Strongly correlated quantum systems out of equilibrium

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Beyond Standard Optical Lattices
&
Disentangling Quantum Many-Body Systems
KITP 2010

Correlated systems out of equilibrium

- Quantum quenches
- Free expansions on a lattice
- Driven correlated systems
- Steady-state out of equilibrium

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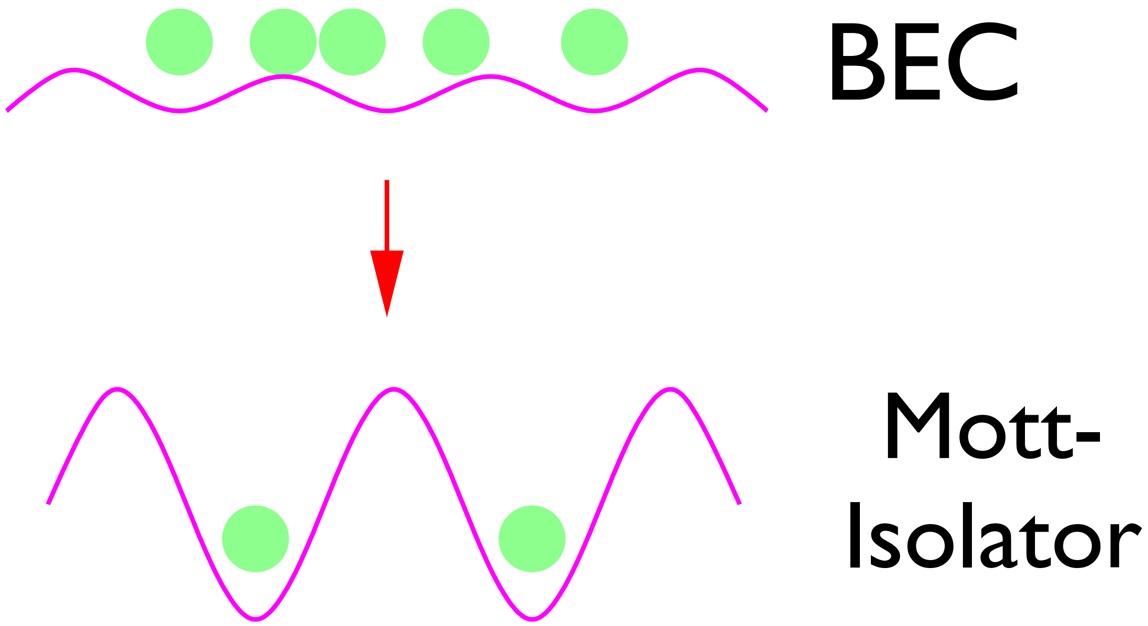
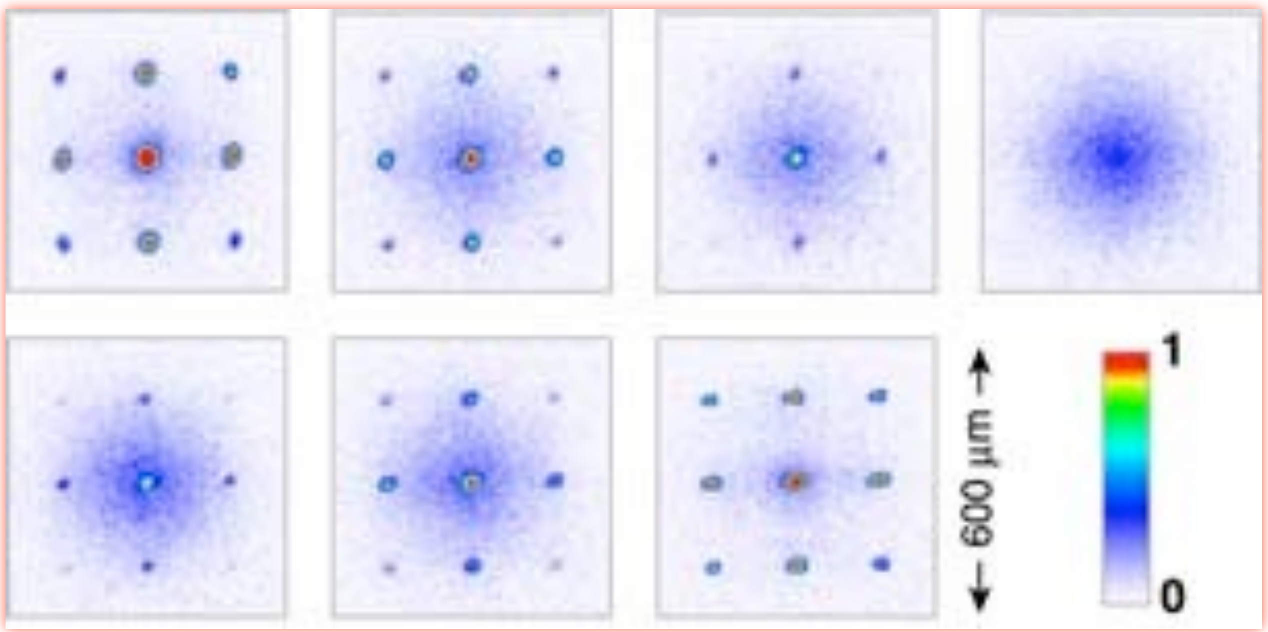
Strongly correlated quantum gases out of equilibrium

Collapse and revival of a BEC

M. Greiner, O. Mandel, T.W. Hänsch, I. Bloch, Nature **419**,51 (2002)





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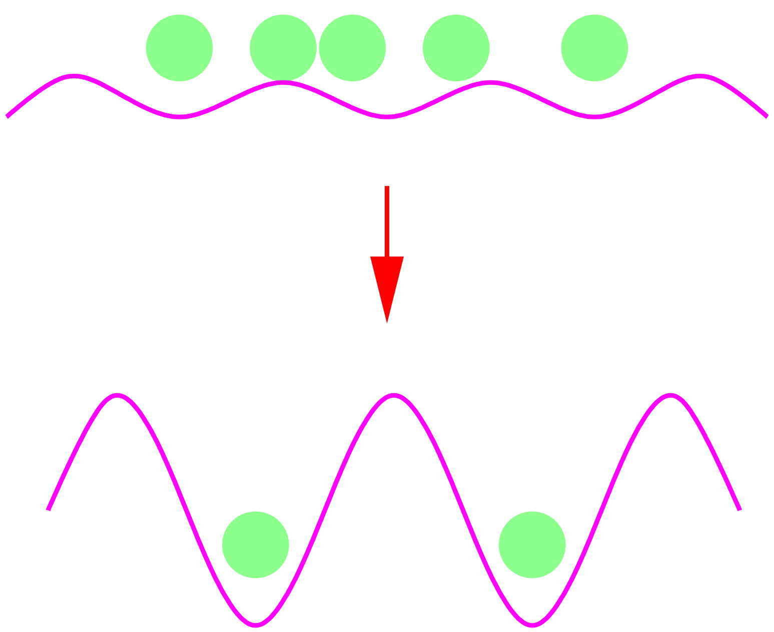
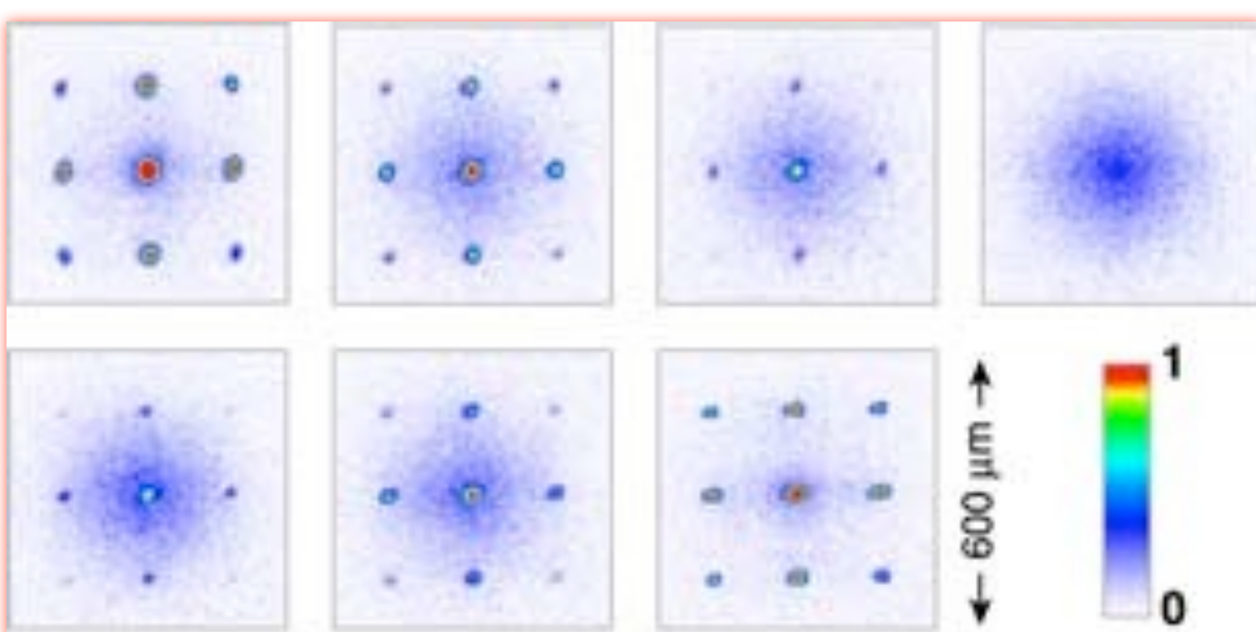


BEC

$$H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1)$$

Mott-
Isolator



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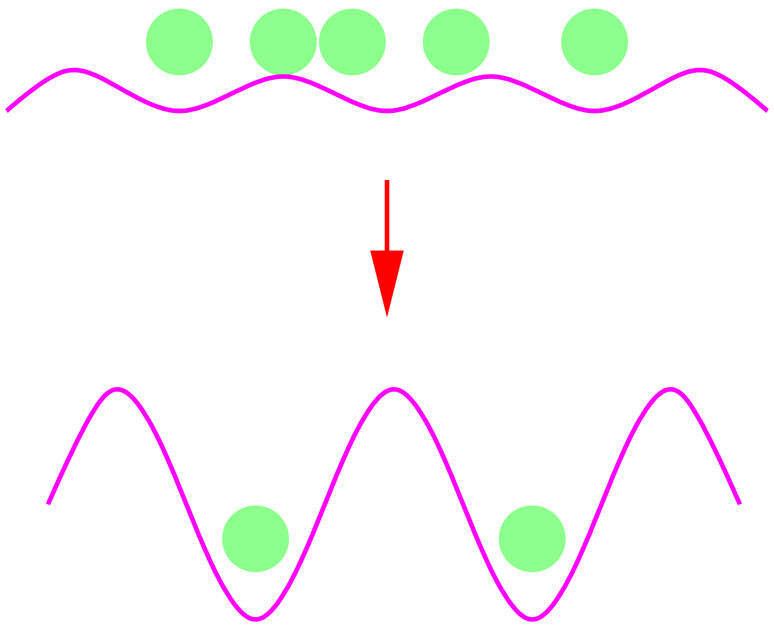

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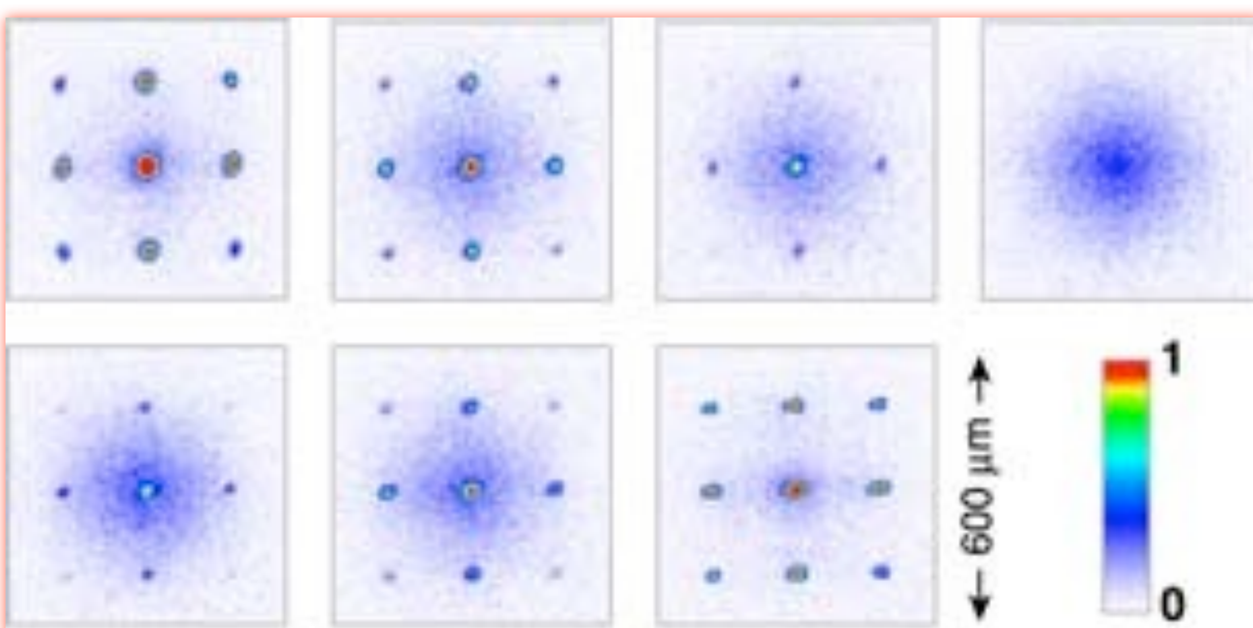
$$H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1)$$

