh Son
Institute for Nuclear Theory, University of Washington

DTS, N. Yamamoto, 1203.2697

Triangle anomalies

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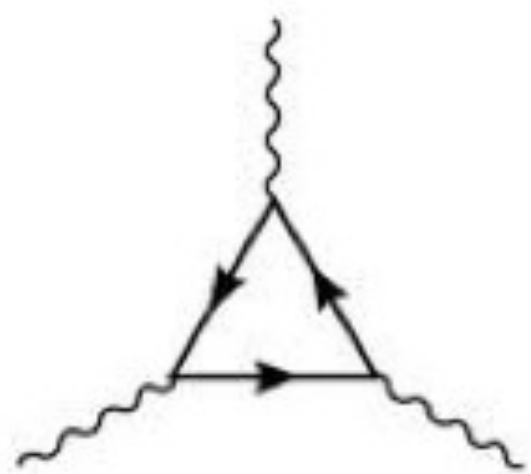
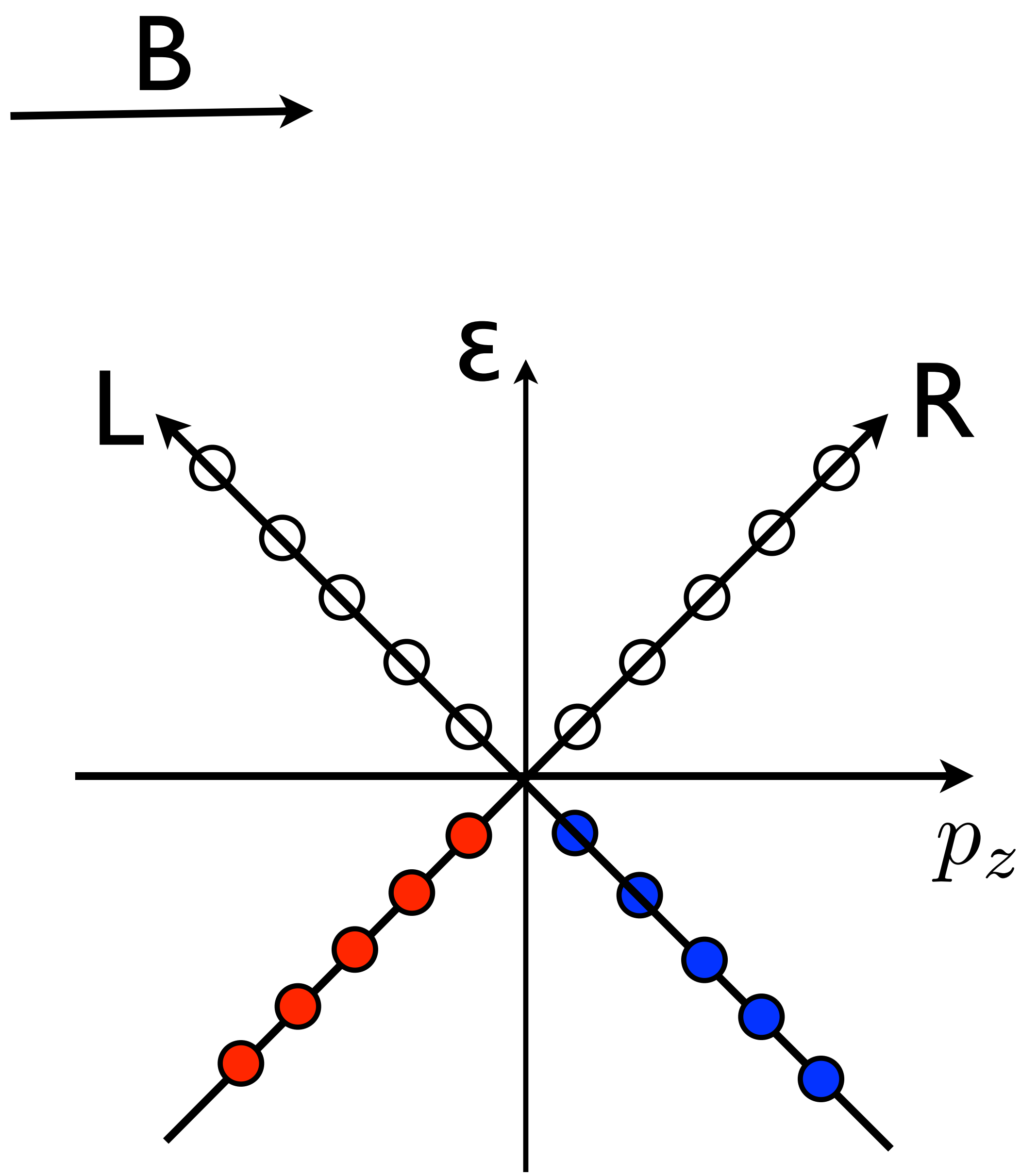


What are triangle anomalies

- Triangle anomalies are inherently quantum features of 4D quantum field theories
- Symmetry of a classical theory, broken by quantum effects
- Deep connections to topology
- First found by Adler, Bell, and Jackiw while considering decay of neutral pions: $\pi^0 \to 2\gamma$

Triangle anomalies

Massless fermions: lowest Landau level is chiral



Triangle anomalies

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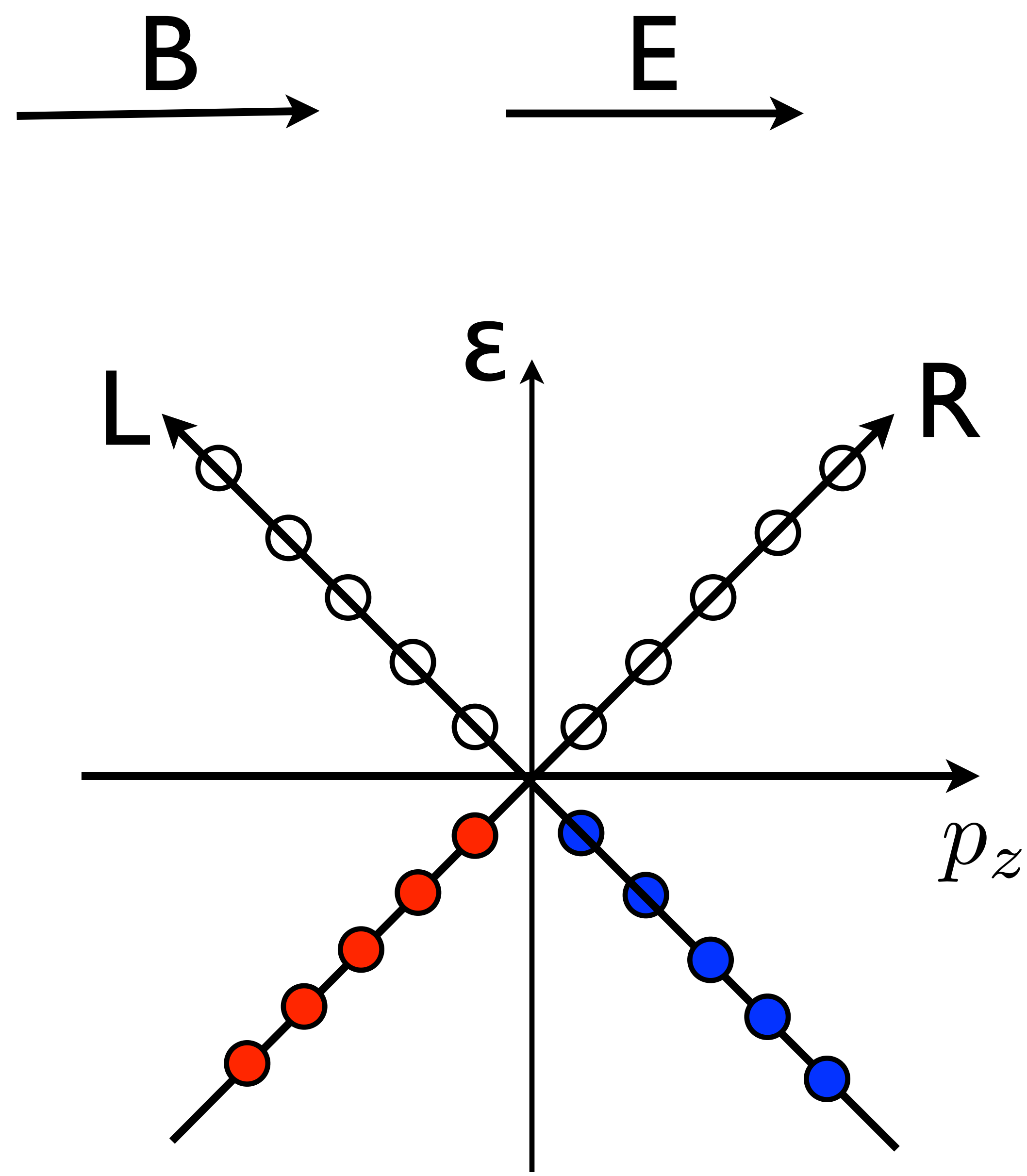

