

# **Fractional Fermi Liquid from doping spin-triplet doublons**

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## Collaborators:



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**Harvard**



**Zheng Zhu**  
**MIT->Harvard->KITs**

## Ref:

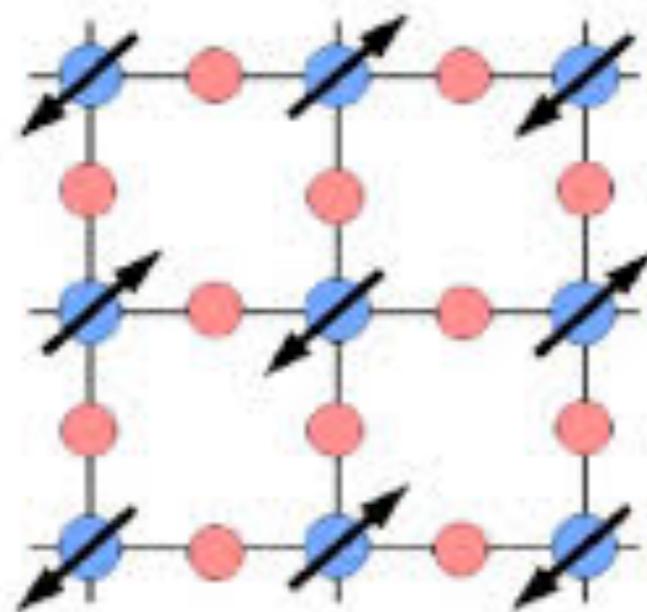
Type II t-J model in superconducting nickelate  $Nd_xSr_{1-x}NiO_2$ , **Ya-Hui Zhang** and Ashvin Vishwanath  
arxiv: 1909.12865

Symmetric Pseudogap metal in a generalized t-J model, **Ya-Hui Zhang** and Zheng Zhu  
arxiv: 2008.11204

