Giant Thermal Conductivity of Low-Dimensional Quantum Magnets

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DAAD-Antorchas

F. Heidrich-Meisner, A. Honecker, D. C. Cabra, and W. Brenig,
PRB 71, 184415 (05)
Physica B, 359 (05)
PRL 92, 069703 (04)
PRB 68, 134436 (03)
PRB 66, R140406 (02)

C. Hess, B. Büchner, U. Ammerahl, L. Colonescu, F. Heidrich-Meisner, W. Brenig, and A. Revcolevschi, PRL 90, 197002 (03)
C. Hess, C. Baumann, U. Ammerahl, B. Büchner, F. Heidrich-Meisner, W. Brenig, and A. Revcolevschi, PRB 64, 184305 (01)

Punch/Outline

Magnetic (heat) transport is

- observed to be very large in spin ladders, chains, ...
- ballistic in integrable systems:
  - ➲ extrinsic scattering relevant

  dissipative in non-integrable system:
  - ➲ intrinsic scattering may set transport
- consistent with intrinsic scattering on spin ladders

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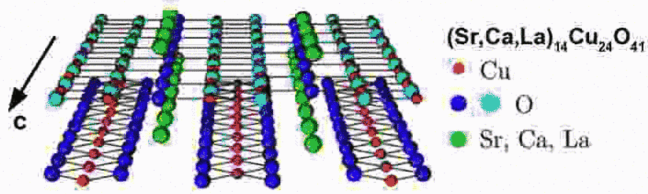
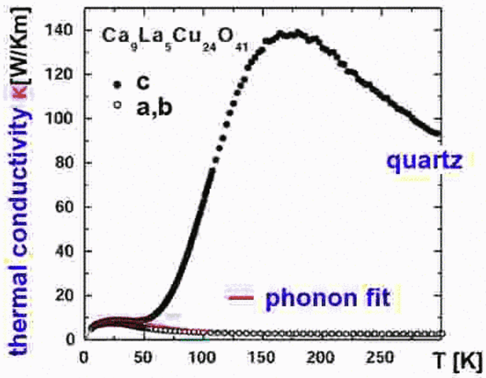
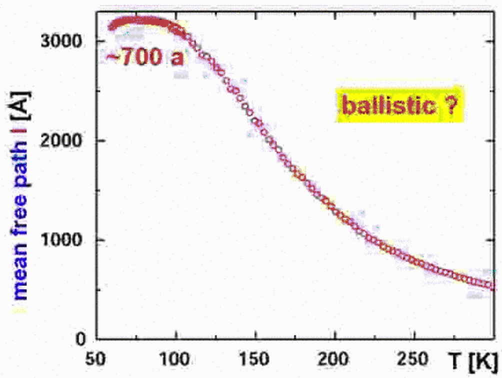
Magnetic Heat Conductivity of Spin Ladders

$J_{\perp} \approx J_{\parallel} \approx 1300K$
$J_{ring}/J_{\perp} \approx 0.1$
$\Delta_{spin} \approx 400K$



● Kinetics: $\kappa = \sum_k C_{Vk} v_k l_k$





Hess et. al. PRB 64, 184305 (01), Sologubenko et al., PRL 84, 2714 (00), Kudo et al., JPSJ 70, 437 (01)

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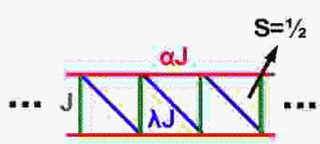
Model: Frustrated Zig-Zag Spin-½ Ladder



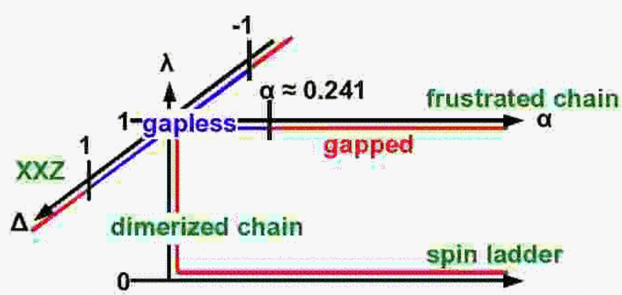
$$H = \sum_l h_{lm}(J, \Delta, \alpha, \lambda, h)$$

$$h_{lm} = J_{lm}\left[\left(S_l^+ S_{l+i}^- + h.c.\right)/2 + \Delta S_l^z S_{l+i}^z + h S_l^z\right]$$

