

# A quantum magnetic analog to the critical point of water

F. Mila

Ecole Polytechnique Fédérale de Lausanne  
Switzerland

PHYSICAL REVIEW LETTERS **121**, 127201 (2018)

---

**Thermal Critical Points and Quantum Critical End Point  
in the Frustrated Bilayer Heisenberg Antiferromagnet**

J. Stapmanns,<sup>1</sup> P. Corboz,<sup>2</sup> F. Mila,<sup>3</sup> A. Honecker,<sup>4</sup> B. Normand,<sup>5</sup> and S. Wessel<sup>1</sup>

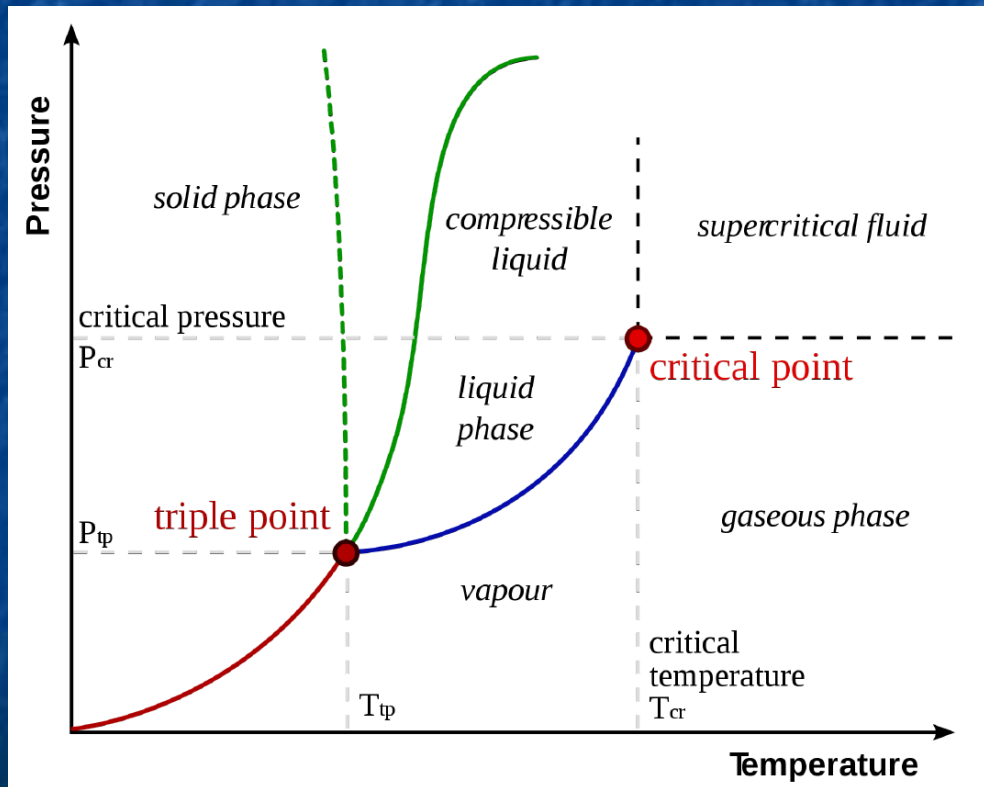
**A quantum magnetic analogue to the critical point of water**

J. Larrea Jiménez,<sup>1,2</sup> S. P. G. Crone,<sup>3</sup> E. Fogh,<sup>2</sup> M. Zayed,<sup>4</sup> R. Lortz,<sup>5</sup> E. Pomjakushina,<sup>6</sup> K. Conder,<sup>6</sup> A. M. Läuchli,<sup>7</sup> L. Weber,<sup>8</sup> S. Wessel,<sup>8</sup> A. Honecker,<sup>9</sup> B. Normand,<sup>10,2</sup> Ch. Rüegg,<sup>10,11,2,12</sup> P. Corboz,<sup>3</sup> H. M. Rønnow,<sup>2</sup> and F. Mila<sup>2</sup>

**SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub> under pressure**

# The critical point of water

1822: Baron Cagniard de la Tour

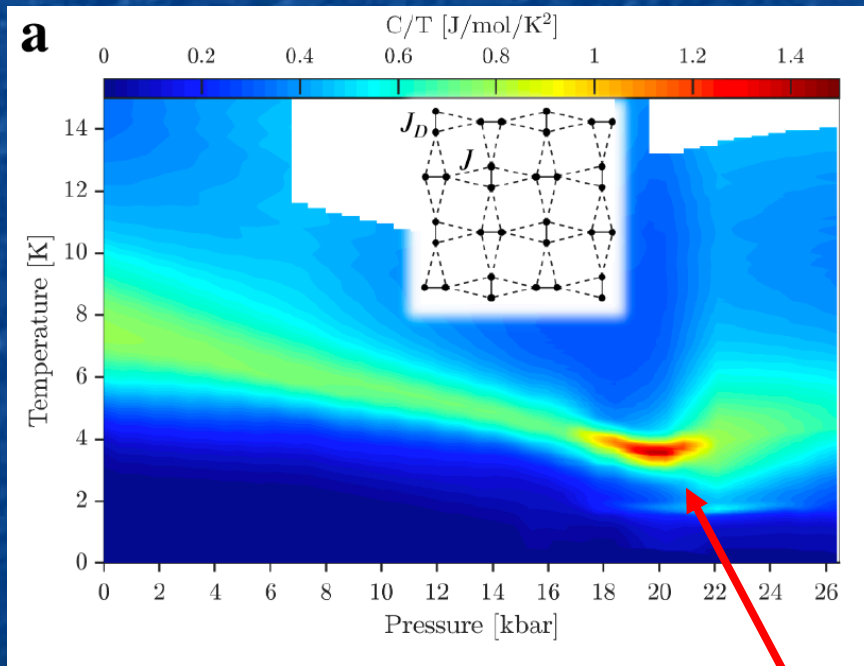


$T_c = 374^\circ \text{ C}$   
 $P_c = 218 \text{ bar}$

Figure from Wikipedia

# $\text{SrCu}_2(\text{BO}_3)_2$

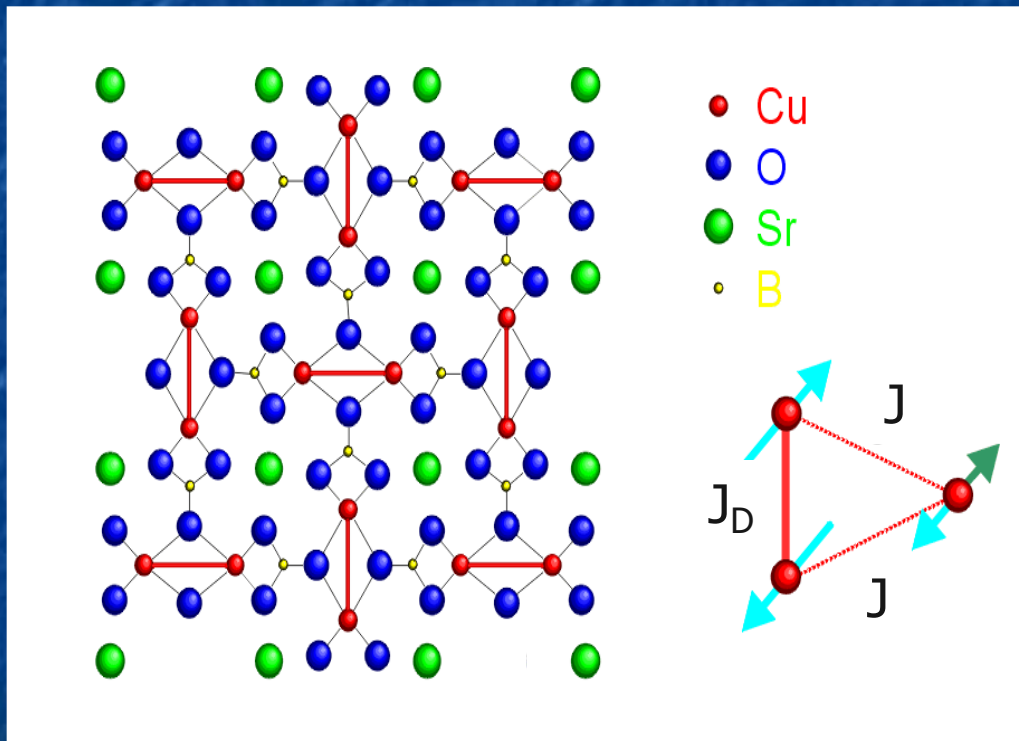
2019: H. Rønnow and collaborators



FM: Critical point around  $P=19$  kbar and  $T=3.3\text{K}$

# SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub>

Smith and Keszler, JSSC 1991



Cu<sup>2+</sup> -> Spin 1/2

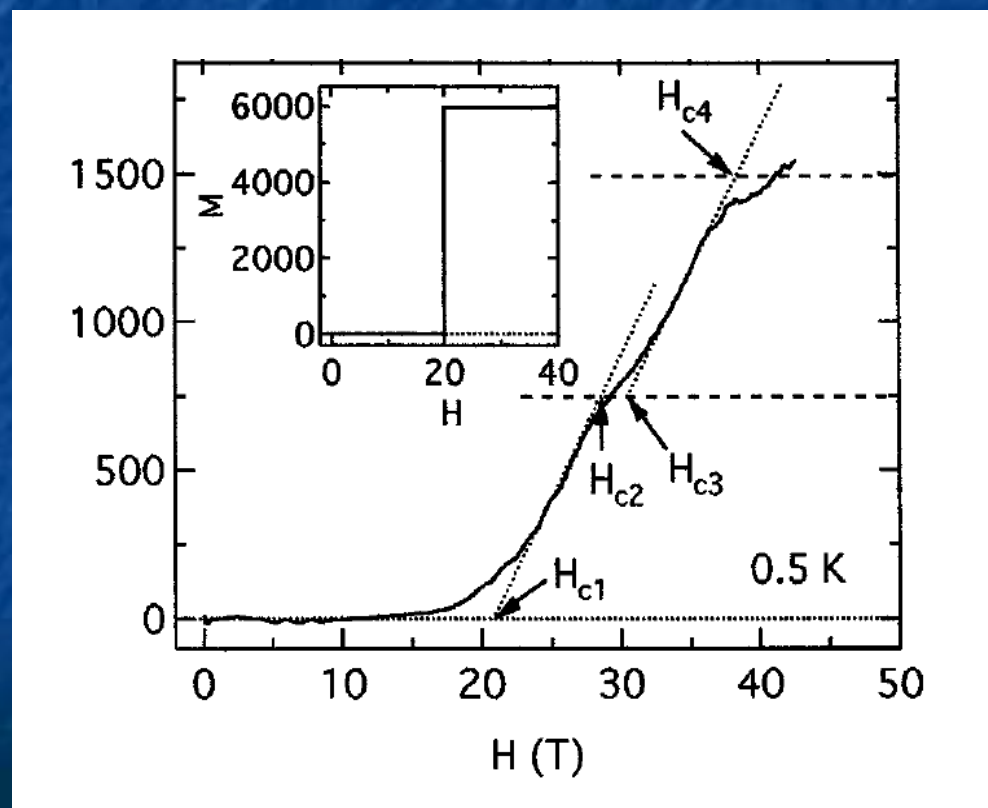
$J_D \approx 85 \text{ K}$

$J/J_D \approx 0.63$

Orthogonal dimer model  $\Leftrightarrow$  Shastry-Sutherland

## Exact Dimer Ground State and Quantized Magnetization Plateaus in the Two-Dimensional Spin System $\text{SrCu}_2(\text{BO}_3)_2$

