Topological Z_2 invariant in Kitaev spin liquids

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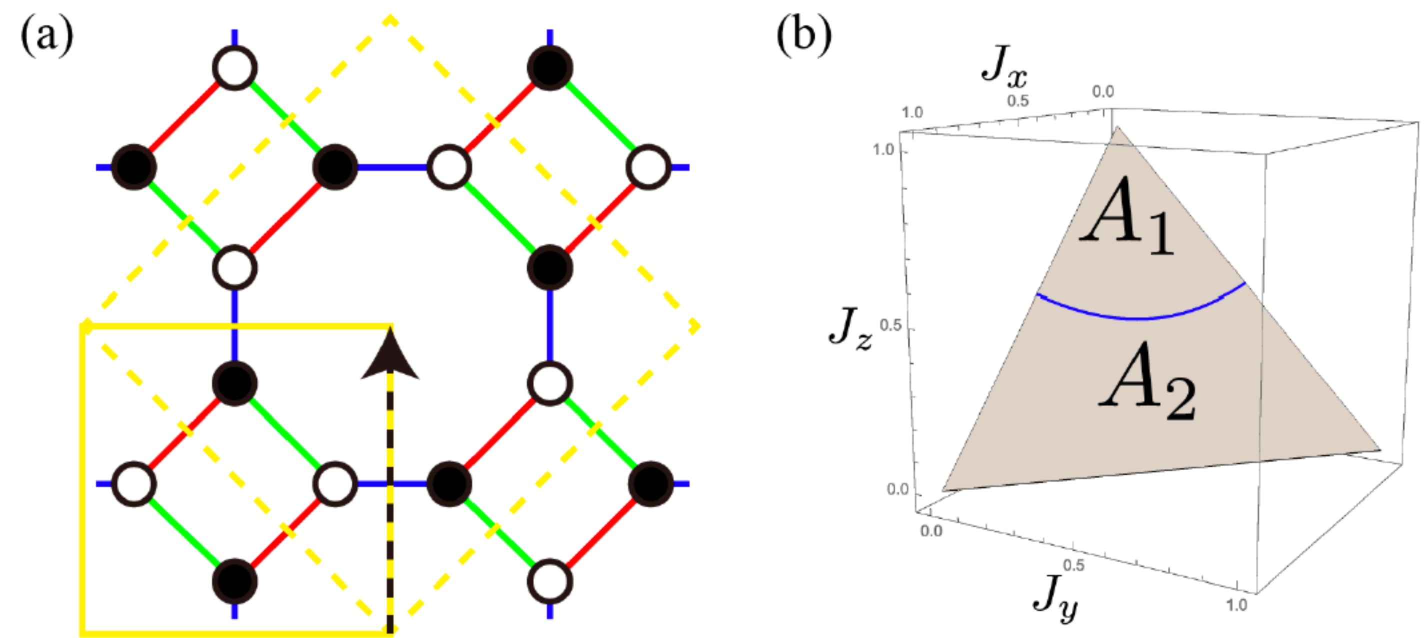
correlated-20 program

arXiv:2005.03399

What is known:

- The Kitaev model on the squareoctagon lattice can be solved exactly.
- From Lieb's theorem the ground state is π-flux.



$$H = -J_x \sum_{\langle jk\rangle \in x} \sigma_j^x \sigma_k^x - J_y \sum_{\langle jk\rangle \in y} \sigma_j^y \sigma_k^y - J_z \sum_{\langle jk\rangle \in z} \sigma_j^z \sigma_k^z,$$

- The phase diagram has two gapped phases A_1 and A_2, separated by the gapless line ($J_x^2 + J_y^2 = J_z^2$).

