

Putting Floquet theory to work: Random hopping systems and topological phases with time dependent Hamiltonians

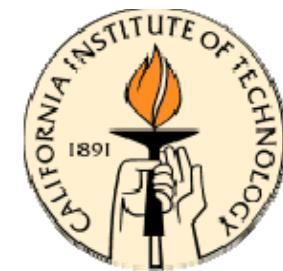
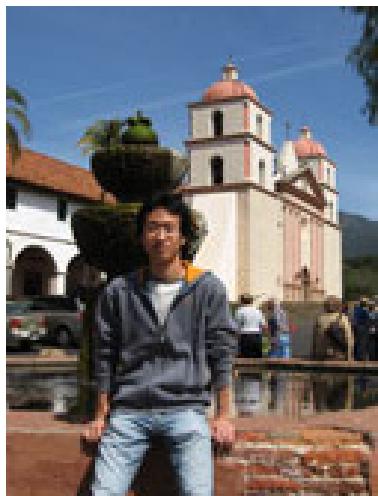
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Packard Foundation,
Sloan Foundation,
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Exotic wave functions seeking parent Hamiltonian

- *Motivation:*
realizing interesting Hamiltonians using temporal modulation.

Inspiring example: *Fractional quantum Hall in optical lattices,*
Sorensen, Demler, Lukin (2004)

- *Personal obsession:*
Dyson singularity in random hopping systems.
- *Current fascination:*
2d Topological insulators.

Outline

Part 1 – Random hopping

- Dyson singularity in random hopping models.
- Dynamic localization.
- Modulated Anderson disorder - properties.
- Experimental realization

Part 2 – Topological phases

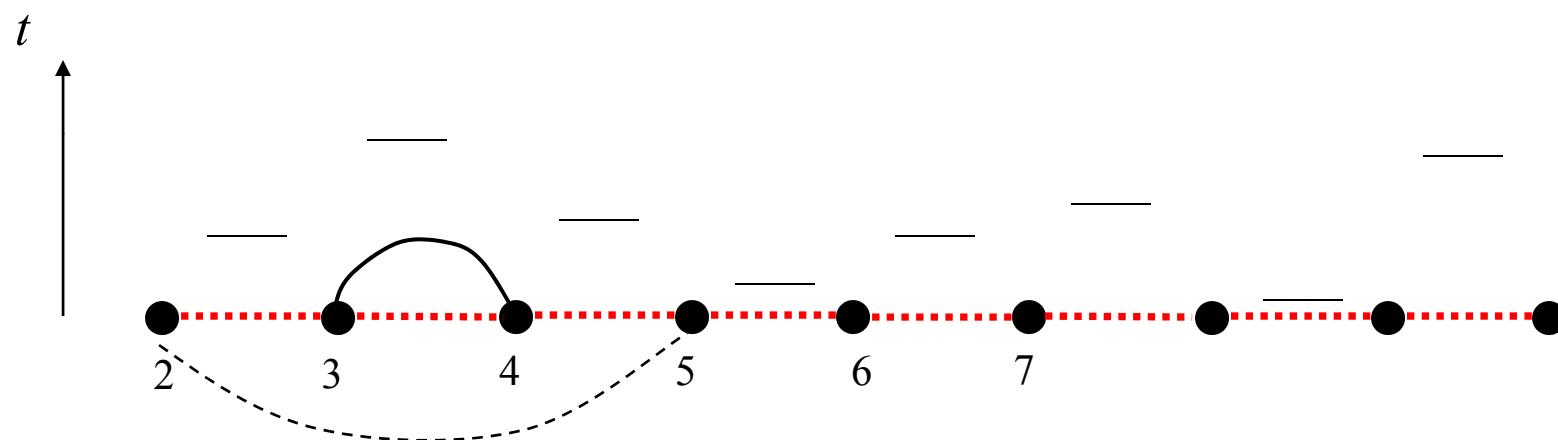
- 2d Topological phases crash course
- How can we topologize the trivial?
- Experimental realization (?)

Random hopping models

Particle-hole symmetric localization

- Random hopping in 1d:

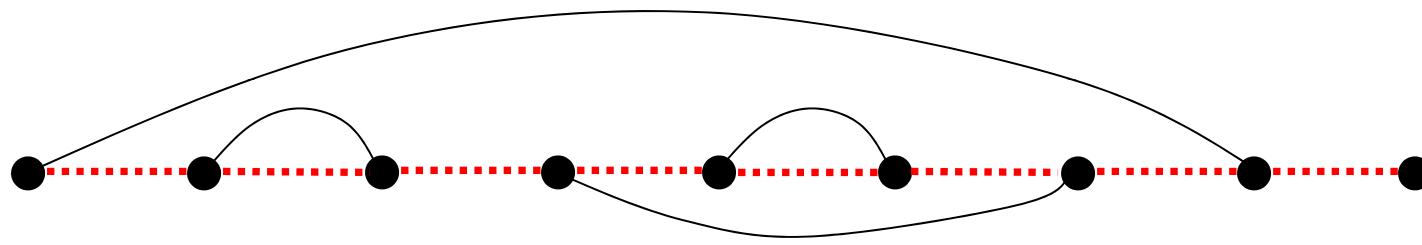
$$H = - \sum_{\langle ij \rangle} t_{ij} c_{i,s}^+ c_{j,s}$$



Particle-hole symmetric localization

- Random hopping in 1d:

$$H = - \sum_{\langle ij \rangle} t_{ij} c_{i,s}^+ c_{j,s}$$



- Chiral (sublattice) symmetry – delocalized state at band center in 1d and 2d.

Gade, (1993), Motrunich, Damle, Huse, (2001).

- Random hopping localization vs. interactions in 2d?

See M. Foster, A. Ludwig. Etc.

- Mott glass phase for random bosons (incompressible *and* gapless).

Altman, Kafri, Polkovnikov, GR (2006), Orignac, Le Doussal, Giamarchi (1998).

Properties of a random hopping model

Dyson (1953)
 Thouless (1972)
 Fisher (1994)

- Density of states:

Pure chain:

$$E_k = -2u \cos k$$

$$\rho(E) = \frac{1}{2\pi\sqrt{4u^2 - E^2}}$$

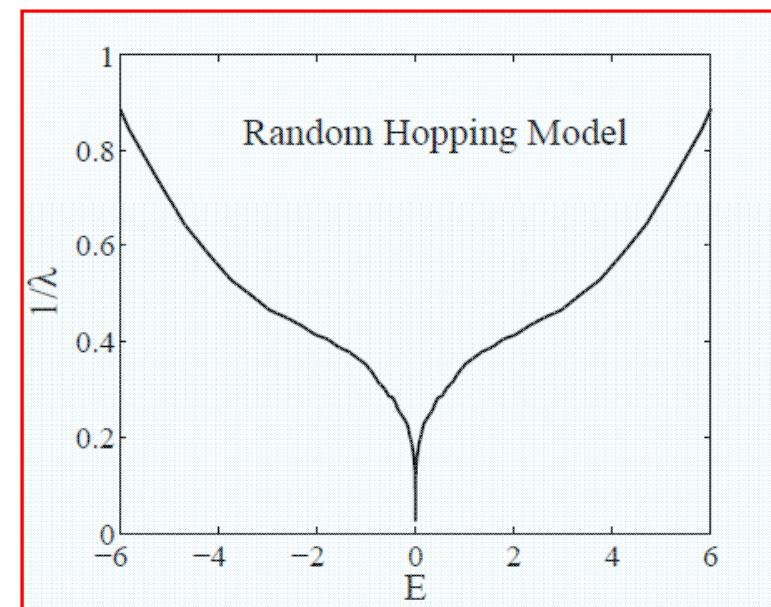
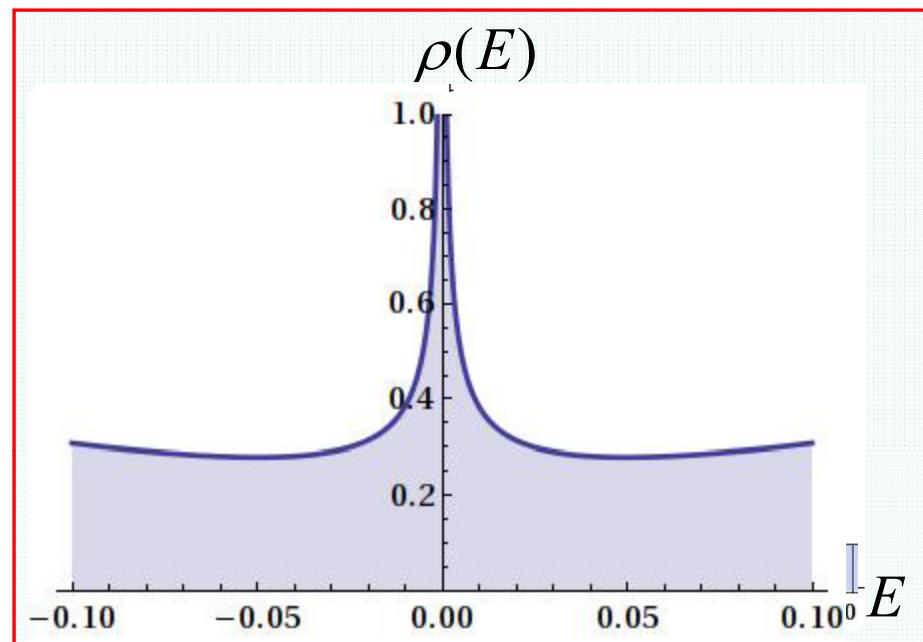
Random hopping chain:

$$\rho(E) \sim \frac{1}{E \ln^3 E}$$

- End-to-end wave function decay:

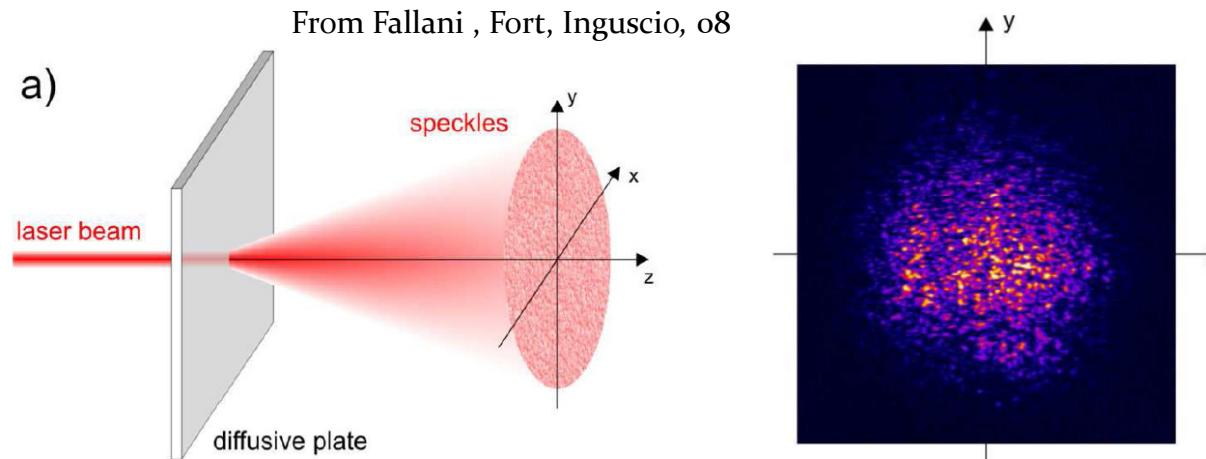
$$\langle \psi_1 \psi_L \rangle \sim e^{-L/\lambda}$$

$$\lambda_E \sim \ln |E|$$



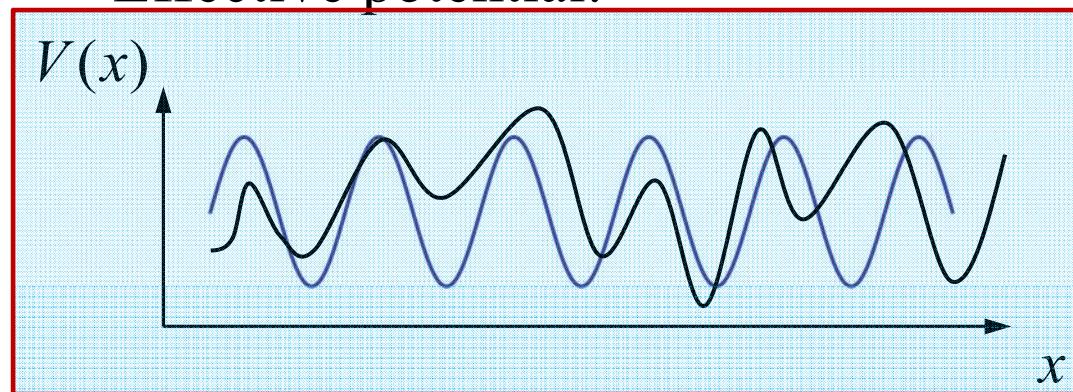
Experimental realization?

- Disorder typically produced by a speckle potential:



See also exp's by:
Aspect,
DeMarco, etc.

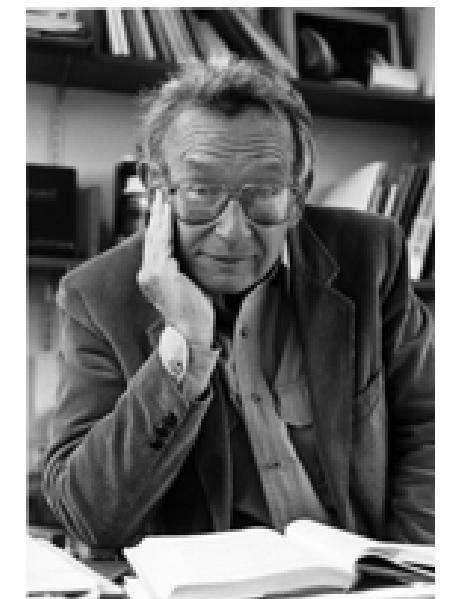
- Effective potential:



$$H = -\sum_{\langle ij \rangle} u_{ij} b_{i,s}^+ b_{j,s} + \sum_i v_i b_i^+ b_i$$

random hopping

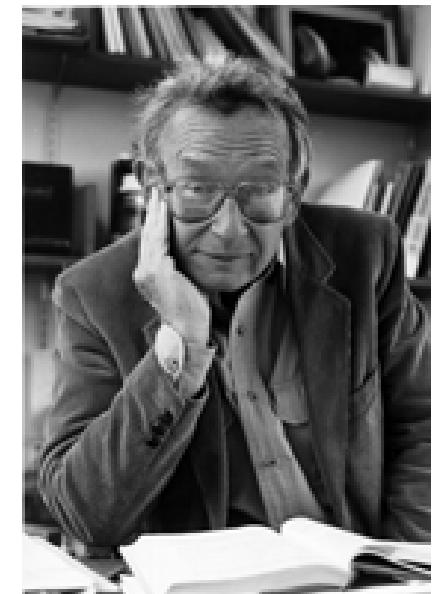
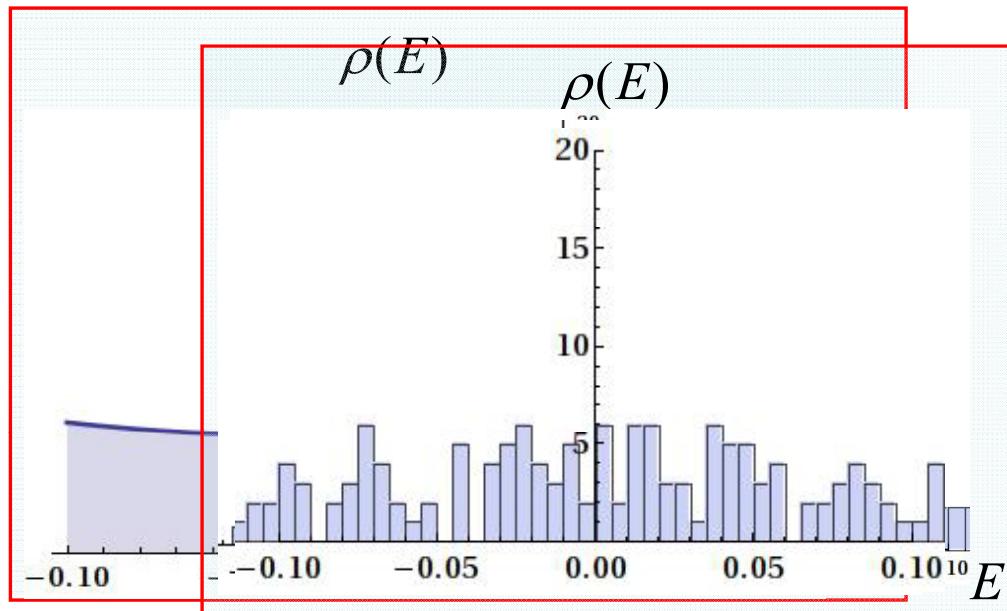
random potential



Problem:
Anderson!