↓

$$U \gg t$$

↓



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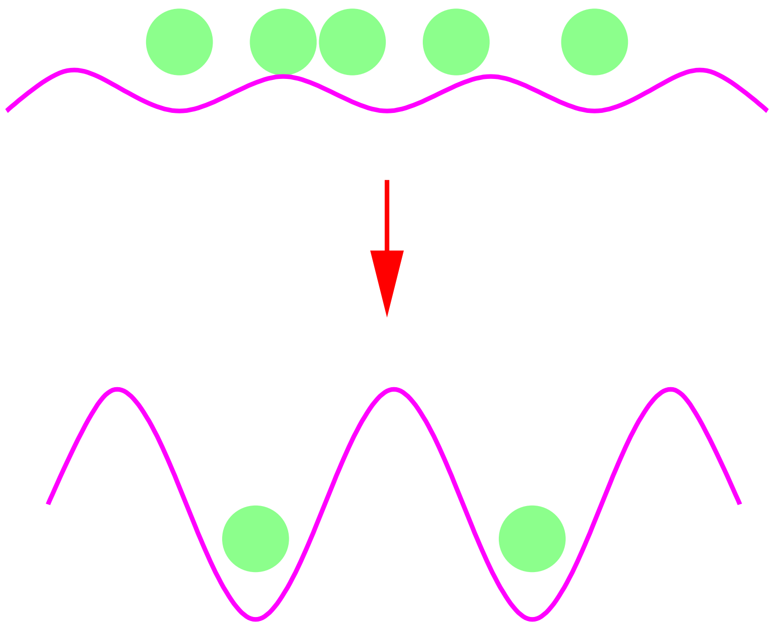


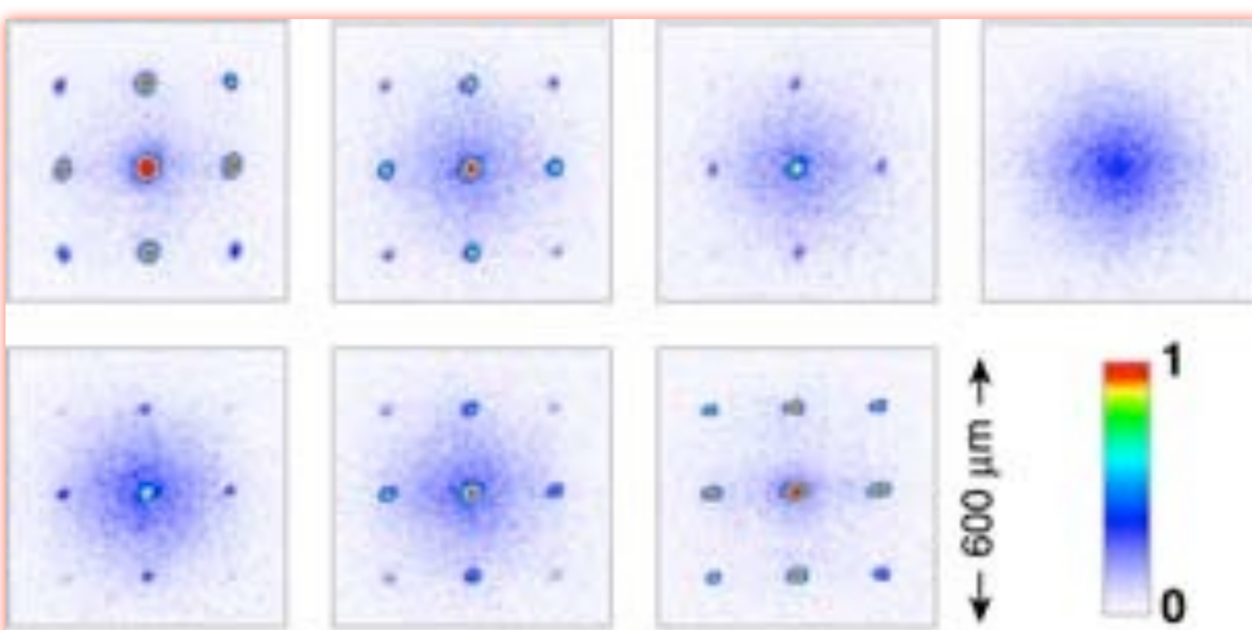
BEC

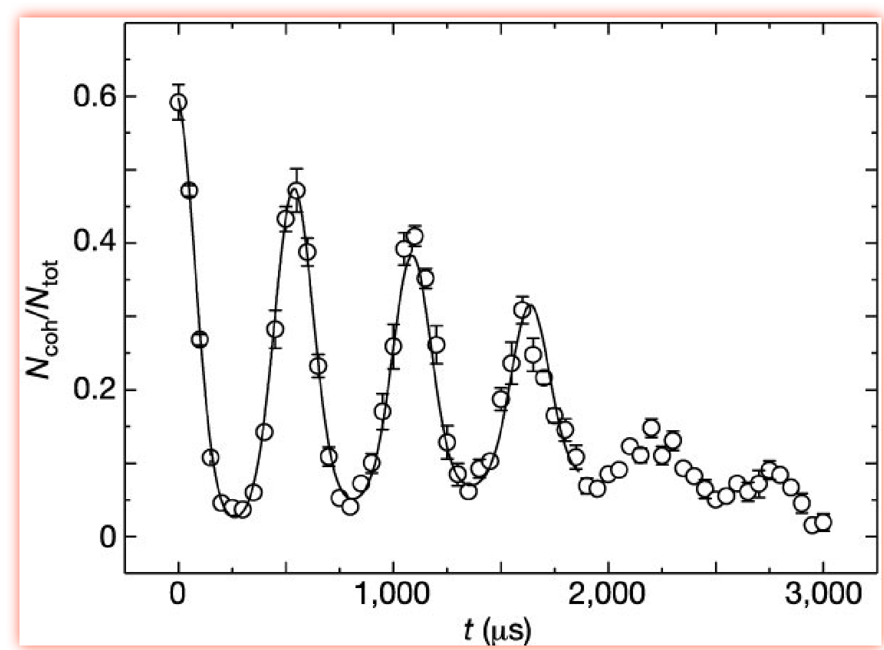
Mott-Isolator

$$H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$

$$U \gg t$$

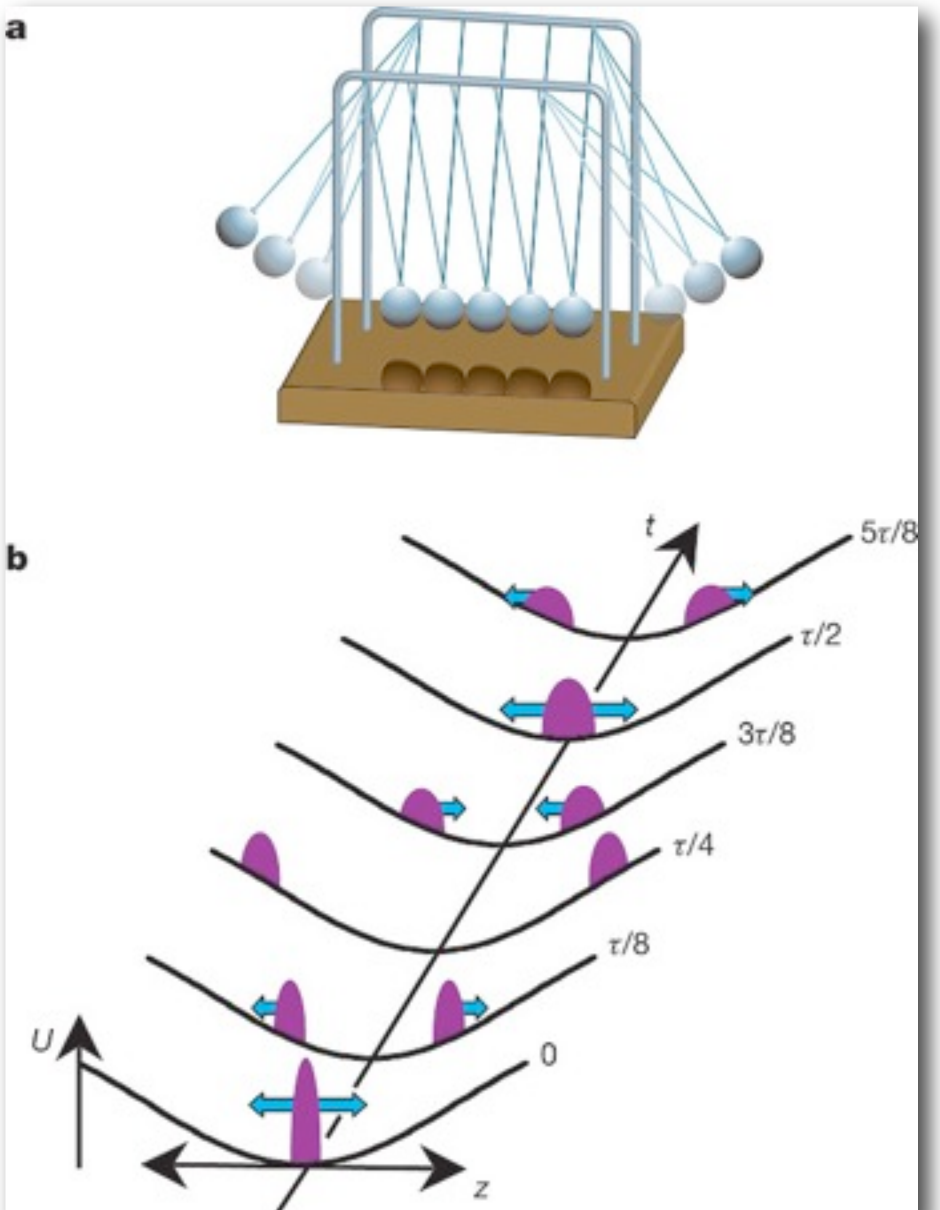
$$| \psi(t) \rangle \simeq \exp \left[-i \left(\frac{U}{2} \sum_i n_i (n_i - 1) \right) t \right] | \psi_0 \rangle$$

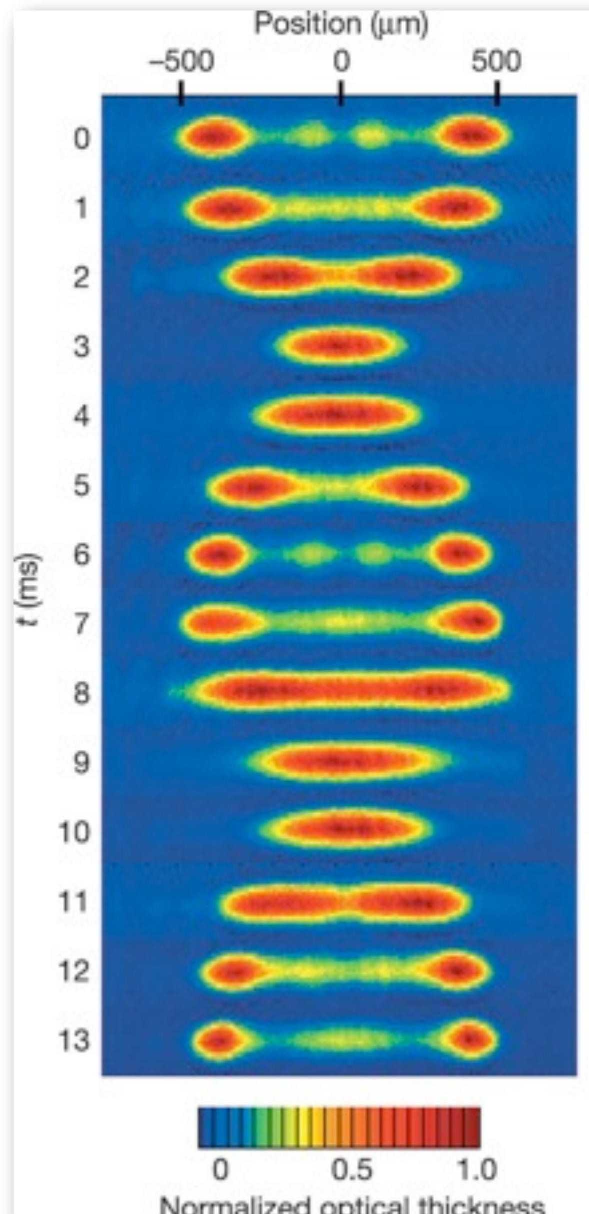


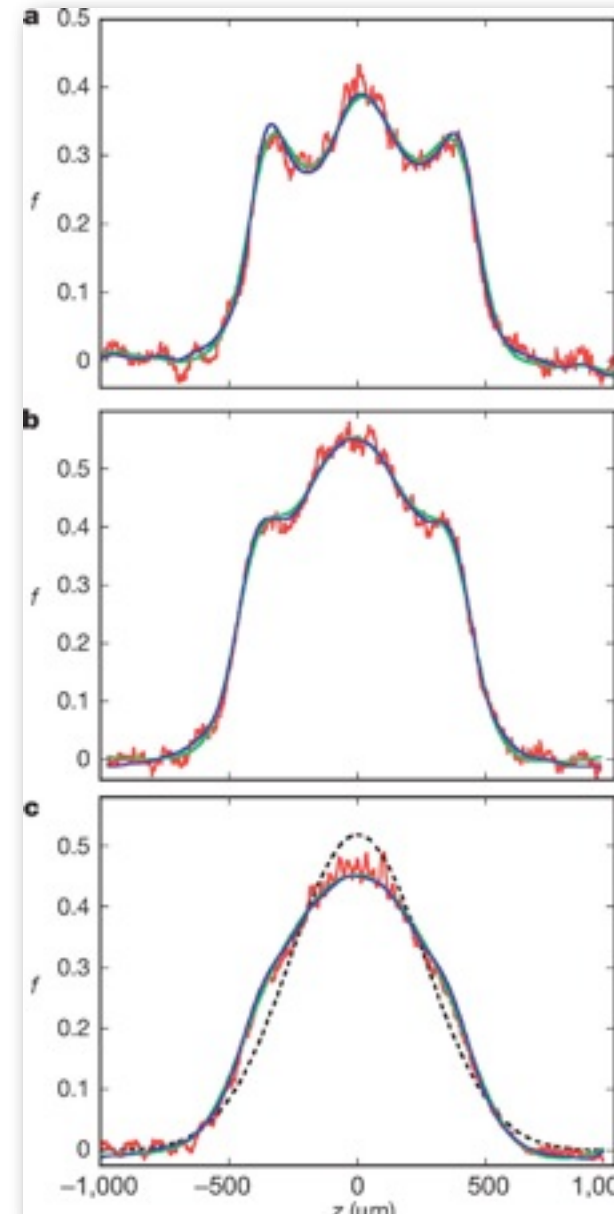


1D interacting bosons out of equilibrium

T. Kinoshita, T. Wenger, and D. Weiss, Nature **440**, 900 (2006)







Even after 15 - 20 periods the momentum distribution function clearly differs from a thermal distribution