Triangle anomalies

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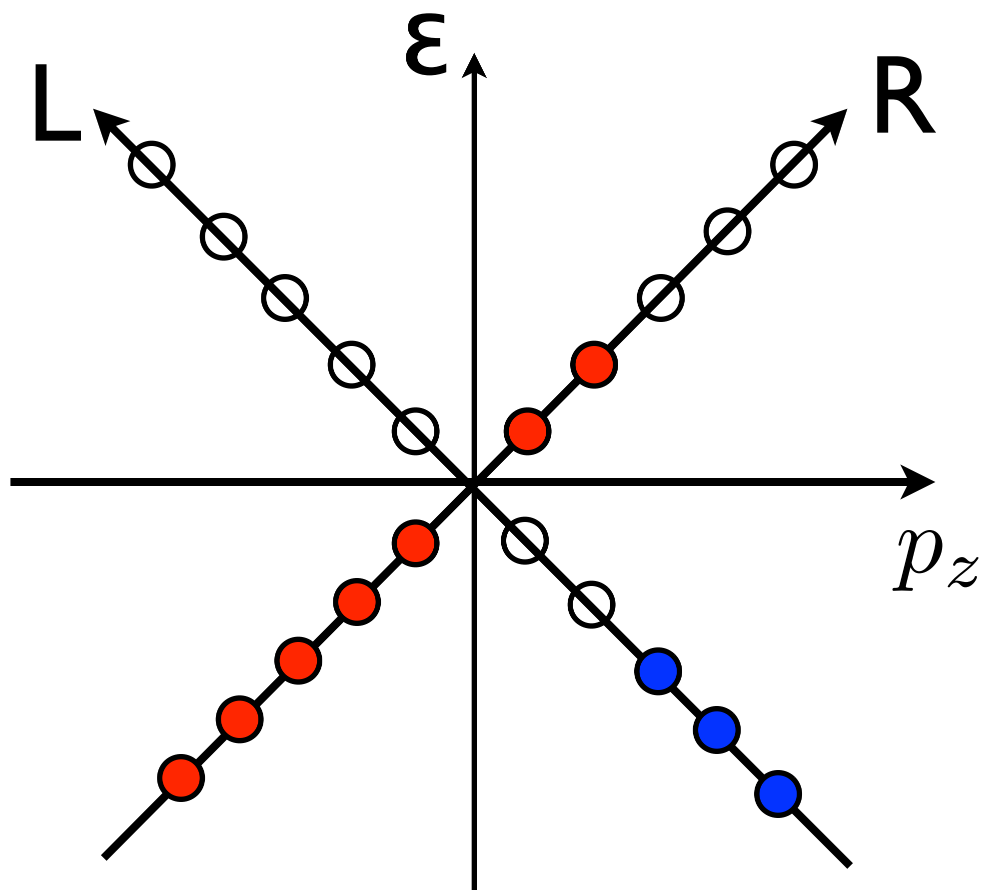
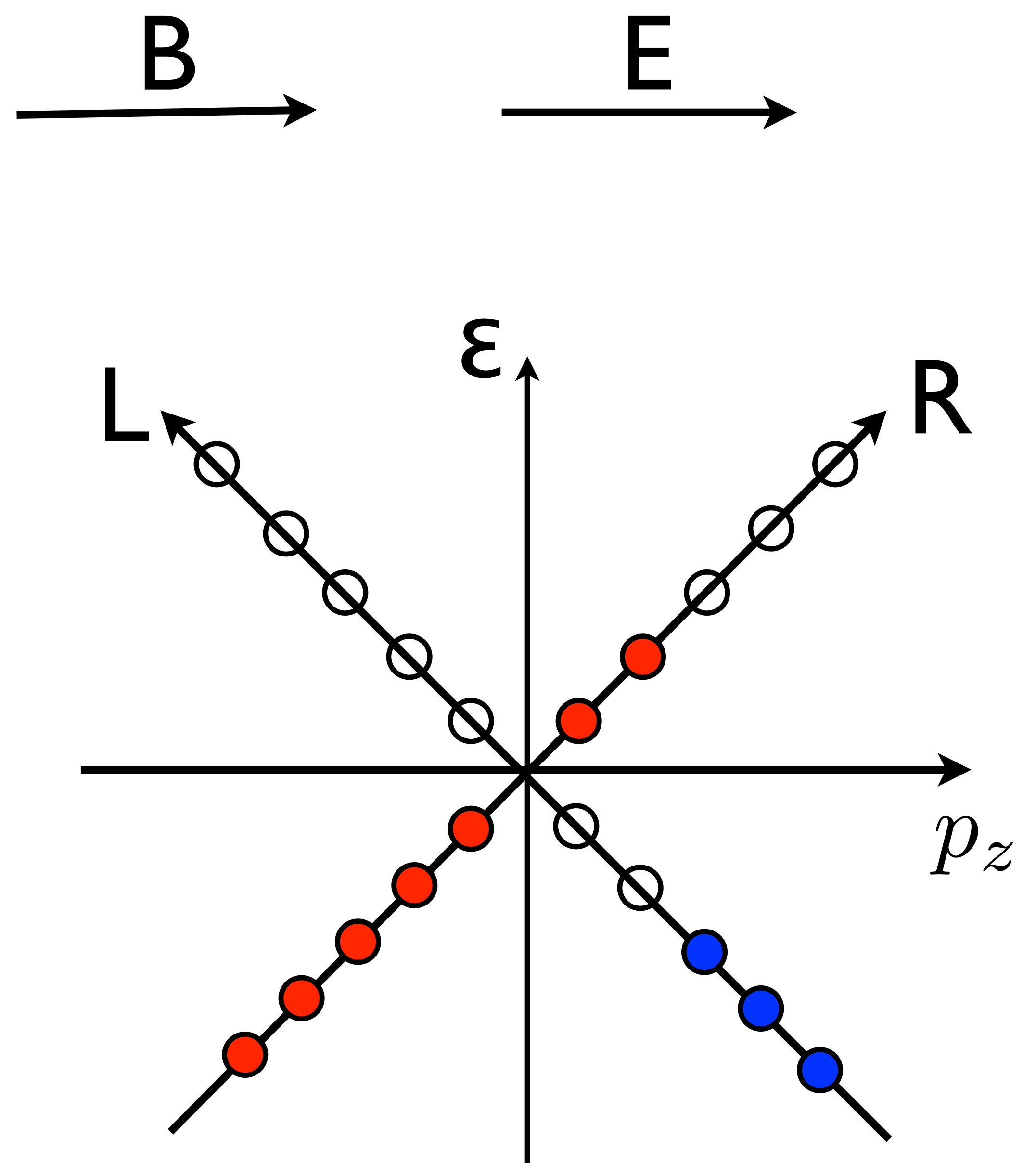


