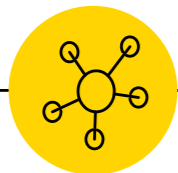


# Shadowed Triplet Pairing in Hund's metal with Spin-Orbit Coupling

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Canadian Institute for  
Advanced Research



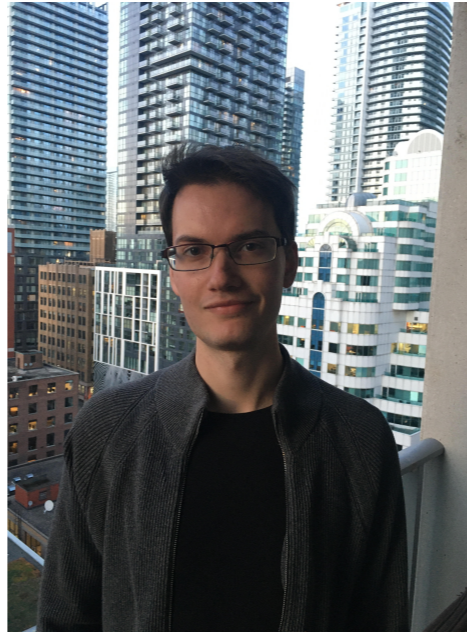
**NSERC**  
**CRSNG**



KITP Correlated Systems with Multicomponent Local Hilbert Spaces,  
2020 November 10



**Austin Lindquist**



**Jonathan Clepkens**



**Christoph Puetter**

## References

[arXiv:2009.08597](#)

Shadowed Triplet Pairings in Hund's Metals with Spin-Orbit Coupling

J. Clepkens, A. Lindquist, HYK

[arXiv:1912.02215](#)

Distinct reduction of Knight shift in superconducting state of  $\text{Sr}_2\text{RuO}_4$  under uniaxial strain, PRR 2, 320 (2020).

A. Lindquist, HYK

[arXiv:1101.4656](#)

Identifying spin-triplet pairing in spin-orbit coupled multi-band superconductors, EPL 98, 27010 (2012).

C. Puetter, HYK

**C. M. Puetter, PhD Thesis, Univ. of Toronto (2012).**

# Significant spin-orbit coupling (SOC)

## Importance of Hund's coupling

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	* 71 Lu	* 72 Hf	* 73 Ta	* 74 W	* 75 Re	* 76 Os	* 77 Ir	* 78 Pt	* 79 Au	* 80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	* * 103 Lr	* * 104 Rf	* * 105 Db	* * 106 Sg	* * 107 Bh	* * 108 Hs	* * 109 Mt	* * 110 Uun	* * 111 Uuu	* * 112 Uub	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo
		* 57 La	* 58 Ce	* 59 Pr	* 60 Nd	* 61 Pm	* 62 Sm	* 63 Eu	* 64 Gd	* 65 Tb	* 66 Dy	* 67 Ho	* 68 Er	* 69 Tm	* 70 Yb		
		* * 89 Ac	* * 90 Th	* * 91 Pa	* * 92 U	* * 93 Np	* * 94 Pu	* * 95 Am	* * 96 Cm	* * 97 Bk	* * 98 Cf	* * 99 Es	* * 100 Fm	* * 101 Md	* * 102 No		

**Sr<sub>2</sub>RuO<sub>4</sub>**

SOC < bandwidth;  $4d^4$

**SOC & Hund's**

**RuCl<sub>3</sub>**

SOC > bandwidth (honeycomb);  $4d^5$

Kitaev & Gamma interaction from SOC & Hund's

# Outline

- $\text{Sr}_2\text{RuO}_4$ ; spin-triplet vs. singlet?
- Even-parity spin-triplet pairing and SOC:  
Shadowed triplet
- Applying to  $\text{Sr}_2\text{RuO}_4$
- Proposed experiment

# Sr2RuO4

	Sr <sub>2</sub> RuO <sub>4</sub>	Sr <sub>3</sub> Ru <sub>2</sub> O <sub>7</sub>	SrRuO <sub>3</sub>
	superconductor ( $T_c = 1.5$ K)	paramagnetic metal	ferromagnetic metal ( $T_c = 165$ K)
$n$	1	2	$\infty$
space group	tetragonal I4/mmm	orthorhombic Bbcb (# 68)	orthorhombic
lattice parameters	$a = 3.862$ Å, $c = 12.729$ Å, $\theta = \phi = 0^\circ$	$a = b = 5.5006$ Å, $c = 20.725$ Å, $\theta = 6.8^\circ, \phi = 0^\circ$	$a = 5.56$ Å, $b = 5.53$ Å $c = 7.84$ Å $\theta \neq 0, \phi \neq 0$
$\rho_c/\rho_{ab}$	$\gtrsim 400$	$\sim 300$	$\sim 1.1$
$\gamma$	$38 \frac{\text{mJ}}{\text{Ru mol K}^2}$	$110 \frac{\text{mJ}}{\text{Ru mol K}^2}$	$29 \frac{\text{mJ}}{\text{Ru mol K}^2}$
$m^*/m_0$	$\sim 4$	—	$\sim 3 - 3.4$
$R_W$	1.7 – 1.8	$\gtrsim 10$	—
$\mu$	—	—	$1.1\mu_B/\text{Ru}$ (in-plane)

Rice and Sigrist, JPCM (1995): spin triplet with

$$\vec{d}(\mathbf{p}) = \hat{z}(p_x + ip_y) - \text{analog He3 A-phase}$$

# Spin Triplet

## A theoretical description of the new phases of liquid $^3\text{He}$

Anthony J. Leggett

Rev. Mod. Phys., Vol. 47, No. 2, April 1975

Cooper pair explicitly in the form

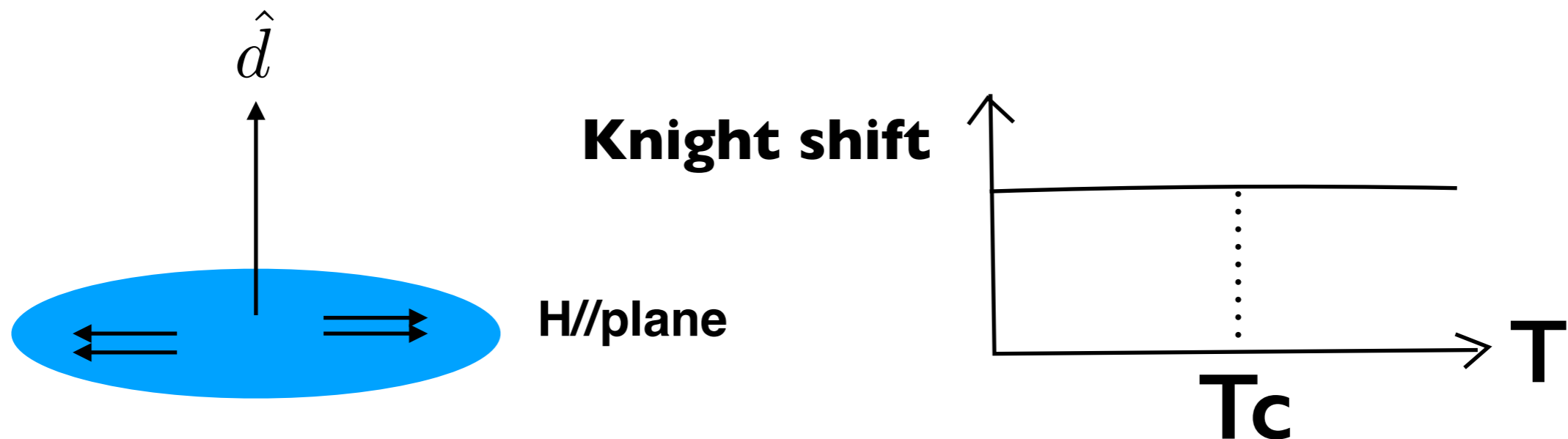
$$\begin{aligned} \Psi(\sigma_1\sigma_2:\mathbf{n}) = & \Psi_{\uparrow\uparrow}(\mathbf{n}) |\uparrow\uparrow\rangle + \Psi_{\uparrow\downarrow}(\mathbf{n}) |\uparrow\downarrow + \downarrow\uparrow\rangle \\ & + \Psi_{\downarrow\downarrow}(\mathbf{n}) |\downarrow\downarrow\rangle \end{aligned} \quad (7.38)$$

and then verify explicitly that for real  $\mathbf{d}(\mathbf{n})$  we have the *operator* relation

$$\mathbf{d}(\mathbf{n}) \cdot \hat{\mathbf{S}}\Psi(\sigma_1\sigma_2:\mathbf{n}) \equiv 0, \quad (7.39)$$

### Introducing d-vector

pairs are condensed in the eigenstates of  $S=1$  and  $S_z=0$

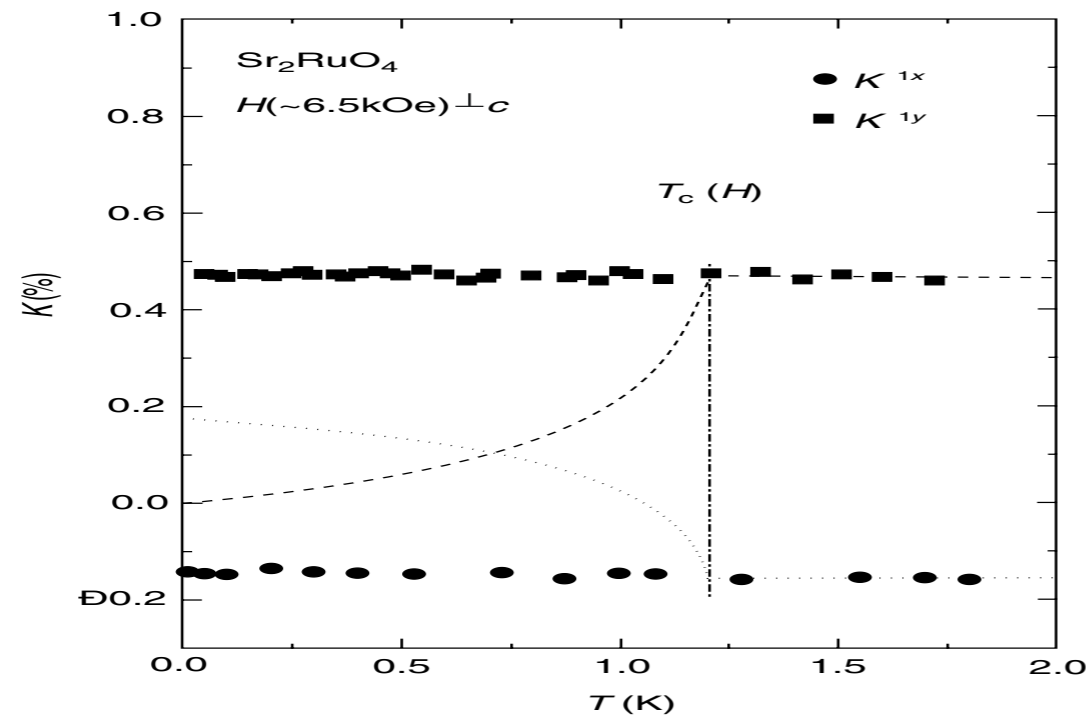


# Review

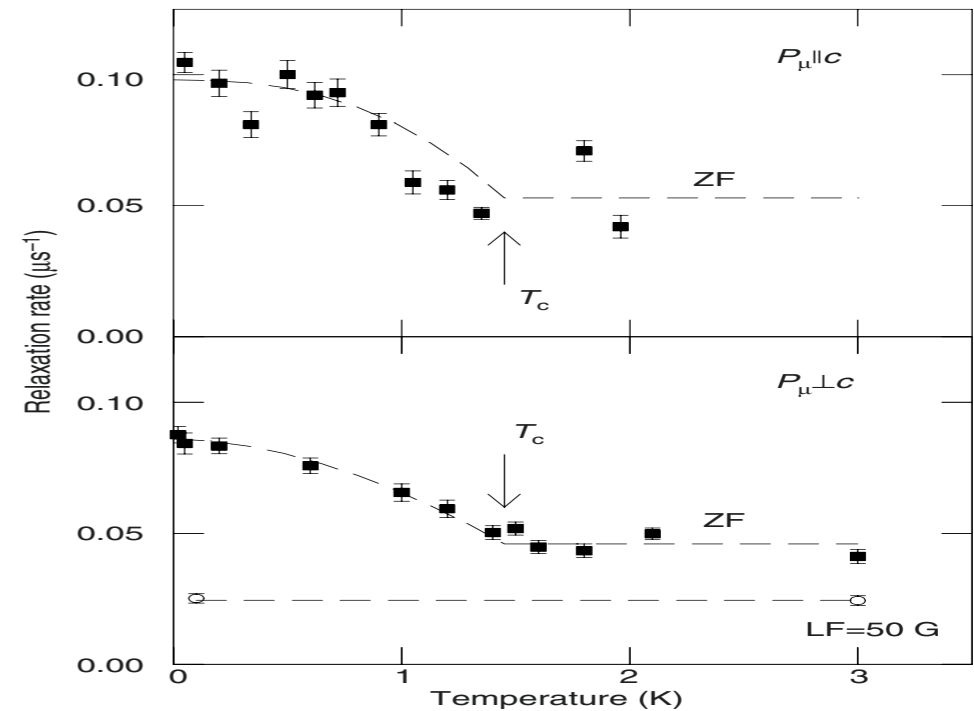
Discovery of SC, Y. Maeno et al , Nature (1994)

