Bi-partite Entanglement in Equilibrium and out-of-equilibrium Many-body systems

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Laboratoire de Physique Théorique, Toulouse

D.P., PRL **105**, 077202 (2010)
D.P., arXiv:1011.2147

OUTLINE

- (Bi-partite) entanglement measures in equilibrium many-body systems: from FQH states to quantum magnets (e.g. spin ladders)
- Some motivations to study out-of-equilibrium systems - Relaxation and thermalization
- Bi-partite entanglement in out-of-equilibrium many-body systems (illustrated for XXZ chain)

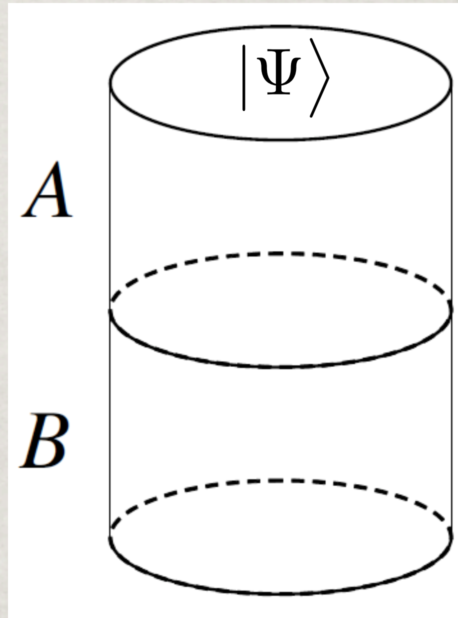
Entanglement measures

Kitaev & Preskill, 2006

Levin & Wen, 2006

Special Issue: Entanglement Entropy in Extended Quantum Systems, J. Phys. A **42**, N° 50, 500301-504012 (2009); Guest Editors: P. Calabrese, J. Cardy and B. Doyon.

Fantastic tools !



RK wf

Stéphan, Furukawa, Misguich & Pasquier, 2009

Topological order

A & B are entangled : (Schmidt decomposition)

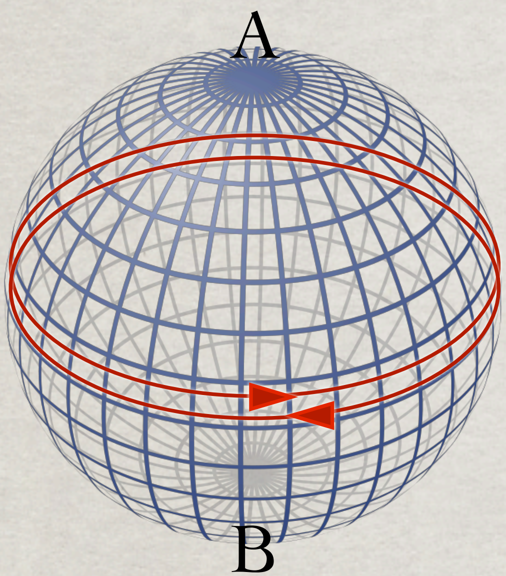
$$|\Psi\rangle = \sum_i \lambda_i |\Psi_i^A\rangle \otimes |\Psi_i^B\rangle$$

Reduced density matrix:

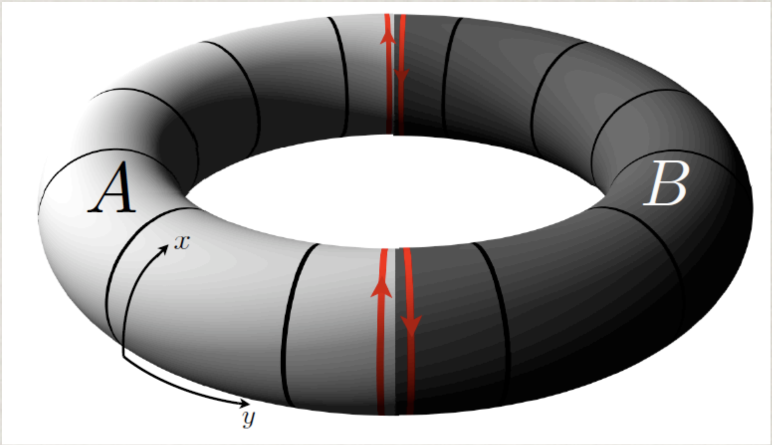
$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

$$S_{\mathrm{entanglement}} = -\mathrm{Tr}\{\rho_A \ln \rho_A\}$$
$$= -\sum_i \lambda_i^2 \ln \lambda_i^2$$

Motivation: entanglement spectra in FQH systems



Li & Haldane, 2008



Lauchli et al., 2009

rewrite the weights as: $\lambda_i = \exp\left(-\xi_i/2\right)$

$$\rho_A = \exp\left(-\hat{\xi}\right)$$

Entanglement spectrum : $\{\xi_i\}$ → Edge states

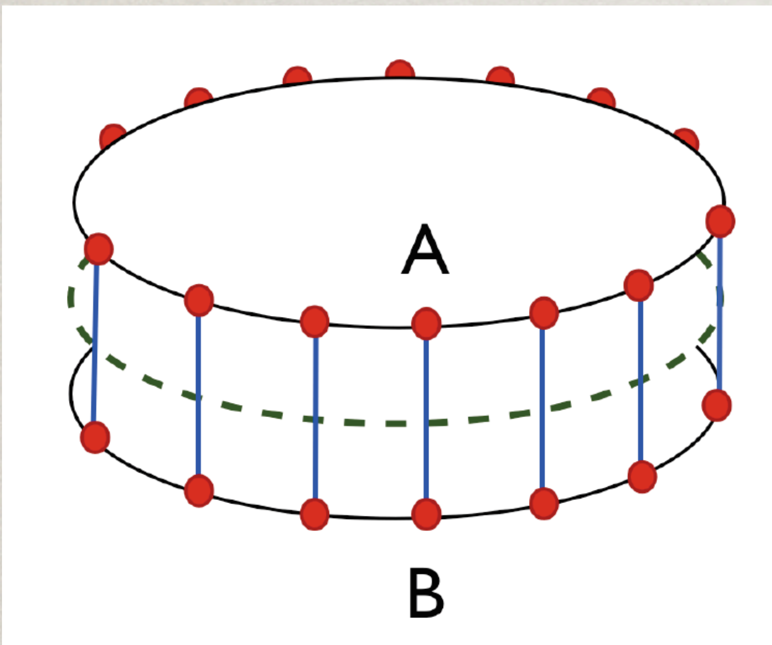
“Haldane” Conjecture:

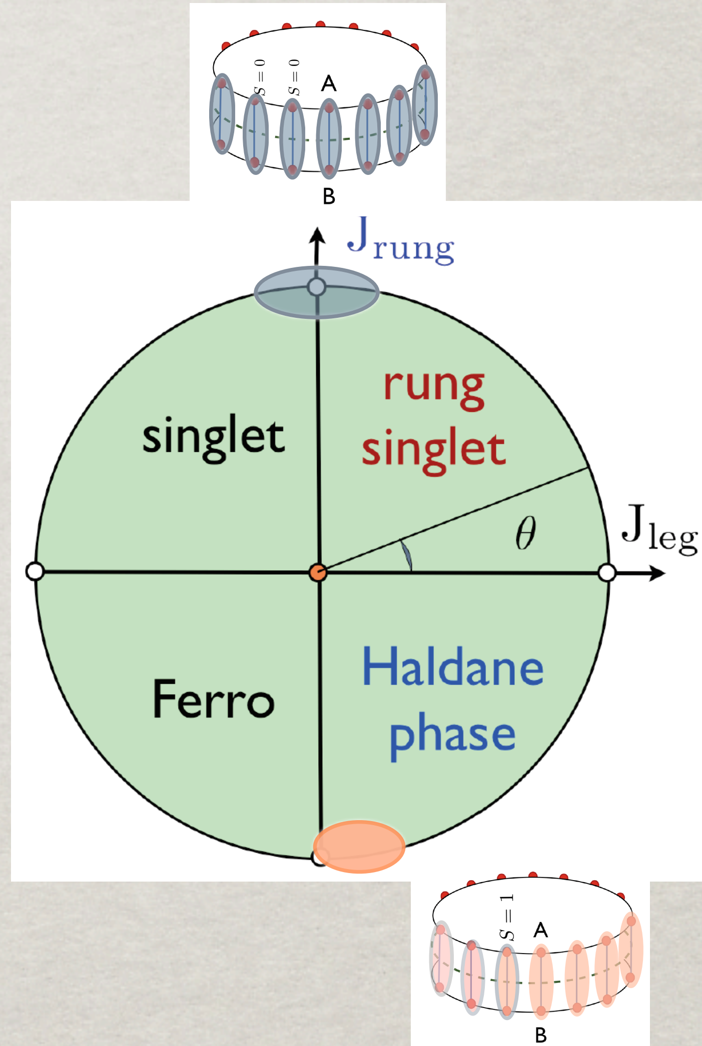
Precise correspondence between the entanglement spectrum of a many-body system partitioned into two sub-systems linked by some “edge” and the true edge spectrum

substantiated in the case of FQH systems

Question #1: universality ?

A simple Quantum S=1/2 magnet:
The 2-leg spin ladder !

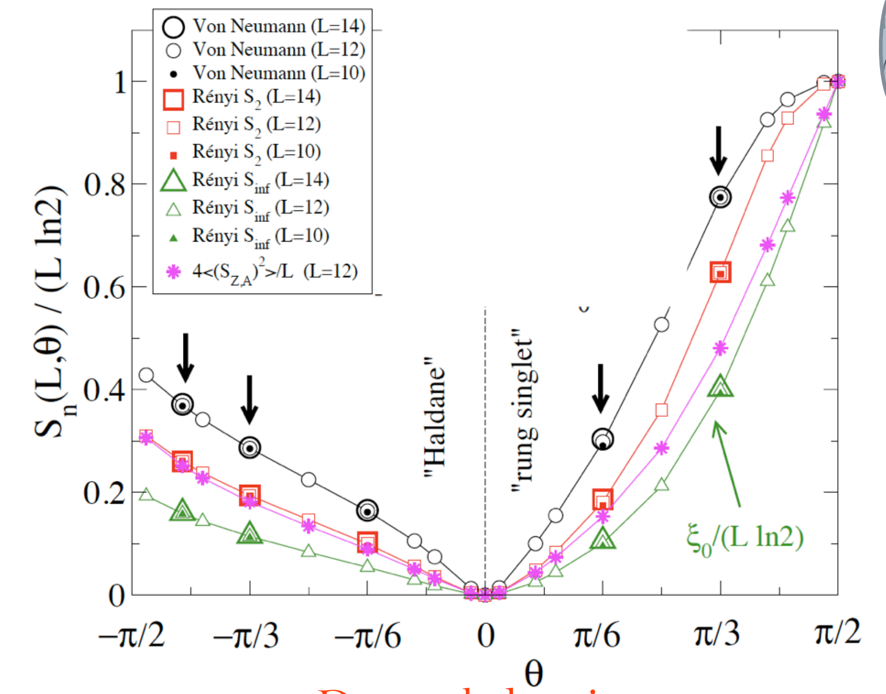


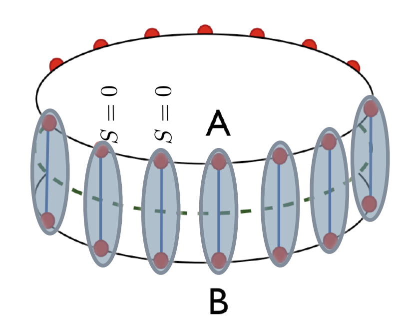


3 gapped (spin-liquid) phases

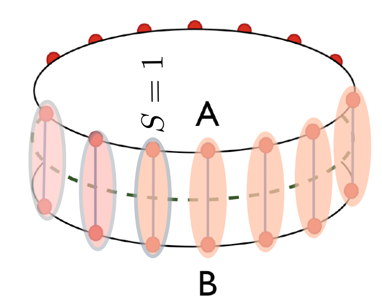
Entanglement entropy

$S(L) \propto L$ when $L \gg l_{\mathrm{mag}}$



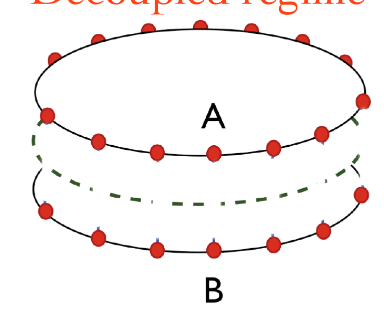


Maximally entangled regime

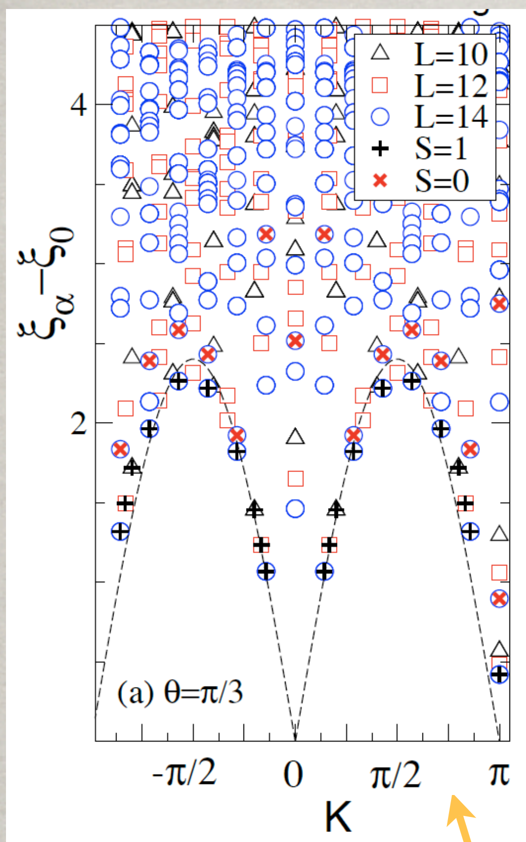


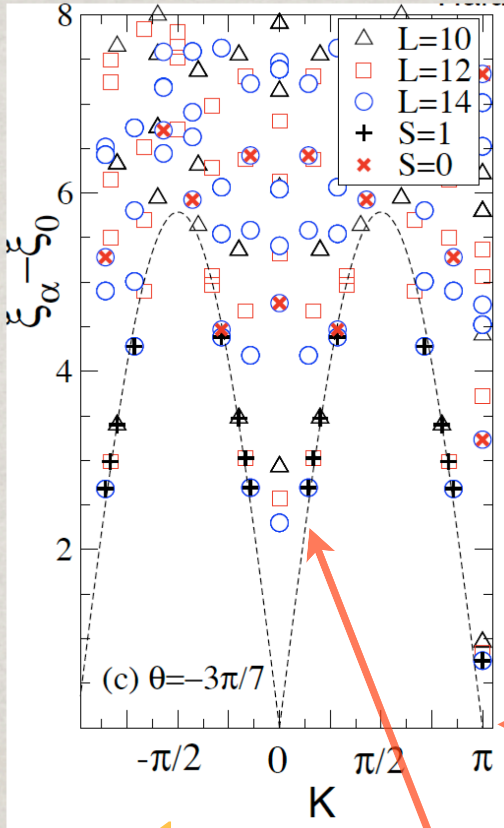
Strongly entangled regime

Decoupled regime



AFM leg coupling

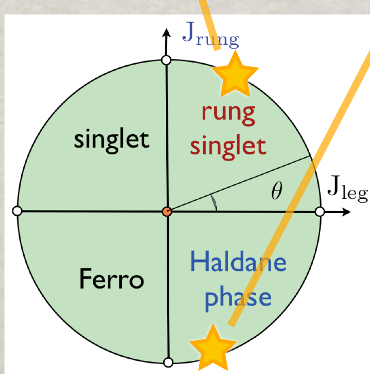




ES consistent with c=1 CFT
(Heisenberg S=1/2 chain)

