Quantum criticality and non-Fermi liquid pairing in Yukawa-SYK model

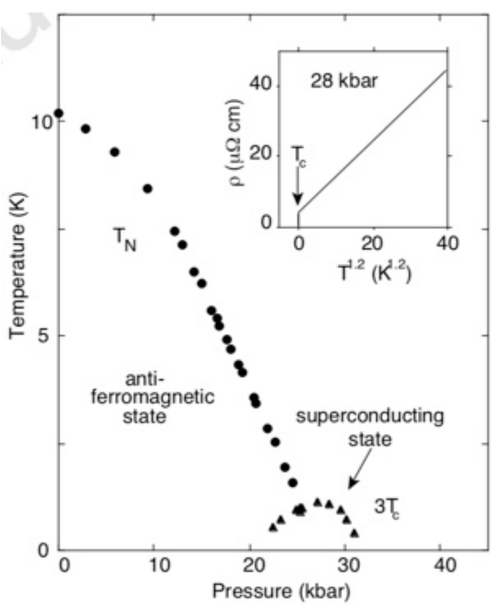
Yuxuan Wang University of Florida

YW, Phys. Rev. Lett. **124**, 017002
Pan, Wang, Davis, **YW**, Zi Yang Meng, arXiv:2001
YW and Chubukov, Phys. Rev. Res. **2**, 033084

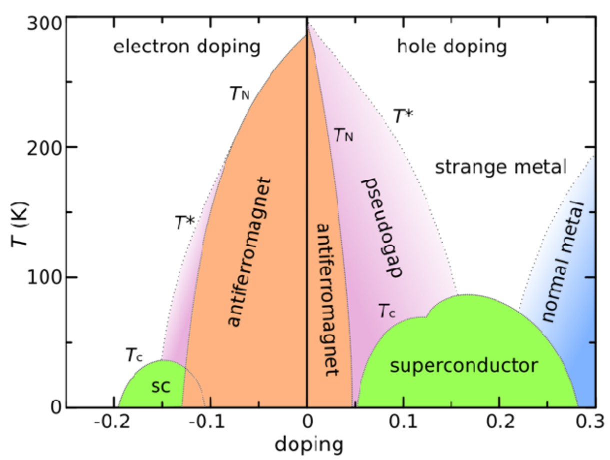
UF UNIVERSITY of FLORIDA

nFL and pairing at quantum-critical points

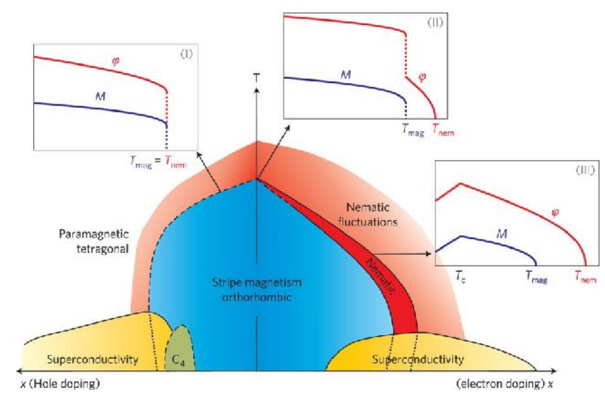
- Paradigm for unconventional SC: pairing by exchanging critical bosons
- Signatures of non-Fermi liquid behavior, such as *T*-linear resistivity, diverging effective mass



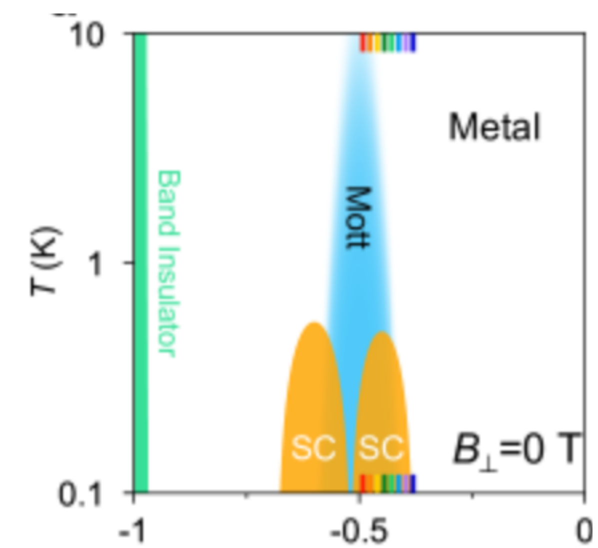
CePd$_2$Si$_2$, Mathur et al



Cuprates, Wikipedia



Pnictides, Fernandes et al



TBG, Cao et al

- A minimal model: $\mathscr{L} = \mathscr{L}_\psi + \mathscr{L}_\phi + g\phi\psi^\dagger\psi$
- Difficult; would be nice to have an exactly solvable model

Yukawa-SYK model

- Inspired by SYK model:

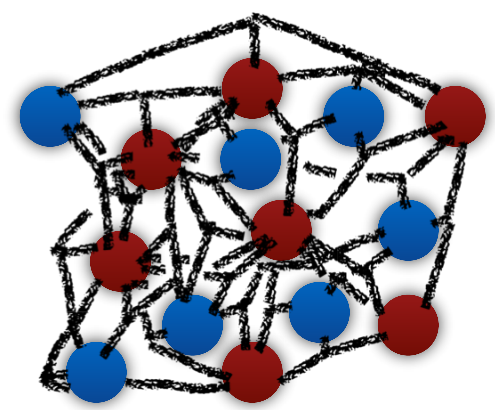
$$\mathscr{H} = \sum_{ij}^{M} \sum_{k}^{N} \left(\frac{i}{\sqrt{N}} t_{ijk}\, \phi_k\, c_i^\dagger c_j \right) + \sum_{k}^{N} \left(\frac{1}{2}\pi_k^2 + \frac{m_0^2}{2}\phi_k^2 \right), \qquad [\phi_k, \pi_k] = i$$