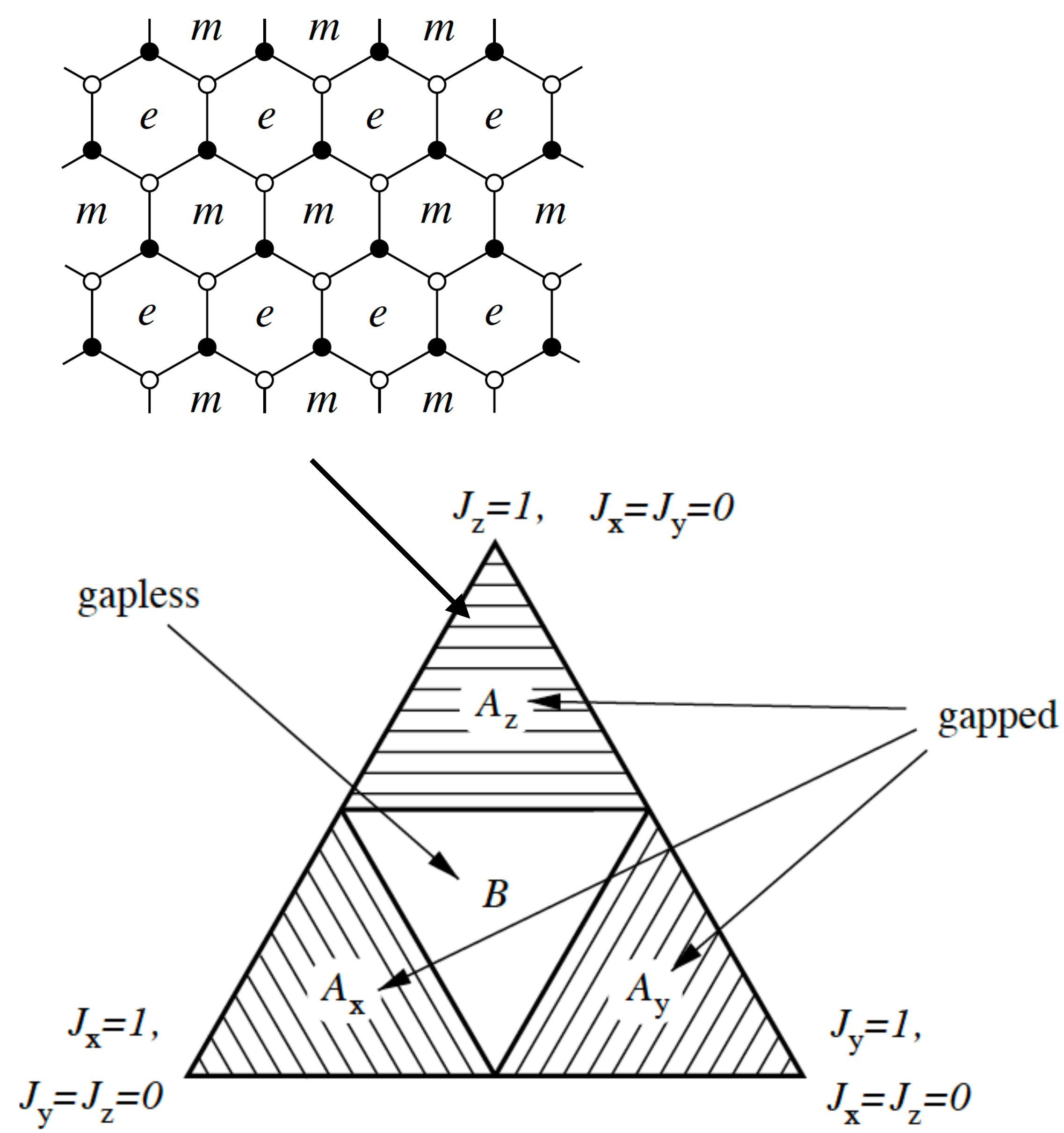
- red: x, green: y, blue: z

This is already known by S. Yang *et al.*, PRB **76**, 180404(R).

- On the gapless line, two Dirac cones appear at (0,0) and (π,π).

1st guess: weak symmetry breaking

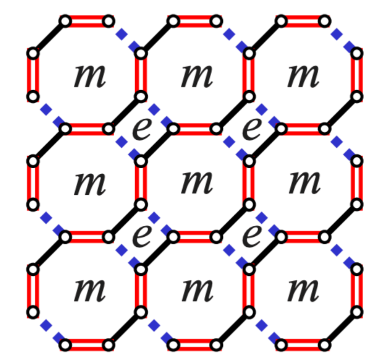
- As for the honeycomb case, Kitaev originally explains the phase difference of A_x, A_y, and A_z using the condensation of anyons.
- The three phases are distinct by the condensation pattern of e- and m-anyons.
- Indeed the three phases are not adiabatiacally connected.



A. Kitaev, Ann. Phys. 321, **2** (2006).

1st guess: weak symmetry breaking

- Kitaev's original theory is not applicable to the squareoctagon lattice because e-m anyons do not break the translation symmetry.



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S. Yang *et al.*, PRB **76**, 180404(R).

- A_1 and A_2 phases are the same in the sense of Kitaev's weak symmetry breaking (anyon condensation).

- The 1st guess failed.

- Another guess is projective symmetry group (PSG). If PSG is different, two phases can be distinct.