Anderson insulator vs. random hopping

- Density of states:



- End-to-end average correlations:

$$\lambda_E \sim \ln |E| \rightarrow \lambda_E \sim \text{const}$$

- Dyson singularity \rightarrow Anderson insulator

- Interacting bosons:

Mott glass \rightarrow Bose glass
(incompressible, gapless)

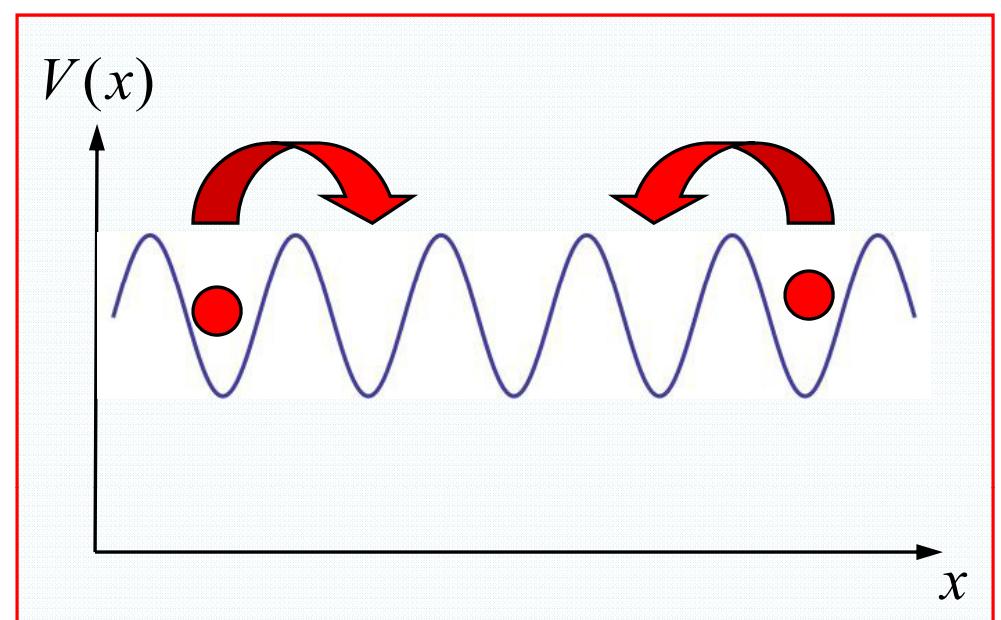
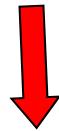
Problem:
Anderson!

Dynamic localization

Holthaus, Arimondo, and others

- Periodically Tilted chain:

$$H = -\sum_i u \left(b_i^+ b_{i+1} + b_{i+1}^+ b_i \right) - \sum_i Fx_i \cos(\omega t) b_i^+ b_i$$



$$H = -u \sum_i \left[\exp\left(i \frac{F}{\omega} \sin \omega t\right) b_i^+ b_{i+1} + h.c. \right]$$

- Use: $e^{ix \sin \omega t} = J_0(x) + \sum_{m \geq 1} J_m(x) \sin(m \omega t)$

$$H \approx \bar{H} = -\sum_i u J_0(F/\omega) \left(b_i^+ b_{i+1} + h.c. \right)$$

$$\begin{aligned} V &\rightarrow V + \frac{\partial f}{\partial t} \\ \vec{A} &\rightarrow \vec{A} + \nabla f \\ f(x, t) &= Fx \frac{\sin \omega t}{\omega} \end{aligned}$$

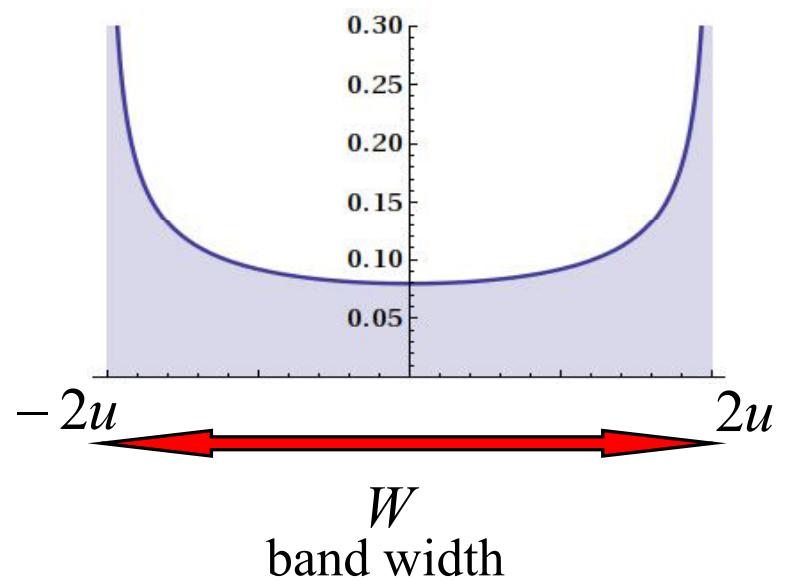
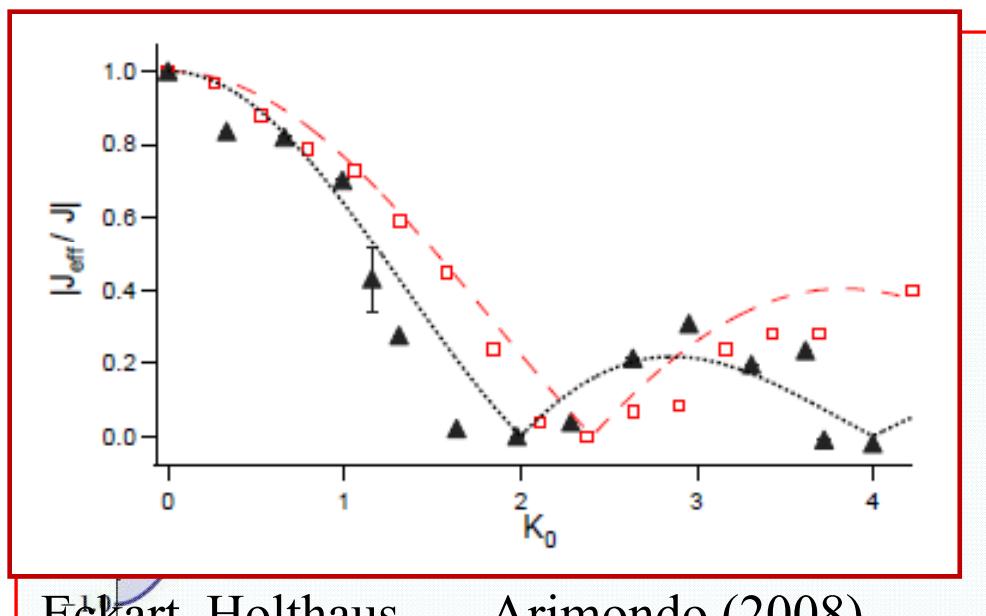
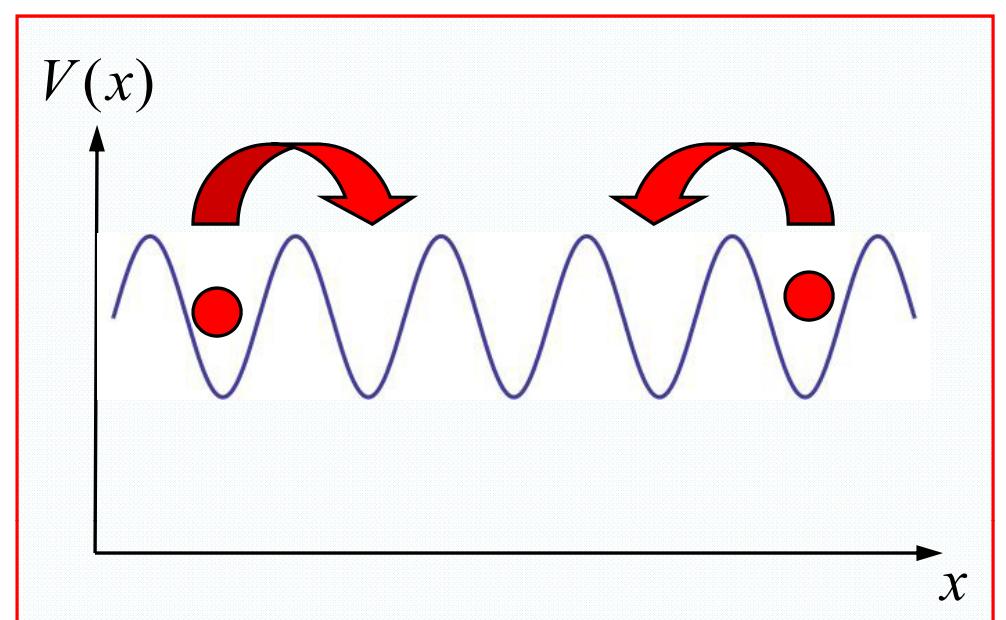
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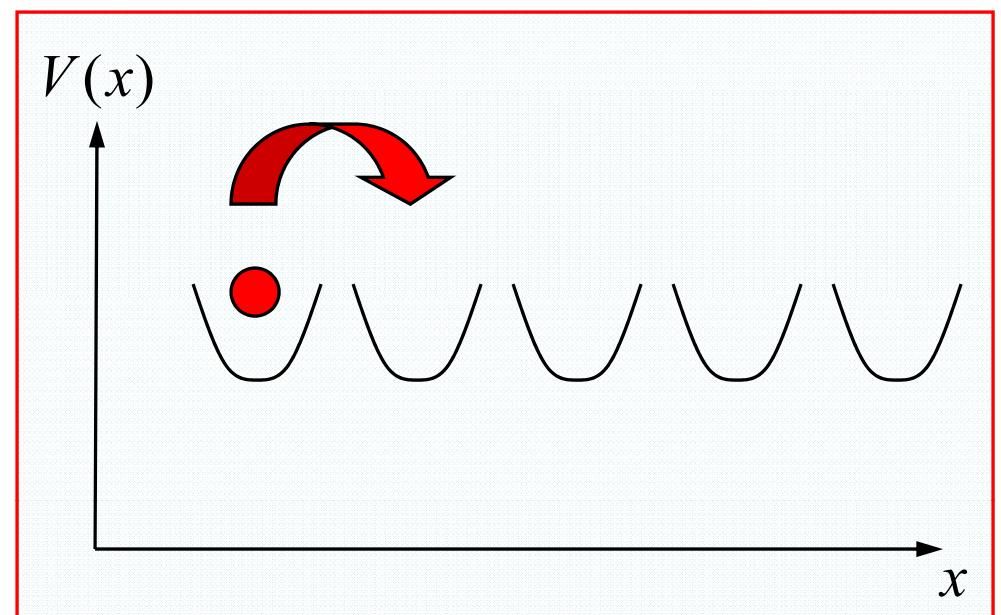


Eckart, Holthaus,..., Arimondo (2008)

Modulated on-site disorder

- Modulated disorder:

$$H = -\sum_i u \left(b_i^+ b_{i+1} + b_{i+1}^+ b_i \right) - \sum_i v_i b_i^+ b_i \cos(\omega t)$$



$$H = -u \sum_i \left[\exp\left(i \frac{(v_{i+1} - v_i)}{\omega} \sin \omega t\right) b_i^+ b_{i+1} + h.c. \right]$$

- Use: $e^{ix \sin \omega t} = J_0(x) + \sum_{m \geq 1} J_m(x) \sin(m \omega t)$

$$H \approx \bar{H} = -\sum_i u J_0((v_{i+1} - v_i)/\omega) (b_i^+ b_{i+1} + h.c.)$$

More honest analysis: Floquet Hamiltonians

- Periodically varying Hamiltonian:

$$H = H_0 + V(t)$$

with $V(t) = V(t + T)$ and: $\int_0^T V(t) dt = 0.$

- Hamiltonian \rightarrow Floquet operator:

$$U(T) = \exp\left(-i \int_0^T dt H(t)\right)$$

- Energy eigenstates \rightarrow Quasi-energy states:

$$U(T)\psi_n = e^{-i\varepsilon_n T}\psi_n$$

$$\varepsilon_n \in [0, \omega] + \varepsilon_0$$

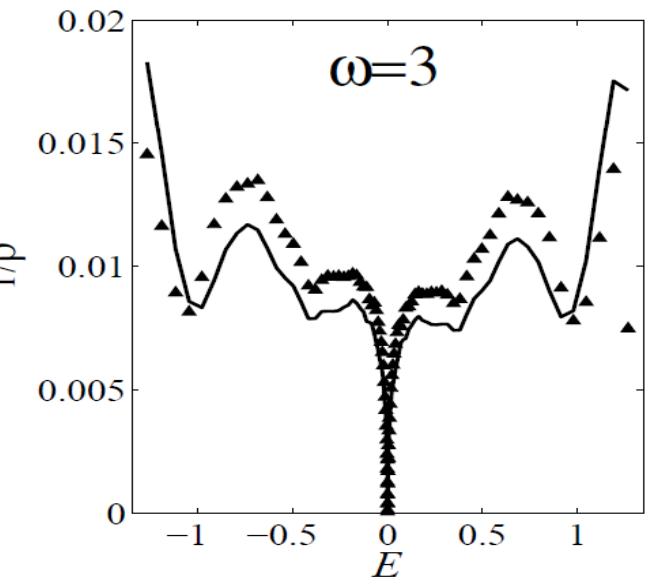
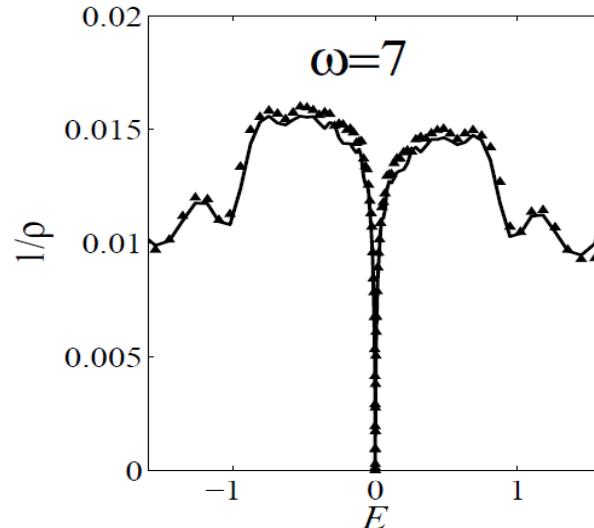
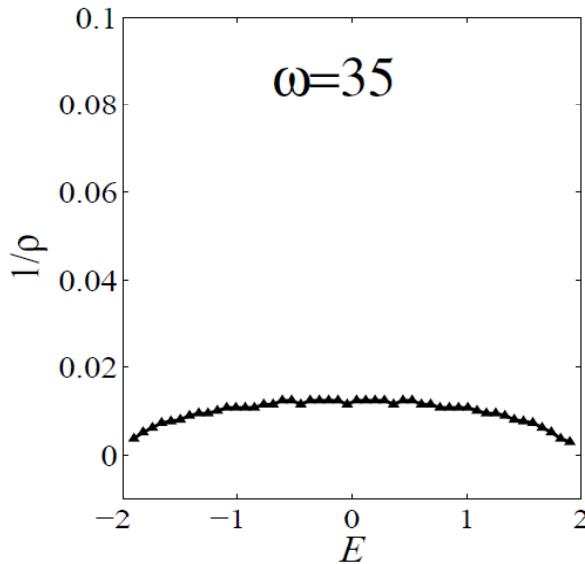
$$\omega = \frac{2\pi}{T}$$