$$\xi_0/L = e_0 + d_1/L^2 + \mathcal{O}(1/L^3)$$

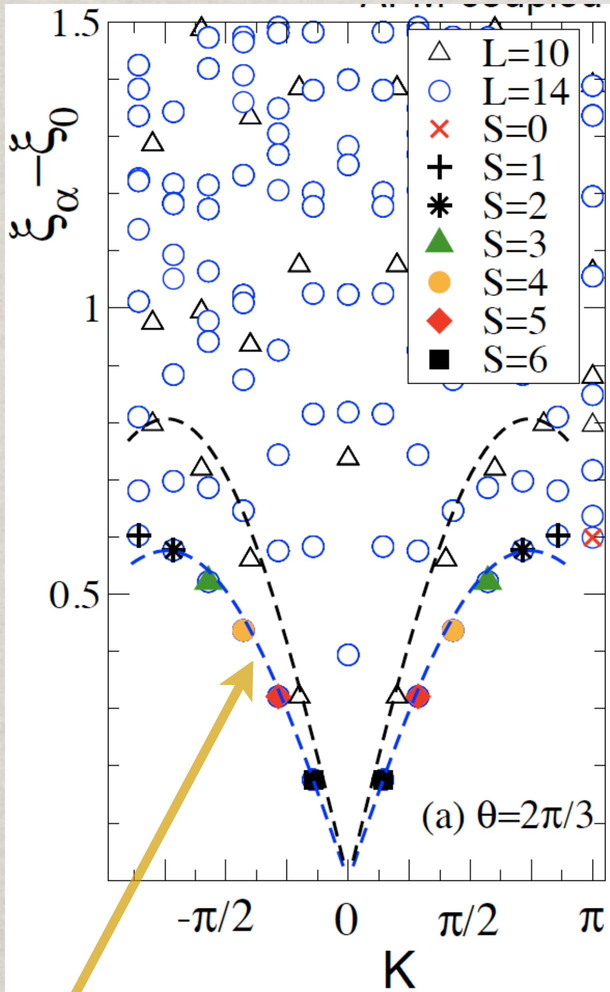
$$d_1 = \pi c v/6$$

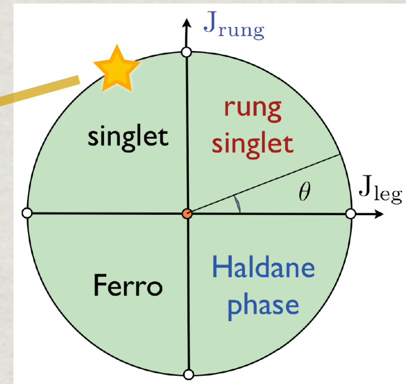


de Cloiseaux-Pearson triplet branch

$$\Delta\xi = v|\sin(K)|$$

FM leg (+ AFM rung) couplings





ES consistent with
1D FM Heisenberg chain

m-magnon bound states:

$$E_m(K) = 2J_{\text{eff}} \sin^2{(K/2)}/m$$

Low-energy envelope :

$$E_{\text{min}}(K) \sim \frac{4\pi}{L} J_{\text{eff}} \sin^2{(K/2)}/K$$

Unified picture in terms of a thermodynamic ensemble

$$\rho_A = \frac{1}{z_\theta} \exp(-\beta_\theta \hat{h})$$

$l_{1D} \sim T_\theta^{-1}$ thermal length

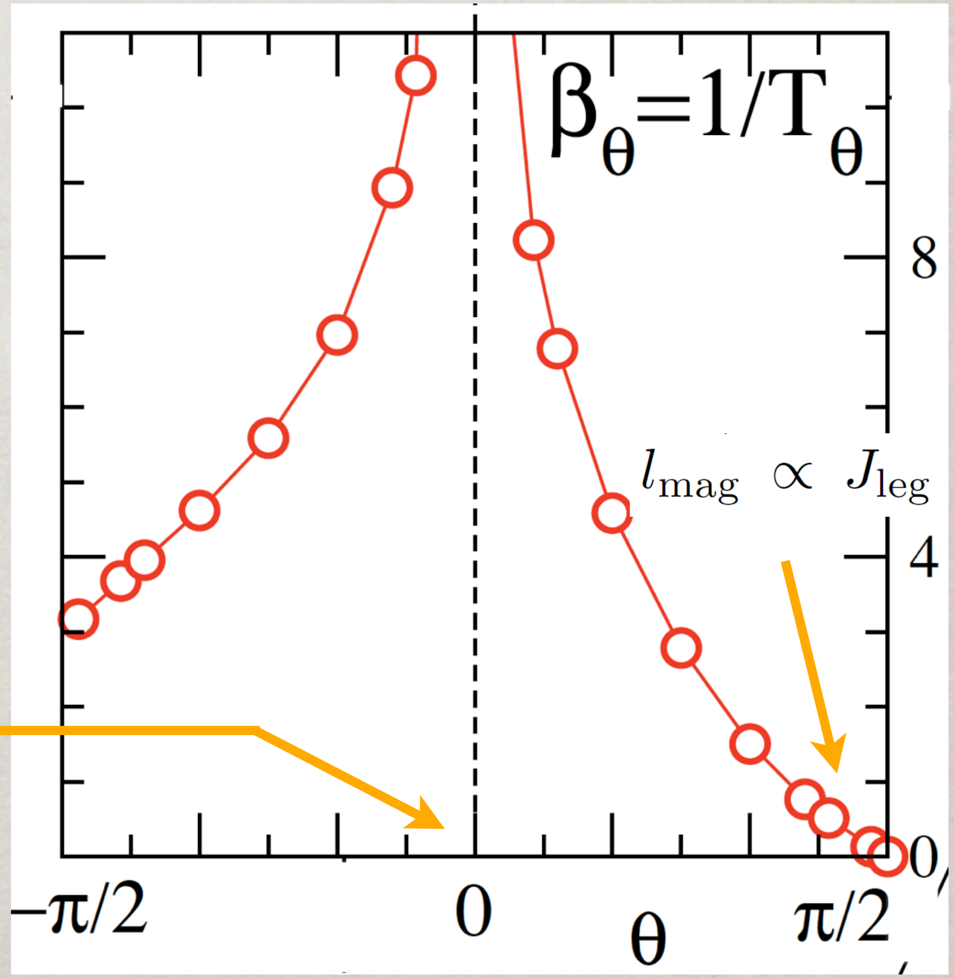
$\sim l_{mag}/2$ spin-correlation length

Thermodynamic entropy of Heisenberg chain

$T_\theta \ll 1$

$S_{VN}/L \sim \pi T_\theta / (3 v_{Heis})$

agrees with numerics



Partial summary

- Extend generality of "Haldane" new conjecture beyond FQH systems
- Open issues:
  - Role of non-local orders: rung singlet & Haldane phase have $\neq$ string OP's (Anfuso & Rosch, 07)
  - QCP ?
  - Extend to more chains ? Practical use for numerics ?

