Composite fermion zero modes:

From the Jain sequence to the integer quantum Hall transition

Srinivas Raghu (Stanford)

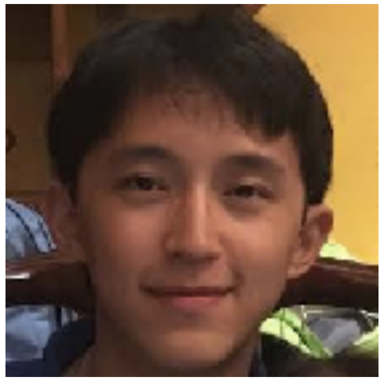
arXiv:1803.07767, 1805.06462, 1903.06297, 1907.13141
2006.11862, 2009.07871.

Prashant Kumar

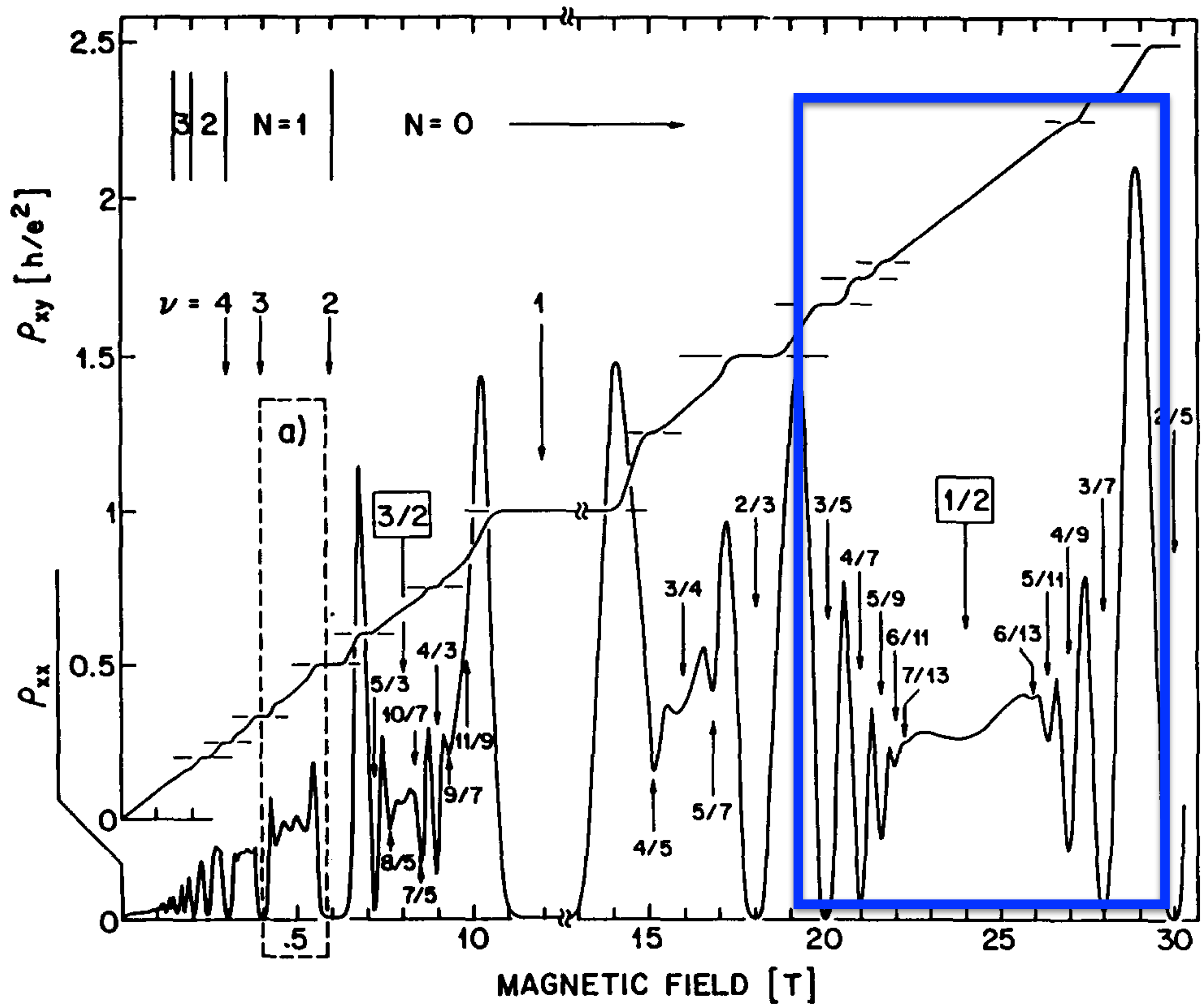
Michael Mulligan

Yong-Baek Kim

Pavel Nosov



Kevin Huang
(Stanford ’21)



Quantum of resistance:

$$\frac{h}{e^2} = 2\pi$$

Quantum of flux:

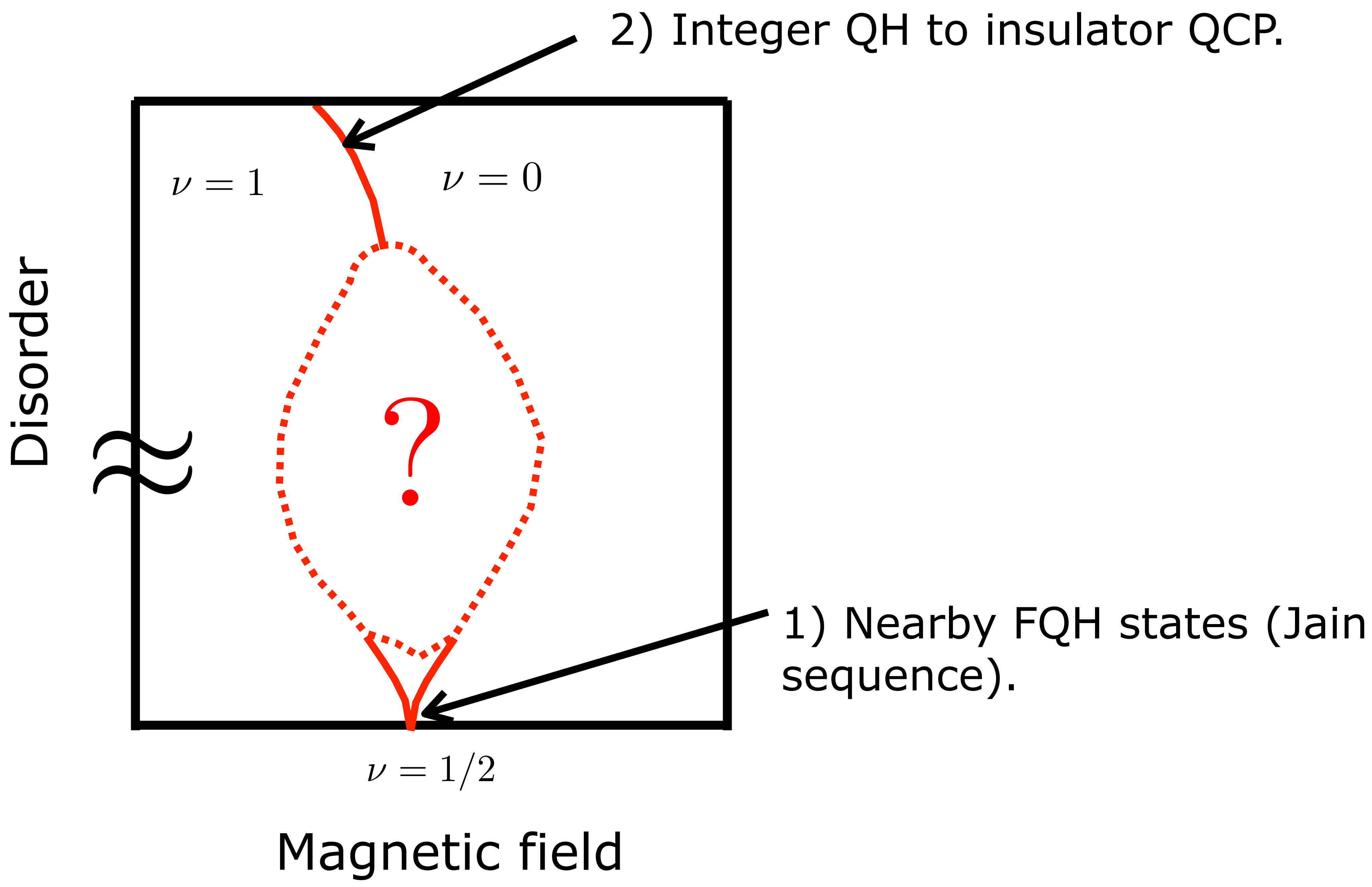
$$\frac{h}{e} = 2\pi$$

Filling fraction:

$$\nu = 2\pi \frac{n}{B}$$

D. Tsui, Phys. B (1990) 59.

Plan for the talk



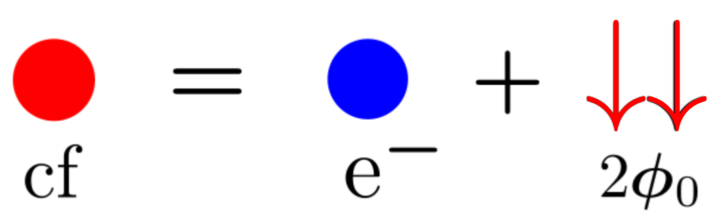
Composite fermions

Lopez, Fradkin; Jain; Halperin,Lee,Read; Kalmeyer, Zhang.

Composite fermions

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi}\underbrace{ada} + \cdots \qquad \hat{K}_A = iD_A^t + \frac{1}{2m}\vec{D}_A^2$$

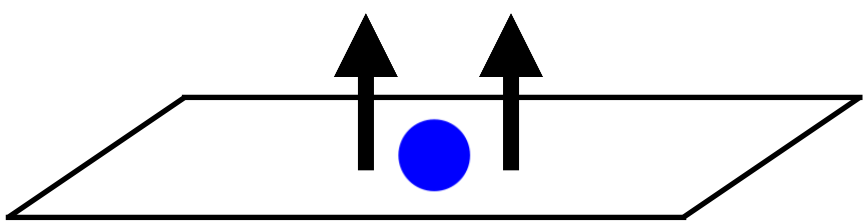
Chern-Simons term: $ada = \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$