Floquet vs. Random hopping: DOS

- Compare exact Floquet quasi-energy states with averaged random hopping H :

$$\bar{H} = -\sum_i u J_0 \left((\nu_{i+1} - \nu_i)/\omega \right) \left(b_i^\dagger b_{i+1} + h.c. \right) \quad \text{vs.} \quad H(t) = H_0 + \sum_i \nu_i \hat{n}_i \cos(\omega t)$$

Density of states:



▲ - Floquet

— - Time averaged H

Hopping:

$$u = 1$$

Disorder width:

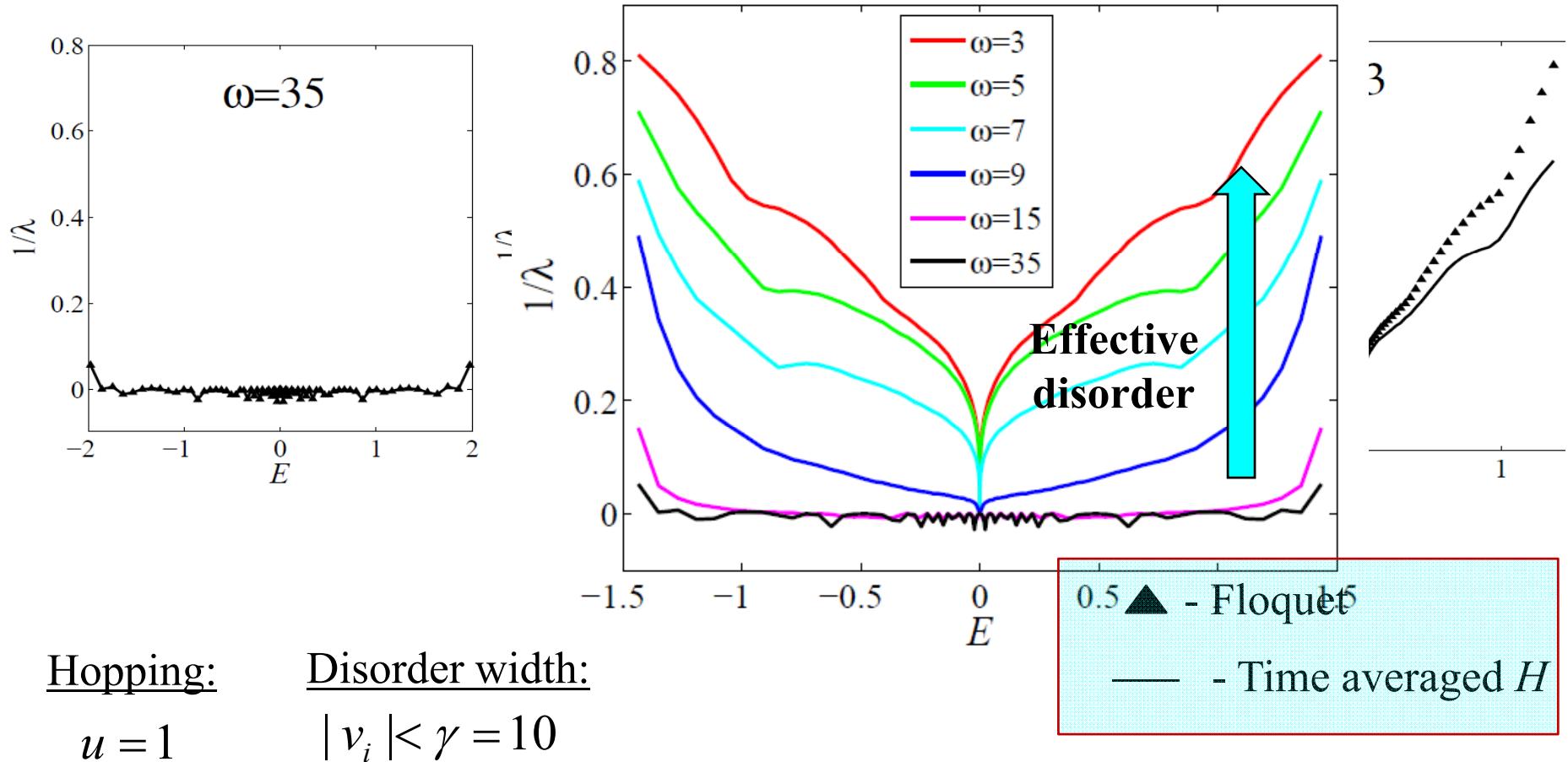
$$|\nu_i| < \gamma = 10$$

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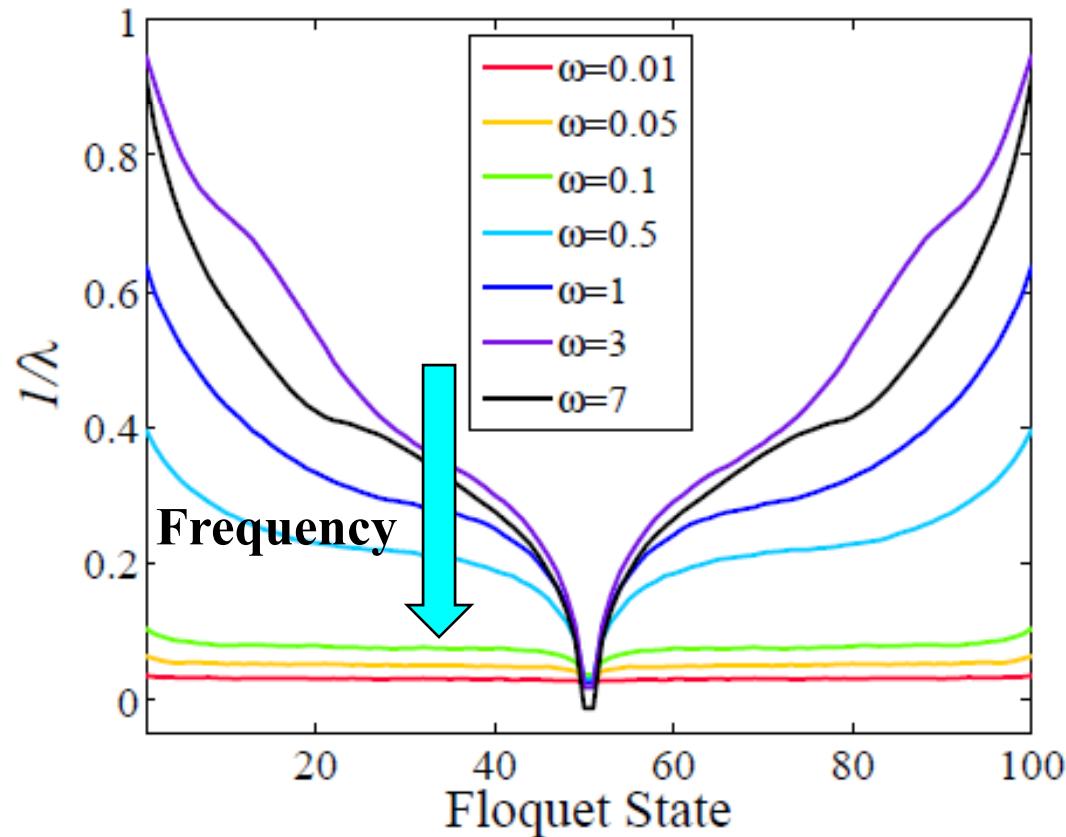
$$\overline{H} = -\sum_i u J_0 \left((\nu_{i+1} - \nu_i)/\omega \right) \left(b_i^\dagger b_{i+1} + h.c. \right) \quad \text{vs.} \quad \overline{H} = H_0 + \sum_i \nu_i \hat{n}_i \cos(\omega t)$$

Localization Length



Low frequency *de*-localization?

- Surprising trend appears in the low frequency regime:



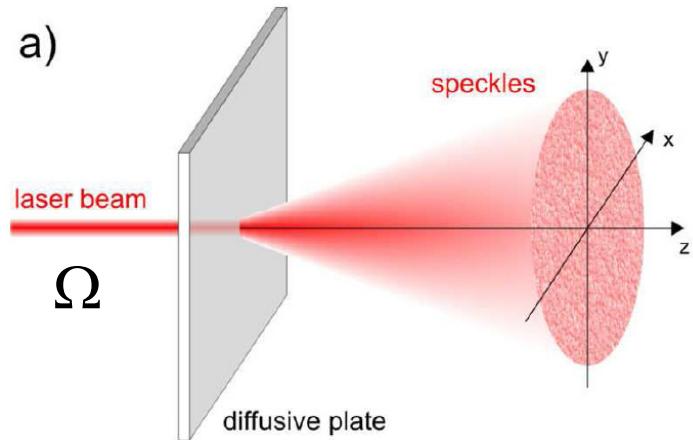
Hopping: Disorder width:
 $u = 1$ $|v_i| < \gamma = 10$

Possible experimental realization

- Speckle potential interaction:

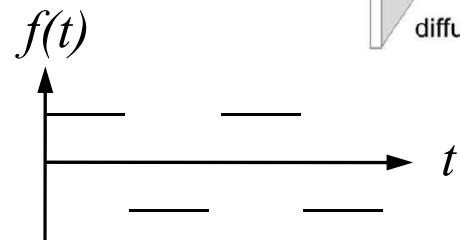
$$V(\vec{r}) = \frac{3\pi c^2}{2\omega_0^3} \left(\frac{\Gamma}{\Delta} \right) I(\vec{r})$$

$$\Delta = \Omega - \Omega_0$$



- To produce modulation:

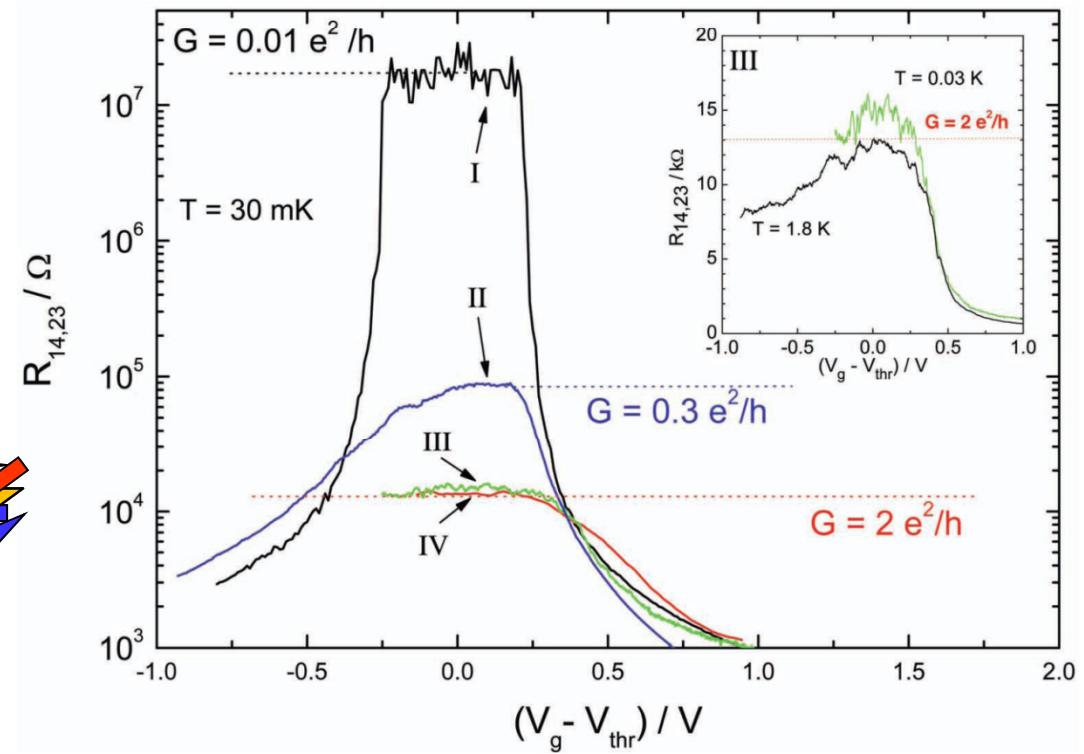
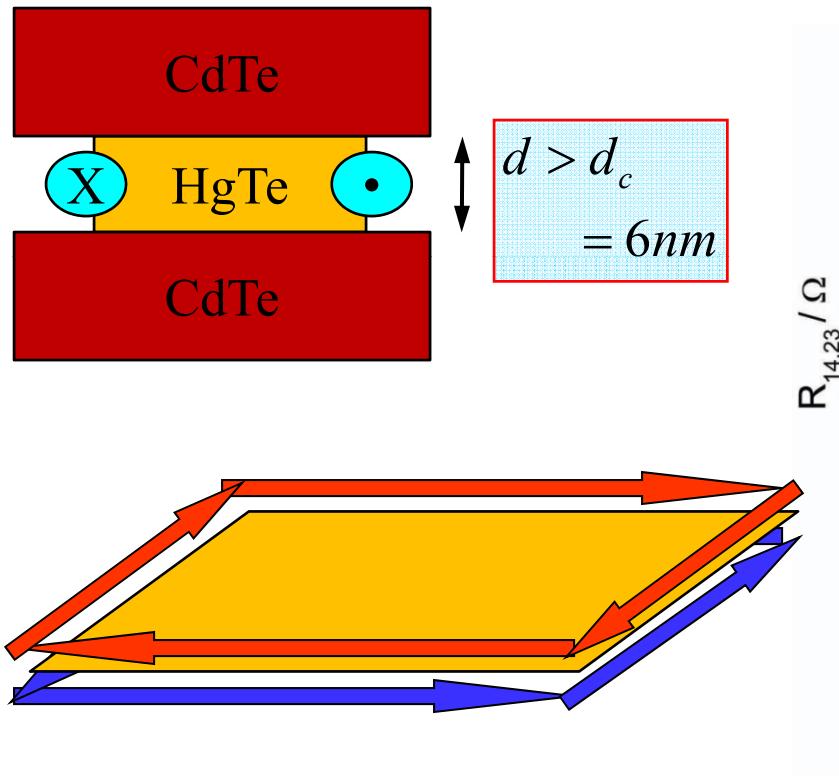
$$\Omega \sim \Omega_0 + \Delta\Omega \ f(t)$$



- Use Bragg spectroscopy to obtain DOS.

