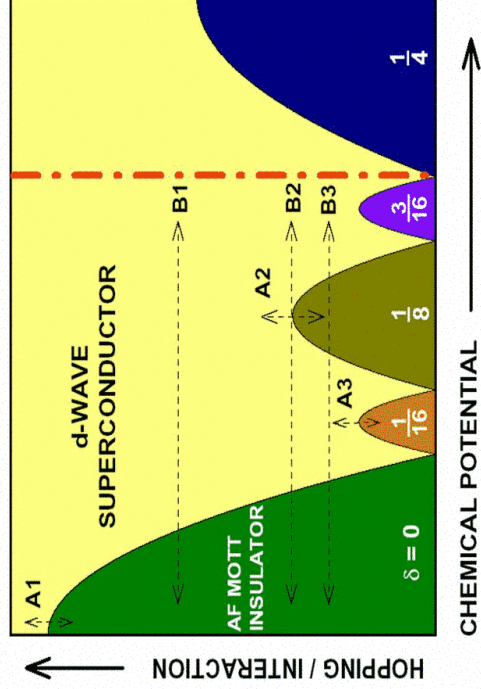


The hole-pair checkerboard state of the high Tc cuprates

Shoucheng Zhang

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Collaborators

H.D.Chen, S. Capponi, O. Vafek

Stanford University

W. Hanke, A.Dorneich, E.Arrigoni, M. Joestingmeier

University of Wuerzburg

A. Auerbach and E. Altman

Technion, Israel

Ali Yazdani group

UIUC

Yoichi Ando group

CRIEPI

[Review of Modern Physics: SO\(5\) theory of antiferromagnetism and superconductivity](#)

- Eugene Demler, Werner Hanke and SCZ

[Antiferromagnetism and hole pair checkerboard in the vortex state of high Tc superconductors](#)
[PRL89, 137004, \(2002\)](#)

[Global Phase Diagram of the High Tc Cuprates](#)
[PRB70, 024516, \(2004\)](#)

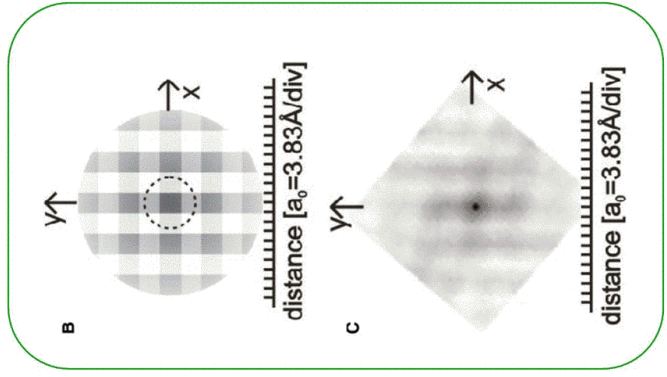
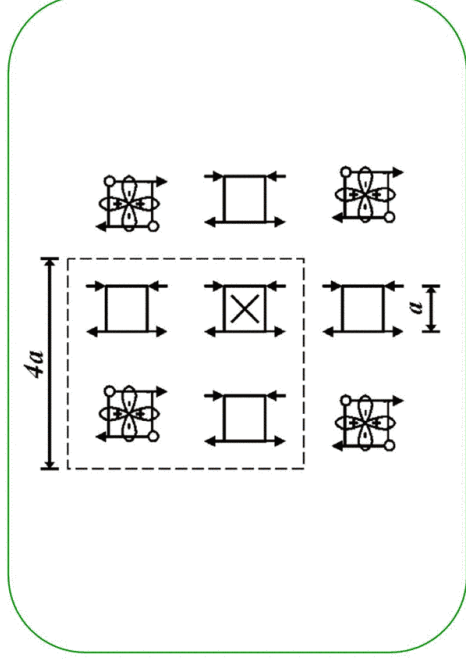
[Pair density wave in the pseudogap state of the high Tc superconductor](#), [PRL 93, 187002, \(2004\)](#).

[Magic doping fractions in the high Tc superconductor](#), [Komiya et al, PRL 94, 207004, 2005](#)

[Key experimental issues in the high Tc problem](#)

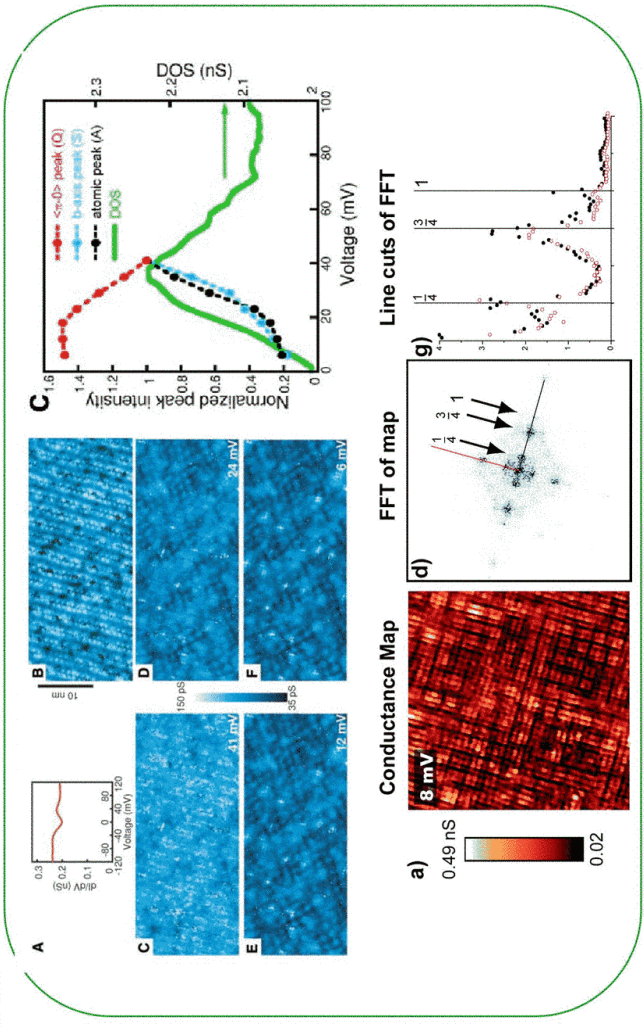
- Nature of the charge order: Stripe or checkerboard?
- Ordered crystal of the holes or hole pairs?
- Supersolid state (pair-density-wave) or insulating hole pair crystal (HPC)?
- Relationship between these competing orders and superconductivity.
- Magic doping levels of LSCO material.
- Charge versus spin orders.
- Fermionic spectra and STM.