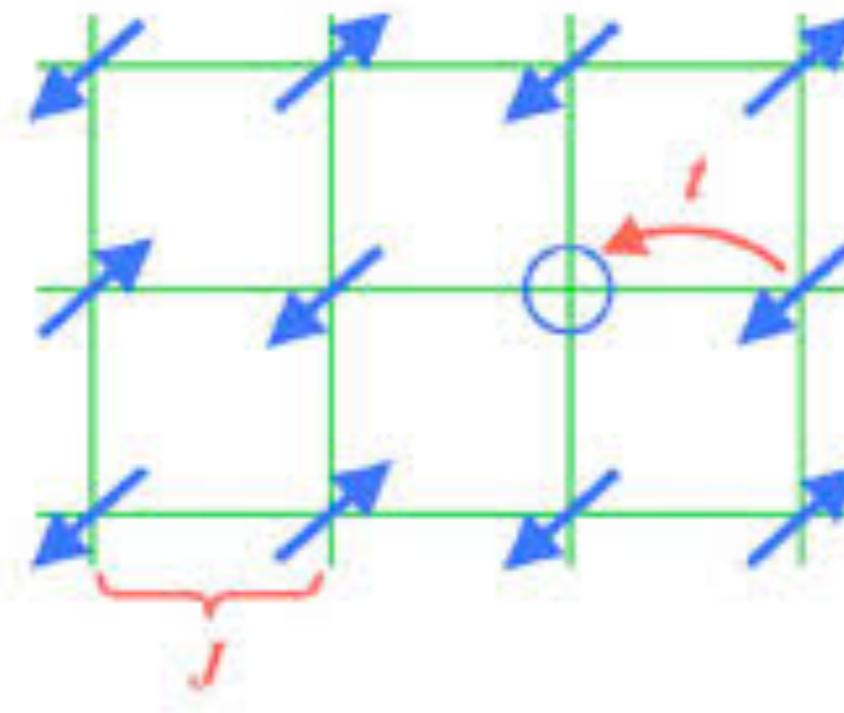
# A new t-J model

## Conventional t-J model

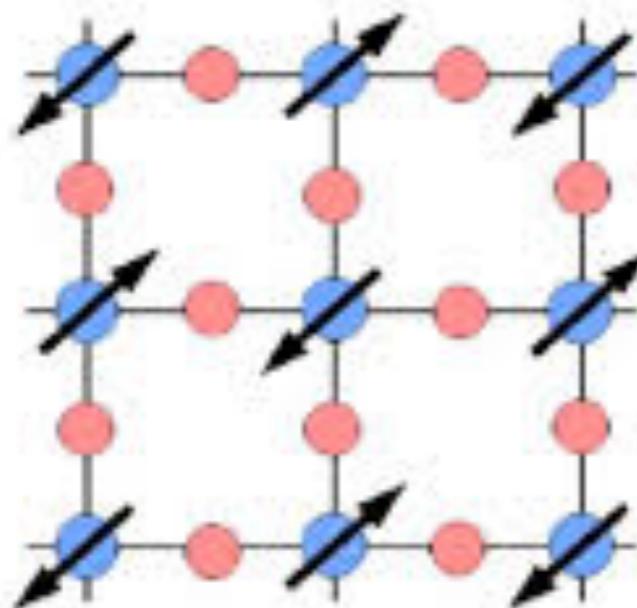
one hole per site



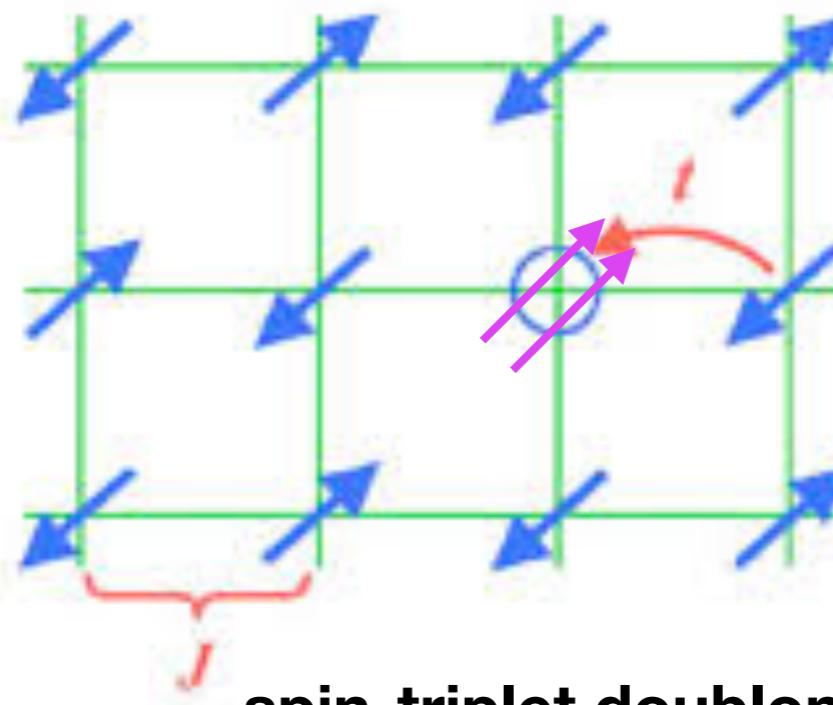
spin-singlet doublon



An unusual case:



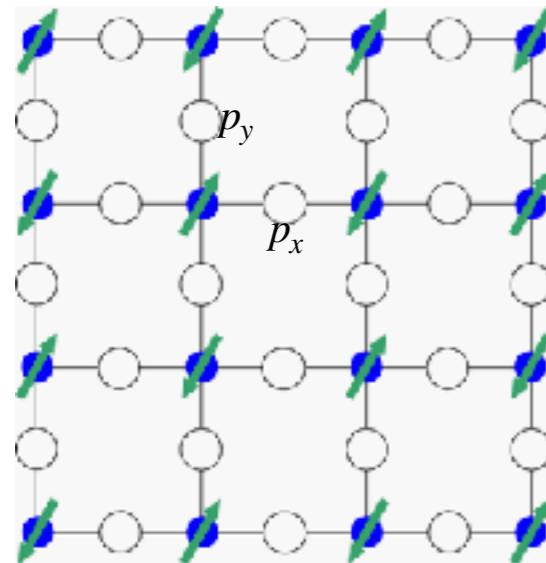
Dope holes  
→



one hole per site

spin-triplet doublon

# Experimental realization: hole doping a spin 1/2 Mott insulator

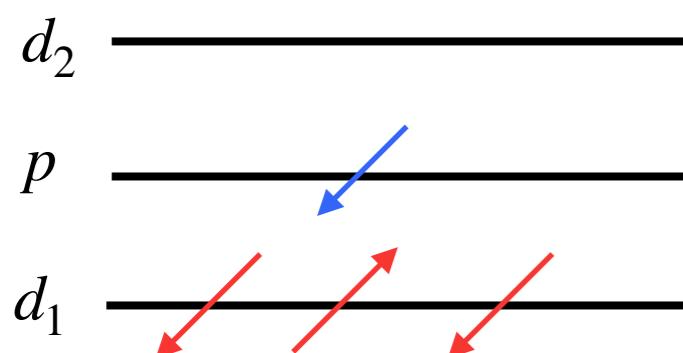


## Spin Singlet

$$ad_{1;\uparrow}^\dagger d_{1;\downarrow}^\dagger |0\rangle + b\epsilon_{\sigma\sigma'}d_{1;\sigma}^\dagger p_{\sigma'}^\dagger |0\rangle$$