Recent fascinations: Topological phases

First observed topological insulator: CdTe, HgTe heterostructures



Bernevig, Hughes, Zhang (2006)

Koenig,.., Molenkamp.., Zhang (2007)

First observed topological insulator: HgTe

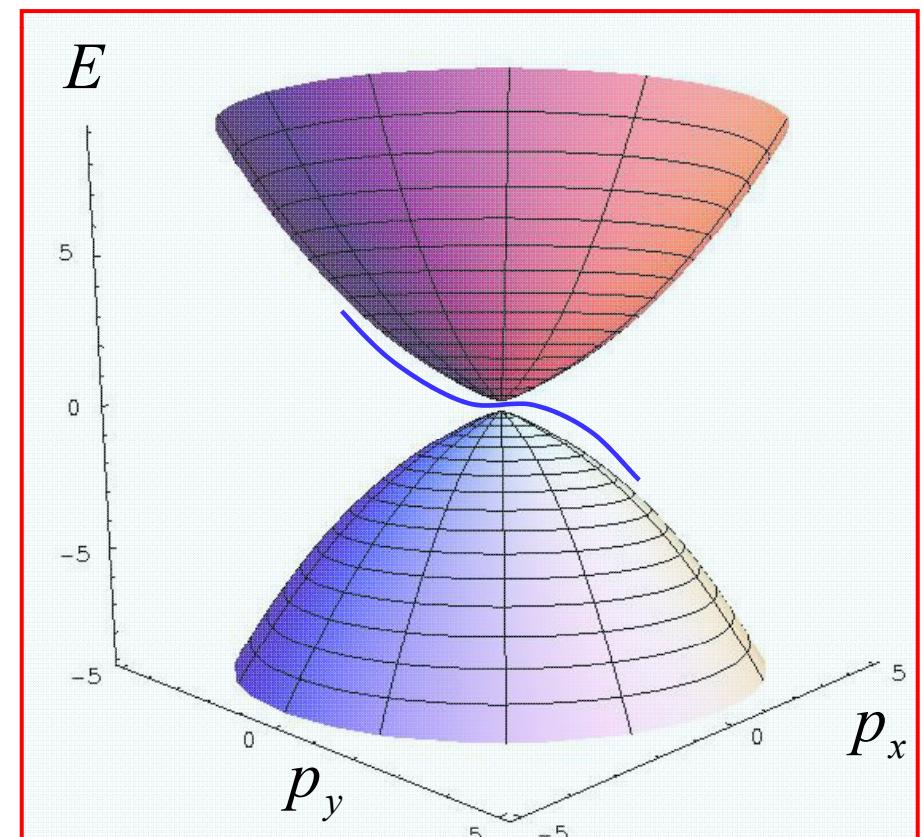
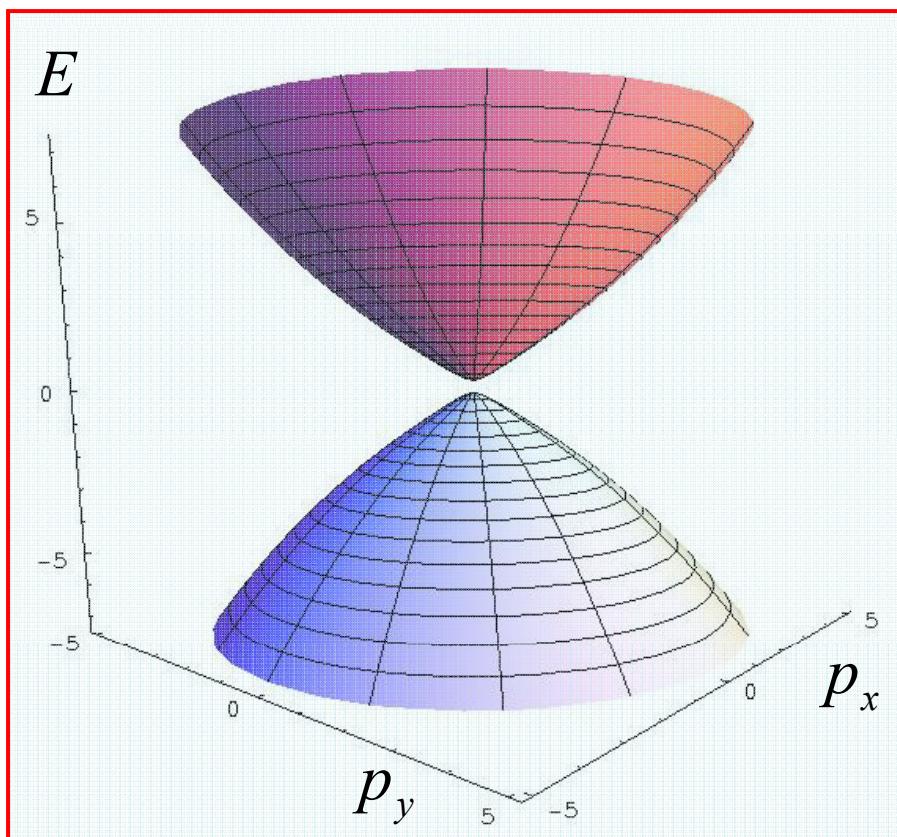
- HgTe dispersion (per spin): Dirac cone with small band gap

$$H = \varepsilon_0 + \vec{d} \cdot \hat{\tau}$$

$$\vec{d} = p_x \cdot \hat{x} + p_y \cdot \hat{y} + \left(m + b(p_x^2 + p_y^2) \right) \cdot \hat{z}$$

$m > 0, b > 0$ (Trivial)

$m > 0, b < 0$ (Topological)



What makes a phase topological?

$$H = \varepsilon_0 + \vec{\tau} \cdot \vec{d}$$

$$\vec{d} = \left(p_x, \quad p_y, \quad m + b(p_x^2 + p_y^2) \right)$$

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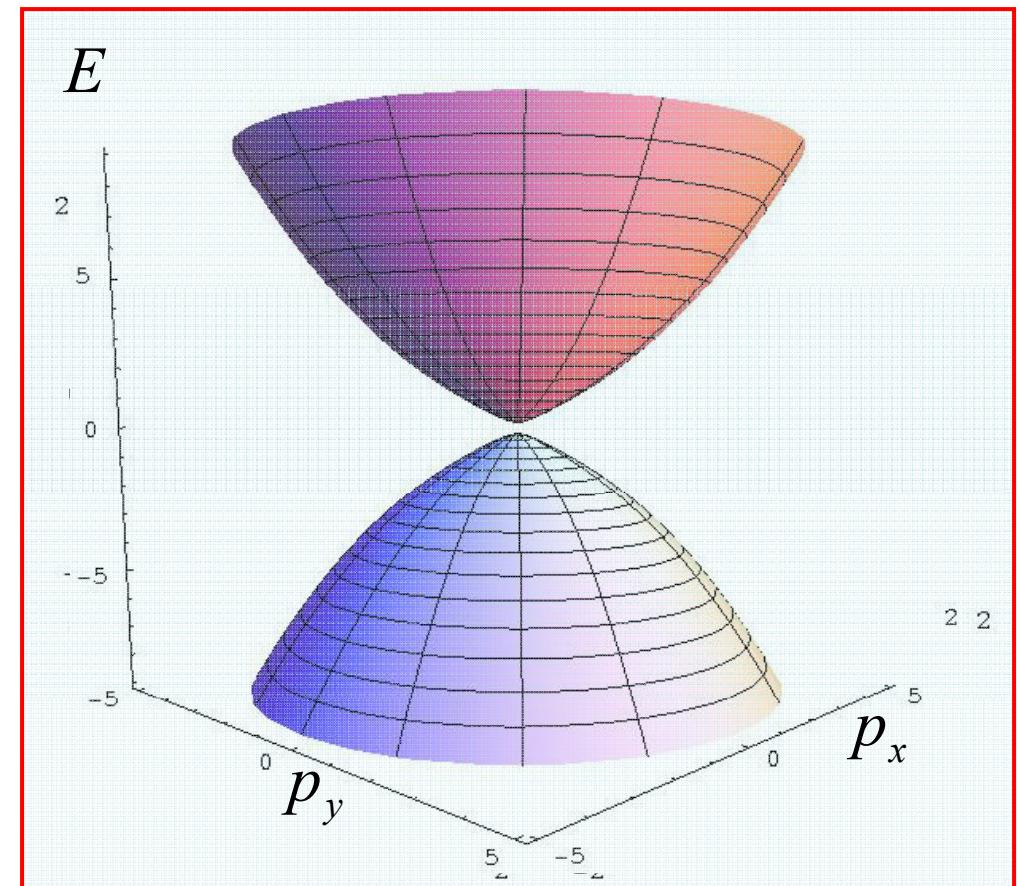
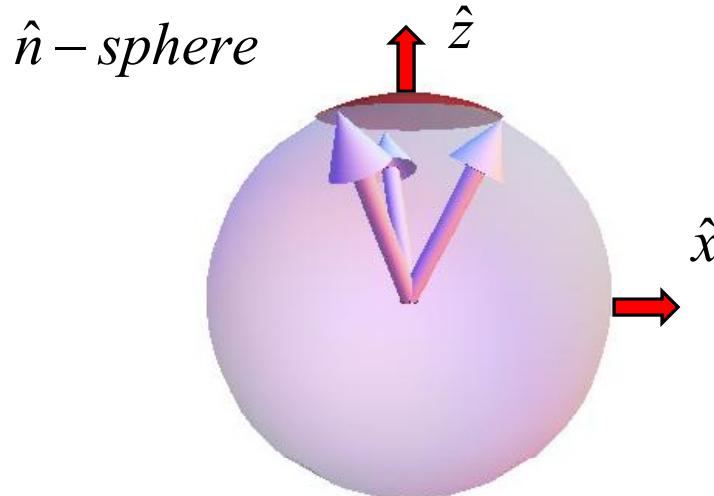
(map BZ **to** unit sphere)

- **TKNN invariant:**

$$\sigma_{xy} = \frac{e^2}{4\pi\hbar} \int_{p \in BZ} dp_x dp_y \hat{n} \cdot \left(\frac{\partial \hat{n}}{\partial p_x} \times \frac{\partial \hat{n}}{\partial p_y} \right)$$

$$m = 0.2, \quad b = 0.3 \quad (\text{non-topo})$$

Wrapping the unit sphere?