Hole pair crystal (or PDW) state and STM
(Chen et al, PRL89, 137004, (2002))



- STM experiments definitely sees 4x4a pattern, both in the vortex core, consistent with a hole pair crystal. It also has rotational symmetry.
- The hole pair crystal state can be realized whenever superconductivity is weakened.

STM in the pseudogap regime and in the absence of magnetic field



Vershinin *et al.*, Science 303, 1995 (2004); Hanaguri *et al.*, Nature 430, 1001, (2004)
See also, J. E. Hoffman *et al.*, Science 295, 466 (2002) and C. Howald *et al.*, Phys. Rev. B 67, 014533 (2003).

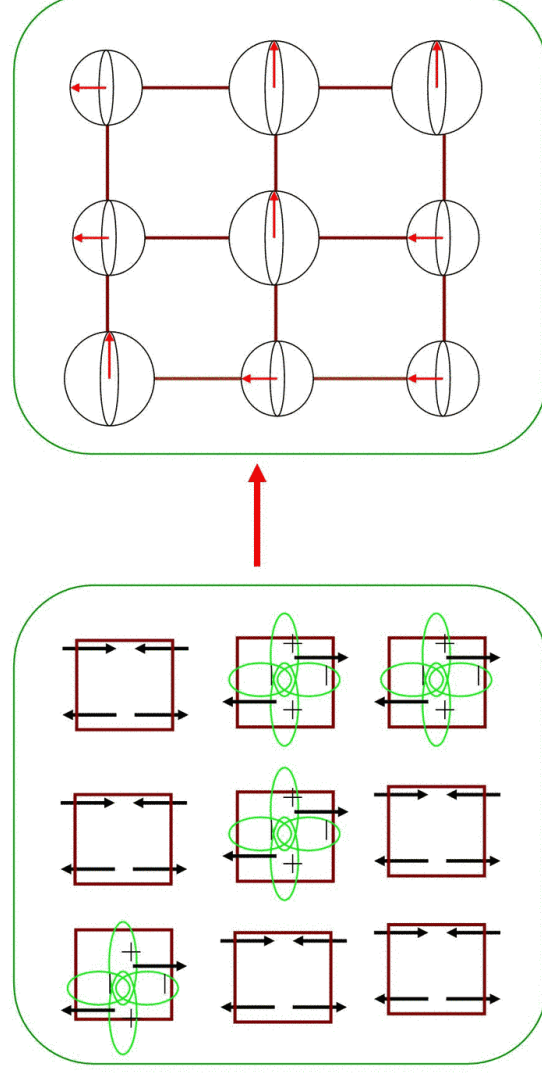
General argument for the hole pair crystal state

- Pre-formed pairs in the pseudogap state (Emery-Kivelson).
- Bosons can either form a superfluid or a solid, or a supersolid state.
- The pair crystal state gains more the short ranged pairing energy compared to the fermi liquid state and the Wigner crystal state of single holes.
- The competition between these states is determined by the ratio of the hole pair kinetic energy and the Coulomb energy.
- Since the pair crystal state is a close competitor of the superconducting state, it can be easily revealed when superconductivity is weakened, i.e. in the vortex core, in the pseudogap regime, and on the surface.
- Approximate symmetry leads to a large pseudogap window. At $D=2$, a model with enhanced symmetry can not order, leading to a strong suppression of T_c .

