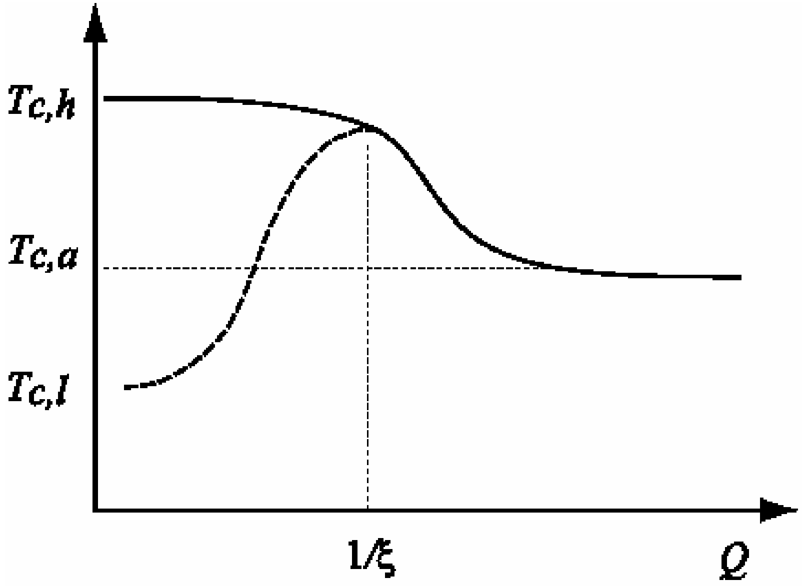
Inhomogeneity can Enhance Superconductivity

Ivar Martin (LANL)
Daniel Podolsky (Berkeley)
Steve Kivelson (Stanford)



cond-mat/0501659, PRB in press

Outline

- Weak coupling observation: inhomogeneity can enhance T_c, quite generically
- Strong-coupling result – suppression of T_c (generic?)
- Speculations and open questions

Proof of principle

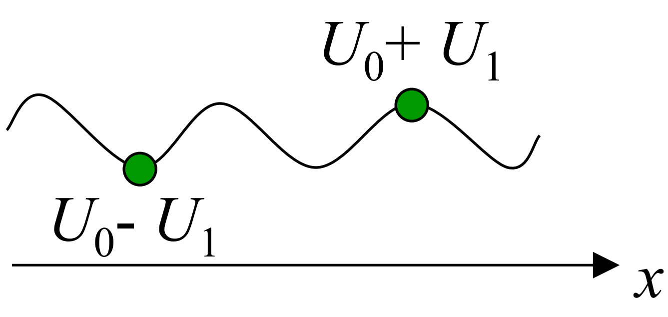
Inhomogeneous negative U Hubbard model:

$$\mathcal{H} = \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}} - \mu) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_i U(x_i) n_{\uparrow}(x_i) n_{\downarrow}(x_i)$$

For constant U → s-wave superconductivity.

Consider effects of weak inhomogeneity

$$U(\mathbf{x}) = U_0 + U_1 \cos \mathbf{Q} \cdot \mathbf{x}$$



Condensation at non-zero momentum

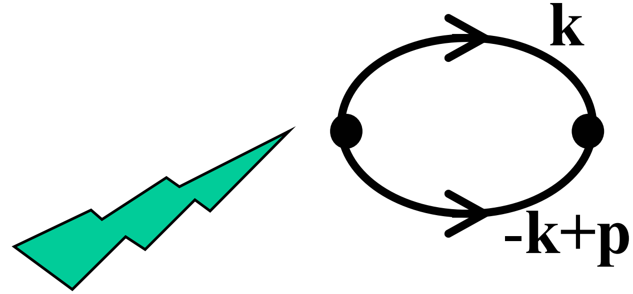
Hierarchy of condensates:

$$\Delta_0, \Delta_{\pm Q}, \Delta_{\pm 2Q} \cdots$$

where…

$$\Delta_{\mathbf{q}} = \sum_{\mathbf{k}\,\mathbf{p}} U(\mathbf{k}) \langle c_{\mathbf{q}/2-\mathbf{k}/2+\mathbf{p}\uparrow} c_{\mathbf{q}/2-\mathbf{k}/2-\mathbf{p}\downarrow} \rangle$$