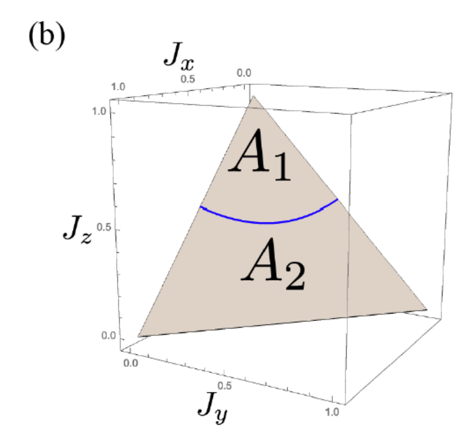
2nd guess: projective symmetry group

- Of course, PSG is a successful theory to classify gapped (or gapless) spin liquids (X. G. Wen 2002 etc.).

- However, Lieb's theorem applies to the whole phase diagram, so the π-flux ansatz state stabilizes in the whole region with the following constant PSG:

$$PSG = \langle\{\hat{G}_0, \hat{G}\hat{T}_1, \hat{G}\hat{T}_2, \hat{G}\hat{\Theta}\}\rangle, \qquad (6)$$

where $G_0(j) = -1$ generates the invariant gauge group.



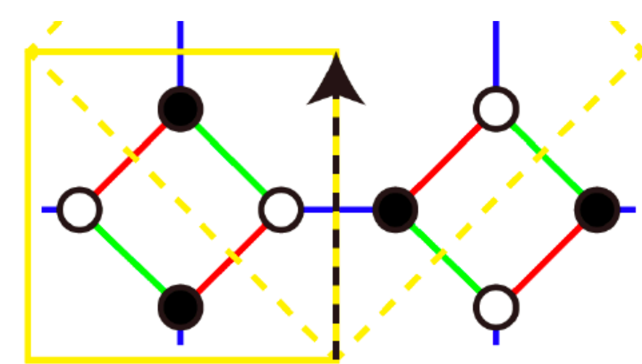
- π-flux ground state and PSG are protected by Lieb's theorem (i.e. same for A_1 and A_2), so PSG also does not work to distinguish them.

Correct answer:
SPT order (topological insulator)
of Majorana fermions!

Remark: PSG still plays an important role

- Naively, the Kitaev model is in class BDI because $\Theta^2 = +1$.
- There is no topological insulator in 2D according to the topological periodic table.
- However, the time-reversal symmetry is implemented projectively, which changes the classification. ←main topic
  - In the Kitaev model the time reversal is related to the sublattice symmetry, and $\Theta = (-1)^j K$ effectively breaks translation in the squareoctagon.

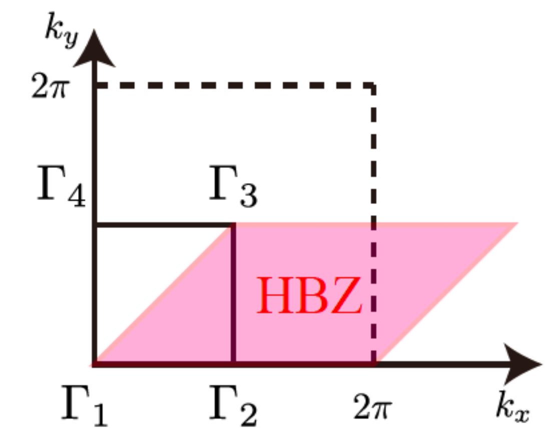
$(-1)^j$: sublattice parity



Answer 1: Fu-Kane symmetry indicator

- Since the sublattice parity is not commensurate with the translation on the squareoctagon lattice, the time reversal connects $\boldsymbol{k}$ to $-\boldsymbol{k}+\boldsymbol{k}_0$.

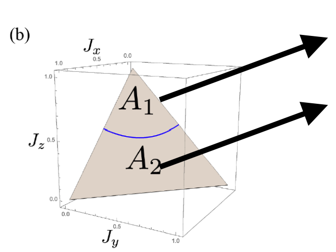
$\boldsymbol{k}_0=(\pi,\pi)$



- This somehow flips the inversion eigenvalue, so the inversion eigenvalue has a relation at IIM (not TRIM): $\xi_\alpha(\Gamma_1)=-\xi_\alpha(\Gamma_3)$, and $\xi_\alpha(\Gamma_2)=-\xi_\alpha(\Gamma_4)$.

IIM: inversion-invariant momentum

- Thus, the topological Z2 invariant is $\delta=\prod_{\alpha=1}^{N}\prod_{i=1}^{2}\xi_\alpha(\Gamma_i)$.



$\delta=-1$
$\delta=1$

L. Fu and C. L. Kane, PRB **76**, 045302 (2007).

Answer 2: Nonlocal Pfaffian invariants

- The topological Z_2 invariant can be defined using a Pfaffian invariant, but is not local in the original BZ.

