

Flying Cryostats: Superfluid Solvation and Molecular Motions in Helium Droplets

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Helium droplets are older than one might guess....

“Experiments on the condensation of helium by expansion”
H. Kammerlingh Onnes. Communication Nr. 105 of the
Physical Laboratory at Leiden (April 24, 1908)

... I had observed in expanding helium, which at a temperature of $-259\text{ }^{\circ}\text{C}$ had been strongly compressed...

At the expansion of the helium a dense gray cloud appeared, from which separated solid masses floating in gaseous helium resembling cotton wool ...

... it is possible that the mist has been a liquid (helium) cloud

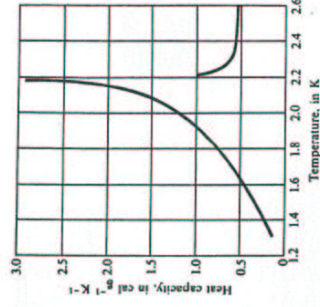
macroscopic sample of liquid helium achieved July 10, 1908
Nobel prize awarded for liquefaction in 1913

superfluidity of ^4He discovered in 1938 (Allen, Misener, Kapitza)

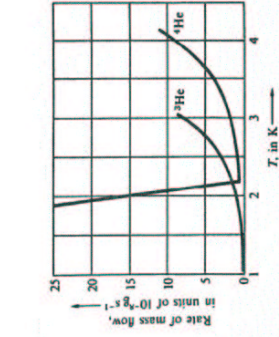
Superfluidity in bulk ^4He



phase diagram

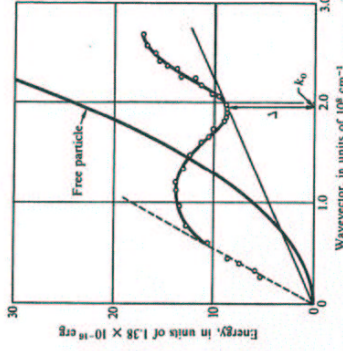


heat capacity



comparison of rates of flow under gravity through a fine hole

elementary excitations



Helium is 'self-cleaning': molecules aggregate, go to boundaries

REVIEW B

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Ultimate fate of a gas of atomic hydrogen in a liquid-helium chamber: Recombination and burial

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(Received 30 September 1983; revised manuscript received 12 October 1983)

In this article we consider the evolution in time of a gas of atomic hydrogen. Gaseous atomic hydrogen is produced in a cell in which the walls are coated with a film of liquid helium to reduce surface recombination. "Stabilized" spin-polarized hydrogen is at best long lived, slowly decaying away by recombination. The resultant H_2 , either in the form of single molecules or molecular clusters, penetrates the surface of the liquid helium and finally attaches to an underlying wall. The experimental technique developed for this study introduces a new method for producing atomic or molecular clusters at low temperature, either in a helium "background" gas or in liquid helium.

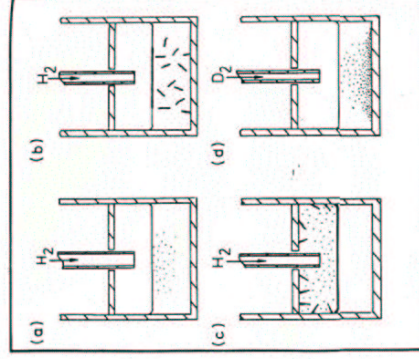
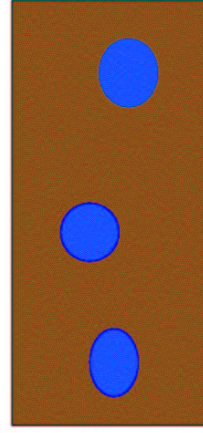
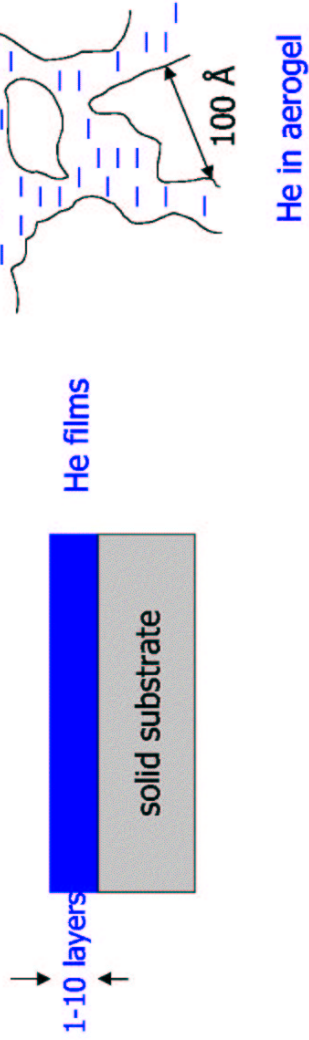