BCS self-consistency for T_c



$$\Delta_{\mathbf{q}} = \int \frac{d^d p}{(2\pi)^d} U(\mathbf{q}-\mathbf{p}) K(\mathbf{p}) \Delta_{\mathbf{p}},$$

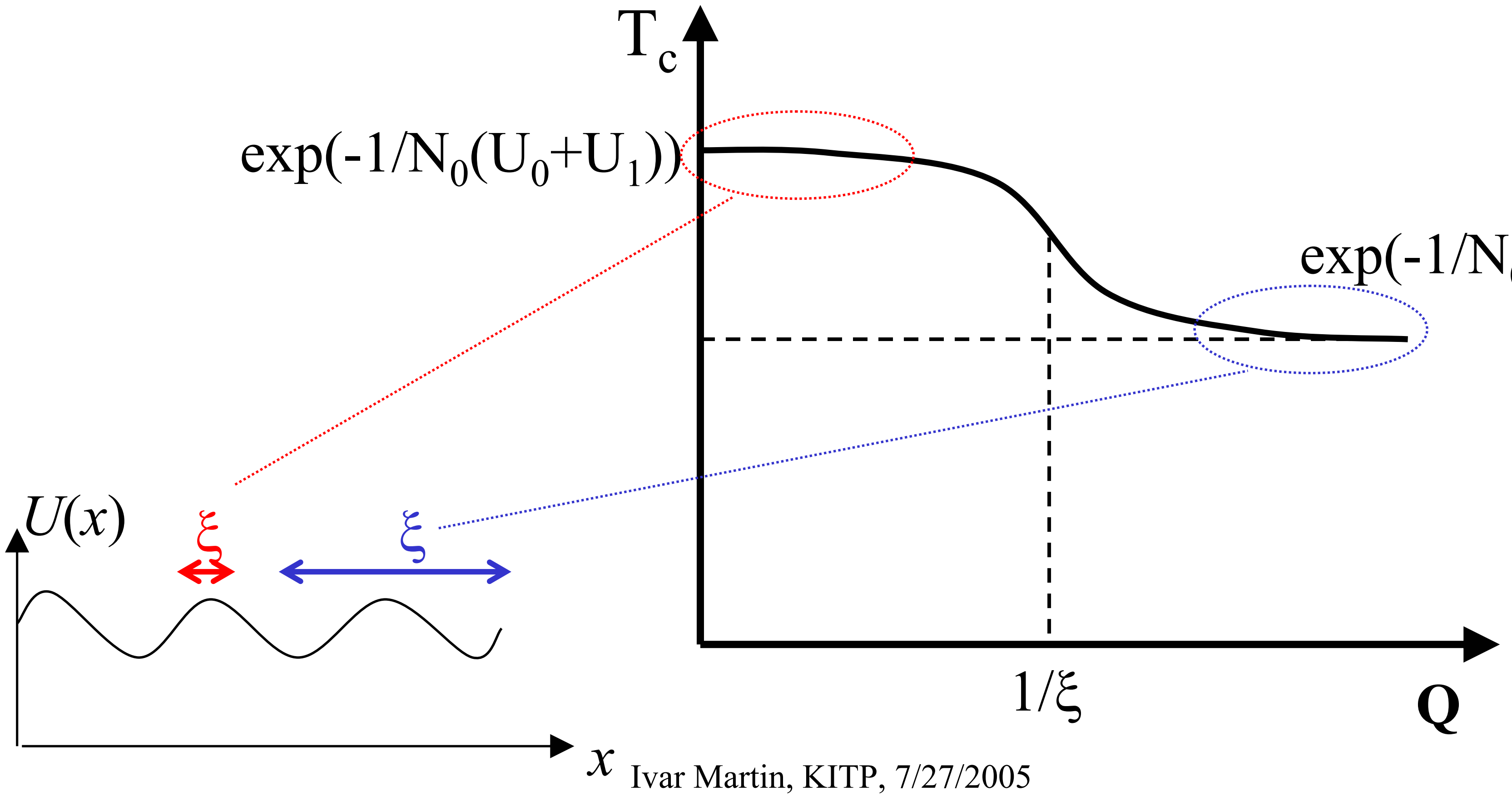
Kernel: $$K(\mathbf{p}) \approx N_f \ln\left[\frac{2\gamma\omega_D}{\pi\sqrt{T^2+(v_f p)^2}}\right]\Theta(\omega_D - |v_f p|).$$