- There is an effective Kramers degeneracy between $\boldsymbol{k}$ and $-\boldsymbol{k}+\boldsymbol{k}_0$. The "Kramers degeneracy" is apparent only after folding the Brillouin zone between $\boldsymbol{k}$ and $\boldsymbol{k}+\boldsymbol{k}_0$.

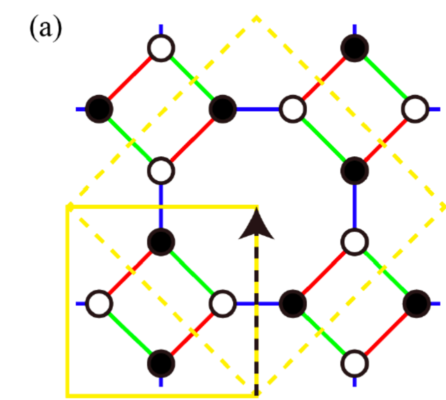
- After that, we can define a new time reversal $\Theta_{\_}$ with $\Theta_{\_}^2 = -1$. Then, the definition gets similar to the one by Fu-Kane-Mele.

$$\delta = \prod_{i=1}^{2} \mathrm{Pf}[w(\Gamma_i)] = \prod_{\alpha=1}^{N} \prod_{i=1}^{2} [-i\xi_\alpha(\Gamma_i)], \qquad (10)$$

L. Fu and C. L. Kane, PRB **76**, 045302 (2007).

Answer 3: Dimensional reduction approach

- Assuming that the time reversal weakly breaks the translation, the unit cell is expanded twice.



- Then, (half translation × time reversal) becomes one of the underlying symmetries without a gauge transformation.

- $\Theta_S = T_{\boldsymbol{D}} K$ obeys $\Theta_S^2 = e^{2\boldsymbol{D}\cdot\boldsymbol{k}i}$, so on the $2\boldsymbol{D}\cdot\boldsymbol{k} = \pi$ line the symmetry class becomes class DIII.

- If we take $\boldsymbol{D} = (1/2, 1/2)$, one dimensional subsystem passing (0,π) and (π,0) is a class DIII superconductor.

C. Fang, M. J. Gilbert, and B. A. Bernevig, PRB **88**, 085406 (2013).

Answer 3: Dimensional reduction approach

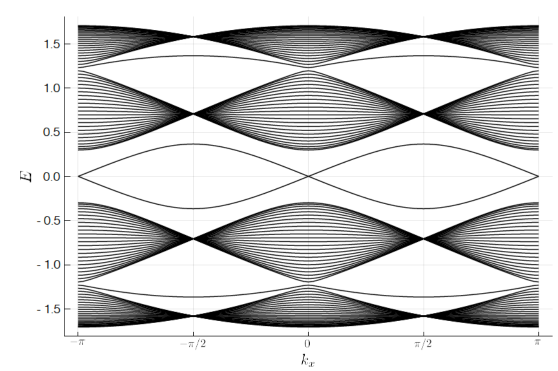
- Thus, we can use a Z_2 invariant for the 1D DIII topological superconductor on this subsystem.

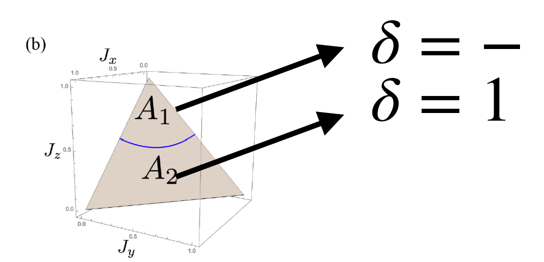
Θ_S action at $(0,\pi)$

$$\delta = \left(\det U^K\right) \frac{\mathrm{Pf}[w_S(0,\pi)]}{\mathrm{Pf}[w_S(\pi,0)]}. \qquad (11)$$

$$U^K_{\alpha\beta} = \langle\tilde{\alpha}| \left(\lim_{n\to\infty} \prod_{j=0}^{n} P_F(\boldsymbol{k}_j) \right) |\beta\rangle\,, \qquad (12)$$

where $\boldsymbol{k}_j = (j\pi/n, \pi - j\pi/n)$ and $P_F(\boldsymbol{k})$ is a spectral projector onto the occupied states at $\boldsymbol{k}$ [23].





- Reproducing δ, and edge states for δ = -1 (A_1 phase).

J. C. Budich and E. Ardonne, PRB **88**, 134523 (2013).

Summary

- We defined the same invariant in three ways.
- A new Z_2 invariant is beyond the topological periodic table (cannot be explained by class BDI).
- A new classification is beyond PSG or anyon condensation.
- Suggesting the existence of a large number of unknown gapped spin liquids: topological crystalline spin liquids, etc.