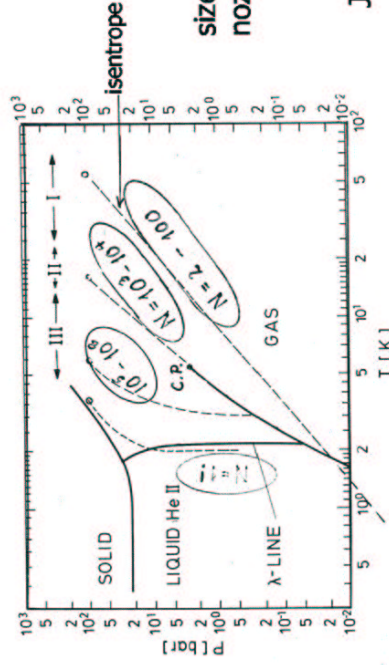
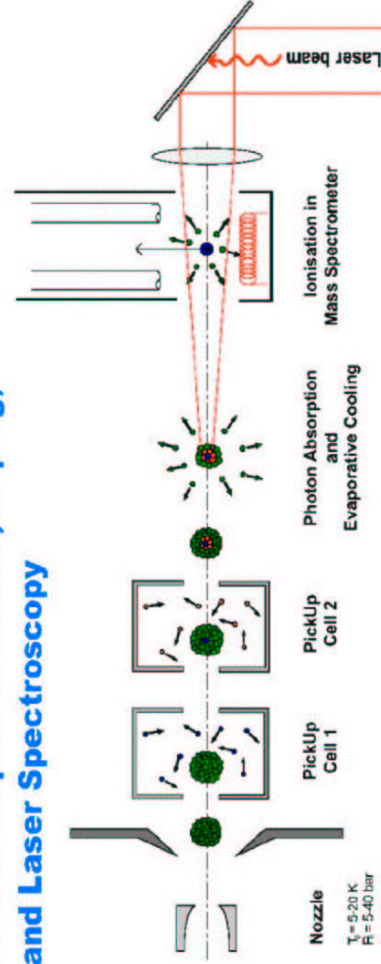
FIG. 5. Results of visual observations: (a), (b), and (c) H_2 flowing into liquid helium under low-, medium-, and high-flow conditions, respectively; (d) D_2 flowing into liquid helium.

Superfluidity in confined geometries:



He bubbles in Cu matrix under pressure

Helium Droplet Formation, Doping, and Laser Spectroscopy

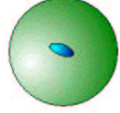


size of cluster, N , controlled by nozzle conditions

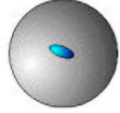
J. P. Toennies, et al. (Goettingen)

The Droplet Zoo

1. Pure ^4He and ^3He Droplets



$N_4 = 10^3 - 10^7$
 $T = 0.38 \text{ K}$



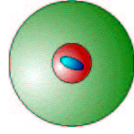
$N_3 = 5 \cdot 10^5 - 10^6$
 $T = 0.15 \text{ K}$

2. Mixed $^4\text{He}/^3\text{He}$ Droplets



$N_3 = 5 \cdot 10^5 - 10^6$
 $N_4 = 0 - 10^3$
 $T = 0.15 \text{ K}$

3. Doping by $p\text{-H}_2$ (>99%), $o\text{-D}_2$ (~96%)



$N_4 \sim 5000$
 $N_{\text{H}_2, \sigma_1} = 0 - 20$



$N_3 \sim 5000$
 $N_{\text{H}_2, \sigma_1} = 0 - 20$



$N_3 \sim 5000$
 $N_4 \sim 500$
 $N_{\text{H}_2, \sigma_1} = 0 - 20$

A. Vilesov

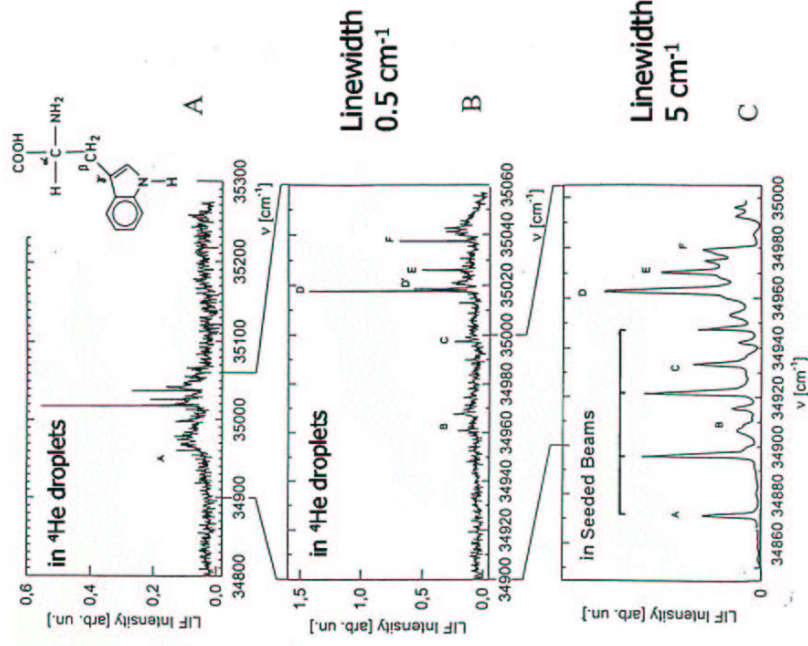
He Droplets

- Cold environment: $T=0.37 \text{ K}$ (0.15 K) and superfluid (^4He)
- Gentle quantum matrix for spectroscopy
- High resolution spectroscopy - probe of solvent effects in simple system (but quantum....)
- He droplets can be used to assemble molecular, metal, and other clusters, to pick up biological molecules, semiconductor nanocrystals, etc., for spectroscopic study...
- Chemical reactions, unstable species - low temperature control

for an overview see Toennies et al., Physics Today, Feb. 2001

Sharp spectra:

LIF spectra of
an amino acid
tryptophan



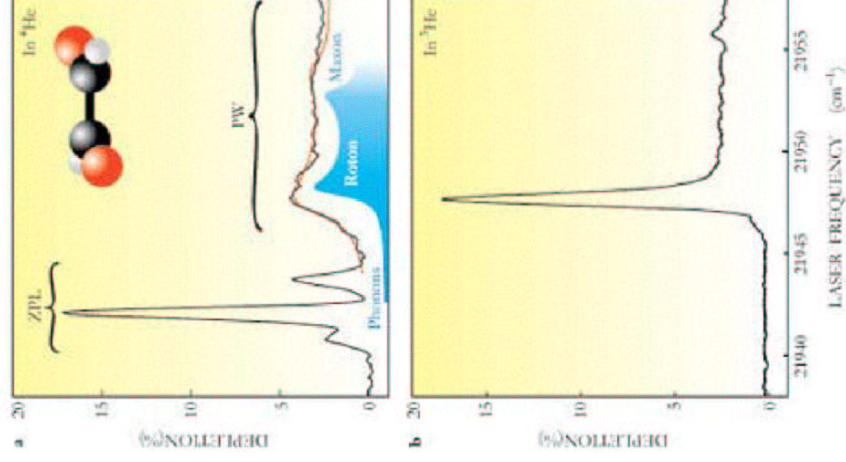
Lindinger et al.
JCP 110, 1429 (1999)