Rice and Sigrist, JPCM (1995): spin triplet with

$$\vec{d}(\mathbf{p}) = \hat{z}(p_x + ip_y) - \text{analog He3 A-phase}$$



K. Ishida et al, Nature (1998)



G. Luke et al, Nature (1998)

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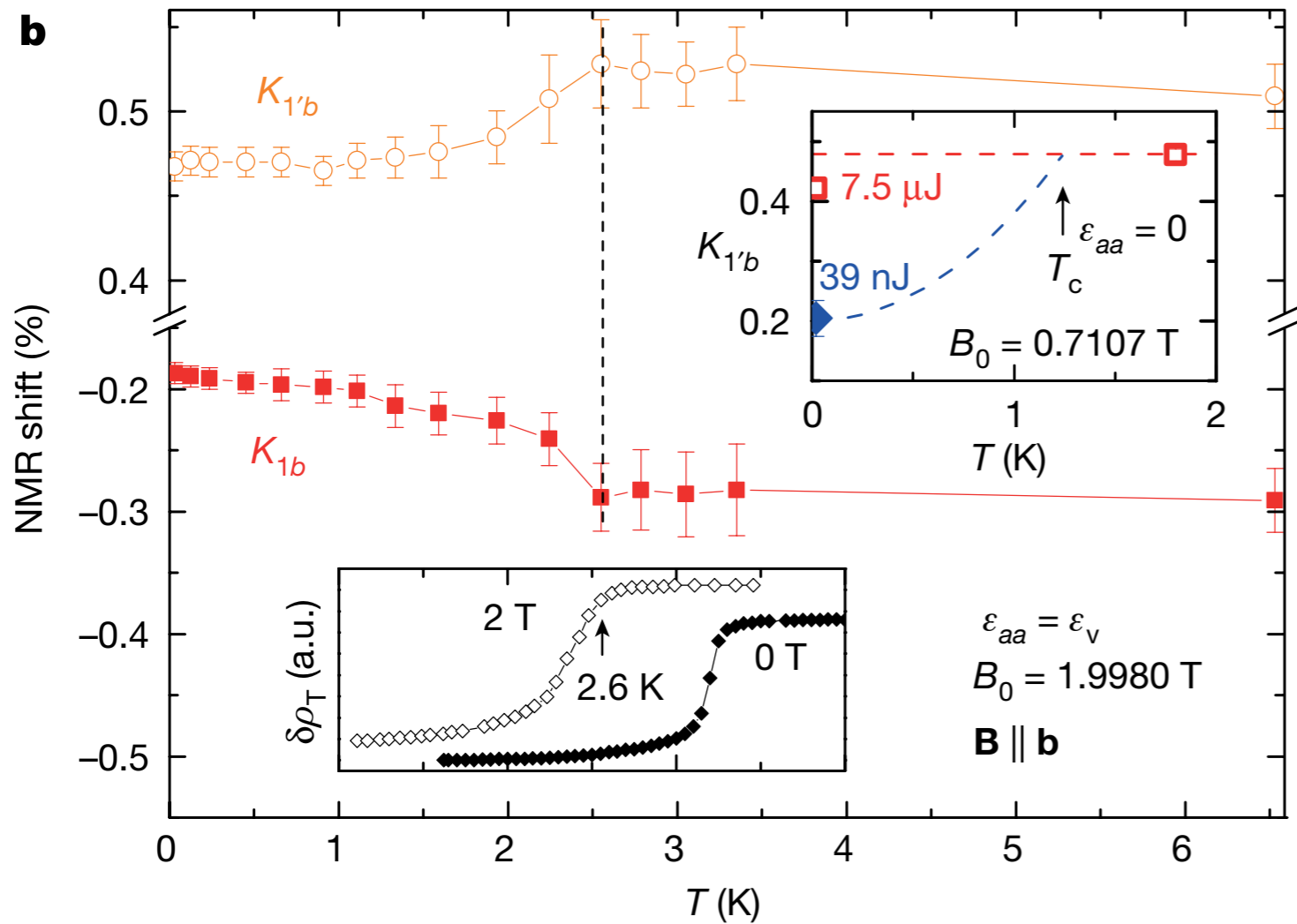
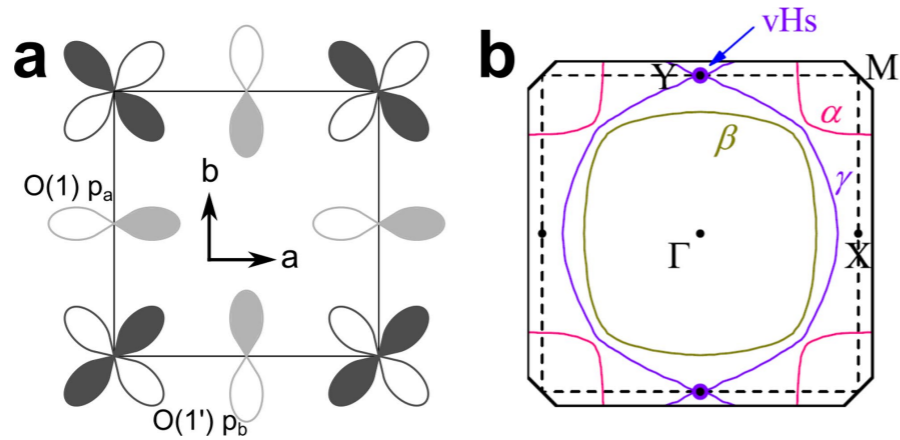
Summer Seminars for Correlated Electrons  
and Frustrated Magnets

[Zoom Link](#)

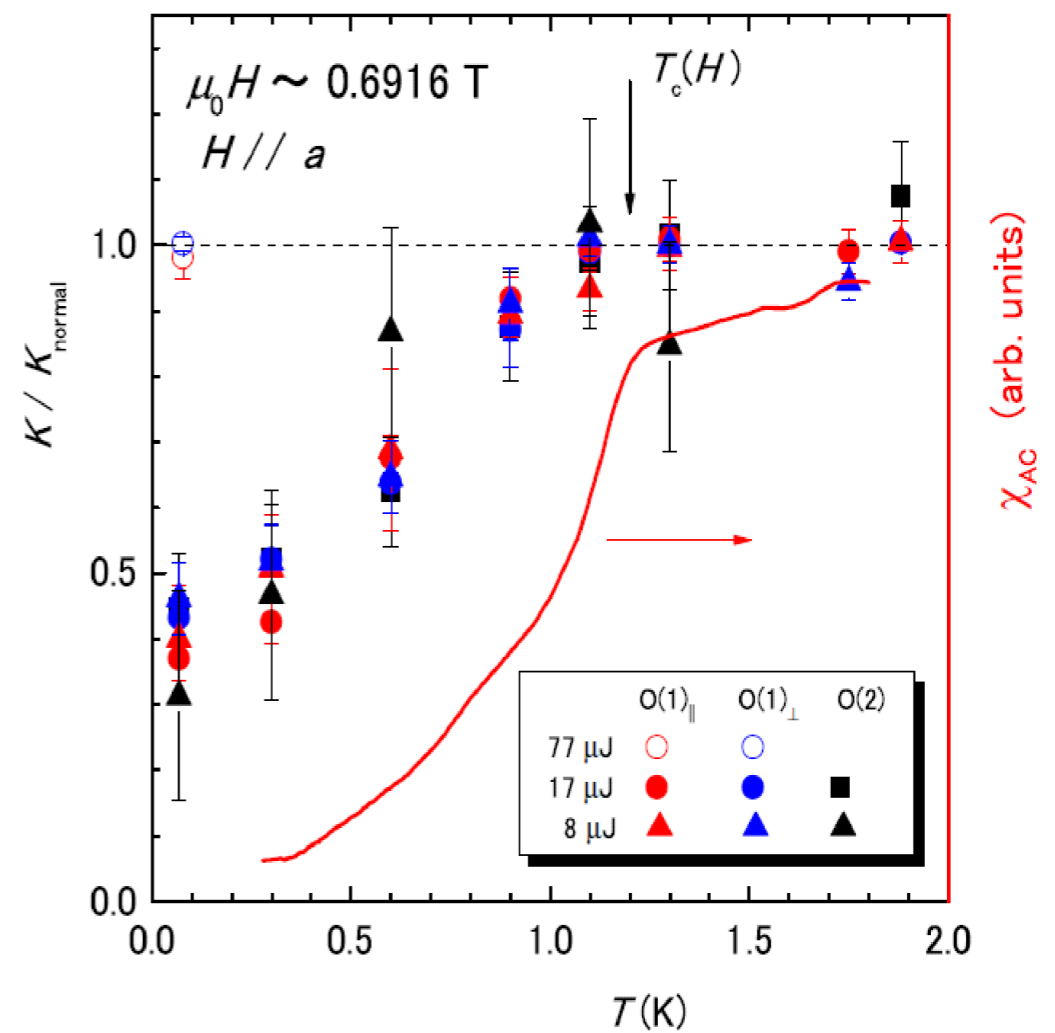
<https://sites.google.com/umn.edu/cm-weekly-seminar/home>