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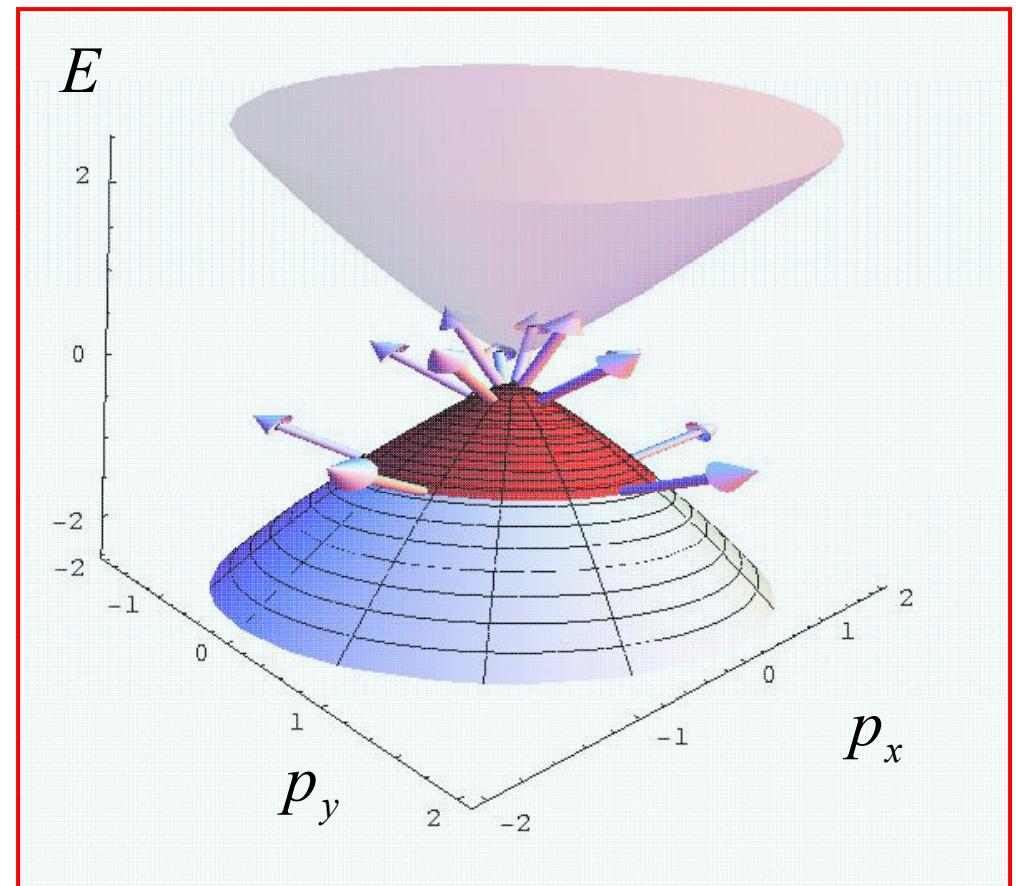
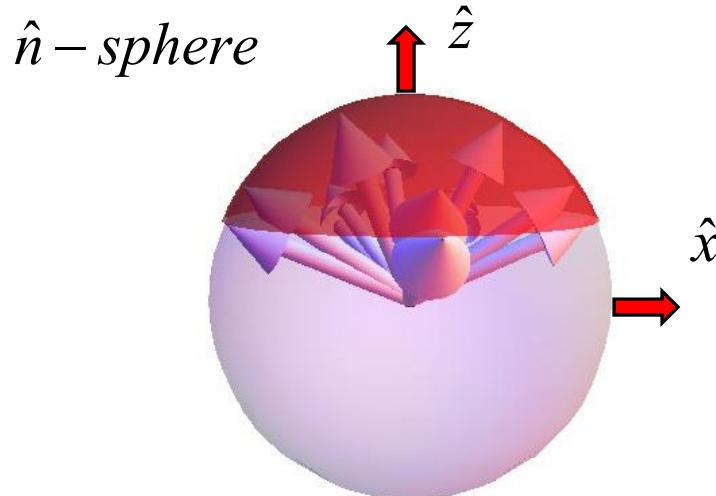
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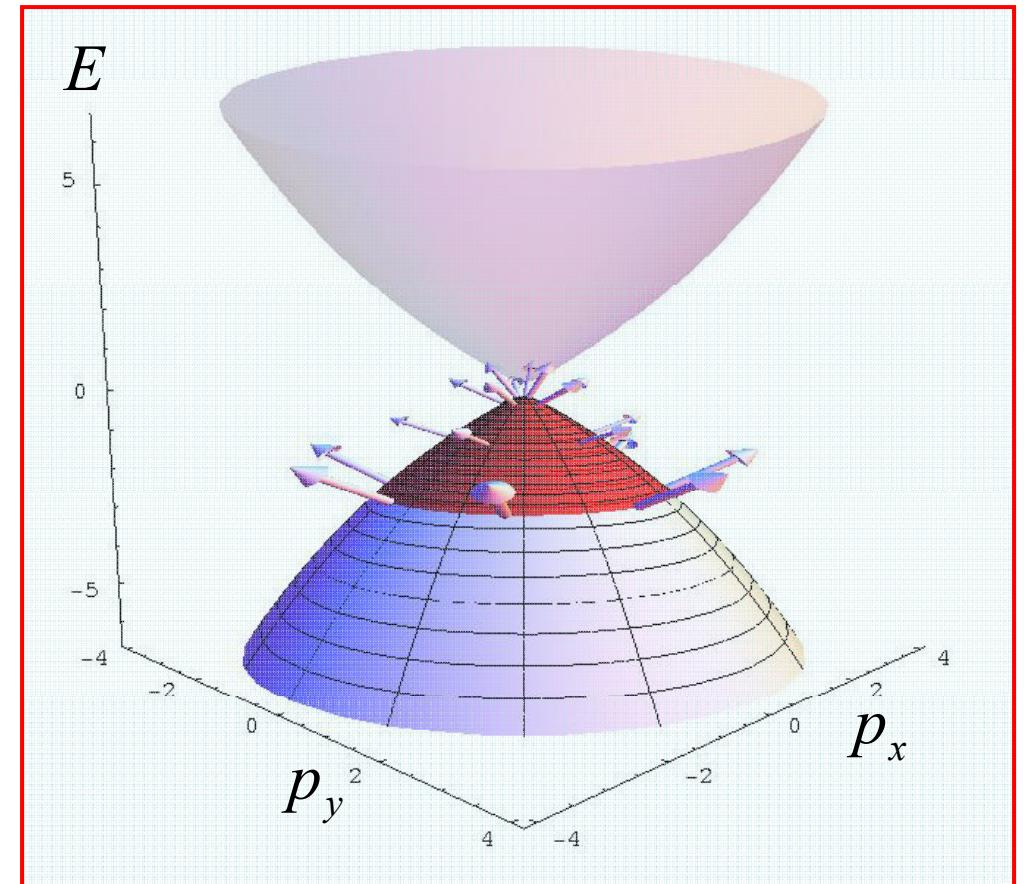
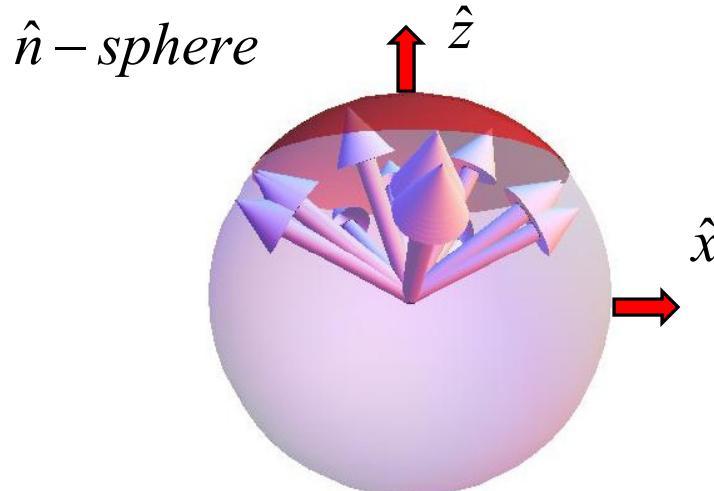
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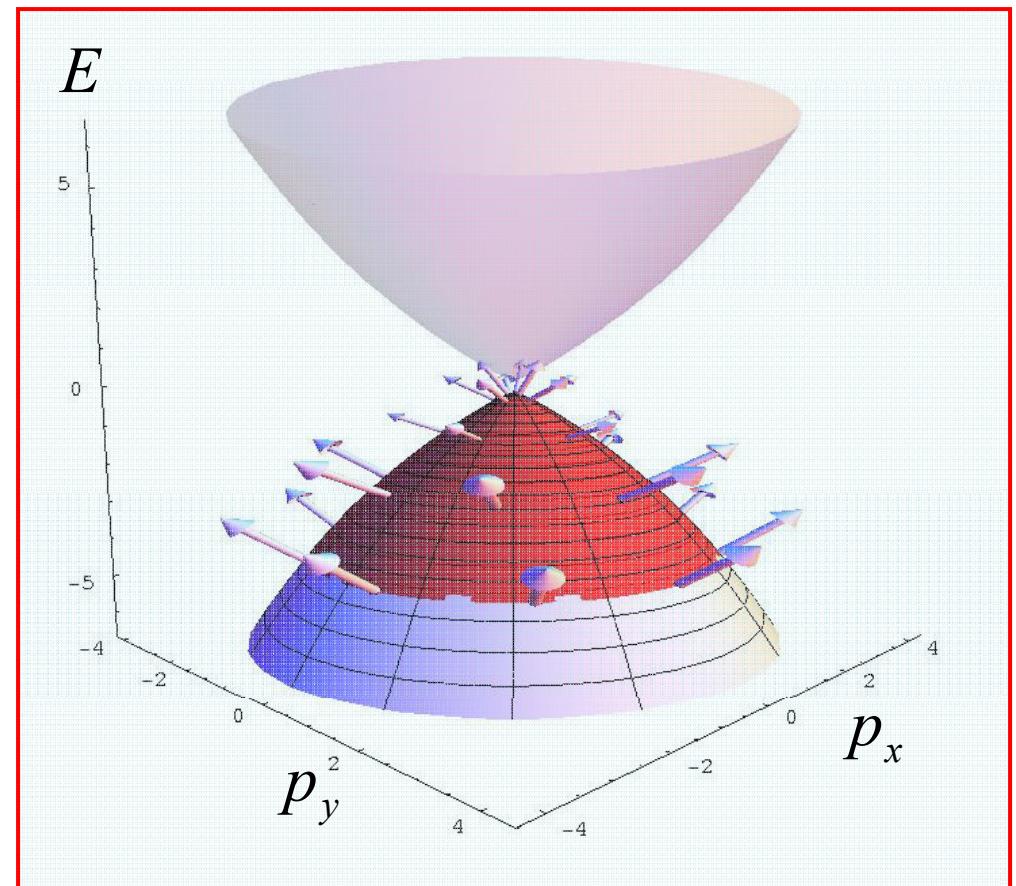
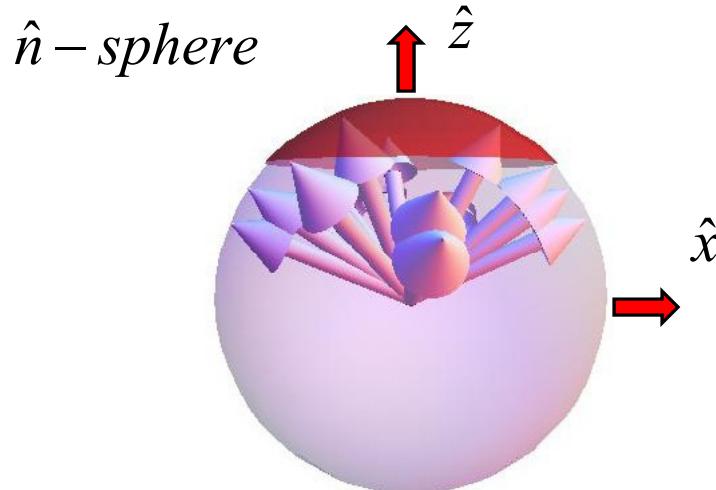
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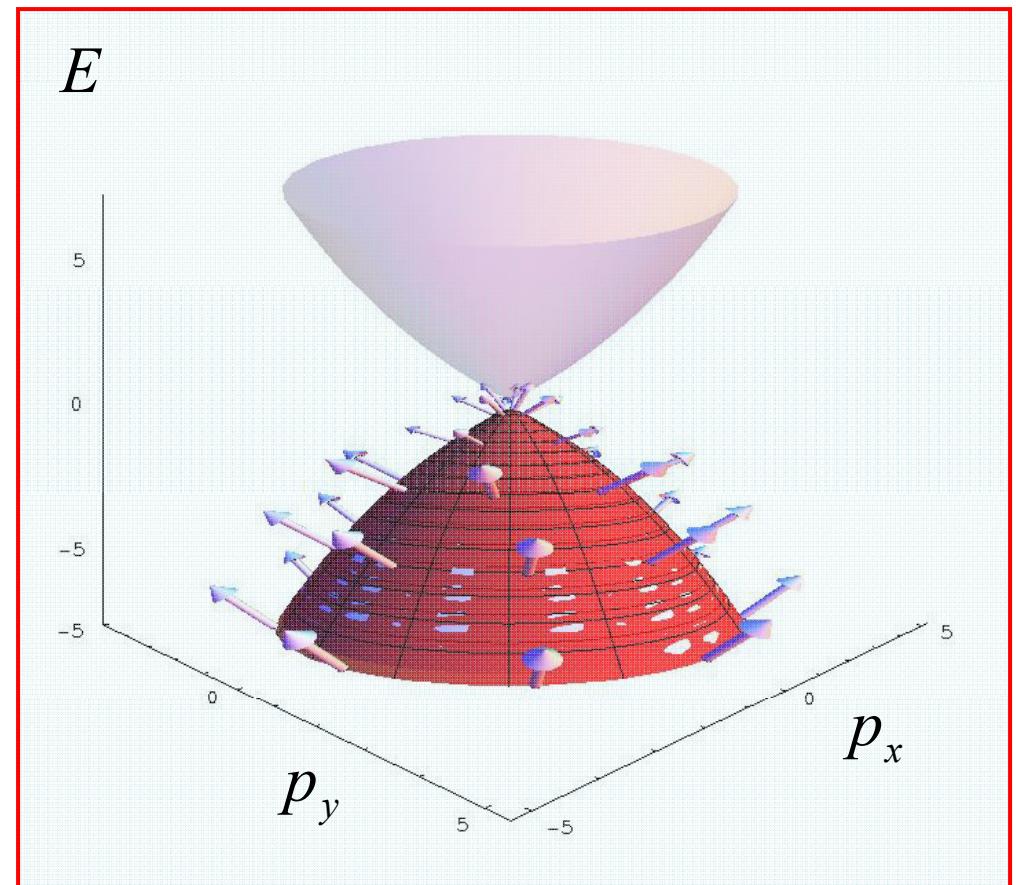
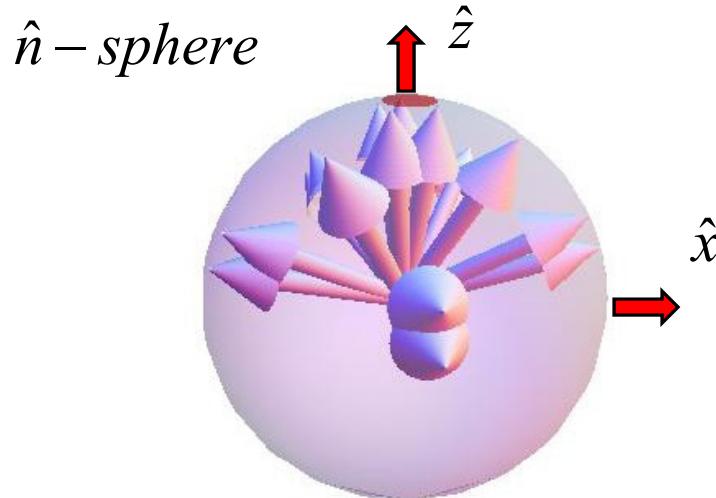
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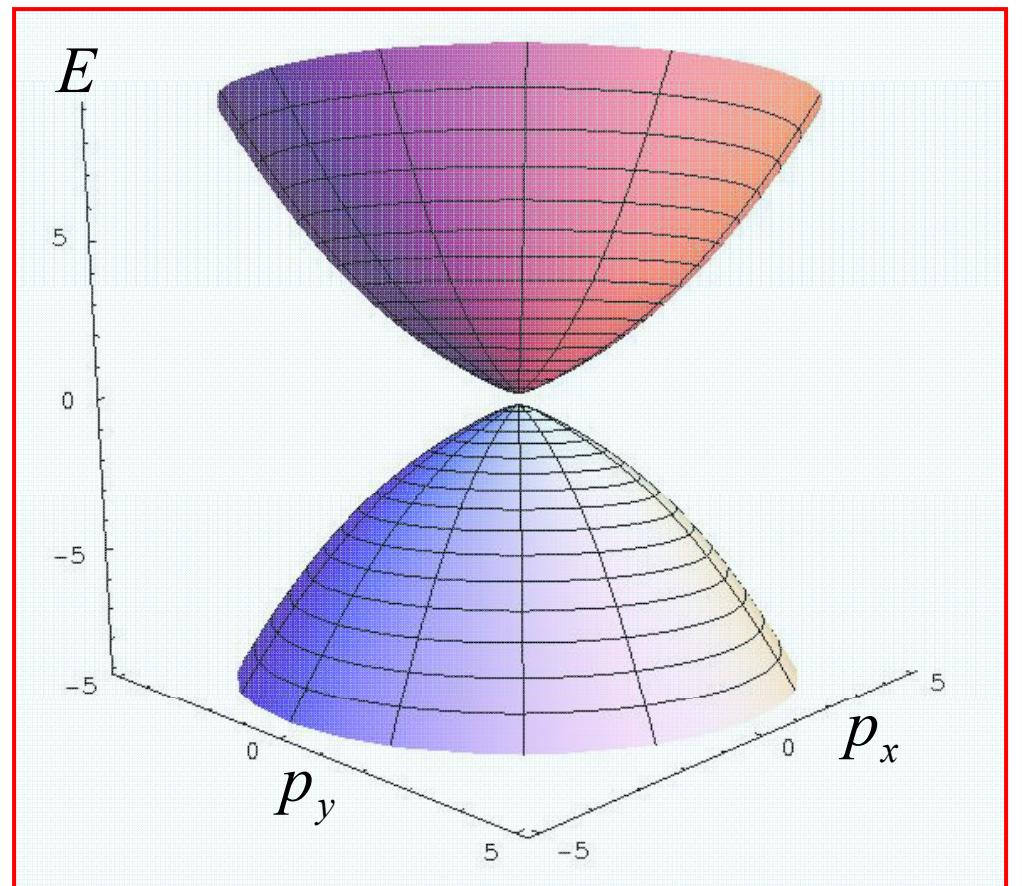
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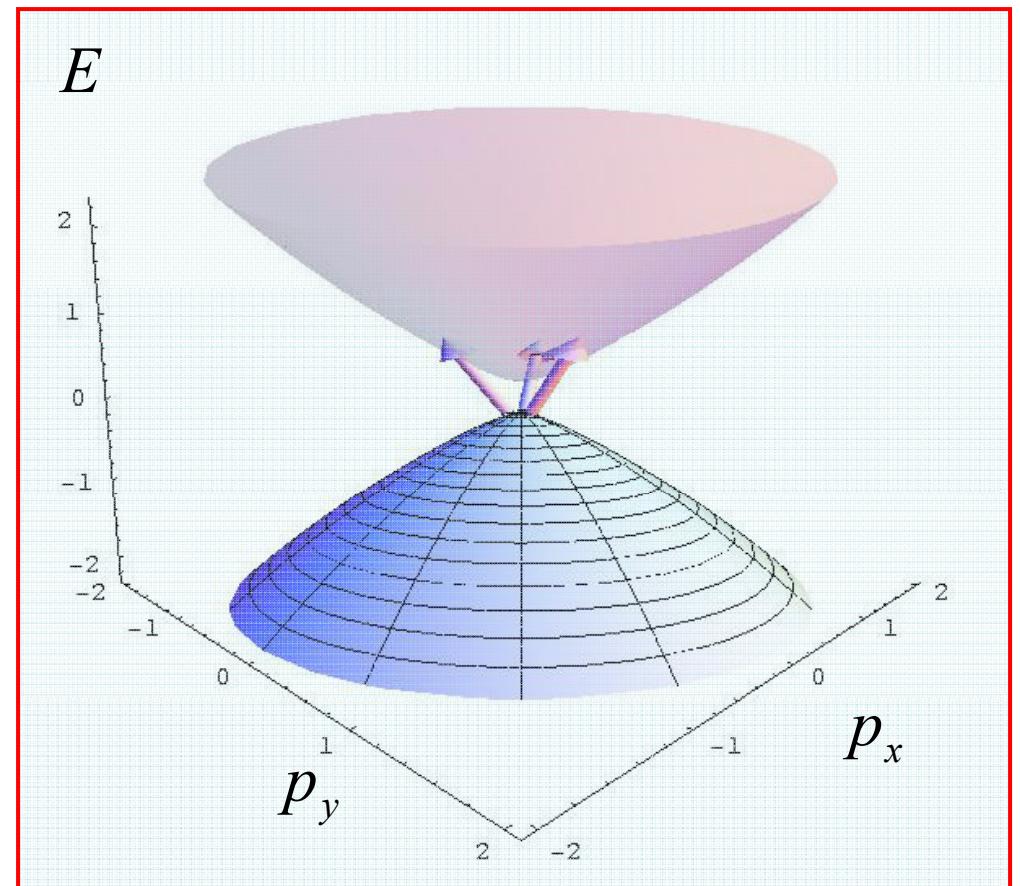
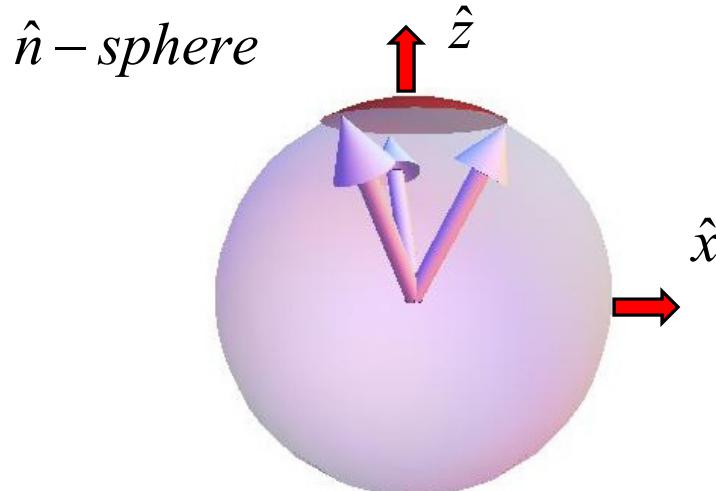
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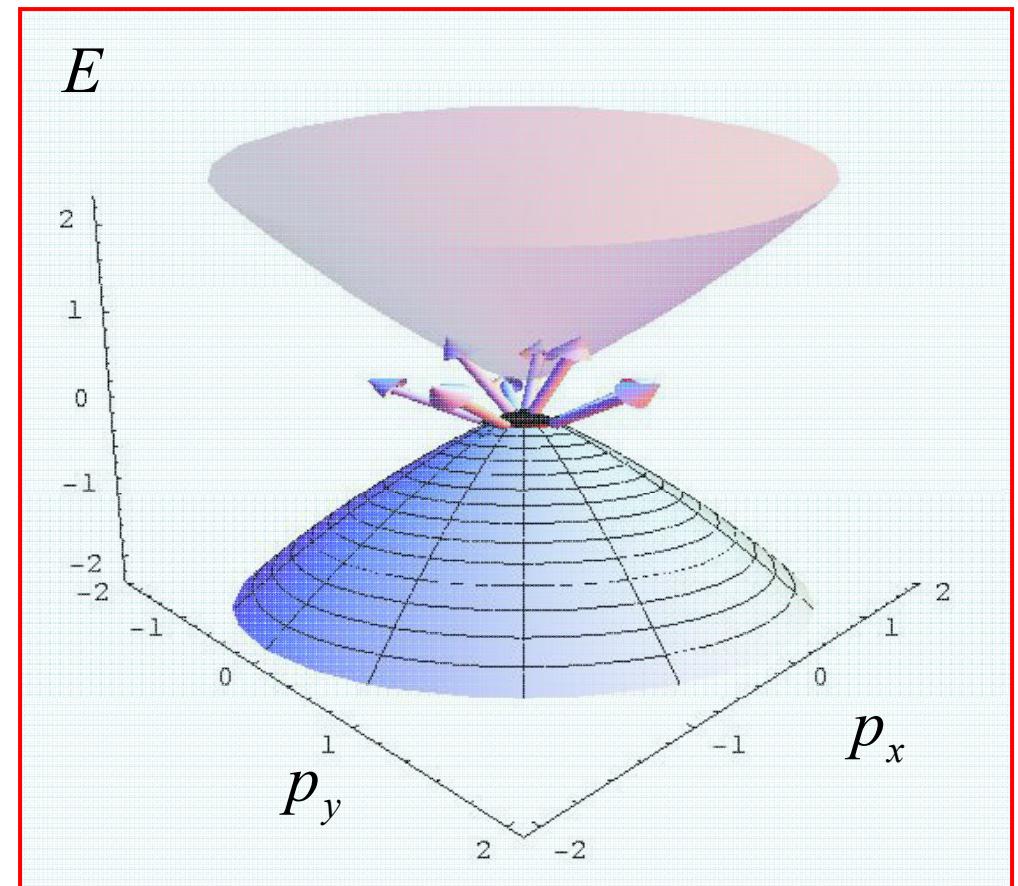
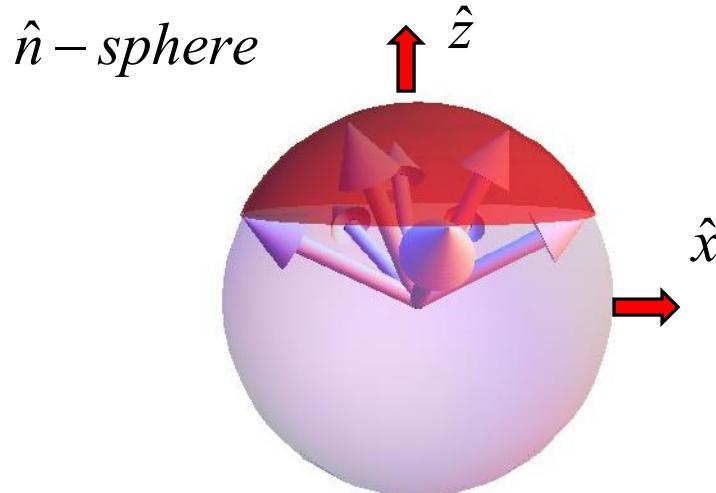
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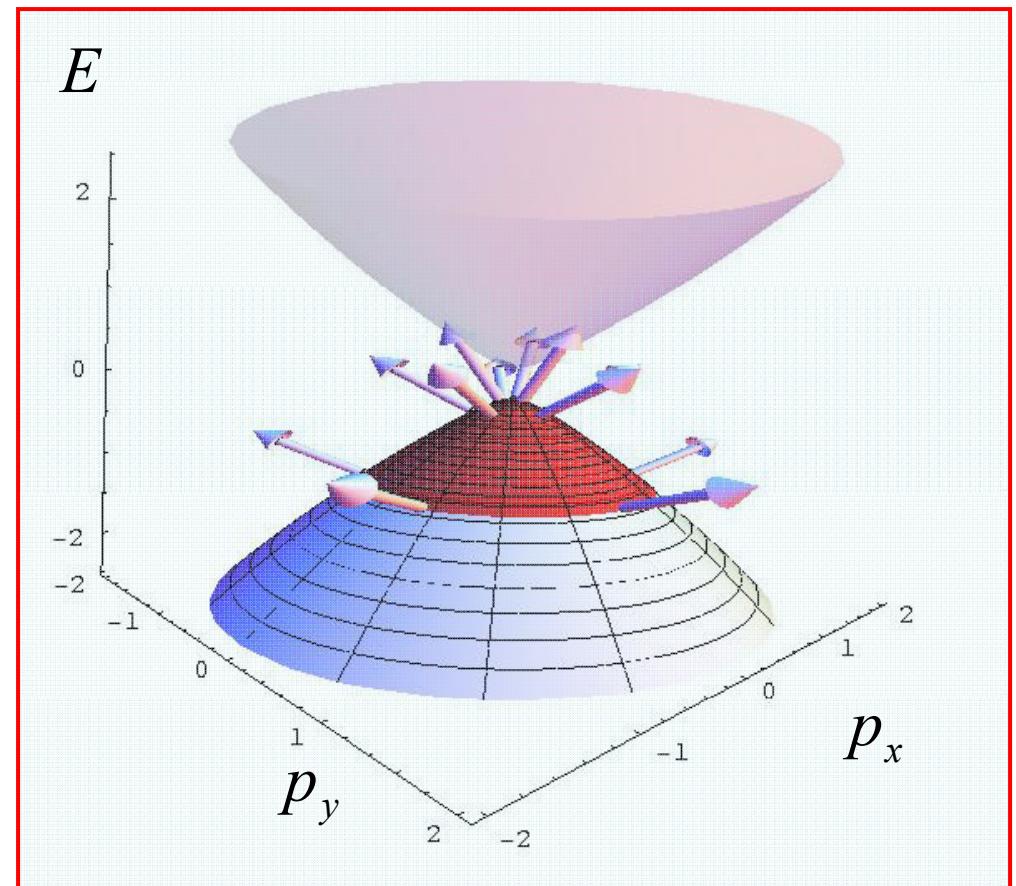
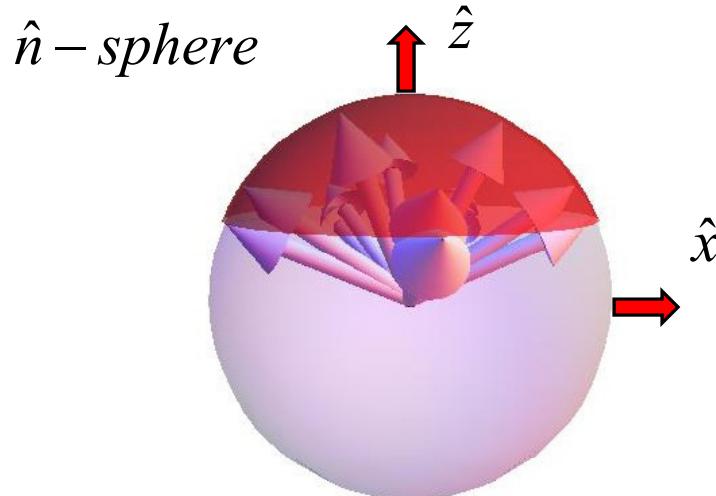
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$$\vec{d} = \left(p_x, \quad p_y, \quad m + b(p_x^2 + p_y^2) \right)$$