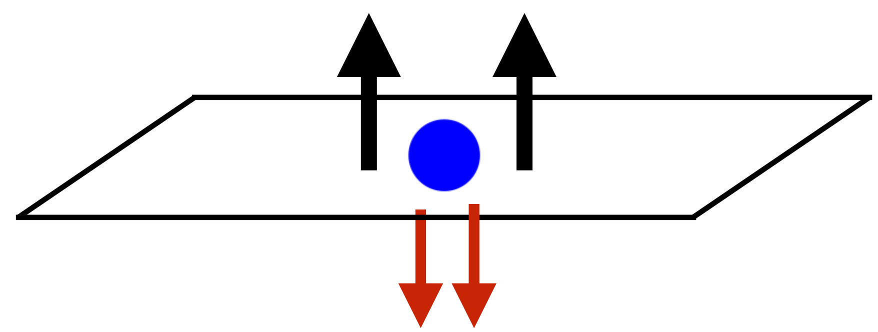


Flux-attachment

Composite fermions and the half-filled LL

$\nu = 1/2$ $\quad\Rightarrow\quad$ $\nu_{cf} = \infty$

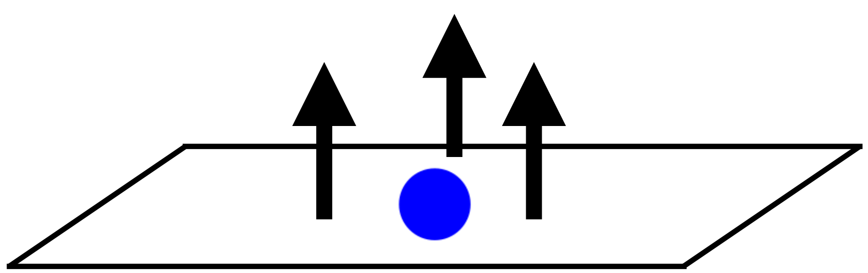


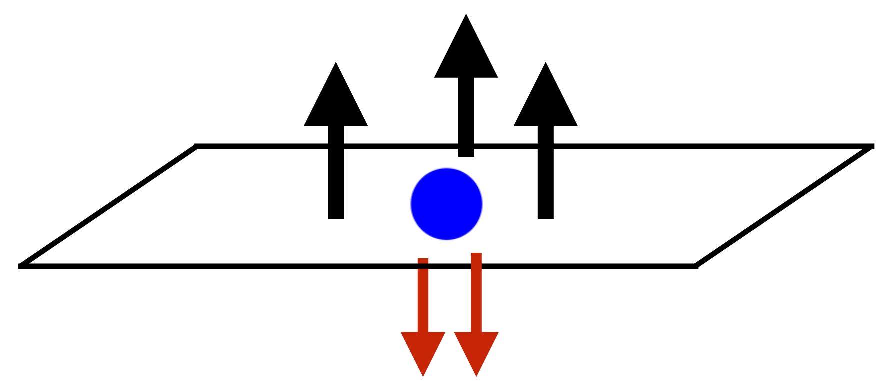


Electrons in a large field $\quad\Rightarrow\quad$ CF Fermi sea

Composite fermions and Jain sequence

$e.g.\ \ \nu = 1/3 \quad \Longrightarrow \quad \nu_{cf} = 1$

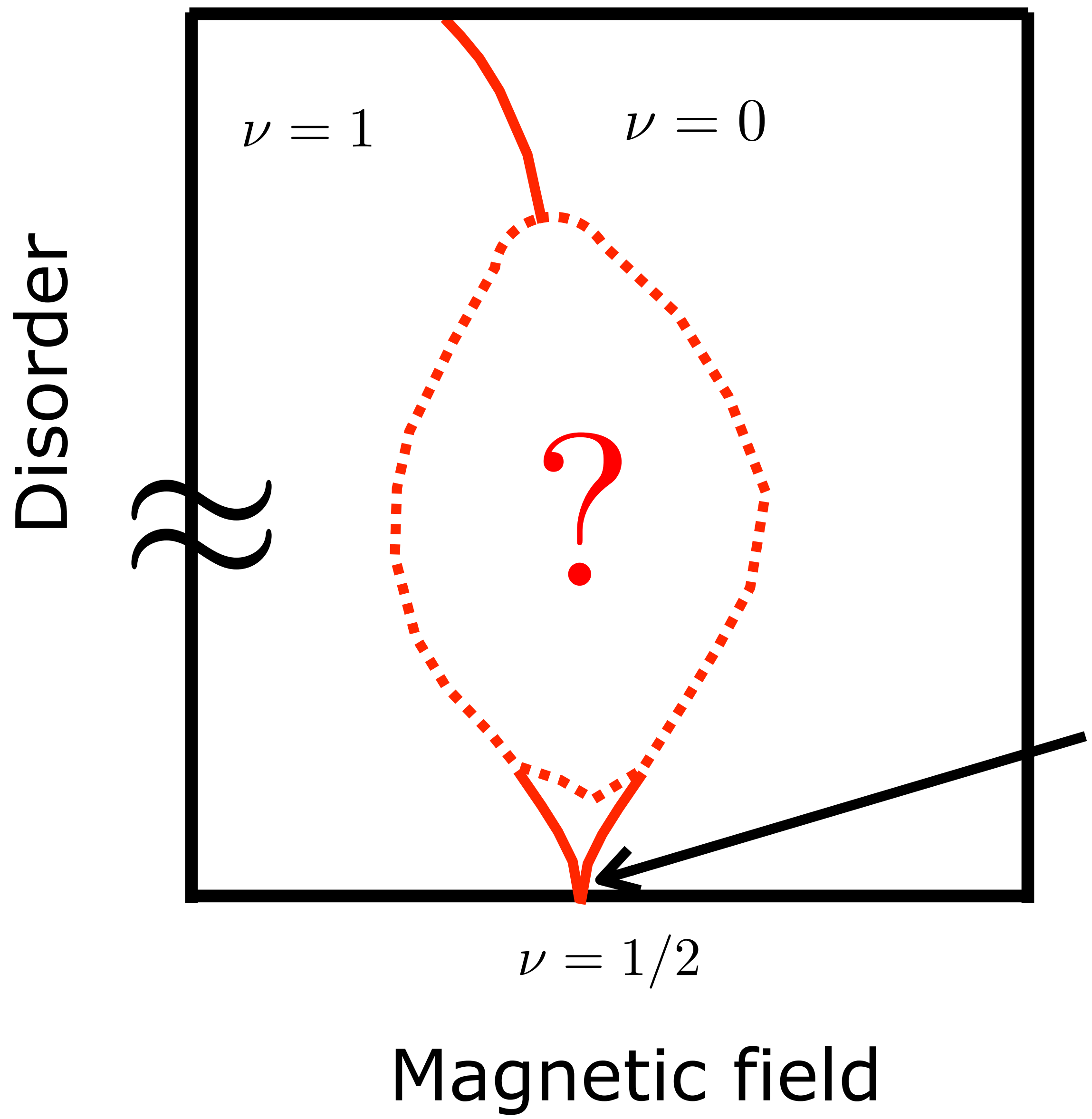




Jain sequence:

$$\nu = \frac{p}{2p+1} \quad \Longrightarrow \quad \nu_{cf} = p$$

Jain sequence: integer quantum Hall states of CFs



I. particle-hole symmetry of Jain sequence.

Prashant Kumar, Michael Mulligan, SR, PRB 2019.

Particle-hole symmetry of Jain sequence

$$\nu = \frac{p}{2p+1} \quad \xRightarrow{\text{ph}} \quad 1 - \nu = \frac{p+1}{2p+1}$$

e.g.

$p = 1$

$$\nu = \frac{1}{3} \quad \xRightarrow{\text{ph}} \quad \nu = \frac{2}{3}$$

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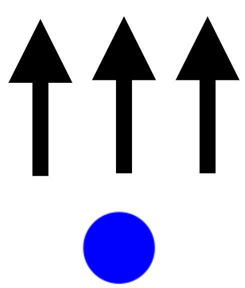
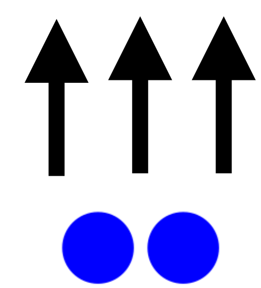


$$\overset{\text{ph}}{\Longrightarrow}$$



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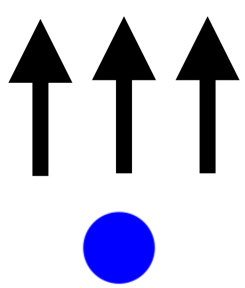

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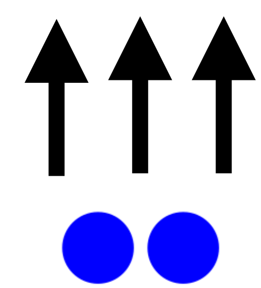
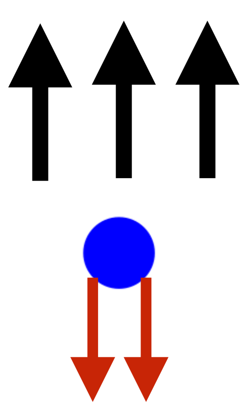
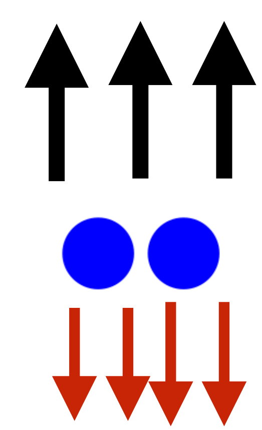


$\nu_{cf} = 1$



$$\overset{\text{ph}}{\Longrightarrow}$$



$\nu_{cf} = -2$

Particle-hole symmetry of Jain sequence

$$\nu = \frac{p}{2p+1} \quad \overset{\text{ph}}{\Longrightarrow} \quad 1 - \nu = \frac{p+1}{2p+1}$$

e.g.

$p = 1$

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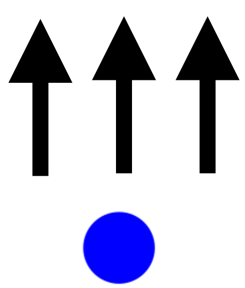
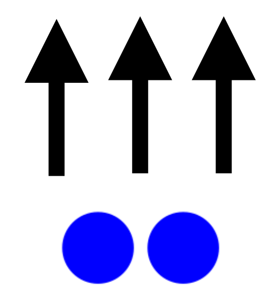
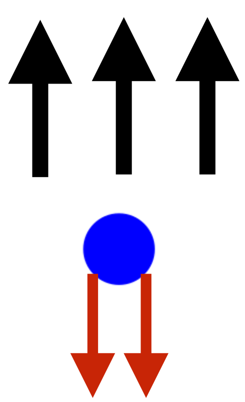


$\overset{\text{ph}}{\Longrightarrow}$



$\nu_{cf} = 1$



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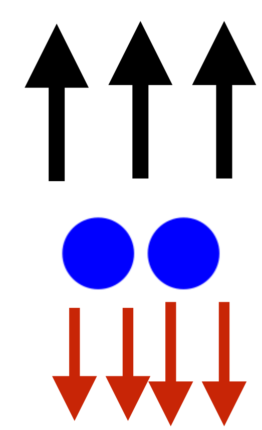

$\nu_{cf} = -2$

Resolution: CF zero mode.

Particle-hole symmetry of CFs

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

$$B = \nabla \times A$$
$$b = \nabla \times a$$

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Deviate slightly from half-filling: $\mu = \mu_{1/2} + V$

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$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu_{1/2} - \frac{b+B}{2m}\right) f + \frac{1}{2}\frac{1}{4\pi} a da + \cdots$$

$$\text{shift} : a \to a - A$$

Particle-hole symmetry of CFs

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} a da + \cdots$$

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$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_a + \mu_{1/2} - \frac{b}{2m}\right) f + \frac{1}{2}\frac{1}{4\pi}(a - A)d(a - A) + \cdots$$

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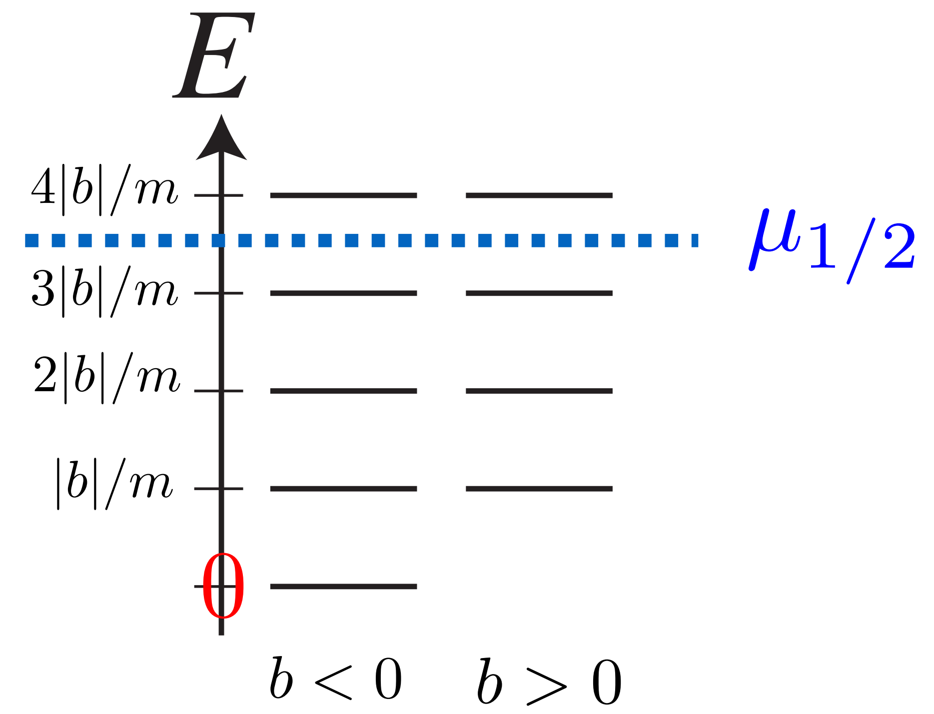
CF Landau Levels (Jain sequence):

$$E_n = \frac{|b|}{m}\begin{cases} n+1, & b > 0 \\ n, & b < 0 \end{cases}$$



CF Zero mode occurs only for b<0.

p filled LLs for b>0: p+1 filled LL for b < 0.