Triangle anomalies

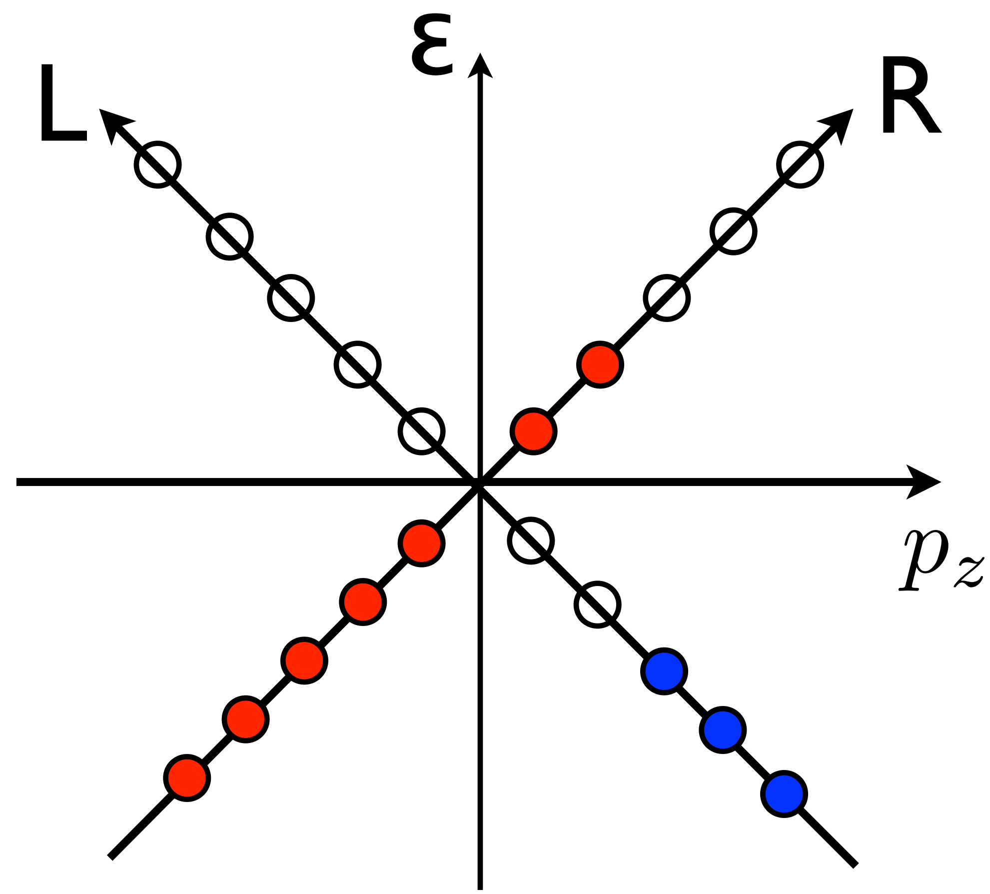
Massless fermions: lowest Landau level is chiral



$$\partial_\mu j^\mu_{\mathrm{R,L}} = \pm \frac{1}{4\pi^2} \mathbf{E} \cdot \mathbf{B}$$

Triangle anomalies

Massless fermions: lowest Landau level is chiral



$$\partial_\mu j^\mu_{\mathrm{R,L}} = \pm \frac{1}{4\pi^2} \mathbf{E} \cdot \mathbf{B}$$

exact

Anomalous hydrodynamics

- Recently anomalies have been found to exhibit themselves in a regime one would normally think as completely classical: the hydrodynamic regime
