Nonperturbative Approaches to String Theory

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1. Introduction
2. Matrix theory
3. String field theory

1. Introduction

String theory as of 1995:

Five consistent superstring theories

(IIA, IIB, I, heterotic $E_8 \times E_8$ and SO(32))

- Describe on-shell scattering of gravitational quanta in 10 space-time dimensions

- Successful quantum theories of gravity, best candidates to unify gravity and QFT of standard model

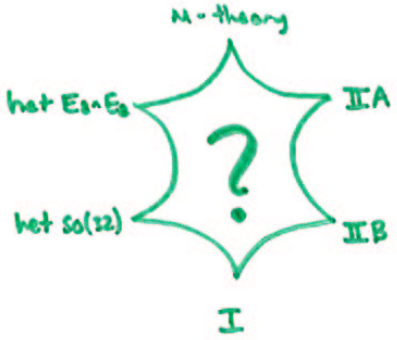
Problems:

- Which of the five theories describes nature?
- Only perturbative expansion—no fundamental description
- No mechanism for choosing specific 4D compactification

"Second superstring revolution" (1995-present)

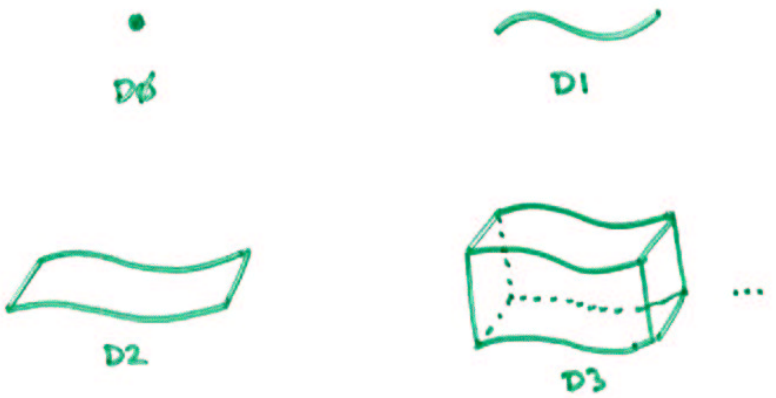
Fueled by two new realizations:

i) Dualities: All 5 string theories are related



- Strings & M-theory limits of unknown underlying theory
- Perturbative/nonperturbative dualities relate theories
- Example: M-theory (in 11D) as strong coupling limit of IIA
  $g \to (R_{11}/l_{11})^{3/2}$.
- Answers: "which theory" question.

ii) Theories contain higher-dimensional branes



IIA: Dp-branes: $p = 0, 2, 4, 6, 8$, NS5-brane, strings

IIB: Dp-branes: $p = 1, 3, 5, 7, 9$, NS5-brane, strings

M-theory: M2-brane, M5-brane (no strings)

- In SUGRA: multi-dimensional charged extremal black holes
- In string theory: D-branes = boundaries for strings
- D-branes: nonperturbative structures in string theory

Duality and D-branes have led to many advances:

- Black holes:

  Extremal and near-extremal black holes built from D-branes

  Entropy and radiation calculations address information puzzle

- Gauge theories:

  New nonperturbative info about SUSY cousins of QCD

- Brane world scenarios:

  Gravity localized on branes

  Models for hierarchy problem, etc.

- Matrix theory and AdS/CFT:

  Nonperturbative descriptions of string/M-theory in fixed backgrounds through field theory/quantum mechanics

- String field theory:

  New progress on nonperturbative, background-independent formulation from applications to D-branes

2. M(atrix) Theory

No fundamental formulation of M-theory known in general 11-dimensional space-time background

But in asymptotically flat space-time

$$g_{IJ} = \eta_{IJ}$$

In light-front frame

$$X^{\pm} = \frac{1}{\sqrt{2}}(X^0 \pm X^{10})$$

We believe M-theory is described by a simple theory:

M(atrix) Theory

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\left[\dot{X}^i\dot{X}^i + \sum_{i<j}[X^i, X^j]^2 + \text{fermions}\right]$$

$X^i(\tau)$ are 9 $N \times N$ Hermitian matrices, $i = 1, \ldots, 9$

Take limit $N \to \infty$

Two Derivations of Matrix Theory:

Derivation 1: Membrane theory (Goldstone/Hoppe '82)

Consider (SUSY) membrane in 11 dimensions

Functions $X^\mu(\Sigma)$ describe membrane configuration

Theory simplifies in light-front time $X^+ = (X^0 + X^{10})/\sqrt{2}$

Regularize $X^\mu(\Sigma)$ w/ $N \times N$ matrices $X^i \to$ Matrix theory.