Including zero mode: crucial for PH symmetry.

Electromagnetic response

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_a + \mu_{1/2} - \frac{b}{2m}\right) f + \frac{1}{2}\frac{1}{4\pi}(a - A)d(a - A) + \cdots$$

Let p Landau levels be filled for b>0, p+1 for b<0.

Integrate out CFs, a, to obtain EM response:

$$\mathcal{L}^{\text{eff}}_{b>0} = \frac{1}{4\pi}\frac{p}{2p+1}AdA, \qquad \nu = \frac{p}{2p+1}$$

$$\mathcal{L}^{\text{eff}}_{b<0} = \frac{1}{4\pi}\frac{p+1}{2p+1}AdA, \qquad \nu = \frac{p+1}{2p+1}$$

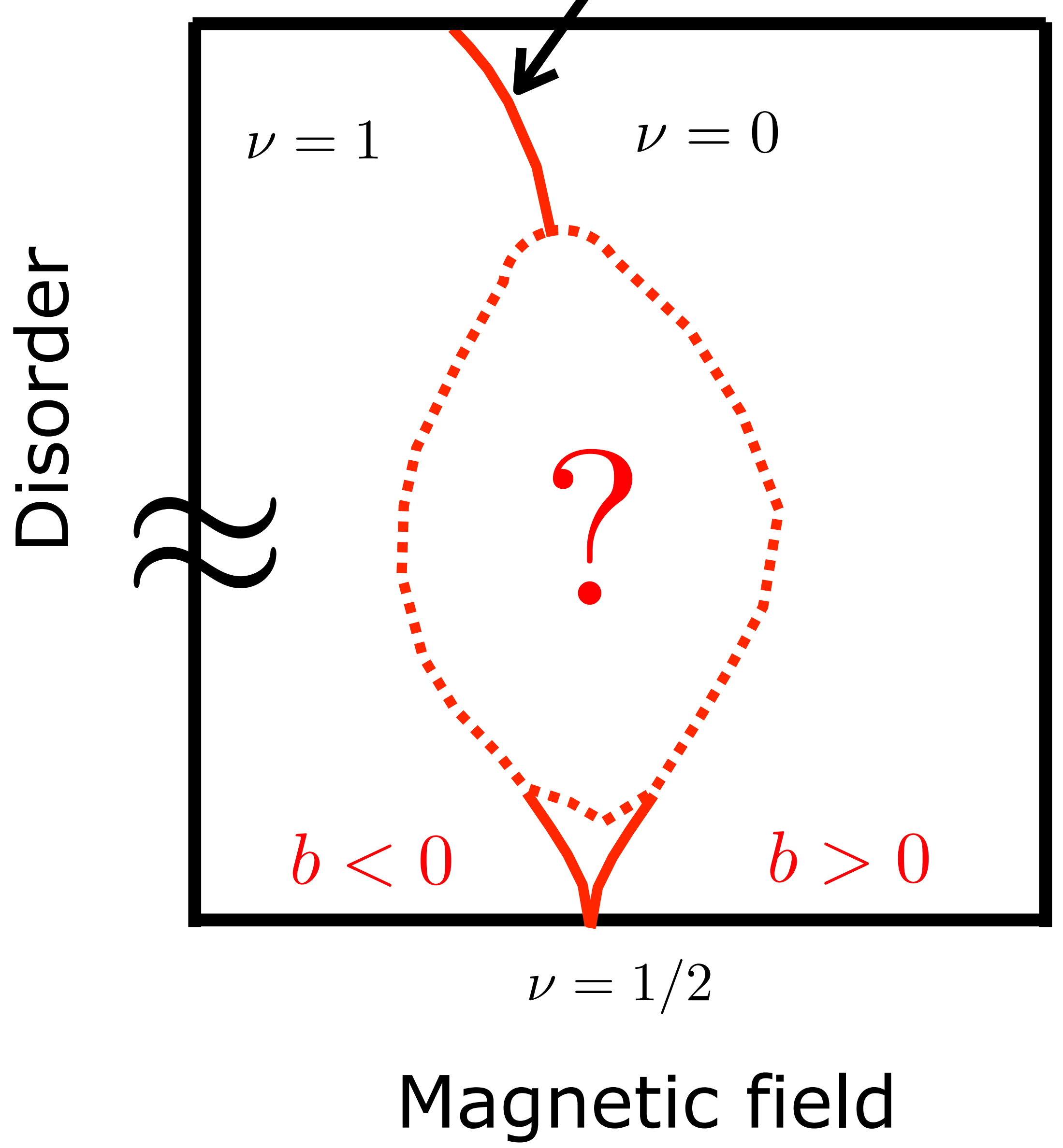
Including the zero mode, we recover ph symmetry.

PH for electrons = T for CFs.

$\nu \to 1 - \nu$ electrons

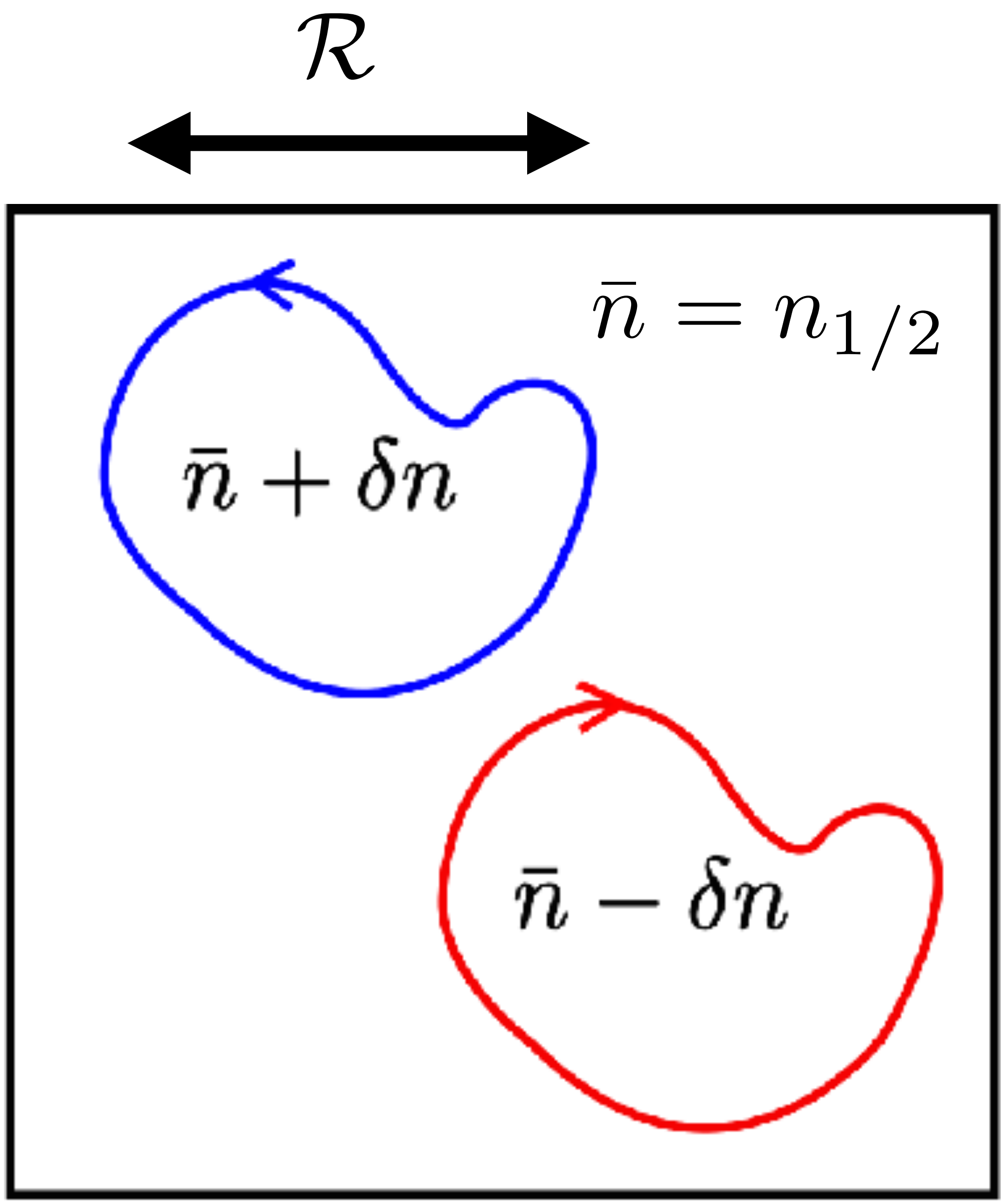
$b \to -b$ cfs

II. The IQH to Insulator transition.



Prashant Kumar, Yong-Baek Kim, SR arXiv:1907.13141.

Disorder of interest



Statistical PH symmetry:

$$\overline{V(r)} = 0$$

$$\overline{V(r)V(r')} = \Delta e^{-\left(\mathbf{x}-\mathbf{x}'\right)^2/\mathcal{R}^2}$$

Long-wavelength disorder:

$$\mathcal{R} \gg \ell_B$$

CFs with disorder

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

Before: tuning away from half-filling:

$$\mu = \mu_{1/2} + V$$

Now: quenched random potential:

$$\mu(r) = \mu_{1/2} + V(r)$$

Before: we studied

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_a + \mu_{1/2} - \frac{b}{2m}\right) f + \frac{1}{2}\frac{1}{4\pi}(a-A)d(a-A) + \cdots$$