random Yukawa coupling *dynamical boson*

$$\langle t_{ijk} \rangle = 0, \quad \langle t_{ijk}^2 \rangle = \omega_0^3$$

$$t_{ijk} = -\, t_{jik}$$

Typical scale $\omega_F = \omega_0^3 / m_0^2$



2-colorable 3-uniform hypergdaph

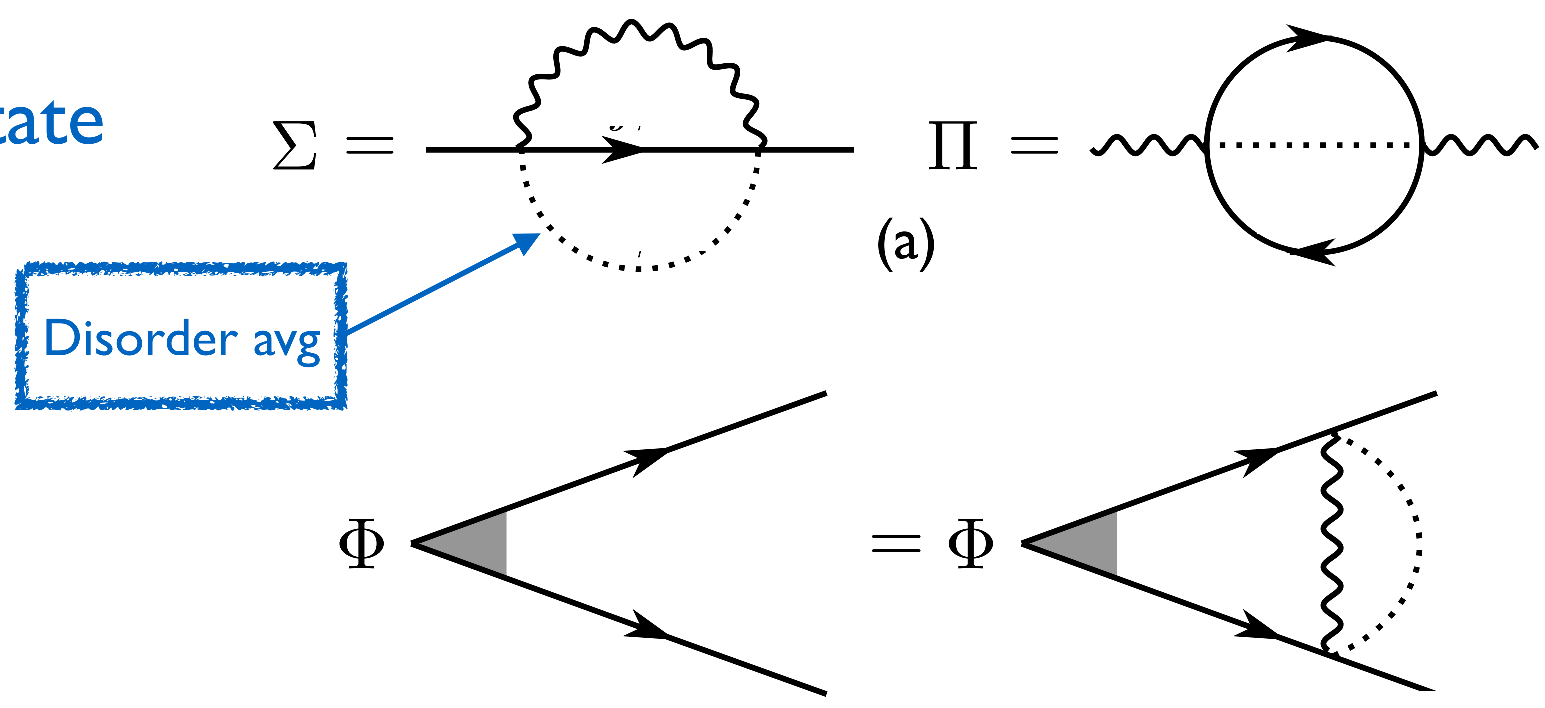
- Related models:
  - SUSY SYK Fu-Maldacena-Sachdev 2017
  - Random electron-phonon interaction Esterlis-Schmalian 2019
  - Low-rank SYK Kim-Cao-Altman 2020

Large-N, M limit

- Taking the $N \to \infty, M \to \infty$ limit dramatically simplifies the diagrams to be included
- Vertex corrections and replica off-diagonal fluctuations $1/N$ suppressed

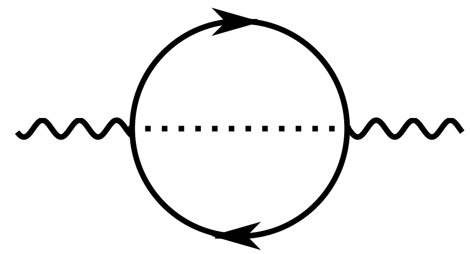
Normal state

Pairing



Renormalization of the boson mass

- Boson self-energy

$\Pi =$ 

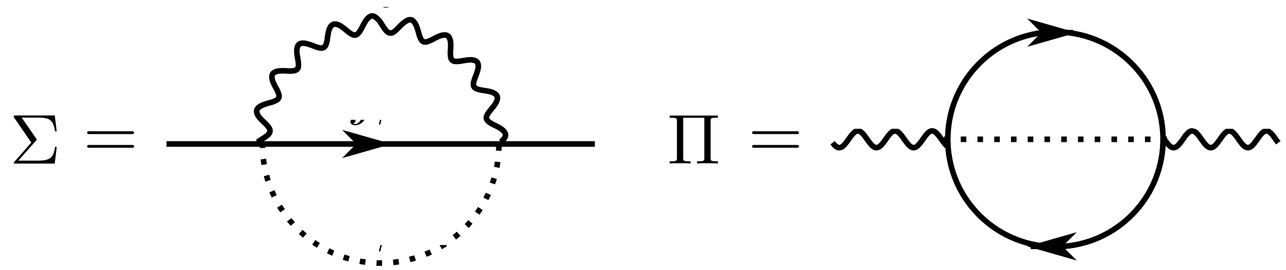
$$\Pi(\Omega = 0) \sim -\omega_0^3 \int \frac{d\omega_m}{(i\omega_m)^2}, \qquad m^2 = m_0^2 - \Pi(\Omega = 0)$$

- Power-law divergence in the IR, driving the boson mass down. What next?
  - Condense the boson? Then the fermions acquire a random hopping amplitude. $\Sigma(\omega_m) \propto i\langle\phi\rangle \mathrm{sgn}(\omega_m)$.
  - But fluctuations of ϕ are a large effect in 0d and destroy $\langle\phi\rangle$.

Fluctuation effects $\propto$ # boson/# fermion $= N/M$

Self-tuning to criticality

- A distinct solution where ϕ stays critical



- Bubble is cut off at the energy scale ω_f , and the boson becomes critical

$$\Pi(\Omega = 0) \sim -\,\omega_0^3 \int_{\omega_f} \frac{d\omega_m}{(i\omega_m)^2}, \qquad m^2 = m_0^2 - \Pi(\Omega = 0) = 0$$

$$\omega_f \sim \omega_0^3 / m_0^2 \equiv \omega_F$$

- Critical boson in turn makes fermions incoherent below ω_f , consistent with the cutoff in the fermion bubble.