$$\begin{pmatrix} \vdots \\ \Delta_{-2Q} \\ \Delta_{-Q} \\ \Delta_0 \\ \Delta_Q \\ \Delta_{2Q} \\ \vdots \end{pmatrix} = \begin{pmatrix} \ddots & & & & & & \\ & U_0K(-2Q) & U_1K(-Q) & 0 & 0 & 0 & \\ & U_1K(-2Q) & U_0K(-Q) & U_1K(0) & 0 & 0 & \\ & 0 & U_1K(-Q) & U_0K(0) & U_1K(Q) & 0 & \\ & 0 & 0 & U_1K(0) & U_0K(Q) & U_1K(2Q) & \\ & 0 & 0 & 0 & U_1K(Q) & U_0K(2Q) & \\ & & & & & & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ \Delta_{-2Q} \\ \Delta_{-Q} \\ \Delta_0 \\ \Delta_Q \\ \Delta_{2Q} \\ \vdots \end{pmatrix}$$

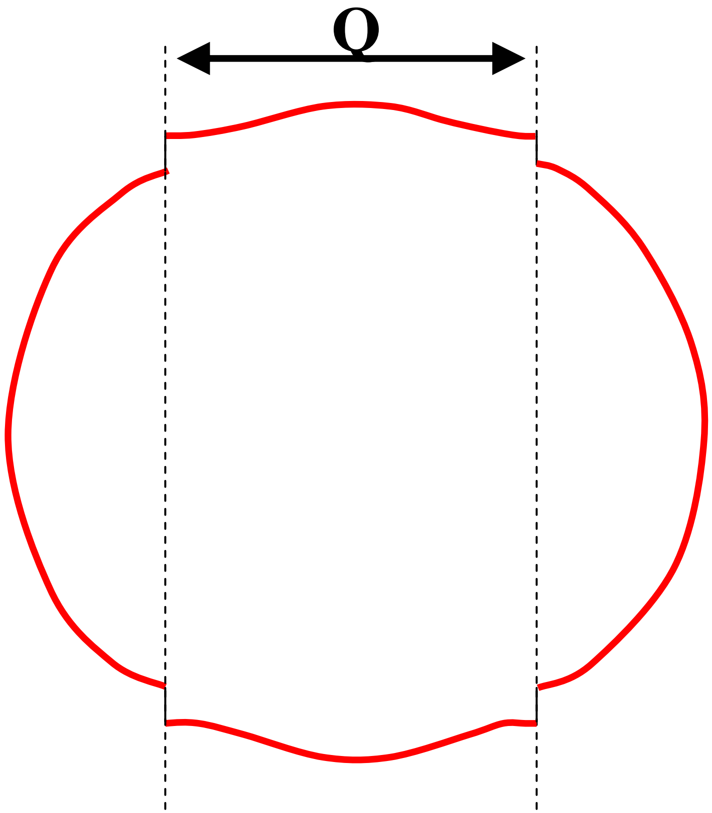
Think of BCS equation as quantum particle on a lattice:
"Energy" can always be lowered by delocalizing wave function ➔ higher T_c

Momentum dependence of MF T_c

Large **Q** and small **Q** limits easy to compute:

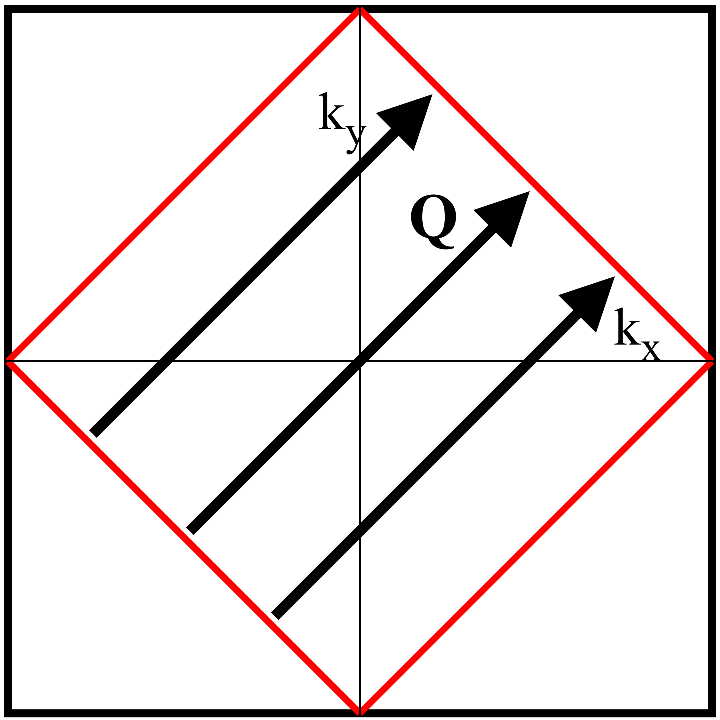


An "External" Charge Density Wave effect on Density of States



Weak effect

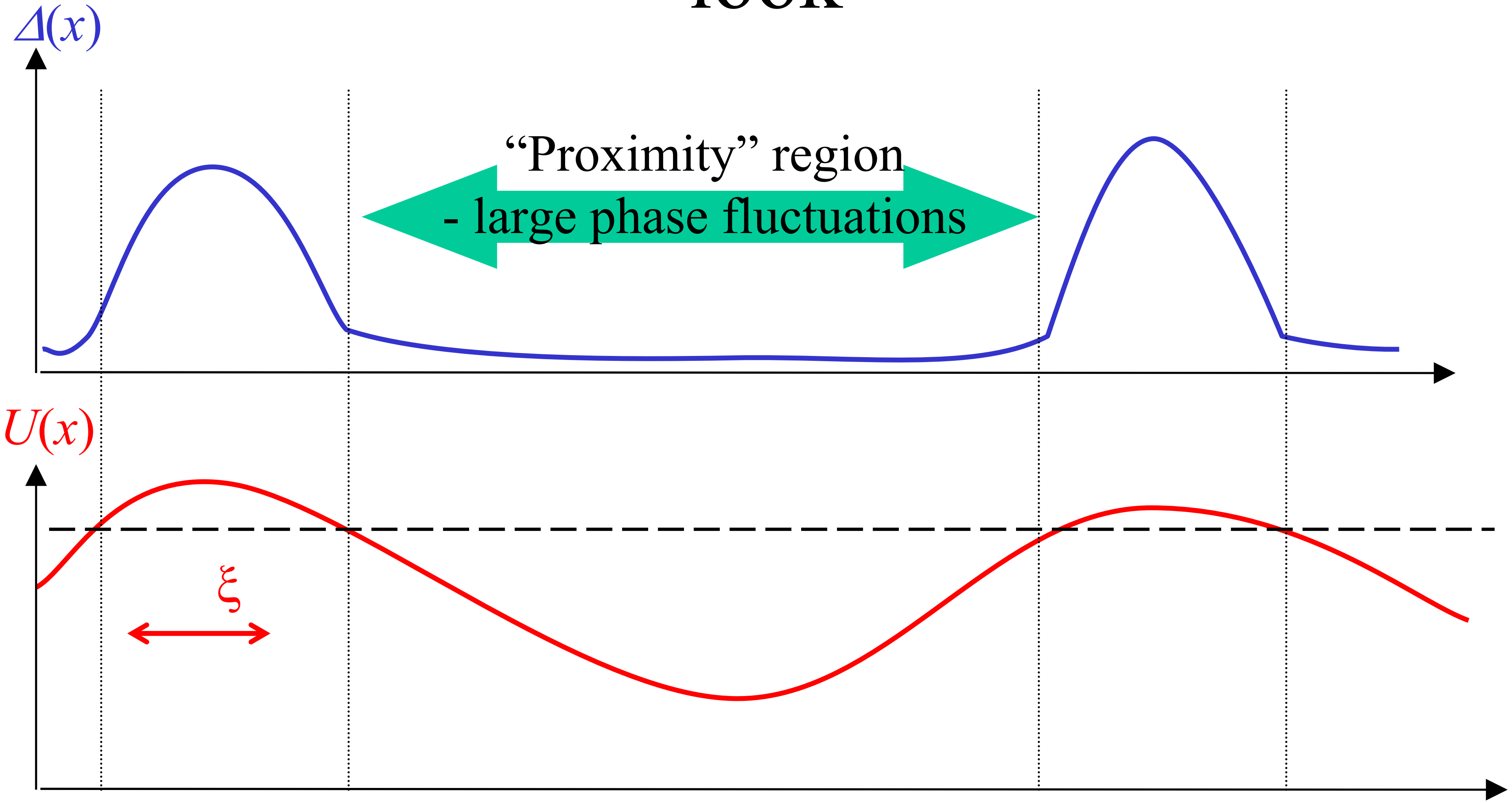
$N_{eff} \approx N_0$



Strong effect

$N_{eff} < N_0$

Small Q limit (Q << 1/ξ) – detailed look



Effect of phase fluctuations in small Q limit

Ginzburg-Landau functional:

$$F = -\int d\mathbf{r}\, d\mathbf{r}' K(\mathbf{r}-\mathbf{r}')\Delta(\mathbf{r})\Delta(\mathbf{r}') + \int d\mathbf{r} \frac{\Delta(\mathbf{r})^2}{U(\mathbf{r})}$$