Evidence of superfluidity in ⁴He_N:

electronic excitation of gloxal
molecule (C₂H₂O₂)

in ⁴He_N - gap, phonon wing fit to
~ bulk DOS
in ³He_N - no gap

S. Grebenev et al., Physica B 280, 65 (2000)



IR Spectrum of OCS in helium droplets

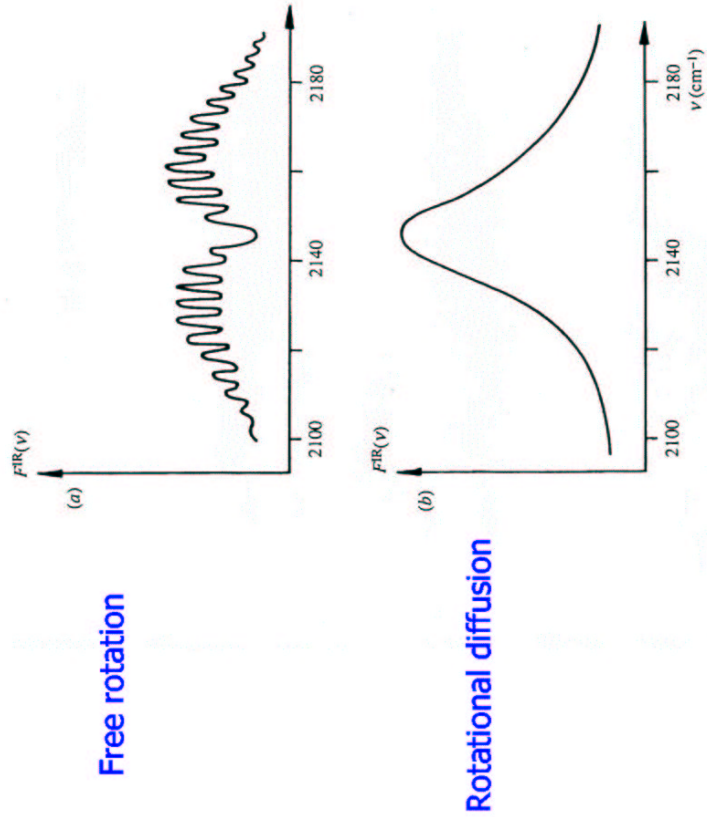
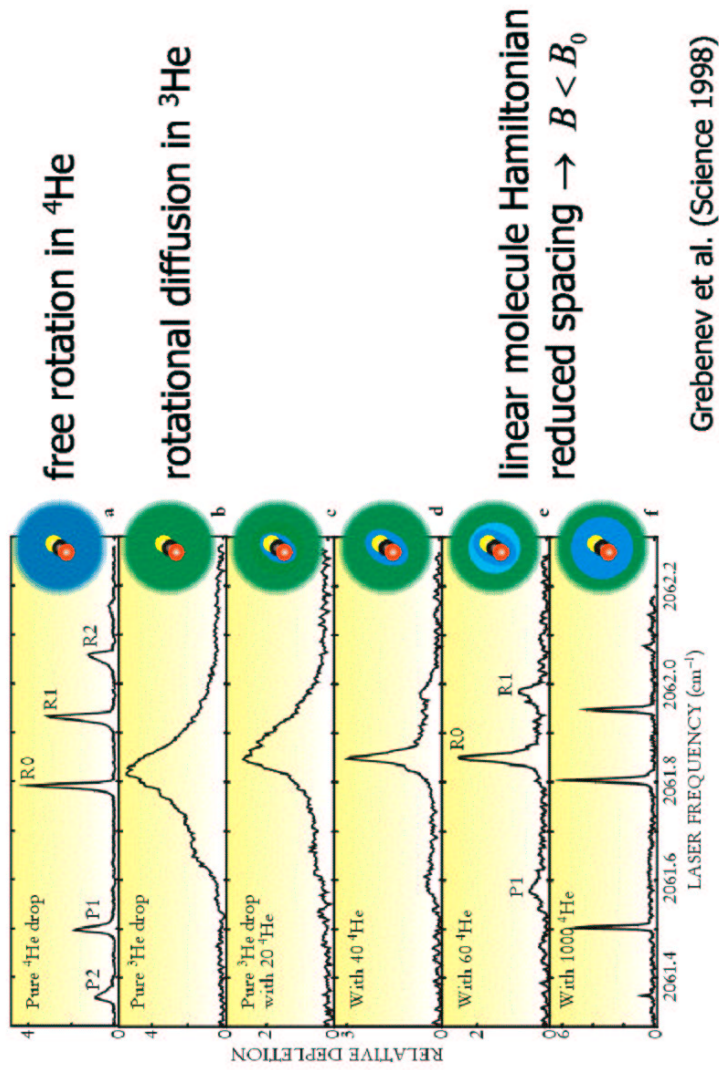
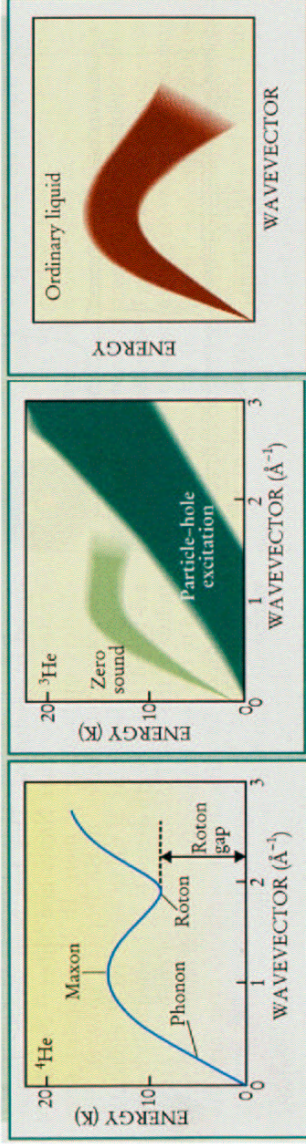