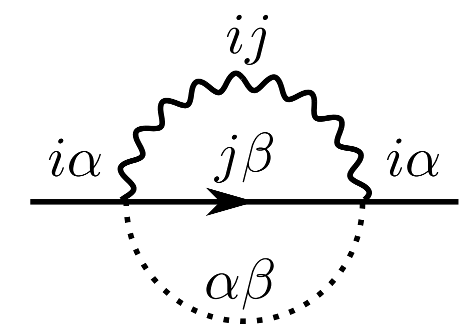
- Self-tuned QC from comparable boson and fermion flucttutations

Schwinger-Dyson equations

$$D(\Omega) = \frac{1}{\Omega^2 + \Pi(\Omega) - \Pi(\Omega = 0)}, \quad G(\omega) = -\frac{1}{i\omega + \Sigma(\omega)}.$$

m_0^2

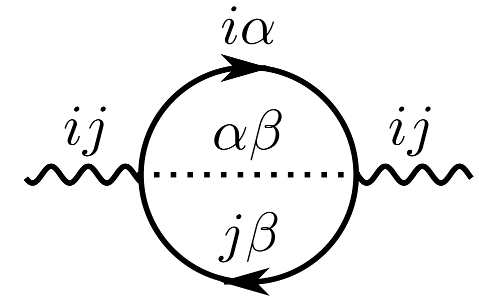
$\Sigma =$



$$\Sigma(\omega) \sim \omega_0^3 \int \frac{d\Omega}{2\pi} D(\Omega) G(\omega + \Omega)$$

$$\Pi(\Omega) \sim \omega_0^3 \frac{M}{N} \int \frac{d\omega}{2\pi} G(\omega - \Omega/2) G(\omega + \Omega/2)$$

$\Pi =$



- Can be solved self-consistently by the power-law (conformal) ansatz

$$\Sigma(\omega_n) = iA\omega_0^{1-x} |\omega|^x \mathrm{sgn}(\omega_n),$$

$$\tilde{\Pi}(\Omega) \equiv \Pi(\Omega) - \Pi(0) = B\omega_0^{1+2x} |\Omega|^{1-2x}.$$

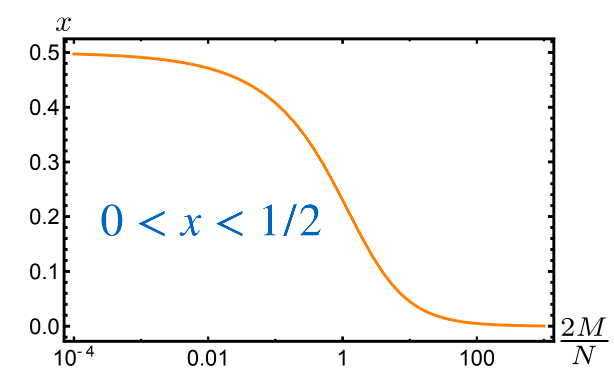
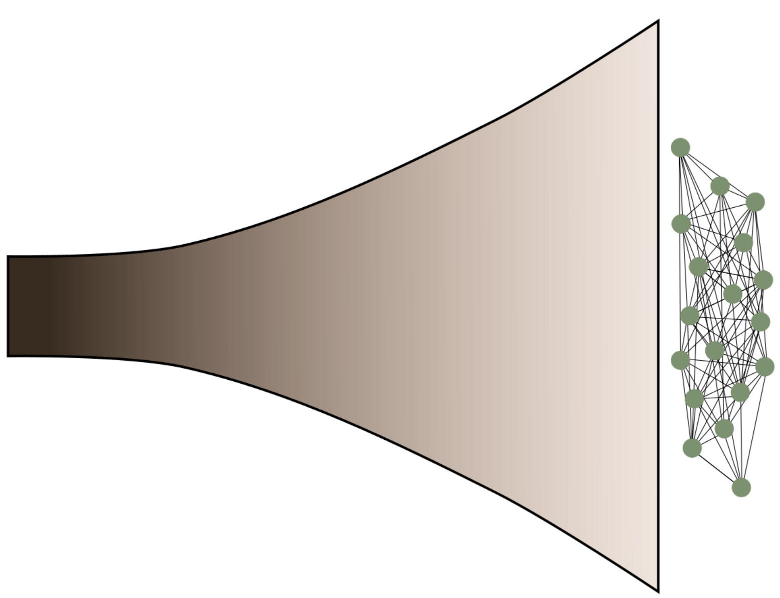


Fig. from: Patel-Sachdev 2018



- Maximally chaotic $\lambda_L = 2\pi k_B T$ just like original SYK Kim-Cao-Altman 2020

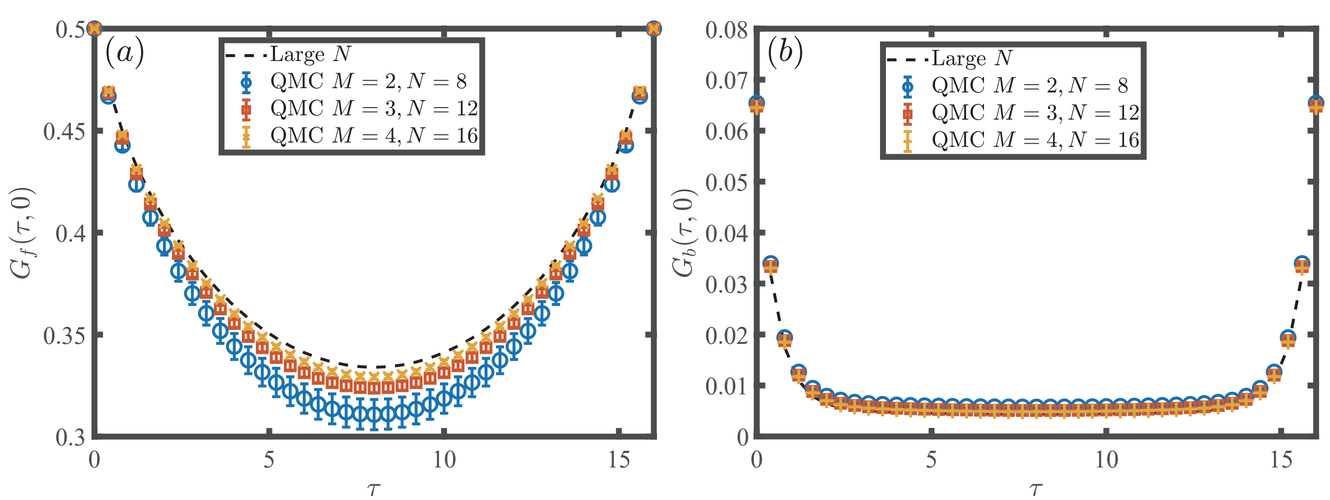
nFL or spin glass?