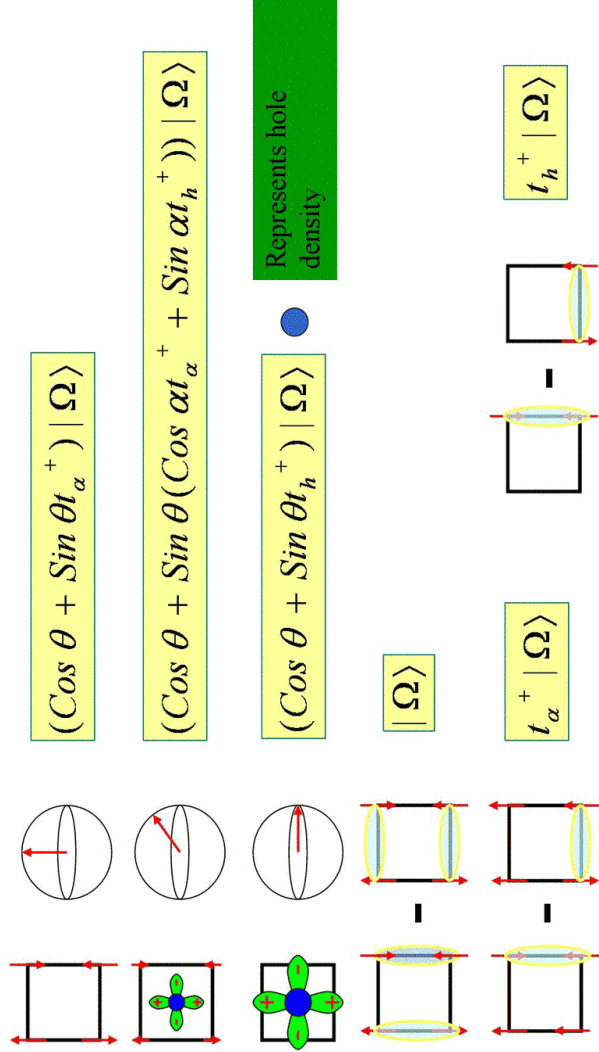
From the t-J model to the SO(5) model

Zhang et al., Altman and Auerbach

- The CORE algorithm

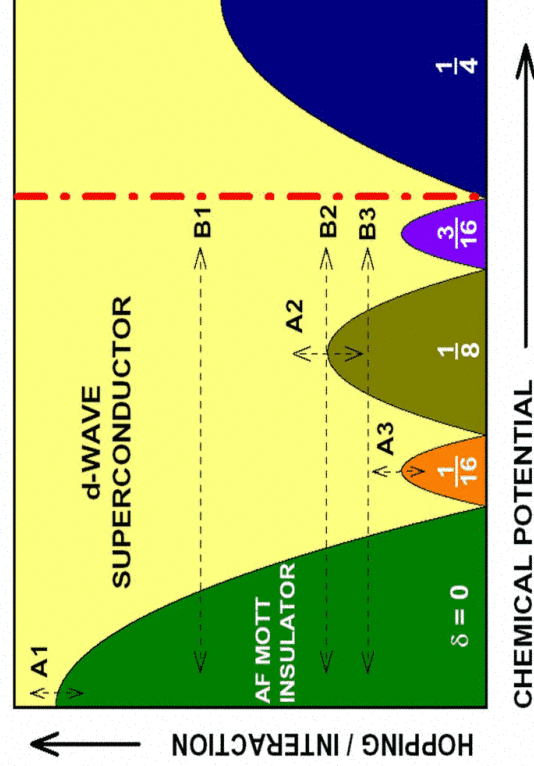


Microscopic basis of the SO(5) postulate: 5 states on a plaquette



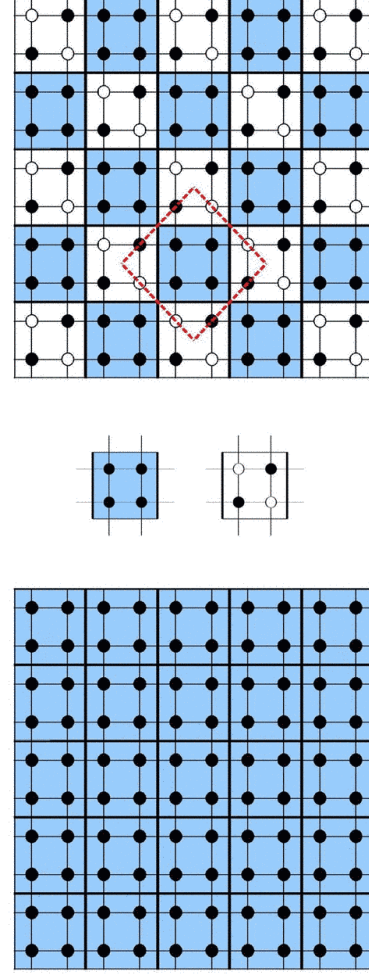
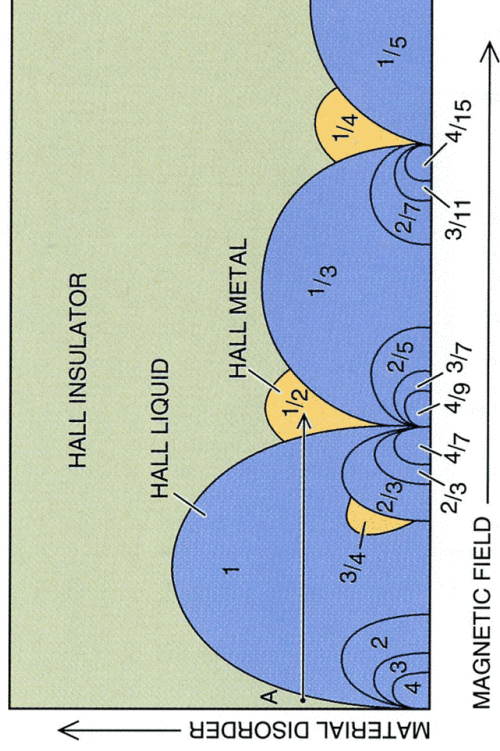
Global phases diagram of the SO(5) model
(PRB70, 024516, (2004))

- Self-similar structure of the hole-pair-crystal phases.
- At the phase boundary between the dSC and the insulating hole-pair-crystal state, one might have the PDW state where both orders coexist.



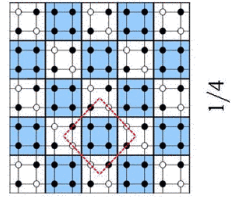
Global phases diagram of the FQHE
 (Kivelson, Lee and Zhang, 1992)

- Different states and phase transitions are related to each other by the Law of Corresponding states, or $SL(2,Z)$ symmetry.