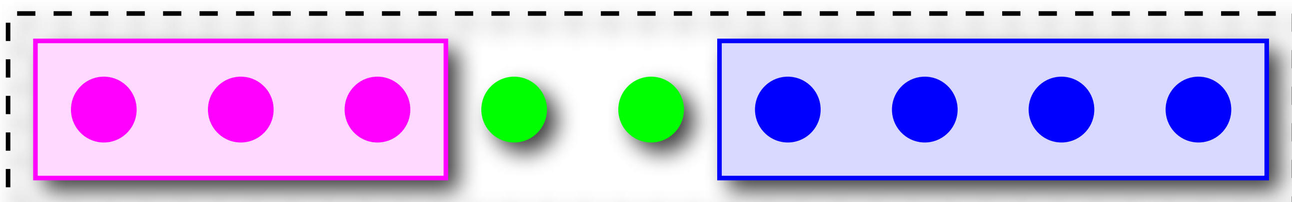
Numerical methods

Time dependent DMRG

S.R. White and A.E. Feiguin, PRL **93**, 076401 (2004)
A.J. Daley et al. J. Stat. Mech.: Theor. Exp. P04005 (2004)

Trotter approximation → $$H = \sum_i H_{i,i+1}$$

$$\mathrm{e}^{-iH\Delta t} = \mathrm{e}^{-iH_{even}\Delta t}\mathrm{e}^{-iH_{odd}\Delta t} + \mathcal{O}\left(\Delta t^2\right)$$



$$\rho_s = \mathrm{Tr}_{\mathrm{env}} \mid \psi_s, \psi_{\mathrm{env}}\rangle\langle\psi_{\mathrm{env}}, \psi_s \mid$$

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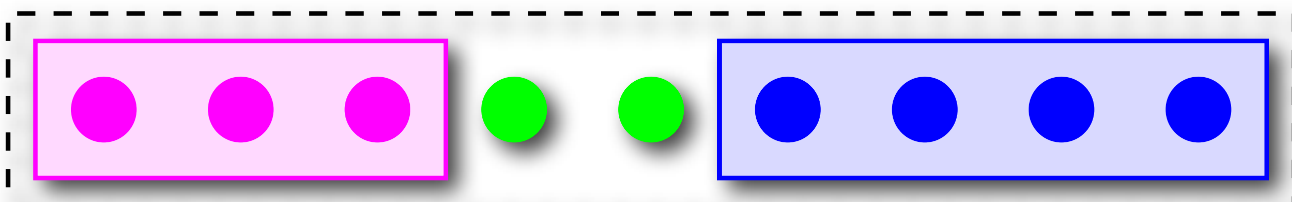

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Lanczos approximation → Krylov subspace

$$\mathrm{e}^{-iH\Delta t} \mid \psi\rangle \simeq V_m \mathrm{e}^{-iL_m\Delta t} V_m^T \mid \psi\rangle \equiv \mid \tilde{\psi}\rangle$$

Exact error bound   M. Hochbruck and Ch. Lubich, SIAM J. Numer. Anal. **34**, 1911 (1997)

$$||\mathrm{e}^{-iH\Delta t} \mid \psi\rangle - \mid \tilde{\psi}\rangle|| \leq 12\mathrm{e}^{(W\Delta t)^2/16m}\left(\frac{eW\Delta t}{4m}\right)^m$$

almost exponential convergence

Exact numerics for hard core bosons in 1D

Hard-core bosons on a 1D lattice

$$H = -t \sum_i \left(b_i^\dagger b_{i+1} + \text{h.c.} \right) + V_\alpha \sum_i x_i^\alpha n_i$$

Jordan-Wigner transformation

$$b_i^\dagger = f_i^\dagger \prod_{j=1}^{i-1} \mathrm{e}^{-i\pi f_j^\dagger f_j}$$

One-particle Green's function for hard-core bosons

$$G_{ij} = \langle \psi_F^G \mid \prod_{\alpha=1}^{i-1} \mathrm{e}^{i\pi f_\alpha^\dagger f_\alpha} f_i f_j^\dagger \prod_{\beta=1}^{j-1} \mathrm{e}^{-i\pi f_\beta^\dagger f_\beta} \mid \psi_F^G \rangle$$

Evolution of a Slater determinant by a bilinear form

$$\mid \psi_F^G \rangle = \prod_{\ell=1}^{N_f} \sum_{i=1}^{N} P_{i\ell} f_i^\dagger \mid 0 \rangle \implies \mathrm{e}^{f_\alpha^\dagger A_{\alpha\beta} f_\beta} \mid \psi_F^G \rangle = \prod_{\ell=1}^{N_f} \sum_{i=1}^{N} \tilde{P}_{i\ell} f_i^\dagger \mid 0 \rangle$$

Quantum quenches

Strongly correlated fermions after a quantum quench

S.R. Manmana, S. Wessel, R.M. Noack, and A. M., PRL **98**, 210405 (2007)

Repulsive spinless fermions

$$H = -t \sum_i \left(f_i^\dagger f_{i+1} + h.c. \right) + V \sum_i n_i n_{i+1} \left(+ V_2 \sum_i n_i n_{i+2} \right)$$

$$V < 2t \rightarrow \text{metal}, \; V > 2t \rightarrow \text{insulator}, \; V_2 \neq 0 \rightarrow \text{nonintegrable}$$

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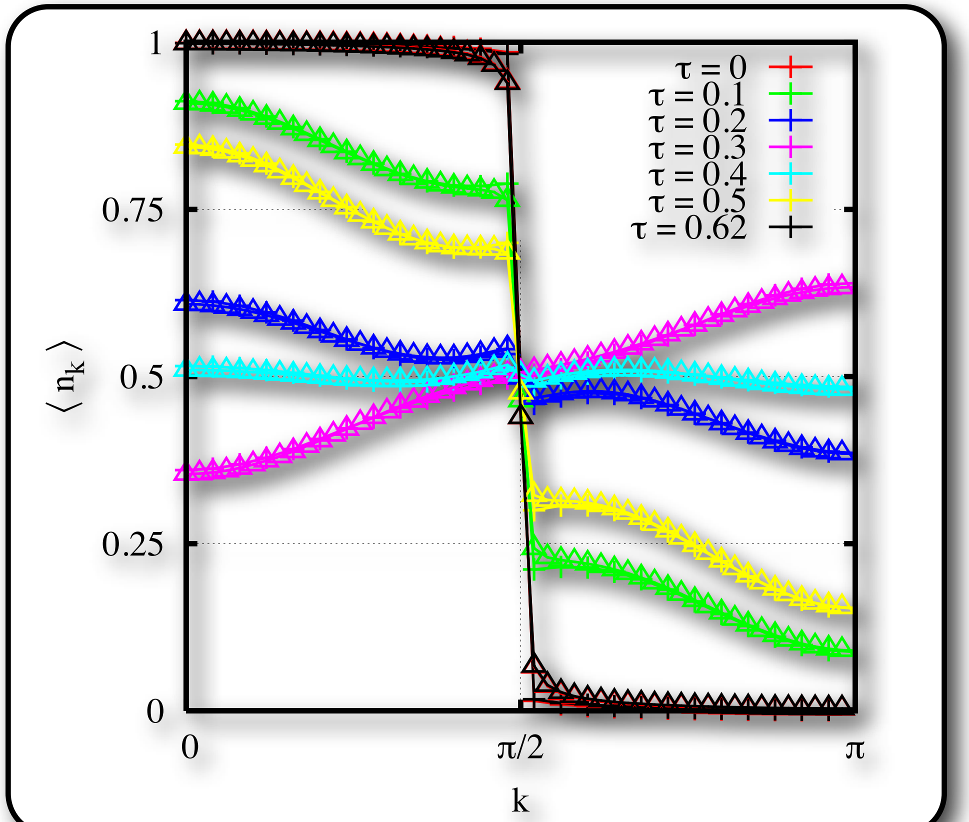
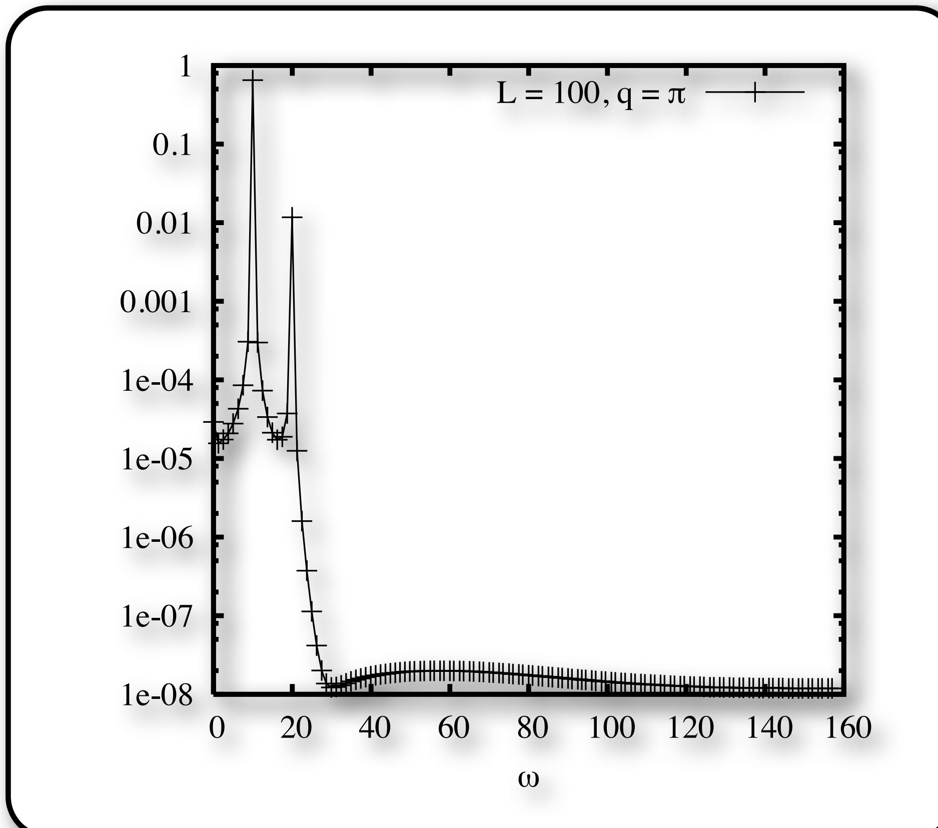