update on  $\text{Sr}_2\text{RuO}_4$ : A. Mackenzie

# NMR: Knight shift

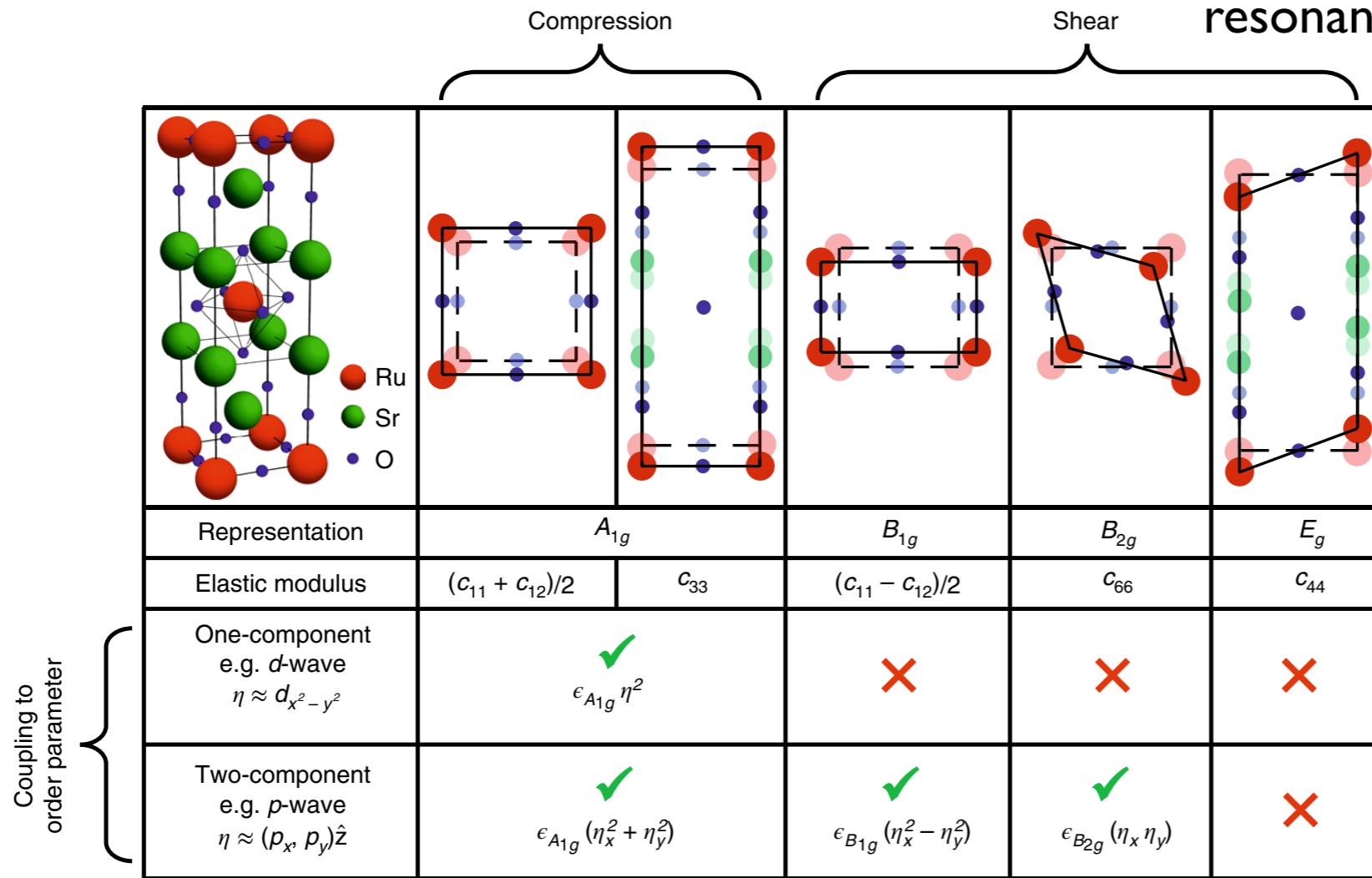


A. Pustogow et al, Nature (2019)

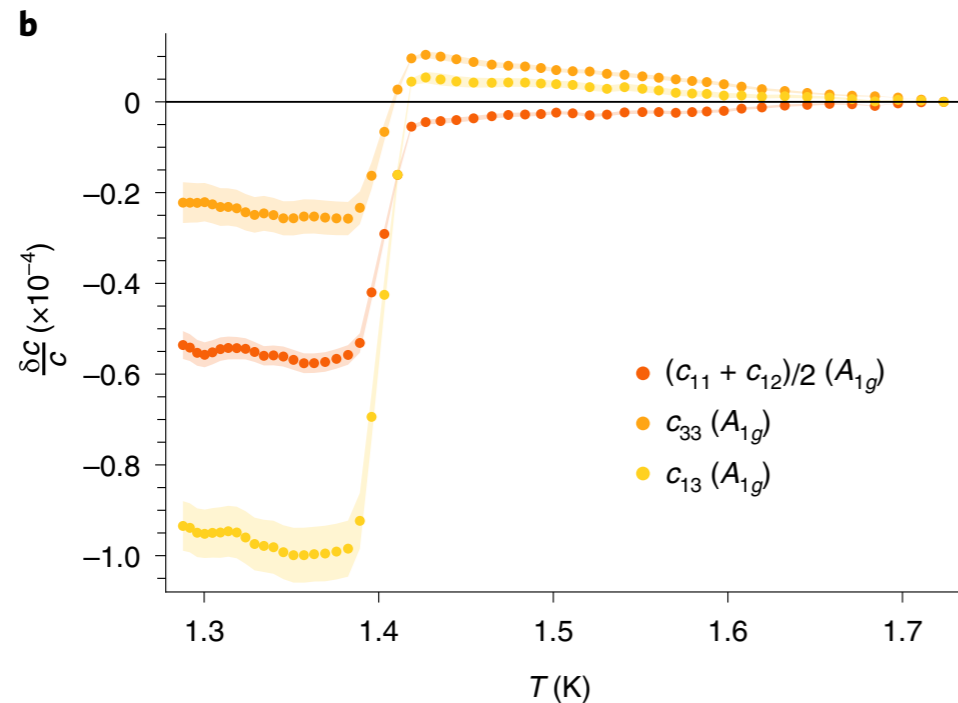


K. Ishida et al, JPSJ 89, 034712 (2020)

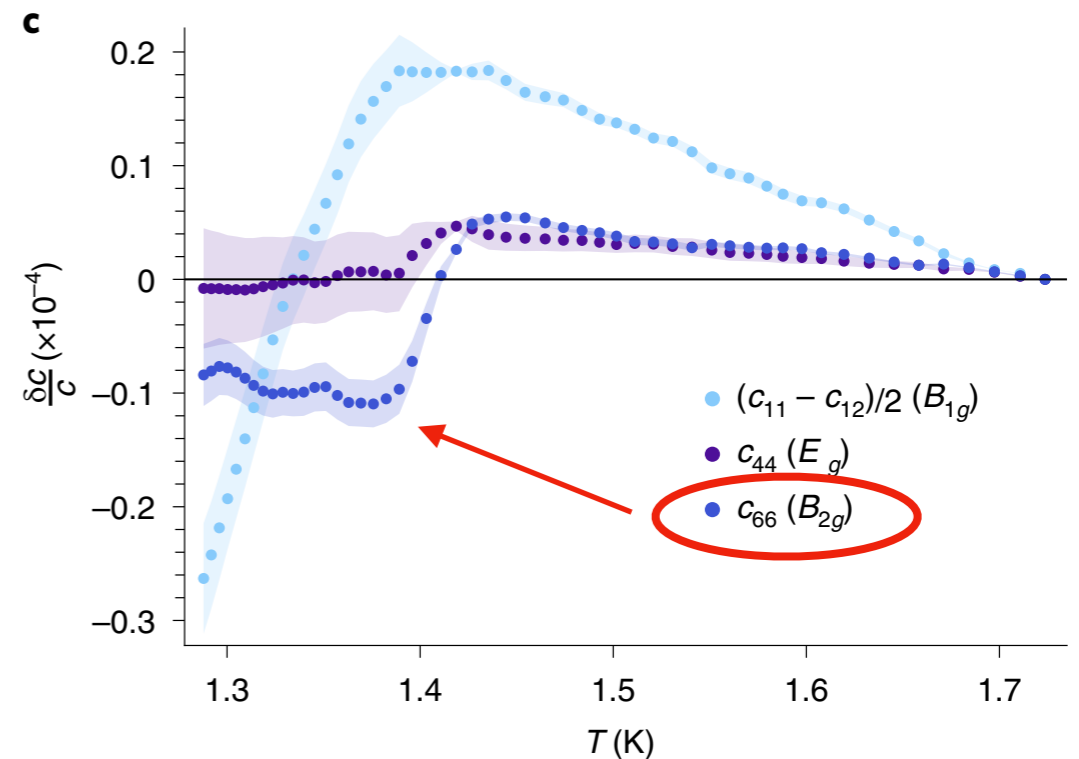




**two-component order parameter**

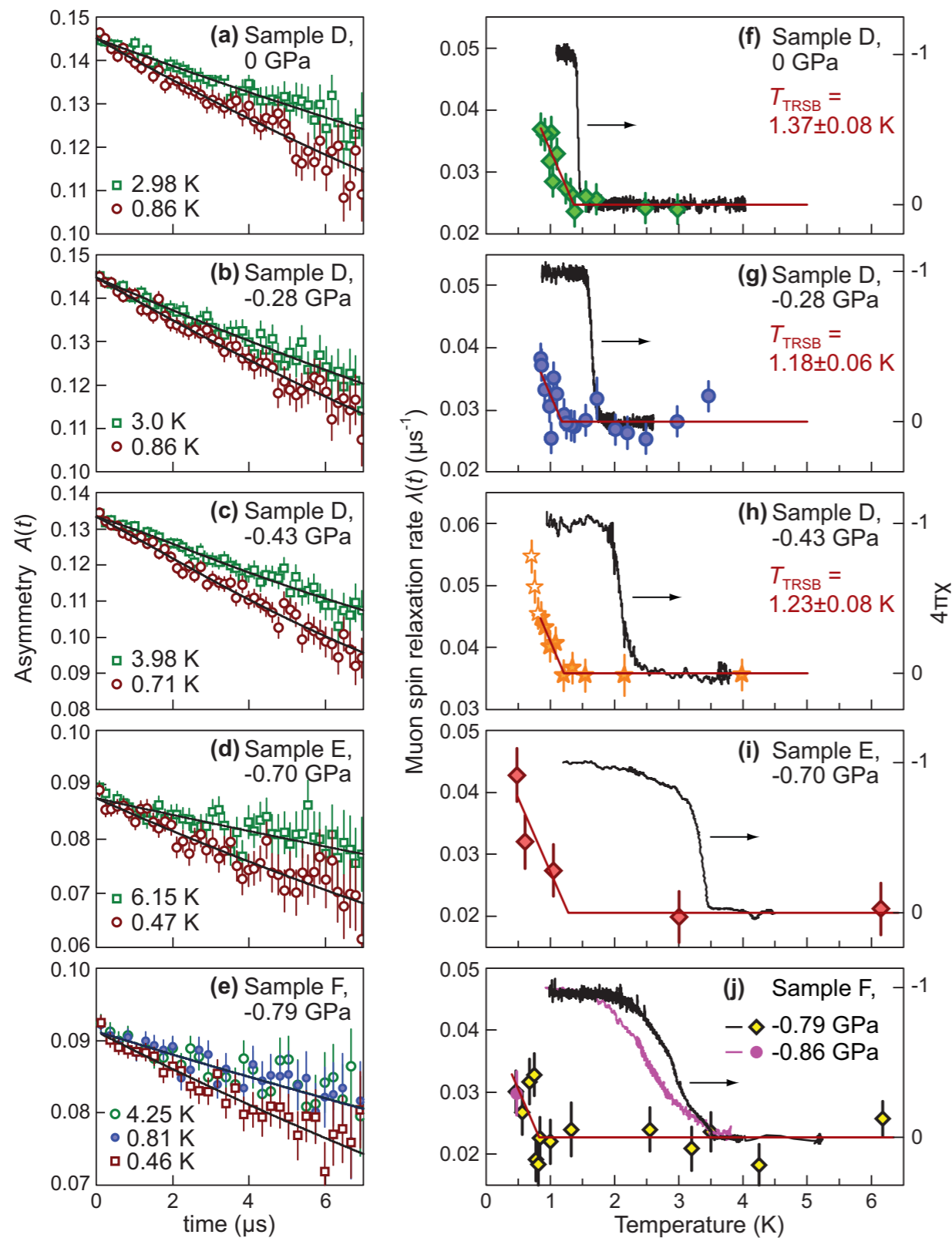


ultrasound: C. Lupien PhD thesis 2002

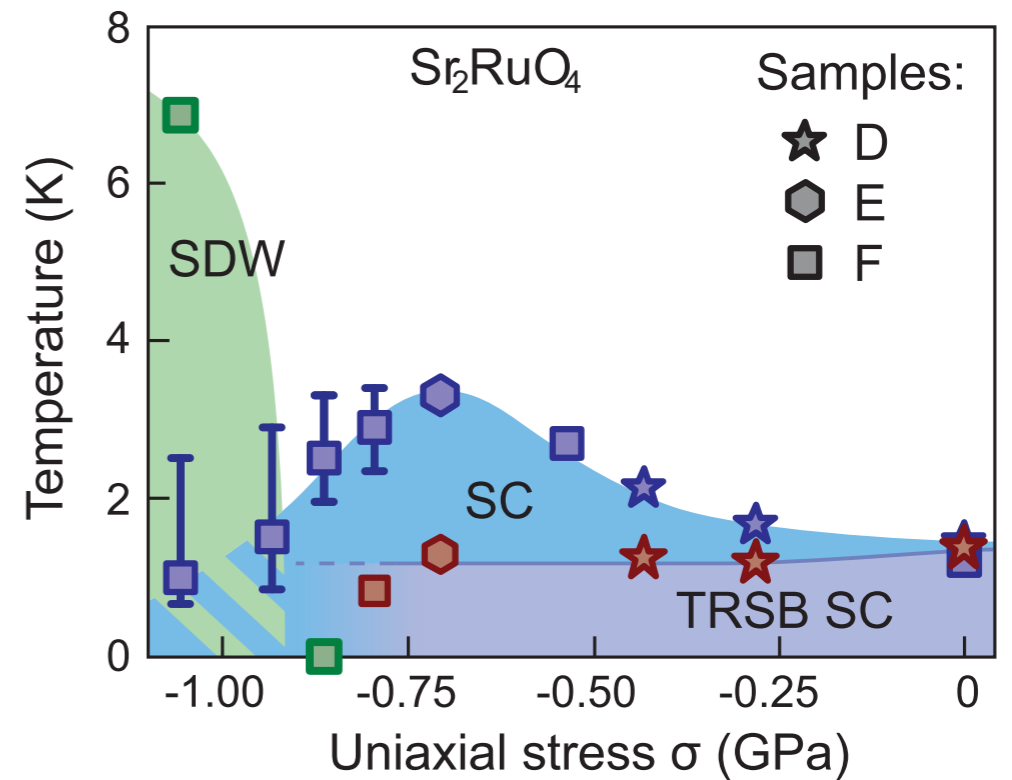


S. Gohsh et al, Nature Physics (2020)

# muSR under strain

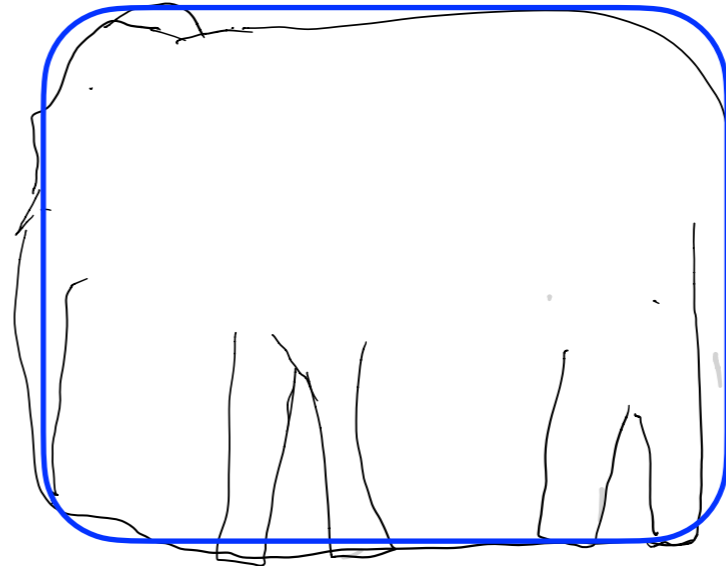
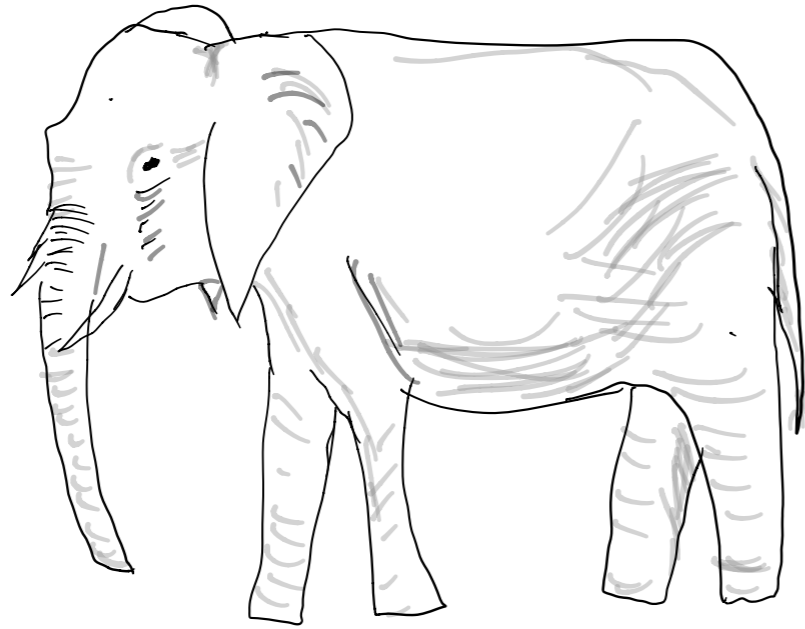


**time reversal  
 symmetry broken  
 SC**

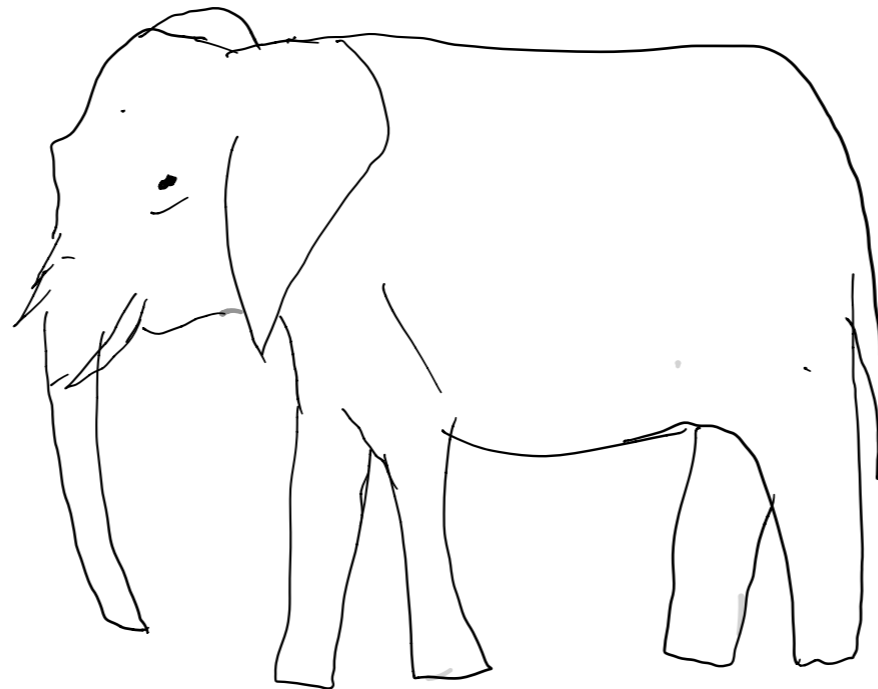


V. Grinenko et al, arXiv:2001.08152

# reality vs. beauty of simplicity

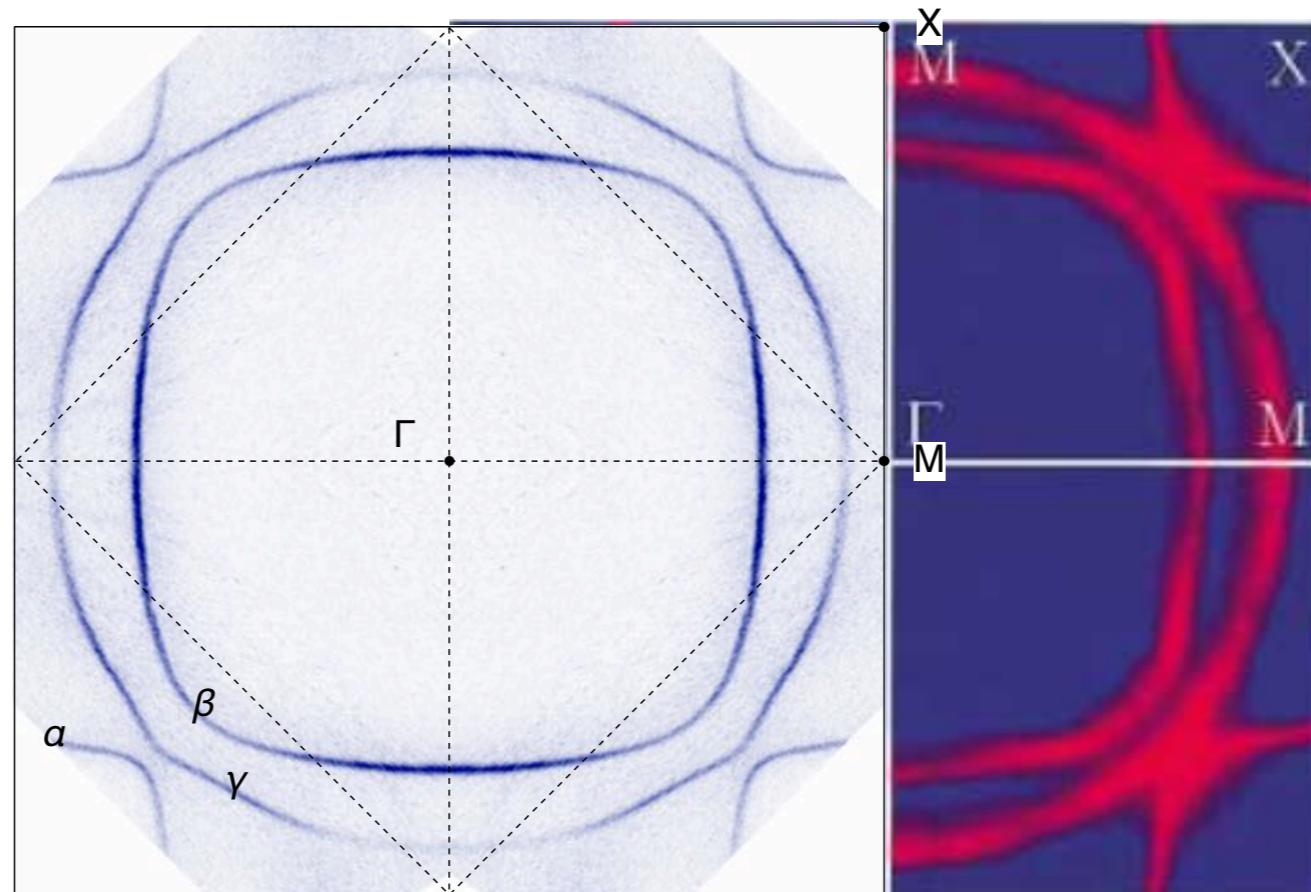


**topology = ball = trivial**



# Reality of multi-orbital systems

## Fermi Surface



A. Tamai et al, PRX (2019)

A. Mackenzie et al, PRL 76, 3786 (1996);  
C. Bergemann et al, PRL 84, 2662 (2000);  
A. Damascelli et al, PRL 85, 5194 (2000);