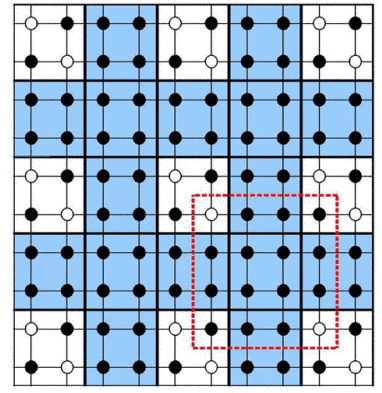
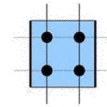
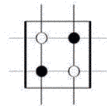


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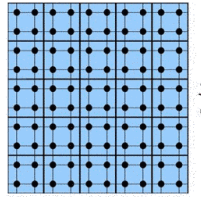
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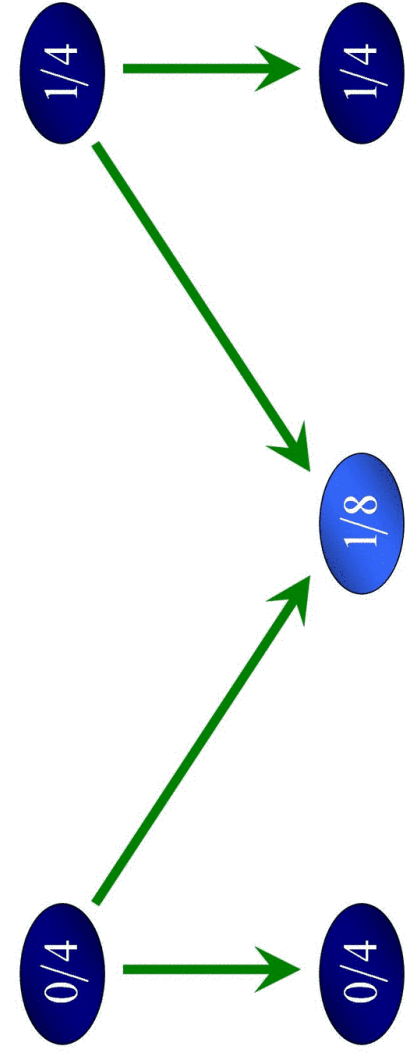
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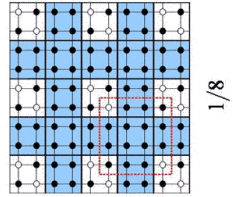


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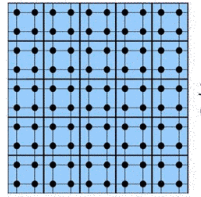


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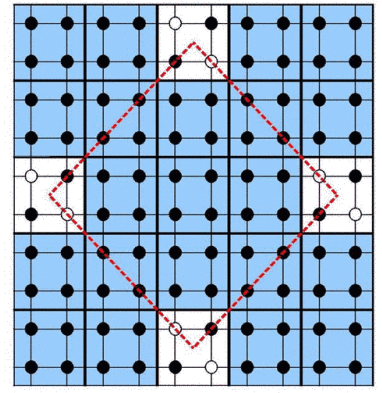
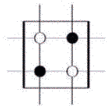
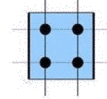




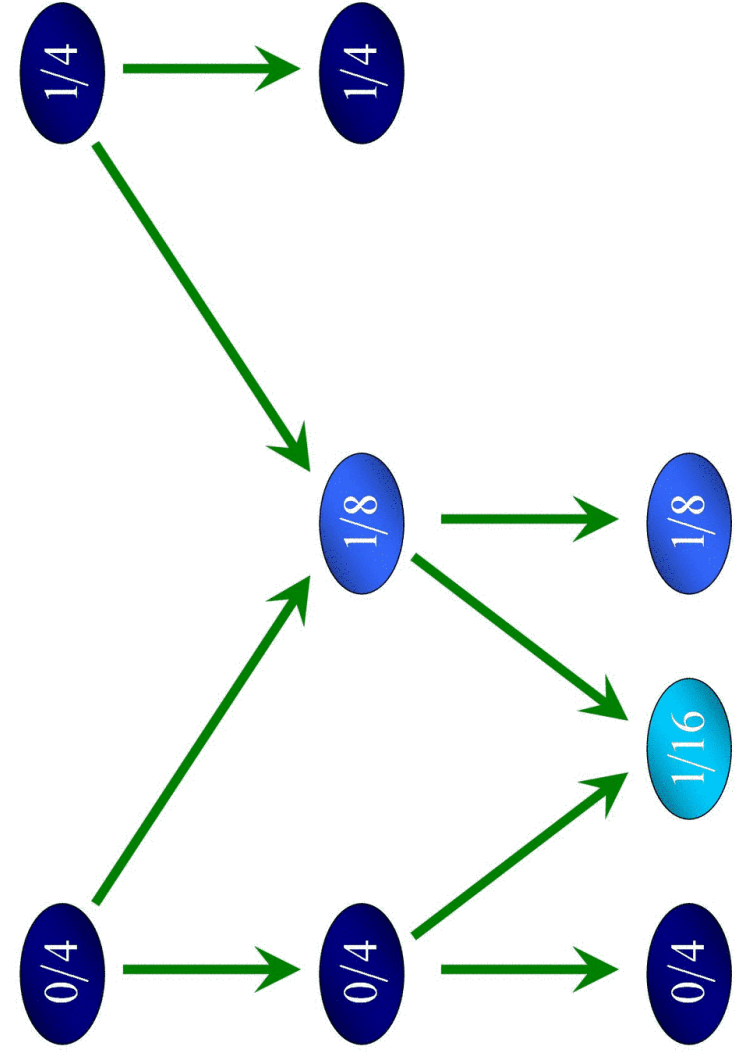
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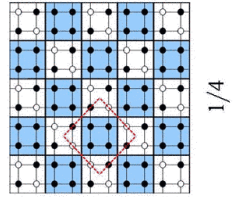