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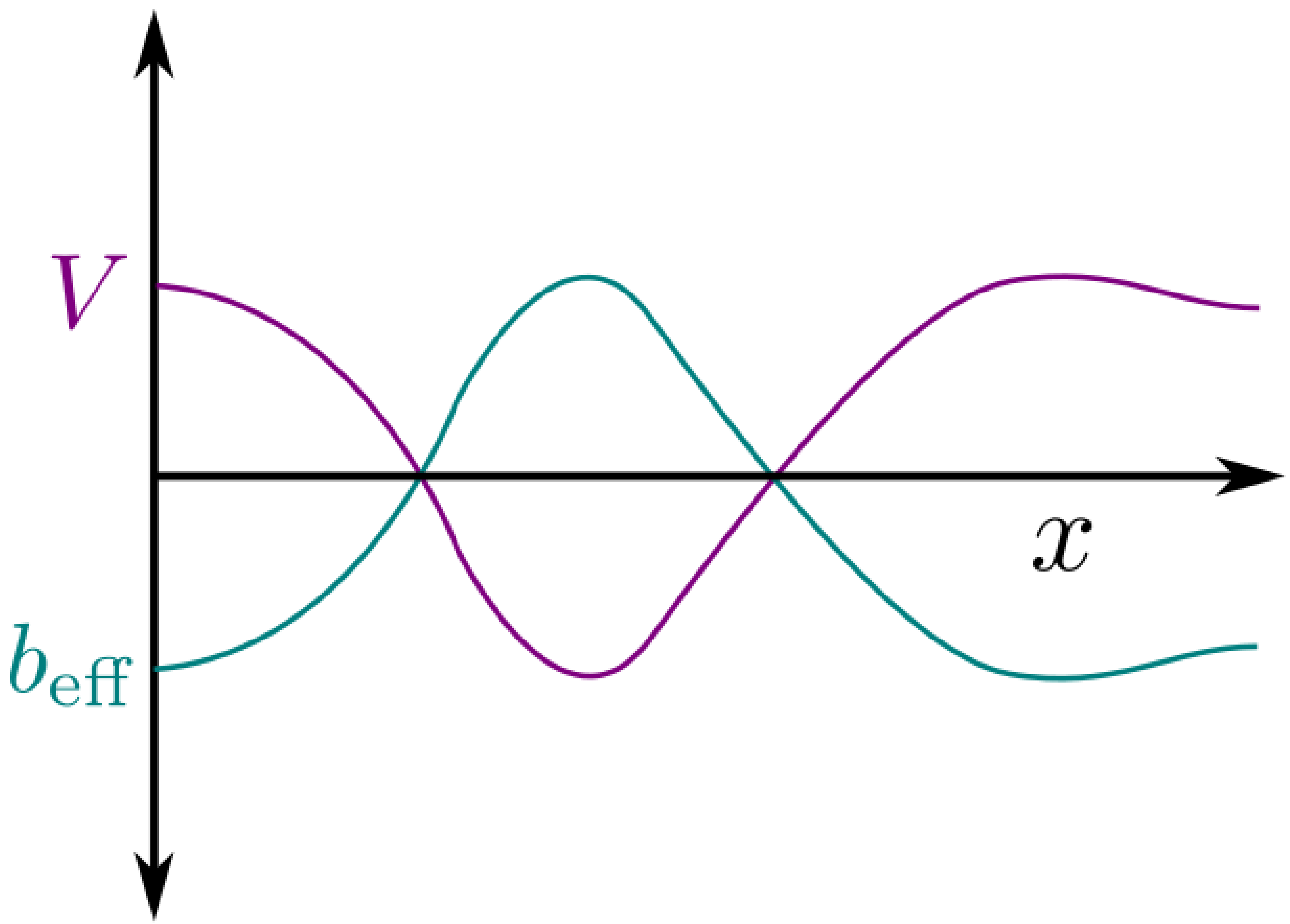
Now: with disorder

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{a} + \mu_{1/2} - \frac{b(r)}{2m}\right) f + \frac{1}{2}\frac{1}{4\pi}(a-A)d(a-A) + \cdots$$

Disorder problem: random potential slaved to random flux.

CFs with disorder



CFs with disorder

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Associated 1st quantized Hamiltonian:

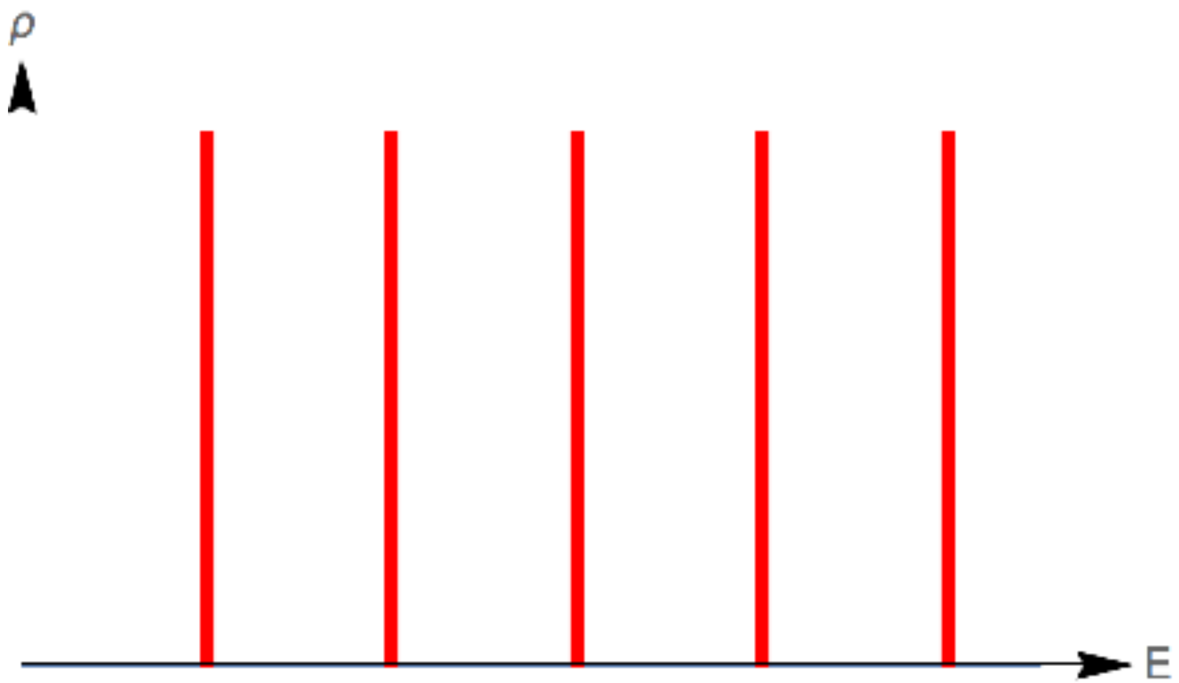
$$\mathcal{H}_{cf} = \frac{1}{2m}\left[(\boldsymbol{p}+\boldsymbol{a})^2 + b(r)\right], \quad b(r) = \nabla \times a(r)$$

Disorder problem: random potential slaved to random flux.

Incomplete LL levitation

Start with slight deviation from half-filling. Increase disorder.

$$\overline{b} > 0$$

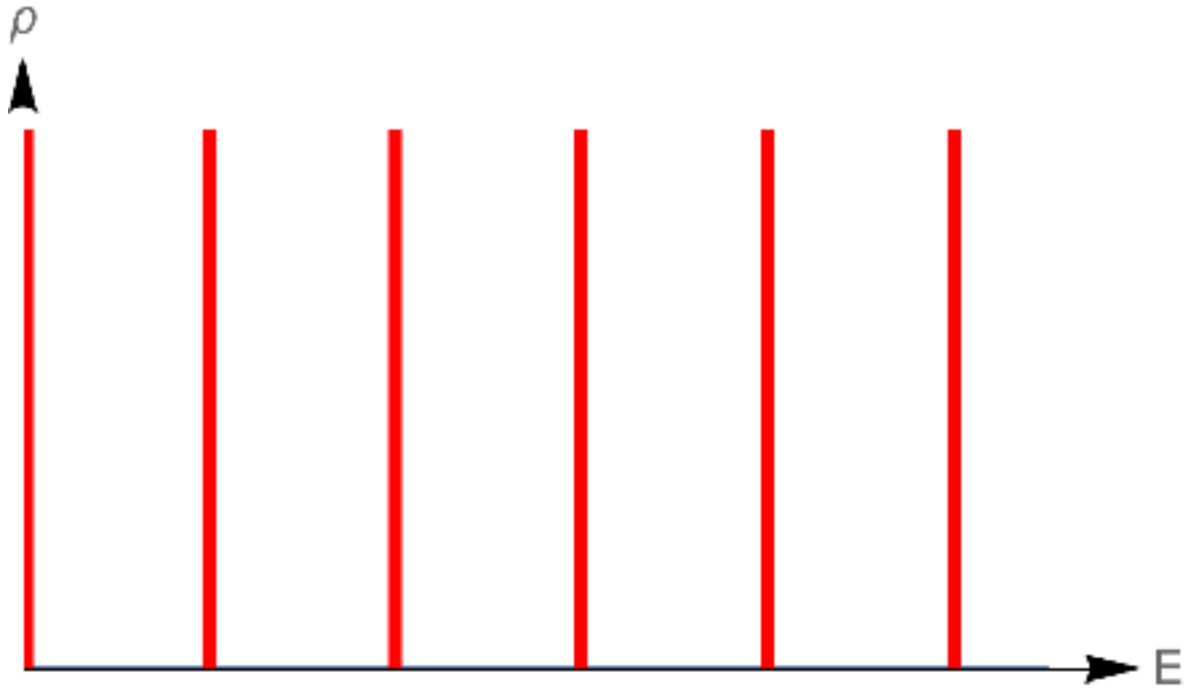


D.E. Khmelnitskii, Phys. Lett. A **106**, 182 (1984).
R.B. Laughlin, PRL **52**, 2304 (1984).

Incomplete LL levitation

Zero mode does not levitate!