$$\hat{n} = \vec{d} / \|\vec{d}\|$$

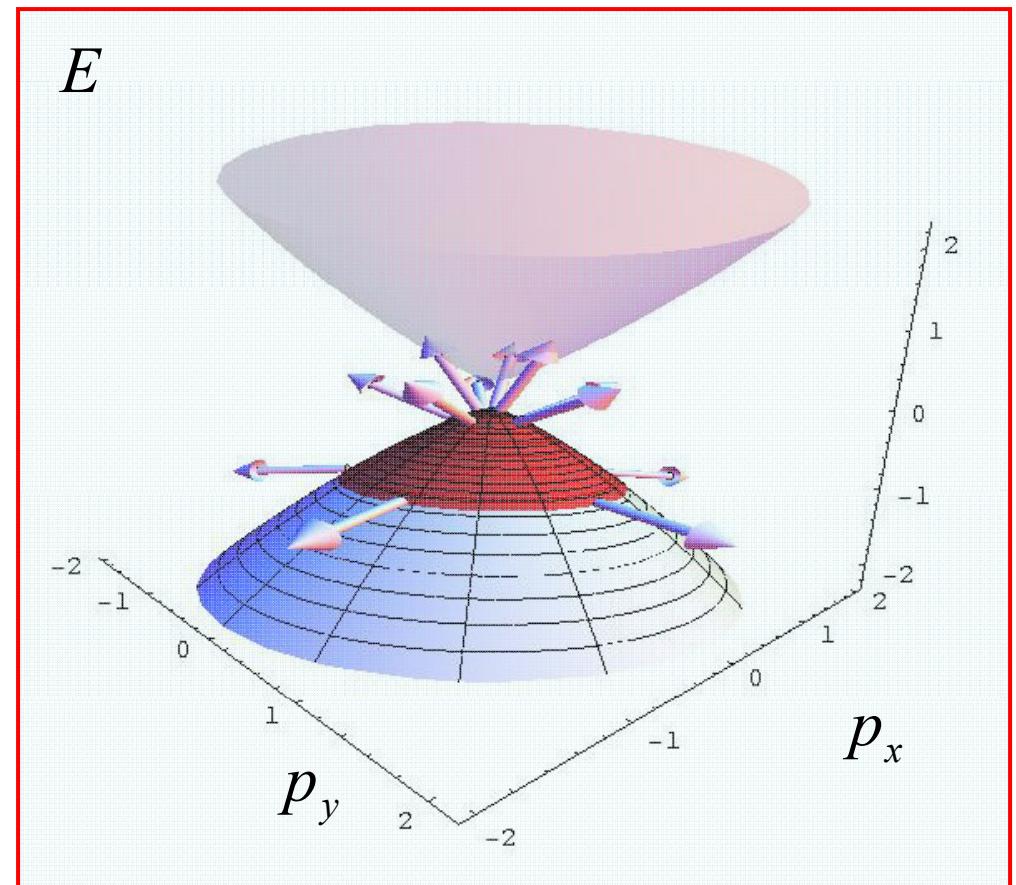
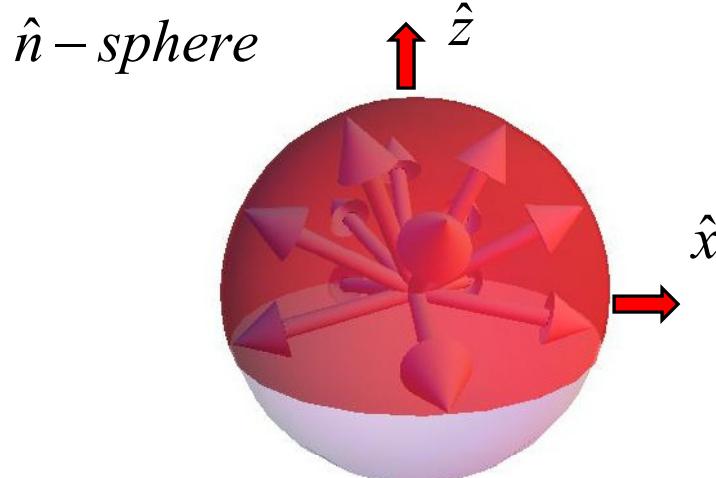
(map BZ **to** unit sphere)

- **TKNN invariant:**

$$\sigma_{xy} = \frac{e^2}{4\pi\hbar} \int_{p \in BZ} dp_x dp_y \hat{n} \cdot \left(\frac{\partial \hat{n}}{\partial p_x} \times \frac{\partial \hat{n}}{\partial p_y} \right)$$

$$m = 0.2, \quad b = -0.3 \quad (\text{topo})$$

Wrapping the unit sphere?