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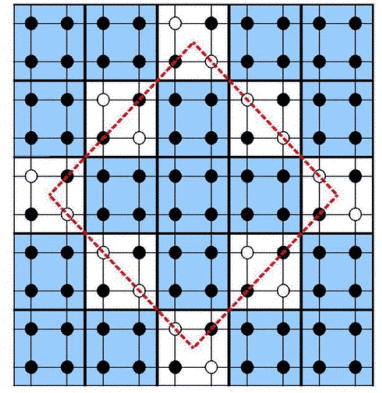
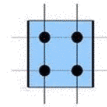
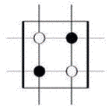


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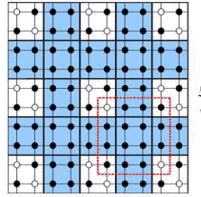




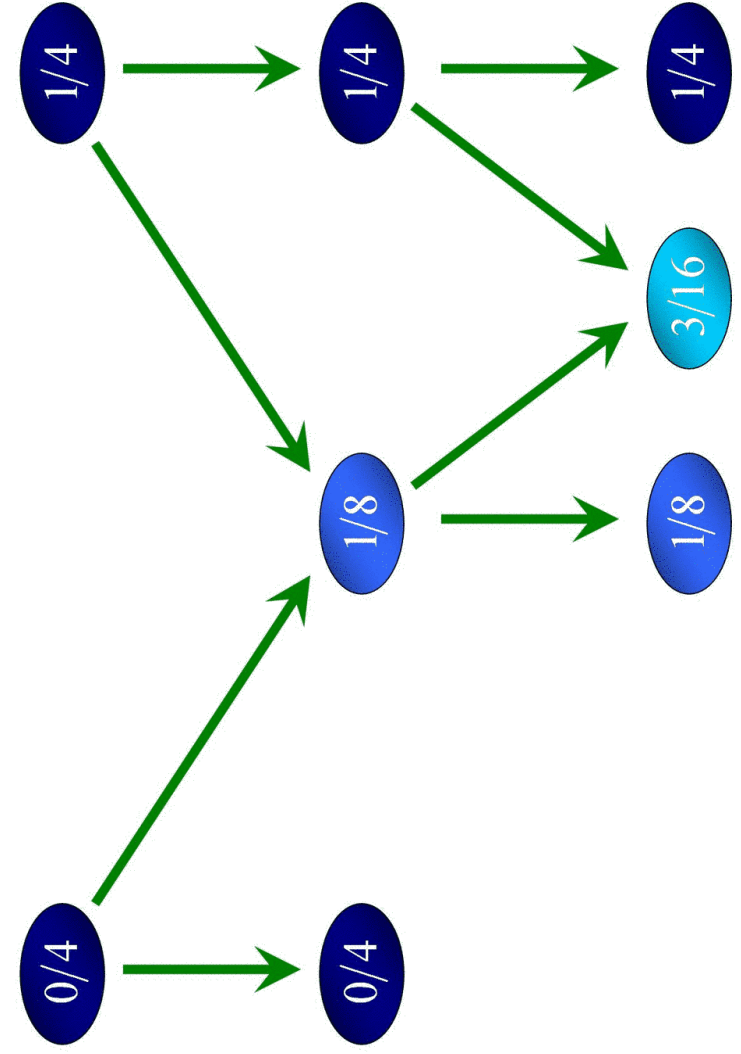
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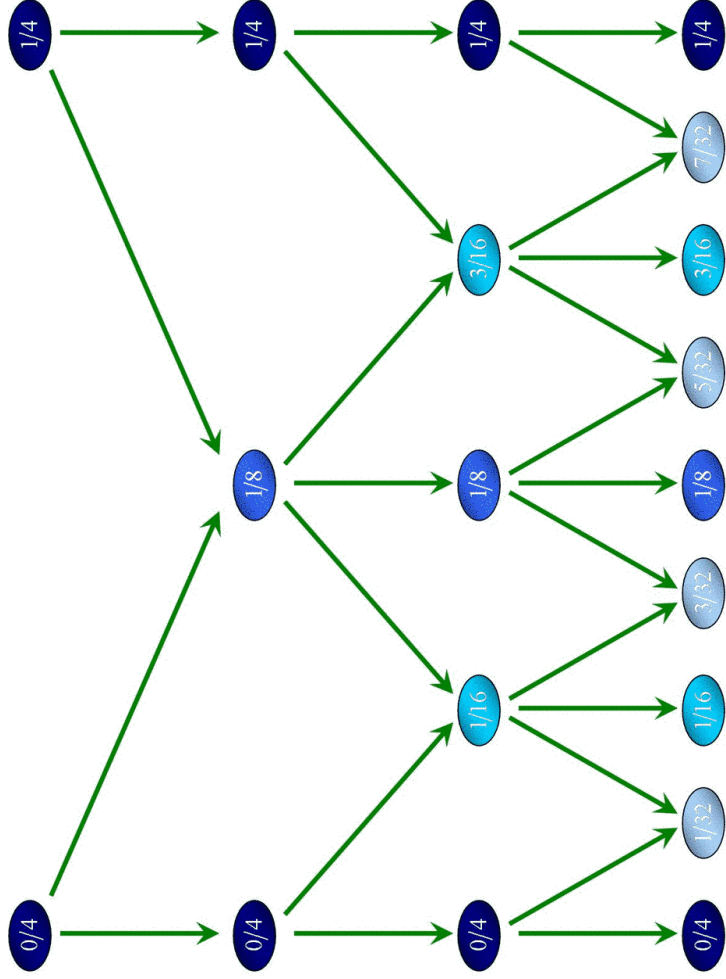