  - finite temperature, length scales >> mean free path
- Largely due to gauge/gravity duality, more concretely: fluid/gravity correspondence

$$j^{5\mu} = n_5 u^\mu - \sigma T (g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \frac{\mu}{T} + \xi \epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

convection diffusion vorticity

If sugar behaved that way

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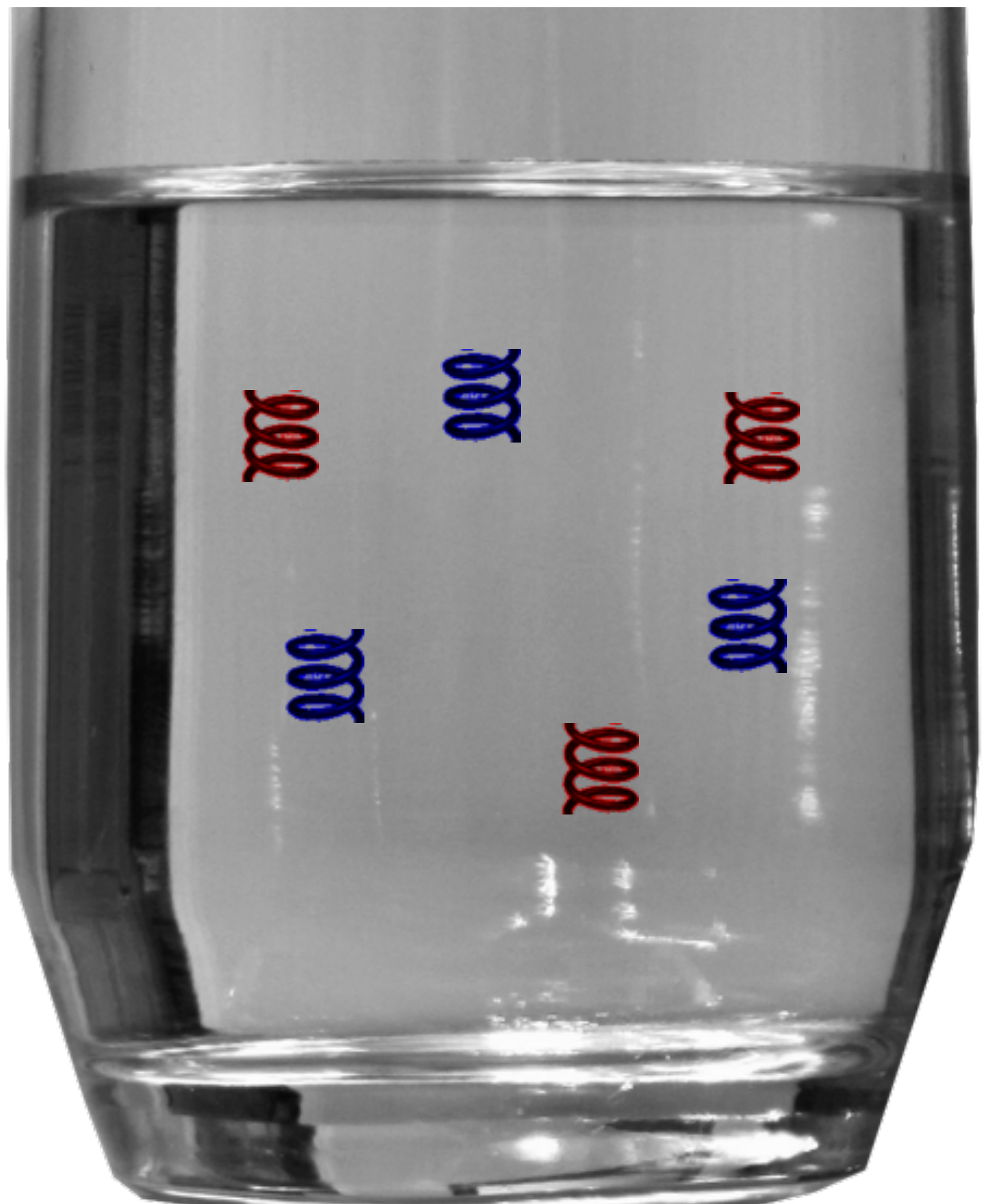
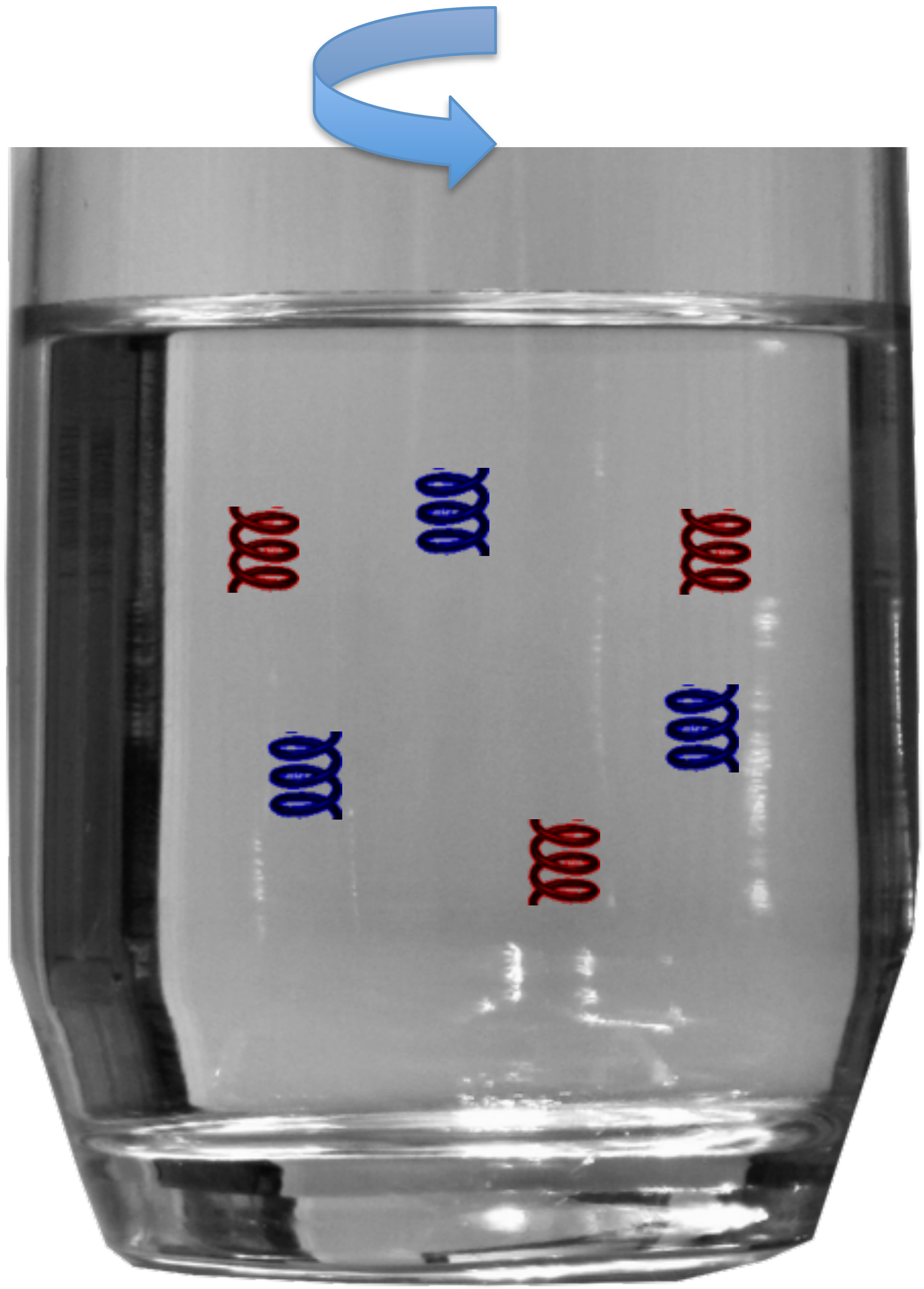
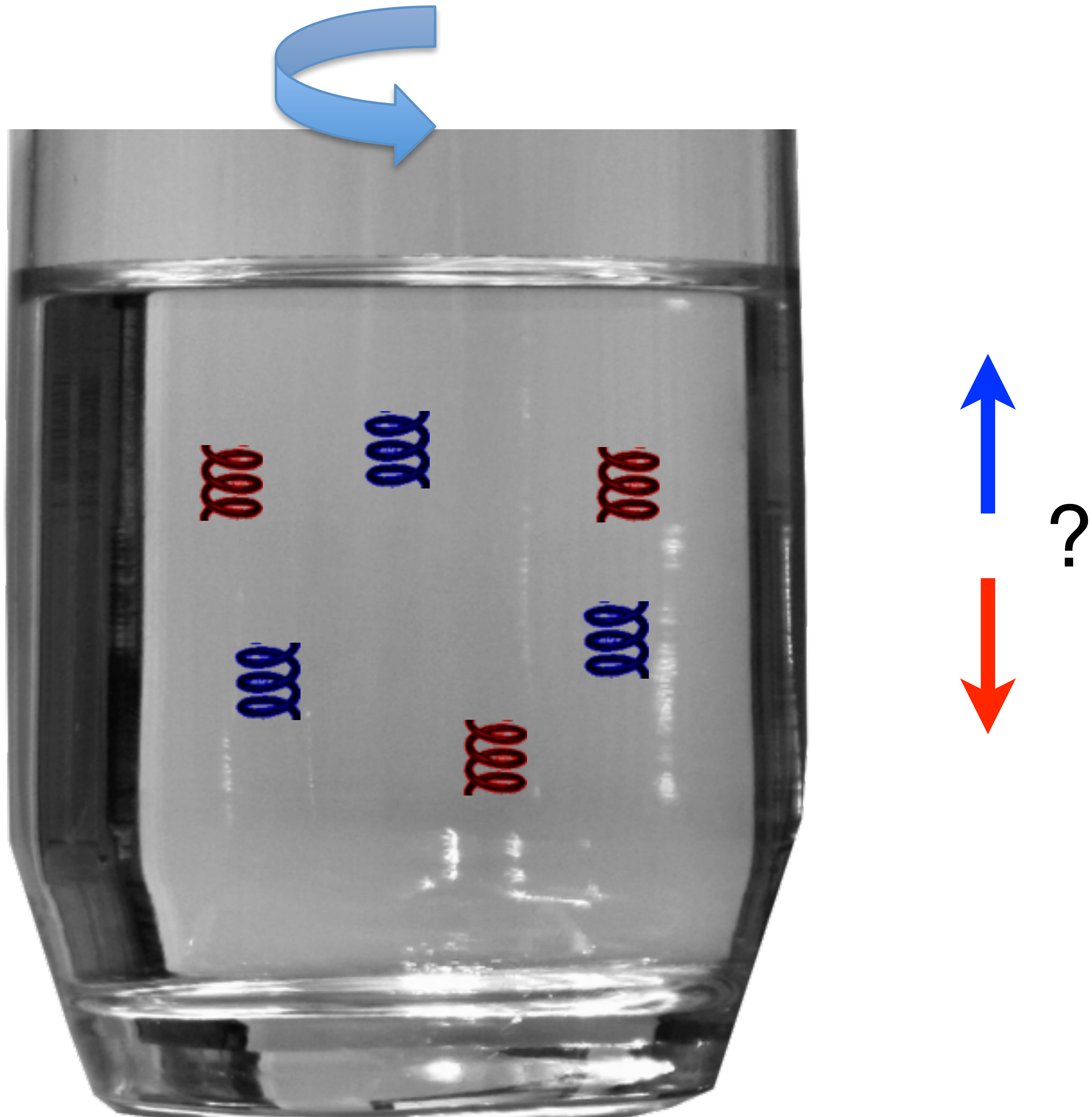