Fig. 0.1. Infrared absorption spectrum of CO in gas (a) and liquid (b) at 80 K

Spectra of elementary excitations



Babichenko and Kagan: PRL 83, 3458 (1999)
 lower spectral density of states coupled to rotational levels of molecule in ⁴He_N

Rotational constants in ⁴He droplets T ~ 0.38 K, N ~ 1000

[B] = cm⁻¹

Molecule	B ₀	B in ⁴ He _N	B/B ₀ (%)	ref.
HF	19.78	19.48	98	1
NH3	9.94	7.5	76	2
HCN	1.478	1.175	79	3
HCCH	1.182	1.048	89	4
OCS	0.20	0.073	37	5
HCCCN	0.15	0.052	33	4
SF6	0.091	0.034	36	6
(HCN) ₂	0.058	0.019	33	4

1) Nauta et al. (JCP 2000) 4) Callegari et al. (PRL 1999)
 2) Behrens et al. (JCP 1998) 5) Grebenev et al. (JCP 2000)
 3) Conjusteau et al. (JCP 2000) 6) Hartmann et al. (PRL 1995)

Theory:

- direct calculation of rotational energy levels
 - DMC, POITSE
- density analysis
 - adiabatic following
- molecular rotation and superfluidity
 - local two-fluid theory
 - role of hydrodynamics

Diffusion Monte Carlo:

$$\frac{\partial \Psi}{\partial t} = -D\nabla^2 \Psi + V\Psi \quad \text{diffusion with source/sink } V$$

$$\Psi(t) = e^{-\hat{H}t} \Psi(0) \quad \Psi(t \rightarrow \infty) \sim \Phi_0(t)$$

importance sampling: $\Psi \rightarrow f = \psi_T \Phi$

$$\frac{\partial f}{\partial t} = -D\nabla^2 f + E_L f + F_Q f, \quad E_L = \psi_T^{-1} \hat{H} \psi_T$$

$$F_Q = \nabla \ln |\psi_T|^2$$

use ψ_T to impose nodal structures \rightarrow excited states

ψ_T has boson symmetry of ${}^4\text{He}_N$

POITSE: (Projection Operator Imaginary Time Spectral Evolution)

spectral information from time evolution

$$\int_0^{\infty} e^{-\omega t} \kappa(\omega) d\omega = C(t) = \langle A(t)A(0) \rangle \quad t = it_r$$

evaluate with Monte Carlo over Ψ_T

cf. absorption spectra from real time

$$\int \mu(0)\mu(t_r)e^{i\omega t_r} dt_r \sim \sigma(\omega)$$

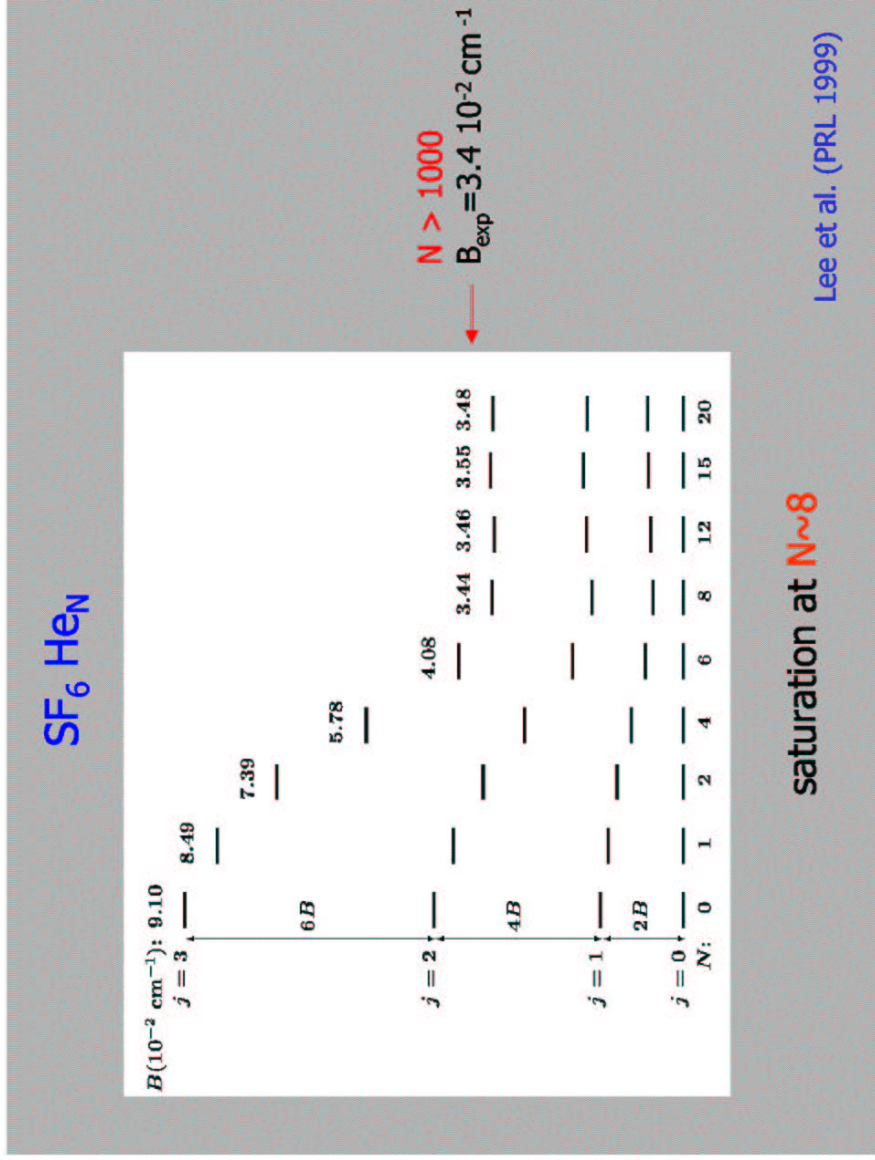
$$\kappa(\omega) = \mathcal{L}^{-1} [C(t)] = \frac{1}{2\pi} \int C(\omega)e^{i\omega t} dt \quad \text{spectral function} \rightarrow \text{excitations}$$