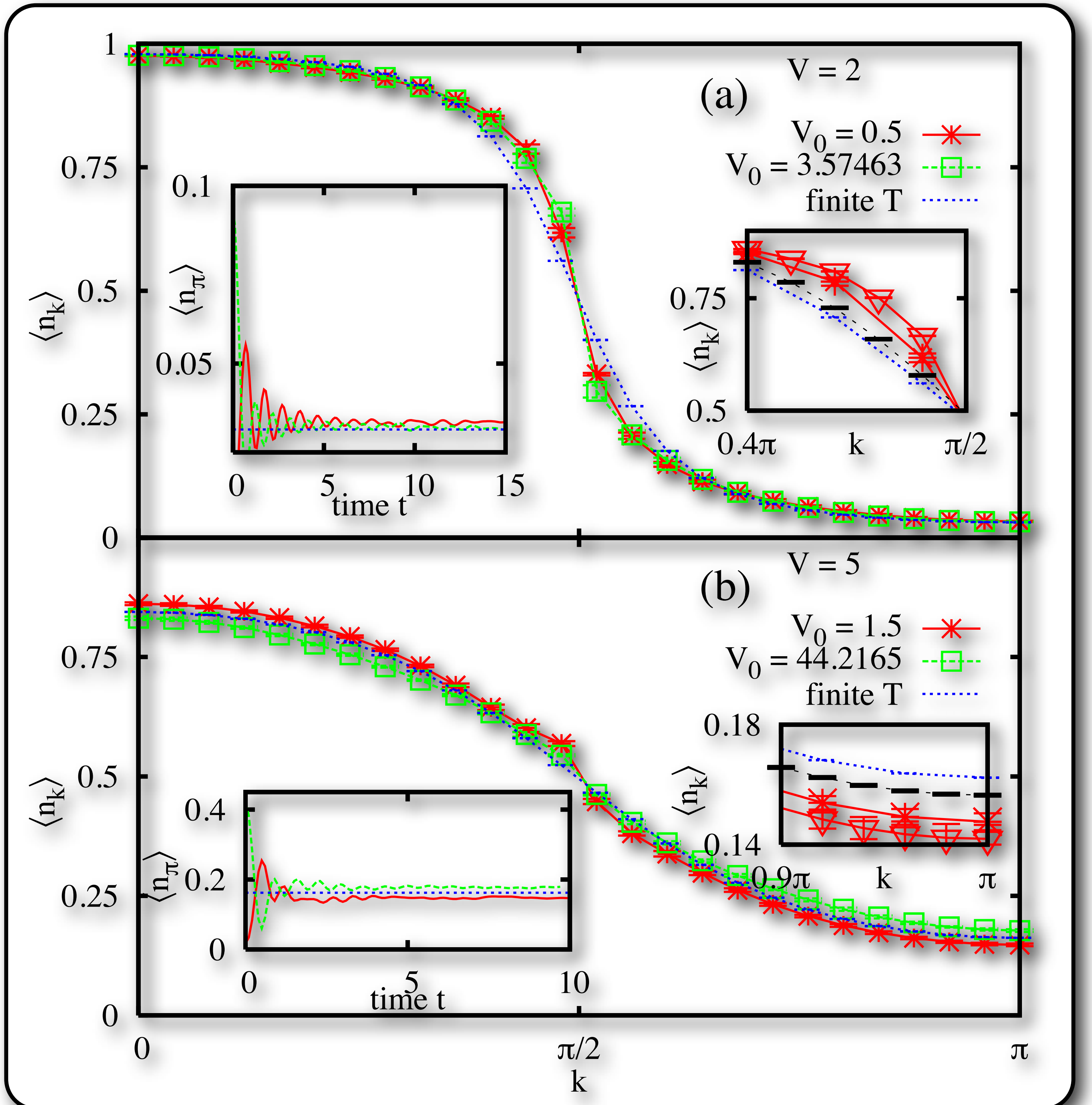
Collaps and revival in the atomic limit

Time scale

V^{-1}



Relaxation after a quantum quench

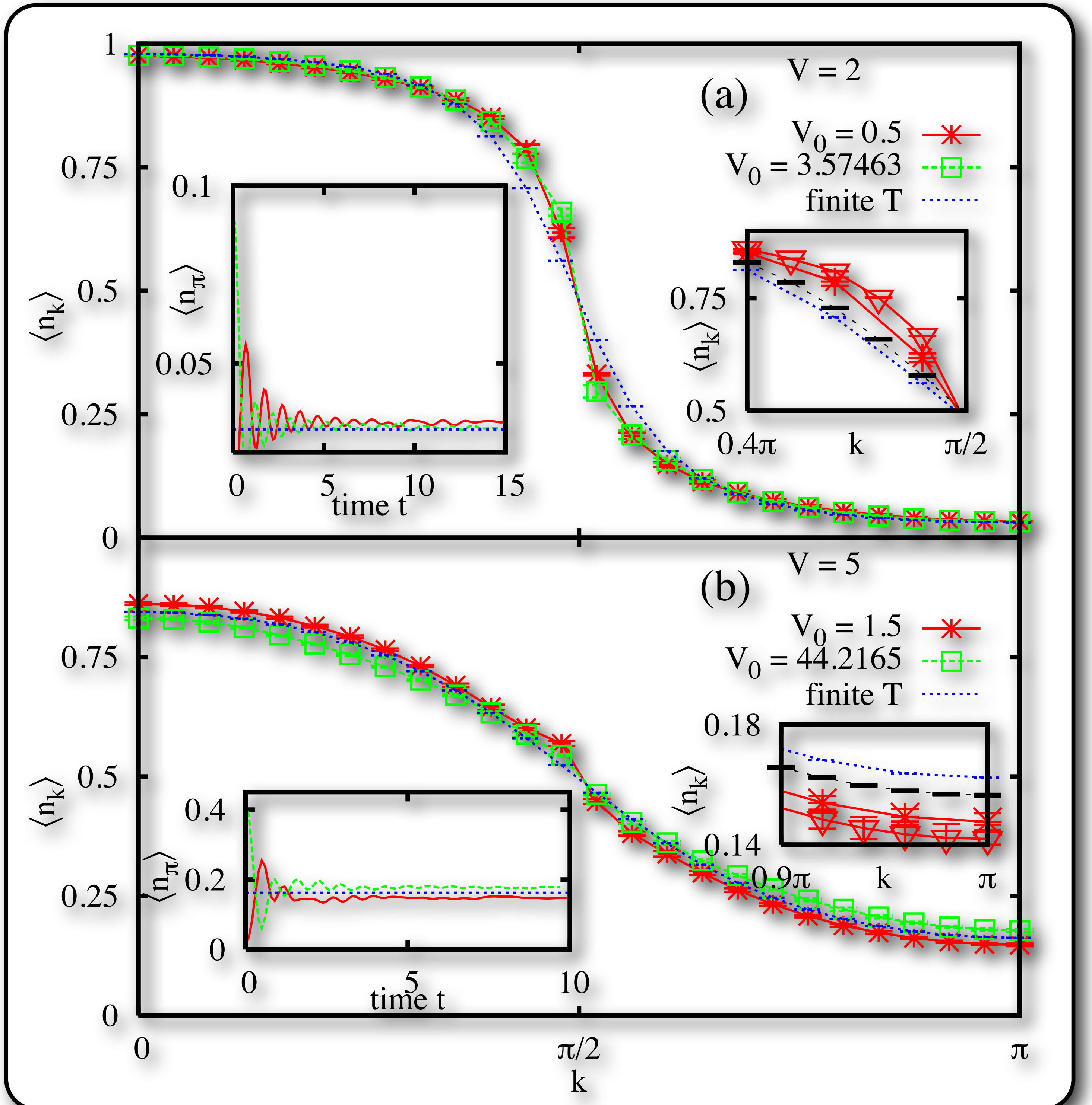


- Evolution from two initial states with same E

(a) Evolution with V=2t. Relax to same n_k but no thermalization

(b) Evolution with V=5t. Initial states far apart. They relax to different n_k. No Thermalization

Relaxation after a quantum quench



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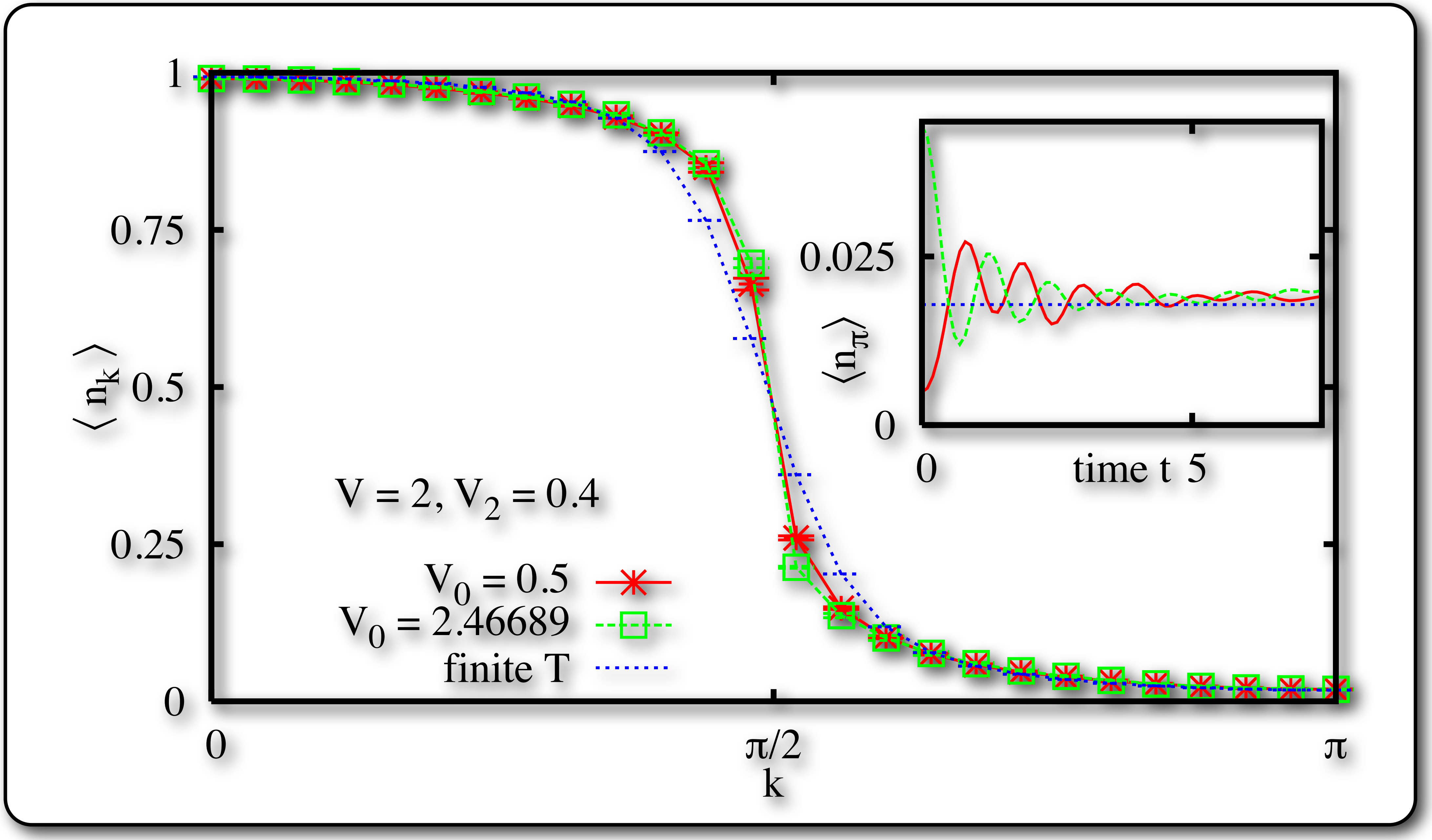
Memory of initial state lost but no thermalization

Role of integrability



The nonintegrable case does not thermalize either

See M. Rigol, PRL **103**, 100403 (2009)

Relaxation in many-body quantum systems

M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, PRL **98**, 050405 (2007)

Assumption: Relaxation to constrained equilibrium

- Von Neumann entropy with constraints imposed by integrals of motion

$$S = -k_B \left(\mathrm{Tr} \hat{\rho} \ln \hat{\rho} + \sum_i \lambda_i \hat{\mathcal{O}}_i \right)$$

- Jayne's principle of maximum entropy

$$\hat{\rho} = \frac{\mathrm{e}^{-\sum_i \lambda_i \hat{\mathcal{O}}_i}}{\mathrm{Tr}\, \mathrm{e}^{-\sum_i \lambda_i \hat{\mathcal{O}}_i}}$$

See also M. Rigol, A. M., and M. Olshanii, PR A **74**, 053616 (2006)

Energy distribution and constrained Gibbs ensemble

Conserved quantities: $\langle \psi_0 \mid H^n \mid \psi_0 \rangle$

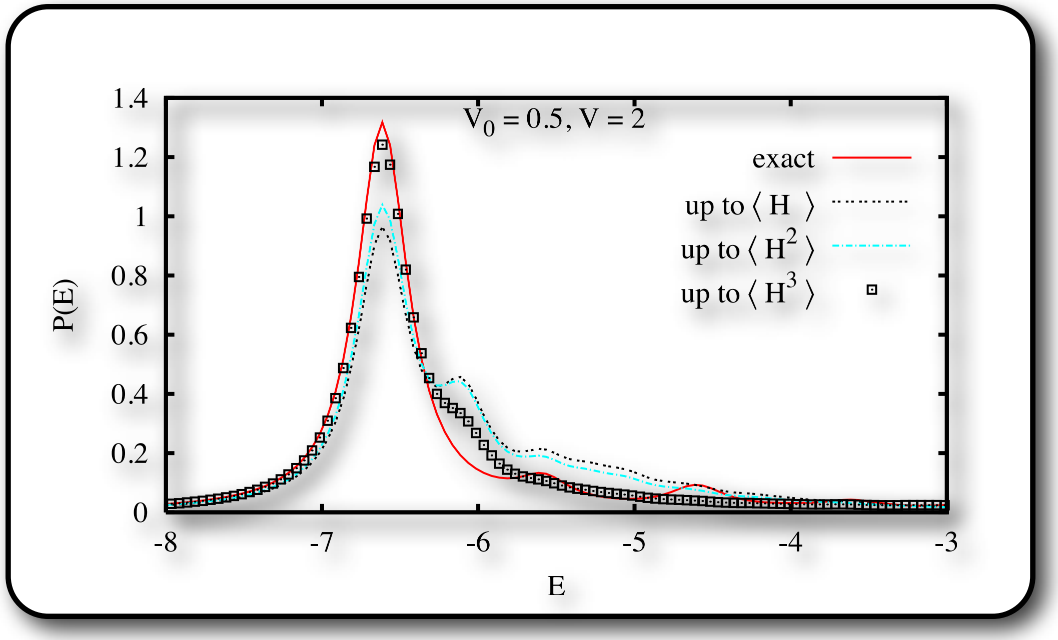
$$\hat{\rho} = \frac{\mathrm{e}^{-\sum_n \lambda_n \hat{\mathcal{O}}_n}}{\mathrm{Tr}\, \mathrm{e}^{-\sum_n \lambda_n \hat{\mathcal{O}}_n}} \quad \text{with} \quad \hat{\mathcal{O}} = H^n$$