Part II

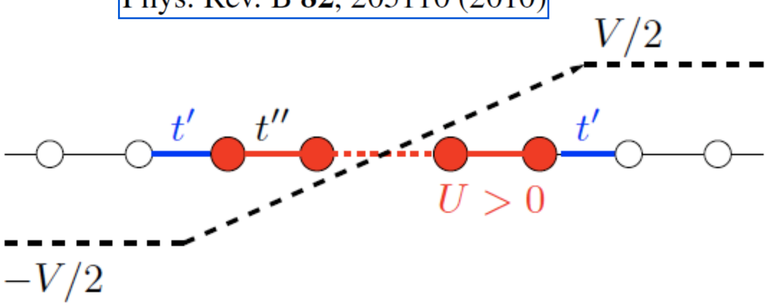
- Exemples of out-of-equilibrium systems - Relaxation and thermalization
- Bi-partite entanglement in out-of-equilibrium many-body systems (illustrated for XXZ chain)

Non-equilibrium electronic transport

Non-equilibrium electronic transport in a one-dimensional Mott insulator

F. Heidrich-Meisner,[1] I. González,[2] K. A. Al-Hassanieh,[3] A. E. Feiguin,[4] M. J. Rozenberg,[5,6] and E. Dagotto[7]

Phys. Rev. B **82**, 205110 (2010)

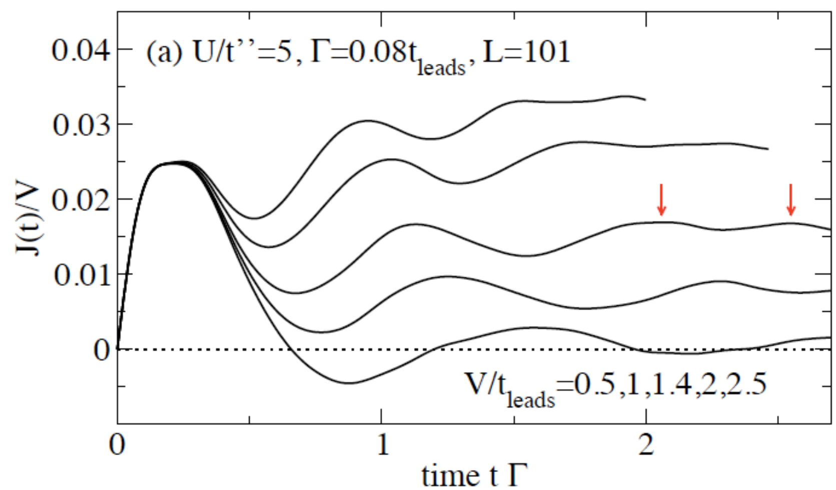


see also:

S. Kirino and K. Ueda, J. Phys. Soc. Jpn **79**, 093710 (2010)

$$H_{\text{bias}} = \Theta(t) \sum_{i=1}^{L} V_i n_i \,,$$

where $\Theta(t)$ is the Heaviside step function



Fondamental issues:

- Relaxation ?
- Characterisation of steady state ?

New Simulators for Condensed Matter !

Cold Atoms in Optical Lattices

Jaksch, Dieter, **Clark**, Stephen A.

Jointly published with Canopus Academic Publishing Limited, UK
1st Edition., 2010, IV, 300 p., Hardcover
ISBN: 978-90-481-8688-4

Due: February 28, 2011

IOP PUBLISHING JOURNAL OF PHYSICS B: ATOMIC, MOLECULAR AND OPTICAL PHYSICS

J. Phys. B: At. Mol. Opt. Phys. **42** (2009) 154009 (27pp) doi:10.1088/0953-4075/42/15/154009

REVIEW

Quantum simulations with cold trapped ions

Michael Johanning, Andrés F Varón and Christof Wunderlich

Fachbereich Physik, Universität Siegen, 57068 Siegen, Germany

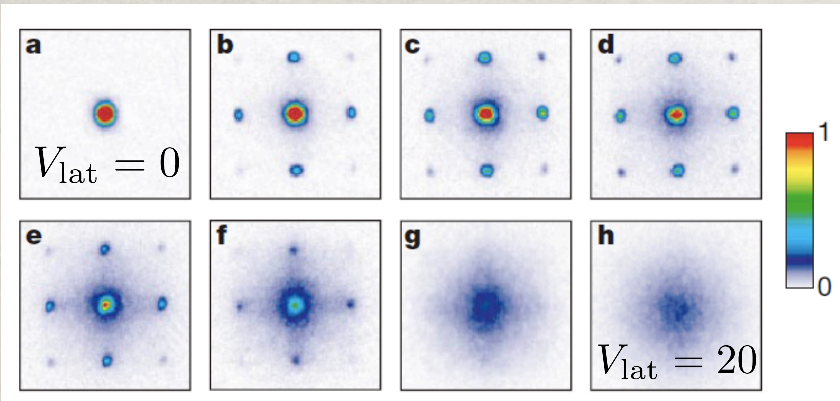
Observation of Quantum Dynamics in Isolated System

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

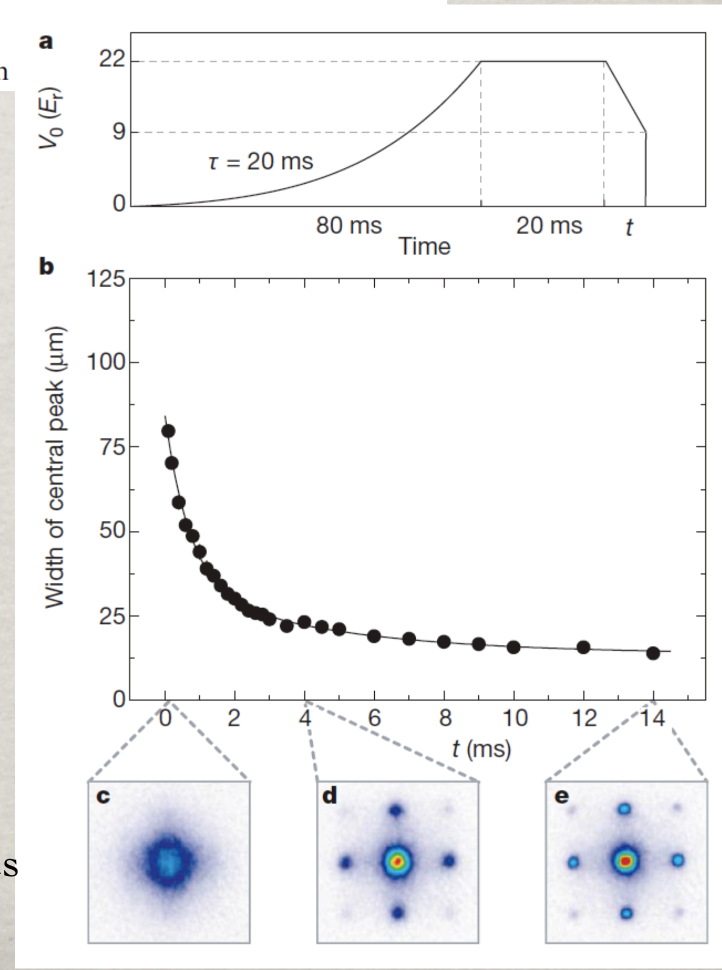
Markus Greiner*, **Olaf Mandel***, **Tilman Esslinger†**, **Theodor W. Hänsch*** **& Immanuel Bloch***

** Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*
† Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland

NATURE | VOL 415 | 3 JANUARY 2002 | www.nature.com



"Equilibrium superfluid-Mott QPT"



"Quantum Quench"

out-of-equilibrium physics

Recent theoretical progress on quantum quenches

in 1D: t-DMRG

* Bosonic Hubbard chain

C. Kollath, A. M. Läuchli, and E. Altman, Phys. Rev. Lett. **98**, 180601 (2007); G. Roux, Phys. Rev. A **79**,021608(R) (2009).