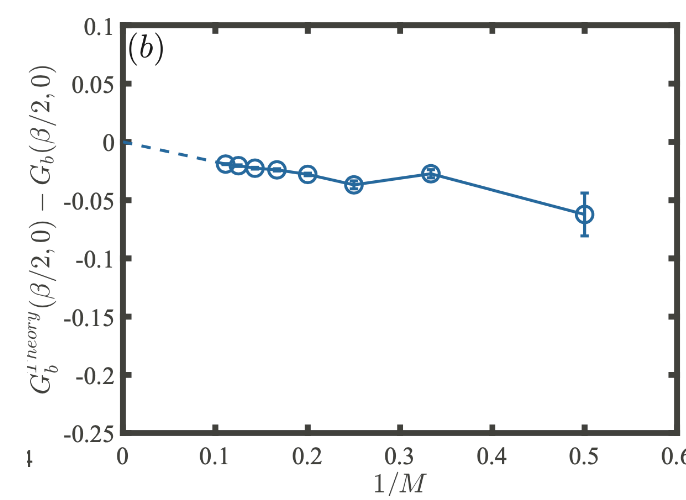
- nFL solution neglects $\mathcal{O}(1/N)$ replica off-diagonal fluctuations
- Summing all $1/N$ diagrams may not be convergent SY 1993, GP 2001, Fu-Sachdev 2016, Baldwin-Swingle 2020

  Fermonic SYK: nFL Bosonic SYK: spin glass at T=0

  Purely bosonic random model: glass at T=0

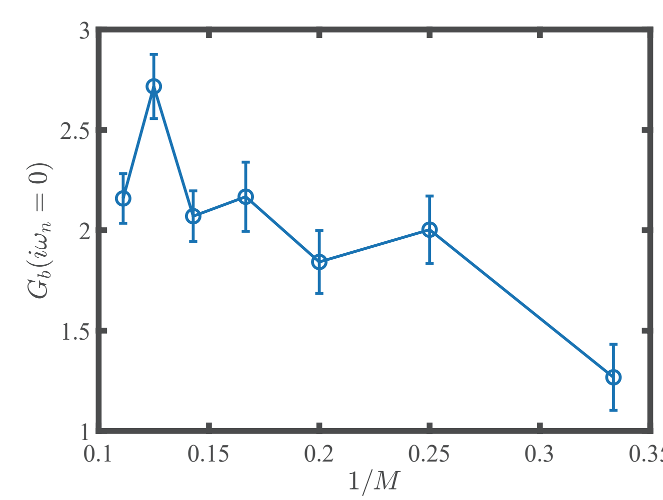
- nFL results confirmed in QMC Pan-Wang-Davis-**YW**-Meng 2020





- A "less random" model shows glassy behavior

$$\mathscr{H} = \sum_{ij}^{N}\sum_{\alpha\beta}^{M}\left(\frac{i}{\sqrt{MN}} t_{\alpha\beta}\,\phi_{ij}\,c_{i\alpha}^{\dagger}c_{j\beta}\right) + \sum_{ij}^{N}\left(\frac{1}{2}\pi_{ij}^{2} + \frac{m_0^2}{2}\phi_{ij}^{2}\right),$$



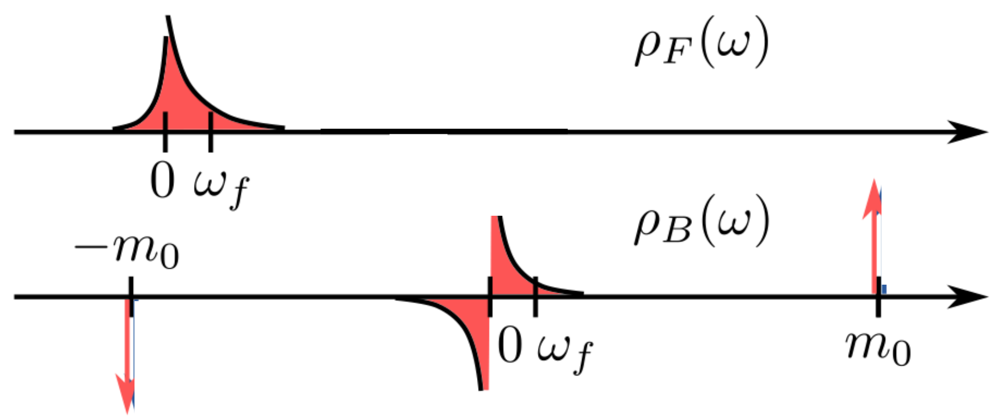
Away from half-filling

- Add a chemical potential term $\mu c_i^\dagger c_i$.
- For small μ see also Georges, Parcollet, Sachdev 2001

$$\Sigma(i\omega) + \mu = \omega_f^{1-x} |\omega|^x (i \mathrm{sgn}(\omega) + \alpha)$$

$$\Pi(i\Omega) + m_0^2 = \beta m_0^2 |\Omega/\omega_f|^{1-x}$$

Spectral asymmetry



- For large μ

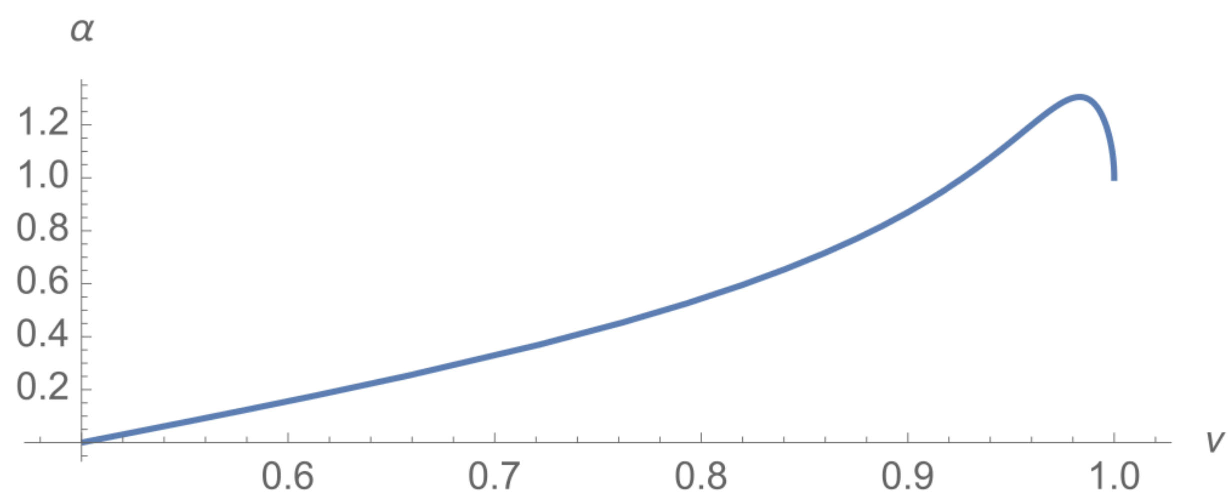
Insulator solution

$$\Pi(\Omega = 0) \sim -\omega_0^3 \int \frac{d\omega}{(i\omega - \mu)^2} = 0! \quad m^2 = m_0^2, \quad \Sigma(\omega) \ll \omega$$

- Transition?