$$+\alpha \int d\mathbf{r} \rho(\mathbf{r})\Delta(\mathbf{r})^2 + \frac{\beta}{2}\int d\mathbf{r}\Delta(\mathbf{r})^4,$$

Procedure:

- First minimize F for real $\Delta == \Delta_{\mathrm{MF}}$
- then allow for phase fluctuations $\Delta(\mathbf{r}) = |\Delta_{MF}(\mathbf{r})|e^{i\theta}$

Effective model (inhom XY):

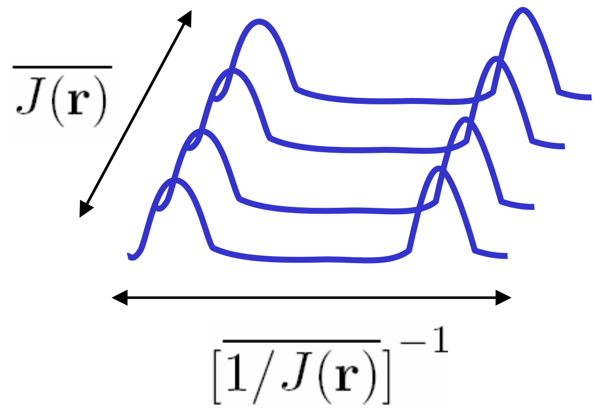
$$F_\theta = \int d\mathbf{r} J(\mathbf{r})(\nabla\theta)^2 \qquad J(\mathbf{r}) = N_f \xi^2 |\Delta_{MF}(\mathbf{r})|^2$$