**cuprate:**  $b \approx 1$   $a \approx 0$

$$J_K \vec{S}_1 \cdot \vec{S}_p \quad J_K \sim \frac{t_{dp}^2}{U}$$



$$d_{i;1} : d_{x^2-y^2}$$

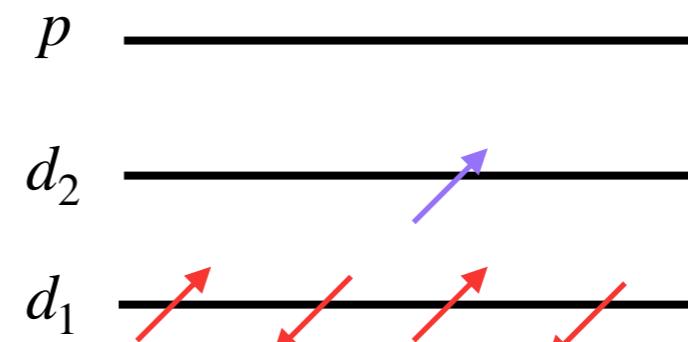
$$d_{i;2} : d_{3z^2-r^2}$$

$$p_i \sim p_{i+\frac{1}{2}\hat{x}} + p_{i-\frac{1}{2}\hat{x}} + p_{i+\frac{1}{2}\hat{y}} + p_{i-\frac{1}{2}\hat{y}}$$

## Spin Triplet

$$\begin{aligned} & d_{1;\uparrow}^\dagger d_{2;\uparrow}^\dagger |0\rangle \quad d_{1;\downarrow}^\dagger d_{2;\downarrow}^\dagger |0\rangle \\ & \frac{1}{\sqrt{2}}(d_{1;\uparrow}^\dagger d_{2;\downarrow}^\dagger + d_{1;\downarrow}^\dagger d_{2;\uparrow}^\dagger) |0\rangle \end{aligned}$$

$$-J_H \vec{S}_1 \cdot \vec{S}_2$$



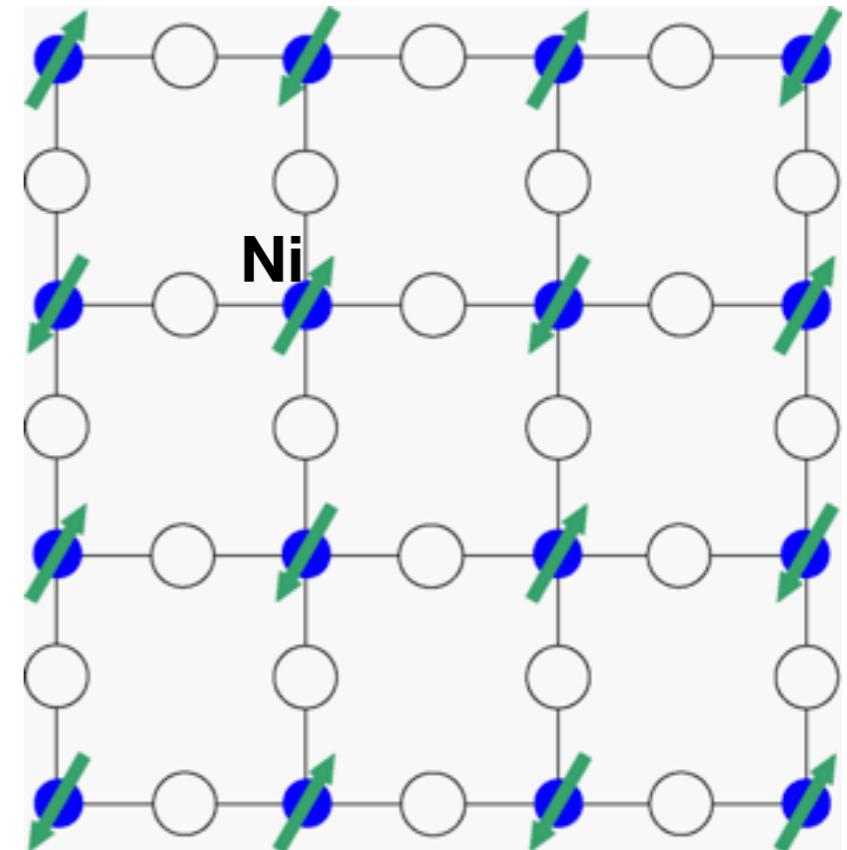
# Possible realization in Nickelate

Danfeng Li,..., Harold Y. Hwang, Nature, 572, 624 (2019)

Cuprate-like structure

Undoped site: one hole on d<sub>1</sub> orbital  $Ni^{1+}$

T<sub>c</sub> around 15 K for hole doping x=0.2



**Key difference from cuprate:**

Oxygen band is further away from Fermi level

**Spin triplet doublon may be favored**

**Current Status:**

It is still not clear whether spin-singlet or spin-triplet doublon wins

Numerical computations (DFT, LDA+U) give opposite results between groups and even within one group.