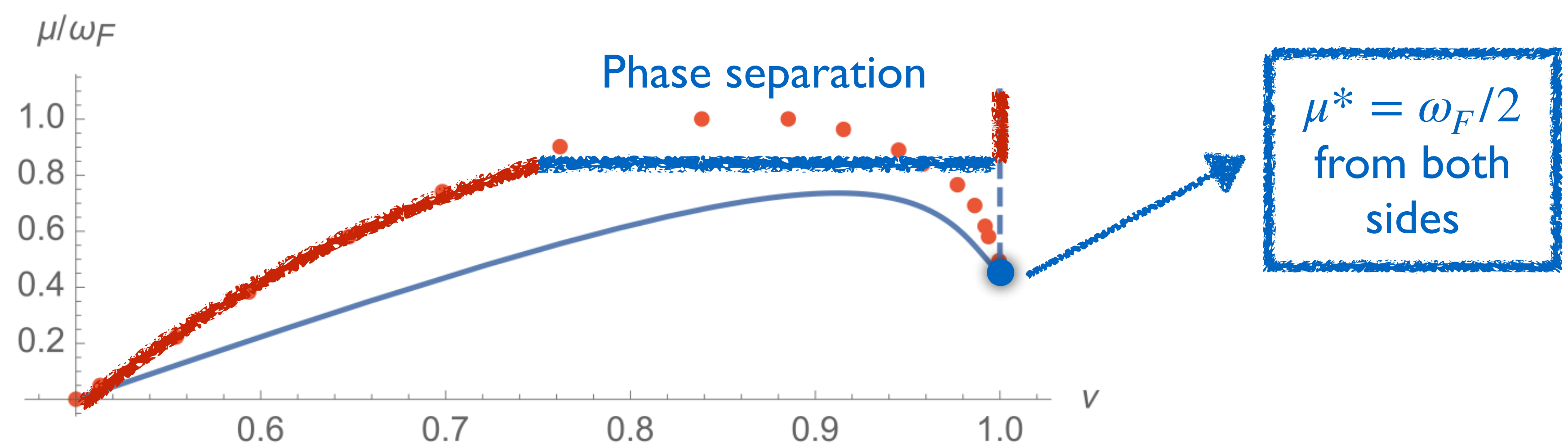
Quantum phase transition varying filling

- Luttinger-like theorem for ν:



YW-Chubukov 2020

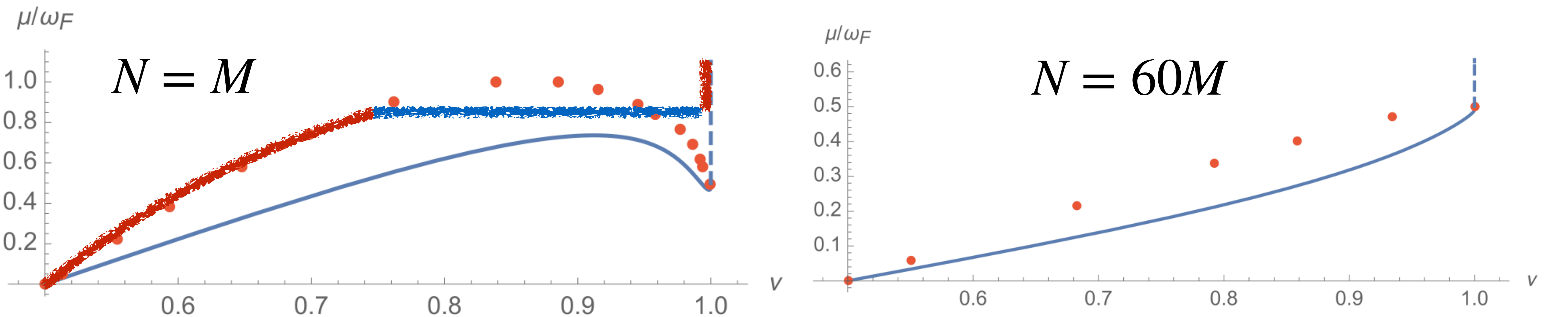
$$\Sigma(i\omega) + \mu = \omega_f^{1-x} |\omega|^x (i\mathrm{sgn}(\omega) + \alpha)$$



- Compressibility $\partial\nu/\partial\mu < 0$; first order phase transition like water-vapor
- Also found numerically in the SYK model Ferrari et al PRL 2017, Patel-Sachdev 2019

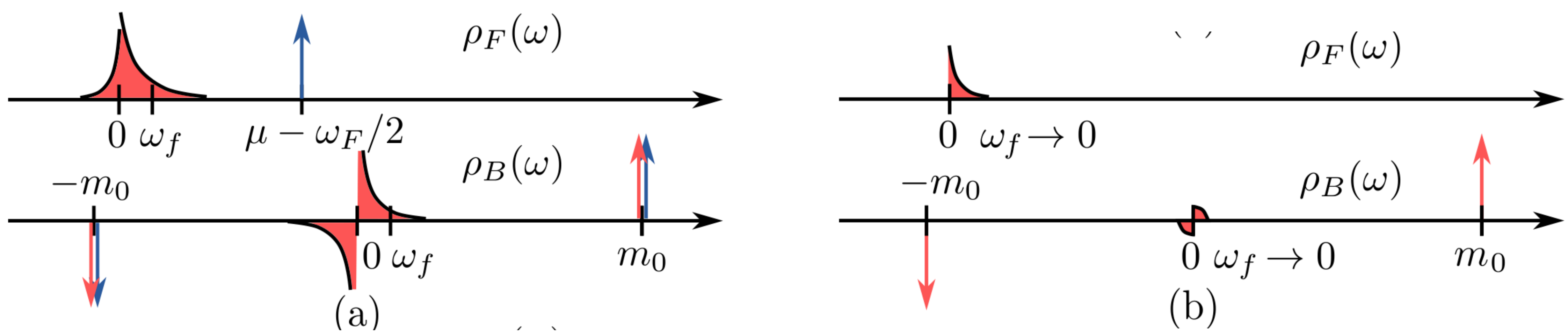
Quantum phase transition varying filling