Energy distribution function for a given state

$$P_\psi(E) = \sum_\mu \delta(E - \epsilon_\mu) |\langle \mu \mid \psi \rangle|^2$$

Energy distribution function in constrained Gibbs ensemble

$$P_G(E) = \mathrm{Tr}\, \delta(E - H)\, \hat{\rho}$$



Energy distribution with exact diagonalization

Distance between distributions

$$\Delta_n = \int \mathrm{d}E\, E^n \mid P(E) - P'(E) \mid$$

Bounded spectrum with bandwidth W

Estimate of $\mid P(E) - P'(E) \mid \sim \Delta_0 / W$

Relative difference between moments of H

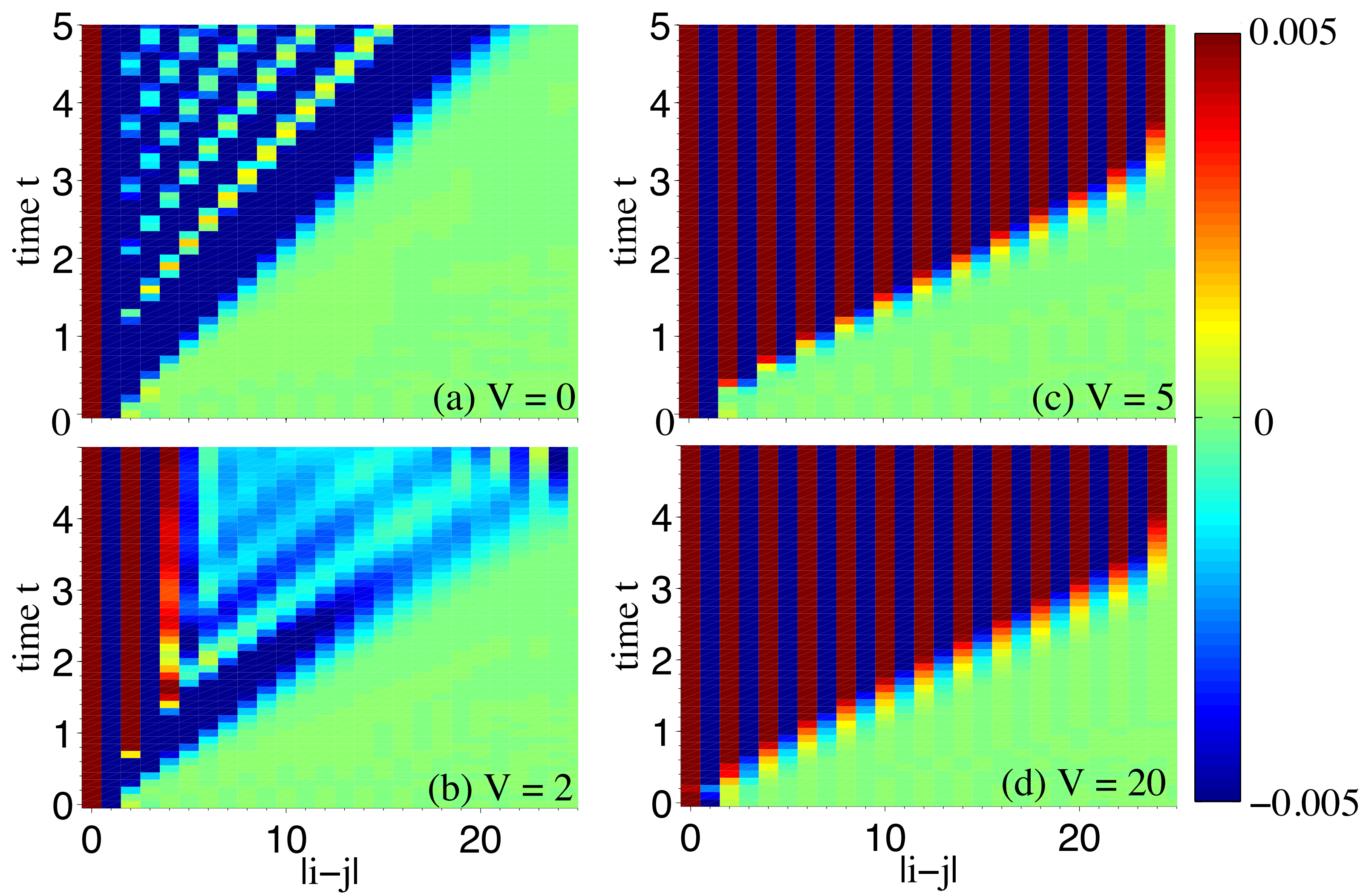
$$\frac{\langle H^n \rangle_P - \langle H^n \rangle_{P'}}{W^n} \leq \frac{\Delta_n}{W^n} \simeq \frac{1}{n+1} \Delta_0$$

Time evolution of correlations

S.R. Manmana, S. Wessel, R.M. Noack, and A. M., PR B **79**, 155104 (2009)

Quenches from an insulator ($V_0 = 10\,t$)

$$C_{ij}(t) = \langle n_i(t) n_j(t) \rangle - \langle n_i(t) \rangle \langle n_j(t) \rangle$$

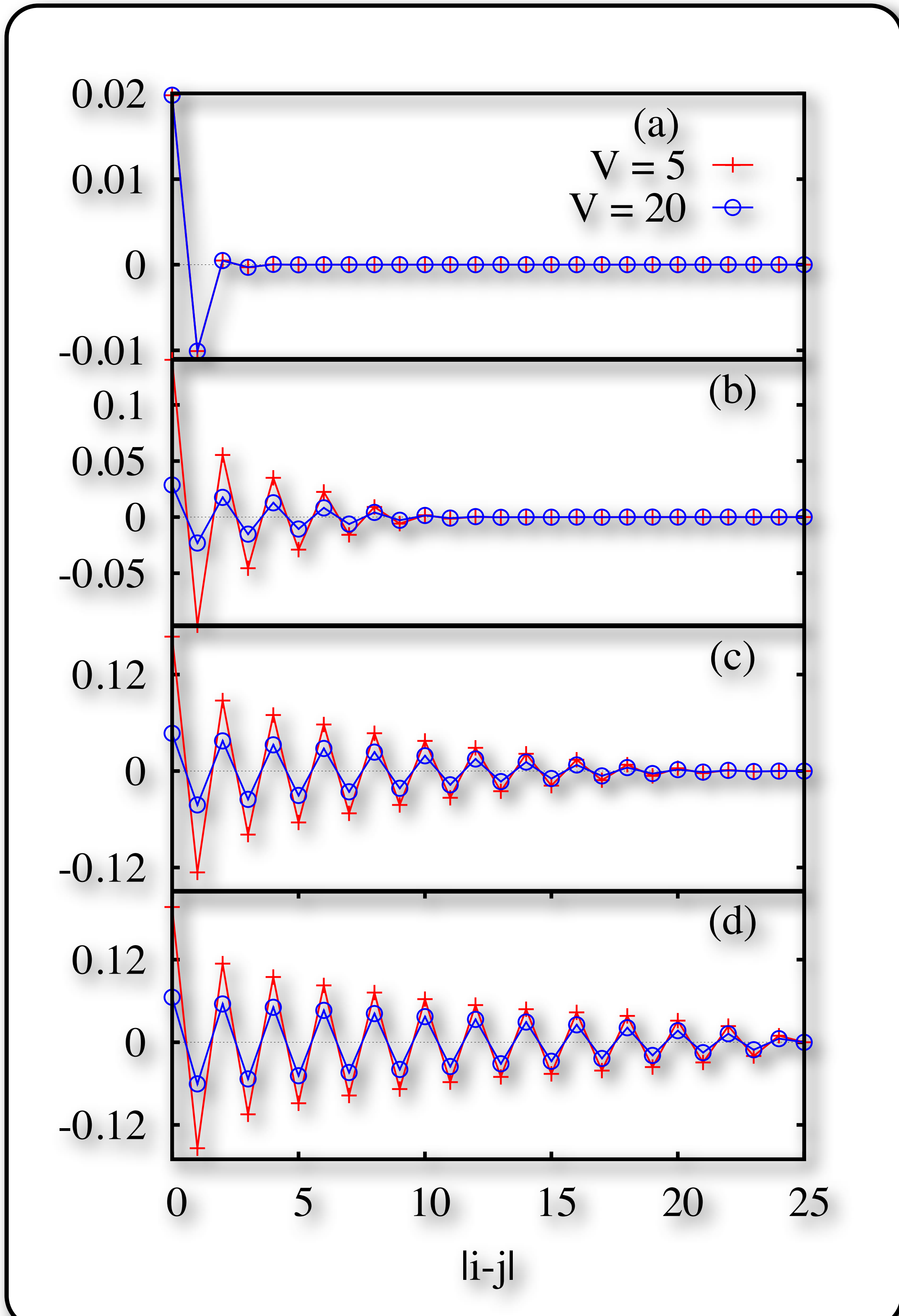


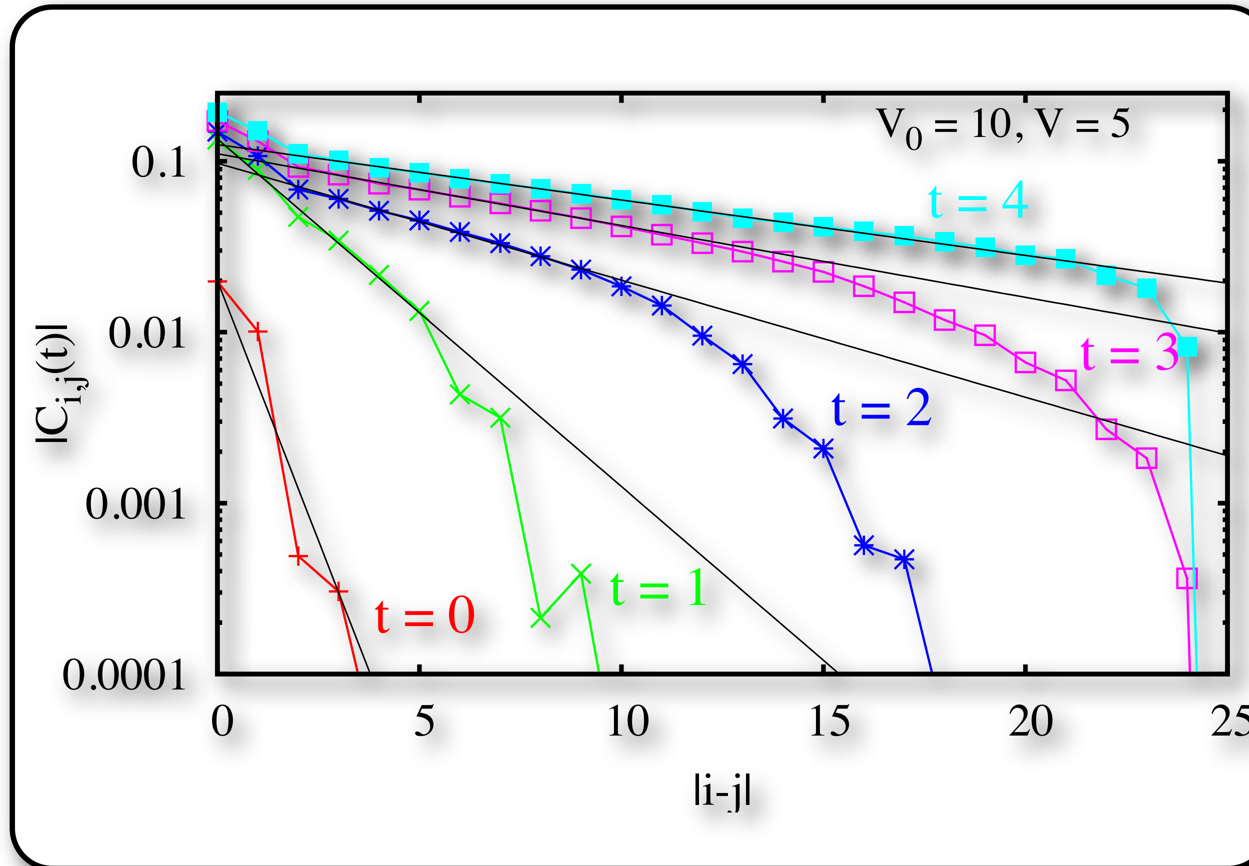
Light-cone effect for propagation of correlations after a quantum quench

$$| \, i - j \, | = 2vt$$

Calabrese and Cardy,
PRL **96**, 136801 (2006);
J. Stat. Mech.: Theory Exp. (2007) P06008