If sugar behaved that way



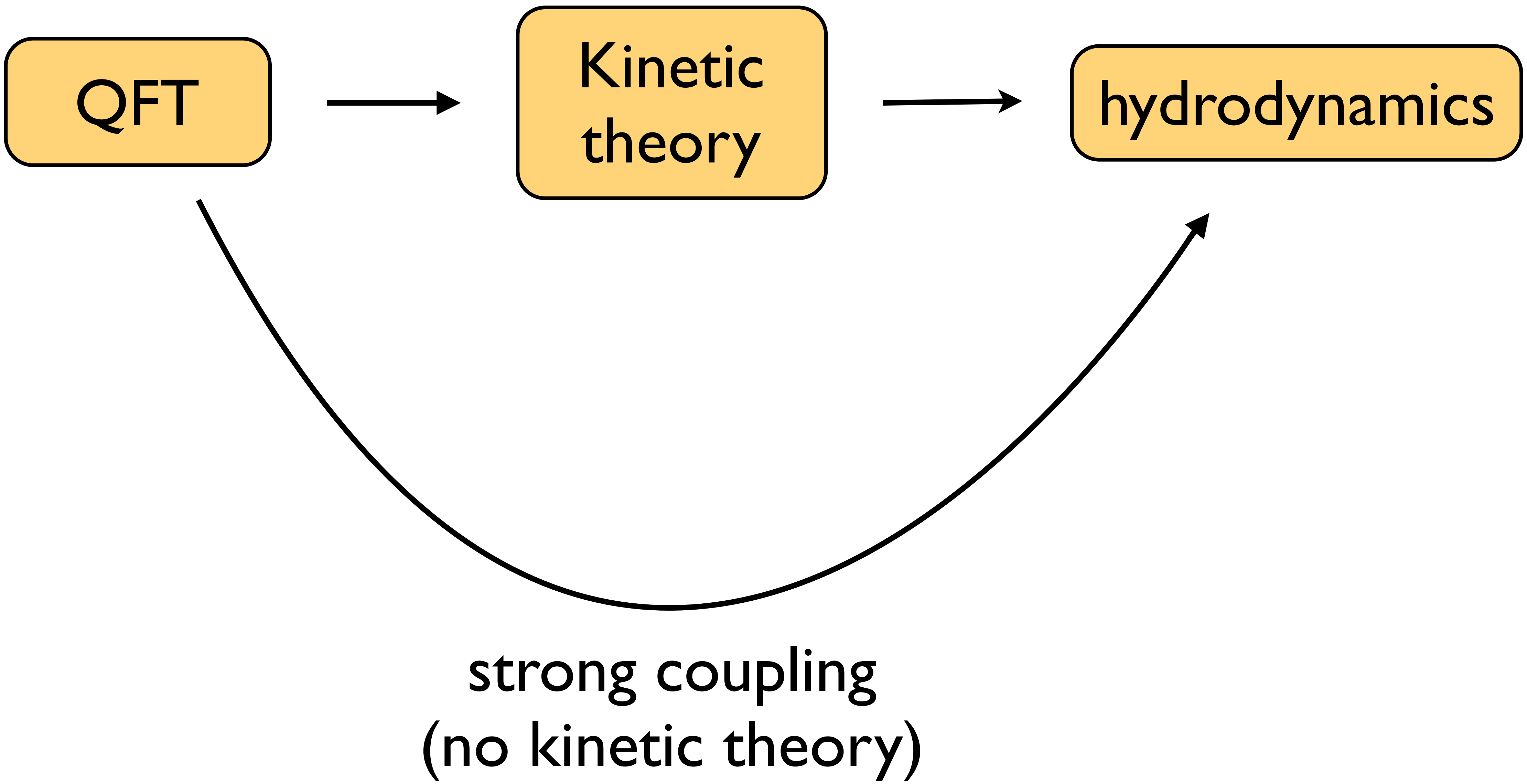
If sugar behaved that way



Beyond gauge/gravity duality

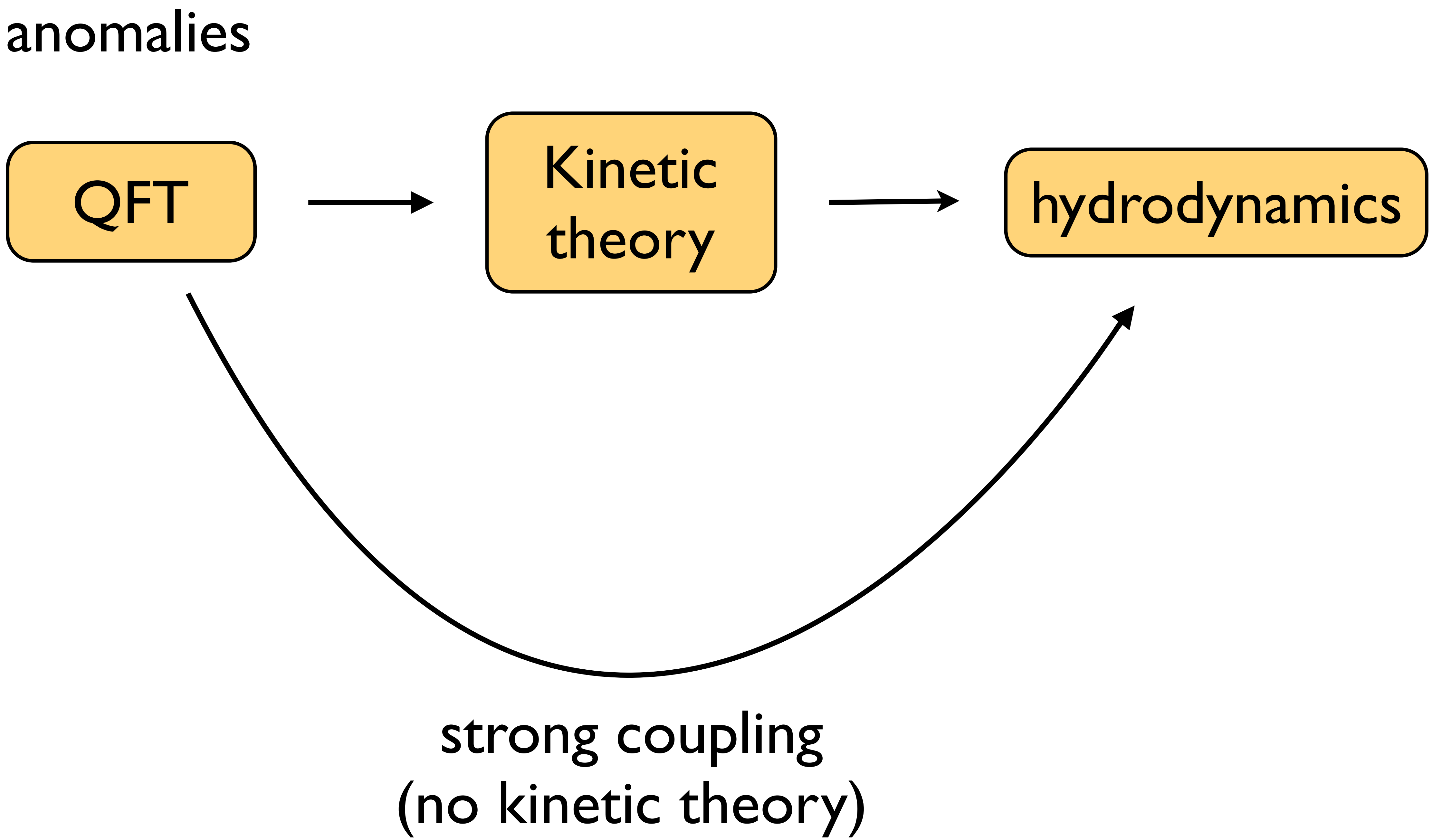
- Although anomalous effects in hydrodynamics were first seen through gauge/gravity duality, we now understand that they exist in a general setting
  - in particular, they do not depend on coupling
  - non only at strong coupling (gauge/gravity duality), but also at weak coupling

Weak coupling



Most important example of kinetic theory: Landau's Fermi liquid theory

Weak coupling



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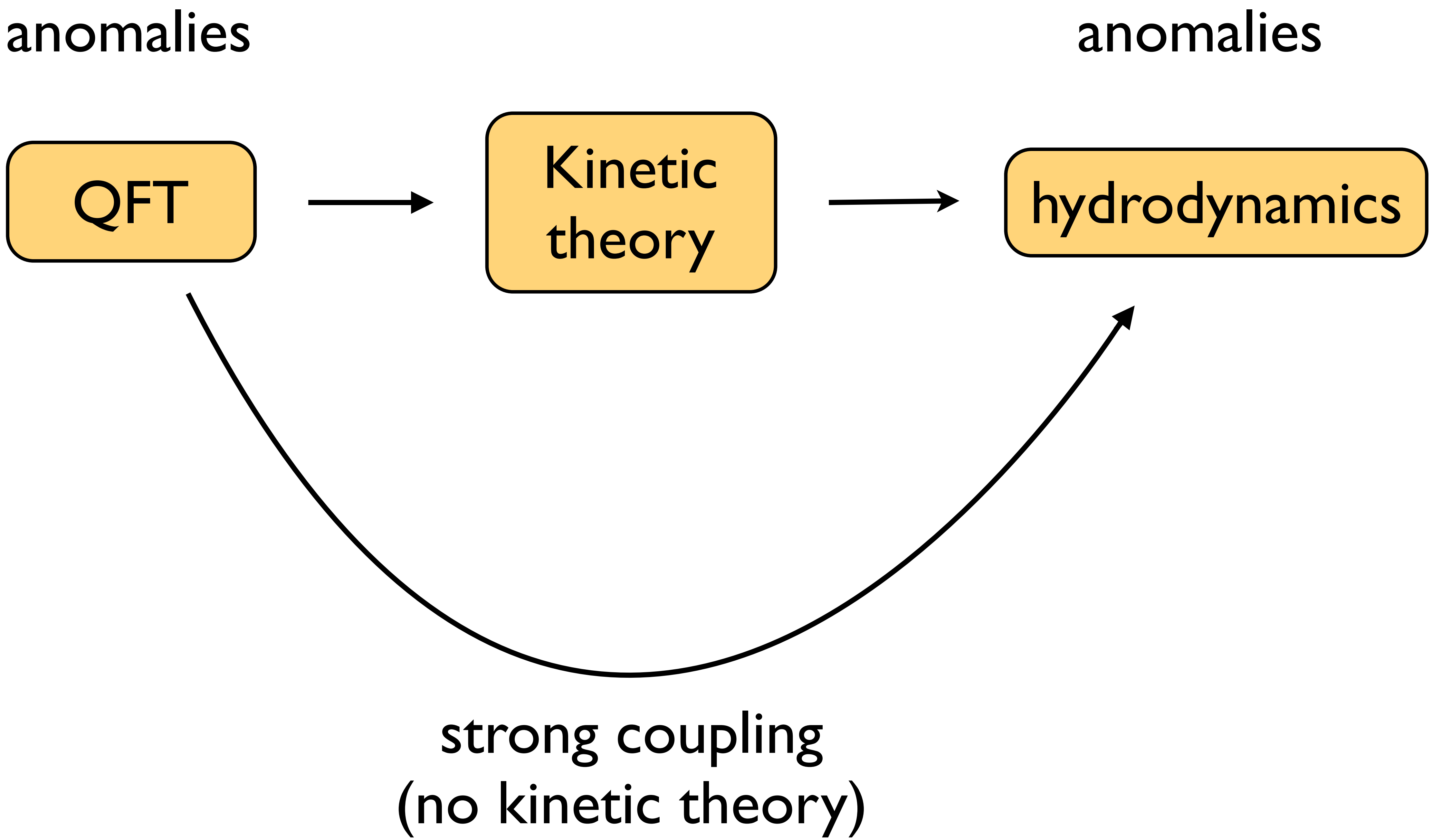