$$\overline{b} < 0$$



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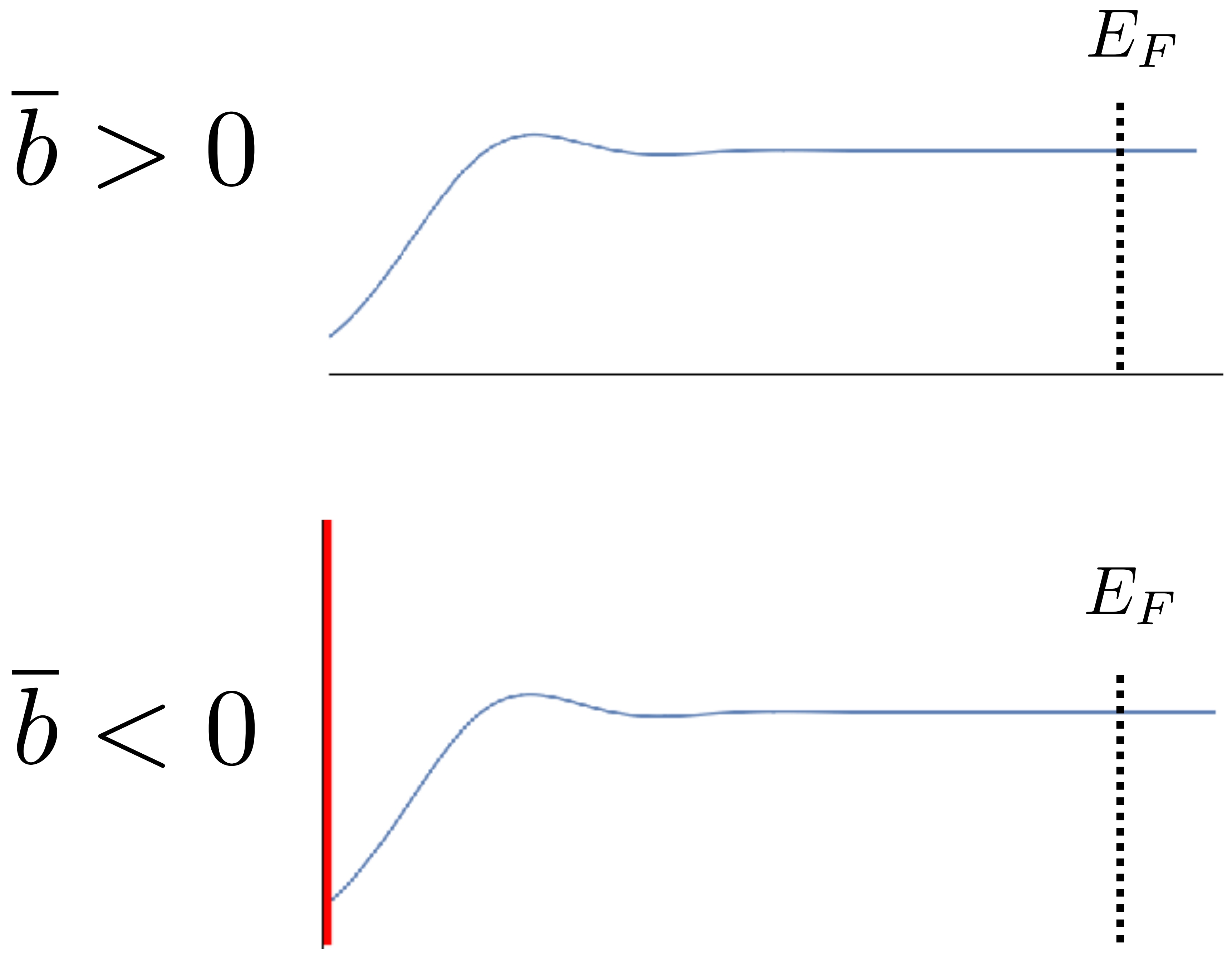
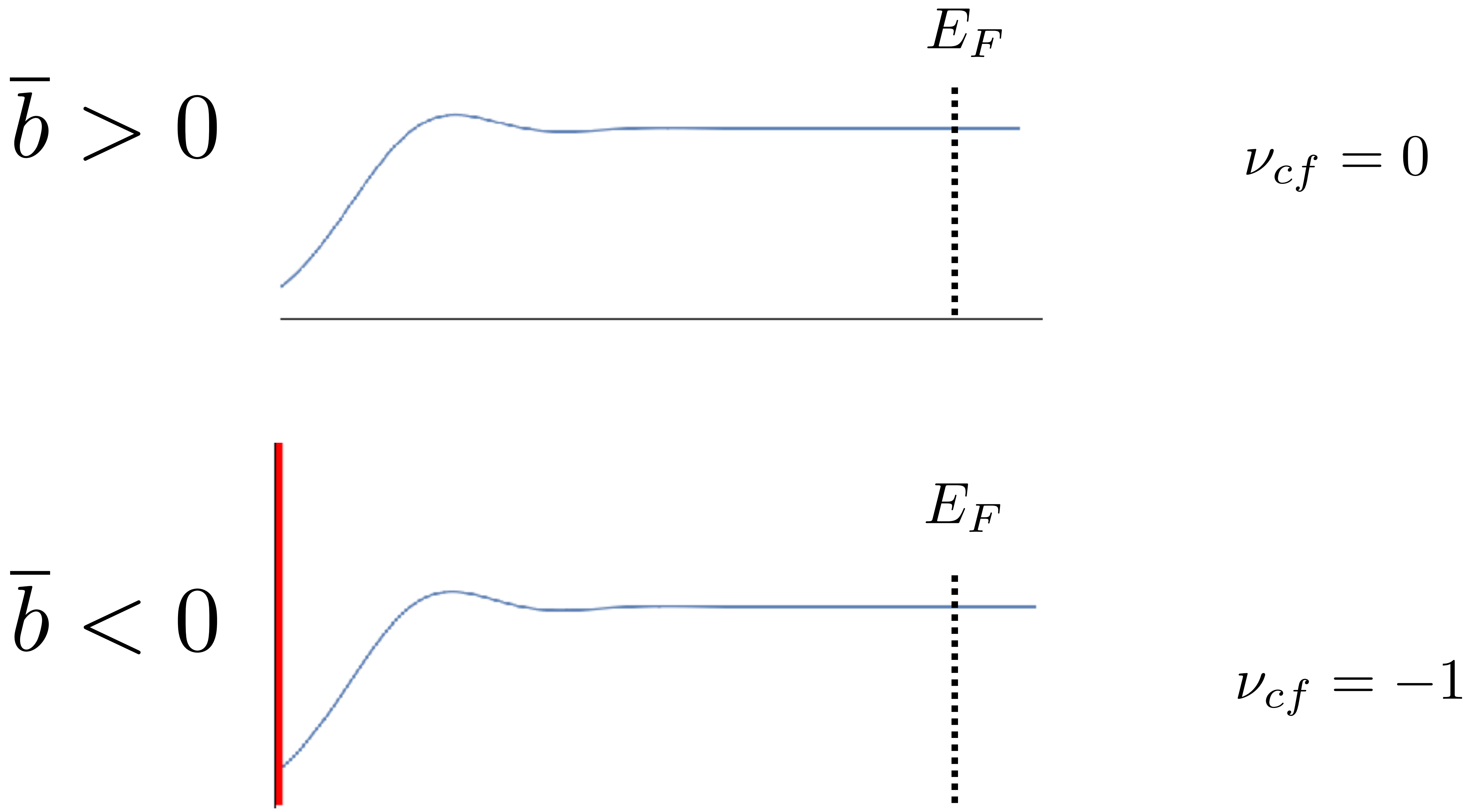
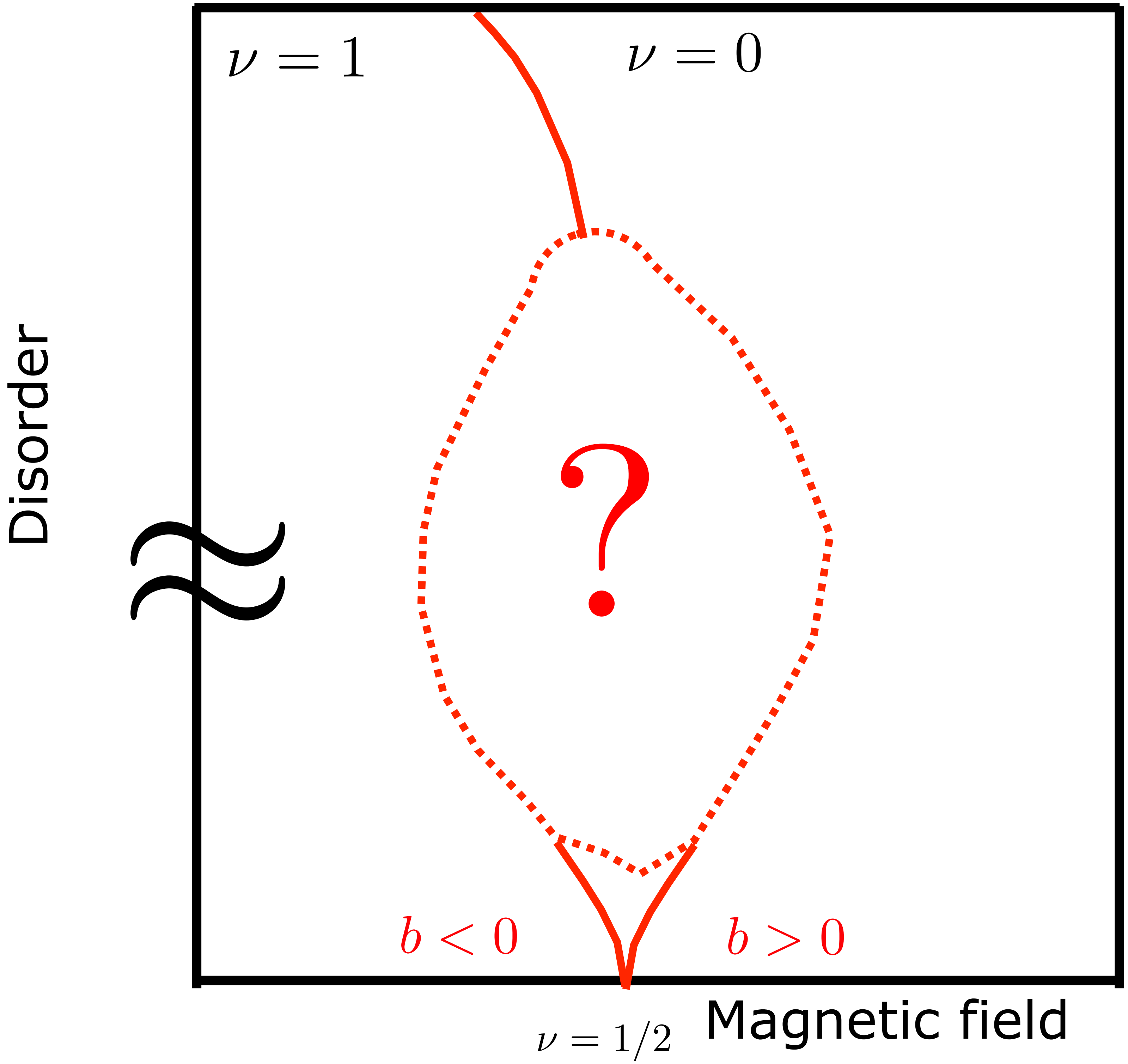


$\overline{b} > 0$
E_F
$\overline{b} < 0$
E_F



$\overline{b} > 0$
E_F
$\nu_{cf} = 0$
$\overline{b} < 0$
E_F
$\nu_{cf} = -1$



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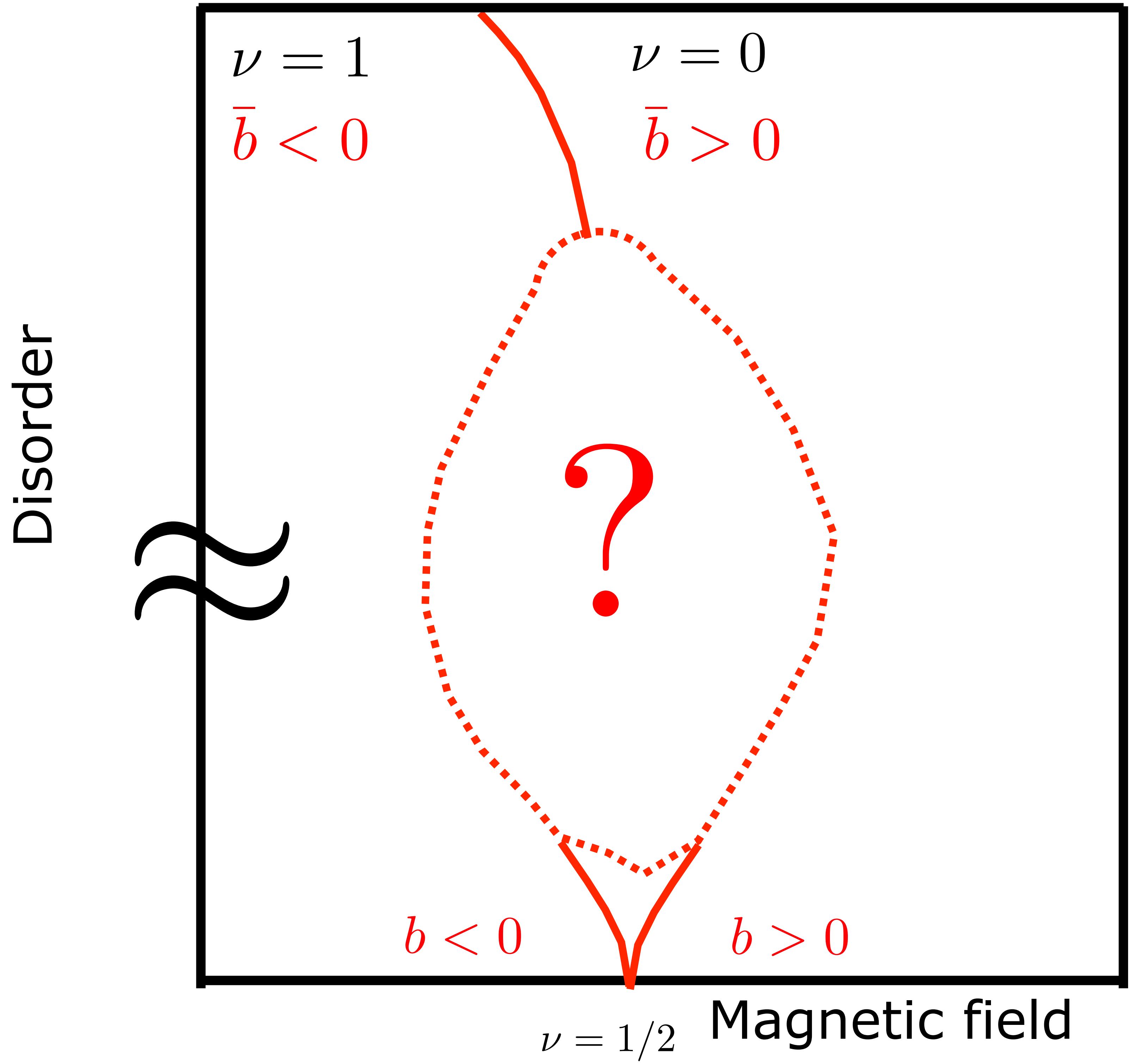