- In the $N \gg M$ limit (boson $\gg$ fermion), becomes second-order



YW-Chubukov 2020

- Gap filling behavior in both cases (no gap closing)



- Finite T critical point? Behavior of quantum chaos across transition?

nFL pairing problem

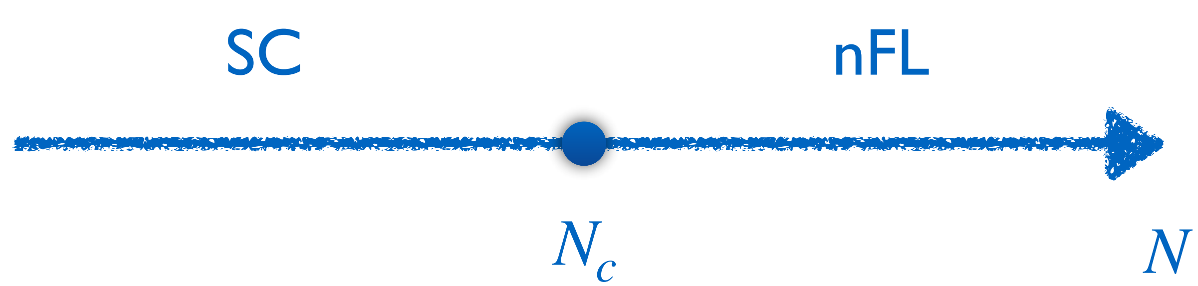
1. Bosons mediate strong attractive interaction in *some* pairing channels; **Good for SC**
2. The same interaction scatters fermions making them incoherent; **Bad for SC**

Winner?

- There is a huge literature on (different versions of) this problem.
  1. Eliashberg equation at $N = 1$
     - SC wins Abanov-Chubukov-Finkel'stein, Wang-Abanov-Altshuler-Yuzbashyan-Chubukov and sometimes loses Andrey's talk on Tuesday
  2. Controlled expansion in large N and small ϵ (dimensional regulator)
     - $1 < N < 1/\epsilon$, pairing preempts nFL. Metlitski-Mross-Senthil-Sachdev
     - $N\epsilon \sim 1$, quantum-critical point in $N_c \sim 1/\epsilon$ separating nFL and SC Raghu-Torroba-Wang, Abanov-Chubukov-Finkel'stein, Chubukov-Schmalian

Pairing near quantum-critical points

- Latter case goes beyond the conventional wisdom of the Cooper instability.
- Requires a fractional spatial dimension ϵ.
- Is such a pairing QCP a robust feature or just an artifact of the dim reg?



- Consider a variant of Yukawa-SYK for pairing

$$\mathscr{H} = \sum_{ij}^{N} \sum_{\alpha\beta}^{M} \left(\frac{i}{\sqrt{MN}} t_{\alpha\beta}^{ij} \phi_{ij} c_{i\alpha}^{\dagger} c_{j\beta} \right) + \sum_{ij}^{N} \left(\frac{1}{2} \pi_{ij}^2 + \frac{m_0^2}{2} \phi_{ij}^2 \right),$$

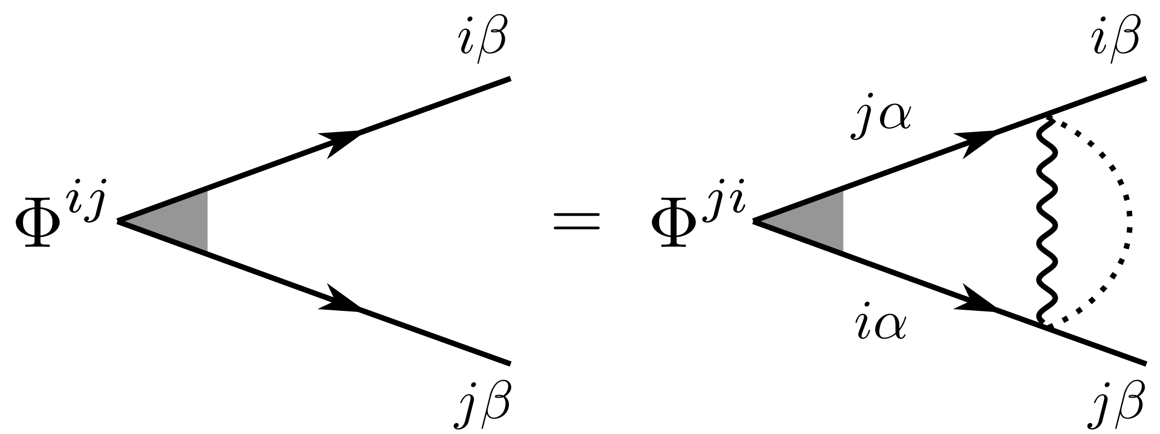
Added a flavor index; N^2 bosons and NM fermions

Pairing in the Yukawa SYK

- Critical boson serves as pairing glue

- Different pairing channels

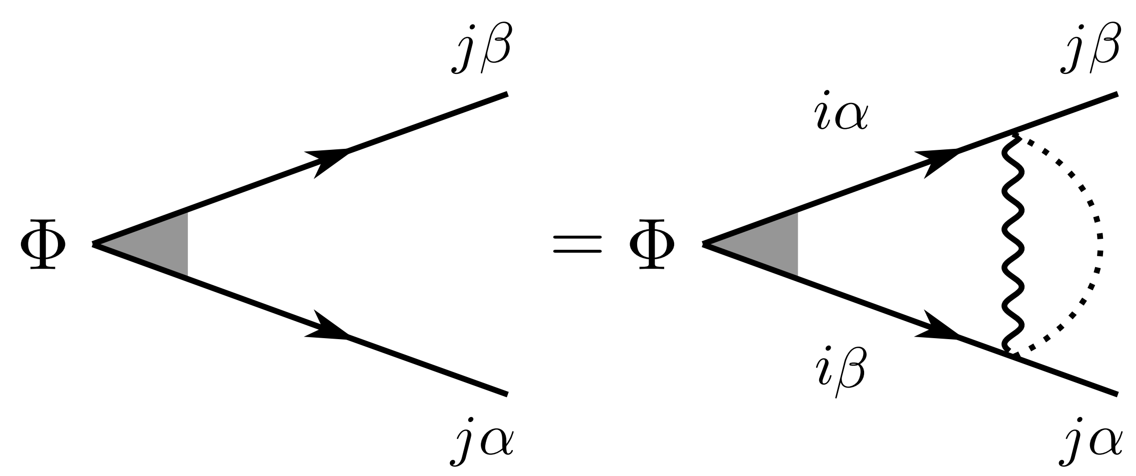
Intra-site,
inter-flavor



$\Phi^{ij} = -\Phi^{ji}$

Repulsive

The pairing problem



- Eliashberg equation for pairing

$$\Phi(\omega) = \frac{\omega_0^3}{M} \int \frac{d\omega'}{2\pi} D(\omega - \omega')\, G(\omega')\, G(-\omega')\, \Phi(\omega')\,.$$