Allows for:

- XXZ-chain: α=0, λ=1
- frustrated chain: α≠0, λ=1
- dimerized chain: α=0, λ≠1
- spin-ladder: α≠0, λ=0

- Ising anisotropy Δ
- magnetic field h



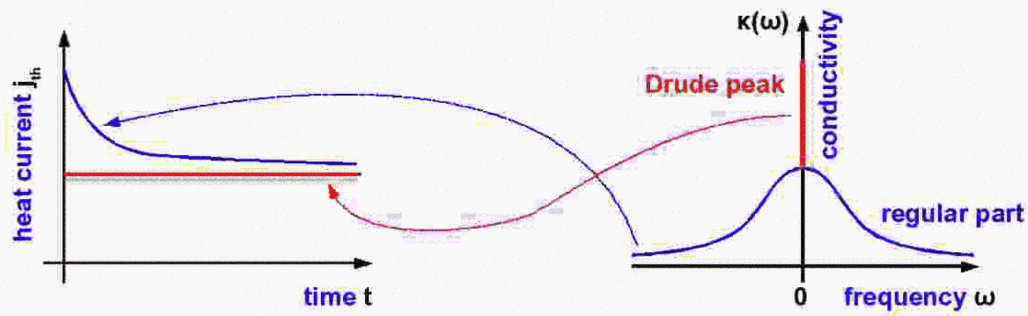
Magnetic Transport: Nutshell

- Time domain

$$j_{th}(t) \sim \langle J^2 \sum_{lmn} \vec{S}_l \cdot (\vec{S}_m \times \vec{S}_n) \rangle$$

- Frequency domain

$$\kappa'(\omega) = D\,\delta(\omega) + \kappa'_{reg}(\omega)$$



If Drude weight D≠0: ballistic transport

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Currents

$$j_{th(s)} = i \sum_{l=1}^{N} \sum_{n,r=0}^{1} \left[H_{l-r-1}, H_{l+n}(S^Z_{l+n}) \right] \Big|_{h=0}$$

$$j_1(h) = j_s \quad j_2(h) = j_{th} - h j_s$$

- Complete exact diagonalization N≤ 20, extrapolate N→∞
- Bosonization & CFT (T«J)
- compare to Bethe-Ansatz
- Jordan-Wigner fermions

$$\begin{matrix} \text{spin} \\ \text{thermal} \end{matrix} \quad \begin{bmatrix} \langle j_1 \rangle \\ \langle j_2 \rangle \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} \nabla h \\ -\nabla T \end{bmatrix}$$

Transport coefficients

$$\mathrm{Re}\, L_{ij}(\omega) = D_{ij}(h,T)\,\delta(\omega) + \mathrm{Re}\, L^{reg}_{ij}(\omega)$$

$$D_{ij} \sim \frac{1}{ZT^p} \sum_{E_m = E_n} e^{-E_n/T} \langle m|j_i|n\rangle \langle n|j_j|m\rangle \qquad \text{Drude}$$

$$\mathrm{Re}\, L^{reg}_{ij}(\omega) \sim \frac{b(\omega)}{ZT^q} \sum_{E_m \neq E_n} e^{-E_n/T} \langle m|j_i|n\rangle \langle n|j_j|m\rangle\, \delta(\omega - \Delta E) \qquad \text{regular}$$