inversion made with maximum entropy method

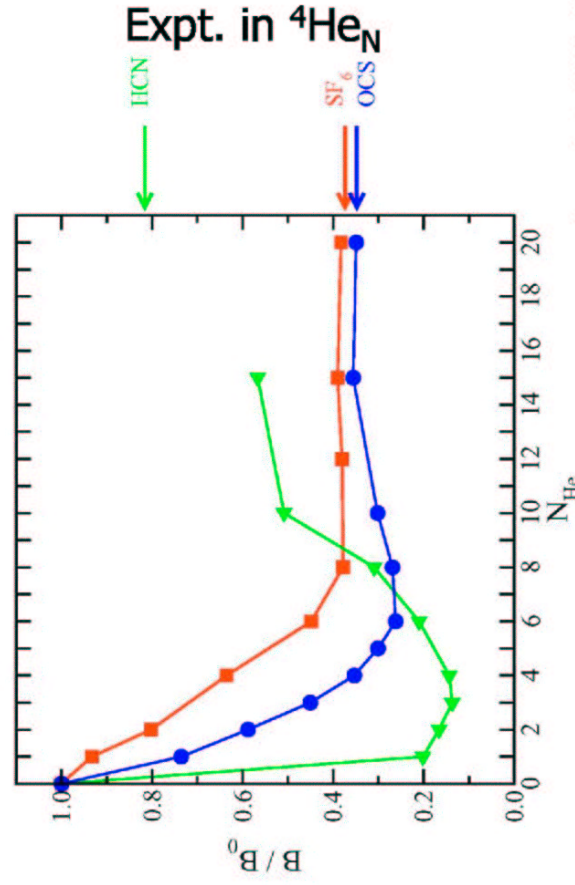
Blume et al. (PRE 1997)

Rotational energy levels of molecules in $^4\text{He}_N$: direct calculations

1. Imaginary time spectral evolution (POITSE)
 - diatomics, SF_6 D. Blume Blume et al. (JCP 1999)
 - HCN A. Viel Viel et al. (JCP 2001)
 - OCS F. Paesani
2. Fixed node DMC
 - rotational importance sampling P. Niyaz Viel et al. (CPC 2002)
 - fixed frame SF_6 E. Lee Lee et al. (PRL 1999)
 - mixed frame HCN A. Viel Viel et al. (JCP 2001)
3. Quasi-adiabatic approximation (Quack & Suhm'91)
 - OCS F. Paesani Paesani et al. (EPL 2001)



Size dependence of rotational constants obtained from direct calculations



SF₆ - fixed node DMC
HCN, OCS - POITSE

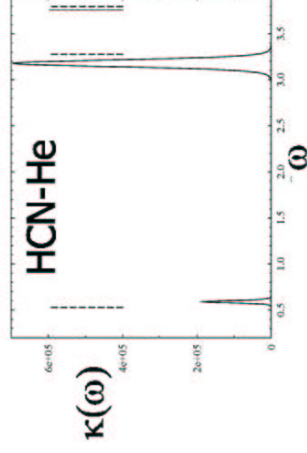
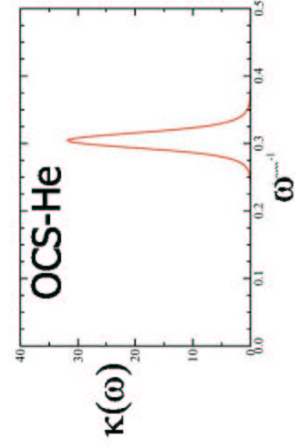
Lee et al. (PRL 1999)
Viel & Whaley (JCP 2001)
Paesani et al. 2002

Nature of nodal surfaces for rotation:

Fixed node:

- SF₆ - free molecule nodal surface ✓
- HCN - free molecule nodal surface ✗

POITSE:



OCS - free rotation projector gives 1 peak
 HCN - free rotation projector gives 2 peaks ⇒
 more complex nodal surfaces

POITSE:

Spectral evolution (projector) Monte Carlo

$$\psi_e = \hat{A} \psi_0$$

$\Delta J = j + l$ $J_0 = 0$

compute spectral function

$$\kappa(\varepsilon) = \sum_f \left| \langle \psi_0 | \hat{A} | \psi_f \rangle \right|^2 \delta(E_f - E_0 + \varepsilon)$$

no nodal constraints

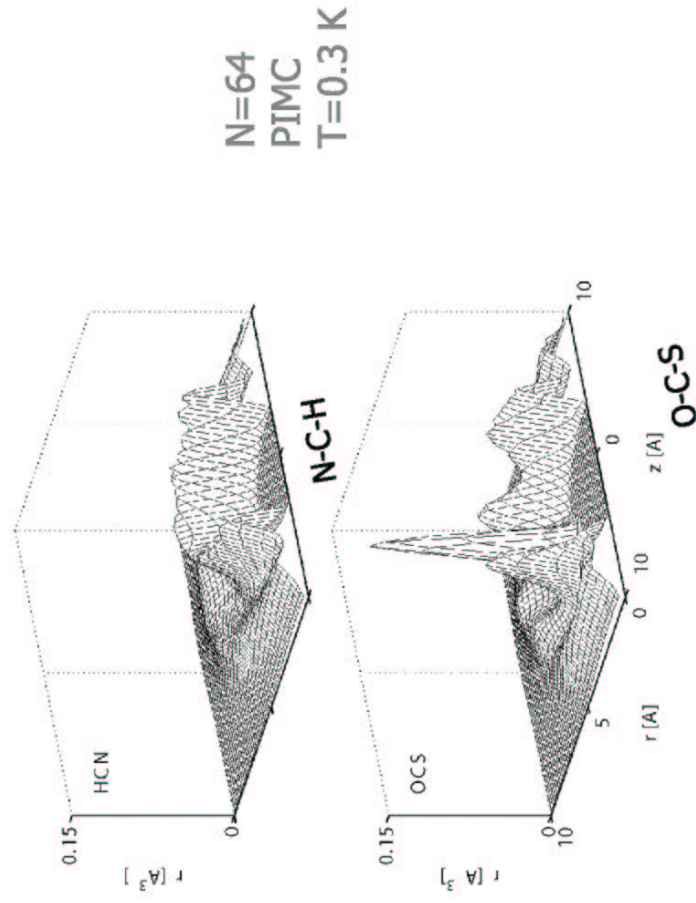
→ Extract excitations with $\Delta J \sim j$

- no contribution to ΔJ from relative motion of He
- allows 'renormalization' of molecular motion via rigidly locked He rotation

DMC helium densities with/without molecular rotation

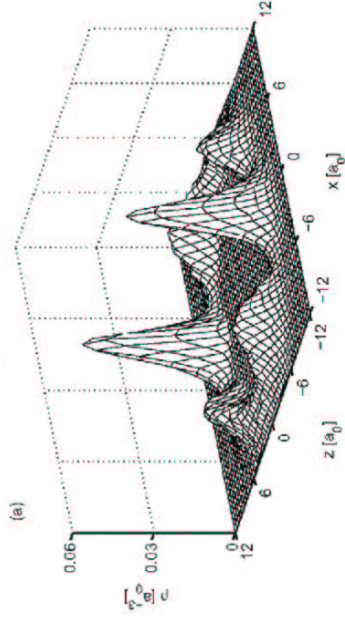
→ analysis of adiabatic following
of molecular rotational motion by helium

Comparison of helium densities around HCN and OCS

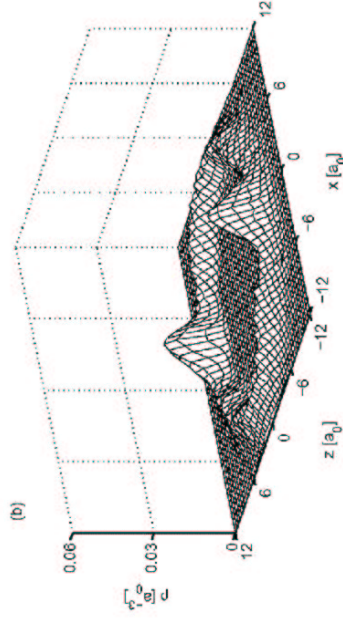


Helium density around benzene

N=14



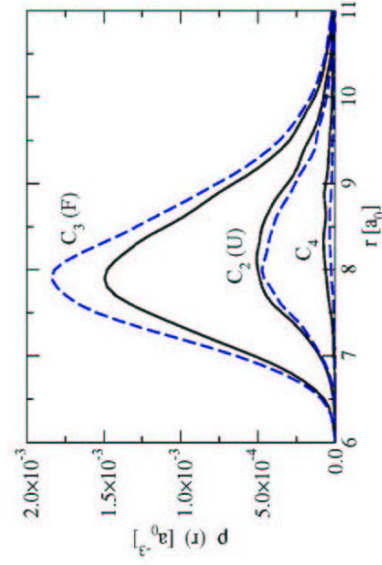
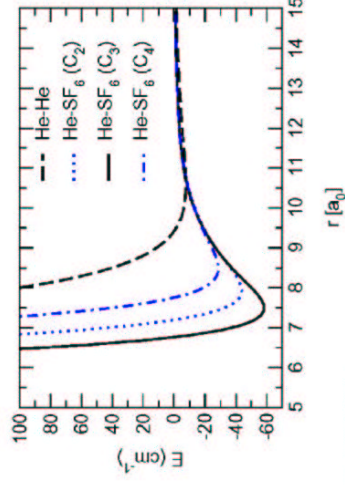
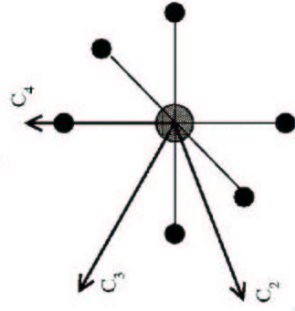
no benzene rotation