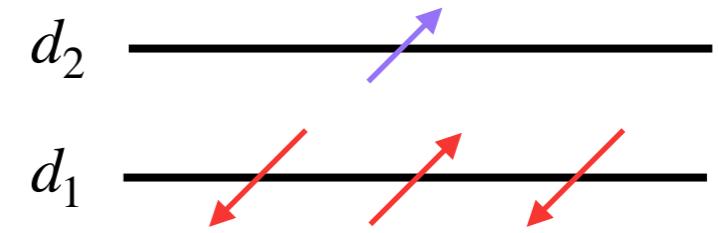
Experimental result (X-ray ) is not conclusive

# Spin-triplet t-J model

Hilbert Space:

- t-J model Hilbert space (5 dimensional)

$$\{| \uparrow \rangle, | \downarrow \rangle, |x\rangle, |y\rangle, |z\rangle\}$$



$$H = H_t + H_J$$

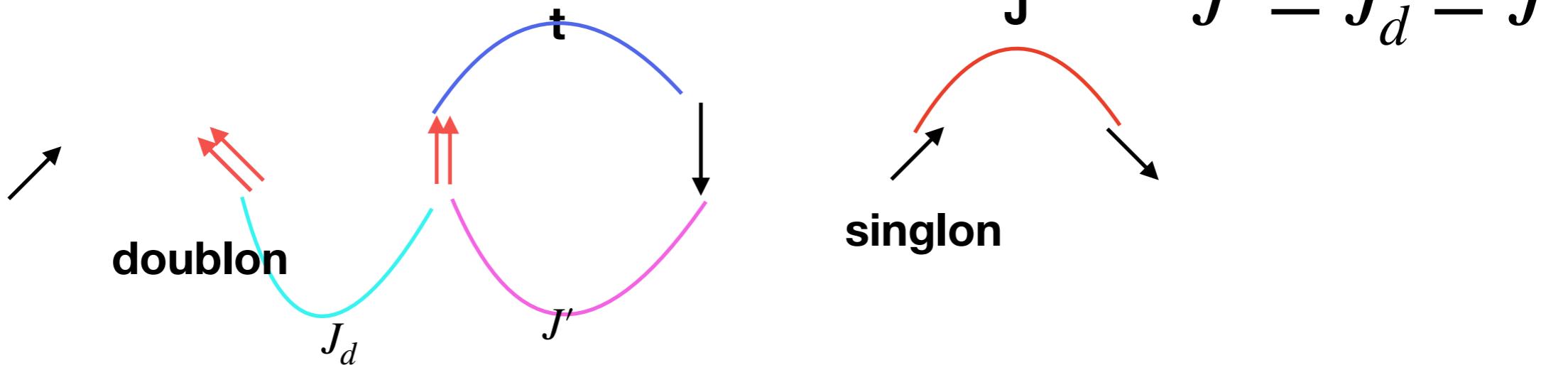
$$H_t = - \sum_{\langle ij \rangle} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}$$

$$H_J = \sum_{\langle ij \rangle} (J s_i \cdot s_j + J_d S_i \cdot S_j + \frac{J'}{2} (\mathbf{s}_i \cdot \mathbf{S}_j + \mathbf{S}_i \cdot \mathbf{s}_j))$$