Effective *x-y J*: Interesting examples

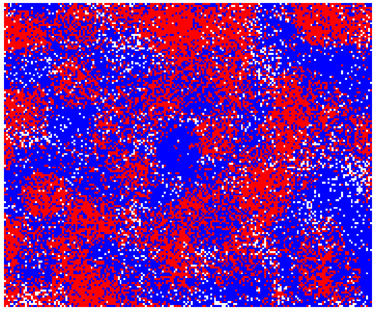
Unidirectional inhomogeneity

$T_{\mathrm{KT}} \sim J_{\mathrm{eff}}$

$$= \sqrt{\overline{J(\mathbf{r})}\,\left[\overline{1/J(\mathbf{r})}\right]^{-1}} \approx \sqrt{J_{min} J_{max}}$$



Same result is likely to hold for a random inhomogeneity (duality argument)

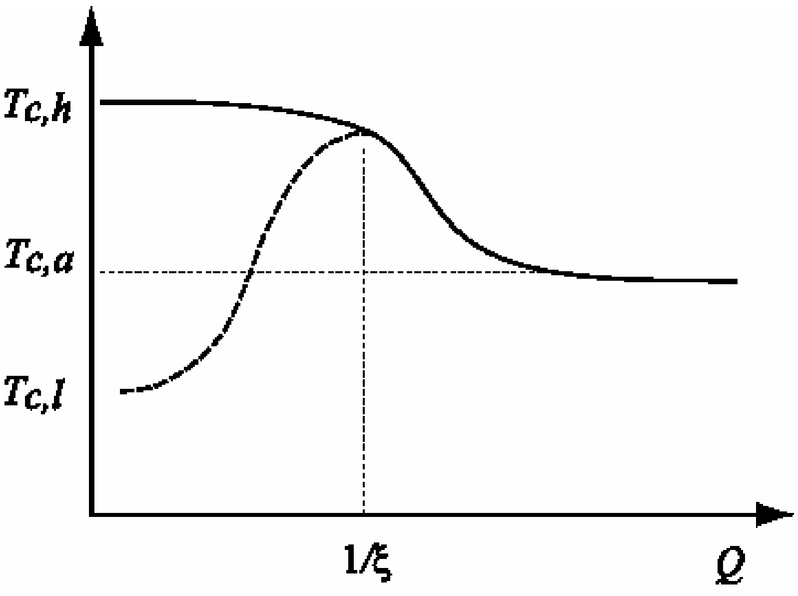


“Phase diagram”

$U_0+U_1 \rightarrow$

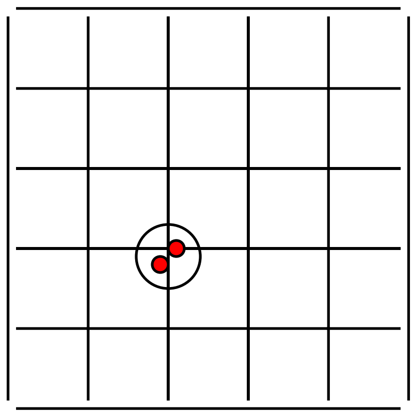
$U_0 \rightarrow$

$U_0-U_1 \rightarrow$



Strong Coupling Case -weak inhomogeneity

$|U| >> t$: electrons pair-up into hard-core bosons with small bandwidth, $\sim t^2/|U|$
Then they Bose condense at $T_{\text{BEC}} \sim t^2/|U|$



Or formally:

Hubbard [-|U|, x, h = 0] ⇔ Hubbard[|U|, x' = 0, h]