## Orbitals are mixed

## SOC: spin direction changes along k-space

# Antisymmetric wave-function condition

$$\vec{d}(\mathbf{k}) = -\vec{d}(-\mathbf{k})$$

Single band/orbitals  
odd-parity pairing;  
example p-wave,  $\sin(kx)$  or  $\sin(ky)$

---

Multi-orbital/bands  
even-parity triplet pairing is allowed;  
orbital (a,b) antisymmetric

$$\vec{d}(\mathbf{k}) = \vec{d}(-\mathbf{k})$$

**eg:**  $\langle c_{\mathbf{k},\sigma,a}^\dagger c_{-\mathbf{k},\sigma,b}^\dagger - c_{\mathbf{k},\sigma,b}^\dagger c_{-\mathbf{k},\sigma,a}^\dagger \rangle$

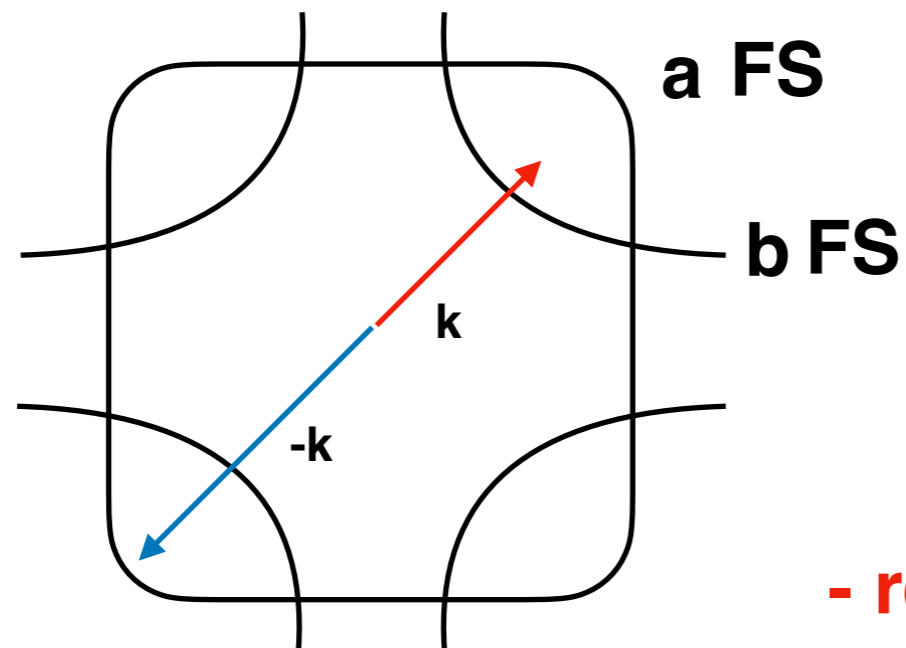
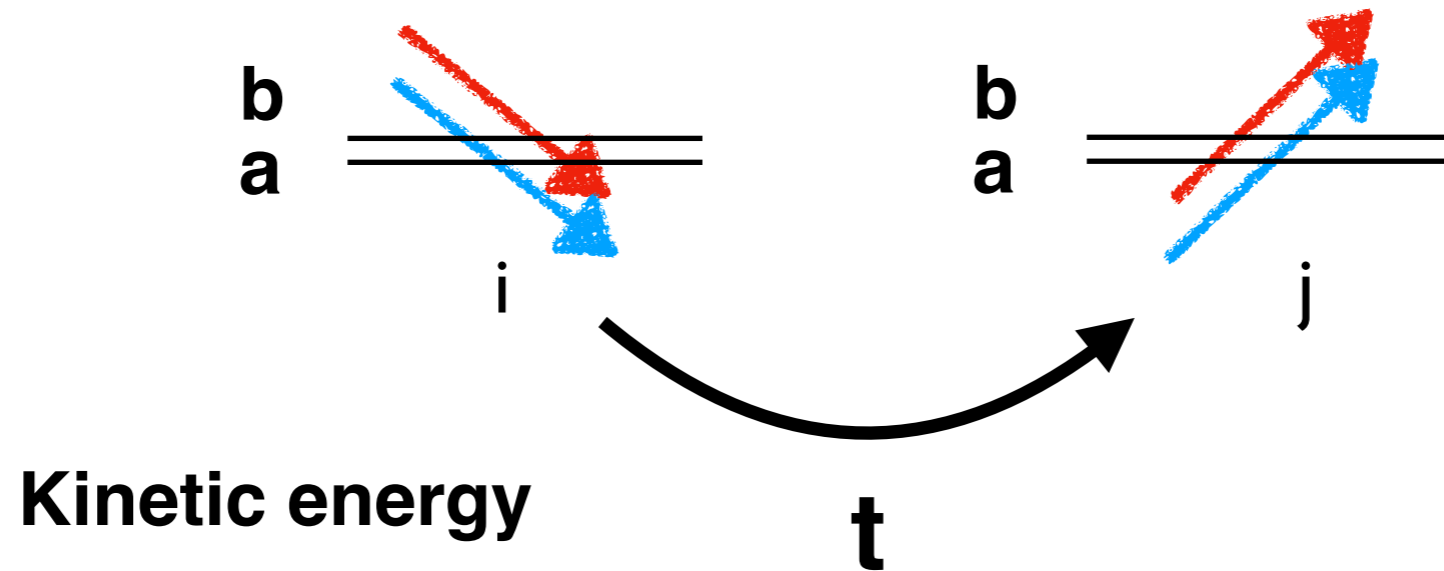
# Multi-orbital Interaction

$$\begin{aligned}
 H_{int} &= \frac{U}{2} \sum_{i,a,\sigma \neq \sigma'} n_{a,i\sigma} n_{a,i\sigma'} + \frac{U'}{2} \sum_{i,a \neq b,\sigma\sigma'} n_{a,i\sigma} n_{b,i\sigma'} \\
 &+ \frac{J_H}{2} \sum_{i,a \neq b,\sigma\sigma'} c_{a,i\sigma}^\dagger c_{b,i\sigma'}^\dagger c_{a,i\sigma'} c_{b,i\sigma} \\
 &+ \frac{J_H}{2} \sum_{i,a \neq b,\sigma \neq \sigma'} c_{a,i\sigma}^\dagger c_{a,i\sigma'}^\dagger c_{b,i\sigma'} c_{b,i\sigma},
 \end{aligned}
 \longrightarrow
 \begin{aligned}
 H_{int} &= \frac{4U}{N} \sum_{a,\mathbf{k}\mathbf{k}'} \hat{\Delta}_{a,\mathbf{k}}^{s\dagger} \hat{\Delta}_{a,\mathbf{k}'}^s \\
 &+ \frac{2(U' - J_H)}{N} \sum_{\{a \neq b\},\mathbf{k}\mathbf{k}'} \hat{\mathbf{d}}_{a/b,\mathbf{k}}^\dagger \cdot \hat{\mathbf{d}}_{a/b,\mathbf{k}'} \\
 &+ \frac{4J_H}{N} \sum_{a \neq b,\mathbf{k}\mathbf{k}'} \hat{\Delta}_{a,\mathbf{k}}^{s\dagger} \hat{\Delta}_{b,\mathbf{k}'}^s \\
 &+ \frac{2(U' + J_H)}{N} \sum_{a \neq b,\mathbf{k}\mathbf{k}'} \hat{\Delta}_{a/b,\mathbf{k}}^{s\dagger} \hat{\Delta}_{a/b,\mathbf{k}'}^s,
 \end{aligned}$$

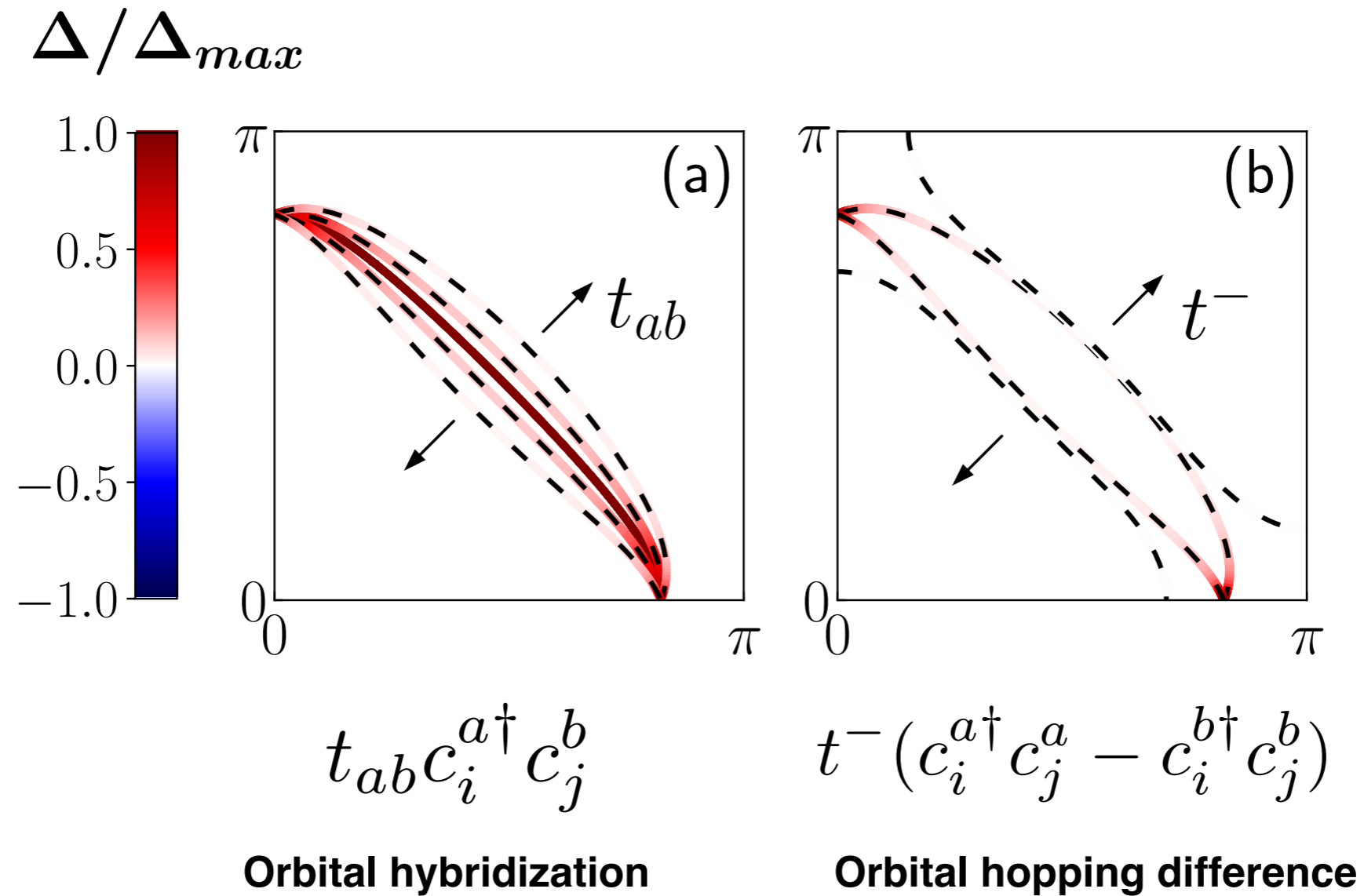
spin triplet

$$\begin{aligned}
 \hat{\mathbf{d}}_{a/b,\mathbf{k}} &= \frac{1}{4} \sum_{\sigma\sigma'} [i\sigma^y \boldsymbol{\sigma}]_{\sigma\sigma'} (c_{a,\mathbf{k}\sigma} c_{b,-\mathbf{k}\sigma'} - c_{b,\mathbf{k}\sigma} c_{a,-\mathbf{k}\sigma'}) \\
 \hat{\Delta}_{a/b,\mathbf{k}}^s &= \frac{1}{4} \sum_{\sigma\sigma'} [i\sigma^y]_{\sigma\sigma'} (c_{a,\mathbf{k}\sigma} c_{b,-\mathbf{k}\sigma'} + c_{b,\mathbf{k}\sigma} c_{a,-\mathbf{k}\sigma'}) \\
 \hat{\Delta}_{a,\mathbf{k}}^s &= \frac{1}{4} \sum_{\sigma\sigma'} [i\sigma^y]_{\sigma\sigma'} c_{a,\mathbf{k}\sigma} c_{a,-\mathbf{k}\sigma'},
 \end{aligned}$$

**Pairing is local:**  $U' < J_H$



**However, it is fragile  
- requires degeneracy of bands**



Hund's rule coupling as the microscopic origin of the spin-triple pairing in a correlated and degenerate band system, A. Klejnberg, J. Spalek, JPCMP 11, 6553 (1999);

X. Dai et al, PRL (2008) on Pnictides

**SOC!**

C. Puetter, HYK, EPL 98, 27010 (2012); O.Vafek, A.V. Chubukov, PRL (2017); .....



# Effects of SOC: 2-orbital model

$$H = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger (H_0(\mathbf{k}) + H_{\text{SOC}}^z(\mathbf{k}) + H_{\text{pair}}(\mathbf{k})) \Psi_{\mathbf{k}}$$

$$\Psi_{\mathbf{k}}^\dagger = (\psi_{\mathbf{k}}^\dagger, T\psi_{\mathbf{k}}^T T^{-1}) \quad \psi_{\mathbf{k}}^\dagger = (c_{\mathbf{k}\uparrow}^{a\dagger}, c_{\mathbf{k}\uparrow}^{b\dagger}, c_{\mathbf{k}\downarrow}^{a\dagger}, c_{\mathbf{k}\downarrow}^{b\dagger})$$

$$H_0(\mathbf{k}) = \rho_3 \left( \frac{\xi_{\mathbf{k}}^+}{2} \sigma_0 \tau_0 + \frac{\xi_{\mathbf{k}}^-}{2} \sigma_0 \tau_3 + t_{\mathbf{k}} \sigma_0 \tau_1 \right)$$

$$\xi_{\mathbf{k}}^\pm = \xi_{\mathbf{k}}^a \pm \xi_{\mathbf{k}}^b, \quad t_{\mathbf{k}} c_{\mathbf{k}}^{a\dagger} c_{\mathbf{k}}^b : \text{orbital hybridization}$$

$$H_{\text{SOC}}^z(\mathbf{k}) = -\lambda_{\mathbf{k}} \rho_3 \sigma_3 \tau_2 \quad \text{momentum dep. SOC}$$

$$H_{\text{pair}} = -d_{a/b}^z \rho_2 \sigma_3 \tau_2.$$

$$d_{a/b}^z \equiv (U' - J_H) \frac{1}{N} \sum_{\mathbf{k}} \langle \hat{d}_{a/b, \mathbf{k}}^z \rangle. \quad \text{spin-triplet}$$

