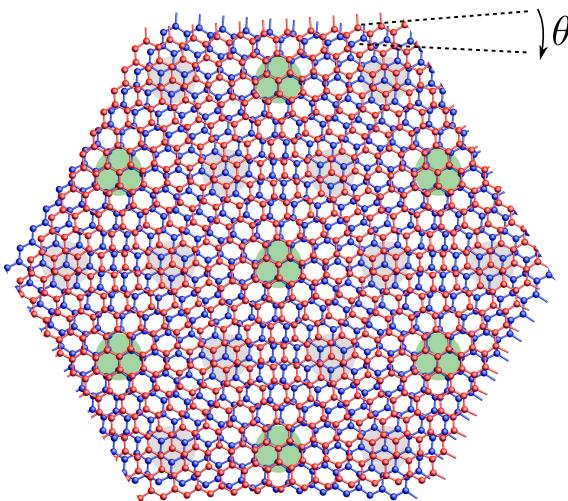


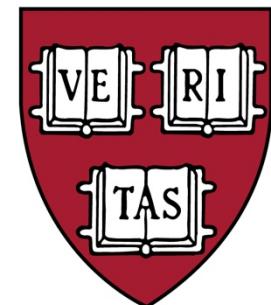
Superconductivity, its relation to the correlated insulators, & field-controlled nematicity in moiré systems



Mathias S. Scheurer

$$N(t) (| \text{Harvard} \rangle + \alpha(t) | \text{UIBK} \rangle)$$

Virtual talk at KITP Program:
“Correlated systems with multi-component local Hilbert spaces”
Monday, 12/14/2020



 universität
innsbruck

Superconductivity, its relation to the correlated insulators, & field-controlled nematicity in moiré systems

Part I: Possible topological connection between superconductivity & correlated insulator via mutual **Wess-Zumino-Witten term**

Ref.: Christos, Sachdev, MSS, PNAS **117**, 29543 (2020).



M. Christos
(Harvard)



S. Sachdev
(Harvard)

connects to: S. Chatterjee's & E. Khalaf's & A. Vishwanath's talks

Superconductivity, its relation to the correlated insulators, & field-controlled nematicity in moiré systems

Part II: Nematic order and its control in twisted double-bilayer graphene

Refs.: Rubio-Verdú *et al.*, arXiv:2009.11645. (experiment); see C. Rubio-Verdú's talk.
Samajdar, MSS *et al.*, (in preparation).

group of A. Pasupathy

C. Rubio-Verdu
S. Turkel
L. Song
K. Watanabe
T. Taniguchi

additional theory:
L. Klebl
H. Ochoa
L. Xian
D. Kennes
A. Rubio



R. Samajdar
(Harvard)

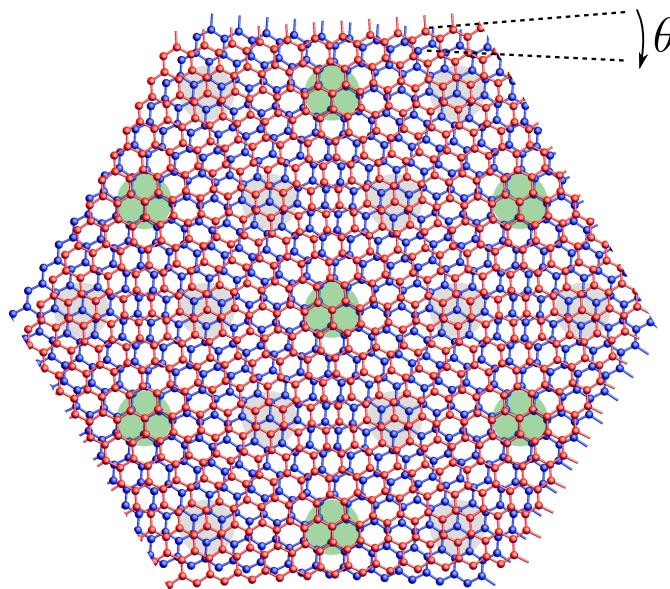


J. Venderbos
(Drexel)

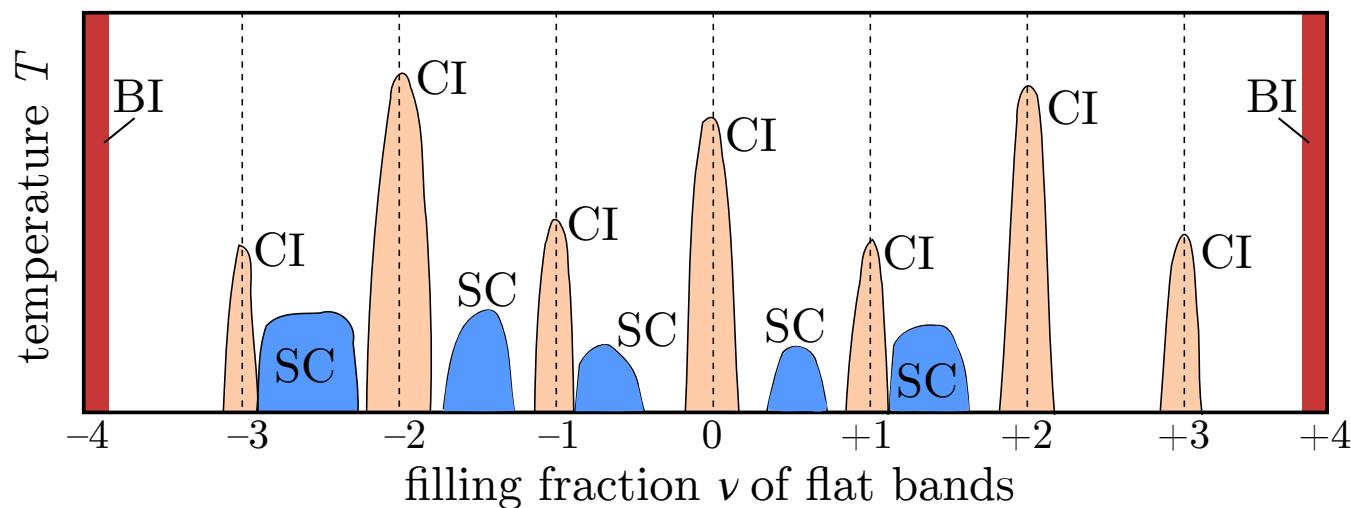


R. Fernandes
(Minnesota)

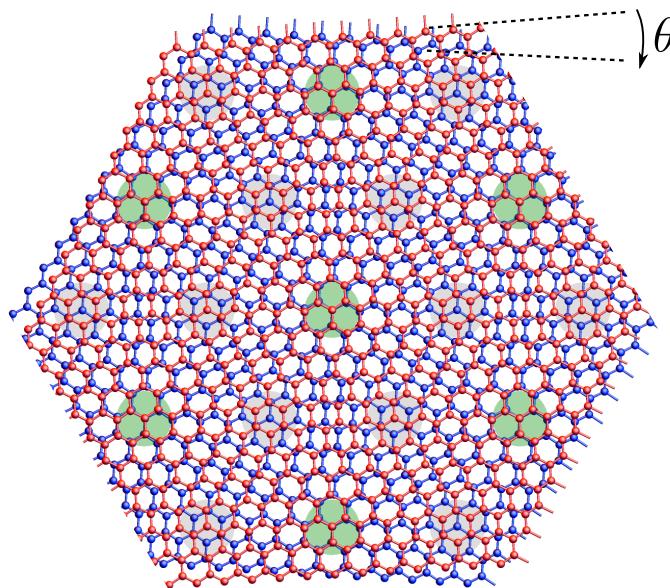
“Twisted bilayer, again?” (L. Balents)



P. Jarillo-Herrero/MIT:
Superconductivity & correlated insulator
Y. Cao *et al.*, Nature 556, 80 (2018).
Y. Cao *et al.*, Nature 556, 43 (2018).