Analogous to quantization of string,

But: matrices contain many-body states

encoded in block-diagonal matrices

$$X^i = \begin{pmatrix} X^i_{(1)} & 0 \\ 0 & X^i_{(2)} \end{pmatrix}$$

So Matrix theory is naturally a second quantized theory.

Derivation 2: Many D0-branes (Banks/Fischler/Shenker/Susskind '96)

Start with N D0-branes in 10D.

Action complicated: nonabelian Born-Infeld

Low energy limit $\to$ Matrix theory

Connection with 11D picture:

D0-branes carry compact momentum

Large N: "Infinite Momentum Frame" (IMF)

Simple classical solution:

$$X^i = \begin{pmatrix} x^i_1 + v^i_1\tau & 0 & 0 & \ddots \\ 0 & x^i_2 + v^i_2\tau & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ \ddots & 0 & 0 & x^i_N + v^i_N\tau \end{pmatrix}$$

Describes N pointlike gravitons

Motion is nonrelativistic in light-front time $\tau = X^+$

Extended objects in Matrix theory

M-theory contains:

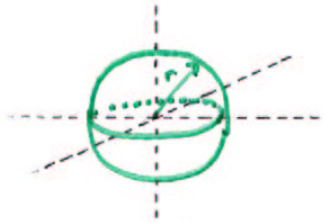
- Supergravitons (pointlike gravitational quanta)
- Membrane (2 dimensions extended in space)
- 5-brane (5 dimensions extended in space)

How are these encoded in $N \times N$ matrices?

Gravitons: pointlike solutions on (classical) linear trajectories

Noncommuting matrices form extended objects

Membranes: Example— spherical membrane



$$X^i = \tfrac{r}{N} J^i, i = 1,2,3$$

$$[X_i, X_j] = \frac{2r}{N}\epsilon^{ijk} X_k \qquad (\mathrm{SU}(2) \text{ generators})$$

satisfy $X_1^2 + X_2^2 + X_3^2 = r^2 \mathbb{1} + \mathcal{O}(1/N^2)$

Couples to 11D supergravity fields as a spherical membrane

5-branes: from similar noncommuting matrix structures

All multipole moments of branes, stress-energy determined.

Gravitational interactions in Matrix theory

Multiple objects from block-diagonal matrices

$$X^i = \begin{pmatrix} X^i_{(1)} & 0 \\ 0 & X^i_{(2)} \end{pmatrix}$$

Classically, no interactions in MT. $\ddot{X}^i = 0 \cdot [[X^i, [X^j], X^j]$

Classical supergravity from loop effects in MT

- One loop: Newtonian (linear) supergravity
- Two+ loops: relativistic (nonlinear) corrections

One loop: complete analysis; Higher loops: partial analysis

First explicit derivation of gravity in $D \geq 4$ from a well-defined quantum theory.

- One perspective: closed strings (GR) from open strings (MT)
- Some technical problems at higher order related to large N

  Quantum bound states are large ($\rho \sim N^{1/3}$)

  Need limit r fixed, $N \to \infty$

  Quantum bound states unknown, even for $N = 2$.

Matrix theory summary

Features

+ Well-defined quantum theory
+ Nonperturbative theory of gravity in flat space-time
+ Classical gravity from quantum effects in MT
+ Describes extended objects (membrane, 5-brane)
+ Simple compactifications possible (tori)

Bugs

− Technically difficult to analyze beyond linear theory
− Light-cone structure simplifies theory
− Asymptotic space-time background fixed

Common aspects of Matrix theory and AdS/CFT

Both Matrix theory and AdS/CFT allow (in principle) calculation of local quantum gravity processes:

Graviton scattering

Brane interactions

Both are nonperturbative definitions of quantum gravity.

But, to address cosmological questions:

What vacua are allowed for string theory?

Why empirical world in 4D?, Why standard model?

we need a background-independent theory.

One approach: String Field Theory (SFT)

3. String Field Theory

Idea of SFT:

- String field $\Psi[X(\sigma)]$ functional on space of string configurations
- $\Psi[X(\sigma)]$ encodes an infinite family of space-time fields
- Action for Ψ gives off-shell nonperturbative definition of string theory.
- Shown to be background independent, for small deformations, although not manifestly

SFT developed heavily in mid 1980's

- Technically complicated
- Action only understood clearly for open bosonic string
- No "killer app", progress stalled.