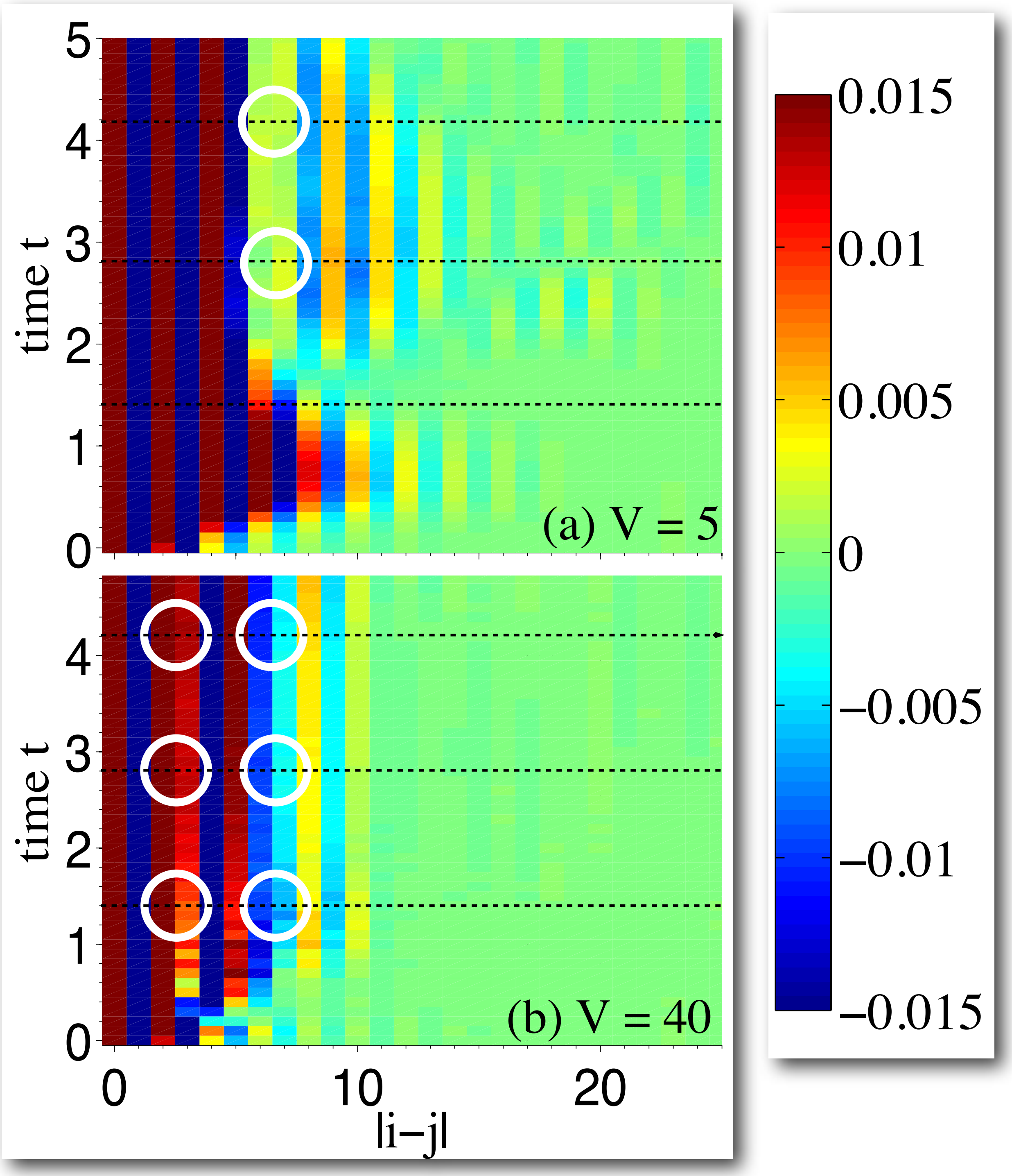
From insulator to insulator

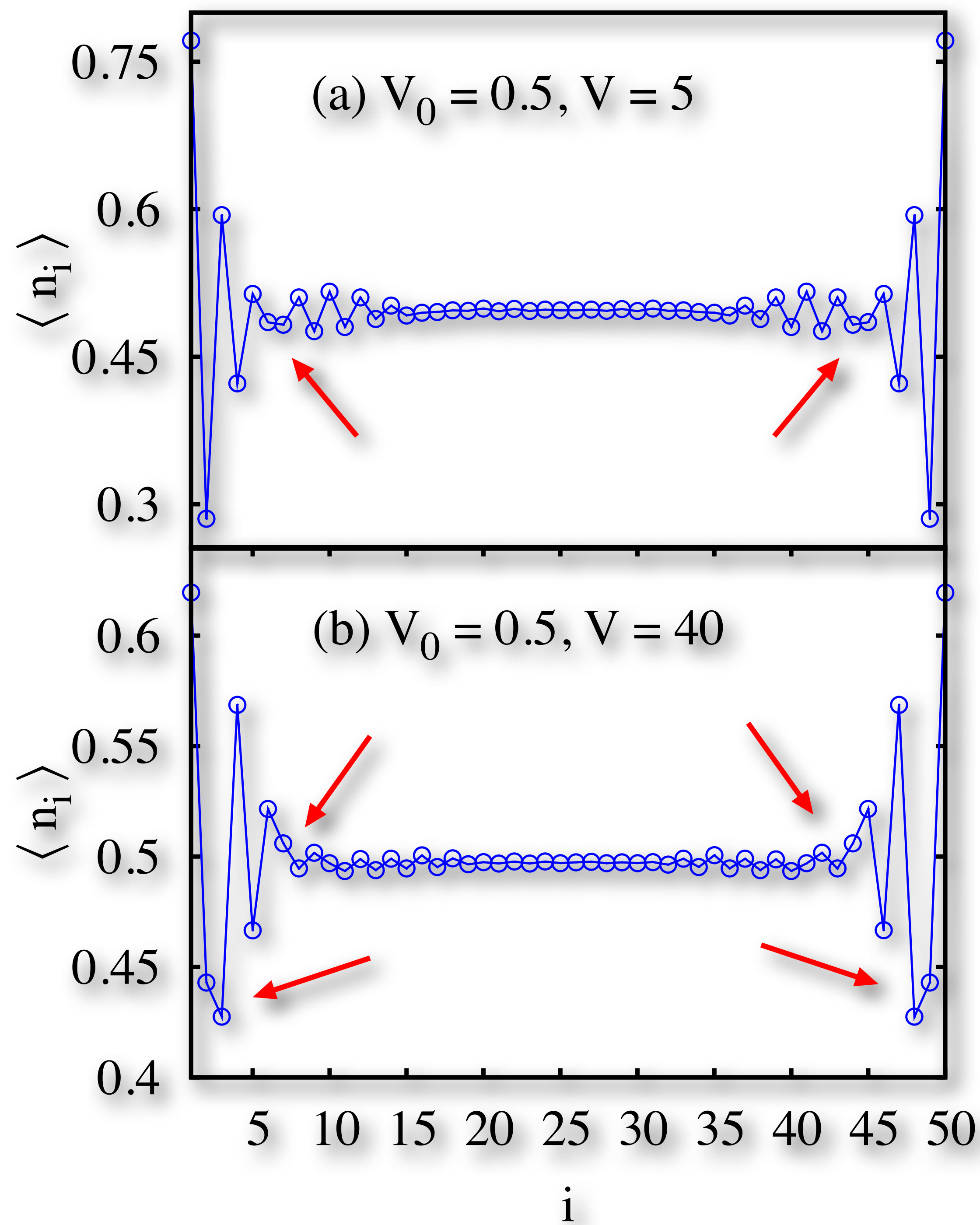




Information goes through the system coherently, increasing the correlation length

From Luttinger liquid to insulator





Phase slips ► domain walls ► Kibble-Zurek mechanism

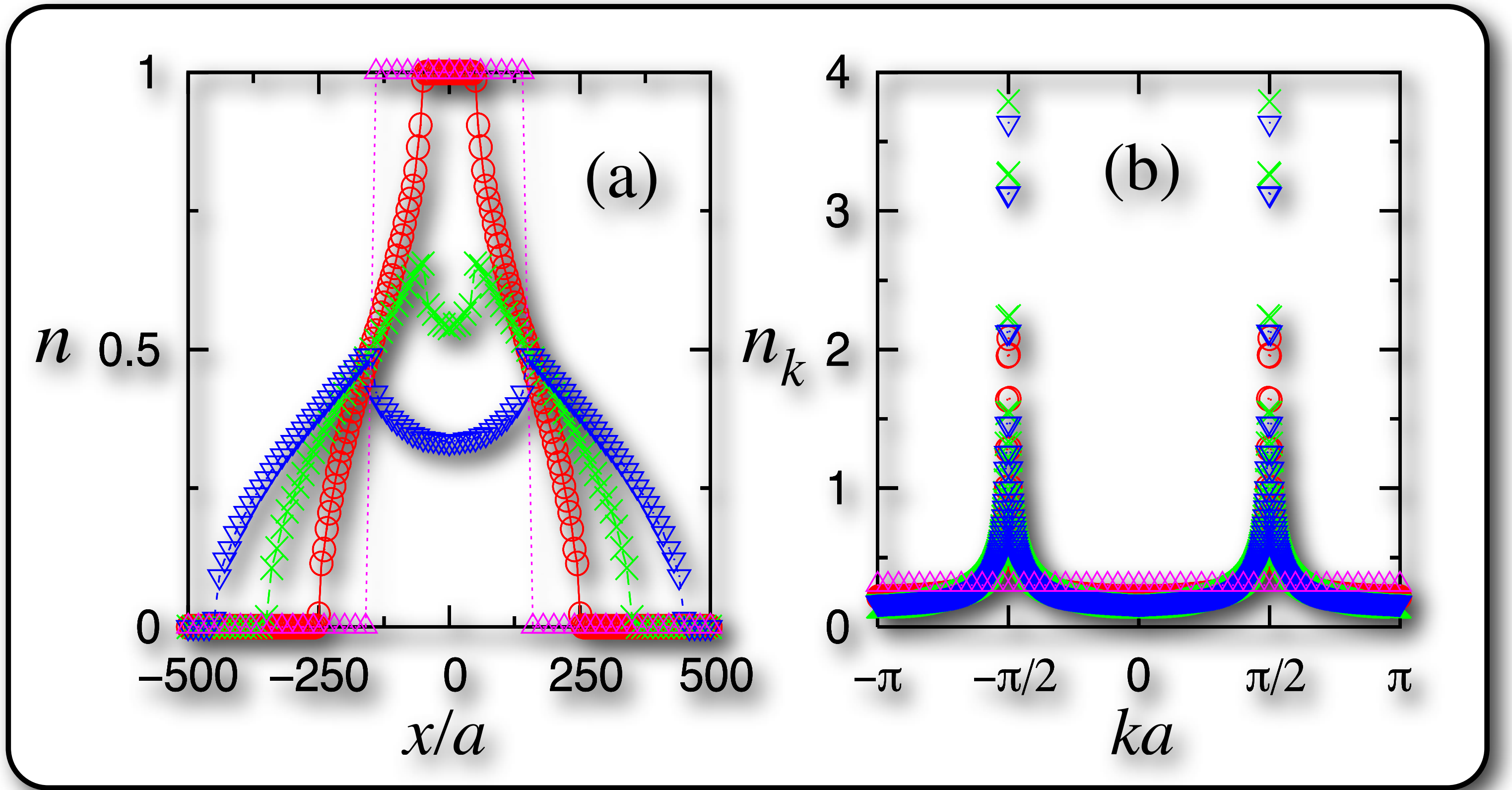
Free expansions in an optical lattice

New features due to the lattice

Free expansion of strongly repulsive bosons

Quasi-condensate at finite momentum out of a Mott-insulator

M. Rigol, A.M., PRL **93**, 230404 (2004)



(a) Densities of an expanding Tonks- Girardeau gas after release
(b) Momentum distribution function

Also for U finite

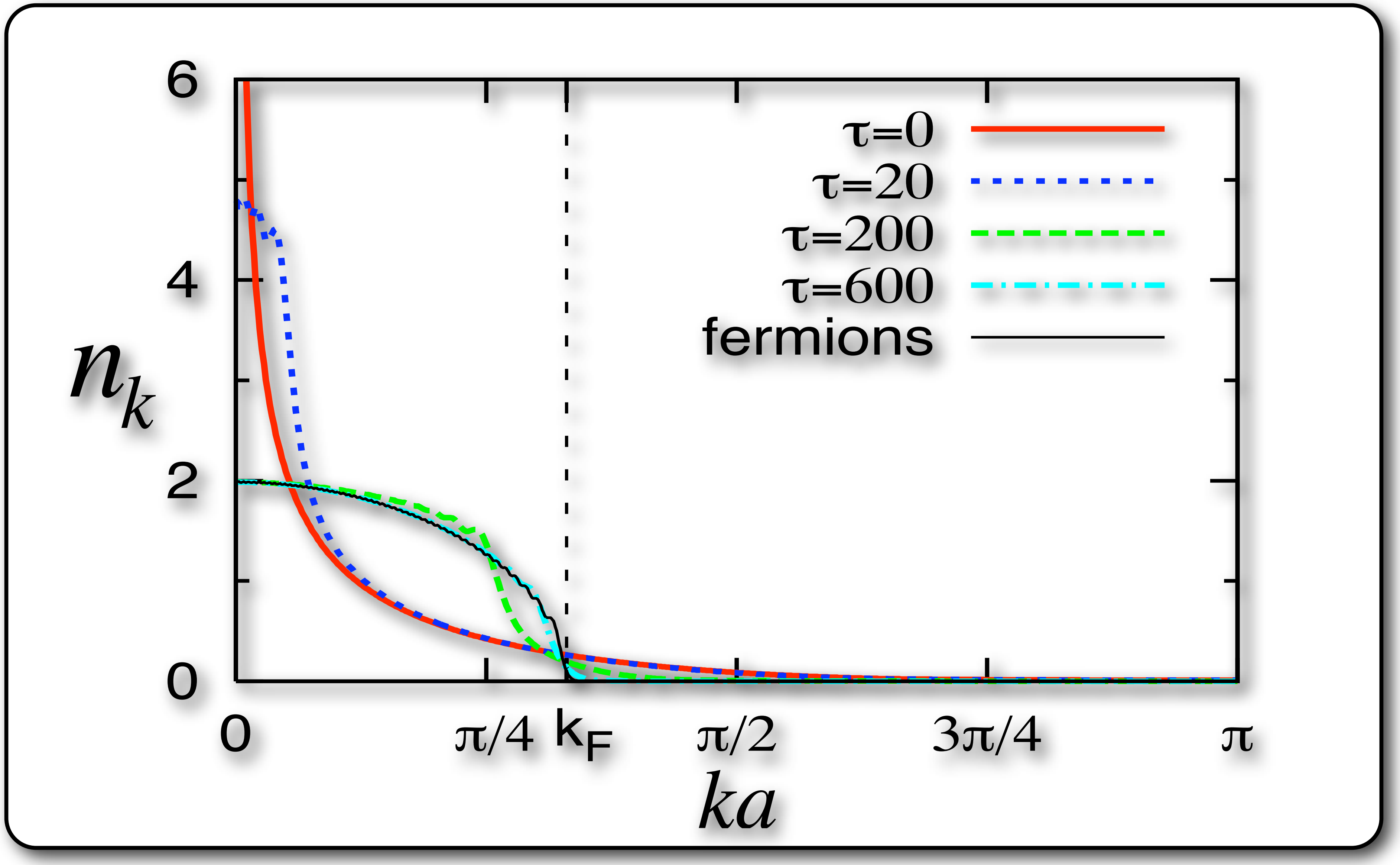
K. Rodriguez, S.R. Manmana, M. Rigol, R.M. Noack, A.M., New J. Phys. **8**, 169 (2006)