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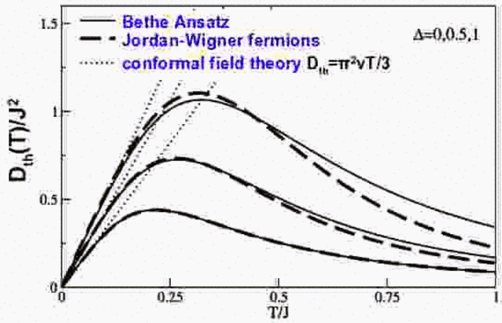
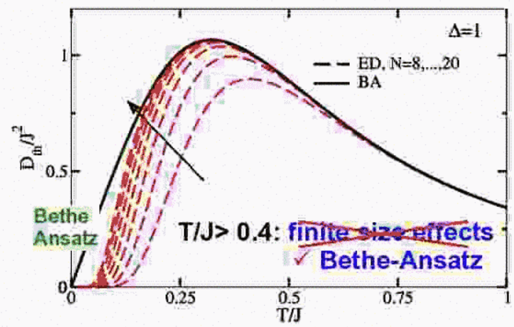
Anistropic Heisenberg Chain

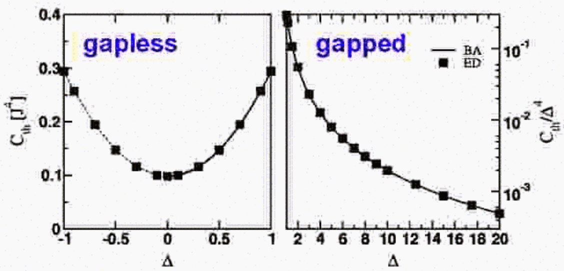


$[H, j_{th}]=0$ ballistic

finite $D_{th}(T)|_{h=0}=D_{22}(T)$



High-T: $D_{th}=C_{th}/T+C_2/T^2+\ldots$



Heidrich-M., et al. PRB 66, 140406 (02) & 68, 134436 (03), Klümper, Sakai, J. Phys. A 35,2173 (02)

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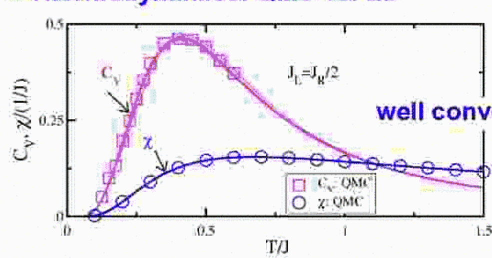
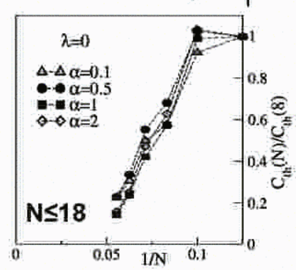
Non-Integrable Systems I: Spin-Ladder & Dimerized Chain

➢ Thermodynamics: QMC vs. ED

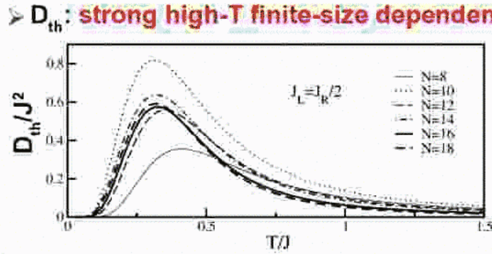


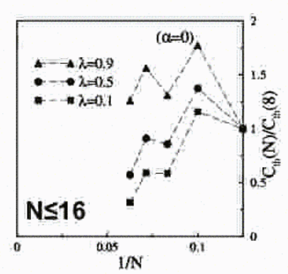
➢ High-T limit: $D_{th}=\frac{C_{th}}{T}+\frac{C_2}{T^2}+\ldots$



$D_{th}(T>0)\to 0$ for $N\to\infty$
dissipative

➢ D_{th}: strong high-T finite-size dependence





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Heidrich-M., et al. PRB 68, 134436 (03)