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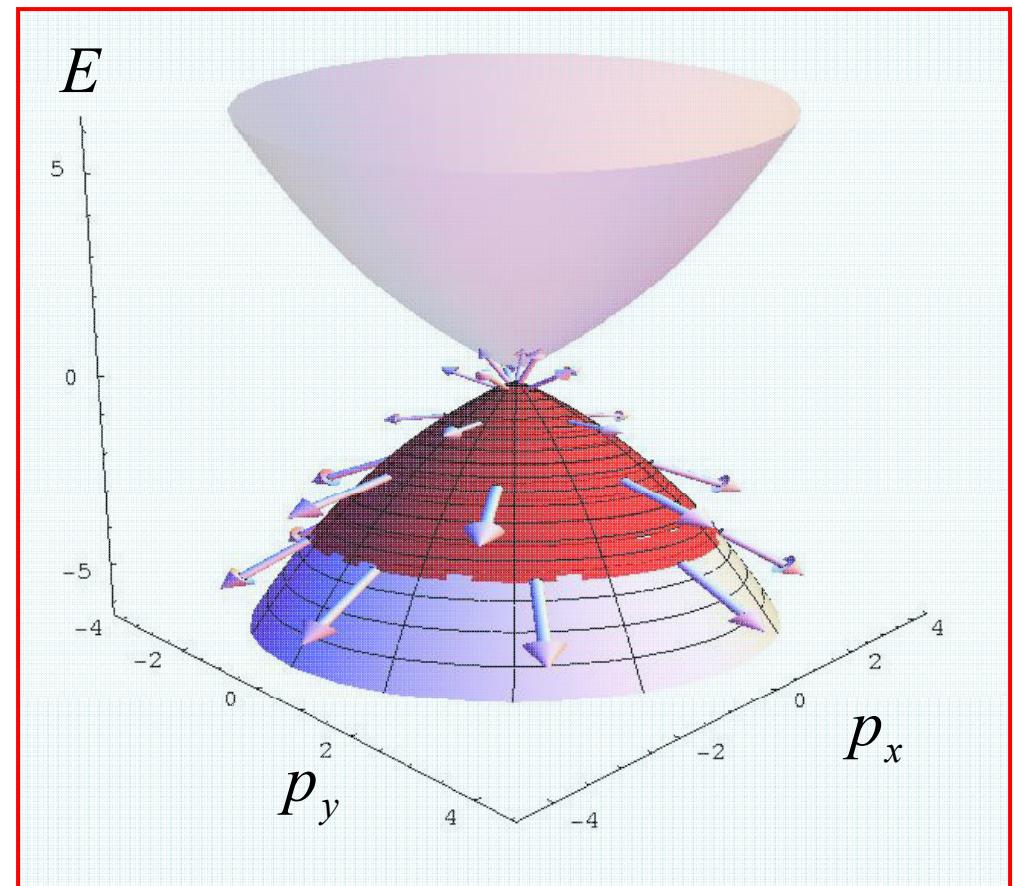
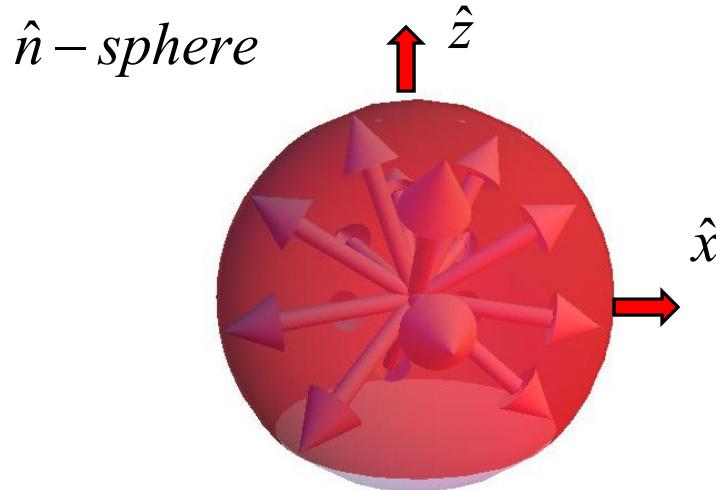
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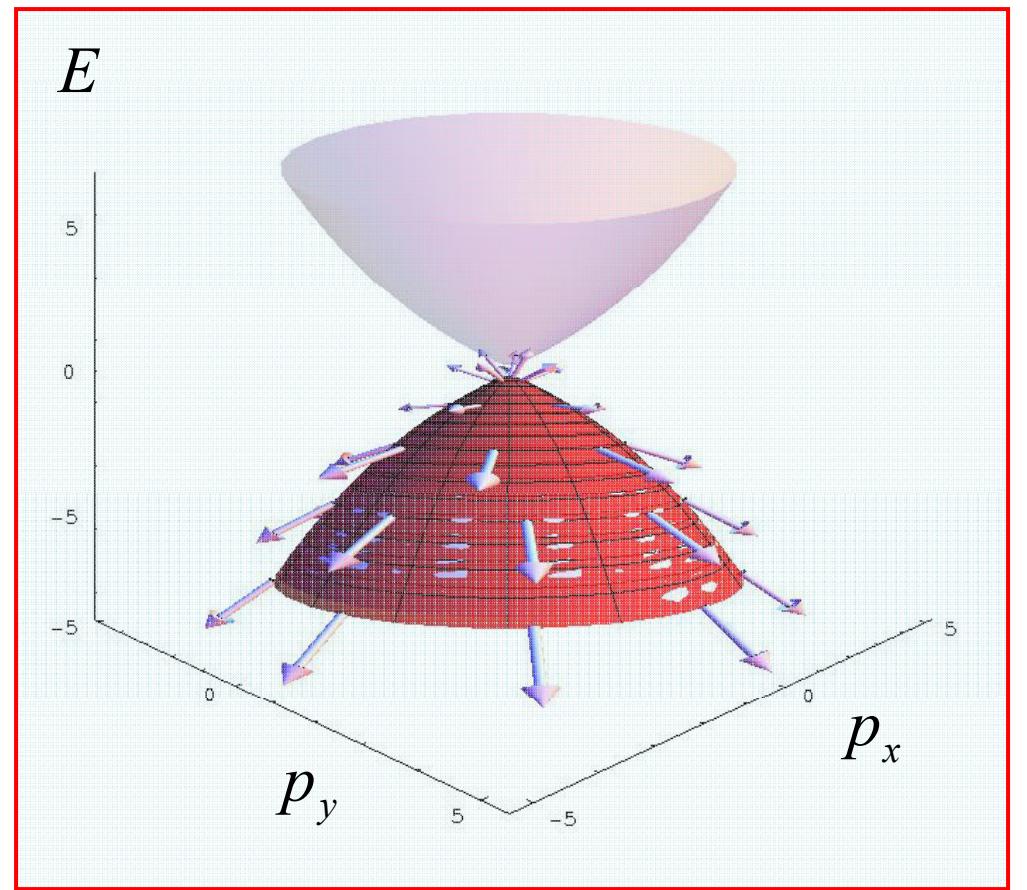
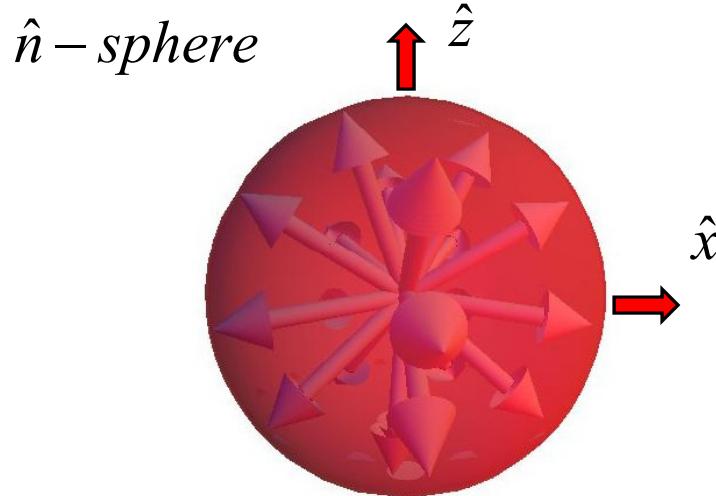
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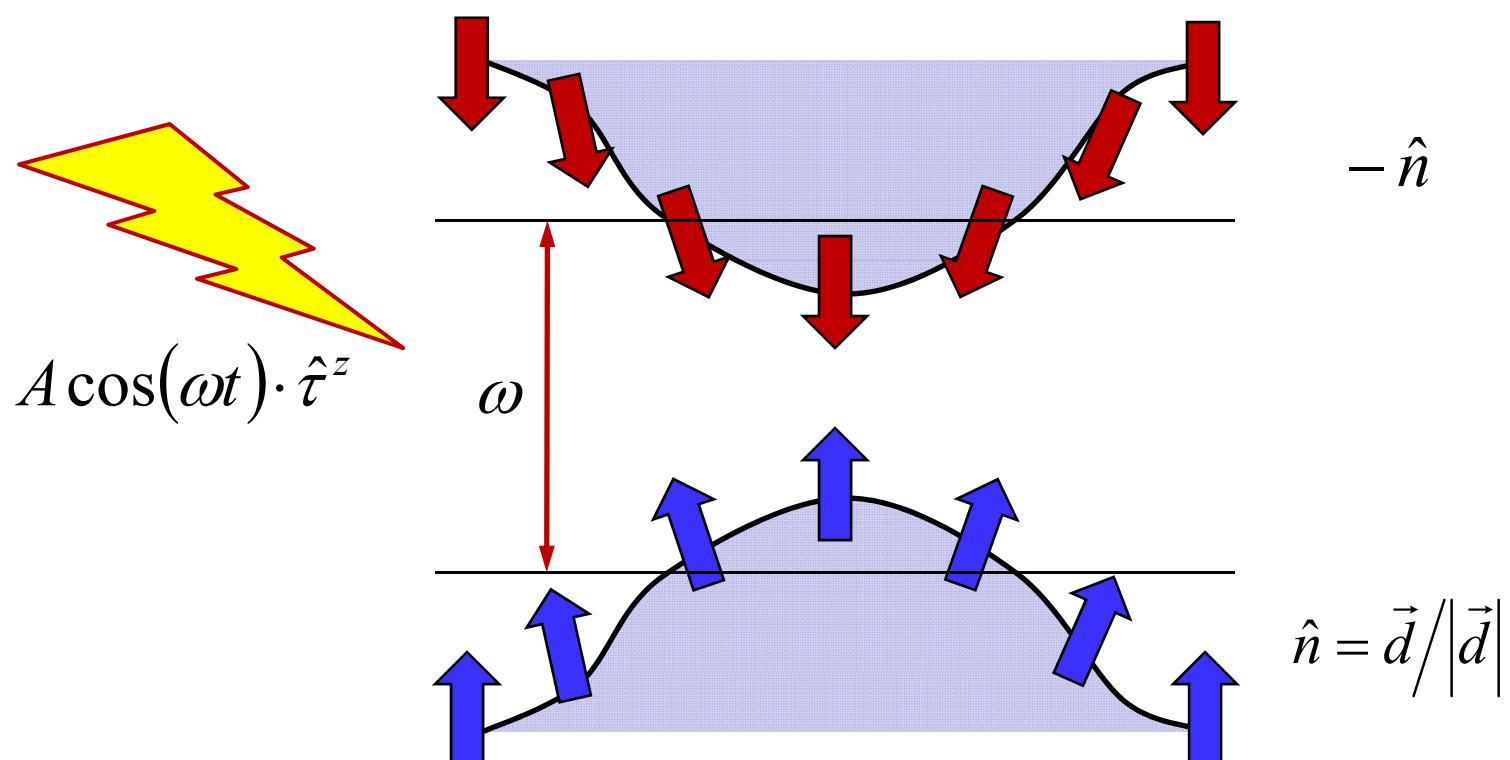
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Wrapping the unit sphere?



Can we induce a topological phase with radiation effects?

- Start with non-topological HgTe wells:



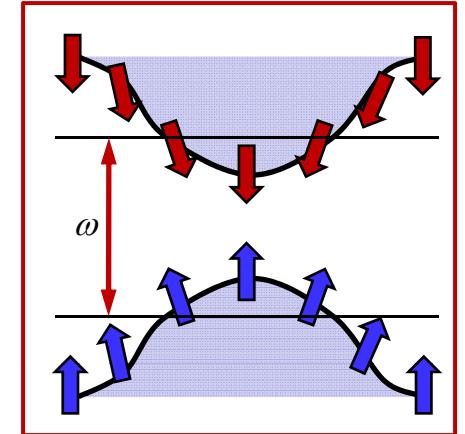
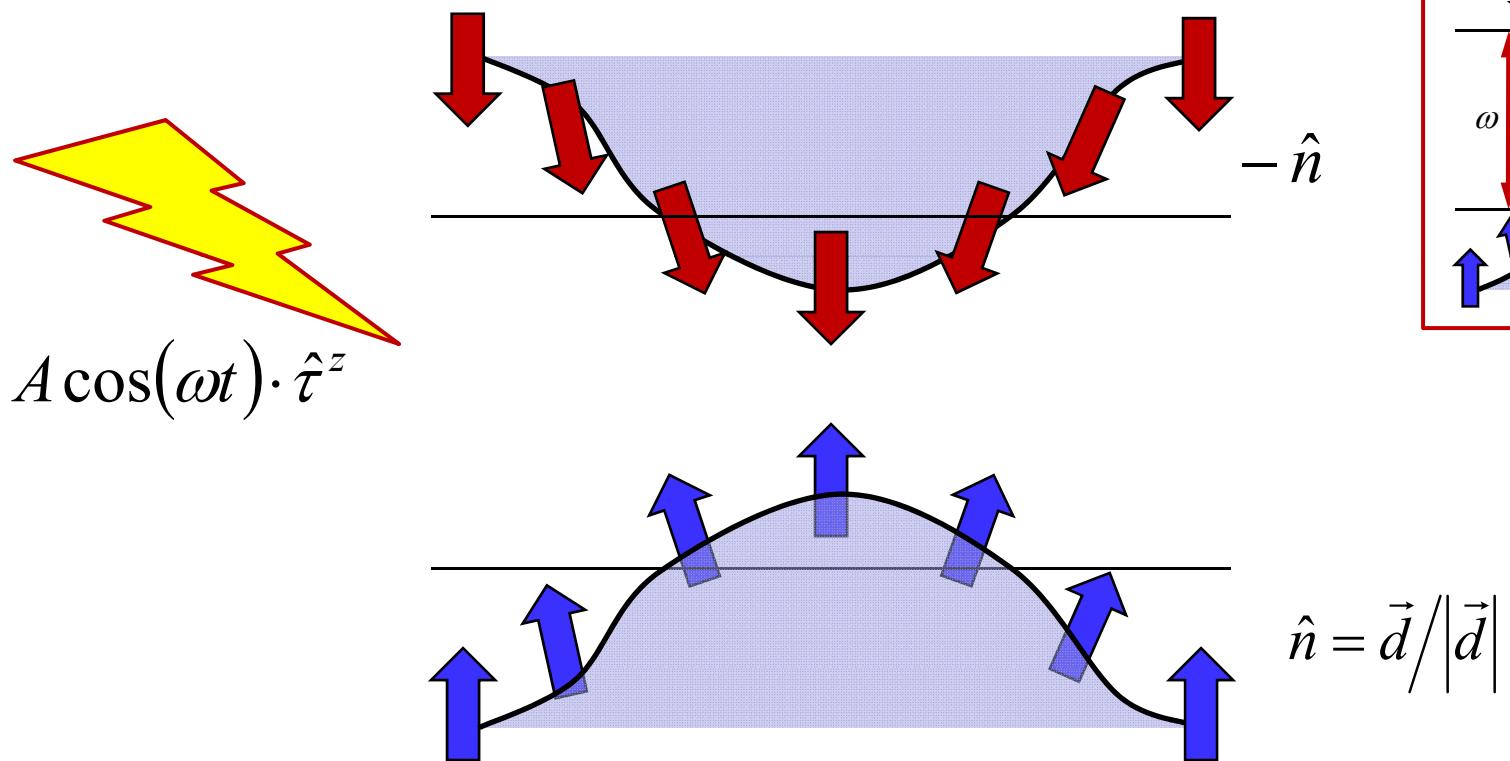
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$$\vec{d} = p_x \cdot \hat{x} + p_y \cdot \hat{y} + \left(m + b(p_x^2 + p_y^2) \right) \cdot \hat{z}$$

$$m, b > 0$$

Irradiation effects

- Start with non-topological HgTe wells:



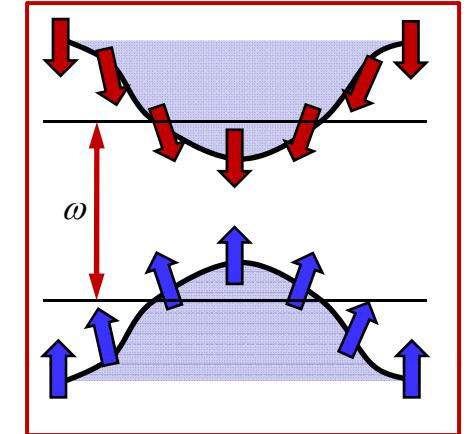
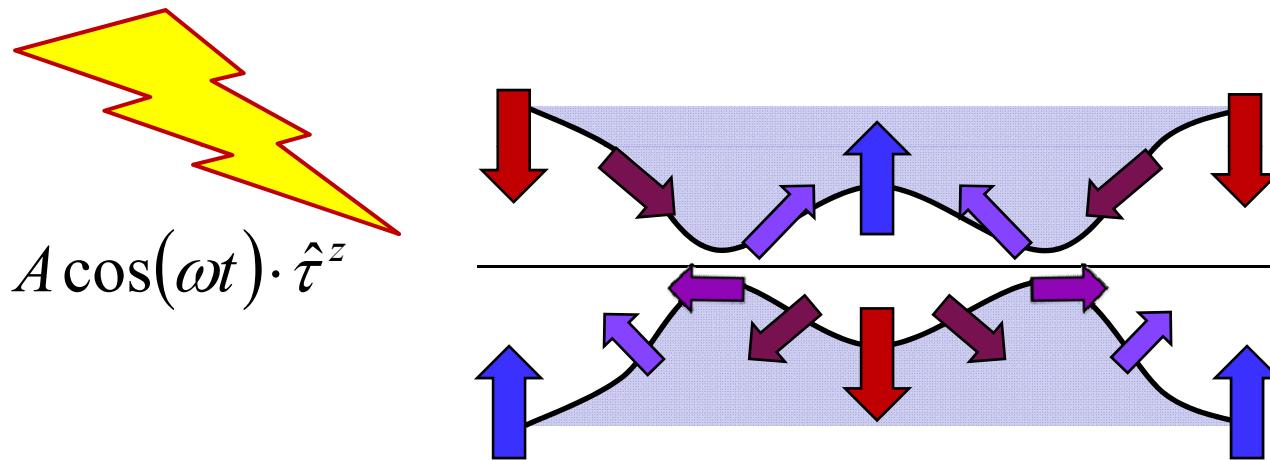
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$$S = \exp \left(-i \int d^T \xi^\rho H^+ \vec{d} \cdot \hat{\tau} \right) = s_1 + s_2 \hat{n}_S(\vec{p}) \cdot \hat{\tau}$$

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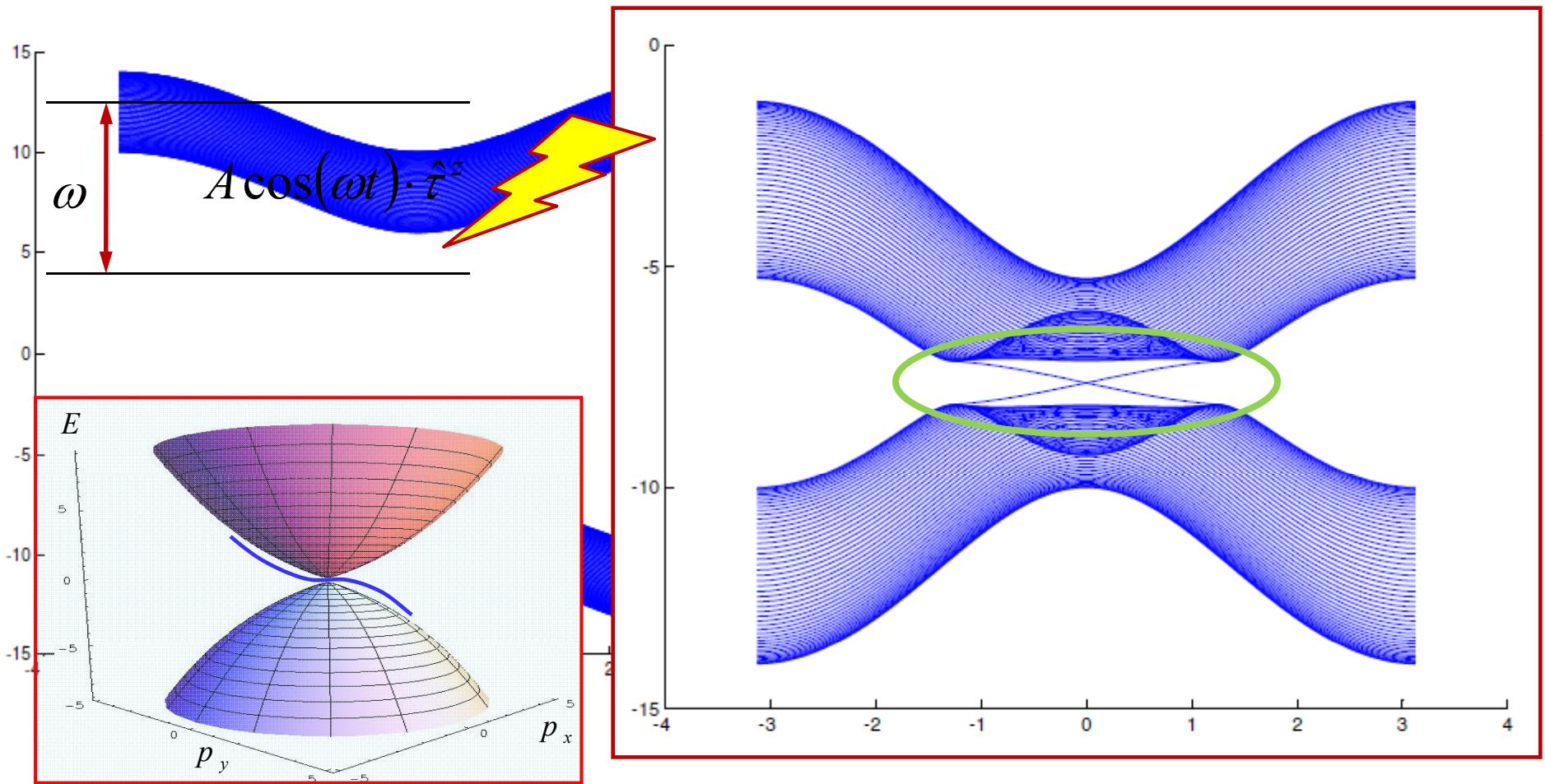


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Floquet topological phase

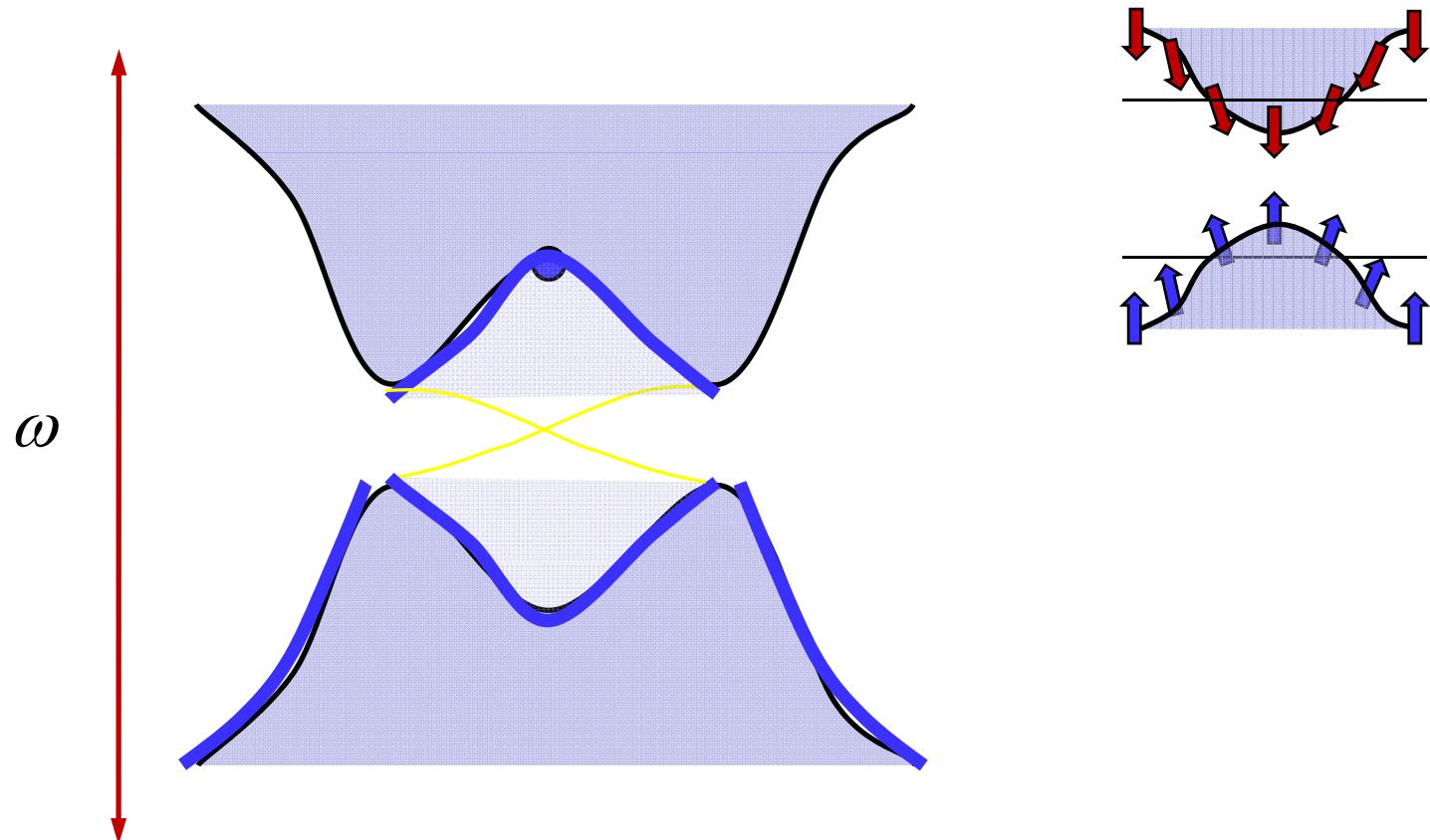
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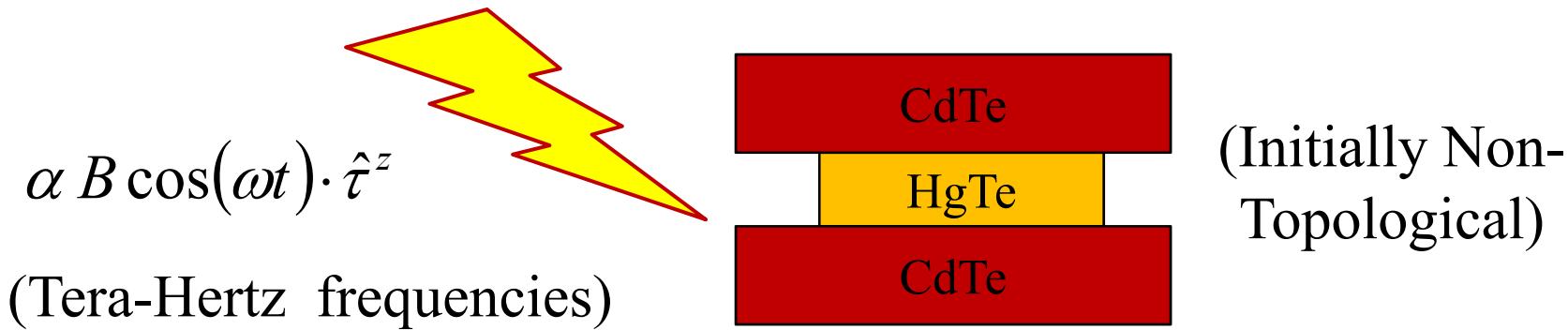


Equilibration in a Floquet topological phase? (Work in progress)

- Phonons can provide bath for low-energy, low momentum relaxation.



Experimental realization In (Hg,Cd)Te wells?



- Using Zeeman coupling:

$$B \sim 10mT$$

↗

$$\Delta \sim 0.1K$$

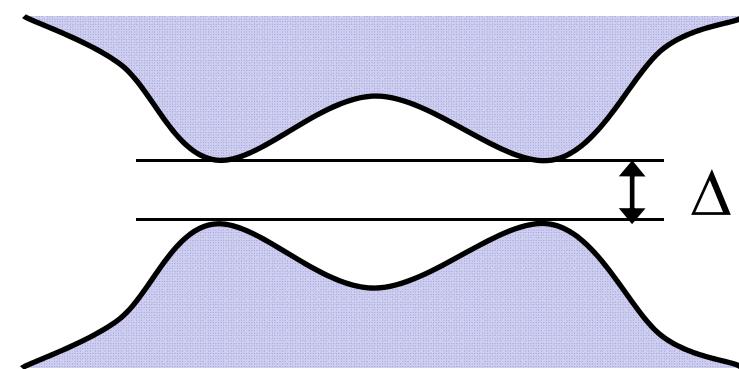
- Electric field:

Linear Stark effect enhancement

$$E \sim 1 V/m$$

↗

$$\Delta \sim 10K$$



$$V(r) = -Er = iE \frac{\partial}{\partial k}$$

Summary and conclusions

- Periodically modulated systems could be used to stabilize unique quantum states.
- Recipe for random hopping: *Modulated speckle potential.*
- Topological insulator on demand: *modulate the subband gap.*
- Open questions:
Relaxation into a steady state – will the unique properties shine out?