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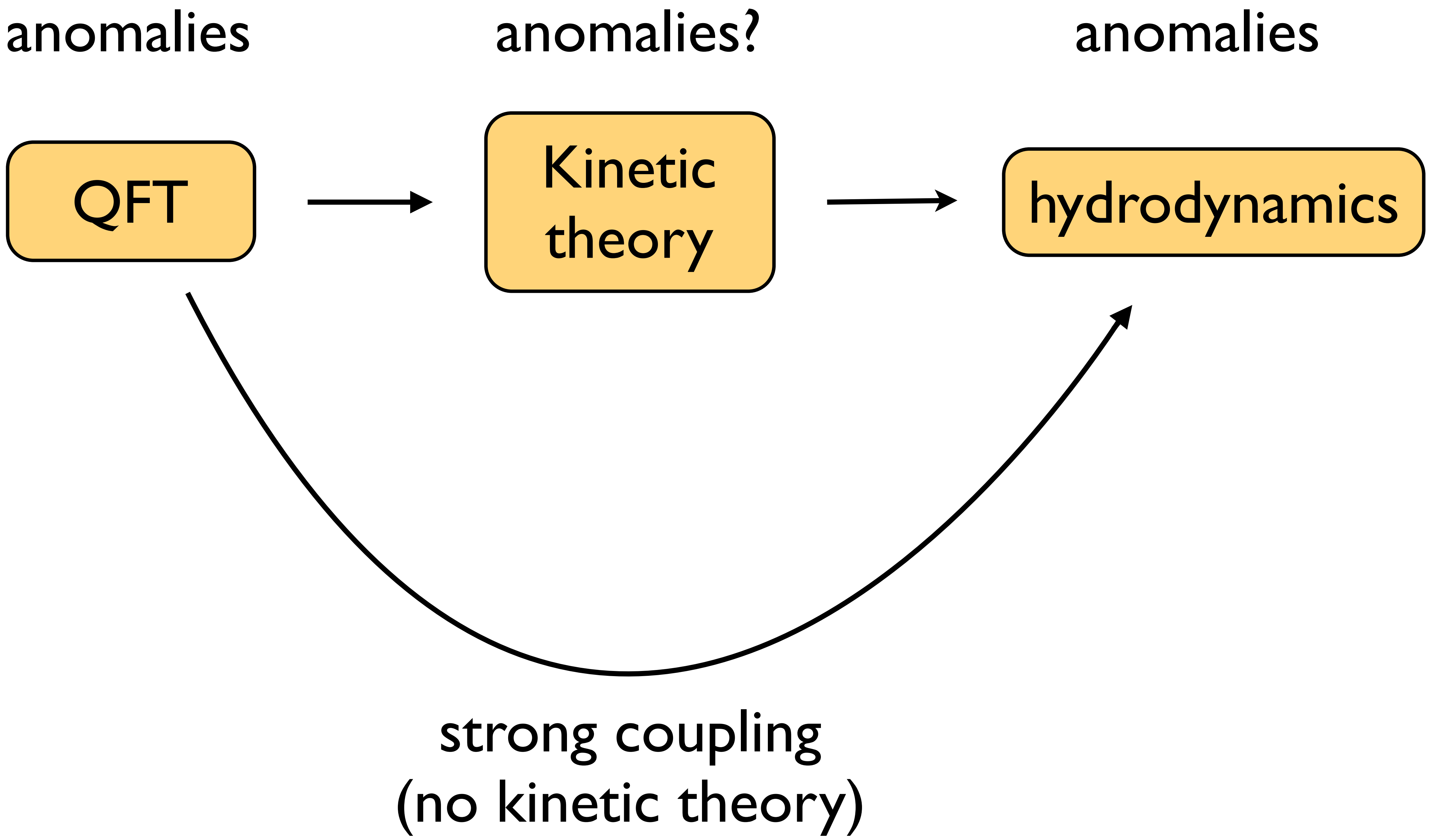
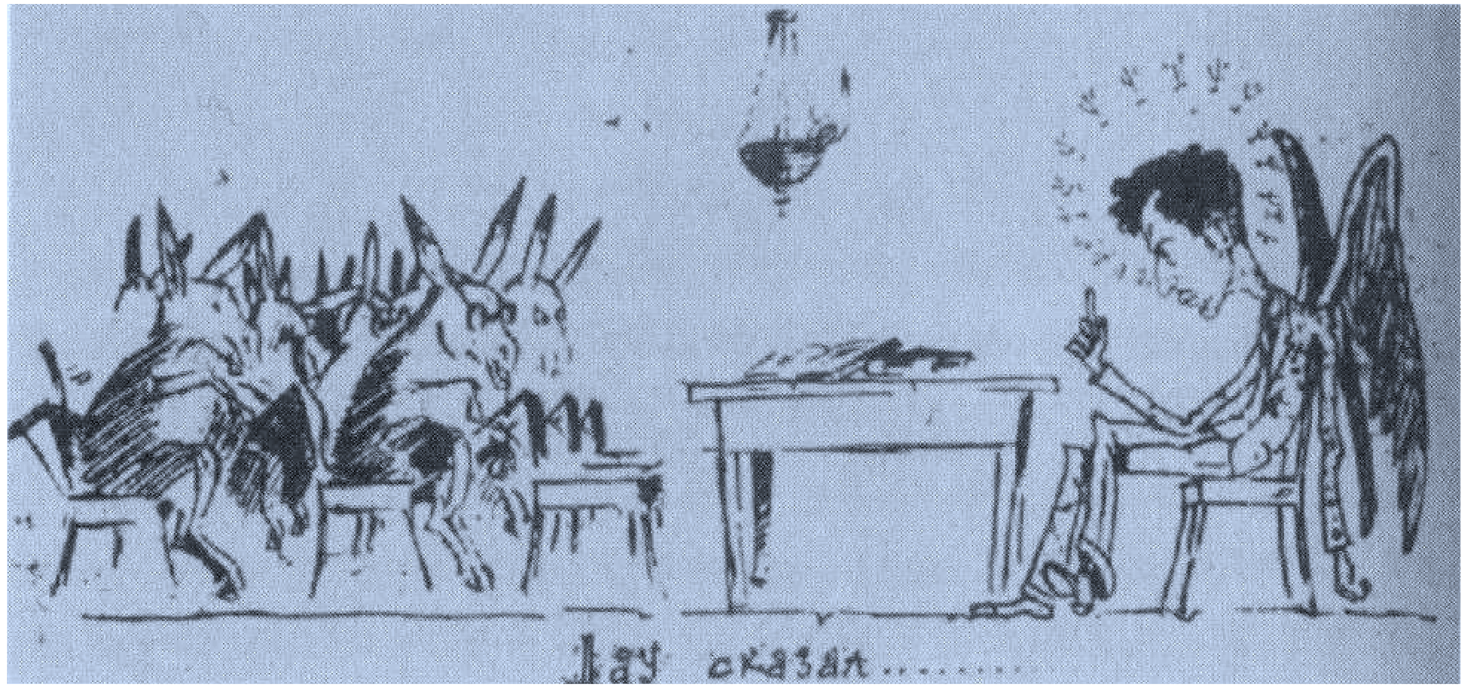


Most important example of kinetic theory: Landau's Fermi liquid theory

Landau’s Fermi liquids

Landau’s Fermi liquids



Landau’s Fermi liquids

- Low-energy degrees of freedom: quasiparticles near Fermi surface
- interaction: forward-scatterings of quasiparticles