**Singlon-doublon coupling**

$$\begin{aligned} d_1 &= 0 \\ c &= d_2 \end{aligned}$$

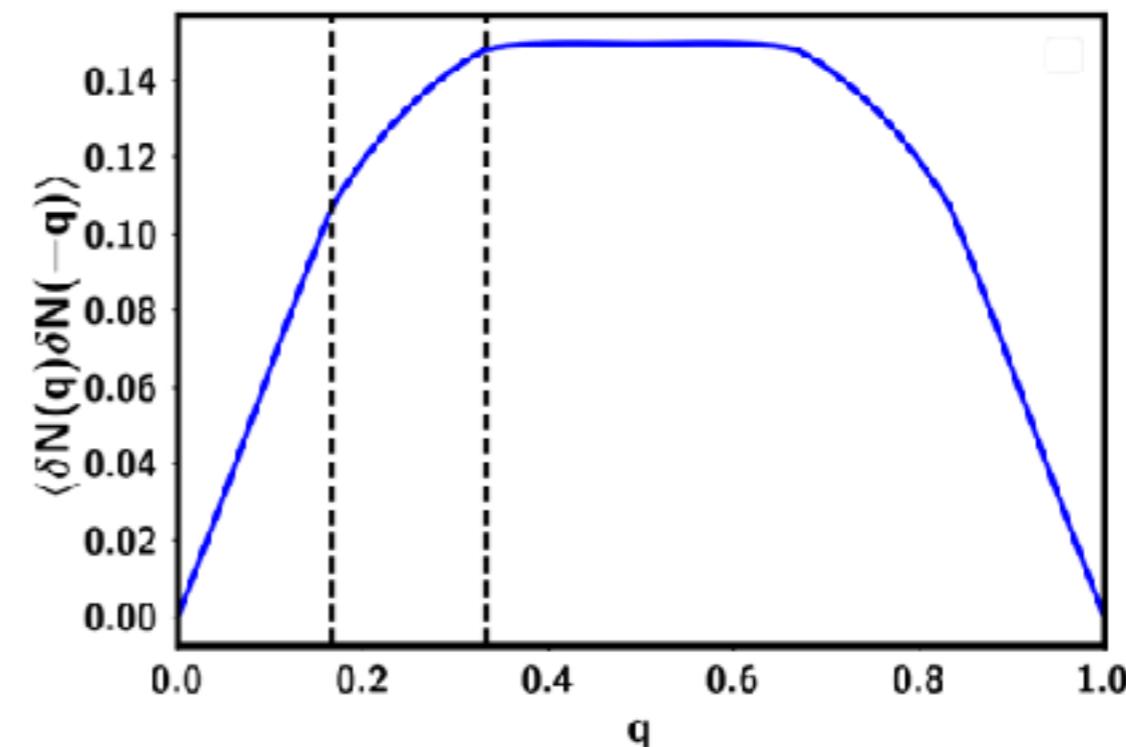