“Twisted bilayer, again?” (L. Balents)



Key **open questions**:

- **Origin and form of superconductivity?**
- What about the **correlated insulator**?
- What is their **relation**?
- **Other phases:**

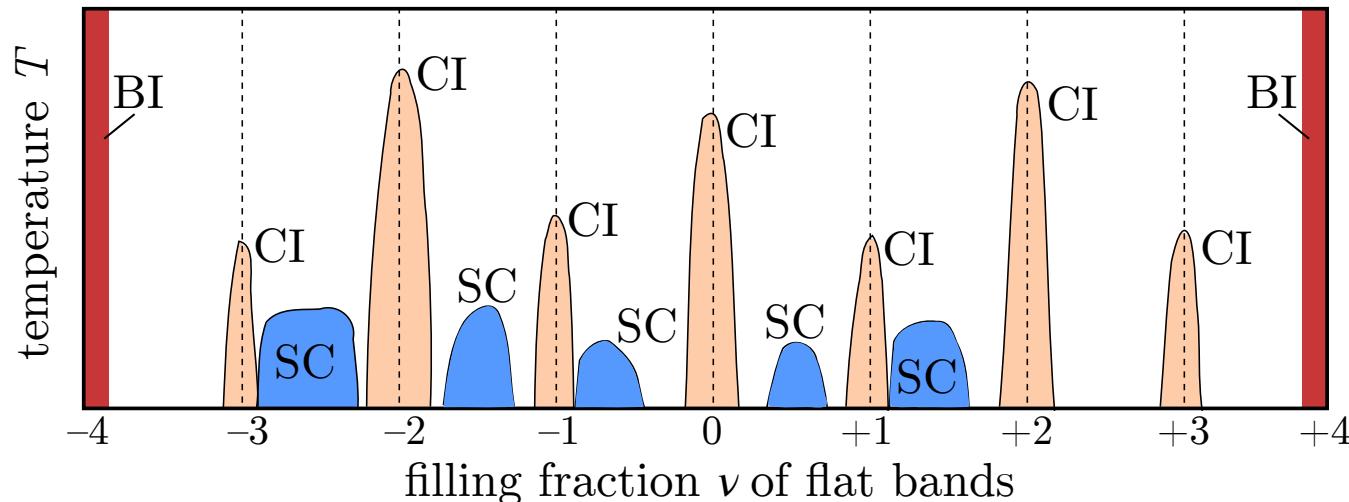
– nematic order

e.g., Jiang *et al.*, Nature **573**, 91 (2020), and many more

– high-temperature “Dirac revivals”

Zondiner *et al.*, Nature **582**, 203 (2020).

Wong *et al.*, Nature **582**, 198 (2020).

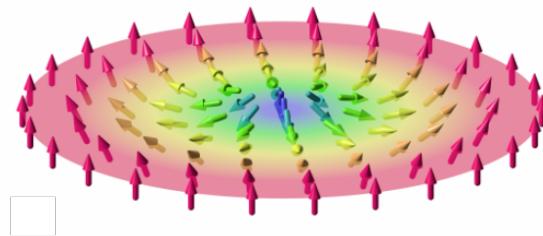


Basic physical picture: common WZW term

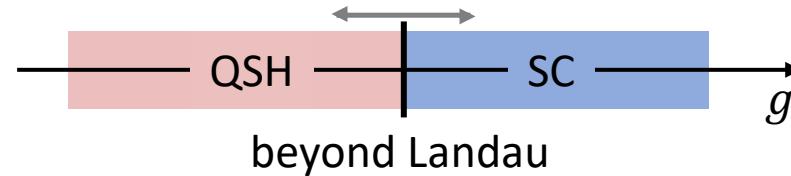
- Many applications of **Wess-Zumino-Witten** (WZW) terms, e.g., for **Néel-VBS**
Senthil, Balents, Sachdev, Vishwanath, Fisher (2004).
- Key **inspiration**: spin Hall in **single-layer graphene** Grover & Senthil, PRL 100, 156804 (2008).

$$S = \int d^3x \bar{\psi} (-i\gamma_\mu \partial_\mu + im\vec{\sigma} \cdot \hat{N})\psi$$

Skyrmion defect in \hat{N} has charge $2e$



Vortex of superconductor has spin $1/2$:



- Our work: classify possible {supercond., insulator} for TBG that allow for “WZW physics”
- Related work: Khalaf *et al.* (arXiv:2004.00638); Chatterjee *et al.* (arXiv:2010.01144).

Twisted bilayer graphene at $\nu = 0$

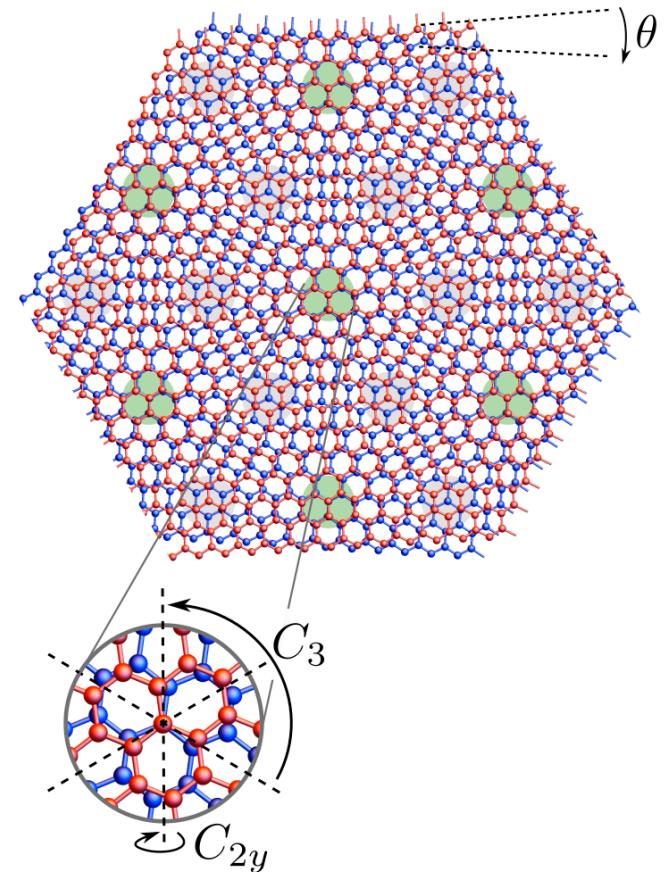
- Low-energy model: $2 \times 2 \times 2 = 8$ Dirac cones

$$H_{\text{LE}} = \nu \sum_{\mathbf{q}}^{\Lambda} f_{\mathbf{q}}^\dagger (q_x \gamma_x + q_y \gamma_y) f_{\mathbf{q}}$$

16 x 16 matrices
(valley/mini-valley/spin/sublattice)

- Superconducting order parameters:

$$H_{\text{SC}} = \sum_{\mathbf{q}} f_{\mathbf{q}}^\dagger \Delta_{\mathbf{q}} T f_{-\mathbf{q}}^* + \text{H.c.}$$



Use emergent point group D_6

Twisted bilayer graphene at $\nu = 0$

- Low-energy model: $2 \times 2 \times 2 = 8$ Dirac cones

$$H_{\text{LE}} = \nu \sum_{\mathbf{q}}^{\Lambda} f_{\mathbf{q}}^\dagger (q_x \gamma_x + q_y \gamma_y) f_{\mathbf{q}}$$

16 x 16 matrices
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- Superconducting order parameters:

$$H_{\text{SC}} = \sum_{\mathbf{q}} f_{\mathbf{q}}^\dagger \Delta_{\mathbf{q}} T f_{-\mathbf{q}}^* + \text{H.c.}$$

Focus on:

- inter-mini-valley (preserve translation)
- inter-valley (due to TRS)

- Particle-hole “partner” orders:

$$H_{\text{PH}} = \sum_{\mathbf{q}} f_{\mathbf{q}}^\dagger m_j f_{\mathbf{q}}$$

- Criteria for WZW term:

$$\begin{aligned} \gamma_i \Delta T &= -\Delta T \gamma_i^T \neq 0, & i &= 1, 2, \\ m_j \Delta T &= \Delta T m_j^T \neq 0, & j &= 1, 2, 3, \\ \text{tr}[\gamma_{i_1} \gamma_{i_2} m_{j_1} m_{j_2} m_{j_3}] &\propto \epsilon_{i_1 i_2 j_1 j_2 j_3}. \end{aligned}$$

Twisted bilayer graphene at $\nu = 0$

➤ Summary of **possible singlet superconductors & partner orders:**

τ	valley
μ	mini-valley
ρ	sublattice/Dirac
σ	spin

Pairing	m_j	Type
A_1	$(\tau_+, \tau_-)\rho_x; \rho_z$	IVC ₊ ; SP
A_1	$(\mu_+, \mu_-)\rho_z; \rho_z\tau_z\mu_z$	MDW ₊ ; VP
A_1	$(\sigma_x, \sigma_y, \sigma_z)\tau_z\rho_z$	QSH
A_2	$(\tau_+, \tau_-)\mu_z\rho_x; \rho_z$	IVC ₋ ; SP
A_2	$(\mu_+, \mu_-)\tau_z\rho_z; \rho_z\tau_z\mu_z$	MDW ₋ ; VP
A_2	$(\sigma_x, \sigma_y, \sigma_z)\tau_z\rho_z$	QSH