H. Kageyama,<sup>1,2,\*</sup> K. Yoshimura,<sup>1,3,†</sup> R. Stern,<sup>3</sup> N. V. Mushnikov,<sup>2</sup> K. Onizuka,<sup>2</sup> M. Kato,<sup>1</sup> K. Kosuge,<sup>1</sup>  
C. P. Slichter,<sup>3</sup> T. Goto,<sup>2</sup> and Y. Ueda<sup>2</sup>



### Anomalies

- $M=0$
  - $M=1/8$
  - $M=1/4$
- and many more...

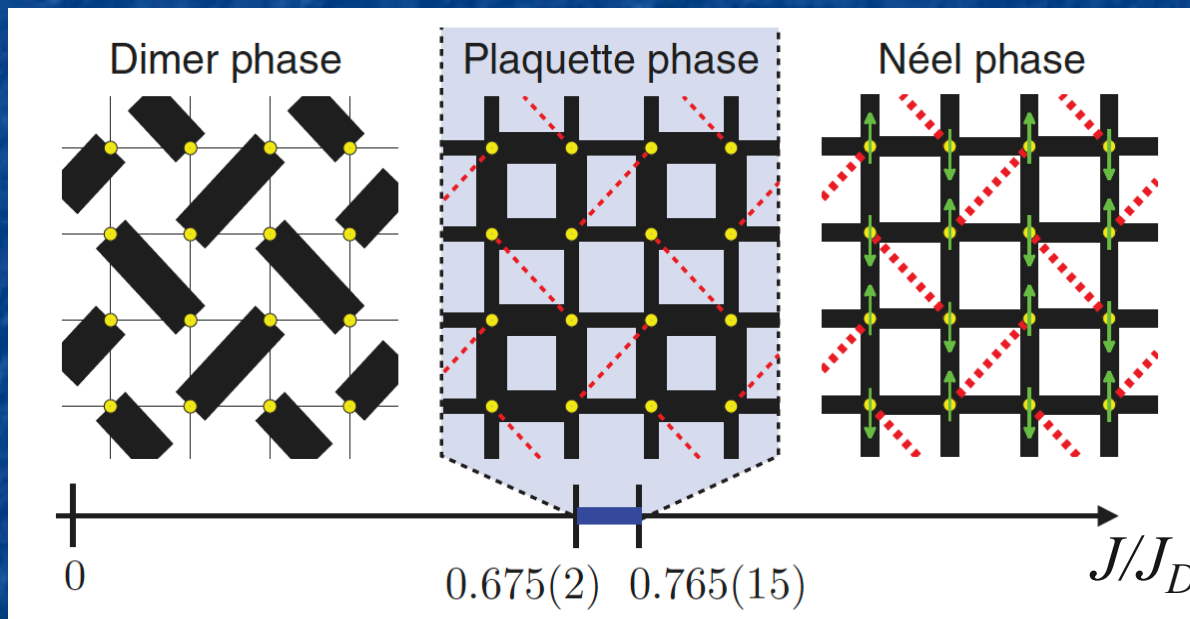
# Tensor network study of the Shastry-Sutherland model in zero magnetic field

Philippe Corboz<sup>1</sup> and Frédéric Mila<sup>2</sup>

<sup>1</sup>Theoretische Physik, ETH Zürich, CH-8093 Zürich, Switzerland

<sup>2</sup>Institut de théorie des phénomènes physiques, École Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland

(Received 13 December 2012; revised manuscript received 27 February 2013; published 27 March 2013)