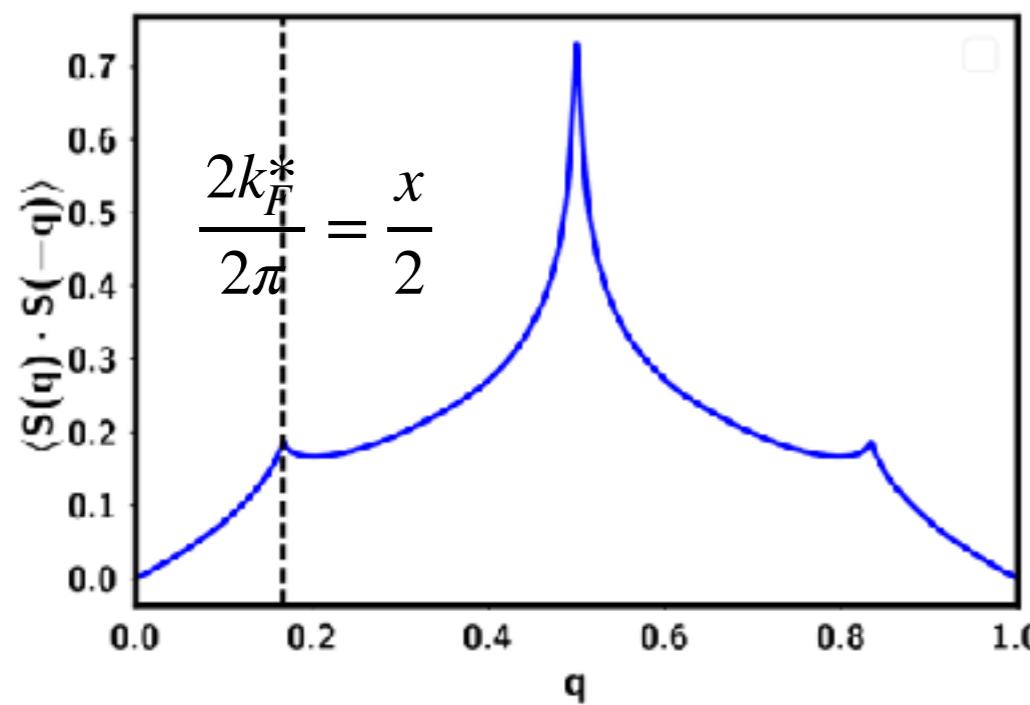
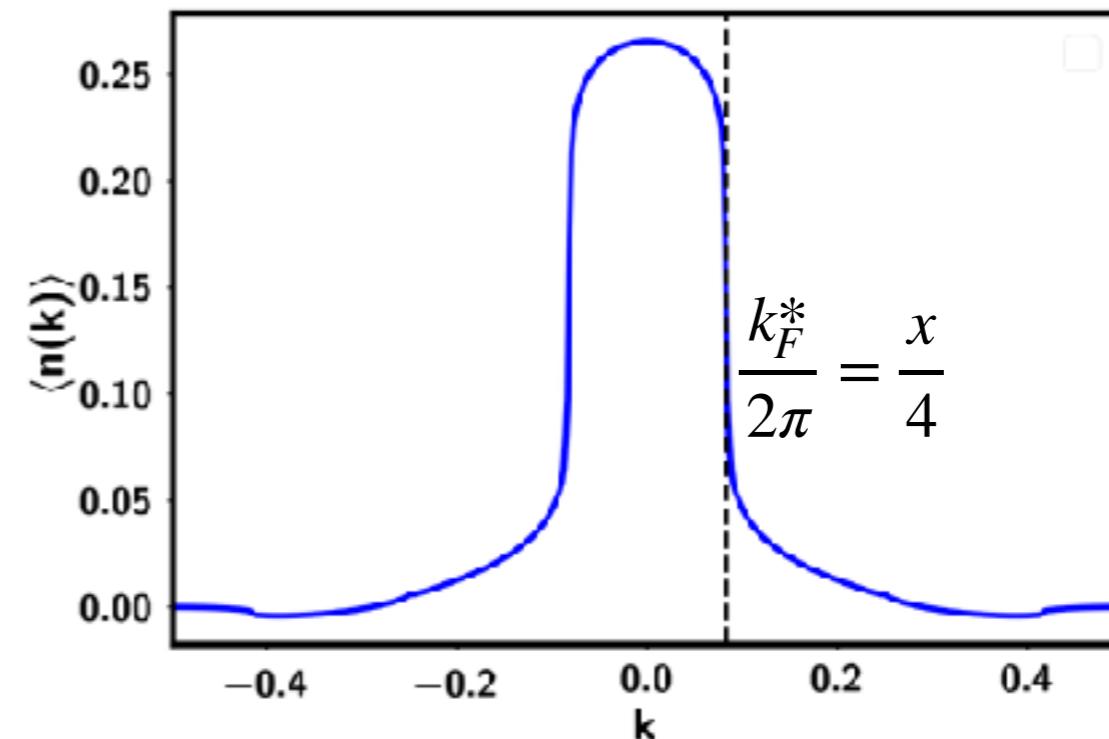
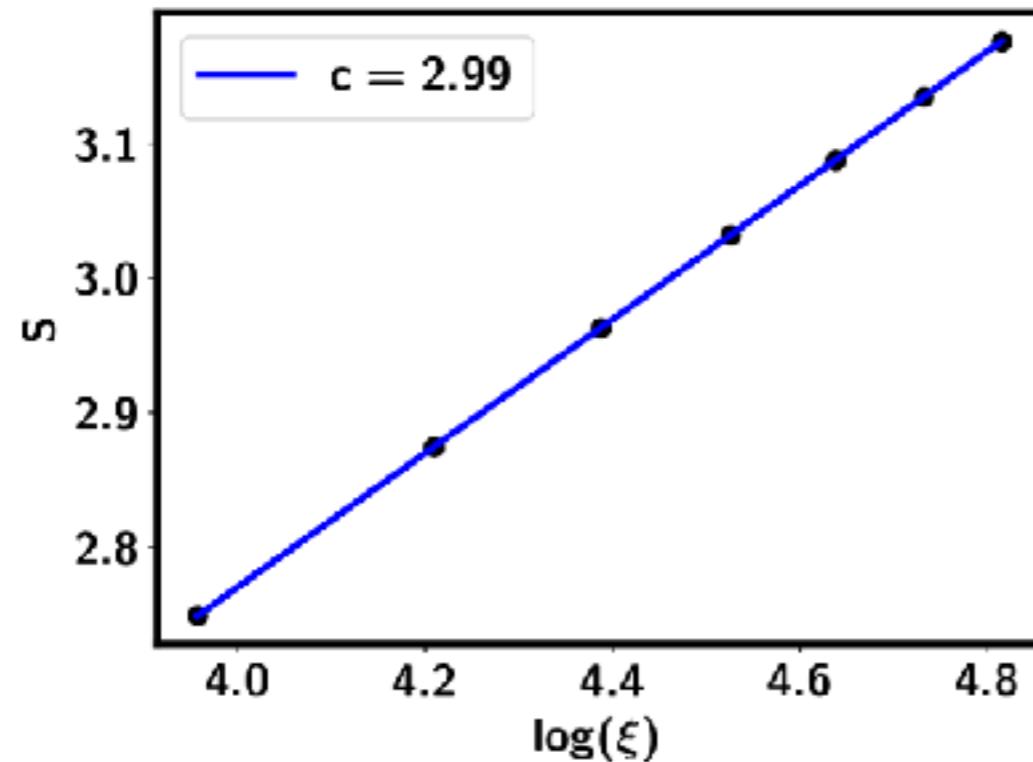
YH. Zhang and A.V - arXiv:1909.12865