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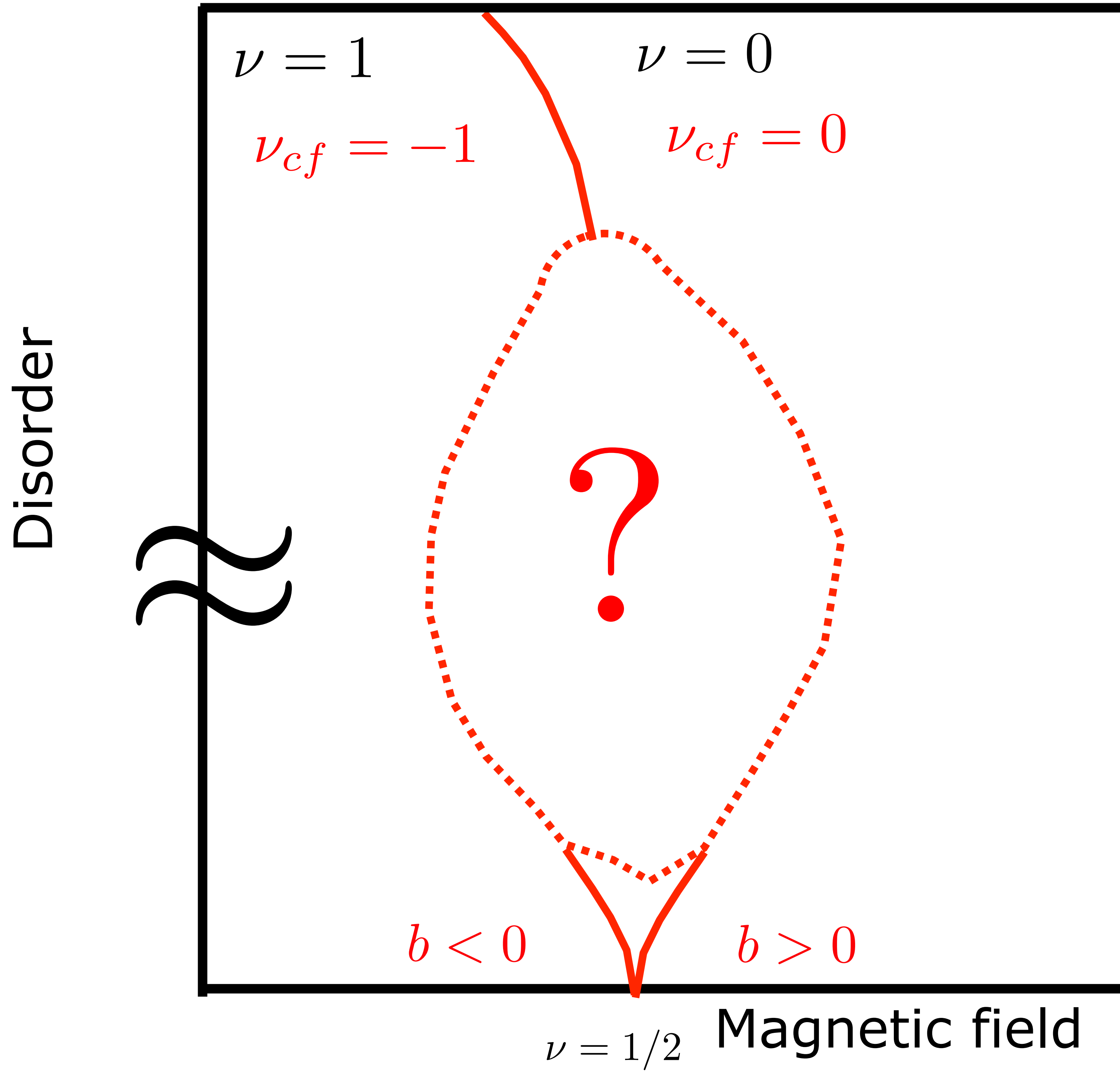

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“Divide and conquer” approach

1) σ_{xy}^{cf} at criticality: analytical calculation using SUSY QM.

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2) σ_{xx}^{cf} at criticality: explicit derivation of NLSM Lagrangian, self-duality.

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1) and 2) uniquely fix the electrical conductivity tensor at criticality.

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3) Numerical study of critical exponents.

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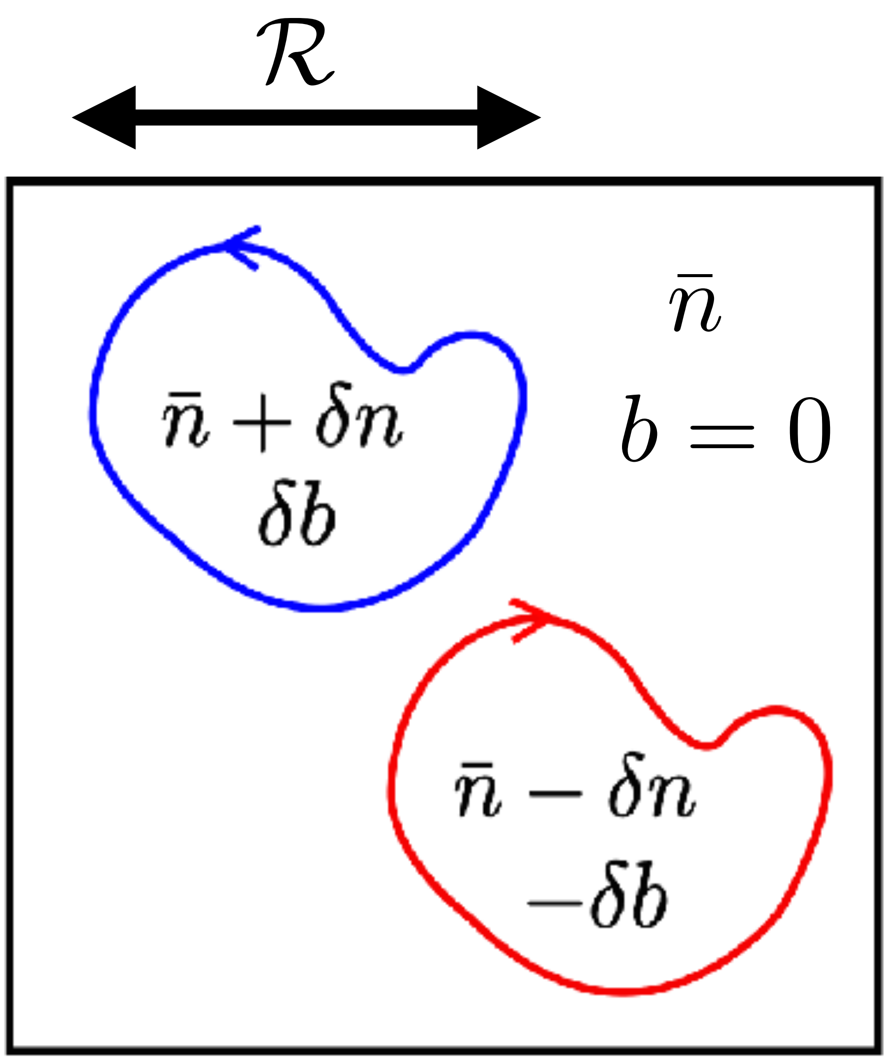
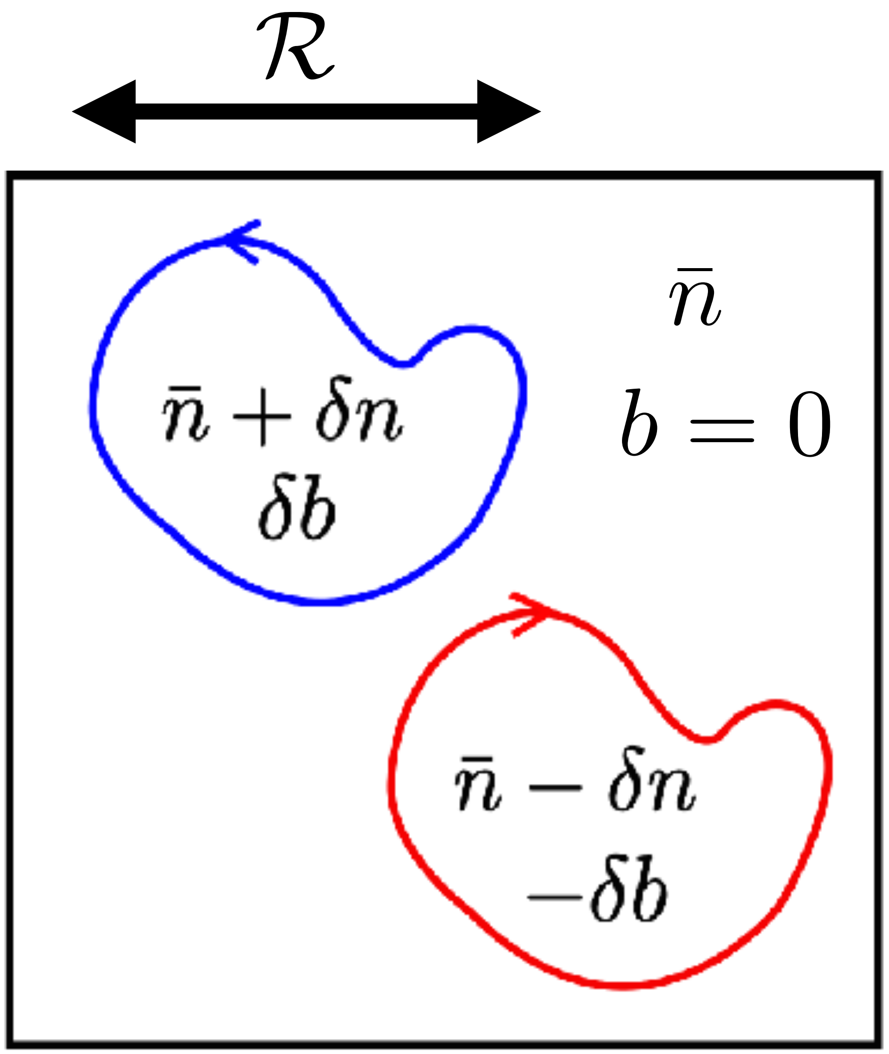
Intuitive argument