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Hierarchy construction

- The real doping x (holes/Cu) of pair crystals

$$x = \frac{2m + 1}{2^n}$$

- The unit cell is $L_a \times L_a$,

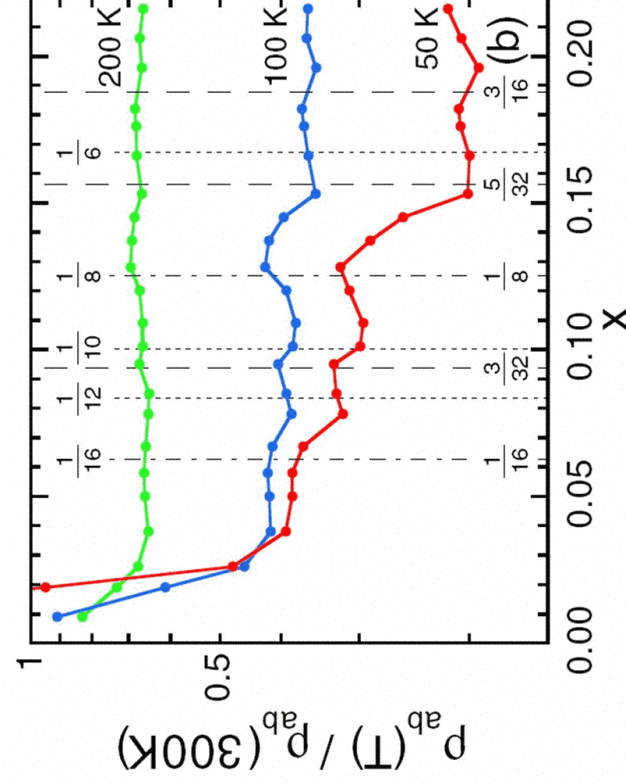
$$L = 2^{(n+1)/2}$$

- Each unit cell has $2m+1$ hole-pairs.
- If n is even, the primitive vectors are along the diagonal directions. If n is odd, the primitive vectors are along the horizontal and vertical directions.

[Experimental predictions for the B3 cut of the phase diagram](#)

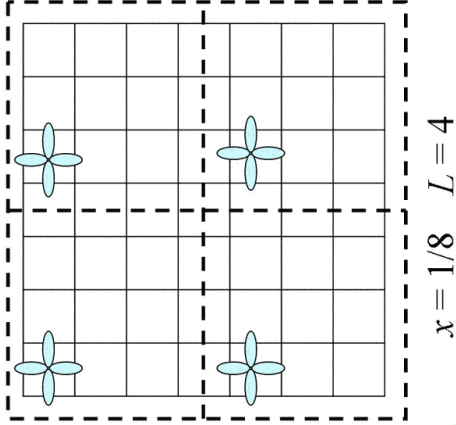
- Magic filling factors
 - Comparison with transport data
- Nature of the charge order
 - Comparison with STM
- Nature of the spin order
 - The mysterious rotation of the neutron peaks at $x=1/16$

[Evidence of 2D charge ordering in \$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4\$ \(Komiya et al, PRL 94, 207004, 2005\)](#)



Hole pair crystal vs hole crystal

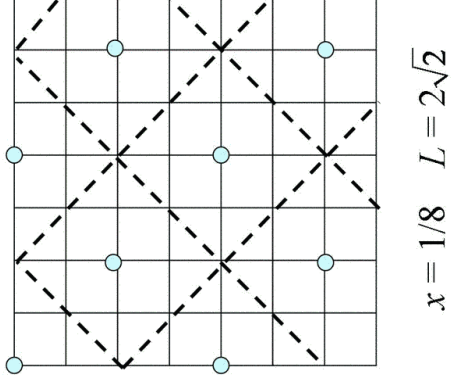
- For **hole pair crystal**, there is 1 hole-pair (2 holes) per unit cell. If the unit cell is $L \times L$, the doping is $x=2/(L \times L)$.



H.D. Chen *et al.*, Phys. Rev. Lett. **89** 137004(2002).