Grover & Senthil

➤ Other IRs of SC (B_1, B_2, E_1, E_2) not possible:

do not lead to gap at Dirac points, $\gamma_i \Delta T \neq -\Delta T \gamma_i^T$

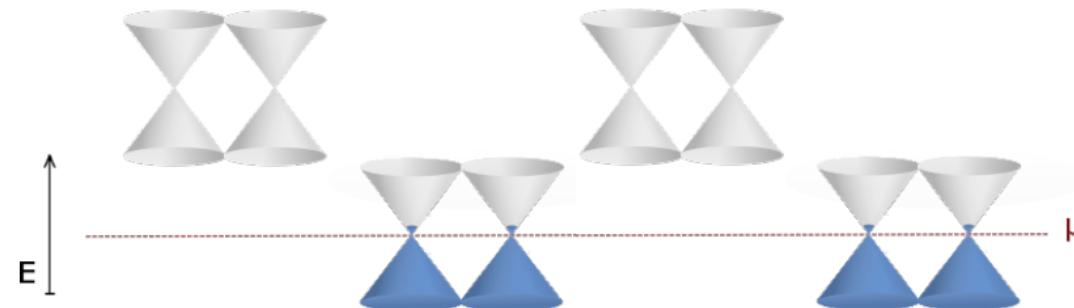
➤ Not possible for **triplet**:

not the right manifold (e.g., unitary triplet has $6 > 5$ real components)

Twisted bilayer graphene at $\nu = \pm 2$

- Key ingredient: ‘**Dirac revival**’ at integer $\nu \neq 0$ at high temperature
Zondiner *et al.*, Nature **582**, 203 (2020). Wong *et al.*, Nature **582**, 198 (2020).

$\nu = -2$:



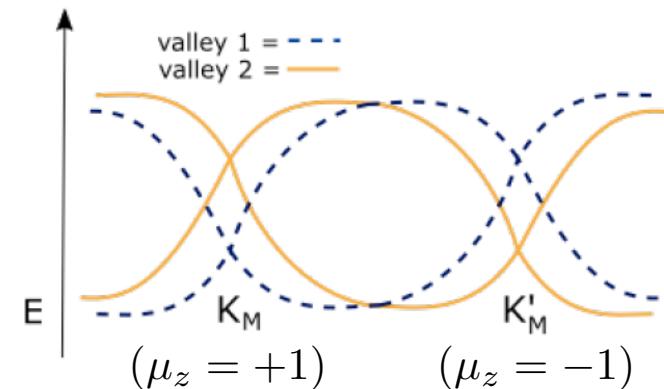
- Modified ‘**parent theory**’:

$$\tilde{H}_{\text{LE}} = \sum_{\mathbf{q}}^{\Lambda} f_{\mathbf{q}}^\dagger [q_x \gamma_x + q_y \gamma_y + M] f_{\mathbf{q}}$$

Conditions:

- **shift** Dirac cones: $[M, \gamma_i] = 0$
- correct **eigenvalues**: $\{+1, -1\}$ with deg. 8
- **not** lead to additional **Fermi surfaces**

Example: $M = \tau_z \mu_z$



Twisted bilayer graphene at $\nu = \pm 2$

➤ All configurations for $\nu = 0$ **remain possible** for some M and thus for $\nu = \pm 2$:

Pairing	m_j	Type	M	
A_1	$(\tau_+, \tau_-) \rho_x; \rho_z$	IVC ₊ ; SP	μ_x	τ valley
A_1	$(\mu_+, \mu_-) \rho_z; \rho_z \tau_z \mu_z$	MDW ₊ ; VP	$\tau_x \rho_y \mu_z; \tau_z \sigma_z$	
A_1	$(\sigma_x, \sigma_y, \sigma_z) \tau_z \rho_z$	QSH	$\mu_x; \tau_x \rho_y \mu_z$	
A_2	$(\tau_+, \tau_-) \mu_z \rho_x; \rho_z$	IVC ₋ ; SP	$\tau_z \mu_x$	μ mini-valley
A_2	$(\mu_+, \mu_-) \tau_z \rho_z; \rho_z \tau_z \mu_z$	MDW ₋ ; VP	$\tau_z \sigma_z; \tau_x \rho_y$	
A_2	$(\sigma_x, \sigma_y, \sigma_z) \tau_z \rho_z$	QSH	$\tau_z \mu_x; \rho_y \tau_x$	

➤ New options for **singlet**, only at $\nu = \pm 2$:

M	Partner Orders m_j	Partner SC	Type
$\mu_x; \tau_z \mu_y \sigma_z$ $\tau_z \mu_x \sigma_z; \tau_z \sigma_{x/y}$ $\rho_y \tau_x \sigma_z; \tau_z \sigma_z$ $\tau_z \mu_x \sigma_z; \rho_y \mu_x \tau_x \sigma_z; \mu_y$ $\rho_y \mu_x \tau_x \sigma_z; \tau_z \sigma_z; \rho_y \tau_y \sigma_z; \tau_y \rho_y \mu_z$ $\rho_y \tau_x \sigma_z; \tau_z \mu_x \sigma_z$	$\rho_x \mu_x (\tau_+, \tau_-); \rho_z$	A_1	IVC-MDW ₊ ; SP
	$\rho_x \mu_z \sigma_z (\tau_+, \tau_-); \rho_z$	A_1	IVC ₊ ; SP
	$\tau_z \rho_z \sigma_z (\mu_+, \mu_-); \rho_z \tau_z \mu_z$	A_1	MDW ₊ ; VP
	$\rho_z \tau_z \mu_y (\sigma_x, \sigma_y); \rho_z \tau_z \sigma_z$	A_1	AFM ₊ [±] ; SpVSP
	$\tau_y \mu_z \rho_x (\sigma_x, \sigma_y); \rho_z \tau_z \sigma_z$	A_1	SBO-AFM ₊ [±] ; SpVSP
	$\rho_z \mu_z (\sigma_x, \sigma_y); \rho_z \tau_z \sigma_z$	A_1	SBO-AFM ₊ [±] ; SpVSP
$\tau_z \mu_y; \mu_x \sigma_z$ $\mu_x \sigma_z; \tau_z \sigma_{x/y}$ $\tau_z \sigma_z$ $\mu_x \sigma_z; \rho_y \mu_y \tau_x \sigma_z; \tau_z \mu_y$ $\rho_y \mu_x \tau_x \sigma_z; \tau_z \sigma_z; \tau_y \rho_y$ $\mu_x \sigma_z$	$\rho_x \mu_x (\tau_+, \tau_-); \rho_z$	A_2	IVC-MDW ₊ ; SP
	$\rho_x \sigma_z (\tau_+, \tau_-); \rho_z$	A_2	IVC ₋ ; SP
	$\rho_z \sigma_z (\mu_+, \mu_-); \rho_z \tau_z \mu_z$	A_2	MDW ₋ ; VP
	$\rho_z \mu_y (\sigma_x, \sigma_y); \rho_z \tau_z \sigma_z$	A_2	AFM ₋ [±] ; SpVSP
	$\tau_x \rho_x (\sigma_x, \sigma_y); \rho_z \tau_z \sigma_z$	A_2	SBO-AFM ₋ [±] ; SpVSP
	$\rho_z \mu_z (\sigma_x, \sigma_y); \rho_z \tau_z \sigma_z$	A_2	SBO-AFM ₋ [±] ; SpVSP

Twisted bilayer graphene at $\nu = \pm 2$

➤ And **triplet states** are now also possible due to **broken spin rotation symmetry**:

Pairing	m_j	Type	M^a	M^b
B_1^a/B_1^b	$(\tau_+, \tau_-) \mu_z \rho_x; \rho_z$	IVC $_-$; SP	$\tau_z \mu_x \sigma_x$	σ_x
B_1^a/B_1^b	$(\mu_+, \mu_-) \rho_z; \tau_z \rho_z \mu_z$	MDW $+$; VP	$\tau_z \sigma_x$	σ_x
B_1^a/B_2^b	$(\tau_+, \tau_-) \rho_x \sigma_x; \rho_z$	spIVC $_-$; SP	—	σ_x
B_1^a/B_2^b	$(\mu_+, \mu_-) \tau_z \rho_z \sigma_x; \tau_z \rho_z \mu_z$	MDW $+$; VP	$\tau_z \sigma_x$	σ_x
B_1^a	$(\sigma_y, \sigma_z) \rho_z \tau_z \mu_z; \tau_z \rho_z \sigma_x$	AFM $^\perp$; SpVSP	$\tau_z \mu_x \sigma_x$	—
B_1^a	$\rho_x \mu_z \tau_x (\sigma_y, \sigma_z); \rho_z \tau_z \sigma_x$	IVC-SBO $^\perp$; SpVSP	$\tau_z \sigma_x$	—
B_2^a/B_2^b	$(\tau_+, \tau_-) \rho_x; \rho_z$	IVC $+$; SP	$\mu_x \sigma_x$	σ_x
B_2^a/B_2^b	$(\mu_+, \mu_-) \tau_z \rho_z; \rho_z \tau_z \mu_z$	MDW $-$; VP	$\rho_y \tau_x \sigma_x; \tau_z \sigma_x$	σ_x
B_2^a/B_1^b	$(\tau_+, \tau_-) \rho_x \sigma_x \mu_z; \rho_z$	IVC $+$; SP	—	σ_x
B_2^a/B_1^b	$(\mu_+, \mu_-) \rho_z \sigma_x; \tau_z \rho_z \mu_z$	MDW $-$; VP	$\tau_z \sigma_x$	σ_x
B_2^a	$(\sigma_y, \sigma_z) \rho_z; \tau_z \rho_z \sigma_x$	AFM $^\perp$; SpSVP	$\rho_y \tau_x \sigma_x$	—
B_2^a	$(\sigma_y, \sigma_z) \rho_z \tau_z \mu_z; \tau_z \rho_z \sigma_x$	AFM $^\perp$; SpVSP	$\mu_x \sigma_x$	—
B_2^a	$\rho_x \tau_x (\sigma_y, \sigma_z); \rho_z \tau_z \sigma_x$	IVC-SBO $^\perp$; SpVSP	$\tau_z \sigma_x$	—

a: unitary, $d \sim (1, 0, 0)$

b: non-unitary, $d \sim (0, 1, i)$

A few implications of “this mess”

- Only SC states transforming under **1D IRs** of D_6 yield WZW options (for singlet $A_{1,2}$ and for triplet $B_{1,2}$; all others don't gap out Dirac cones)
- **SC state** around $\nu = 0$ has to be **singlet**
- If, for any of $\nu = 0, \pm 2$, m_j **break TRS**, the SC **cannot be the A_1 singlet**

- **Nematic order** parameter cannot be partner order (preserves key symmetry $C_2\theta$: shifts rather than gaps out Dirac cones)
- The additional presence of **nematic order/strain** does not alter the results

Superconductivity, its relation to the correlated insulators, & field-controlled nematicity in moiré systems

Part II: Nematic order and its control in twisted double-bilayer graphene

Refs.: Rubio-Verdú *et al.*, arXiv:2009.11645. (experiment); see C. Rubio-Verdú's talk.
Samajdar, MSS *et al.*, (in preparation).

group of A. Pasupathy

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additional theory:
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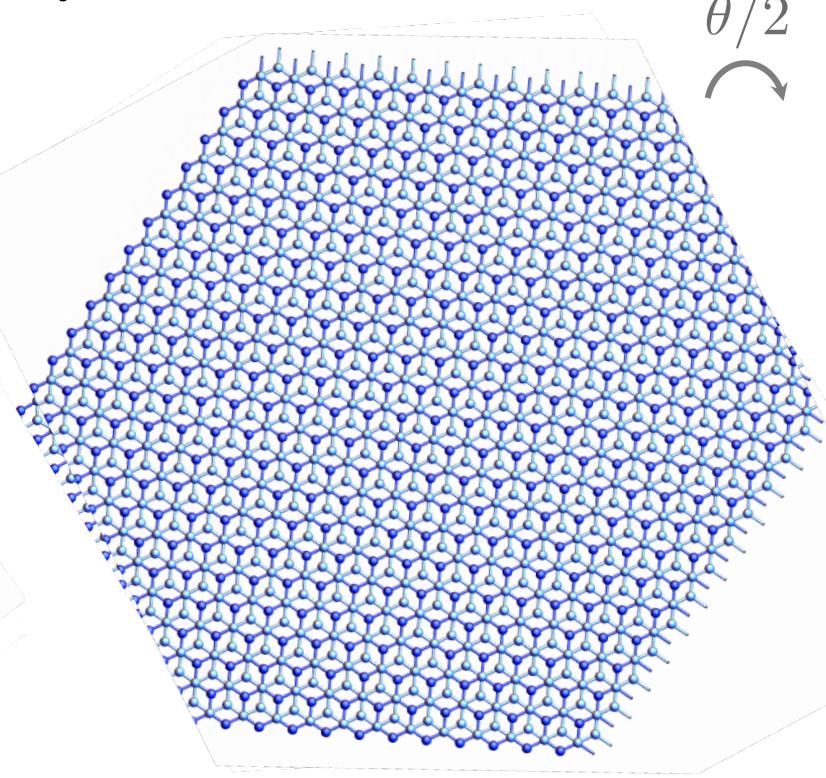
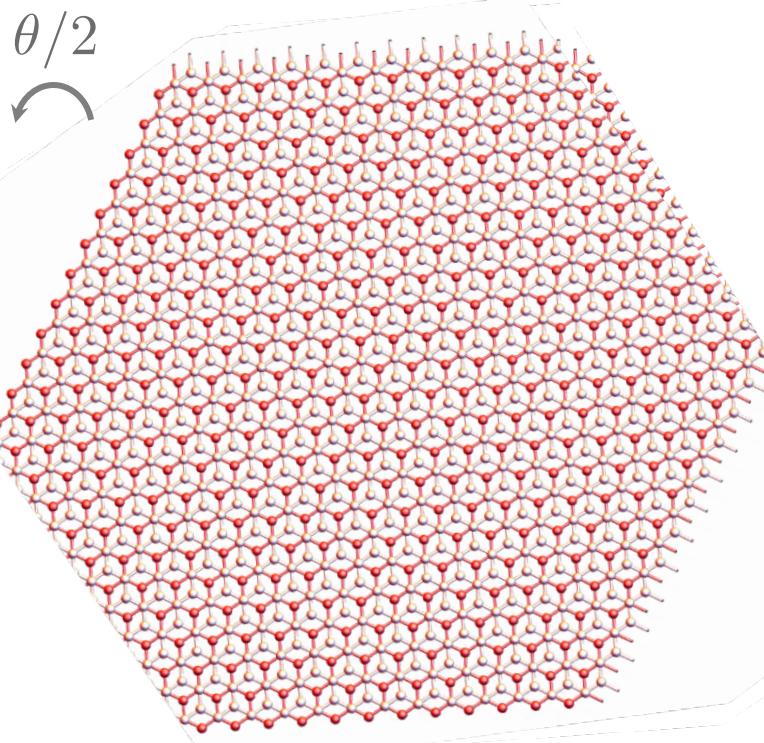


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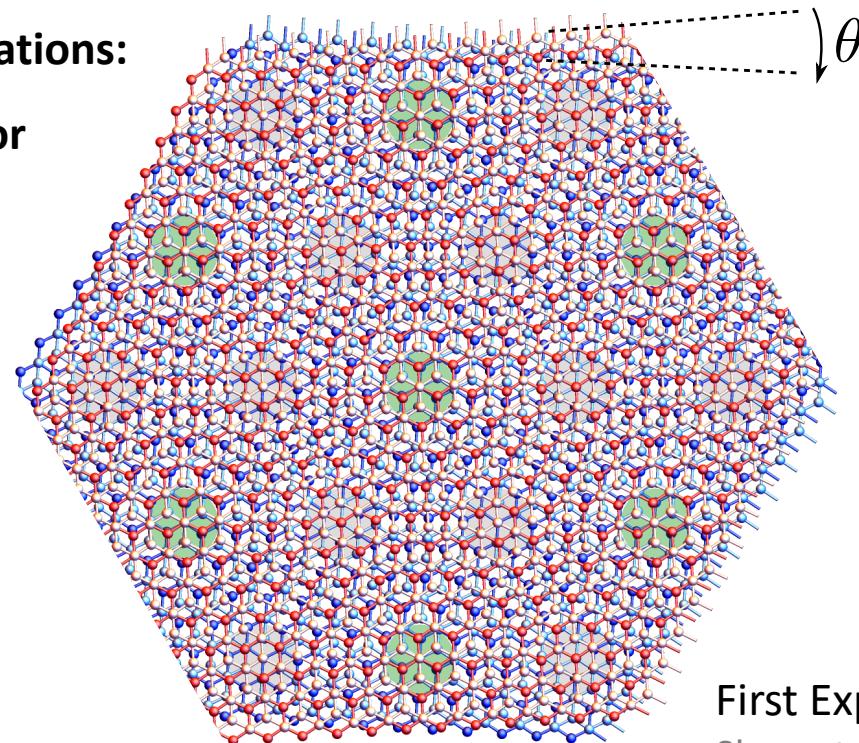
Twisted double-bilayer graphene