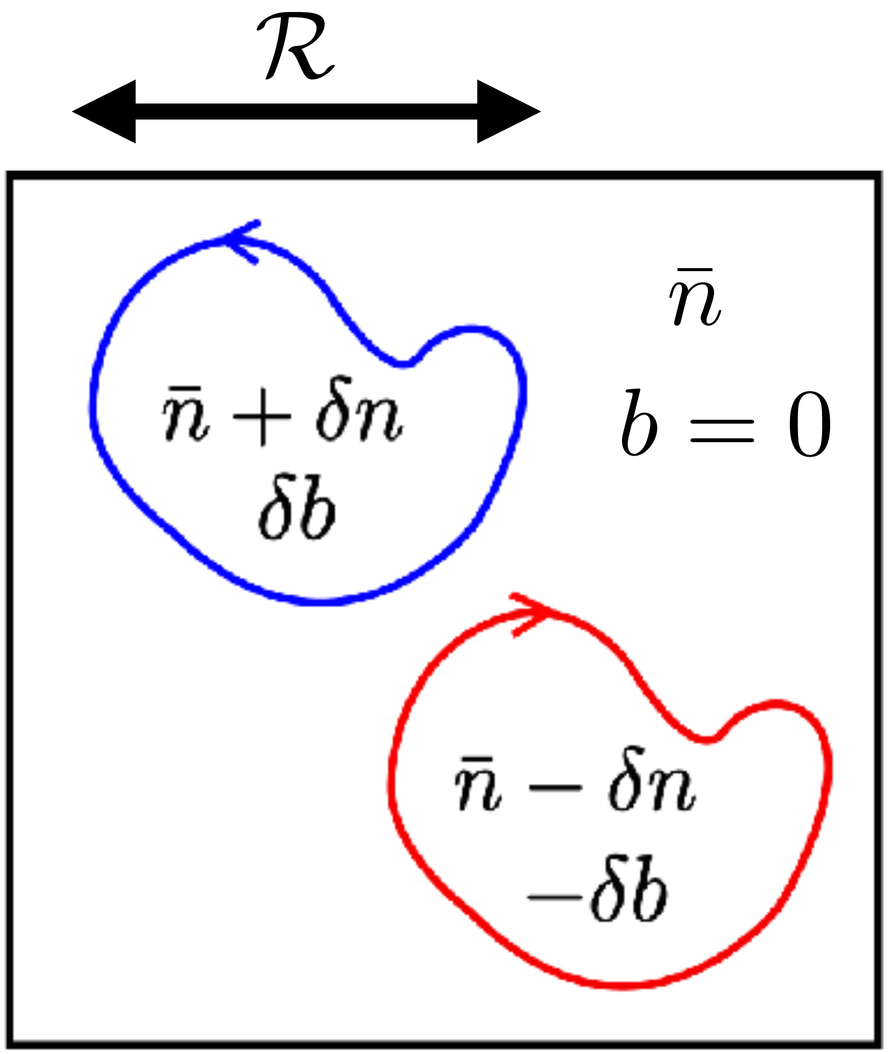


Intuitive argument



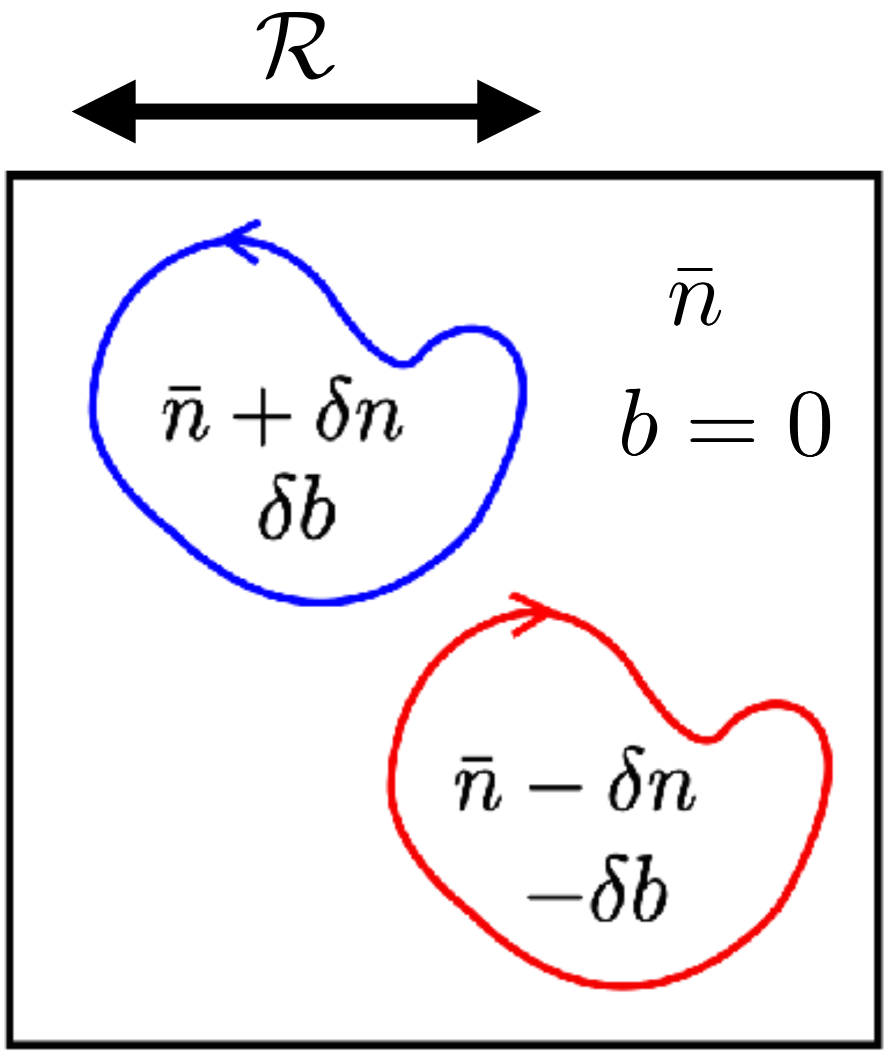
$$\nu_{eff} = 2\pi \frac{(\bar{n} + \delta n) - (\bar{n} - \delta n)}{2\delta b} = 2\pi \frac{\delta n}{\delta b}$$

Intuitive argument



$$\nu_{eff} = 2\pi \frac{(\bar{n} + \delta n) - (\bar{n} - \delta n)}{2\delta b} = 2\pi \underbrace{\frac{\delta n}{\delta b}}_{-\frac{1}{4\pi}}$$

Intuitive argument



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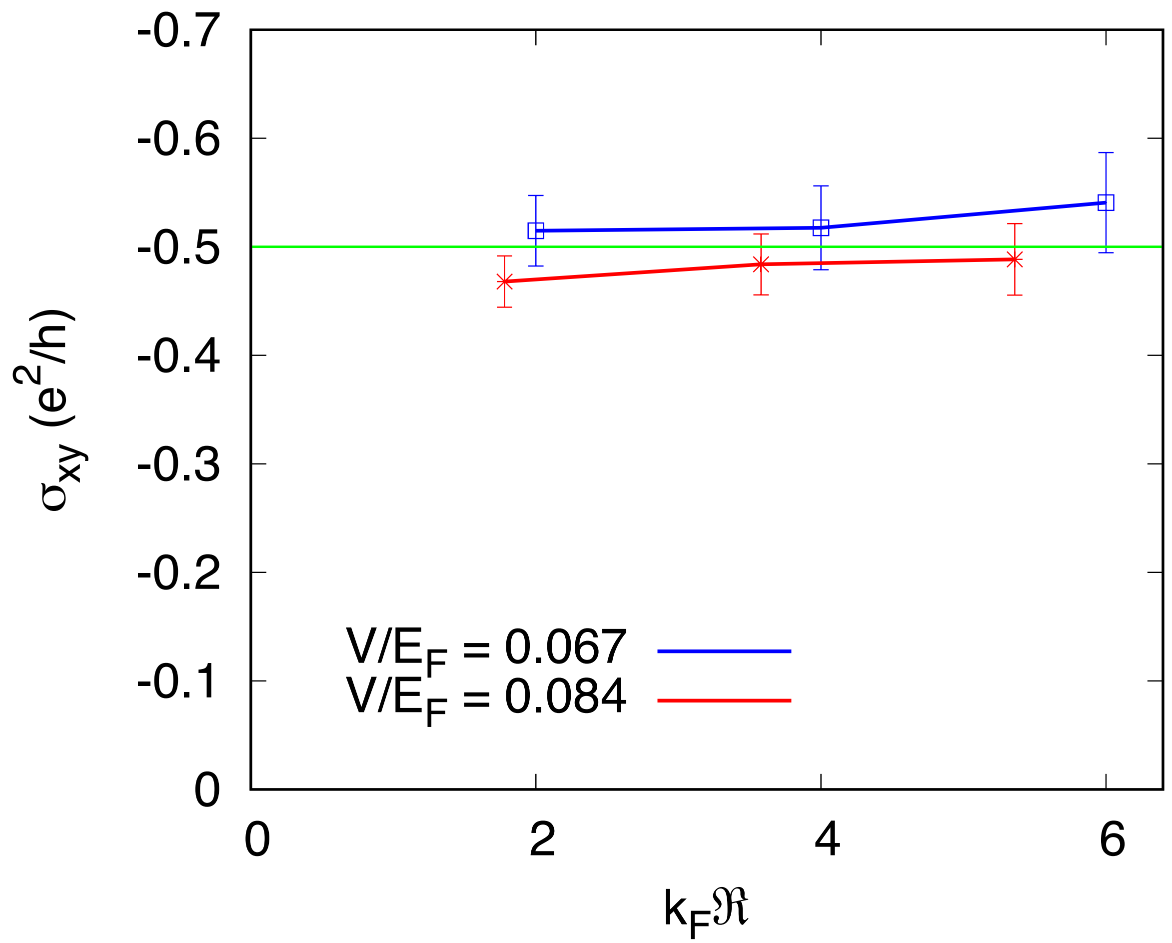
$$\nu_{eff} = -\frac{1}{2}$$

$$\sigma_{xy}^{cf} = -\frac{1}{4\pi}$$

Analytic proof using SUSY QM: P. Kumar, M. Mulligan, SR, 1805.06462.

CFs with disorder

$$\mathcal{H}_{cf} = \frac{1}{2m}\left[(\boldsymbol{p}+\boldsymbol{a})^2 - b(r)\right], \quad b(r) = \nabla \times a(r)$$



Numerical result:

$$\sigma_{xy}^{(cf)} = -\frac{1}{4\pi}$$

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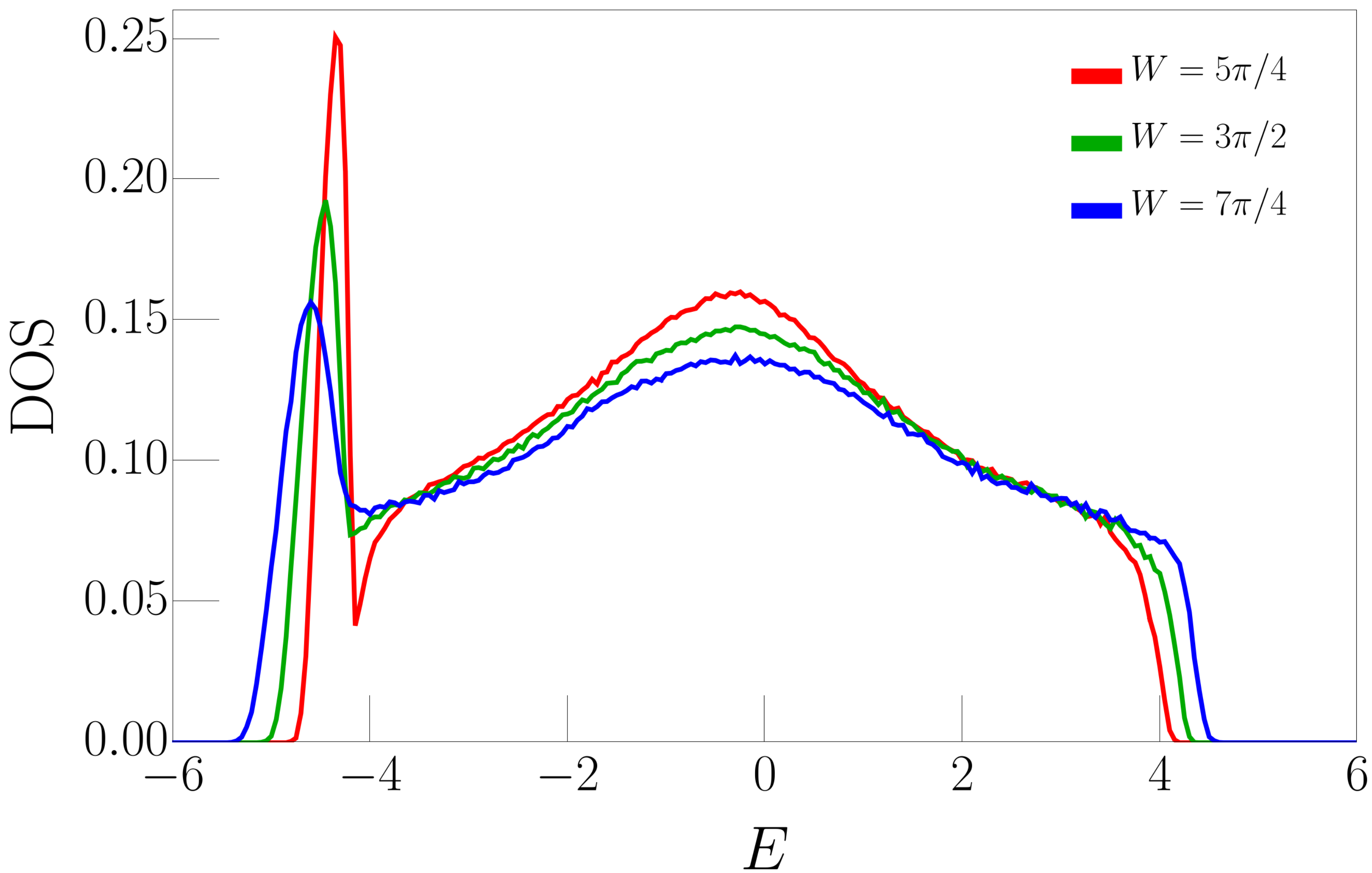
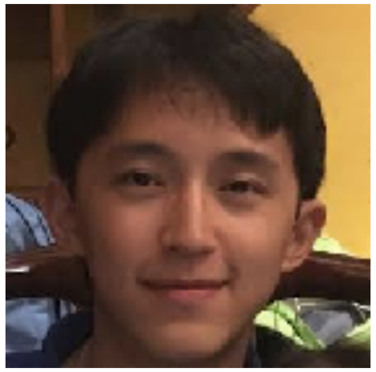


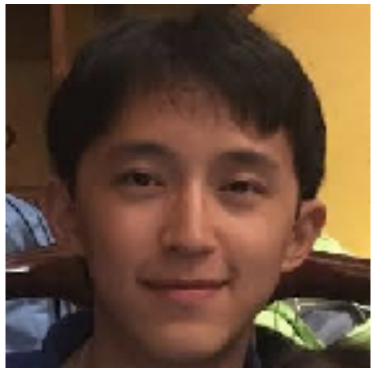


Kevin Huang, SR, Prashant Kumar
Arxiv:2009.07871

Zero modes clearly visible in numerics.

Mean-field exponents

$$\xi \sim |b_0|^{-\nu} \qquad \nu = 2.56 \pm 0.02$$



Kevin Huang, SR, Prashant Kumar
Arxiv:2009.07871

Previous work (Chalker-Coddington model):

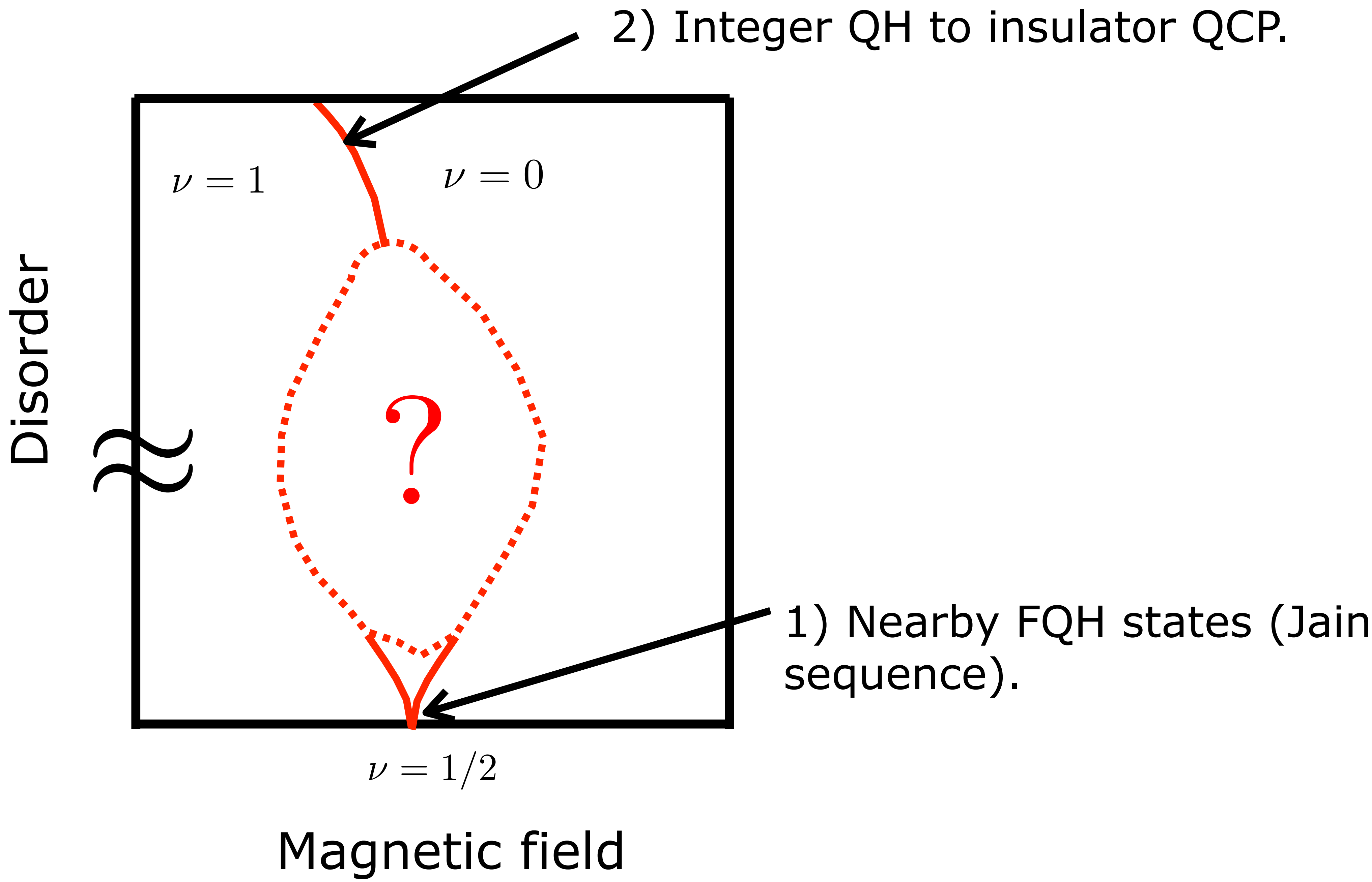
$$\nu = 2.593 \pm 0.01$$

Multifractal wave-functions: $P_q \equiv L^d \langle |\psi|^{2q} \rangle \propto L^{-2(q-1)-\Delta(q)}$

$$\Delta(q) \approx 2q(1-q)\gamma, \gamma = 0.129 \pm 0.005$$

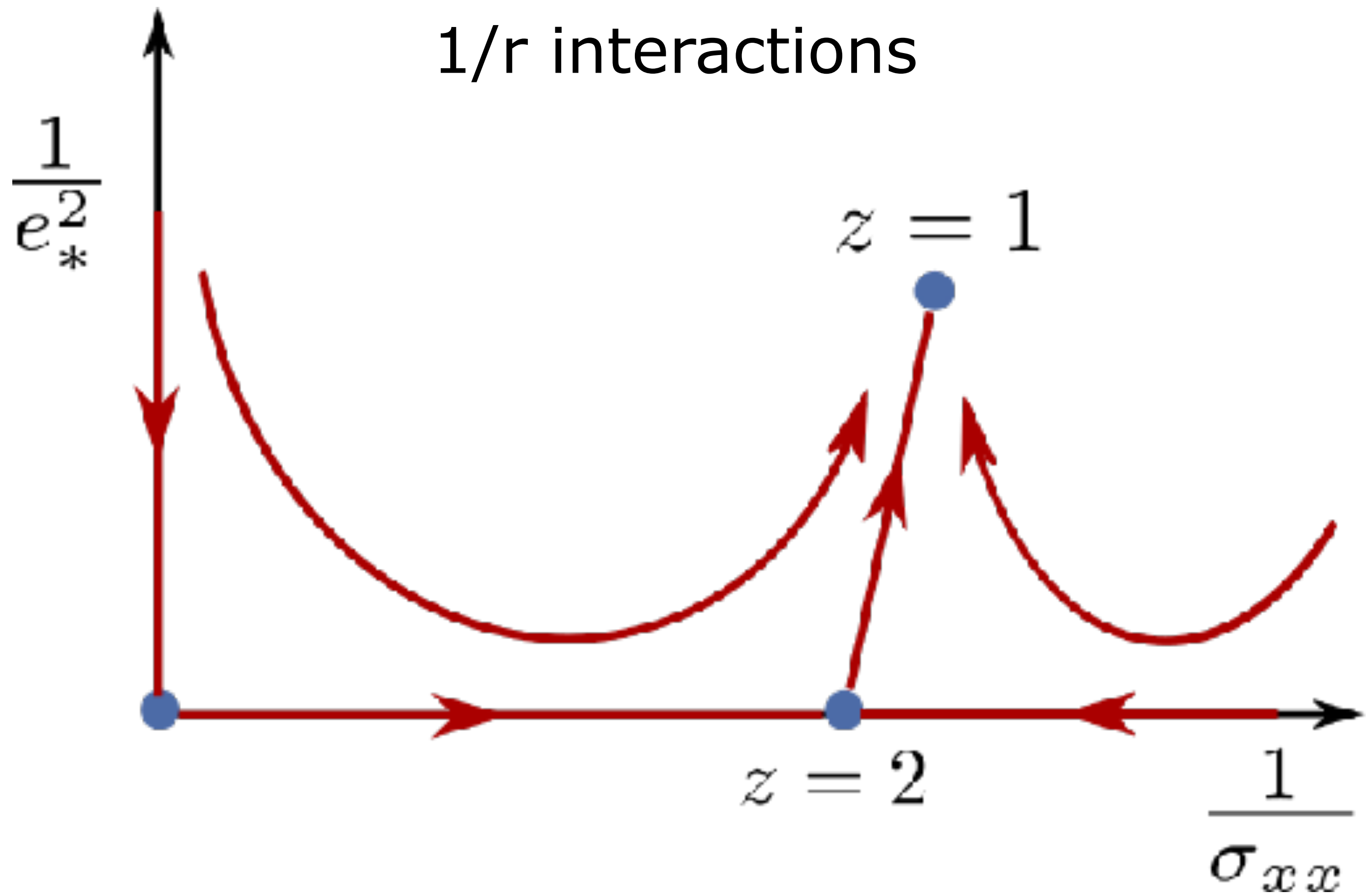
Analytical prediction for Chalker-Coddington model: $\gamma = \frac{1}{8}$ M. Zirnbauer, Nucl. Phys. B 941, 458-506 (2019).

Summary



CF zero modes: crucial for both 1) and 2).

Looking ahead..



Theme: Composite fermion viewpoint of QH critical points.