Landau's Fermi liquids

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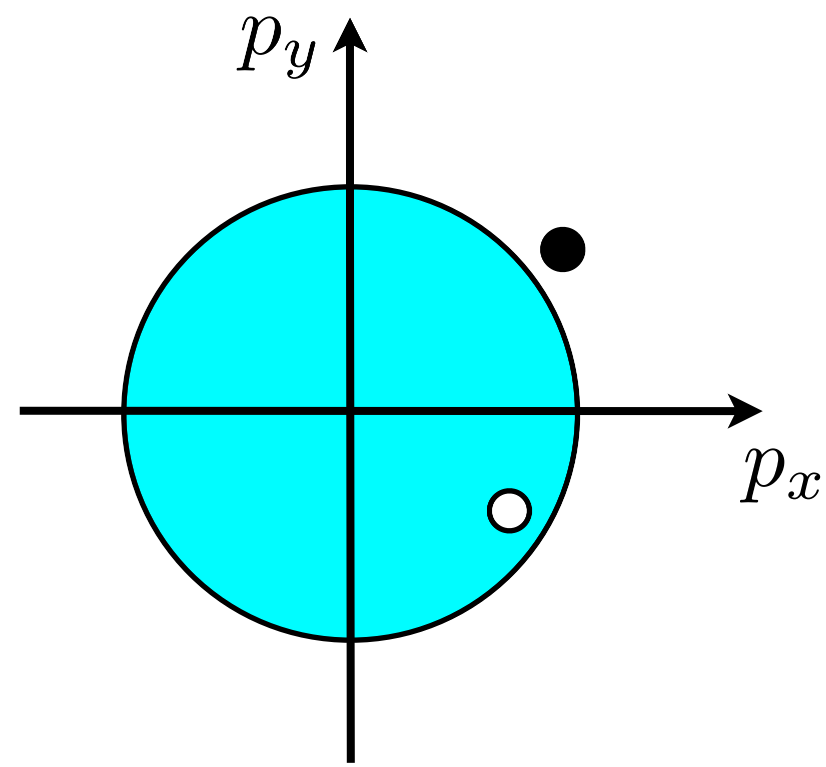


RG interpretation of Fermi liquid theory

- An effective action for quasiparticle
- power counting
- BCS interaction is the only marginally relevant interaction (Polchinski, Shankar)

Fermi liquids

- Dynamics: kinetic equation

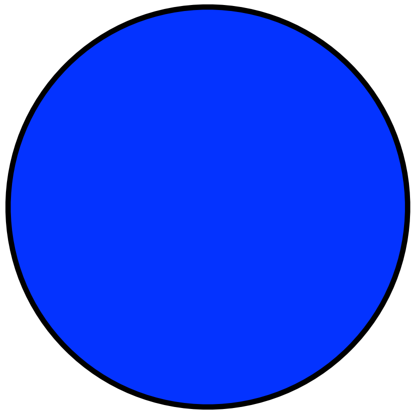
$$\frac{\partial n_{\mathbf{p}}(\mathbf{x})}{\partial t} + \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{x}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = 0$$

$$\epsilon_{\mathbf{p}} = \epsilon^0_{\mathbf{p}} + \delta\epsilon_{\mathbf{p}}$$

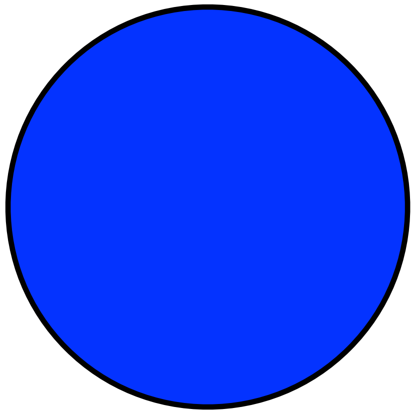
$$\epsilon^0_{\mathbf{p}} = v_{\mathrm{F}}(|\mathbf{p}| - p_{\mathrm{F}}) \qquad \delta\epsilon_{\mathbf{p}} = \int \frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\, f(\mathbf{p},\mathbf{q})\delta n_{\mathbf{q}}(\mathbf{y})$$

Predictions: heat capacity, spin susceptibility, zero sound...

Anomalies in Fermi's liquids



Fermi sphere of
left-handed fermions



Fermi sphere of
right-handed fermions

Anomalies in Fermi's liquids

Fermi sphere of
left-handed fermions

Fermi sphere of
right-handed fermions

$$\vec{E} \cdot \vec{B} \neq 0$$

Anomalies in Fermi's liquids

Fermi sphere of left-handed fermions

Fermi sphere of right-handed fermions

$$\vec{E} \cdot \vec{B} \neq 0$$

Anomalies in Fermi liquids

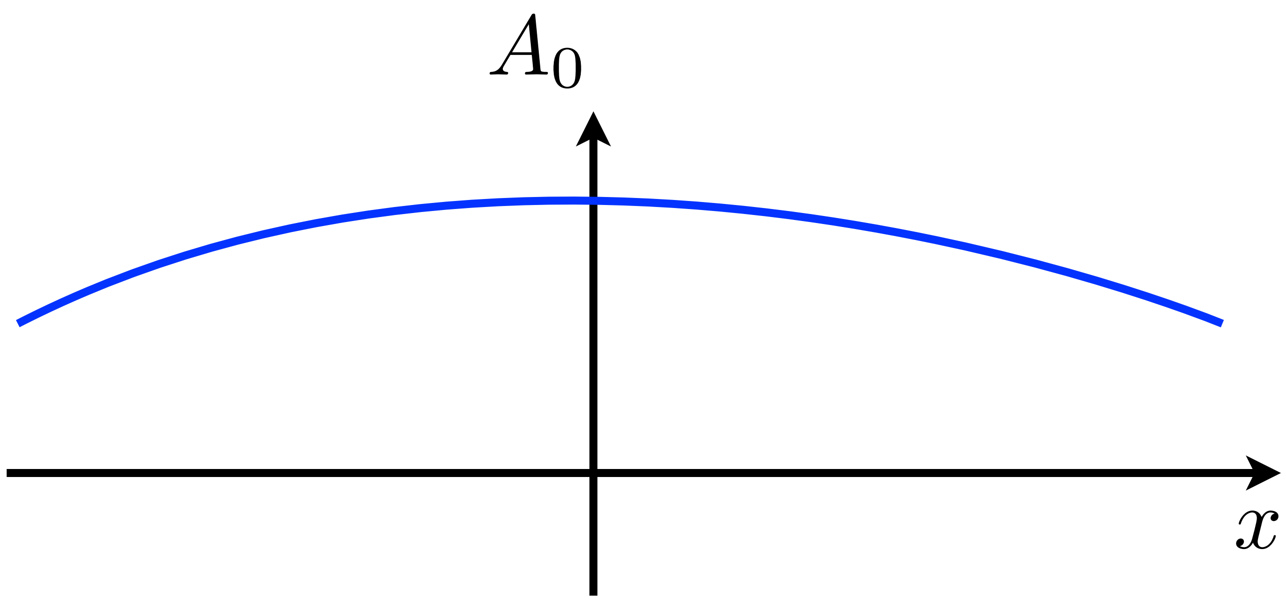
- How does Landau's Fermi liquid theory discriminate left- and right-handed quasiparticles?
- Through magnetic moment?

$$\epsilon_{\mathbf{p}} = |\mathbf{p}| - \gamma \hat{\mathbf{p}} \cdot \mathbf{B}$$

But magnetic moment cannot explains anomalies

Chiral magnetic effect

Son, Zhitnitsky; Metlitskii; Kharzeev et al.



put our system in B field and slowly varying scalar potential (static)

chemical potential traces A_0

$$\cancel{\partial_0 j^0} + \boldsymbol{\nabla} \cdot \mathbf{j} = \pm \frac{1}{4\pi^2} \mathbf{B} \cdot \nabla A_0$$

$$\mathbf{j} = \pm \frac{1}{4\pi^2} \mu \mathbf{B}$$

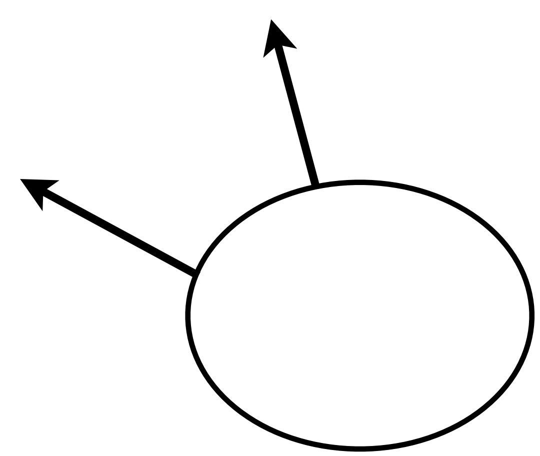
Nonzero current in ground state! contradicts basic tenets of Landau's Fermi liquid theory

Currents in ground state

$$\mathbf{j} = \int d\mathbf{p}\, n_{\mathbf{p}} v_{\mathbf{p}} = \int d\mathbf{p}\, n_{\mathbf{p}} \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} = -\int d\mathbf{p} \epsilon_{\mathbf{p}} \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}$$

μ

=0



Hamiltonian reformulation of Fermi liquid theory

$$\partial_t n_{\mathbf{p}} = i[H,\, n_{\mathbf{p}}]$$

$$H = \int \frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{x}}{(2\pi)^3}\,\epsilon^0_{\mathbf{p}}\delta n_{\mathbf{p}} + \frac{1}{2}\int \frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{q}\,\mathrm{d}\mathbf{x}}{(2\pi)^6}\,f(\mathbf{p},\,\mathbf{q})\delta n_{\mathbf{p}}\delta n_{\mathbf{q}},$$

$$[n_{\mathbf{p}}(\mathbf{x}),\, n_{\mathbf{q}}(\mathbf{y})] = -\mathrm{i}(2\pi)^3\frac{\partial}{\partial \mathbf{p}}\delta(\mathbf{p}-\mathbf{q})\cdot\frac{\partial}{\partial \mathbf{x}}\delta(\mathbf{x}-\mathbf{y}) \times [n_{\mathbf{p}}(\mathbf{y}) - n_{\mathbf{q}}(\mathbf{x})].$$

where does it come from?