* Hard-core bosons chain

M. Rigol, Phys. Rev. Lett. **103**, 100403 (2009); M. Rigol and L. F. Santos, Phys. Rev. A **82**, 011604(R) (2010).

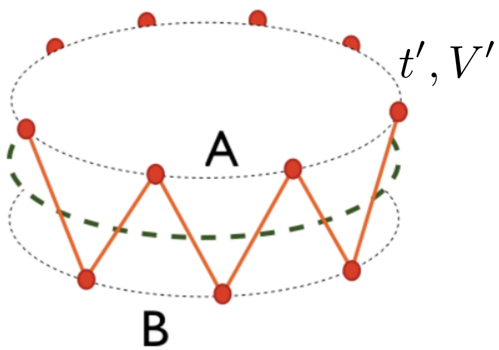
* Fermionic Hubbard chain

F. Heidrich-Meisner et al., Phys. Rev. A **80**, 041603(R) (2009).

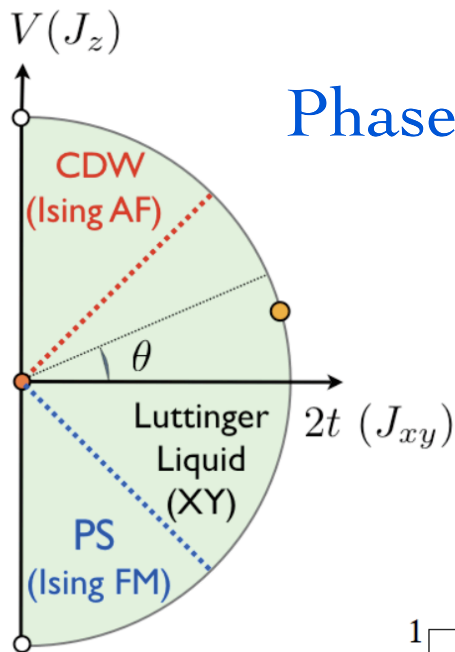
Still many remaining issues on thermalization...

XXZ-chain ⟺ Hardcore bosons

$N = 2L$ sites



A/B bipartition



Phase diagram

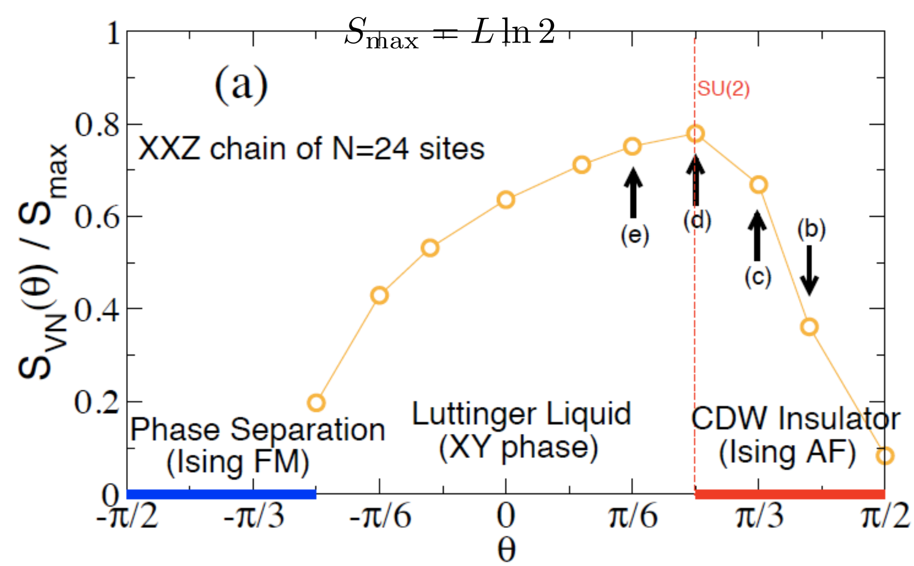
Haldane 1981

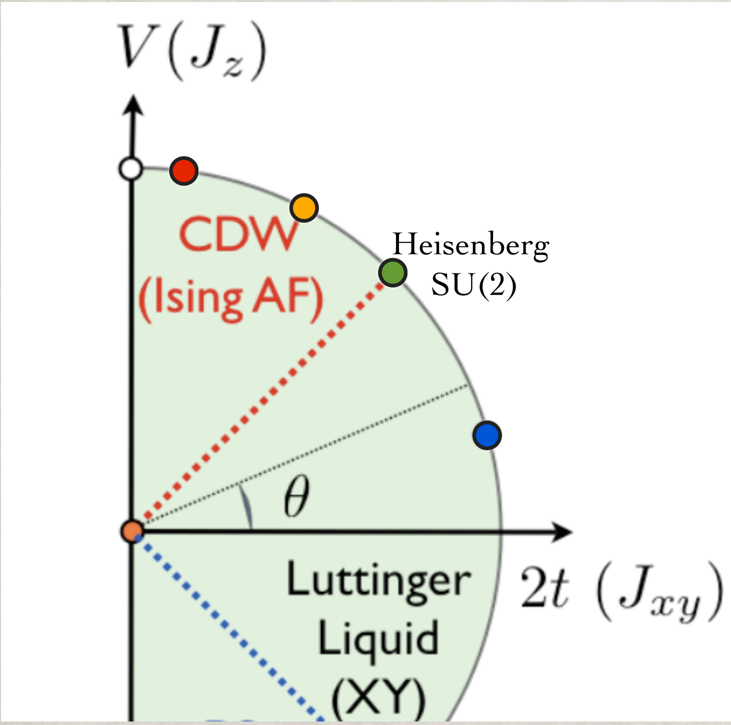
$$H = J_z \sum_i S_i^z S_{i+1}^z + \frac{1}{2} J_{xy} (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+)$$

$$J_{xy} = \cos\theta$$

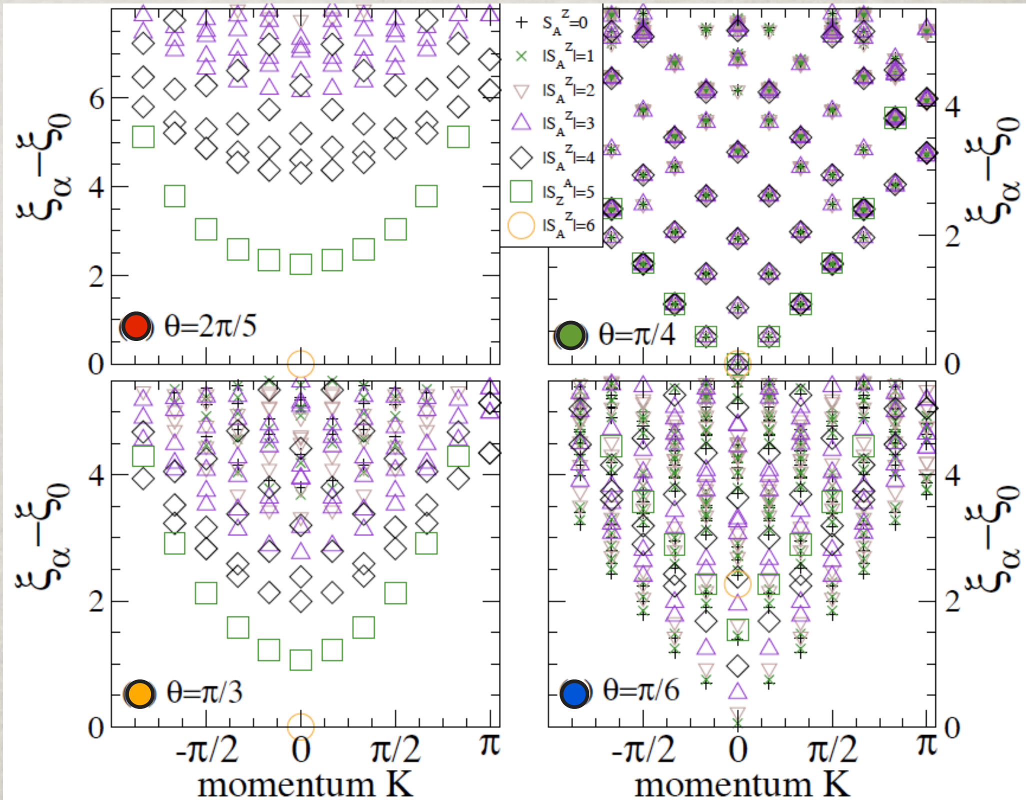
$$J_z = \sin\theta$$

Entanglement entropy /site





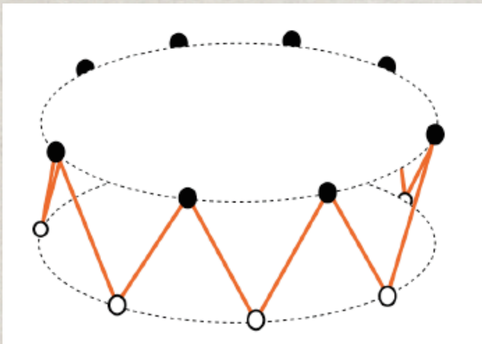
Entanglement spectra



"Equilibrium spectra"