→ Heisenberg + h <~> classical xy

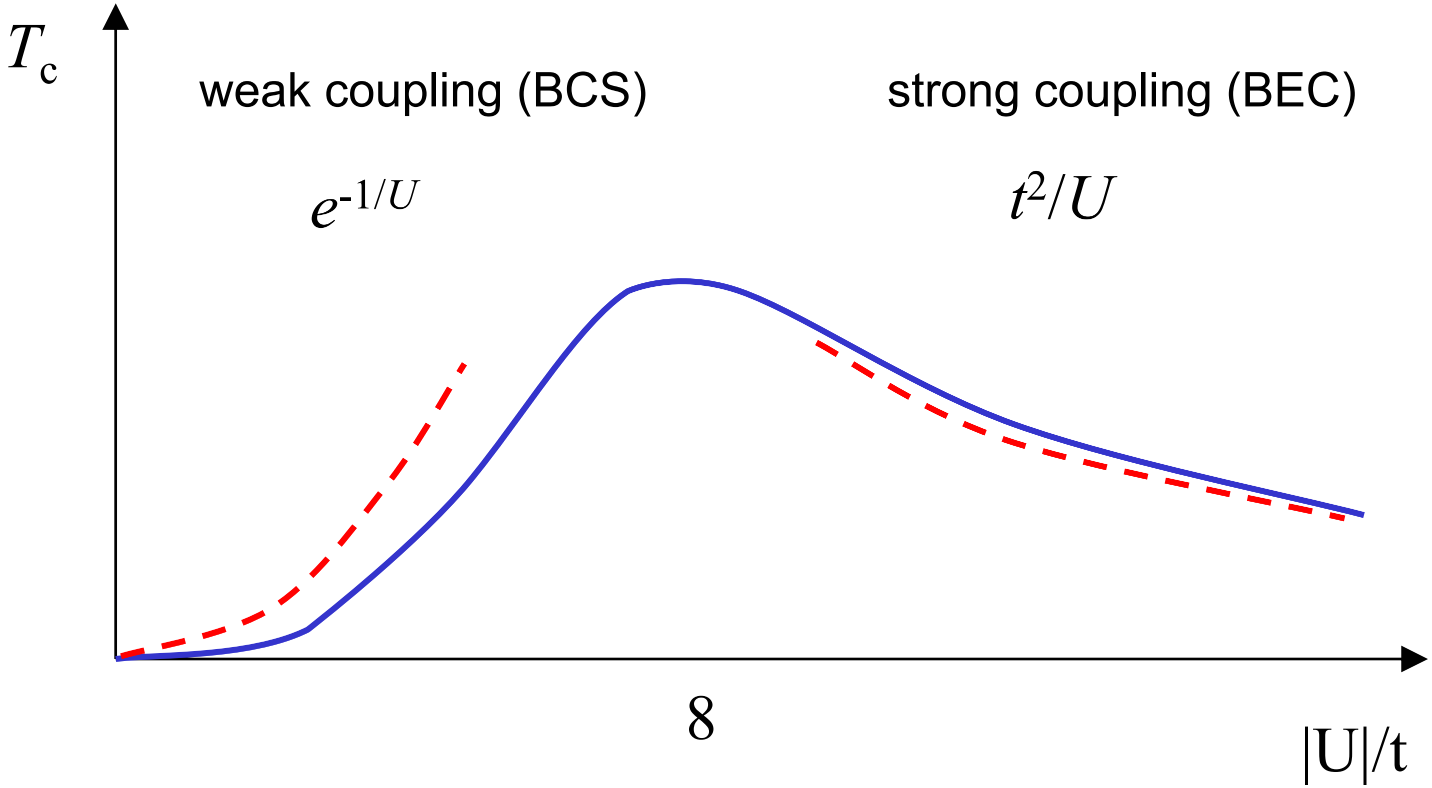
$$J(r) = \frac{4t^2}{U + \delta U(r)}\left[1 - \left(x + \frac{\delta U(r)}{\bar{J}}\right)^2\right] \approx \bar{J}[1 - 2x\delta U(r)/\bar{J}]$$

(*Scalettar et al 1989*)

- $$J_{eff} = \sqrt{\overline{J(r)}\ \overline{[1/J(r)]}^{-1}} < \bar{J}$$

→ Subtle suppression of T_c

U < 0 Hubbard – effect of inhomogeneity



Speculations/open questions

- Can inhomogeneous systems *locally* exceed pairing strength that in bulk could cause structural transition?
- Connection to stripes: can stripes enhance pairing (e.g. "edge" phonons, magnons, or their combinations)?
  - Importance of topological nature of stripes (not of nesting origin)?
- Relation to SC inhomogeneous *repulsive* models (See Arrigoni et al 2005)?