## change to band basis

$$\begin{pmatrix} c_{\mathbf{k}\sigma}^a \\ c_{\mathbf{k}\sigma}^b \end{pmatrix} = \begin{pmatrix} \frac{\eta_\sigma + 1}{2} f_{\mathbf{k}} - \frac{\eta_\sigma - 1}{2} f_{\mathbf{k}}^* & -g_{\mathbf{k}} \\ g_{\mathbf{k}} & \frac{\eta_\sigma + 1}{2} f_{\mathbf{k}}^* - \frac{\eta_\sigma - 1}{2} f_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} \alpha_{\mathbf{k}, s} \\ \beta_{\mathbf{k}, s} \end{pmatrix}$$

# In the band basis

two bands  $\alpha, \beta$

**pseudo-spin singlet (intra-band)**

$$\begin{aligned} \tilde{H}_{\text{pair}}(\mathbf{k}) = & i\Delta^s(\mathbf{k}) [(\alpha_{\mathbf{k},+}^\dagger \alpha_{-\mathbf{k},-}^\dagger - \alpha_{\mathbf{k},-}^\dagger \alpha_{-\mathbf{k},+}^\dagger) - (\beta_{\mathbf{k},+}^\dagger \beta_{-\mathbf{k},-}^\dagger - \beta_{\mathbf{k},-}^\dagger \beta_{-\mathbf{k},+}^\dagger)] \\ & + i\Delta_{\alpha\beta}^s(\mathbf{k}) [(\alpha_{\mathbf{k},+}^\dagger \beta_{-\mathbf{k},-}^\dagger - \alpha_{\mathbf{k},-}^\dagger \beta_{-\mathbf{k},+}^\dagger) + (\beta_{\mathbf{k},+}^\dagger \alpha_{-\mathbf{k},-}^\dagger - \beta_{\mathbf{k},-}^\dagger \alpha_{-\mathbf{k},+}^\dagger)] \\ & + d_{\alpha\beta}^z(\mathbf{k}) [(\alpha_{\mathbf{k},+}^\dagger \beta_{-\mathbf{k},-}^\dagger + \alpha_{\mathbf{k},-}^\dagger \beta_{-\mathbf{k},+}^\dagger) - (\beta_{\mathbf{k},+}^\dagger \alpha_{-\mathbf{k},-}^\dagger + \beta_{\mathbf{k},-}^\dagger \alpha_{-\mathbf{k},+}^\dagger)]. \end{aligned}$$

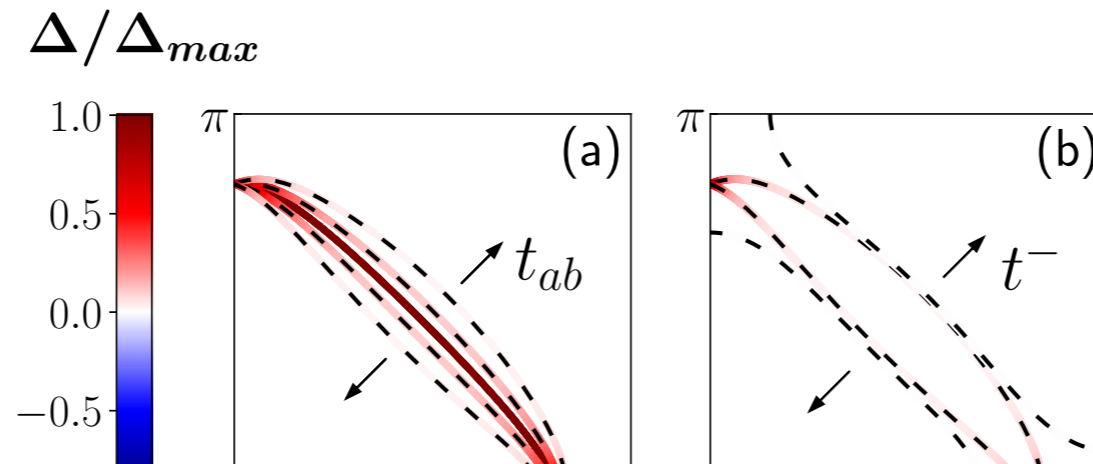
**pseudo-spin triplet (inter-band)**

$$\Delta^s(\mathbf{k}) = -2d_{a/b}^z \text{Im}(f_{\mathbf{k}}) g_{\mathbf{k}} = \frac{-2d_{a/b}^z \lambda_{\mathbf{k}}}{\sqrt{\xi_{\mathbf{k}}^{-2} + 4(t_{\mathbf{k}}^2 + \lambda_{\mathbf{k}}^2)}} \quad \text{spin-triplet}$$

$$\Delta_{\alpha\beta}^s(\mathbf{k}) = -d_{a/b}^z \text{Im}(f_{\mathbf{k}}^2) = -2d_{a/b}^z |f_{\mathbf{k}}|^2 \frac{t_{\mathbf{k}} \lambda_{\mathbf{k}}}{t_{\mathbf{k}}^2 + \lambda_{\mathbf{k}}^2}$$

$$d_{\alpha\beta}^z(\mathbf{k}) = d_{a/b}^z (g_{\mathbf{k}}^2 + \text{Re}(f_{\mathbf{k}}^2)) = d_{a/b}^z (g_{\mathbf{k}}^2 + |f_{\mathbf{k}}|^2 \frac{t_{\mathbf{k}}^2 - \lambda_{\mathbf{k}}^2}{t_{\mathbf{k}}^2 + \lambda_{\mathbf{k}}^2}).$$

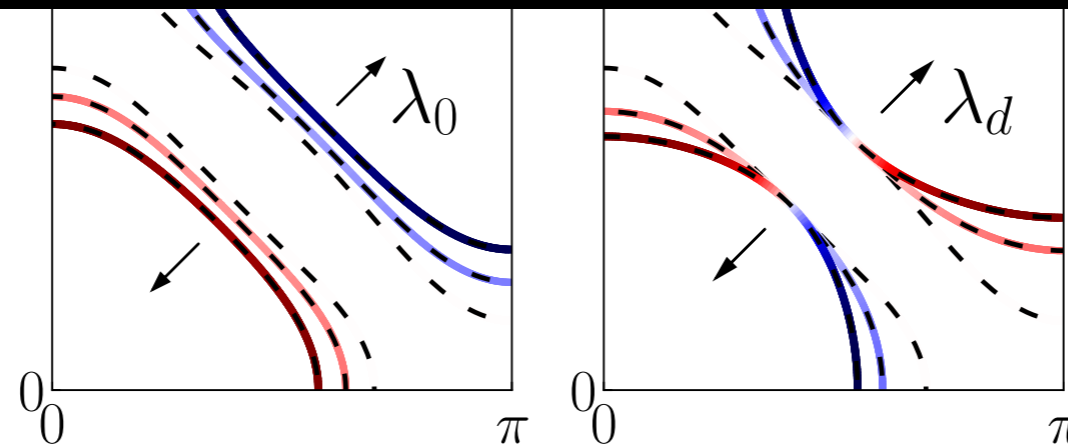
# Significance of SOC on the pairing $\Delta^s(\mathbf{k})$



battle bet. degeneracy breaking terms vs. (SOC & Hund's)

$$\frac{2d_{a/b}^z \lambda_{\mathbf{k}}}{\sqrt{\xi_{\mathbf{k}}^{-2} + 4(t_{\mathbf{k}}^2 + \lambda_{\mathbf{k}}^2)}}$$

SOC enhances pairing



SOC determines pairing symmetry

$$i(\cos k_x - \cos k_y) \left( \sigma_{\sigma\sigma'}^z c_{\mathbf{k}\sigma}^{a\dagger} c_{\mathbf{k}\sigma'}^b \right)$$

Figure from arXiv:2009.08597, J. Clepkens, A. Lindquist, HYK

# Back to Sr<sub>2</sub>RuO<sub>4</sub>: t<sub>2g</sub> orbitals

## Energy Scale

### Correlation

$$U \sim 2 - 3 \text{ eV}$$

$$J \sim 0.4 - 0.7 \text{ eV}$$

Correlated metal

orbital degeneracy breaking terms,  
e.g, orbital hybridization  $\sim 0.01 - 0.1 \text{ eV}$

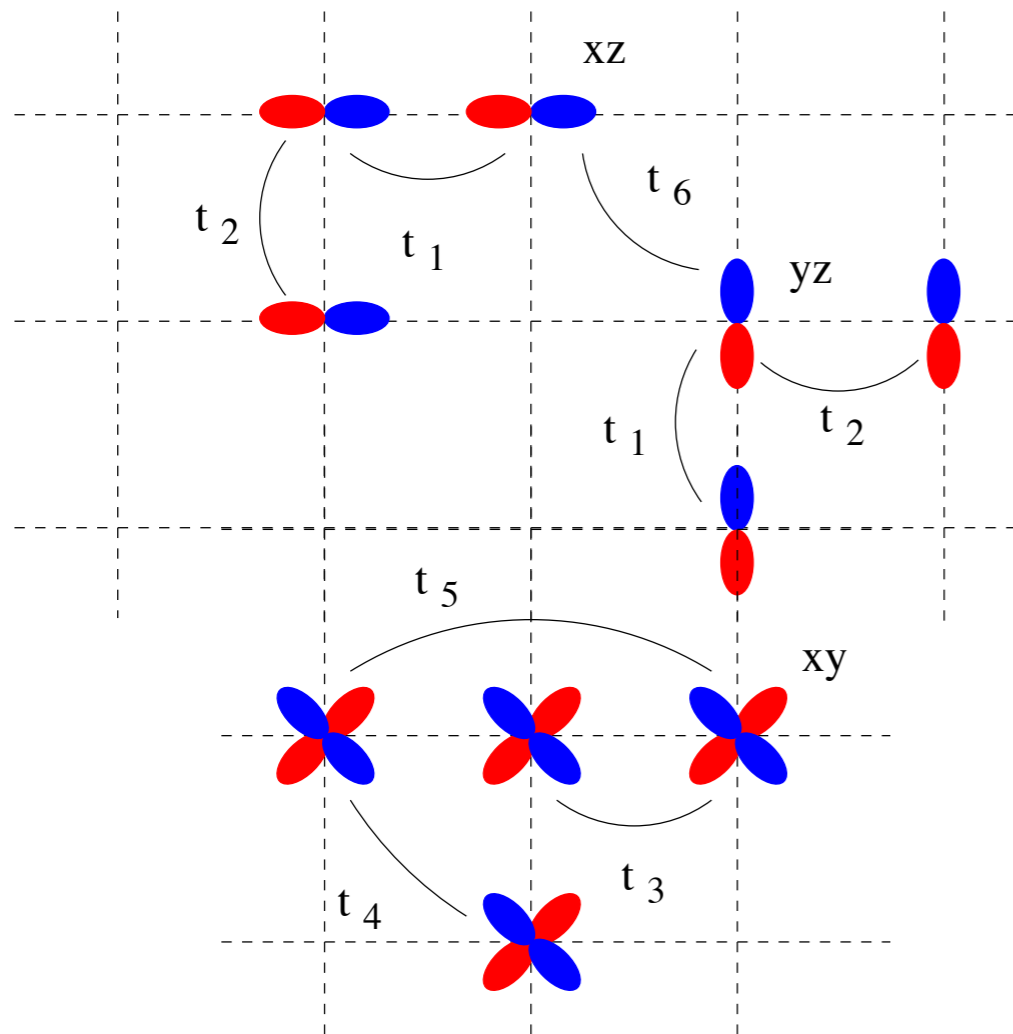
$$\text{SOC} \sim 0.05 - 0.16 \text{ eV}$$

crystal field:  $d_{xy} - d_{xz/yz} \sim 0.08 \text{ eV}$

$$T_c \sim 1.5 - 3 \text{ K}$$

$$H = H_{\text{int}} + H_{\text{kin}} + H_{\text{soc}}$$

$$H_{\text{kin}} + H_{\text{SO}} = \sum_{\mathbf{k}, \sigma} C_{\mathbf{k}\sigma}^\dagger \begin{pmatrix} \varepsilon_{\mathbf{k}}^{yz} & \varepsilon_{\mathbf{k}}^{1d} + i\lambda & -\lambda \\ \varepsilon_{\mathbf{k}}^{1d} - i\lambda & \varepsilon_{\mathbf{k}}^{xz} & i\lambda \\ -\lambda & -i\lambda & \varepsilon_{\mathbf{k}}^{xy} \end{pmatrix} C_{\mathbf{k}\sigma}, \quad C_{\mathbf{k}\sigma}^\dagger = (c_{\mathbf{k}\sigma}^{yz\dagger}, c_{\mathbf{k}\sigma}^{xz\dagger}, c_{\mathbf{k}-\sigma}^{xy\dagger})$$



$$\varepsilon_{\mathbf{k}}^{yz} = -2t_1 \cos k_y - 2t_2 \cos k_x - \mu_{1D},$$

$$\varepsilon_{\mathbf{k}}^{xz} = -2t_1 \cos k_x - 2t_2 \cos k_y - \mu_{1D},$$

$$\varepsilon_{\mathbf{k}}^{xy} = -2t_3 (\cos k_x + \cos k_y) - 4t_4 \cos k_x \cos k_y - 2t_5 (\cos(2k_x) + \cos(2k_y)) - \mu_{xy},$$

$$t_{\mathbf{k}} = -4t_{ab} \sin k_x \sin k_y$$

$\lambda$  atomic spin-orbit coupling (SOC)

$\mu_{1D}$

$\mu_{xy}$

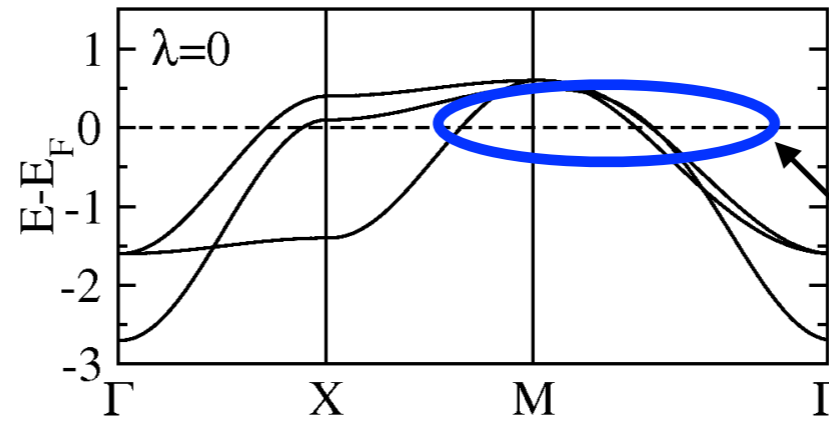
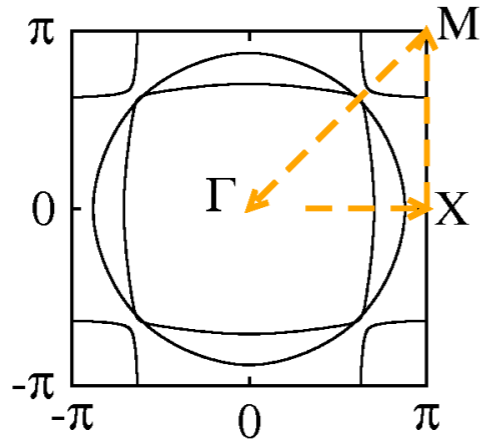
atomic potential