Twisted double-bilayer graphene

Similar signs of correlations:

- Correlated **insulator**
- **Superconductivity**
(under debate)



First Experiments:

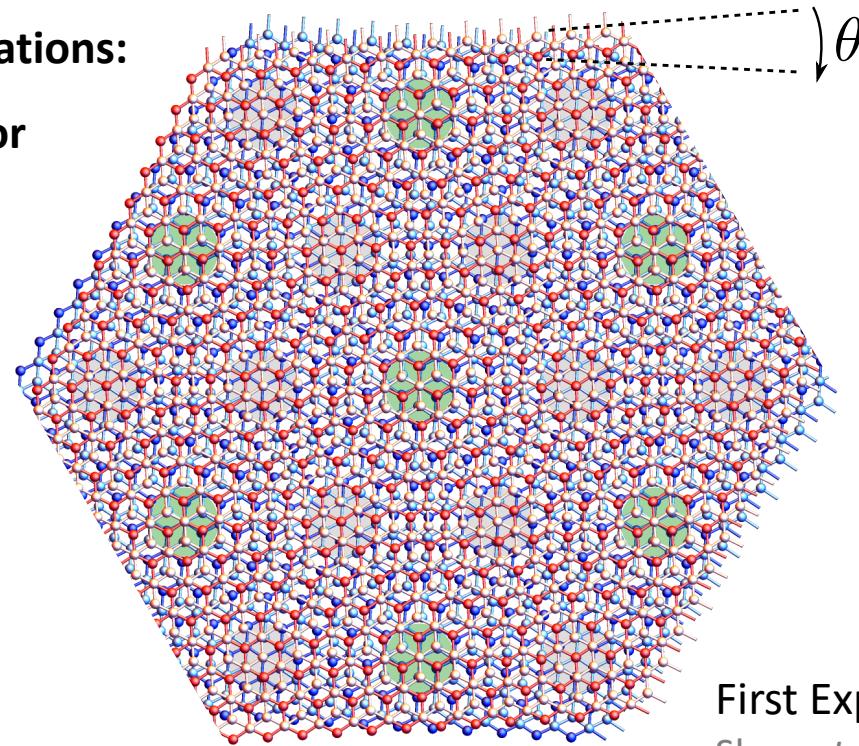
- Shen *et al.*, Nat. Phys. **16**, 520 (2020).
X. Liu *et al.*, Nature **583**, 221 (2020).
Y. Cao *et al.*, Nature **583**, 215 (2020).

Additional tunability:
via electric field

Twisted double-bilayer graphene

Similar signs of correlations:

- Correlated **insulator**
- **Superconductivity**
(under debate)



Still many **open questions**, e.g.:

- **Origin and form of correlated insulator?**
- What about the superconductivity?
- Additional phases: **nematicity**?
- How to further **exploit field-tunability**?

First Experiments:

- Shen *et al.*, Nat. Phys. **16**, 520 (2020).
X. Liu *et al.*, Nature **583**, 221 (2020).
Y. Cao *et al.*, Nature **583**, 215 (2020).

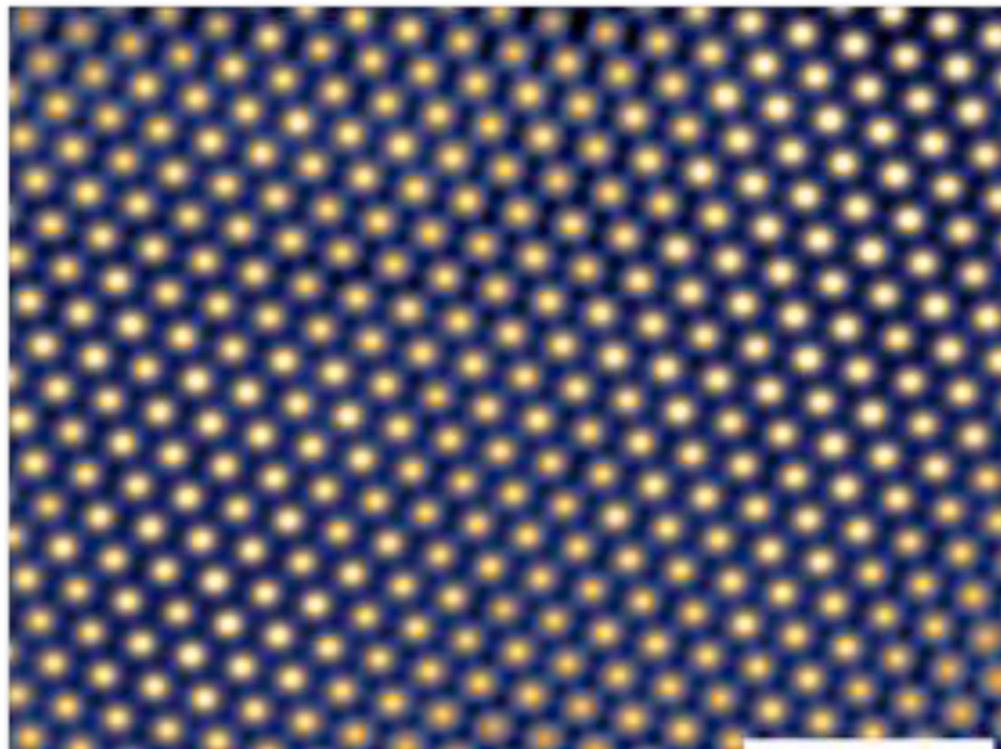
Additional tunability:
via electric field

STM in twisted double-bilayer graphene

Rubio-Verdú *et al.*, arXiv:2009.11645.
see C. Rubio-Verdú's talk.



Topography: moiré pattern



100nm

A. Pasupathy

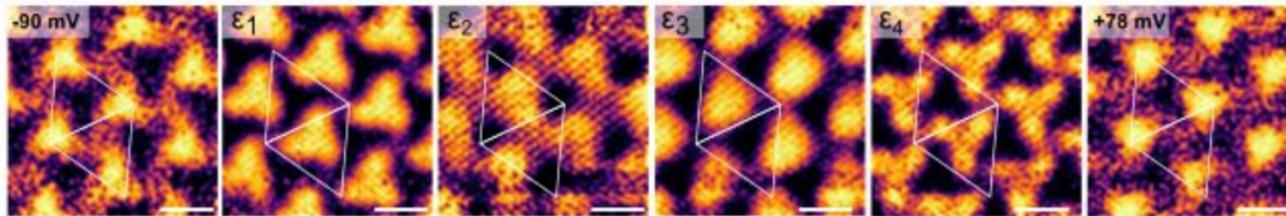
twist-angle 1.05 deg

**very low strain ($\varepsilon = 0.05 \pm 0.05 \%$)
and highly homogeneous**

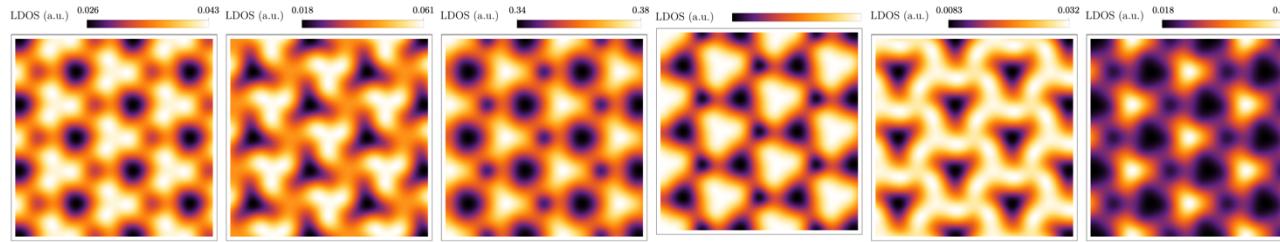
STM in twisted double-bilayer graphene

Energy-dependent LDOS at CNP:

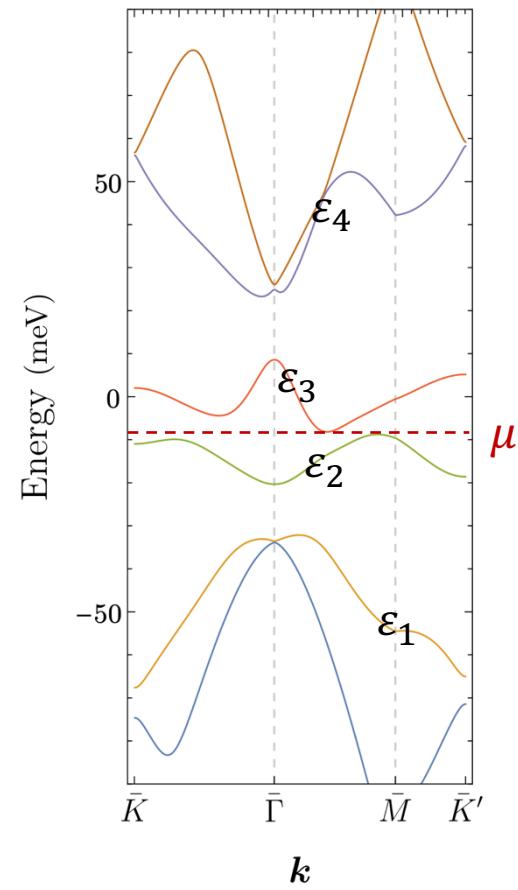
Experiment:



Theory: (continuum model)

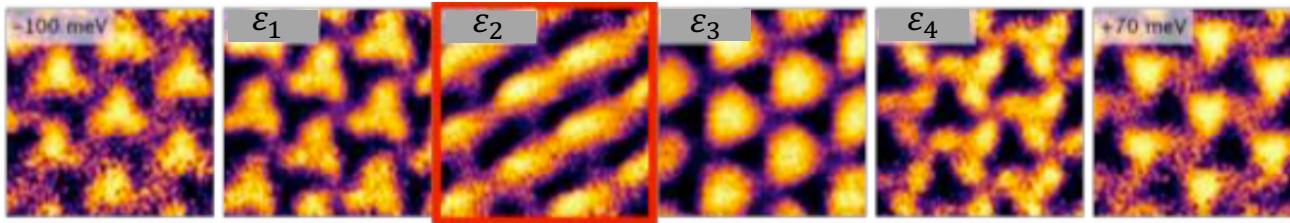


➤ data is C_3 symmetric



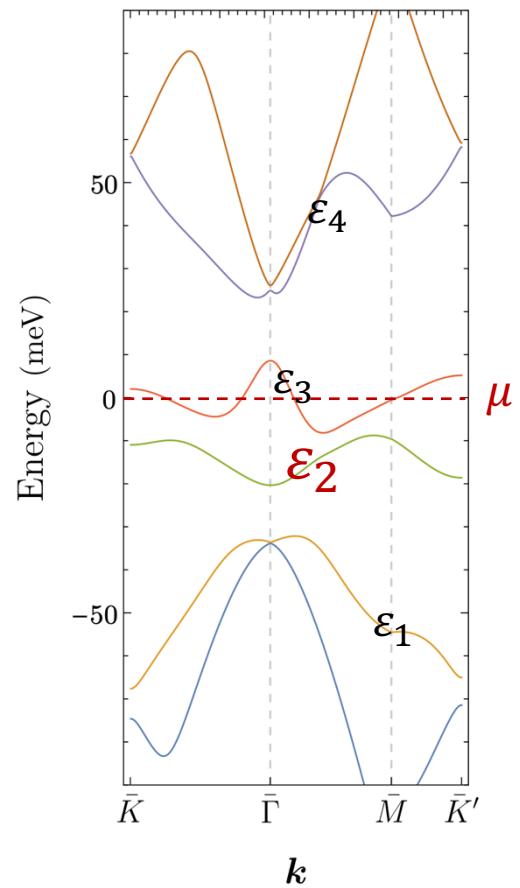
STM in twisted double-bilayer graphene

Energy-dependent LDOS at half-filling of conduction band:



- nematic order at half-filled cond. band
- What is the microscopic form of nematicity here?
order parameter: $\Phi = \Phi_0(\cos 2\theta, \sin 2\theta)$

$$\mathcal{H}_\Phi = \int d\mathbf{r} \int d\delta\mathbf{r} \ \Phi \cdot \phi_{s\ell\tau; s'\ell'\tau'}(\mathbf{r}, \delta\mathbf{r}) c_{\sigma s\ell\tau}^\dagger(\mathbf{r} + \delta\mathbf{r}) c_{\sigma s'\ell'\tau'}(\mathbf{r})$$



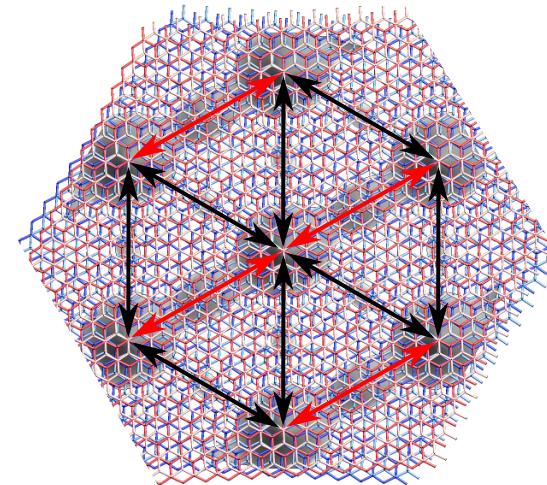
Electronic nematic order

consider **two simple limiting cases**:

“Moiré nematic”:

$$\int d\mathbf{r} \sum_{\mathbf{R} \in \text{NN}} \Phi \cdot \phi(\mathbf{R}) c_{\sigma s \ell \tau}^\dagger(\mathbf{r} + \mathbf{R}) c_{\sigma s \ell \tau}(\mathbf{r})$$

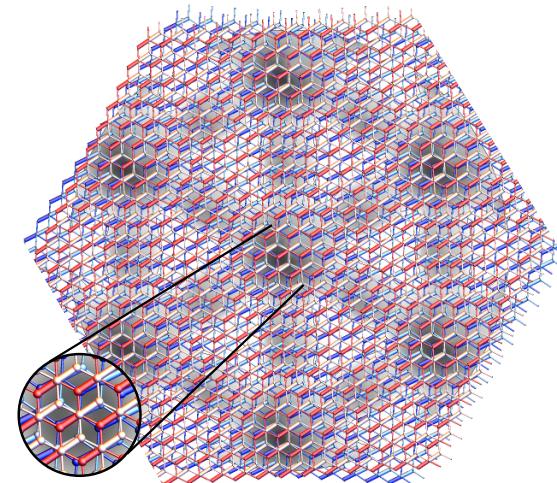
reproduces exp. features



“Graphene nematic”:

$$\int d\mathbf{r} \Phi \cdot \begin{pmatrix} (\rho_x)_{ss'} \\ \tau(\rho_y)_{ss'} \end{pmatrix} c_{\sigma s \ell \tau}^\dagger(\mathbf{r}) c_{\sigma s' \ell \tau}(\mathbf{r})$$

inconsistent with exp.