- Plugging in the low-energy forms of the propagators ($0 < 2x < 1$)

$$\Phi(\omega) = \frac{1}{\alpha(x) M} \int_{|\omega'|>\Delta}^{\omega_{\rm NFL}} \frac{d\omega'}{2\pi} \frac{\Phi(\omega')}{|\,\omega - \omega'\,|^{1-2x}\,|\,\omega'\,|^{2x}}\,.$$

Integral cut off by SC gap Δ and nFL energy $\omega_{\rm NFL}$.

Solving the Eliashberg equation

- Without cutoffs, the equation is solved by a power law ansatz

$$\Phi(\omega) = \frac{1}{\alpha(x)M} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\Phi(\omega')}{|\omega - \omega'|^{1-2x} |\omega'|^{2x}}$$

$$\Phi(\omega) = |\omega|^{-y}, \quad y = y(x, M) = y(M, N)\,.$$

- $\Phi(\omega)$ needs to satisfy boundary conditions at Δ and $\omega_{\rm NFL}$

$$\Phi(\omega) = \frac{1}{\alpha(x)M} \left[\int_{-\infty}^{\infty} - 2\int_{\omega_{\rm NFL}}^{\infty} - 2\int_{0}^{\Delta} \right] \frac{d\omega'}{2\pi} \frac{\Phi(\omega')}{|\omega - \omega'|^{1-2x} |\omega'|^{2x}}$$

- The latter two integrals need to vanish. Only if $\Phi(\omega)$ is oscillatory. Abanov-Chubukov-Finkel'stein, Chubukov-Schmalian, YW-Wang-Torroba
- $\Phi(\omega) = |\omega|^{-y}$, requires a complex $y = y' + iy''$

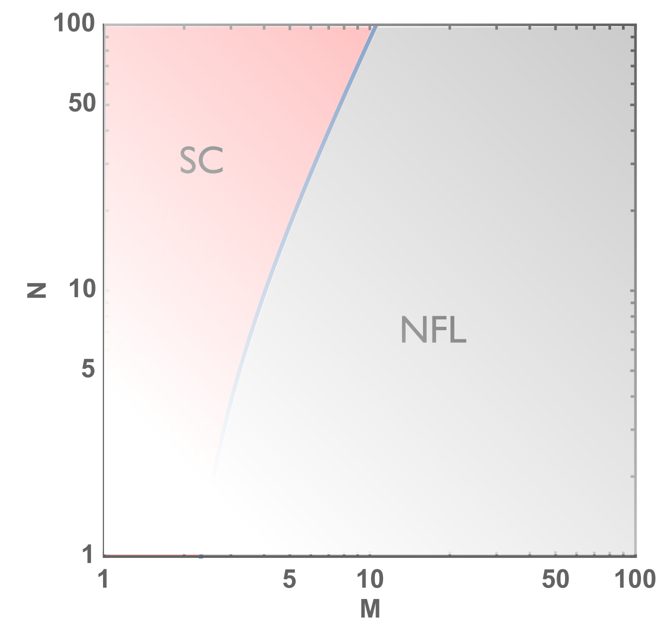
$$\Phi(\omega) = |\omega|^{-y'} \cos(y'' \log|\omega| + \phi)$$

Quantum-critical point for pairing

- Now, when does y become complex?

$$\Phi(\omega) = \frac{1}{\alpha(x)M}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Phi(\omega')}{|\omega-\omega'|^{1-2x}|\omega'|^{2x}}$$

$$\Phi(\omega) = |\omega|^{-y},\ \boxed{y = y(x,M) = y(M,N)}.$$



- For $M \to \infty, N \to \infty$ we need

$$\left(M^2/N\right) \leq 2$$

- SC gap size

$$\Delta \propto \omega_{\text{NFL}} \exp\left(-\frac{\#M^{3/4}N^{-1/4}}{\sqrt{2-M^2/N}}\right)$$

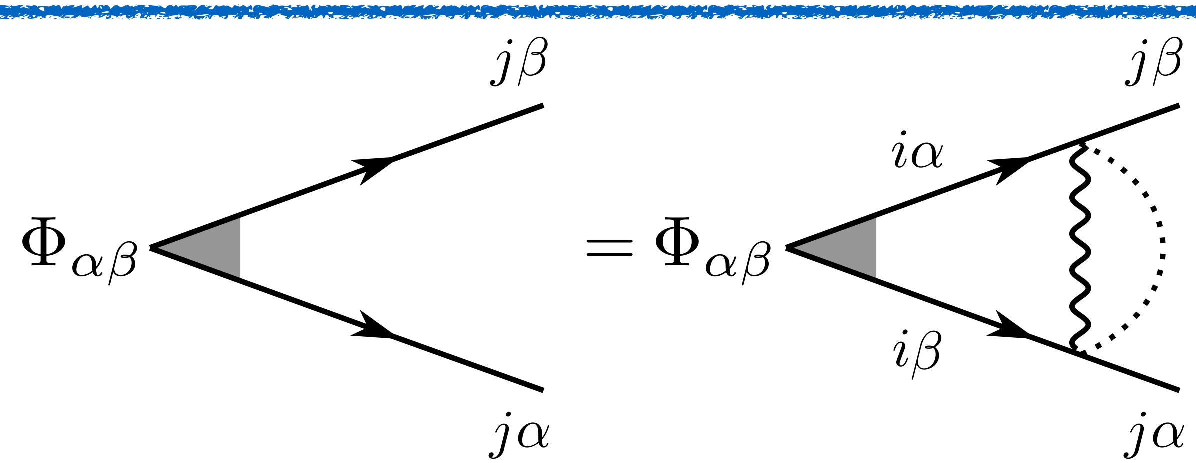
An infinite order transition -- a'la BKT scaling. RG interpretation by Raghu et al

- For $N \sim M$, the system remains a critical metal at $T = 0$, with strong attraction, contrasting with BCS.

Does pairing survive fluctuation effects?

- Previously we argued that the ϕ_{ij} fields cannot order, because fluctuation effects are $\mathcal{O}(N_f/N_b) = \mathcal{O}(N/M)$.
- Near $M^2/N = 2, N \gg M$; this fluctuation effect is especially strong.