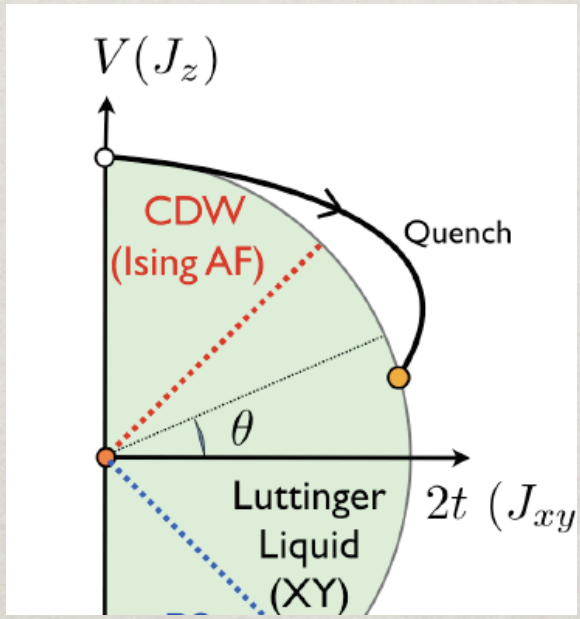
(Boltzmann-Gibbs)

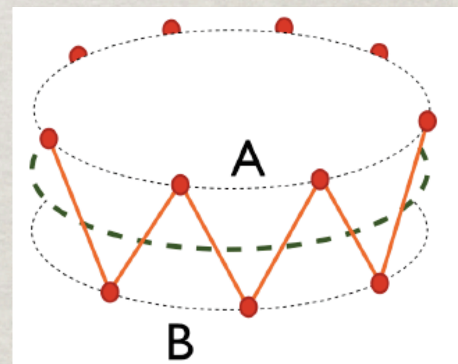
Quantum quench



Initial state

(symmetrized)



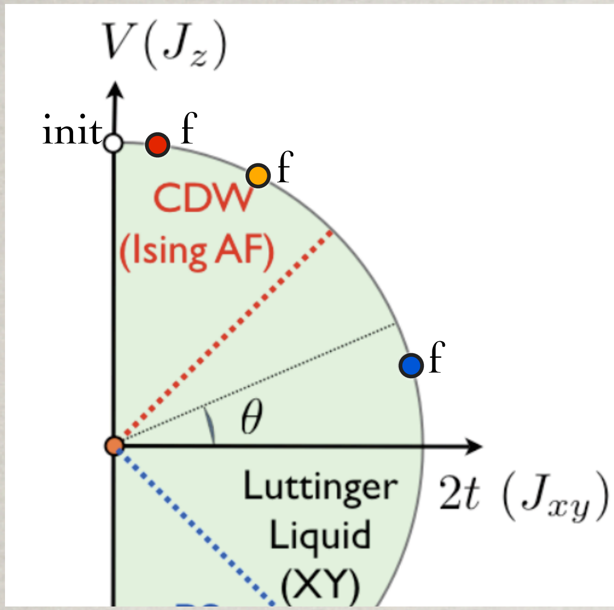


Final state

$$|\phi(\tau)\rangle = \exp\left(-i\tau H(\theta_{\mathrm{f}})\right)|\phi(0)\rangle$$

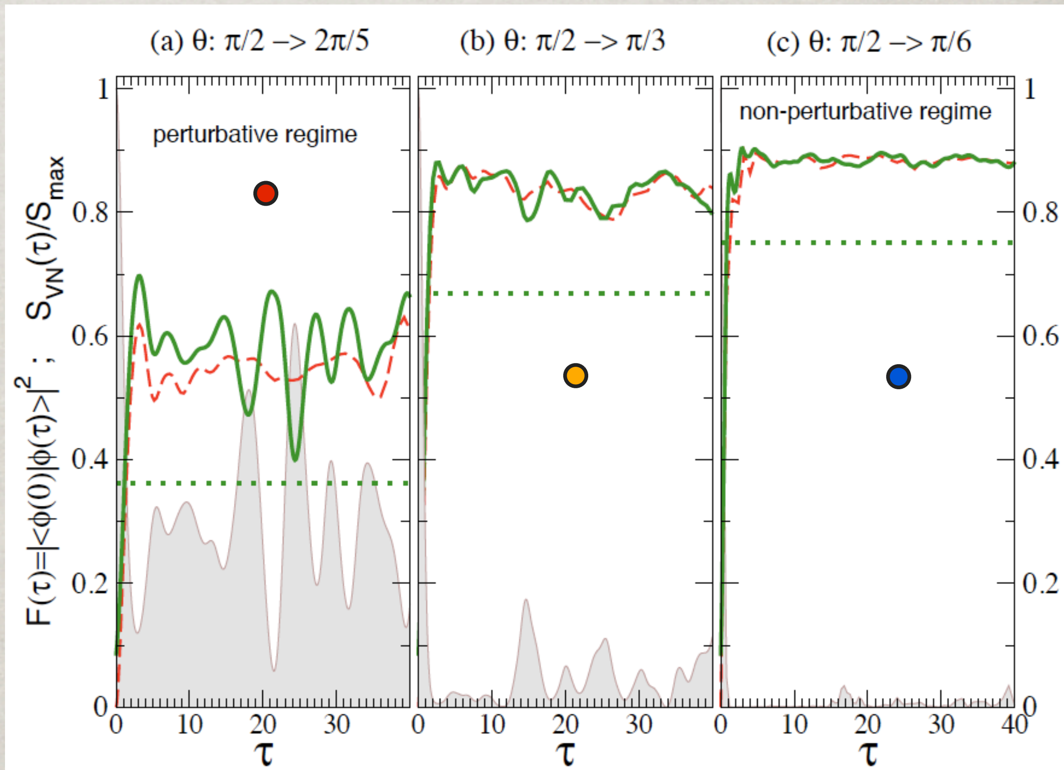
Taylor expansion of $\exp\left(-i\delta\tau H(\theta_{\mathrm{f}})\right)$

arbitrary good precision (typically $\sim 10^{-16}$)