with benzene rotation

P. Huang

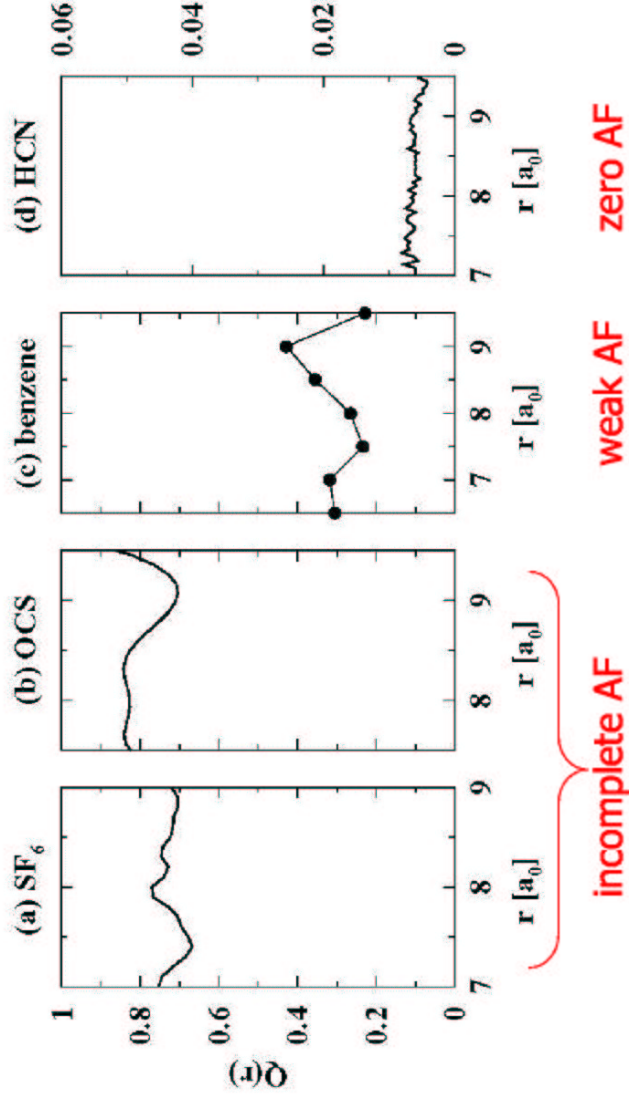
Quantification of Adiabatic Following: SF_6He_N



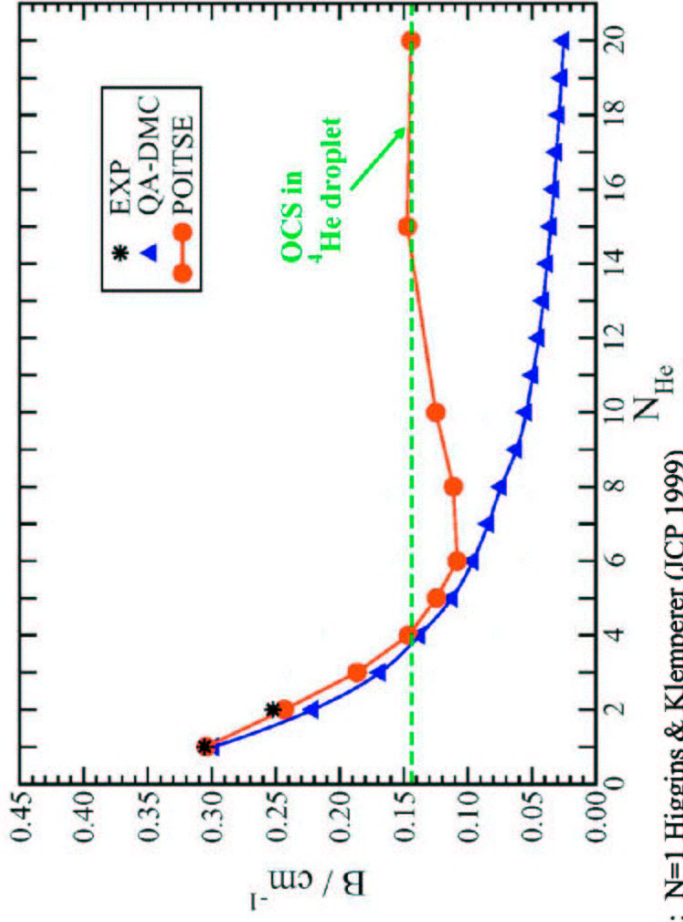
$$Q(r) = \frac{[\rho_U / \rho_F]_{no\ rot}}{[\rho_U / \rho_F]_{rot}}$$

helium density along symmetry axes for n=1

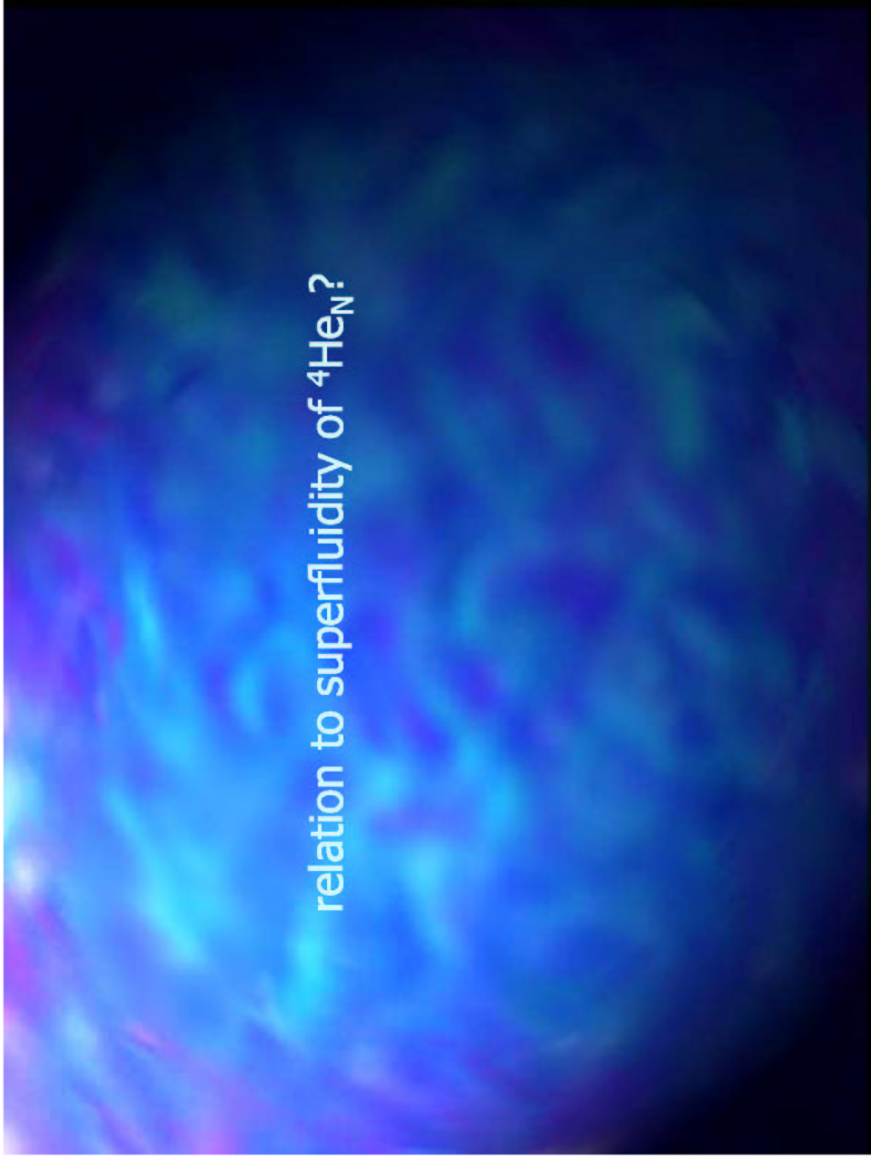
Quantification of adiabatic following of Helium with molecular rotation: $Q(r)$ for $N=1$ complexes