Commutators and Poisson brackets

$$\hat{A} = \int \frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{x}}{(2\pi)^3}\, A(\mathbf{p},\mathbf{x}) n_{\mathbf{p}}(\mathbf{x}), \quad \hat{B} = \int \frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{x}}{(2\pi)^3}\, B(\mathbf{p},\mathbf{x}) n_{\mathbf{p}}(\mathbf{x})$$

$$[\hat{A},\, \hat{B}] = -\mathrm{i} \int \frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{x}}{(2\pi)^3}\, \{A,\, B\} n_{\mathbf{p}}(\mathbf{x})$$

$$\frac{\partial A}{\partial \mathbf{p}} \cdot \frac{\partial B}{\partial \mathbf{x}} - \frac{\partial A}{\partial \mathbf{x}} \cdot \frac{\partial B}{\partial \mathbf{p}}$$

this fixes the commutator between $n_p(x)$

Berry curvature

- Standard Fermi liquid theory: does not distinguish left- and right-handed quasiparticles
- Dirac wavefunction for right-handed particles

$$(\boldsymbol{\sigma} \cdot \mathbf{p}) u_{\mathbf{p}} = |\mathbf{p}| u_{\mathbf{p}}$$

$$\mathcal{A}(\mathbf{p}) = u_{\mathbf{p}}^{\dagger} \partial_{\mathbf{p}} u_{\mathbf{p}}$$

Single quasiparticle

- Motion of a classical particle in an external field

$$\dot{\mathbf{x}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \mathbf{\Omega}$$

$$\dot{\mathbf{p}} = \mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B}$$

(Xiao, Shi, Niu;
Balents, Shindou)

right-handed

$$\mathbf{\Omega} = \pm \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

left-handed

Magnetic monopole
in momentum space

$$\mathbf{\Omega} = \boldsymbol{\nabla}_{\mathbf{p}} \times \mathcal{A}(\mathbf{p})$$

Berry curvature

Berry phase

Single-quasiparticle physics

$$S = \int dt\,(p^i\dot{x}^i - \epsilon(p) + A_0 + A_i\dot{x}^i - \mathcal{A}_i(p)\dot{p}^i) = \int dt\,(-\omega_a\dot{\xi}^a - H(\xi))$$

equation of motion: $\omega_{ab}\dot{\xi}^b + \partial_a H = 0 \qquad \omega_{ab} = \partial_a\omega_b - \partial_b\omega_a$

Hamiltonian interpretation:

$$\dot{\xi}^a = \{H,\, \xi^a\} \qquad\qquad \{\xi^a,\, \xi^b\} = \omega^{ab}$$

$$\{p_i,\, p_j\} = -\frac{\epsilon_{ijk}B_k}{1+\mathbf{B}\cdot\mathbf{\Omega}} \qquad \{x_i,\, x_j\} = \frac{\epsilon_{ijk}\Omega_k}{1+\mathbf{B}\cdot\mathbf{\Omega}}$$

$$\{p_i,\, x_j\} = \frac{\delta_{ij} + \Omega_i B_j}{1+\mathbf{B}\cdot\mathbf{\Omega}}$$

Example: B=0

$$\{p_i,\, p_j\} = 0 \qquad \{p_i,\, x_j\} = \delta_{ij} \qquad \{x_i,\, x_j\} = \epsilon_{ijk}\Omega_k$$

$$\mathbf{L} = \mathbf{x} \times \mathbf{p} \qquad \{L_i,\, L_j\} \neq -\epsilon_{ijk} L_k$$

$$\mathbf{J} = \mathbf{L} \pm \frac{\mathbf{p}}{2|\mathbf{p}|} \qquad \{J_i,\, J_j\} = -\epsilon_{ijk} J_k$$

spin

Modified Fermi liquid theory

$$\mathrm{d}\Gamma = \sqrt{\omega}\,\mathrm{d}\xi = (1 + \mathbf{B}\cdot\mathbf{\Omega})\frac{\mathrm{d}\mathbf{p}\,\mathrm{d}\mathbf{x}}{(2\pi)^3}$$

$$\hat{A} = \int \mathrm{d}\xi\,\sqrt{\omega}\,A(\xi)n(\xi), \quad \hat{B} = \int \mathrm{d}\xi\,\sqrt{\omega}\,B(\xi)n(\xi)$$

$$[\hat{A},\,\hat{B}] = -\frac{\mathrm{i}}{2}\int \mathrm{d}\xi\,\sqrt{\omega}\,\omega^{ab}(A\partial_a B - B\partial_a A)\partial_b n(\xi)$$

Together with a Hamiltonian $H[n_p(x)]$, it determines the equation of motion

Anomalous commutator

$$n(\mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sqrt{\omega}\, n_{\mathbf{p}}(\mathbf{x})$$

$$[n(\mathbf{x}),\, n(\mathbf{y})] = -\mathrm{i}\left(\boldsymbol{\nabla} \times \boldsymbol{\sigma} + \frac{k}{4\pi^2}\mathbf{B}\right) \cdot \boldsymbol{\nabla}\delta(\mathbf{x}-\mathbf{y})$$

Anomalous Hall current

anomaly

$$\sigma_i(\mathbf{x}) = -\int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\, p_i \Omega_k \frac{\partial n_{\mathbf{p}}(\mathbf{x})}{\partial p_k}$$

(~0 for isotropic Berry curvature)

$$k = \frac{1}{2\pi}\int \mathrm{d}\mathbf{S} \cdot \mathbf{\Omega}.$$

+1 right-handed
-1 left-handed

Current (E=0)

$$\dot{n} = \mathrm{i}[H,\, n] = -\boldsymbol{\nabla}\cdot\mathbf{j}$$

$$\mathbf{j} = \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3}\left[-\epsilon_{\mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} - \left(\boldsymbol{\Omega}\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}\right)\epsilon_{\mathbf{p}}\mathbf{B} - \epsilon_{\mathbf{p}}\,\boldsymbol{\Omega}\times\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}}\right]$$

$n_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}$

extra contributions

Anomaly from commutator

- Anomalies are related to non-commutativity of number density at 2 different points

$$H' = H + \int \mathrm{d}\mathbf{x}\, \phi(\mathbf{x}) n(\mathbf{x})$$

$$\partial_t n(\mathbf{x}) = \mathrm{i}[H',\, n(\mathbf{x})] = -\boldsymbol{\nabla}\cdot\mathbf{j} - \left(\boldsymbol{\nabla}\times\boldsymbol{\sigma} + \frac{k}{4\pi^2}\mathbf{B}\right)\cdot\boldsymbol{\nabla}\phi(\mathbf{x})$$

$$\partial_t n + \boldsymbol{\nabla}\cdot\mathbf{j}' = \frac{k}{4\pi^2}\mathbf{E}\cdot\mathbf{B} \qquad \mathbf{j}' = \mathbf{j} + \mathbf{E}\times\boldsymbol{\sigma}$$

anomalous Hall current

anomaly

Chiral magnetic effect

$$\mathbf{j} = \int \frac{d\mathbf{p}}{(2\pi)^3} \left[-\epsilon_{\mathbf{p}} \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} - \left(\boldsymbol{\Omega} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} \right) \epsilon_{\mathbf{p}} \mathbf{B} - \epsilon_{\mathbf{p}} \boldsymbol{\Omega} \times \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}} \right]$$

=0 in thermal equilibrium

$$\frac{k}{4\pi^2} \mu \mathbf{B}$$

Conclusion

- Anomalies can be incorporated into Landau's Fermi liquid theory
  - Berry curvature with nonzero flux
- Relevance for real materials? (doped Weyl semimetals)