# Spin-Orbit Coupling

SOC  $\sim 0.05 - 0.16$  eV

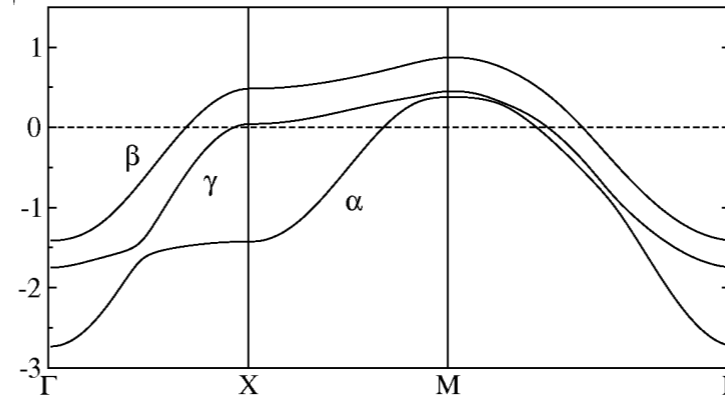
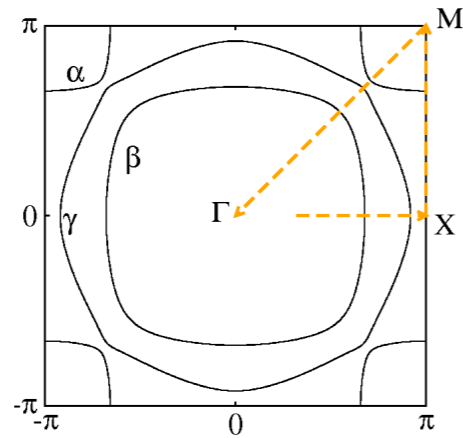


**SOC**

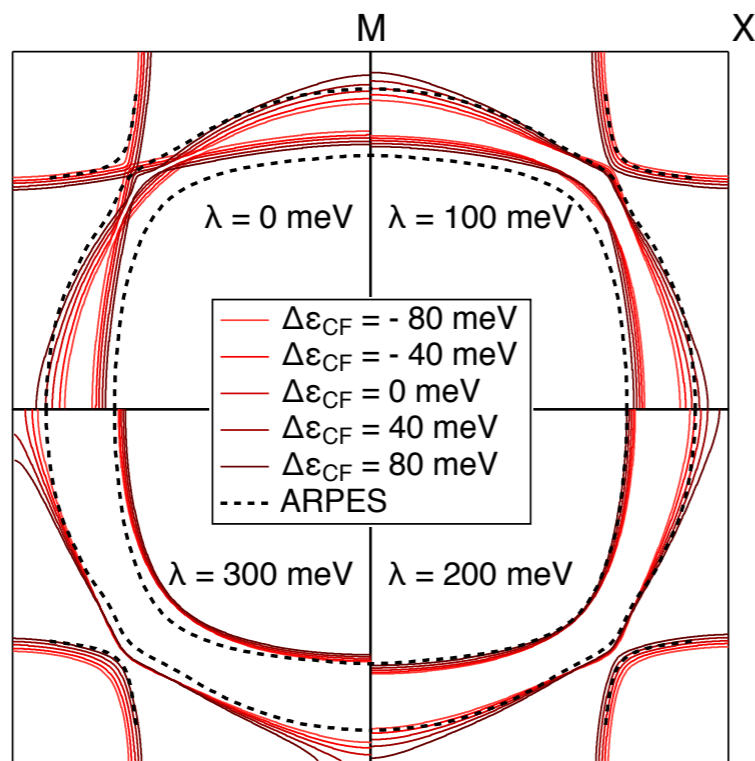


almost degenerate

degeneracy breaking  
vs. SOC



C. M. Puetter, PhD Thesis (2012).



effective SOC  $>$  SOC<sub>LDA</sub>

A. Tamai et al, PRX (2019)

# Multi-orbital Interaction

$$\begin{aligned}
 H_{int} &= \frac{U}{2} \sum_{i,a,\sigma \neq \sigma'} n_{a,i\sigma} n_{a,i\sigma'} + \frac{U'}{2} \sum_{i,a \neq b,\sigma\sigma'} n_{a,i\sigma} n_{b,i\sigma'} \\
 &+ \frac{J_H}{2} \sum_{i,a \neq b,\sigma\sigma'} c_{a,i\sigma}^\dagger c_{b,i\sigma'}^\dagger c_{a,i\sigma'} c_{b,i\sigma} \\
 &+ \frac{J_H}{2} \sum_{i,a \neq b,\sigma \neq \sigma'} c_{a,i\sigma}^\dagger c_{a,i\sigma'}^\dagger c_{b,i\sigma'} c_{b,i\sigma},
 \end{aligned}
 \longrightarrow
 \begin{aligned}
 H_{int} &= \frac{4U}{N} \sum_{a,\mathbf{k}\mathbf{k}'} \hat{\Delta}_{a,\mathbf{k}}^{s\dagger} \hat{\Delta}_{a,\mathbf{k}'}^s \\
 &+ \frac{2(U' - J_H)}{N} \sum_{\{a \neq b\},\mathbf{k}\mathbf{k}'} \hat{\mathbf{d}}_{a/b,\mathbf{k}}^\dagger \cdot \hat{\mathbf{d}}_{a/b,\mathbf{k}'} \\
 &+ \frac{4J_H}{N} \sum_{a \neq b,\mathbf{k}\mathbf{k}'} \hat{\Delta}_{a,\mathbf{k}}^{s\dagger} \hat{\Delta}_{b,\mathbf{k}'}^s \\
 &+ \frac{2(U' + J_H)}{N} \sum_{a \neq b,\mathbf{k}\mathbf{k}'} \hat{\Delta}_{a/b,\mathbf{k}}^{s\dagger} \hat{\Delta}_{a/b,\mathbf{k}'}^s,
 \end{aligned}$$

spin triplet

$$\hat{\mathbf{d}}_{a/b,\mathbf{k}} = \frac{1}{4} \sum_{\sigma\sigma'} [i\sigma^y \boldsymbol{\sigma}]_{\sigma\sigma'} (c_{a,\mathbf{k}\sigma} c_{b,-\mathbf{k}\sigma'} - c_{b,\mathbf{k}\sigma} c_{a,-\mathbf{k}\sigma'})$$

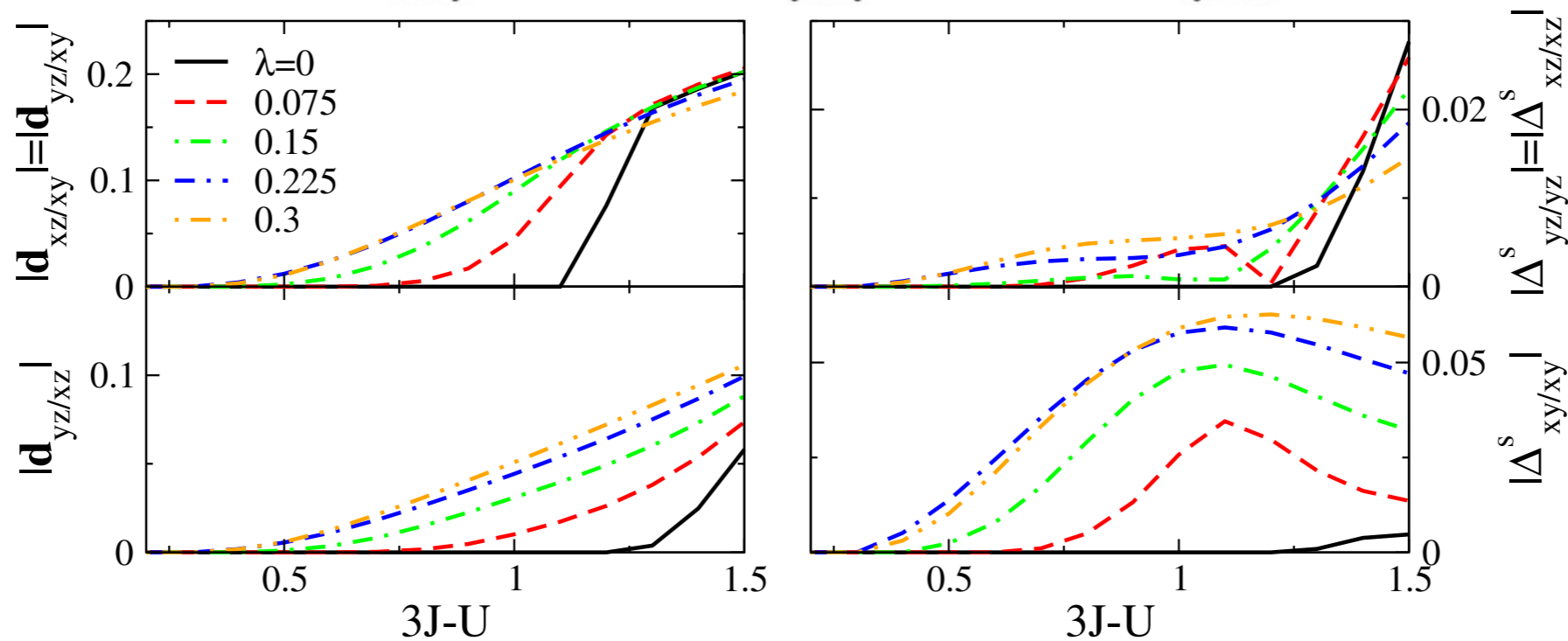
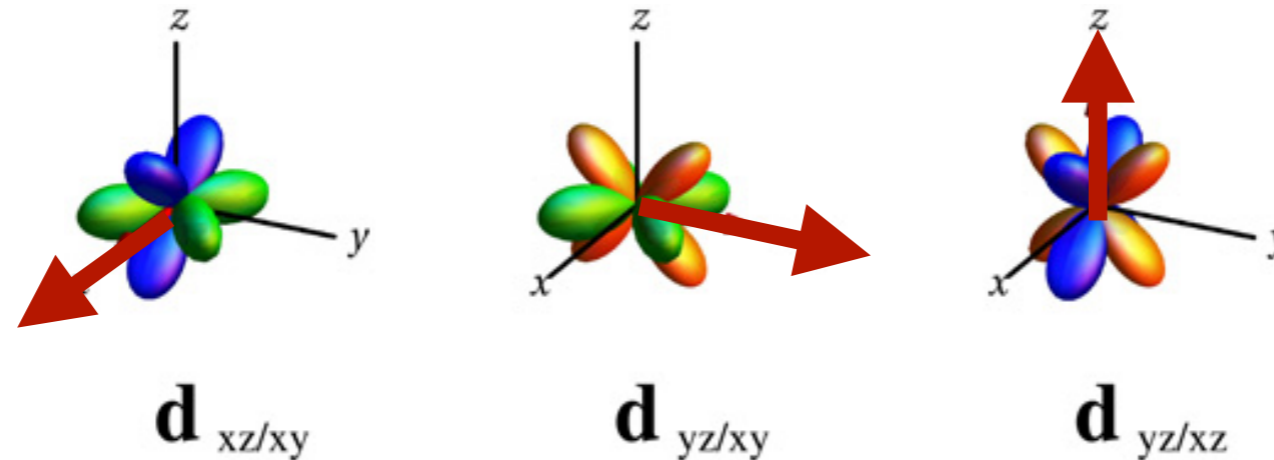
$$\hat{\Delta}_{a/b,\mathbf{k}}^s = \frac{1}{4} \sum_{\sigma\sigma'} [i\sigma^y]_{\sigma\sigma'} (c_{a,\mathbf{k}\sigma} c_{b,-\mathbf{k}\sigma'} + c_{b,\mathbf{k}\sigma} c_{a,-\mathbf{k}\sigma'})$$

$$\hat{\Delta}_{a,\mathbf{k}}^s = \frac{1}{4} \sum_{\sigma\sigma'} [i\sigma^y]_{\sigma\sigma'} c_{a,\mathbf{k}\sigma} c_{a,-\mathbf{k}\sigma'},$$

# Identifying spin-triplet pairing in spin-orbit coupled multi-band superconductors

EPL 98, 27010 (2012) arXiv:1101.4656.

CHRISTOPH M. PUETTER<sup>1</sup> and HAE-YOUNG KEE<sup>1,2(a)</sup>

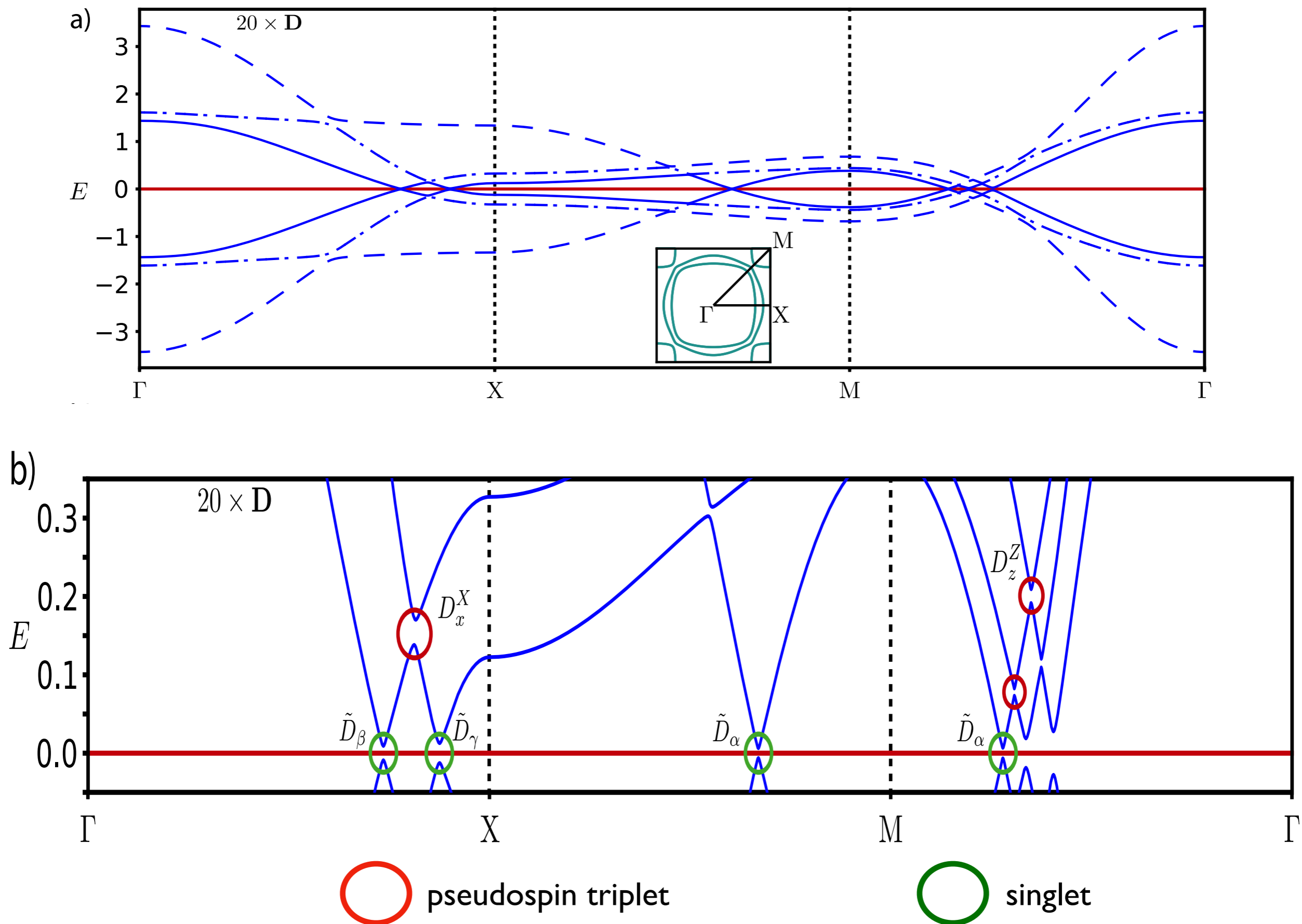


**SOC enhances inter-orbital (orbital-singlet) even-parity spin-triplet & pins d-vector direction (varies in momentum space)**