Field tunability from symmetry

No displacement field:

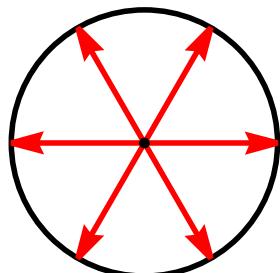
$$\hat{\Phi} = \Phi_1 + i \Phi_2 = |\Phi_0| e^{2i\theta}$$

$$C_3 : \hat{\Phi} \longrightarrow e^{i\frac{2\pi}{3}} \hat{\Phi},$$
$$C_{2x} : \hat{\Phi} \longrightarrow \hat{\Phi}^*$$

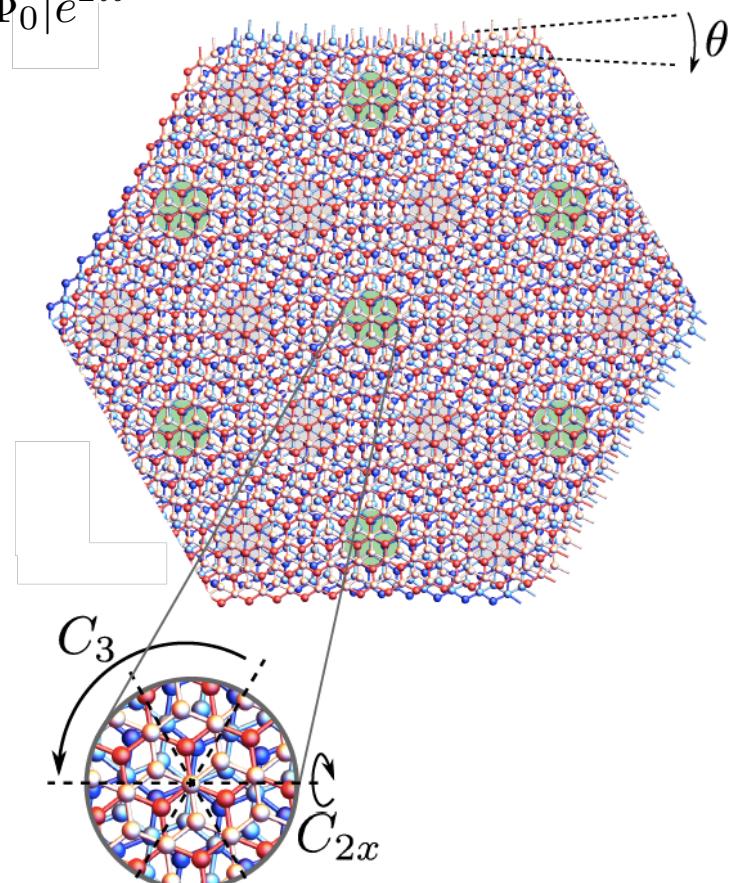
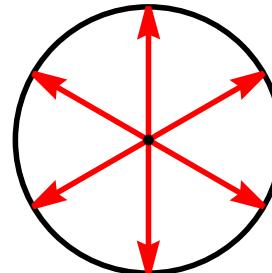
free energy:

$$\mathcal{F} \sim a|\hat{\Phi}|^2 + b \operatorname{Re} \hat{\Phi}^3 + c|\hat{\Phi}|^4$$

only **discrete orientations** of director:



or



Field tunability from symmetry

With displacement field:

$$\hat{\Phi} = \Phi_1 + i \Phi_2 = |\Phi_0| e^{2i\theta}$$

$$C_3 : \hat{\Phi} \longrightarrow e^{i\frac{2\pi}{3}} \hat{\Phi},$$

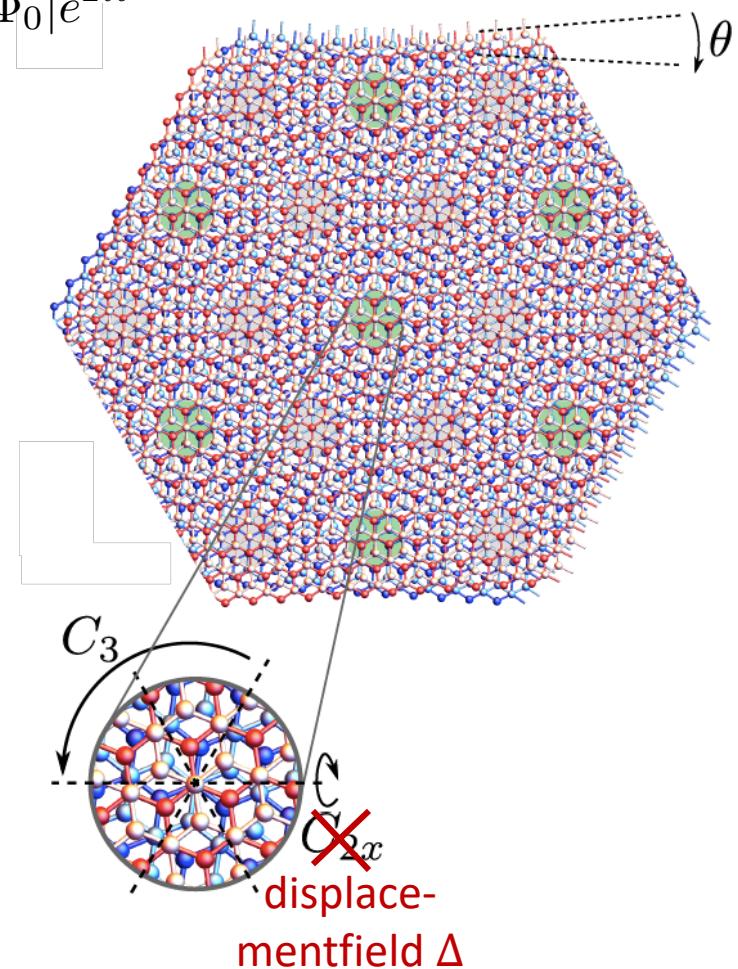
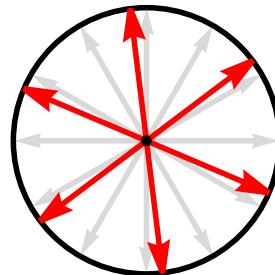
 ~~$C_{2x} : \hat{\Phi} \longrightarrow \hat{\Phi}^*$~~

free energy:

$$\mathcal{F} \sim a|\hat{\Phi}|^2 + b \operatorname{Re} \hat{\Phi}^3 + \tilde{b} \operatorname{Im} \hat{\Phi}^3 + c|\hat{\Phi}|^4$$

with $\tilde{b}(\Delta) = -\tilde{b}(-\Delta)$

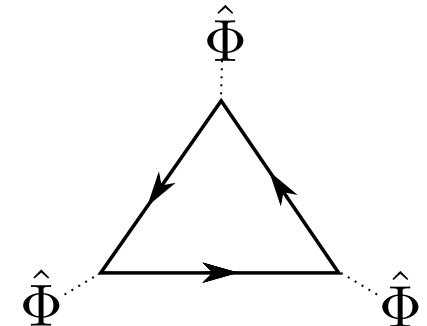
director will rotate continuously



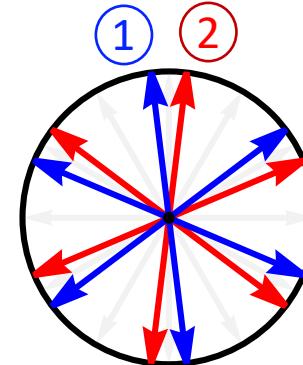
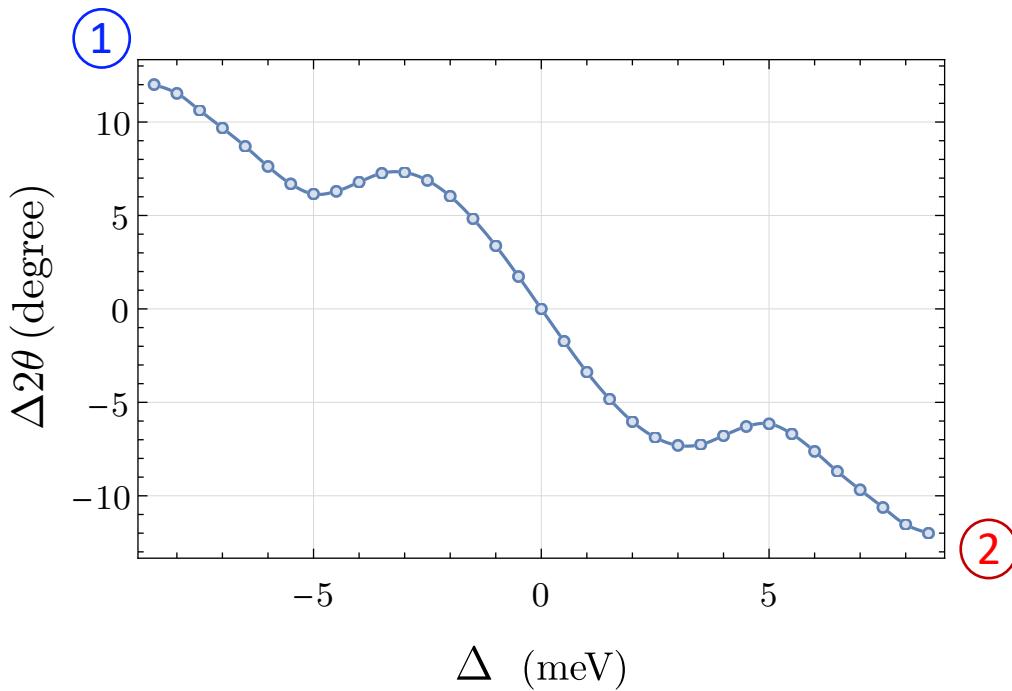
Field tunability microscopically

$$\mathcal{F} \sim a|\hat{\Phi}|^2 + b \operatorname{Re} \hat{\Phi}^3 + \tilde{b} \operatorname{Im} \hat{\Phi}^3 + c|\hat{\Phi}|^4$$

evaluate



for moiré nematic, in continuum model (Koshino, 2019):