Early results:  
Koga & Kawakami, 2000  
Läuchli et al, 2002

iPEPS with various setups and  
bond dimension up to 10

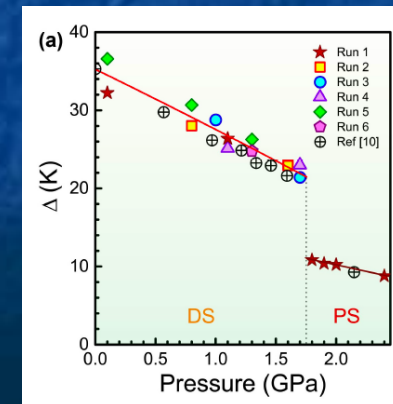
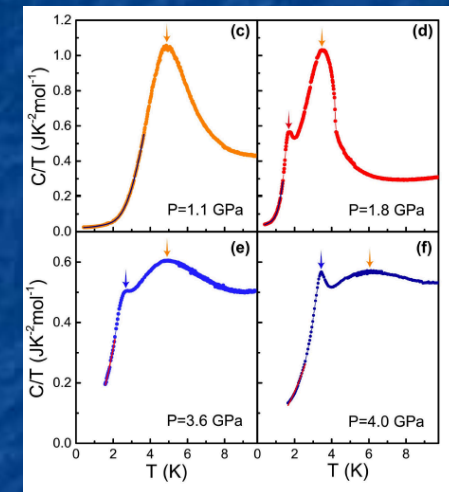
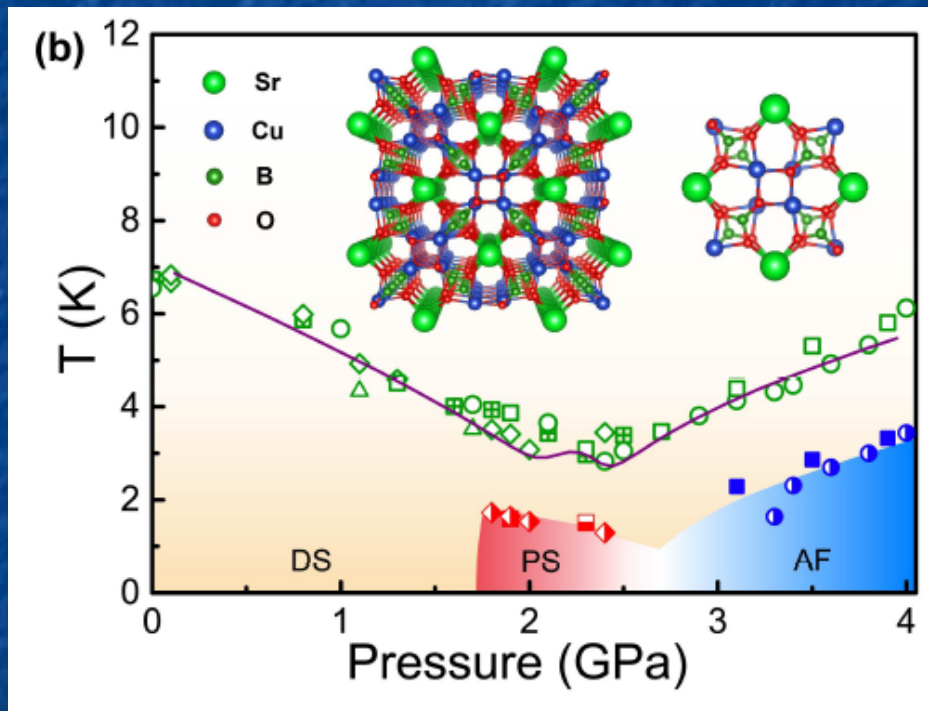
# SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub> under pressure

- Pressure: expected to **change  $J/J_D$**  and found to **increase it**
- NMR (Waki et al, 2007): **intermediate phase** around 24 kbar
- Confirmed later on by **magnetization** (Haravifard et al, 2016) **neutron scattering** (Zayed et al, 2017), **ESR** (Sakurai et al, 2018), and **specific heat** (Guo et al, 2020)



## Quantum Phases of $\text{SrCu}_2(\text{BO}_3)_2$ from High-Pressure Thermodynamics

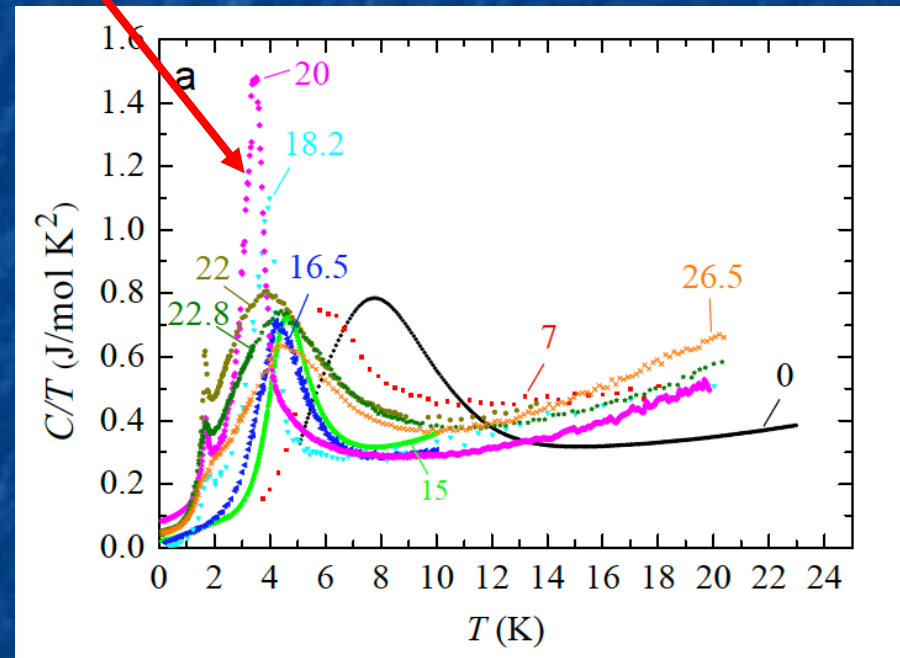
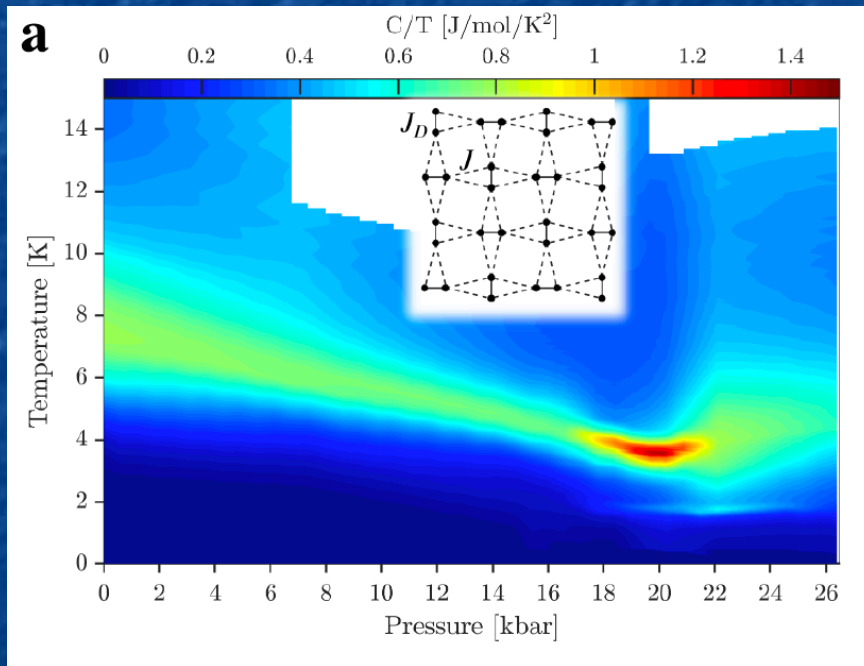
Jing Guo<sup>1</sup>, Guangyu Sun<sup>1,2</sup>, Bowen Zhao<sup>3</sup>, Ling Wang<sup>4,5</sup>, Wenshan Hong<sup>1,2</sup>, Vladimir A. Sidorov<sup>6</sup>, Nvsen Ma<sup>1</sup>, Qi Wu<sup>1</sup>, Shiliang Li<sup>1,2,7</sup>, Zi Yang Meng<sup>1,8,7,\*</sup>, Anders W. Sandvik<sup>3,1,†</sup> and Liling Sun<sup>1,2,7,‡</sup>