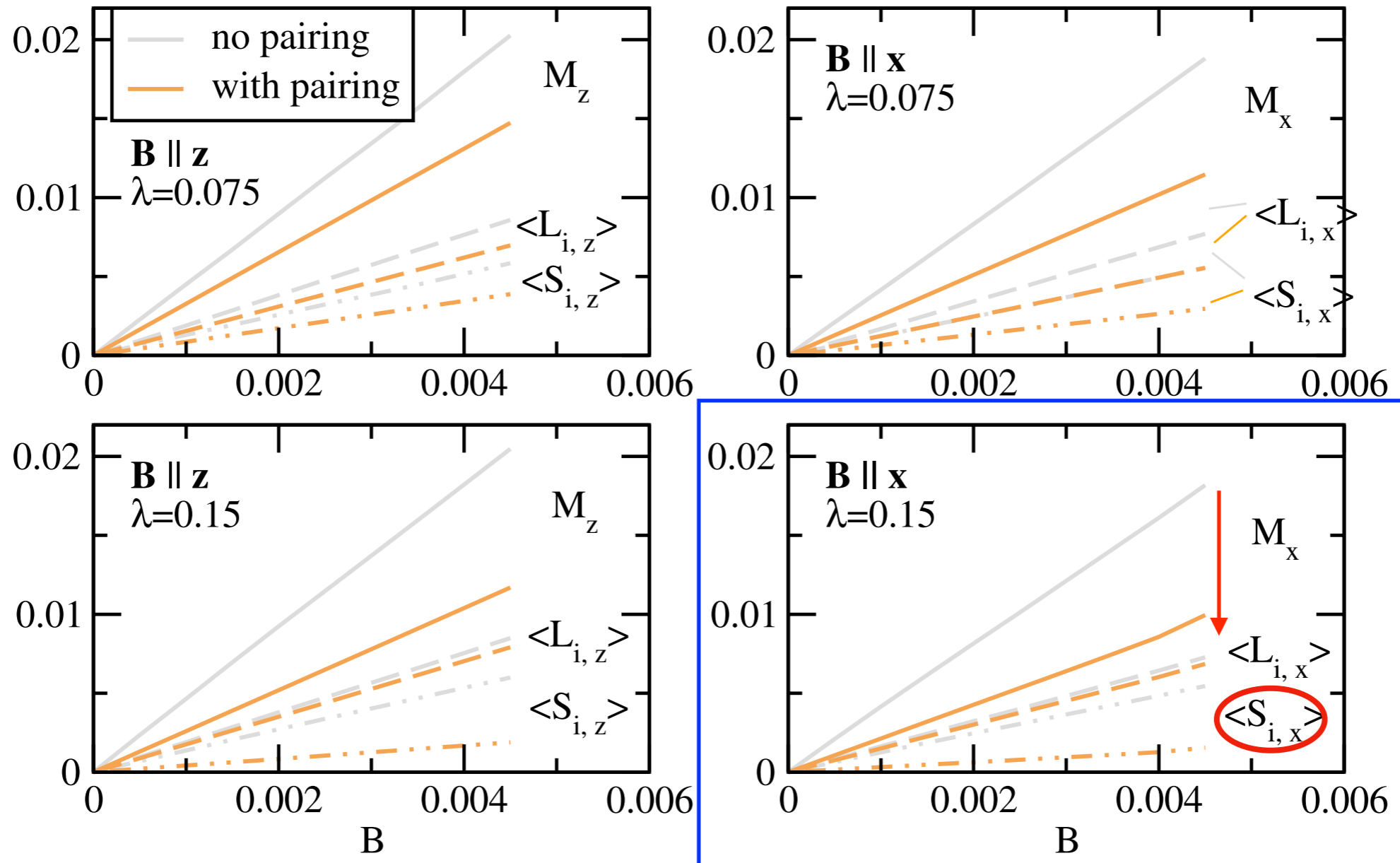
**atomic SOC: anisotropic S-wave**



# QP dispersion



# magnetization



C. Puetter, HYK, EPL 98, 27010 (2012) ; arXiv:1101.4656

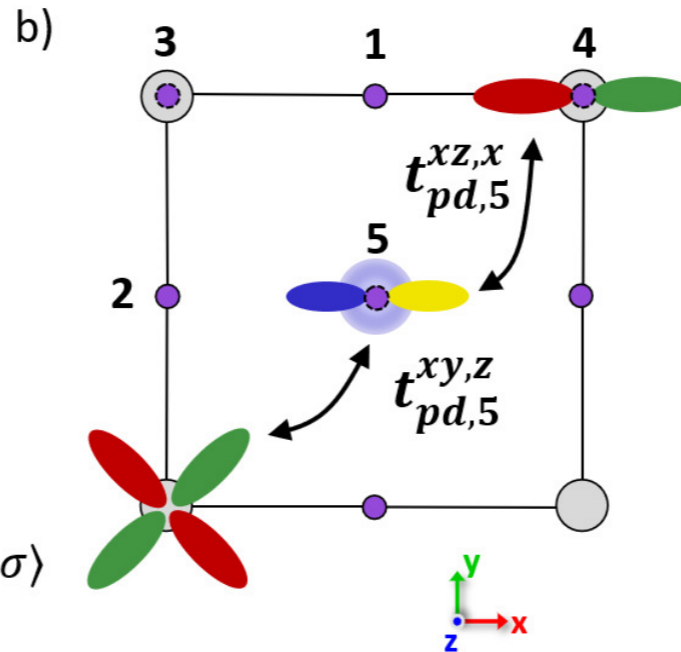
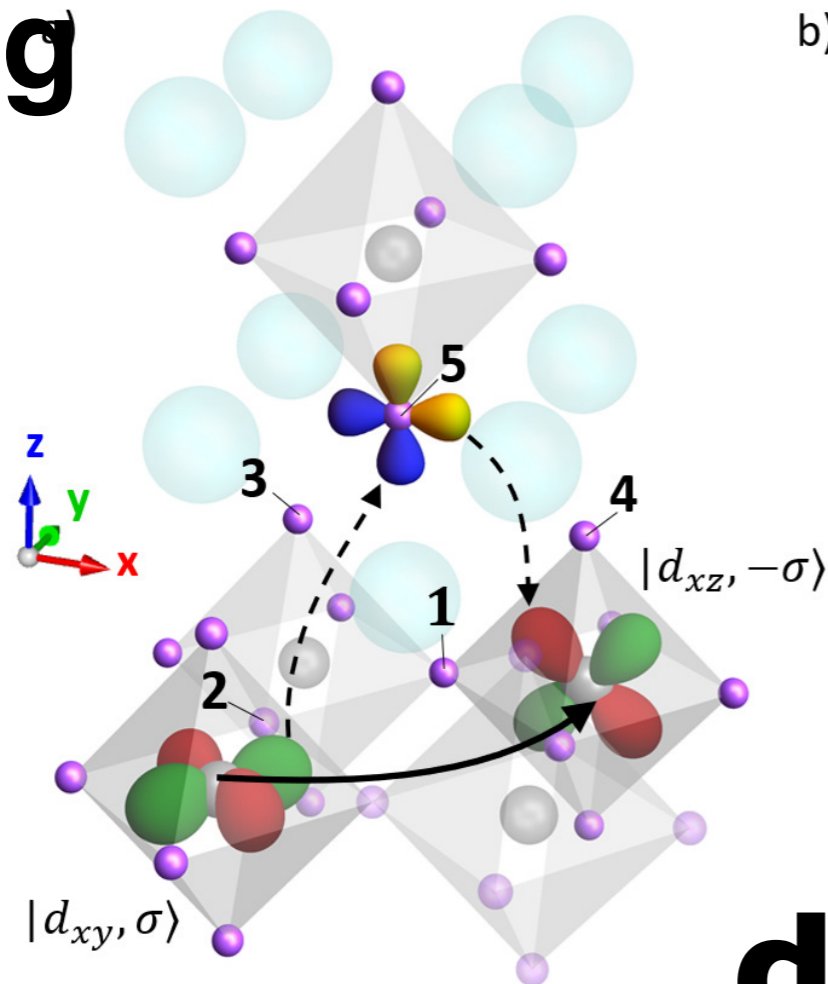
Deduction of Knight shift for all field directions

**How to get two-component OP?**

**momentum dependent SOC  
beyond atomic SOC**

# momentum-dependent SOC: $\lambda(\mathbf{k})$

## B<sub>2g</sub>



$$H_{SOC}^{B_{2g}} = \sum_{p_{\pm}} \frac{H^0 |p_{\pm}\rangle \langle p_{\pm}| H^0}{E_d - E_{p_{\pm}}},$$

## d<sub>xy</sub>

$$-8i\lambda^{B_{2g}} \sum_{\mathbf{k}\sigma\sigma'} \sin k_x \sin k_y \left( \sigma_{\sigma\sigma'}^y c_{\mathbf{k}\sigma}^{xz\dagger} c_{\mathbf{k}\sigma'}^{xy} - \sigma_{\sigma\sigma'}^x c_{\mathbf{k}\sigma}^{yz\dagger} c_{\mathbf{k}\sigma'}^{xy} \right) + h.c.$$

$$\lambda^{B_{2g}} = \frac{\lambda_p}{(E_{pd} + \frac{\lambda_p}{2})(E_{pd} - \lambda_p)} \sum_i t_{pd,i}^{a_i} t_{pd,i}^{b_i}$$

**SOC determines pairing;**  $\Delta^s(\mathbf{k}) \propto d_{a/b} \times \lambda(\mathbf{k})$

**When atomic  $\lambda$  & momentum-dep. SOC  $\lambda^{B2g}$  present**

pseudo-spin singlet  $s + id_{xy}$

$$s \propto d_{xz/yz}^x \lambda, d_{yz/xy}^y \lambda, d_{xz/yz}^z \lambda$$

$$d_{xy} \propto d_{yz(xz)/xy}^{x(y)} (\sin k_x \sin k_y) \lambda^{B2g}$$

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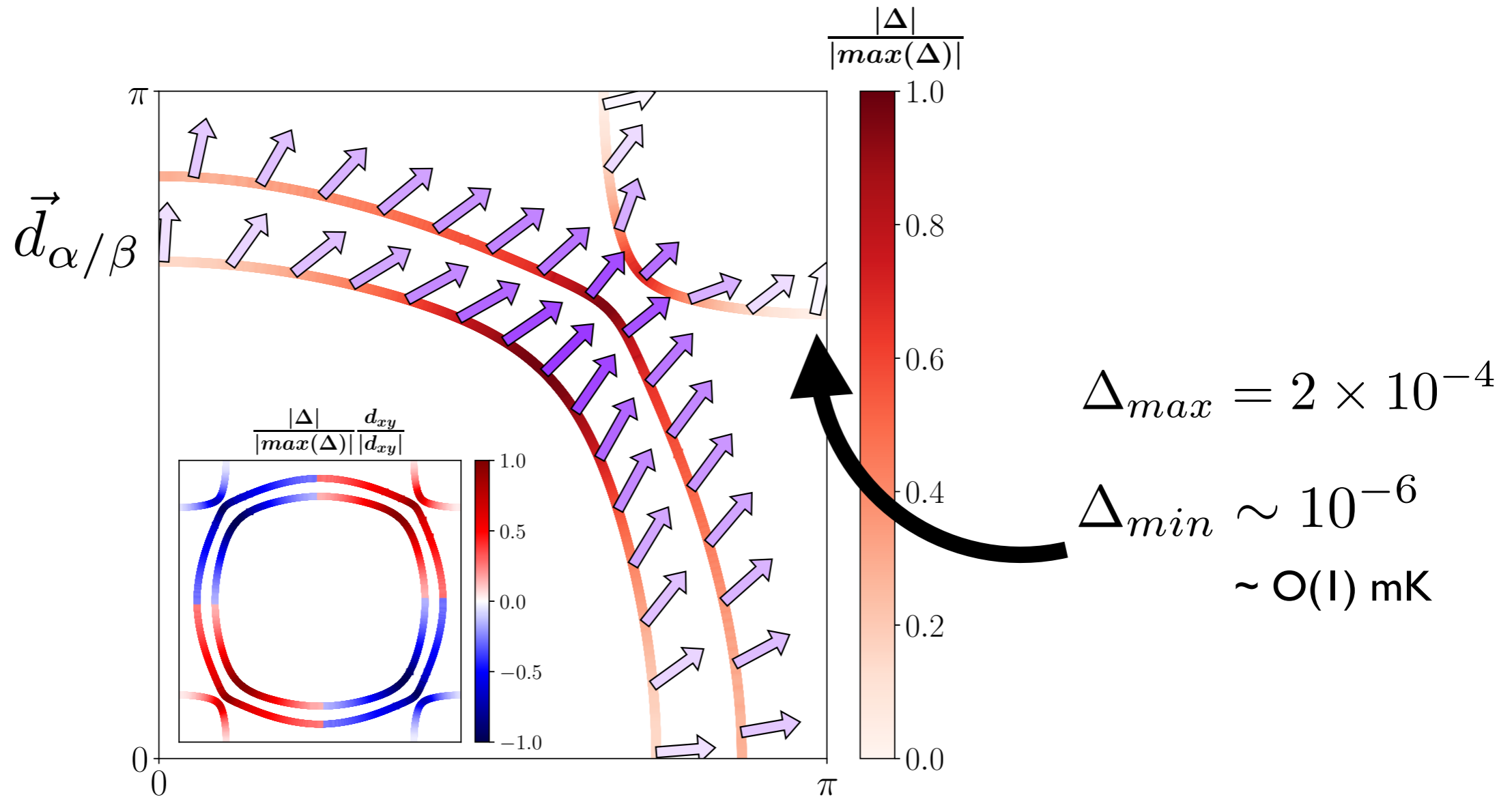

$$J_H - U' = 0.7 \quad \lambda = 0.05 \quad \lambda^{B2g} = 0.038 \quad \lambda^{Eg} = 0.005$$

$t_1$	$t_2$	$t_3$	$t_4$	$t_{ab}$	$\mu_{1d}$	$\mu_{xy}$
0.45	0.05	0.5	0.2	0.025	0.54	0.64