Tachyon condensation

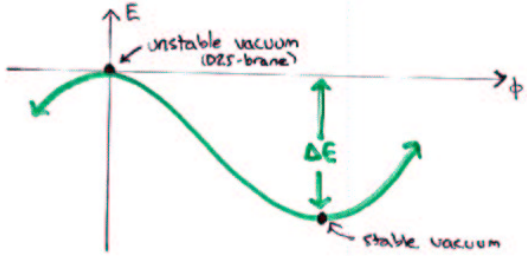
The bosonic open string lives in 26 dimensions

Lightest field is a tachyon: $M_\phi^2 = -1/\alpha'$

Indicates improper choice of vacuum.

2 years ago, Sen proposed:

- Open bosonic string has D25-brane in background
- Tachyon arises from D-brane instability
- Should be able to see decay, find true vacuum using SFT



Concrete predictions:

i) $\Delta E = -T_{25} V$

ii) Lower-dimensional D-branes arise as solitons

iii) Open strings decouple in true vacuum

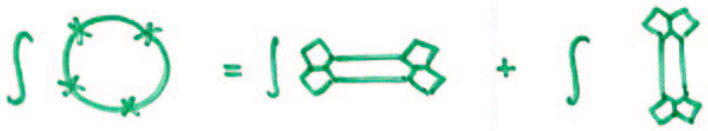
Witten's cubic open bosonic string field theory

Cubic SFT action:

$$S = -\frac{1}{2}\int \Psi \star Q\Psi - \frac{g}{3}\int \Psi \star \Psi \star \Psi$$

Describes tachyon, massless gauge field A_μ, massive fields

In fixed gauge, diagrams cover string moduli space



Cubic interaction is complicated, connects generic 3 fields

Nonlocal interaction includes terms $\sim e^{p^2}$

No nontrivial analytic solutions known.

Approximation technique: "level truncation"

—drop all but finite number of fields in Ψ

- Calculations in level truncation confirm Sen conjectures

level	# of fields	E/T_{25}	
(0, 0)	1	-0.68462	
(2, 4)	3	-0.94855	
(4, 8)	10	-0.98640	(Kostelecký/Samuel '88 Sen/Zwiebach '99)
(6, 12)	31	-0.99514	
(8, 16)	91	-0.99777	
(10, 20)	252	-0.99912	(Moeller/WT '00)

i) Vacuum well-determined numerically

ii) Good numerical evidence for D-brane solitons

iii) Evidence for decoupling of open strings

- Despite substantial work, still no analytic solution.
- Analytic approaches: split strings, simplified (ghost) Q
- Partial understanding of D-brane states

—related to projection operators in $*$-algebra

Further speculations

Evidence indicates that SFT accurately describes D25-brane.

Theory in true vacuum has no open strings.

In true vacuum, only BRST operator Q changes.

How general is theory around true vacuum?

Further conjectures:

iv) Solutions exist with N D25-branes, $E/V = NT_{25}$
—Spontaneously generated $U(N)$ symmetry

v) Closed string states are natural DOF in true vacuum
—Theory is secretly closed string field theory

No concrete evidence yet for (iv) or (v).

Work on both in progress.

If correct, (v) means SFT sees moduli space of vacua.

Possibly fulfills goal of gravitational background independence
—but not explicitly; complex background encoding

- Program must still be generalized to superstring.

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Synthesis & Conclusions

Two nonperturbative approaches to string/M-theory:

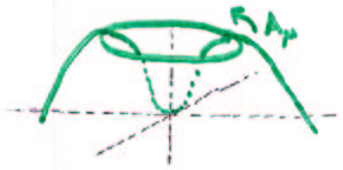
Matrix Theory

String Field Theory

Asymptotic background fixed in MT, not fixed in SFT.

Is background topology fixed?

Suggestive analogy:



Disappearance of brane gauge field in true vacuum

Fields like $A_\mu, g_{\mu\nu}$ only have simple interpretation locally.

Infinite number of fields may encode many topologies

Need more powerful tools for SFT.

Lessons from matrix theory may help:

- A second derivation of SFT from 11 dimensions?
- Connection on light-cone: Matrix string theory?

Goal: Develop formalism and tools to study space of vacua

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