Rotational constant for OCS in 4He_N - POITSE



* expt: $N=1$ Higgins & Klemperer (JCP 1999)
 $N=2$ Xu & Jäger (CPL 2001)
 ----- $N > 1000$ Grebenev et al. (JCP 2000)



Kwon & Whaley (PRL 1999)
Kwon et al. (JCP 2000)

Microscopic two-fluid theory:

- local two-fluid decomposition of quantum solvation density
 - PIMC analysis
- molecule-induced non-superfluid density in first solvation shell
- two-fold helium response to molecular rotation
- adiabatic following of local non-superfluid $\rightarrow I_n$
 - rigid coupling
- adiabatic following of local superfluid $\rightarrow I_s$
 - possible hydrodynamic contribution

effective molecular moment of inertia $I = I_0 + I_n + I_s$
 \rightarrow rotational constant(s) B

Path integral Monte Carlo:

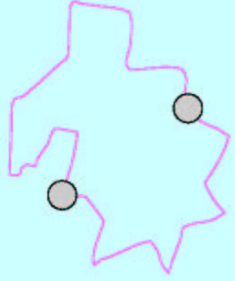
$$\langle O \rangle = Z^{-1} \int dR dR' \rho(R, R'; \beta) \langle R | \hat{O} | R' \rangle$$

$$\int \dots \int dR_1 dR_2 \dots dR_{M-1} \rho(R, R_1; \tau) \dots \rho(R_{M-1}, R; \tau)$$

$$\tau = \beta / M$$



$$\text{Bose symmetry: } \rho(R, R'; \beta) = \frac{1}{N!} \sum_P \rho(R, PR'; \beta)$$



generalized metropolis algorithm:

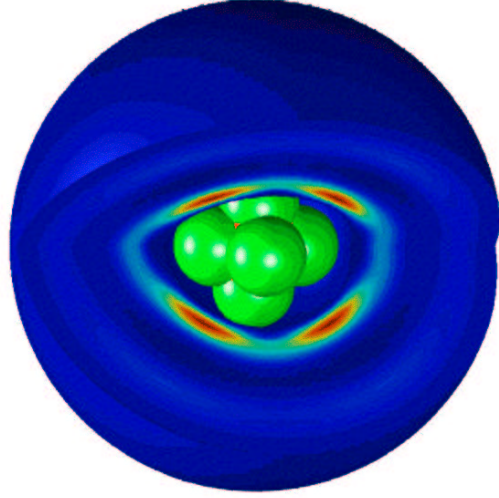
Ceperley and Pollock, PRB (1986)

$$\langle O \rangle \approx \sum_{\text{paths } \alpha} \langle R_\alpha | \hat{O} | R'_\alpha \rangle$$

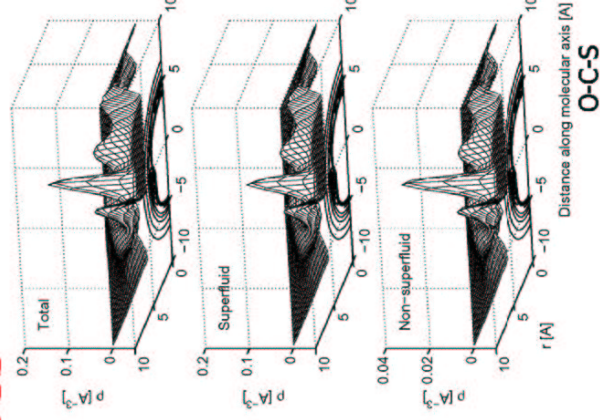
$X^4\text{He}_N$: input V, T, N

local two-fluid density decomposition: $\rho_{tot}(r) = \rho_s(r) + \rho_n(r)$

SF_6



OCS



$\rho_n(r)$ molecular-interaction induced non-superfluid density

I. Adiabatic Following of local non-superfluid:

$$KE(\rho_{ns}) < V_b$$

	$KE(\rho_{ns})$	V_b	$V_b(\text{rigid rot.})$	He follows ?
SF ₆	0.42	21	43	✓
OCS	1.15	42	118	✓
HCN	29	14	25	✗

(energies in K)