Intermediate phase with critical temperature around 2K

# SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub>

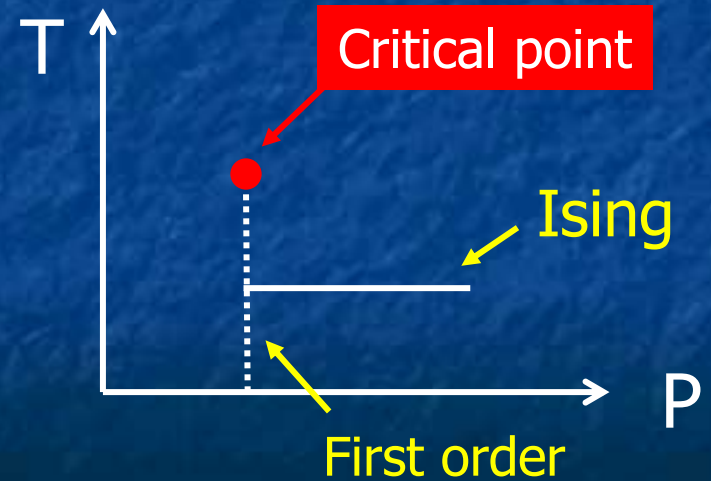
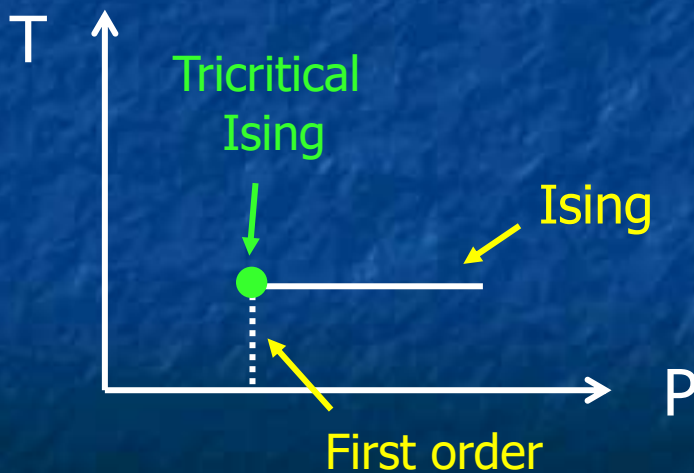
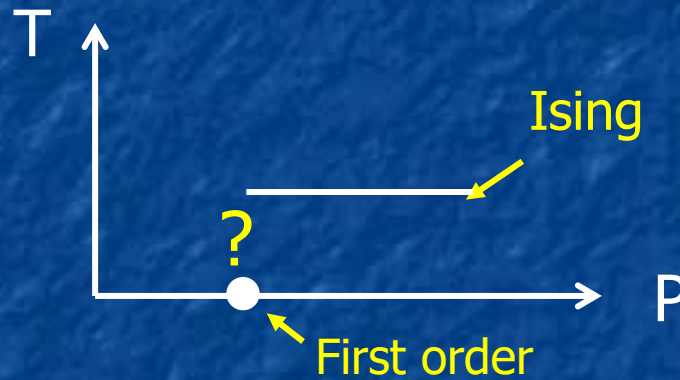
Extremely narrow peak between 18 and 20 kbar



J. Larrea ... P. Corboz ... H. Ronnow, FM, unpublished

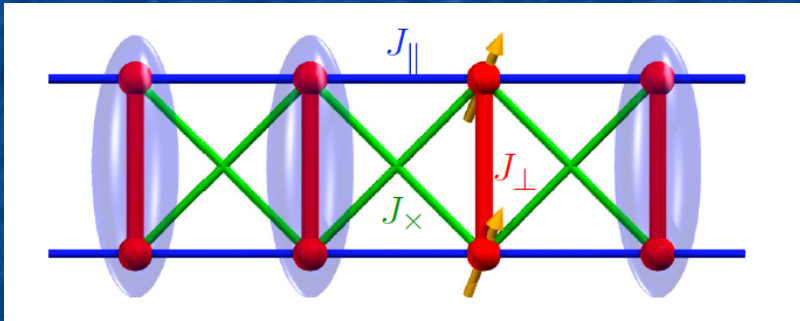
NB: transition into plaquette phase at much lower  $T$ , around 2K