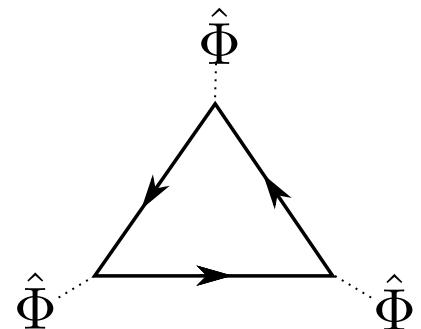


amplification via “wavefunction effect”

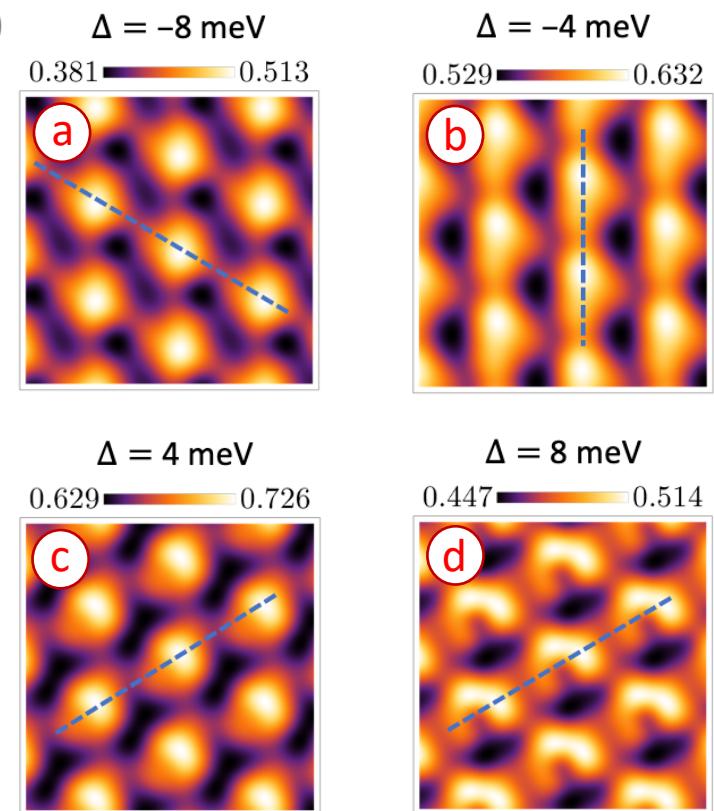
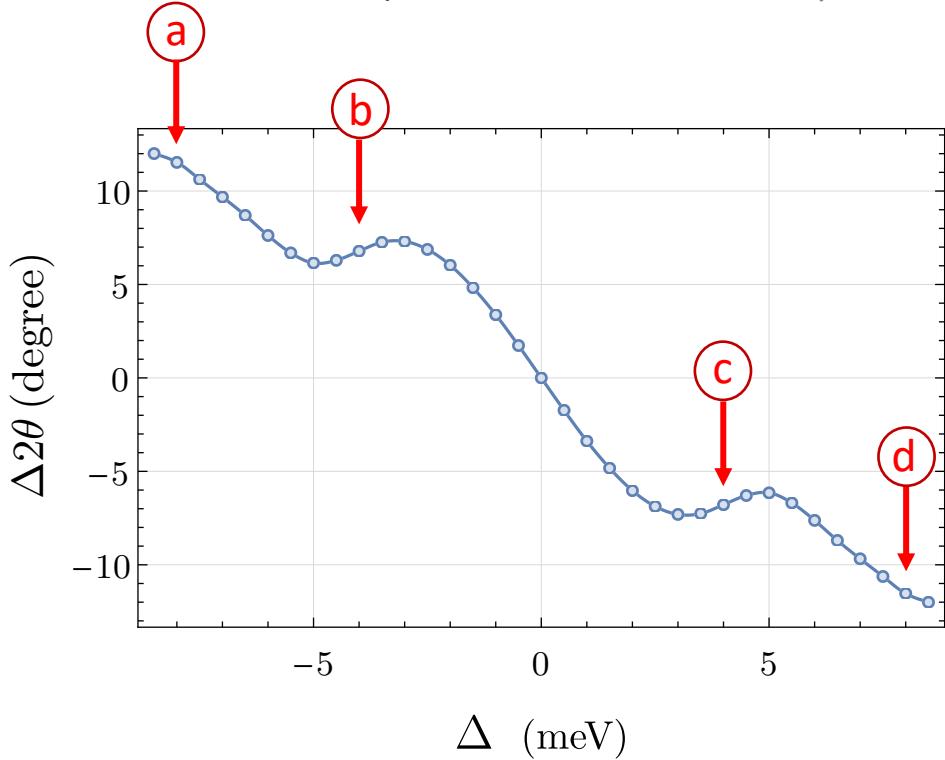
Field tunability microscopically

$$\mathcal{F} \sim a|\hat{\Phi}|^2 + b \operatorname{Re} \hat{\Phi}^3 + \tilde{b} \operatorname{Im} \hat{\Phi}^3 + c|\hat{\Phi}|^4$$

evaluate



for moiré nematic, in continuum model (Koshino, 2019)

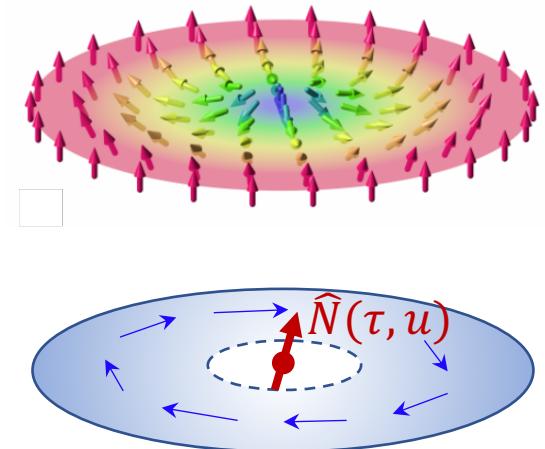


Conclusion

Wess-Zumino-Witten terms in twisted bilayer graphene

Christos, Sachdev, & MSS, PNAS 117, 29543 (2020).

- Classification of “partner” orders {SC,CI} with WZW term and high-temperature orders
- Constraints for microscopic realization

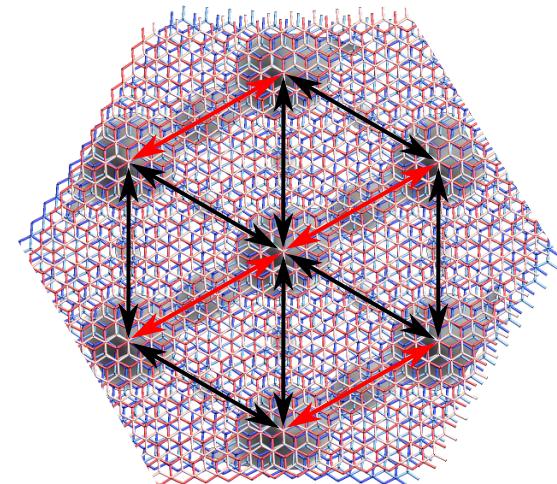


Nematicity in twisted double-bilayer graphene

Rubio-Verdú *et al.*, arXiv:2009.11645.

Samajdar, MMS *et al.* (in preparation).

- STM data indicates **electronic nematic order**
- Order parameter pre-dominantly **moiré nematic**
- Electric field can be used to **rotate nematic director**



Thank you!