Inter-site,
intra-flavor



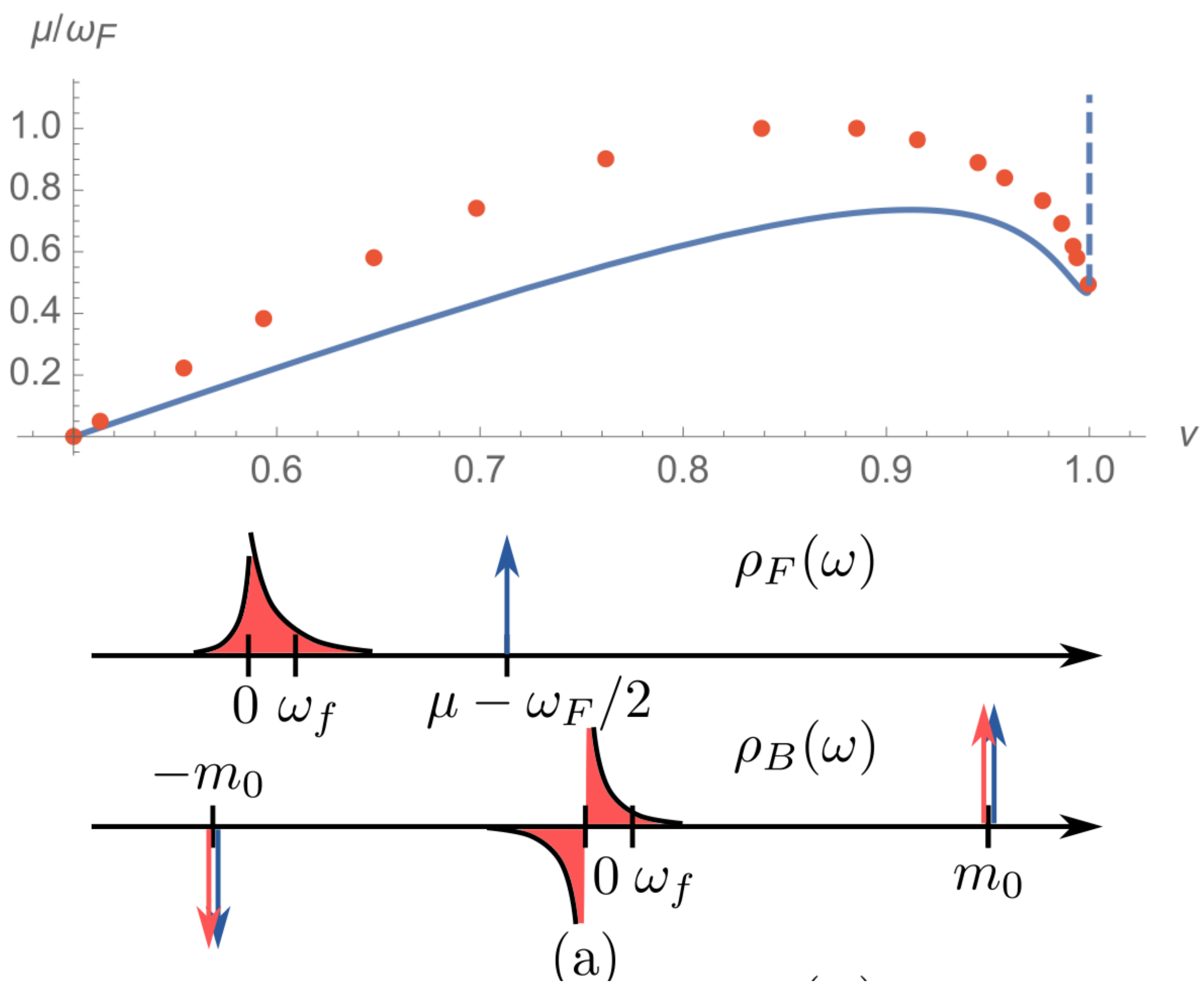
$\Phi_{\alpha\beta} = -\Phi_{\beta\alpha}$

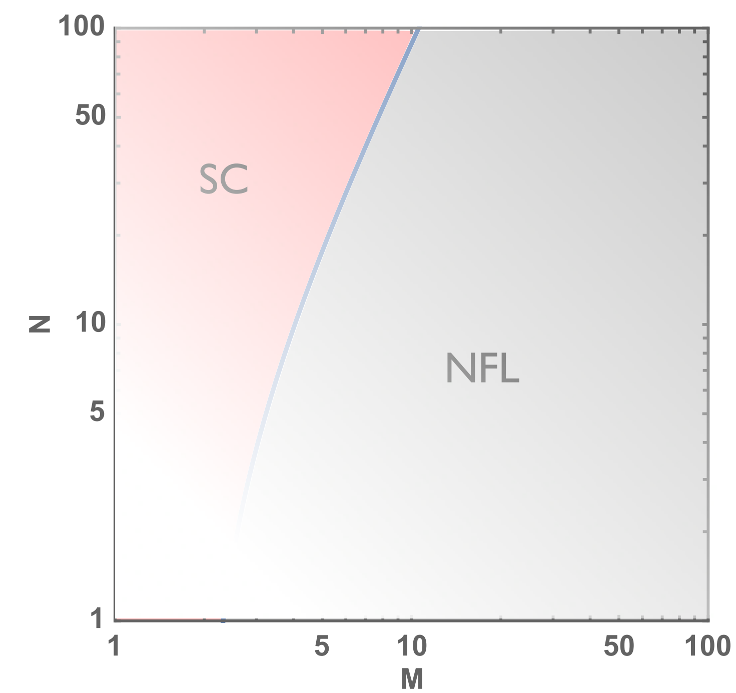
Attractive

- Fluctuation effects for $\Phi_{\alpha\beta}$ are $\mathcal{O}(N_f/N_b') = \mathcal{O}(M/N)$. This is suppressed for the region we are interested in ($N \gg M$). Pairing is mean-field like.

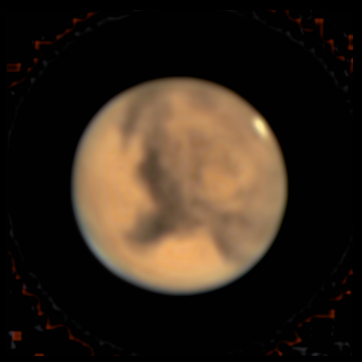
Summary

- We constructed a 0+1d SYK-like model that is solvable in the large-N, M limit.
- Quantum phase transition between nFL and an insulator
- Quantum critical point for Cooper pairing





Thank you!



Mars opposition 2020, with 11-inch backyard telescope