# First-order vs Ising



# Fully frustrated dimer models

- Example: fully-frustrated ladder



$$J_{\times} = J_{\parallel}$$

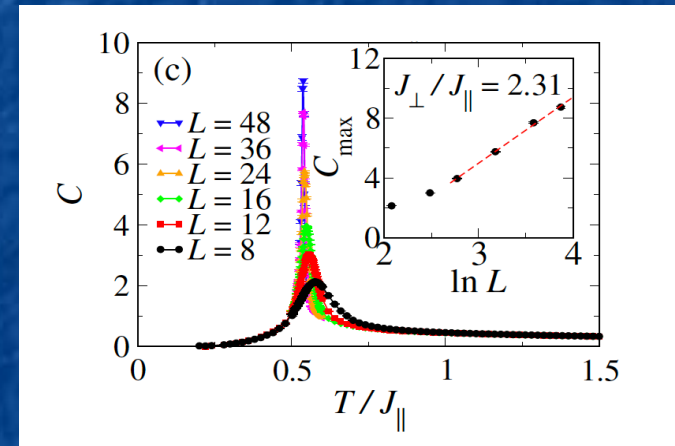
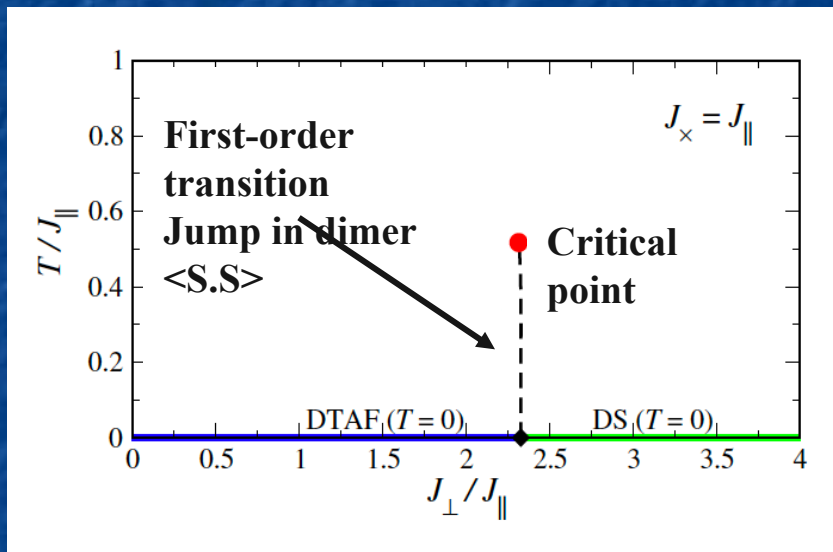
$$H = J_{\parallel} \sum_{i=1}^L \vec{T}_i \cdot \vec{T}_{i+1} + J_{\perp} \sum_{i=1}^L \left( \frac{1}{2} \vec{T}_i^2 - S(S+1) \right)$$

$\vec{T}_i = \vec{S}_i^1 + \vec{S}_i^2$  Total spin on a rung is a **good quantum number**

# Fully Frustrated Bilayer



QMC simulations in dimer basis (Stapmans,...,Wessel, PRL 2018)



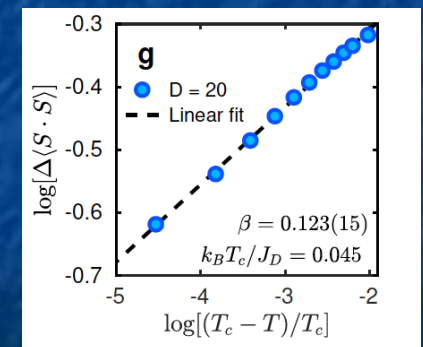
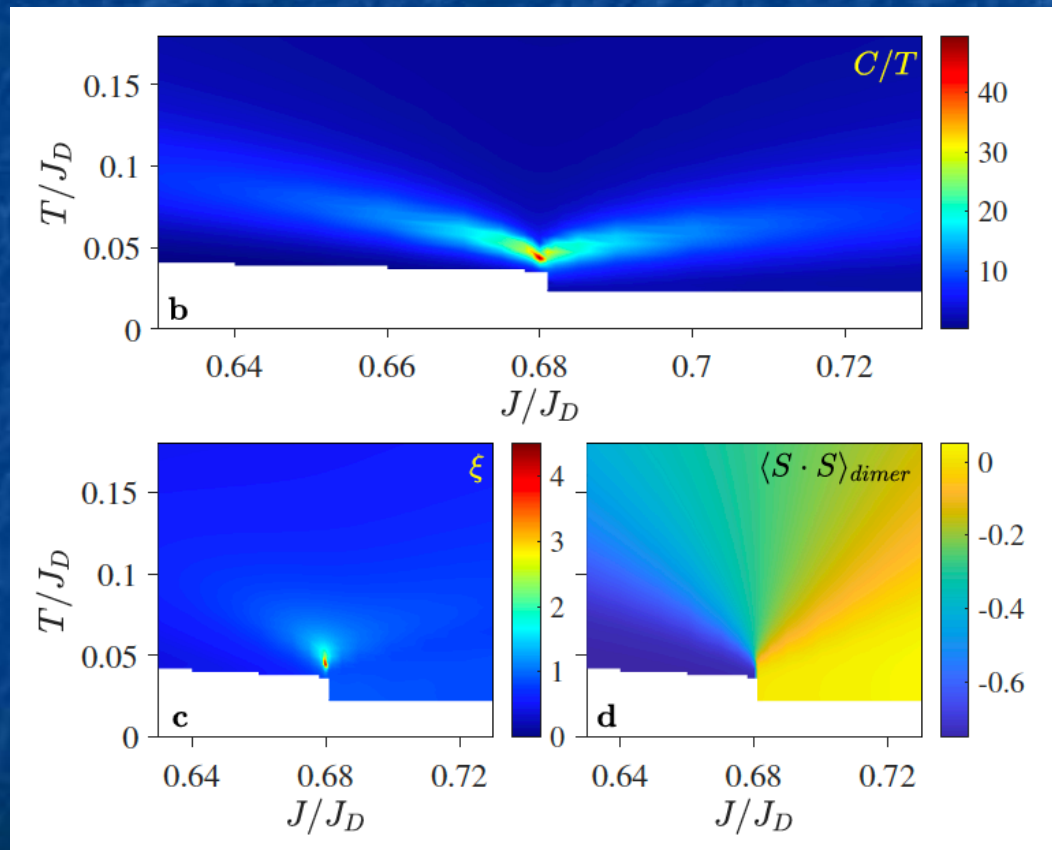
Ising 2D:  $\alpha = 0$ ,  $C \propto \ln L$