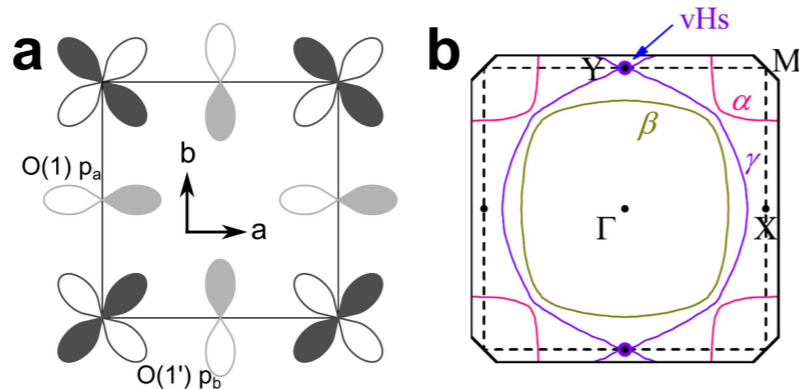
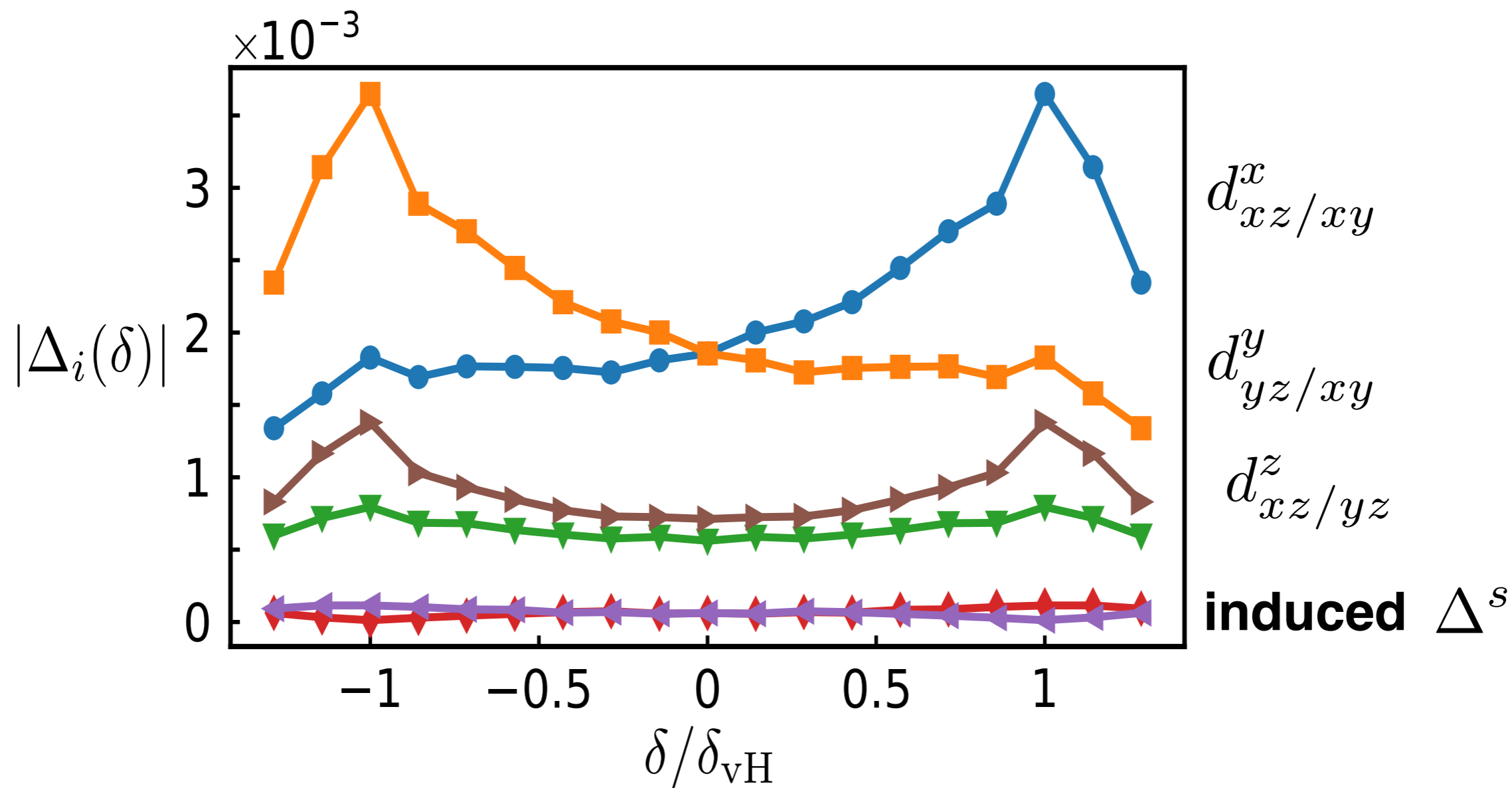
# Shadowed Triplet Pairing

$s + id_{xy}$  pseudo-spin singlet

$\vec{d}_{\alpha/\beta}$  pseudo-spin triplet pairing finite away from the Fermi energy

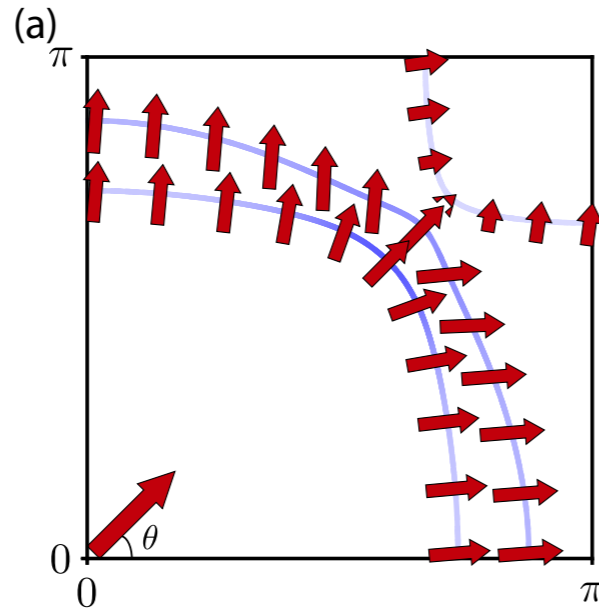


# Effects of Strain

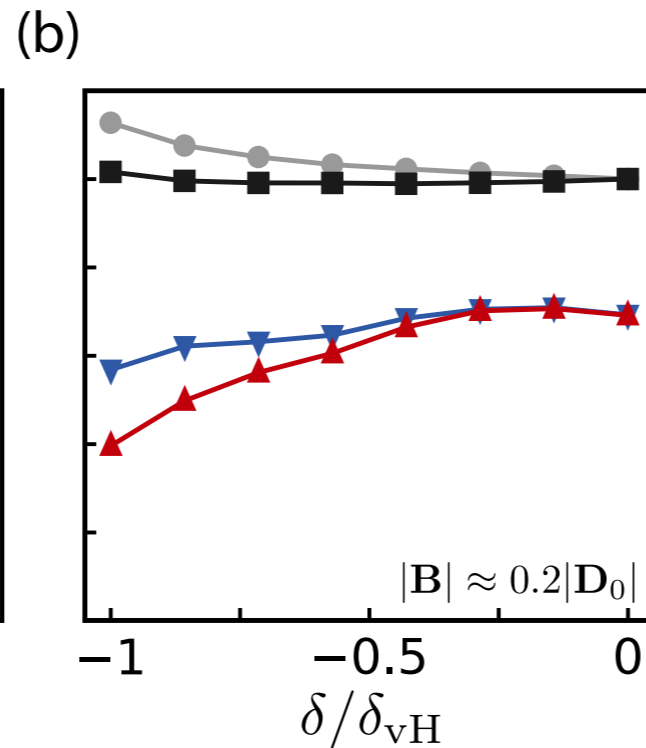
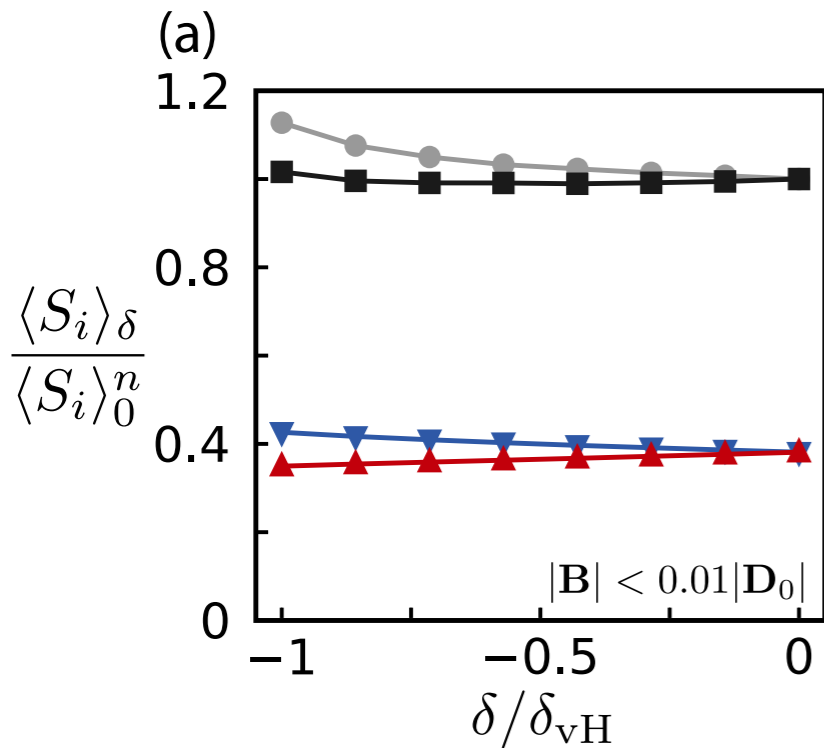
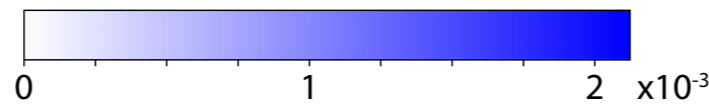
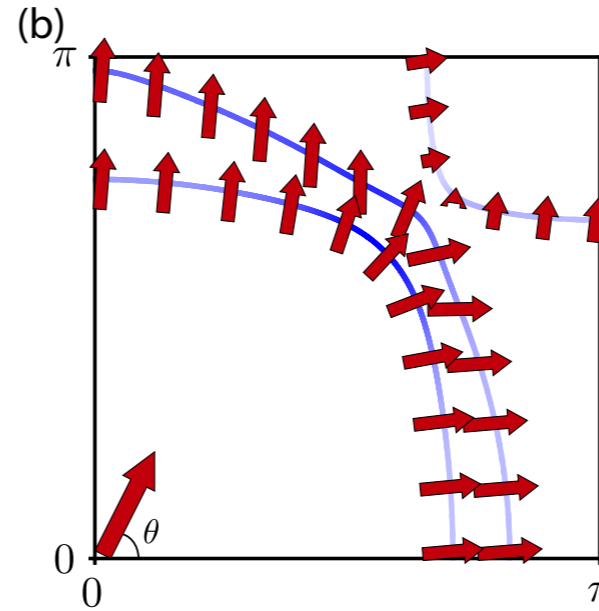


# Proposal to test the theory

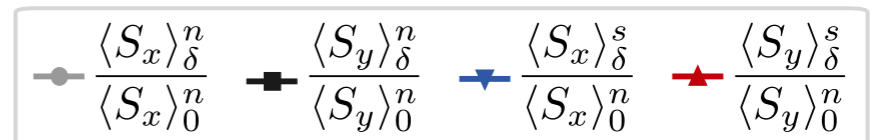
## NMR under strain



## Uniaxial strain along a-axis

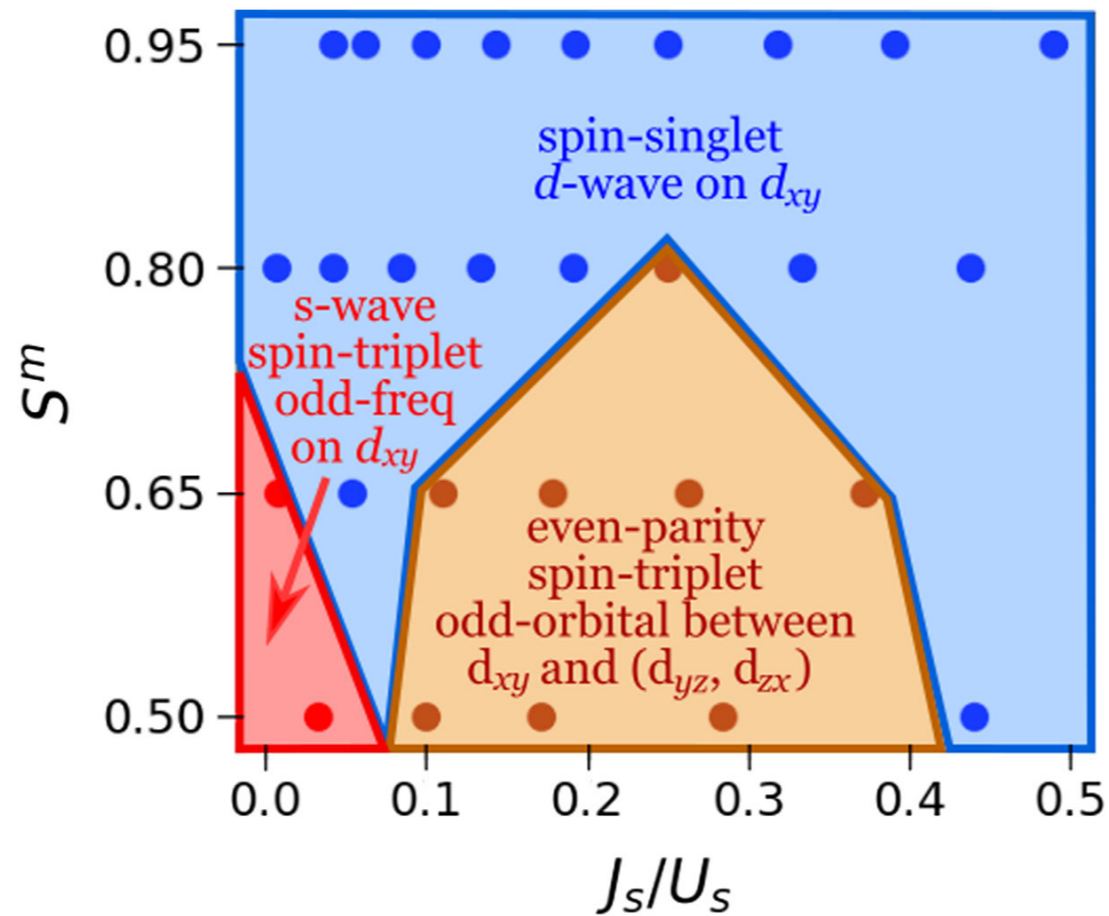


**NMR deduction is more when the field is perpendicular to the uniaxial strain direction (occurs when the field is large)**





# Beyond MF



LDA+ DMFT: O. Gingras et al, PRL 123, 217005 (2019)

## Application to Pnictides

O.Vafek, A.V. Chubukov, PRL (2017); Hund's + SOC on 2-orbitals

# Conclusion

**Within a MF of Kanamori +  $t_{2g}$  + SOC:  
applicable Hund's metal with SOC**

- SOC determines the gap size and k-dependence of pairing
- pseudospin singlet + pseudospin triplet + i induced singlet
- d-vector changes in k-space
- For  $\text{Sr}_2\text{RuO}_4$ :  $s+i d$  (TRSB SC) & pseudospin triplet & induced singlet

**beyond on-site interaction?  $T_c$ ? HQV?**