Entanglement entropy after quench

$$S_{\rm VN}(\tau) \;=\; -{\rm Tr}\{\rho_A(\tau)\ln\rho_A(\tau)\}$$



Fidelity

$$F(\tau) \;=\; |\langle\phi(0)|\phi(\tau)\rangle|^2$$

$$F(\tau) \;\sim\; 2^{-N} \;\rightarrow\; 0$$

Finite-size scaling

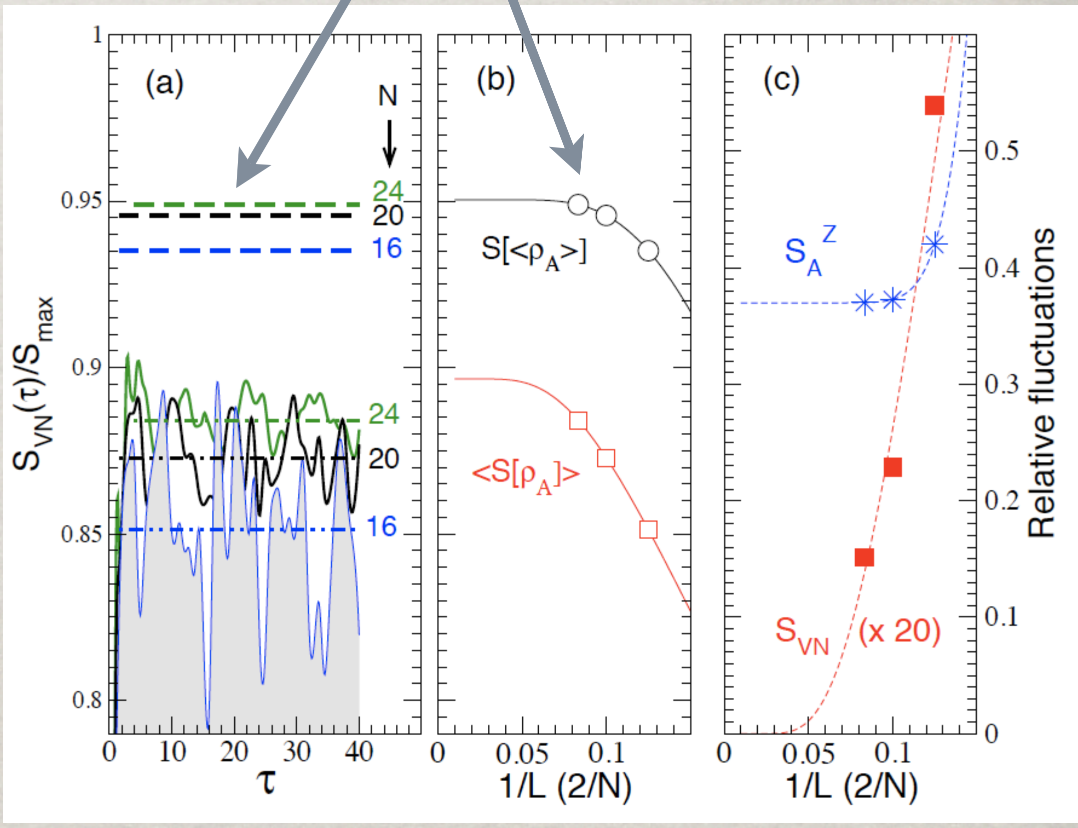
N=16, 20 & 24 sites

$$S_{\mathrm{VN}}^{\mathrm{ave}} = -\mathrm{Tr}\{\rho_A^{\mathrm{ave}} \ln \rho_A^{\mathrm{ave}}\}$$

$$\tilde{O}(\tau) = \mathrm{Tr}(\rho_A(\tau) O)$$

$$\langle G \rangle \equiv \lim_{T\to\infty} \frac{1}{T-T_{\mathrm{f}}} \int_{T_{\mathrm{f}}}^{T} G(\tau)\, d\tau$$

$$\bar{O} = \langle \tilde{O} \rangle = \mathrm{Tr}(\rho_A^{\mathrm{ave}} O)$$

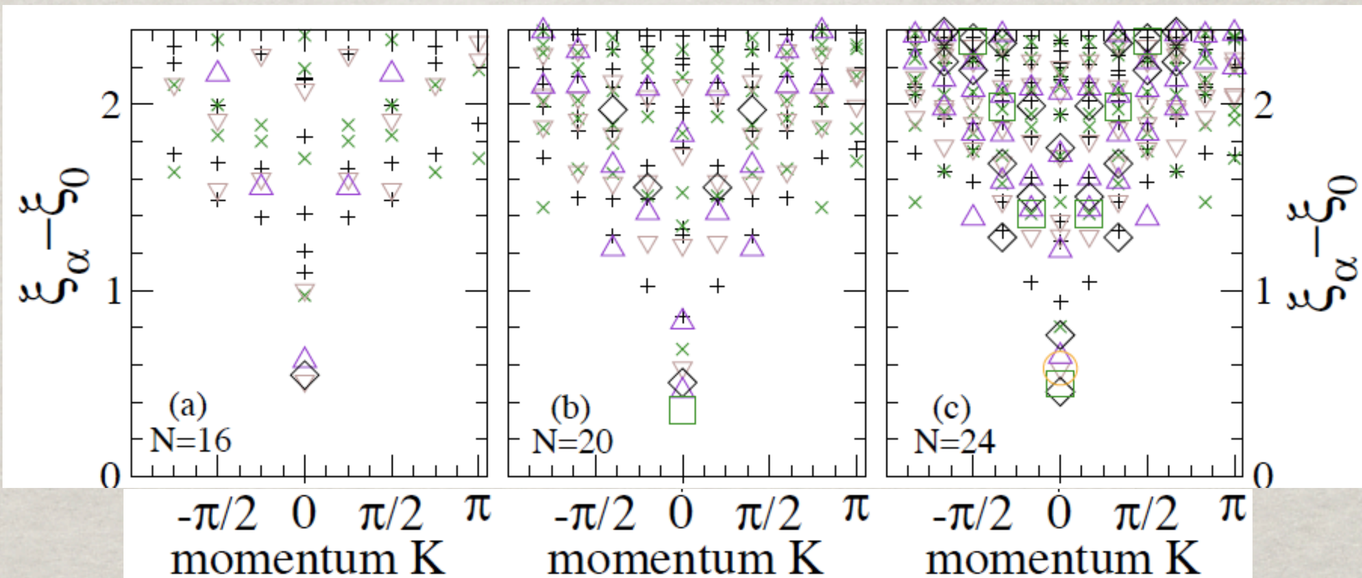


Out-of-equilium entanglement spectra

$$\rho_A^{\mathrm{ave}} = \exp\left(-\Xi_{\mathrm{non-equil}}\right)$$

cannot be described by an effective system in its groundstate

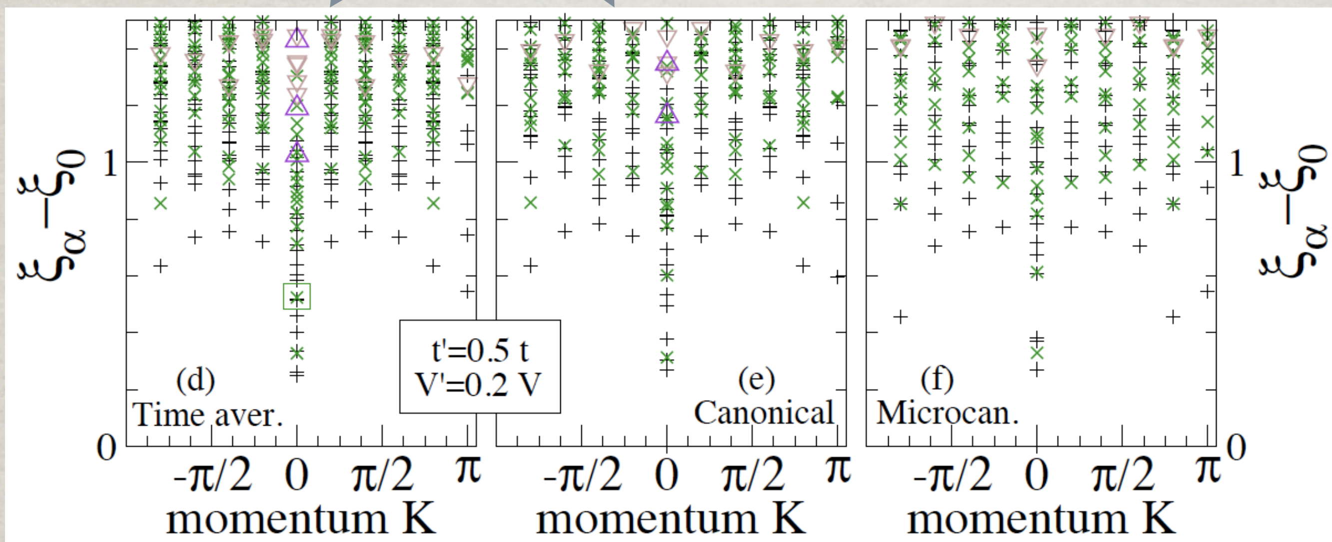
Finite-size scaling



Can it be described by a thermal ensemble ?