Square lattice of dimers with fully frustrated couplings

- Large intra-dimer coupling: dimer singlets
- Larger inter-dimer coupling: dimer triplets and  $S=1$  square lattice AF

# iPEPS for Shastry-Sutherland

## Ising critical point

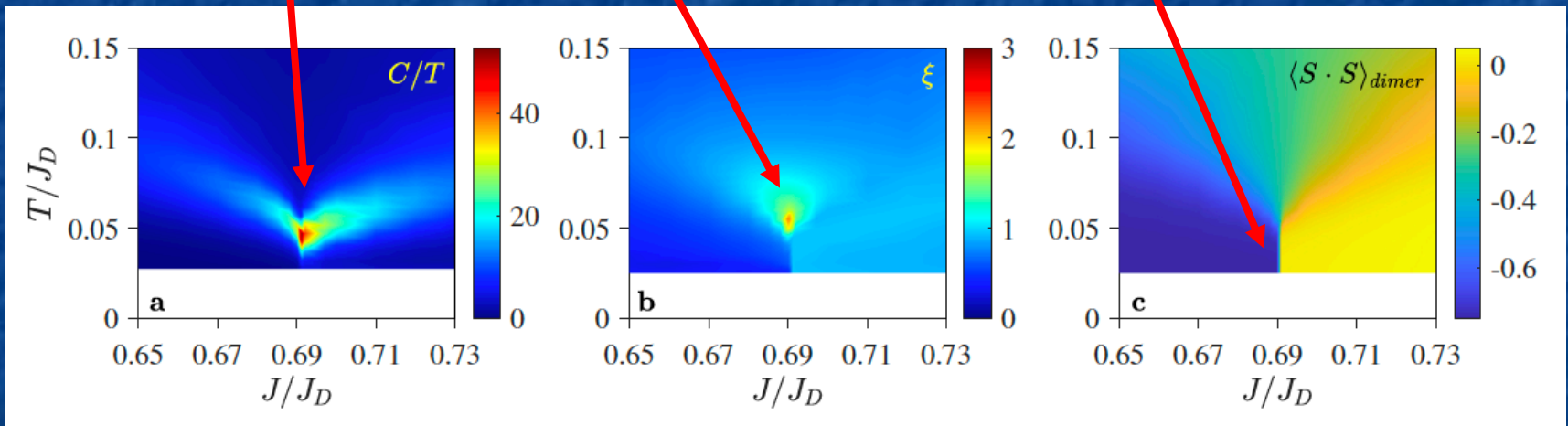


J. Larrea ... P. Corboz ... H. Ronnow, FM, unpublished

# Shastry-Sutherland with intra-dimer DM interaction

Ising critical point

Jump in dimer  $\langle S \cdot S \rangle$



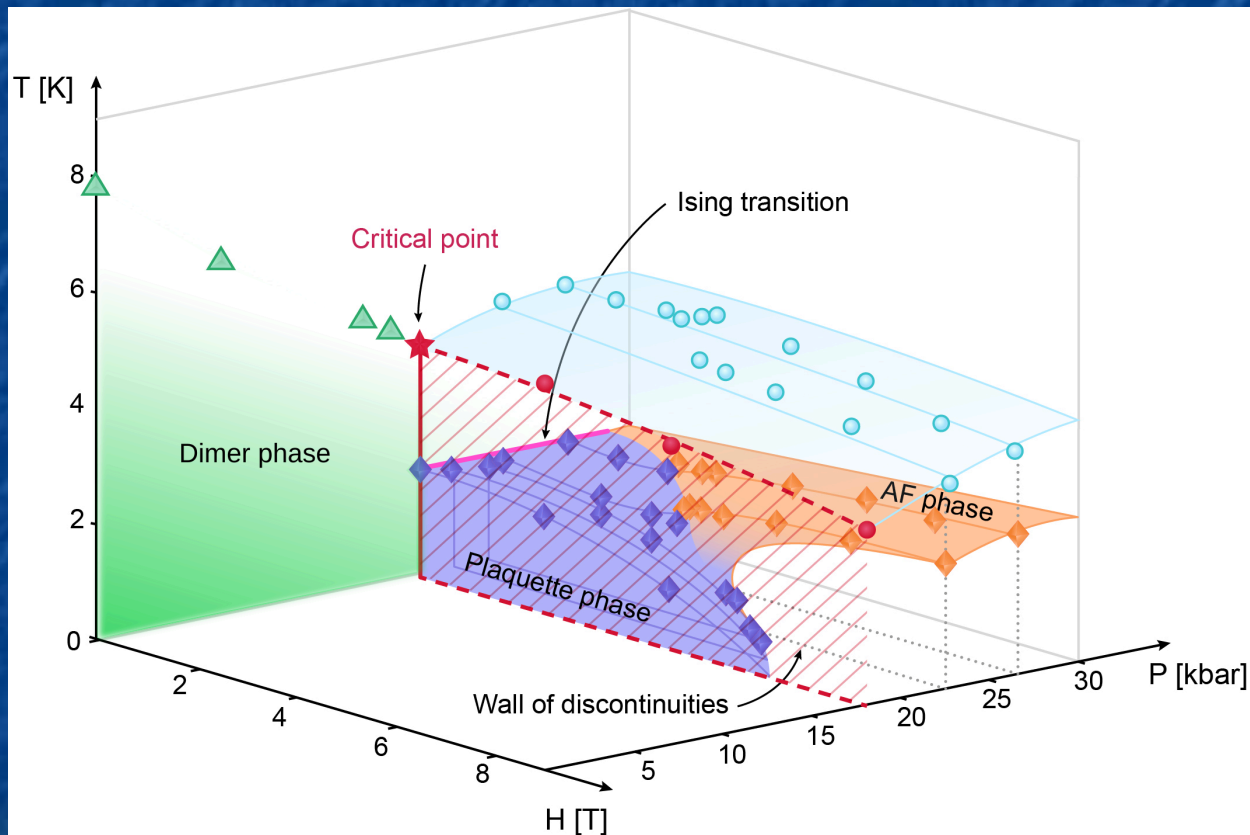
More realistic model for  $\text{SrCu}_2(\text{BO}_3)_2$

# Effect of magnetic field

- First-order transition between two **gapped phases**
  - should be **insensitive to magnetic field**
- Transition between plaquette and Néel phases
  - **Néel phase should be favored** by magnetic field



# (P,T,H) phase diagram



J. Larrea ... P. Corboz ... H. Ronnow, FM, unpublished