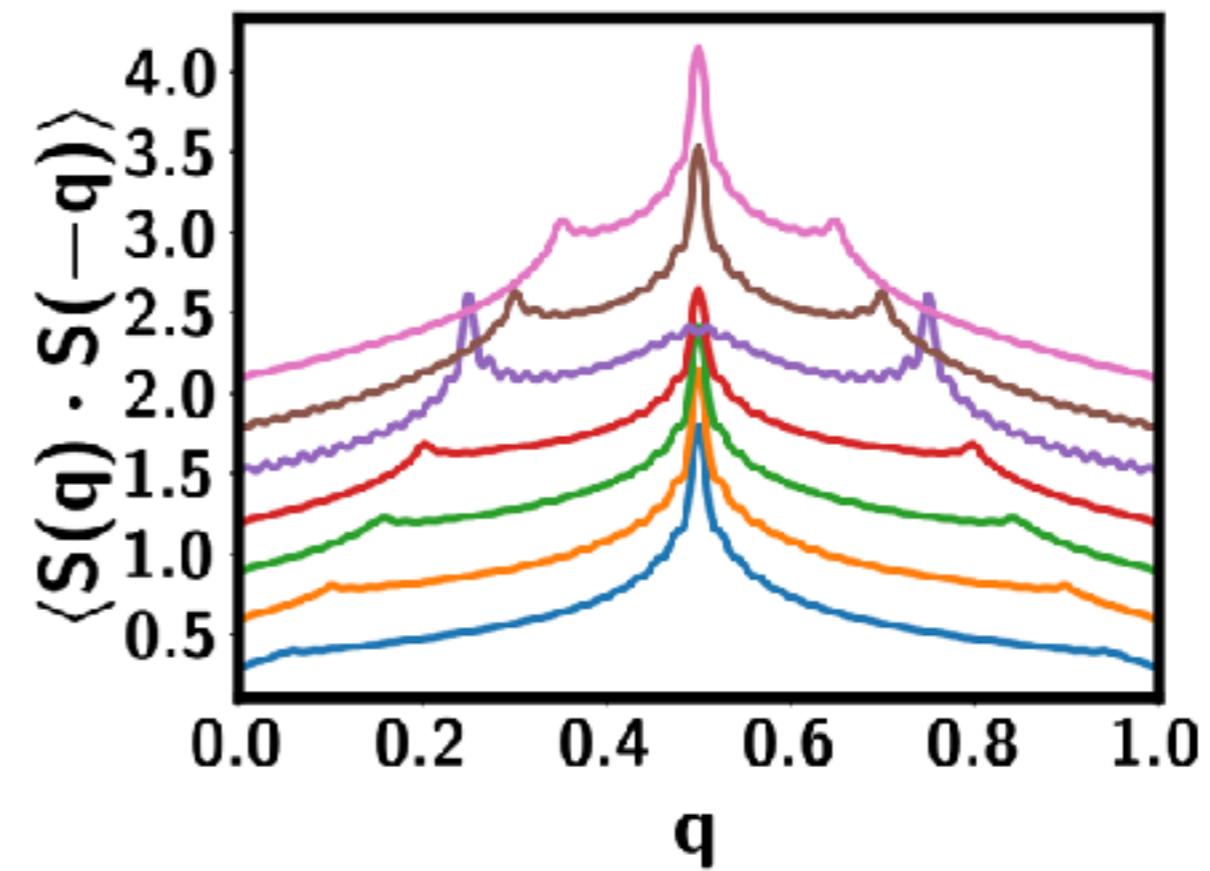
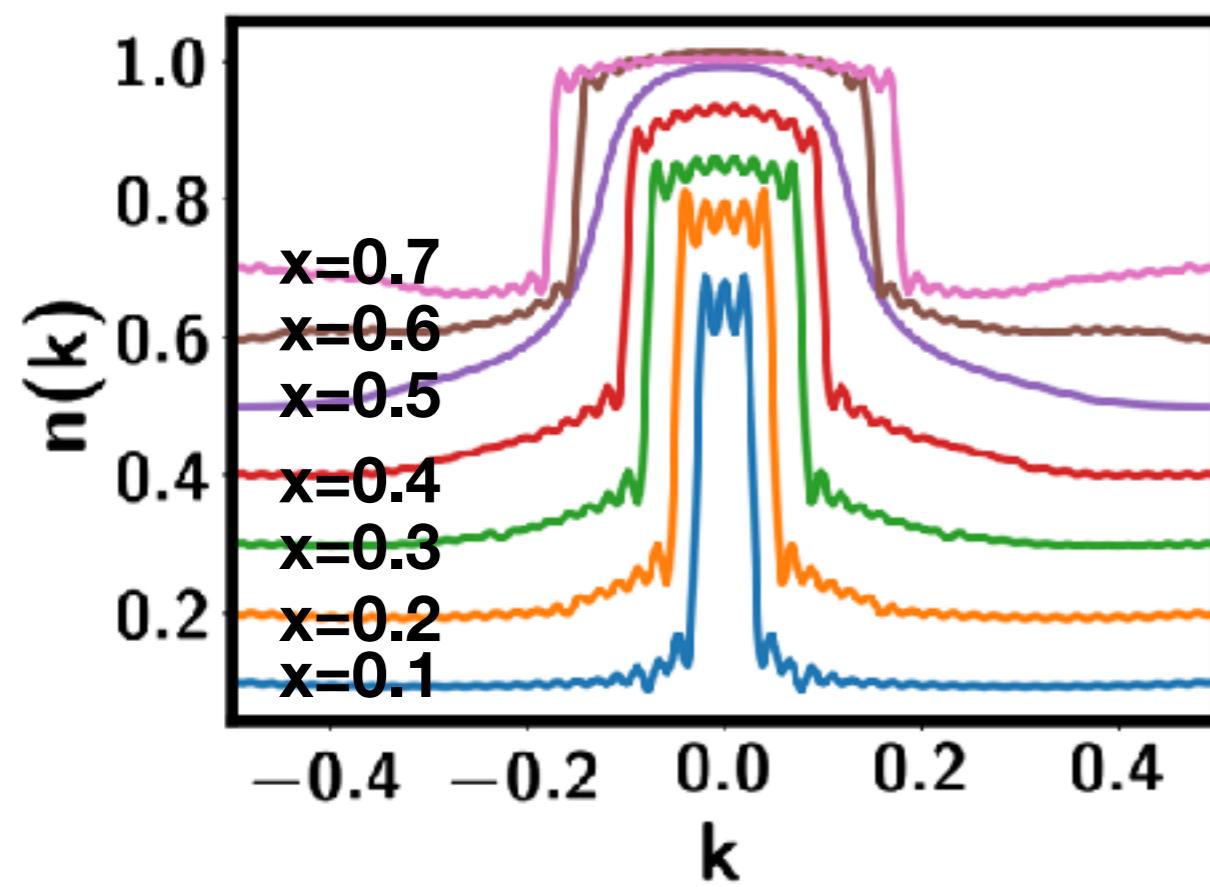