Infinite time-average vs thermal averages

Remarkably similar !



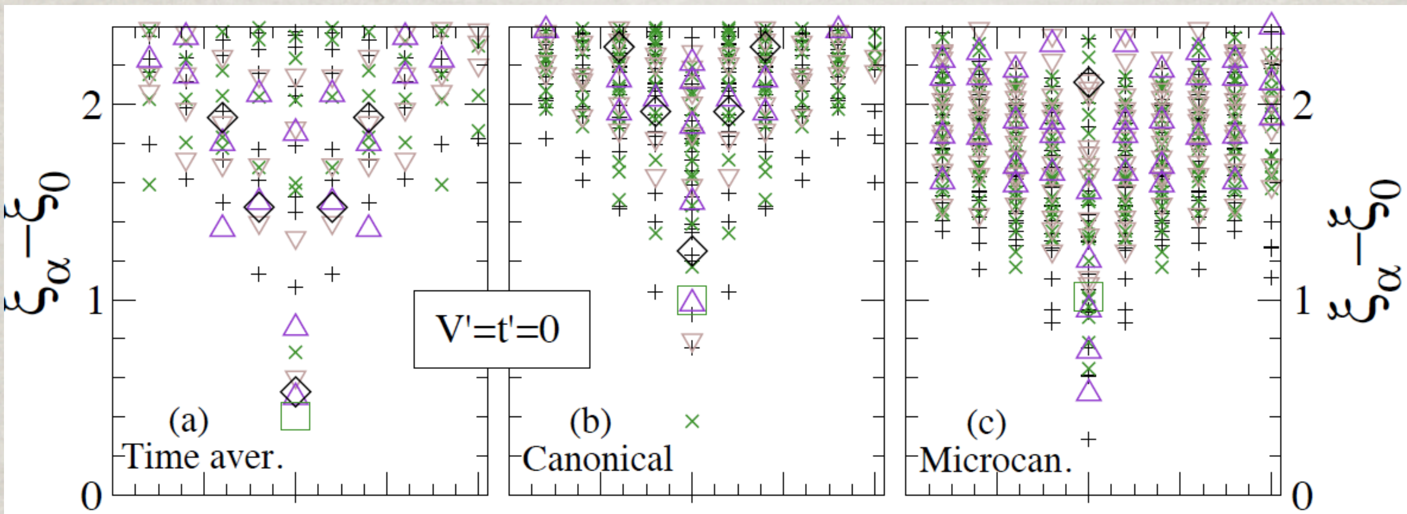
NON-INTEGRABLE SYSTEM

Thermalization occurs even for extensive observables !

Extention of "Eigenstate Thermalisation Hypothesis"

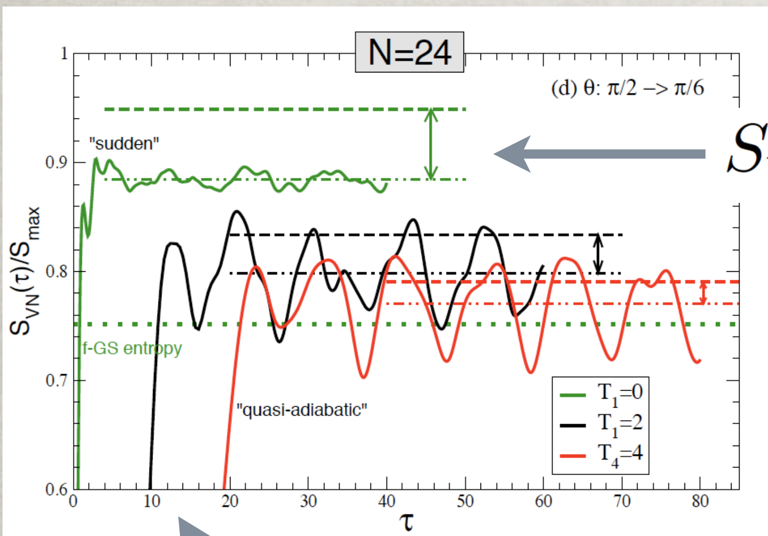
M. Rigol, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008)

INTEGRABLE
SYSTEM



Important differences might be due to:
1) fondamental issues like integrability
and/or
2) technical issues like smaller # of states to average ?

Quasi-adiabatic quenches



$S_{\mathrm{VN}}^{\mathrm{ave}} - \langle S_{\mathrm{VN}} \rangle$

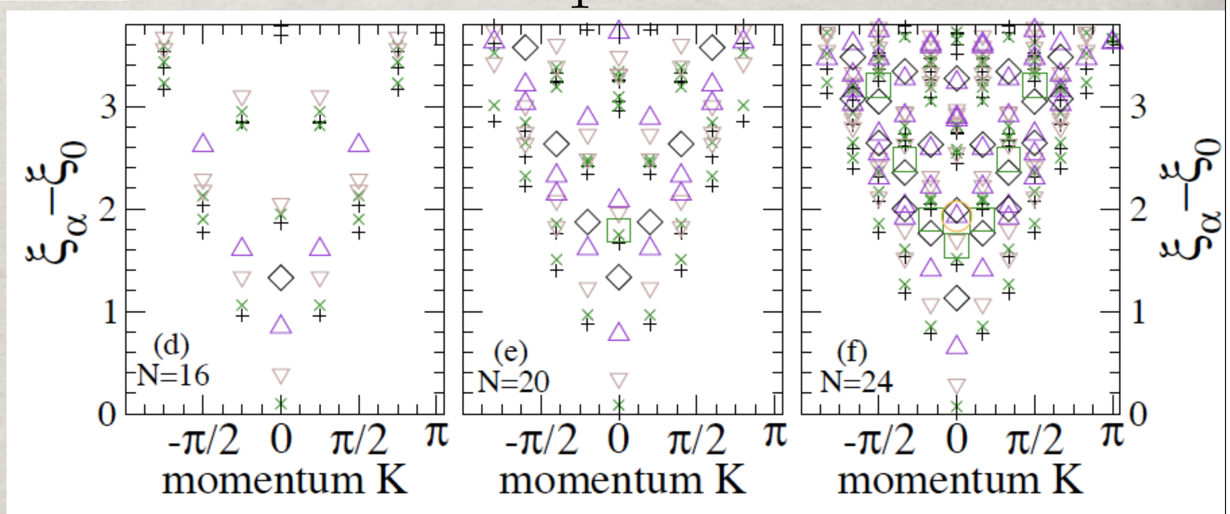
"measures"
deviation from groundstate

$0 < \tau < 10T_1$

$\theta(\tau) = (1 - w_\tau)\theta_{\mathrm{init}} + w_\tau \theta_{\mathrm{f}}$

$w_\tau = 1/(1 + \exp((5T_1 - \tau)/T_1))$

$T_1 = 4$



Summary

* Generically, out-of-equilibrium state cannot be described by an effective system in its groundstate

* (Non-local) bipartition setup provides stringent tests of thermalization

* Thermalization at the level of extensive observable occurs for non-integrable systems

* Deviation between time-average and statistical average of entanglement entropy: $S_{\rm VN}^{\rm ave} - \langle S_{\rm VN} \rangle$