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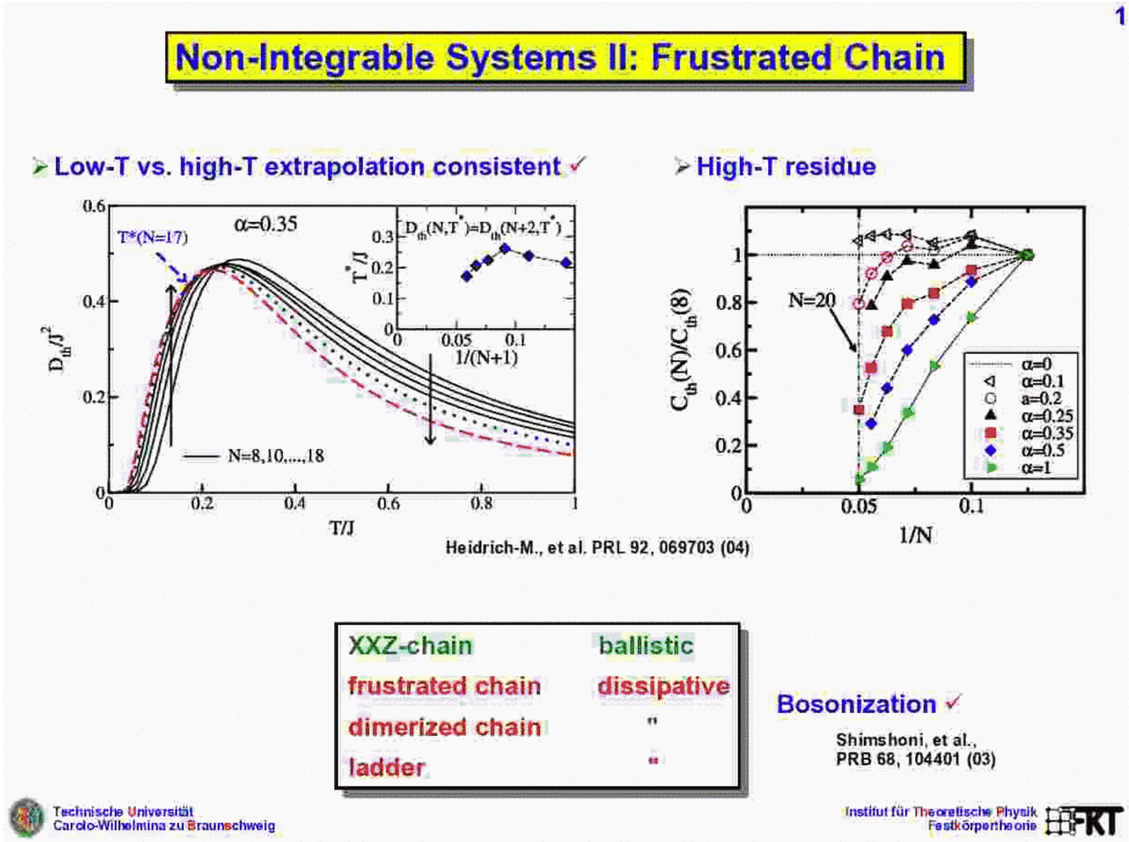
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Non-Integrable Systems II: Frustrated Chain
Low-T vs. high-T extrapolation consistent ✓
High-T residue
α=0.35
T*(N=17)
$D_{th}(N,T^*)=D_{th}(N+2,T^*)$
N=8,10,....,18
D_{th}/J^2
T/J
T^*/J
1/(N+1)
$C_{th}(N)/C_{th}(8)$
1/N
N=20
α=0
α=0.1
a=0.2
α=0.25
α=0.35
α=0.5
α=1
Heidrich-M., et al. PRL 92, 069703 (04)
XXZ-chain ballistic
frustrated chain dissipative
dimerized chain "
ladder "
Bosonization ✓
Shimshoni, et al., PRB 68, 104401 (03)
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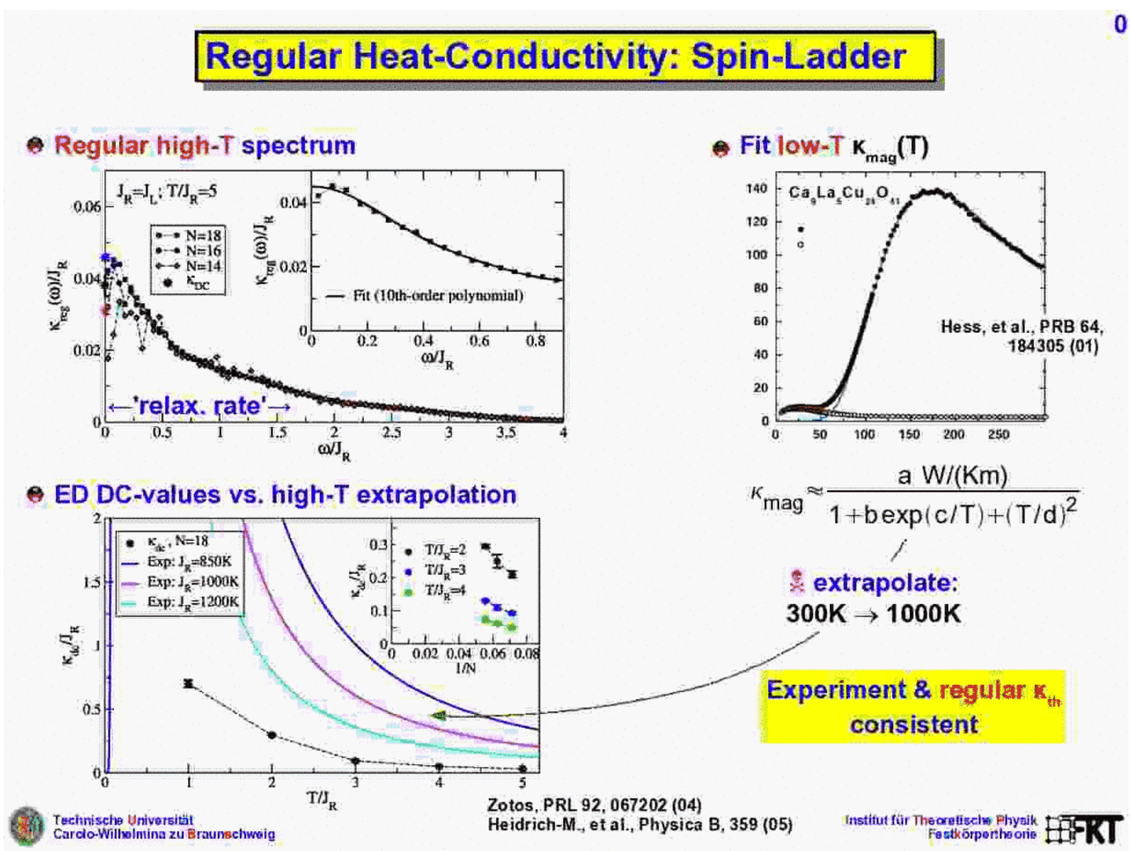
0
Regular Heat-Conductivity: Spin-Ladder
Regular high-T spectrum
Fit low-T $\kappa_{mag}(T)$
$J_R=J_L$; $T/J_R=5$
N=18
N=16
N=14
κ_{DC}
Fit (10th-order polynomial)
$\kappa_{reg}(\omega)/J_R$
ω/J_R
←'relax. rate'→
$Ca_9La_5Cu_{24}O_{41}$
Hess, et al., PRB 64, 184305 (01)
ED DC-values vs. high-T extrapolation
κ_{dc}, N=18
Exp: J_R=850K
Exp: J_R=1000K
Exp: J_R=1200K
T/J_R=2
T/J_R=3
T/J_R=4
κ_{dc}/J_R
T/J_R
1/N
$\kappa_{mag} \approx \frac{a\ W/(Km)}{1+b\exp(c/T)+(T/d)^2}$
extrapolate: 300K → 1000K
Experiment & regular κ_{th} consistent
Zotos, PRL 92, 067202 (04)
Heidrich-M., et al., Physica B, 359 (05)
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Conclusions

- Finite-system ED is a well-controlled approach to thermal transport in (non)-integrable low-D quantum magnets
- XXZ-chain:
  - ballistic transport
  - ED agrees with Bethe-Ansatz where available
  - Jordan-Wigner MF reasonable approximation for Δ,h<<1 and $h \geq h_{sat}$
  - universal low-T behavior for $h=h_{sat}$
  - strong h-dep. of K_{th}
- Non-integrable ladders/chains:
  - thermal Drude weight $\rightarrow 0$ for $N \rightarrow \infty$: dissipative
  - regular response is relevant
  - thermal transport on ☎-compound: regular/intrinsic

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