For 3D see I. Hen and M. Rigol, PRL **105**, 180401 (2010)

Fermionization in an expanding 1D gas of hard-core bosons

M. Rigol and A.M., PRL **94**, 240403 (2005)

Evolution from a quasi-condensate



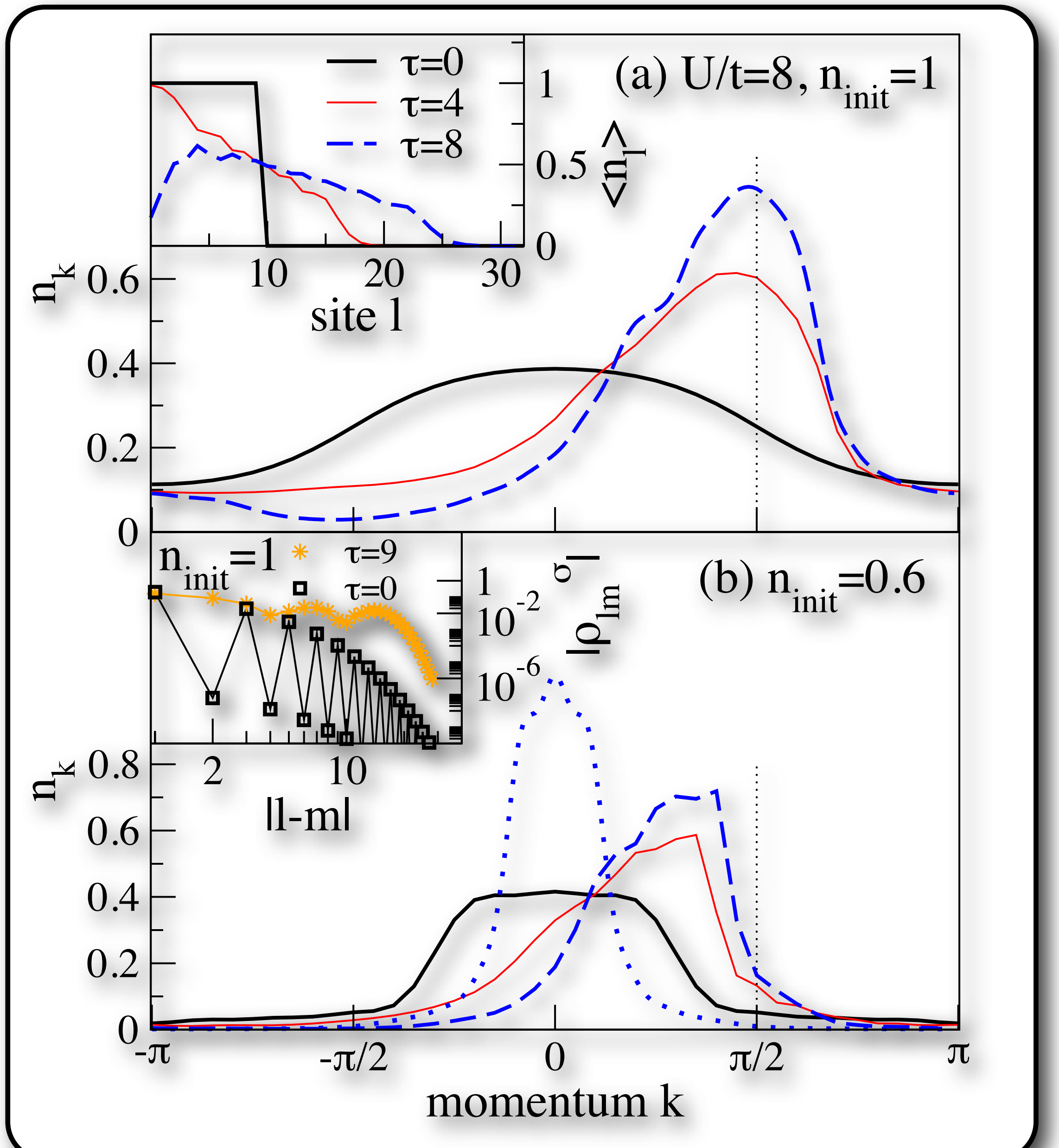
See also A. Minguzzi and D.M. Gangardt, PRL **94**, 240404 (2005)

Free expansion of strongly repulsive fermions

$$H = -t \sum_{i,\sigma} c^{\dagger}_{i+1,\sigma} c_{i,\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- ## Ground-state reference system

F. Heidrich-Meisner, M. Rigol, A.M., A.E. Feiguin, E. Dagotto, PR A **78**, 013620 (2008)



(a) Expansion from a Mott-insulator. n_k peaks at $\pi/2$.

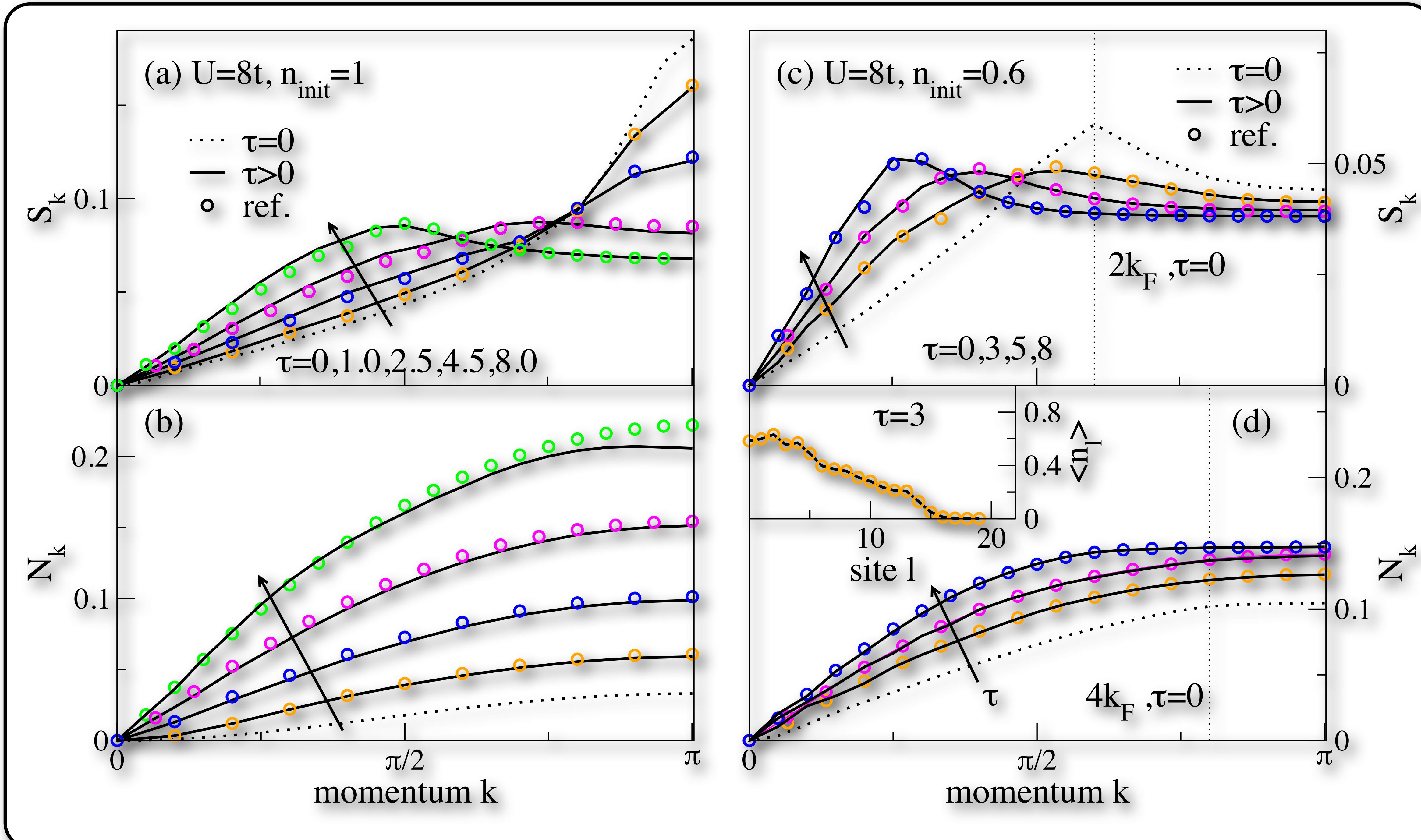
(b) Expansion from a Luttinger liquid.

Edge at large momentum ~

Fermi edge of reference system.

Reference system: system in equilibrium with the same density profile as the expanding one at given times.

Spin and density structure factors

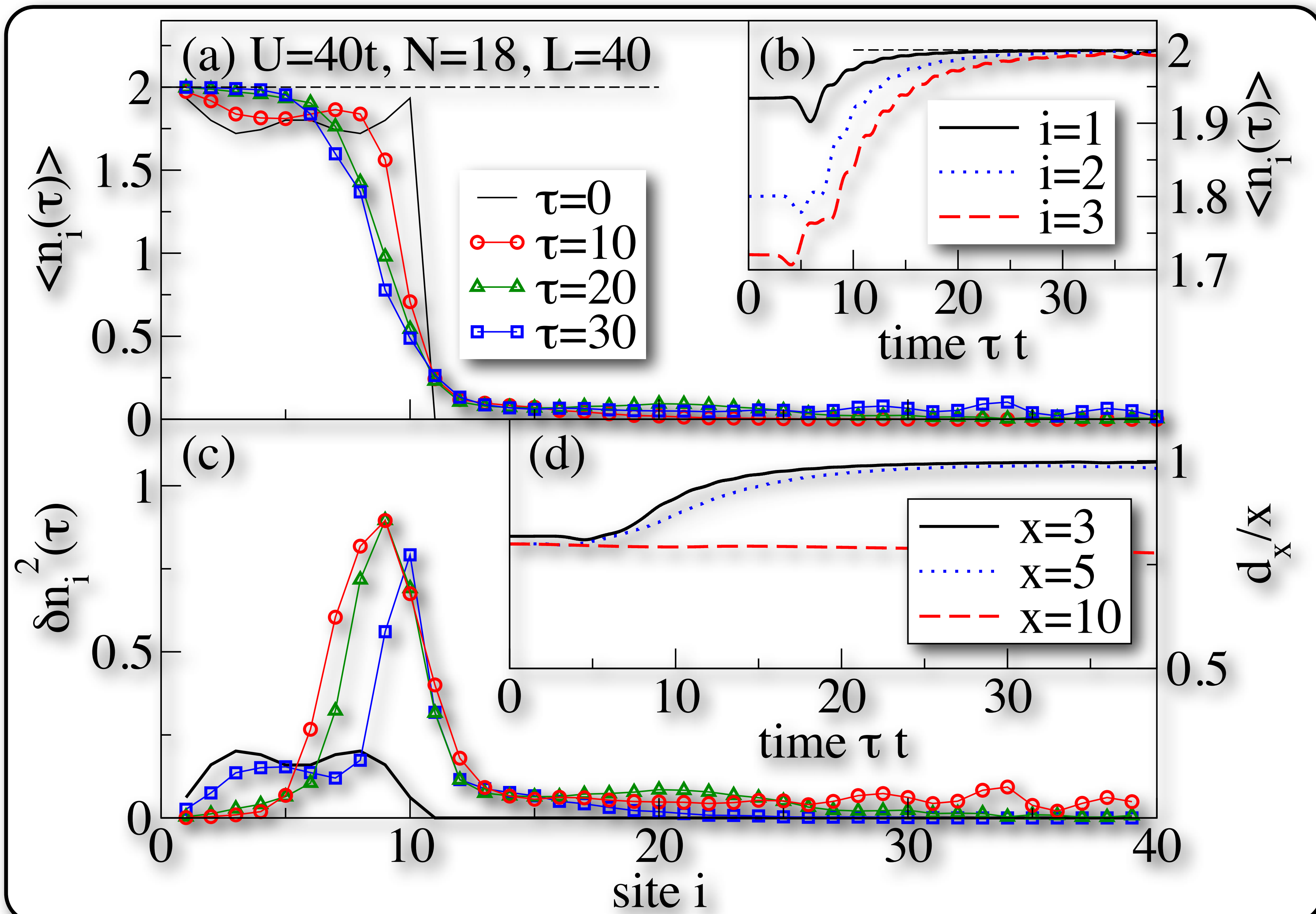


Correlations out of equilibrium accurately described by a reference system in its ground-state