# Conclusion

- $\text{SrCu}_2(\text{BO}_3)_2$  is one of the **most interesting quantum antiferromagnets** ever synthesized
  - **Spin gap**
  - Unique series of **plateaus at  $1/8, 2/15, 1/6, 1/4, 1/3, 1/2$**  (and maybe  $2/5$ )
  - **Topological magnetic excitations** in a small field
  - Probably **spin nematic** and **spin supersolid phases**
  - A **critical point** analogous to that of water
- Main challenge: most of the action takes place above 27 Tesla or above 20 kbar at very low temperature

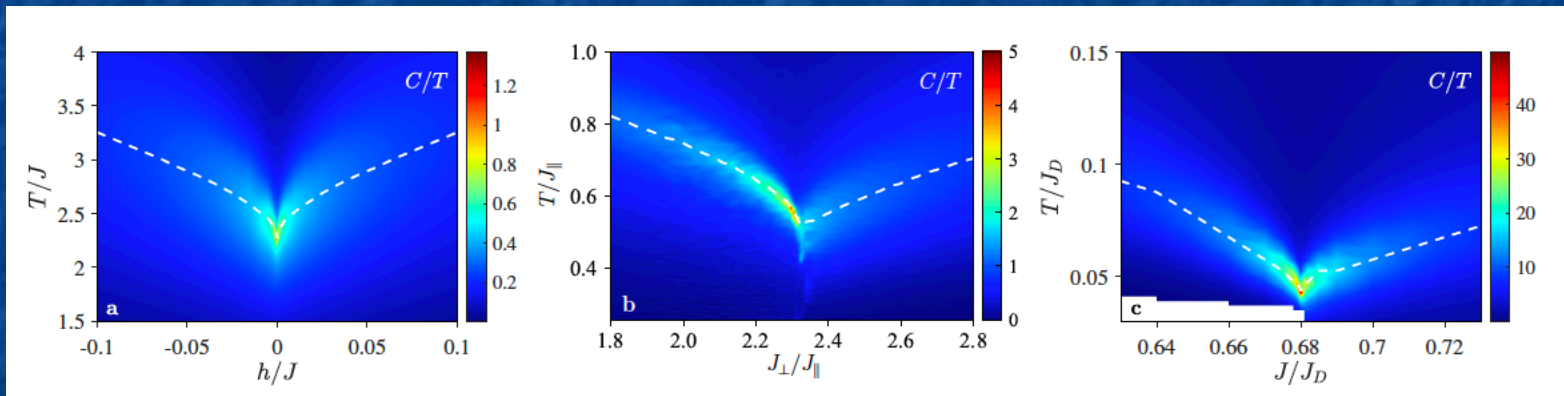
# Perspectives

- Check the presence of **first-transition line** below  $T=3.3\text{K}$
- Probe the **supercritical regime** above  $T_c$  present in that and related models

Ising in a field

FFB

Shastry-Sutherland



First order quantum transitions not so boring after all !