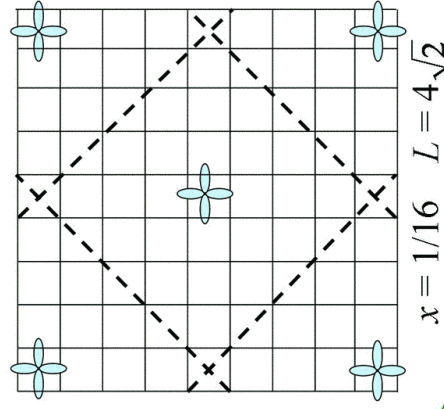
H.C. Fu *et al.*, cond-mat/0403001

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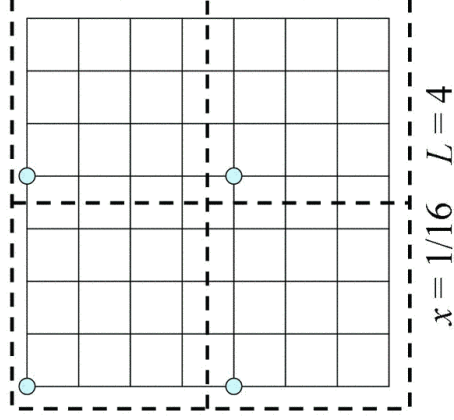
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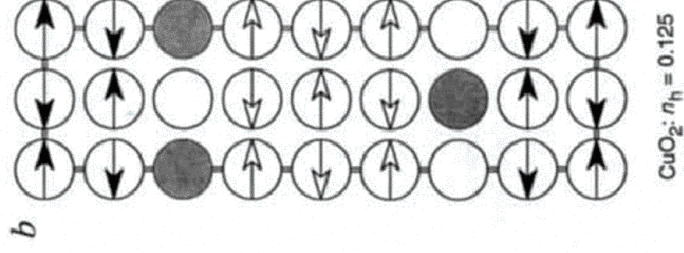
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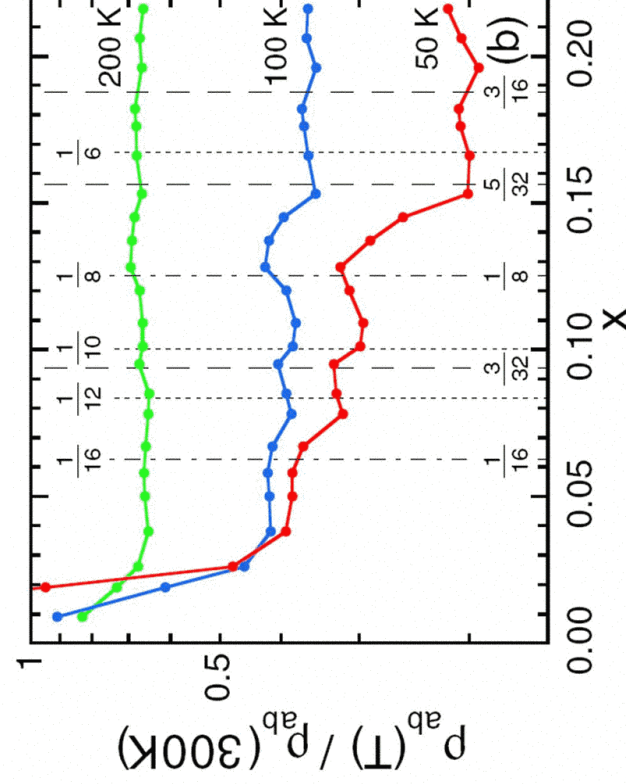


[Pair checkerboard versus stripes](#)

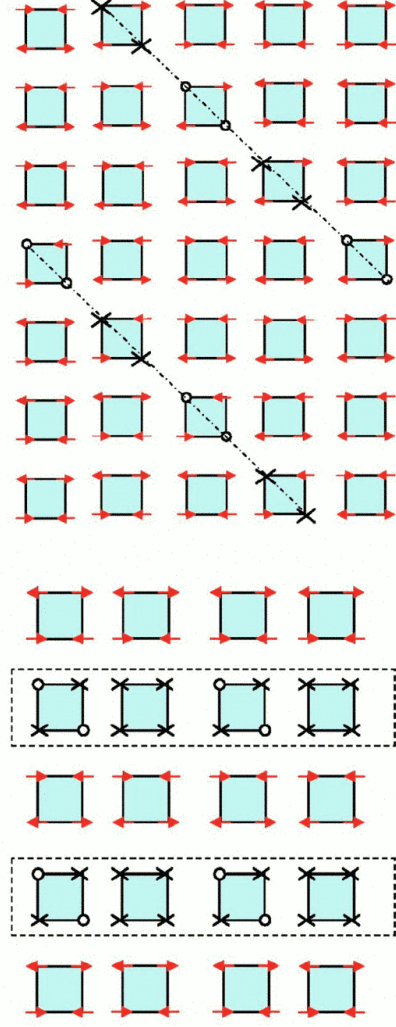
- The standard model for stripes involves a fixed hole density on the stripe. Different doping is accommodated by varying the spacing between stripes.
- This picture leads to "magic fractions" at $1/2p$, i.e. $x=1/6, 1/8, 1/10$ etc. Besides $1/8$, other fractions, which a priori should be equally stable, have not been seen.
- The checkerboard model has a distinct hierarchy, $1/8$ is stronger than $1/16$, which is stronger than $3/32$.



[Evidence of 2D charge ordering in \$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4\$ \(Komiya et al, PRL 94, 207004, 2005\)](#)



Magnetic order



x=1/8, anti-phase domain

x=1/16, anti-phase domain

Checkerboard charge order naturally explains the magic rotation of the neutron scattering peaks at x=1/16. Search for a weak peak at x=3/16, coexisting with the x=1/8 peak, but rotated by 45 degree!

STM Theorems for the mean field BCS/pair density wave state

$$H = \sum_{\langle i,j \rangle} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + \sum_{\langle i,j \rangle} \Delta_{ij} c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger + h.c. + \sum_i V_i c_{i\sigma}^\dagger c_{i\sigma} + h.c.$$

For V=0, there is an exact particle-hole symmetry, generated by the transformation:

$$c_{i\sigma} \Rightarrow (-1)^i c_{i\sigma} \quad H \Rightarrow -H$$

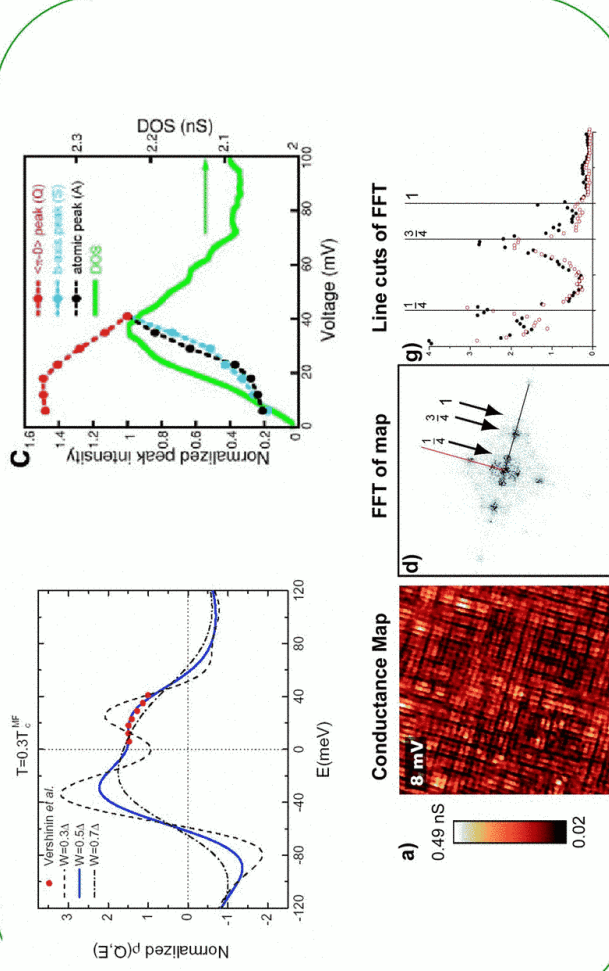
For arbitrary distribution of the d-wave pair amplitude. Assuming particle-hole symmetry and the pseudogap state being dominated by classical (time-independent) phase fluctuations, we show that:

- for **d-wave pair-density-wave** with wavevector Q

$$\rho(Q, E) = \rho(Q, -E)$$

Note: particle-hole symmetry breaking does not change the results qualitatively.

Comparison with STM results



Vershinin *et al.*, Science 303, 1995 (2004); Hanaguri *et al.*, Nature 430, 1001, (2004)
See also, J. E. Hoffman *et al.*, Science 295, 466 (2002) and C. Howald *et al.*, Phys. Rev. B 67, 014533 (2003).

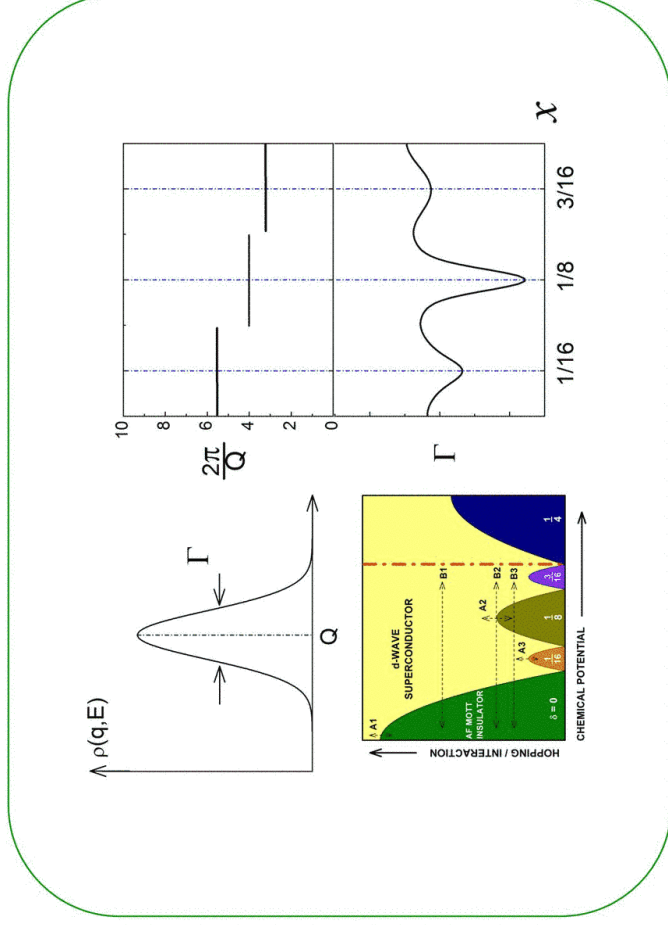
Comparison with STM experiments

- Pair-density-wave is consistent with BSCCO experiments.
 - The non-dispersive peak picks up intensity when the pseudogap opens[1].
 - Consistent with **no** π phase difference at high energy for the new non-dispersive peak at $(2\pi/4.5a, 0)$
- Non-superconducting hole-pair-crystal state could be consistent with recent NCCOC experiment.
 - The periodicity of the new peak is commensurate $4a \times 4a$ [2], first predicted theoretically in H.D. Chen *et al.*, Phys. Rev. Lett. 89 137004(2002).
 - Is there a π phase difference at high energy for the $4a \times 4a$ peak? If so, this may indicate that the hole-pair-crystal state is **not** co-existing with superconductivity.

[1] Vershinin *et al.*, Science (2004).

[2] Hanaguri *et al.*, Nature (2004).

[More predictions on STM and neutron peaks](#)



[Other theoretical developments](#)

Tesanovic, cond-mat/0405235

Anderson, cond-mat/0406038

Franz, Science 305, 1410, (2004)

Vojta, prb66, 104505, (2002)

White and Scalapino, cond-mat/0408249

PHYSICS

Crystalline Electron Pairs

Marcel Franz

Conclusions:

- **Comparison with experiments.**
 - Global phase diagram, with prediction of a new charge-ordered states at $x=1/16, 3/32, 3/16$.
 - STM experiment of 4x4a checkerboard state.
 - Charge ordering in the pseudogap state.
 - Transport anomalies at magic fractions.
- **Theoretical predictions.**
 - Enhanced transport anomalies at magic fractions in high magnetic field.
 - STM and neutron spectra for $x=1/16$ and $x=3/16$.
- **Hole-pair crystal state and the pairing mechanism in the cuprate.**
 - Strong, local pairing.
 - Spectroscopic measurement of the local hole pair.