Quantum distillation of low-entropy states

F. Heidrich-Meisner, S.R. Manmana, M. Rigol, A.M., A.E. Feiguin, E. Dagotto, PR A **80**, 041603(R) (2009)

Expansion from initial states with strong admixture of doubly occupied sites



(a) Increase of density at the left edge

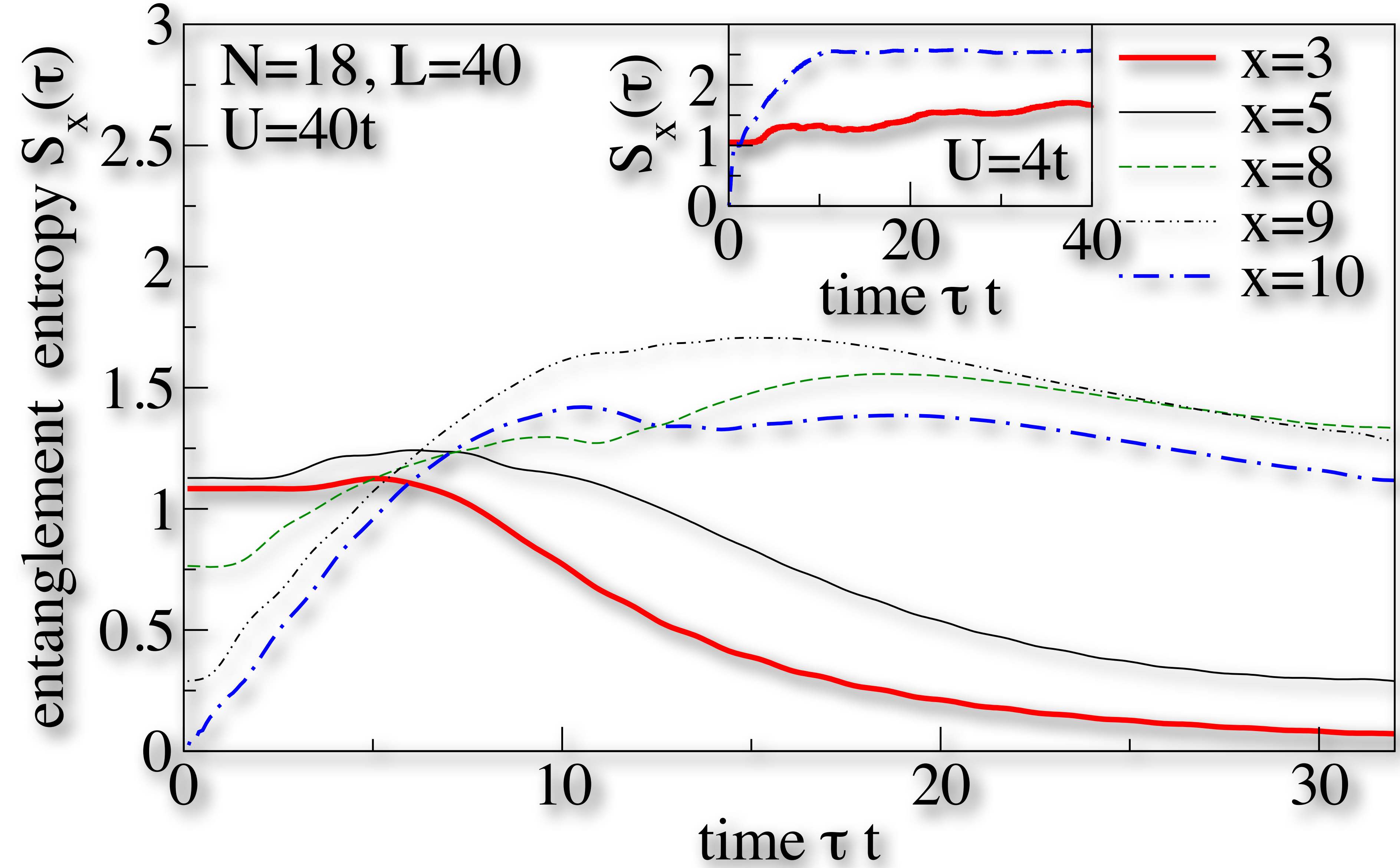
(b) Reach n=2 at the left edge

(c) Suppresion of fluctuation

(d) Double occupancy for 1 ≤ x ≤ 5

Reduction of entanglement entropy

- Expansion from a box



Reduction of entanglement entropy by approaching a Fock-state with <n> = 2

Summary

- Quantum quench from one phase to another
  - Memory of initial state lost but no thermalization.
  - Light cone for propagation of information also in a gapped phase.
  - Kibble-Zurek mechanism for quench into a broken symmetry phase.

Summary

- Free expansions on optical lattices
  - Bosons: condensate at finite momentum starting from a Mott-insulator
  - Fermions: maximum of n_k at $\pi/2$ starting from a Mott-insulator
  - Correlations accurately described by the ground-state of reference systems
  - Quantum distillation of low entropy states by expansion from n > 1, U >> W

Collaborators

- E. Dagotto (ORNL)
- A.E. Feiguin (University of Wyoming)
- F. Heidrich-Meisner (LMU-München)
- S.R. Manmana (JILA, University of Colorado)
- R.M. Noack (Universität Marburg)
- M. Rigol (Georgetown University)
- S. Wessel (Universität Stuttgart)

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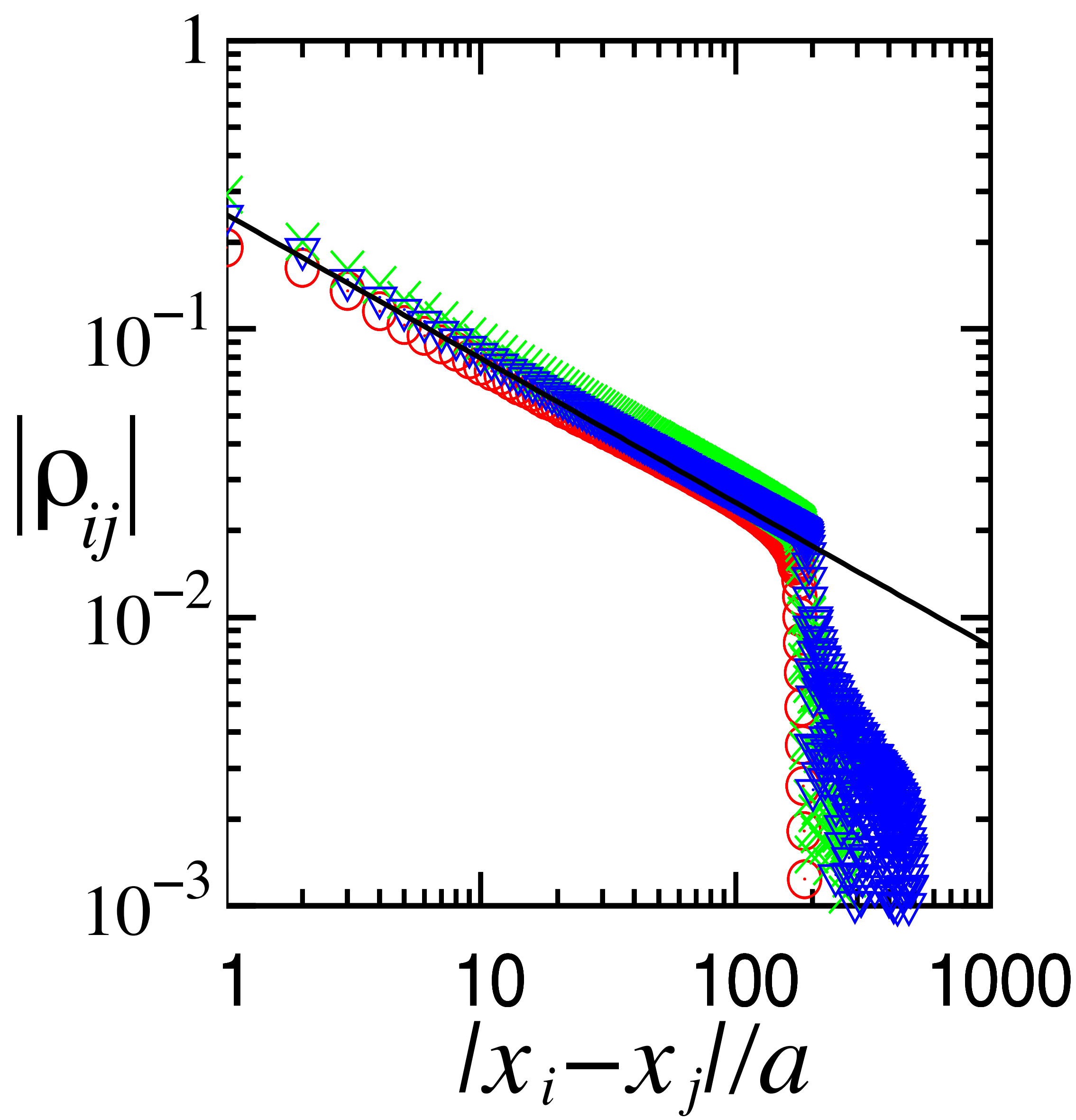
Quasi-long range order at finite momentum

One-particle density matrix



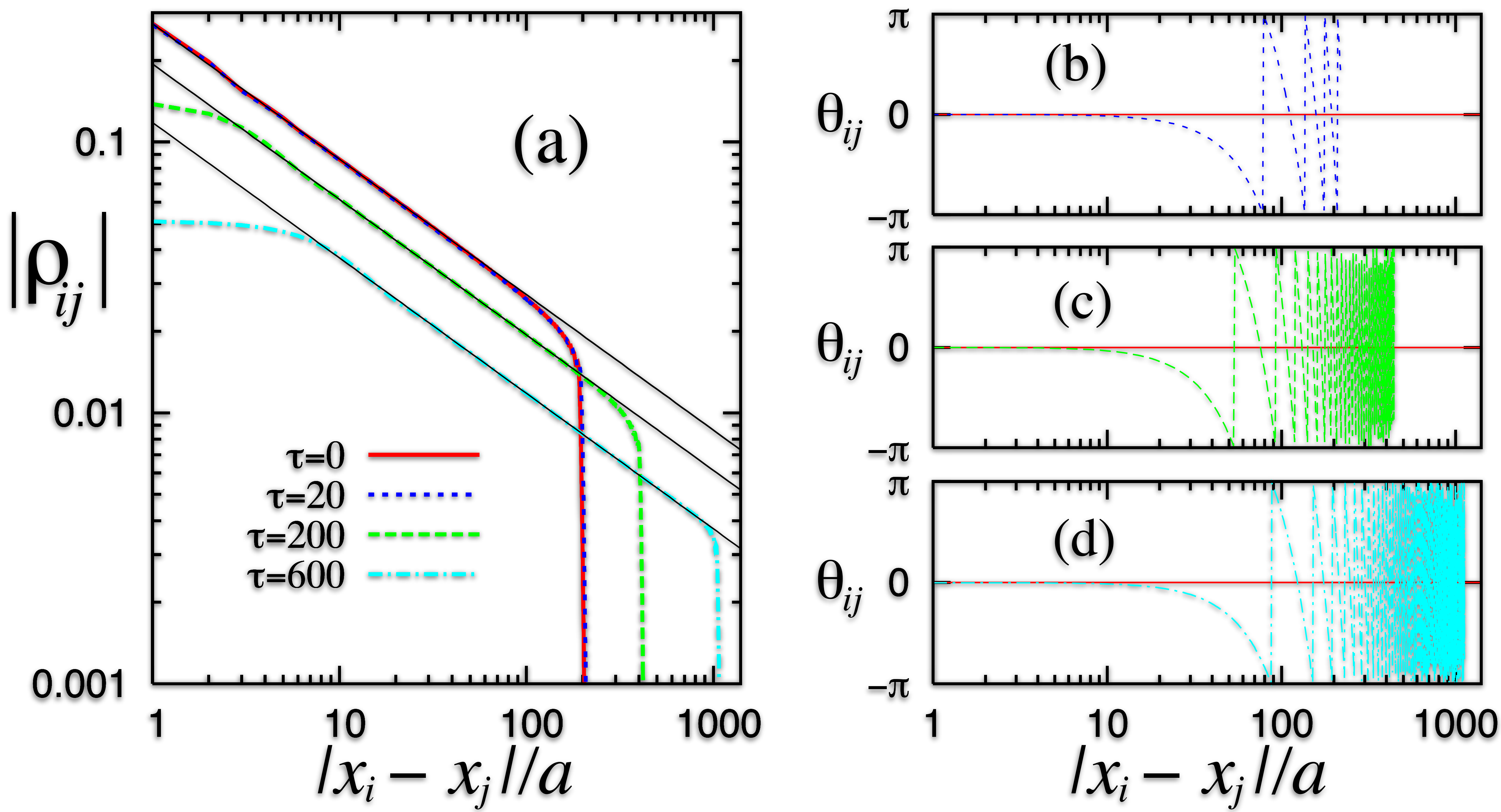
$$\rho_{ij} \sim \frac{1}{\sqrt{|\ i - j\ |}}$$

$$\Rightarrow \lambda_0 \sim \sqrt{N}$$

$\Rightarrow$ Quasi-condensate

Modulus and phase of the 1-P density matrix

Modulus | Phase



Speed for propagation of information