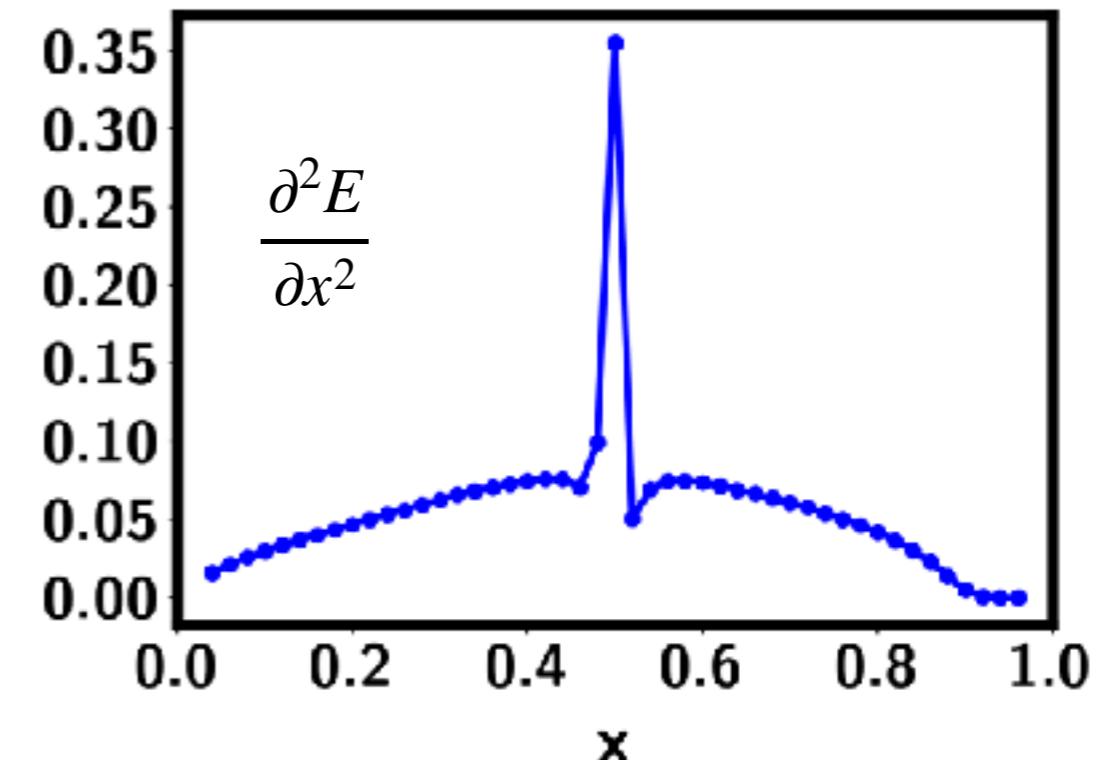
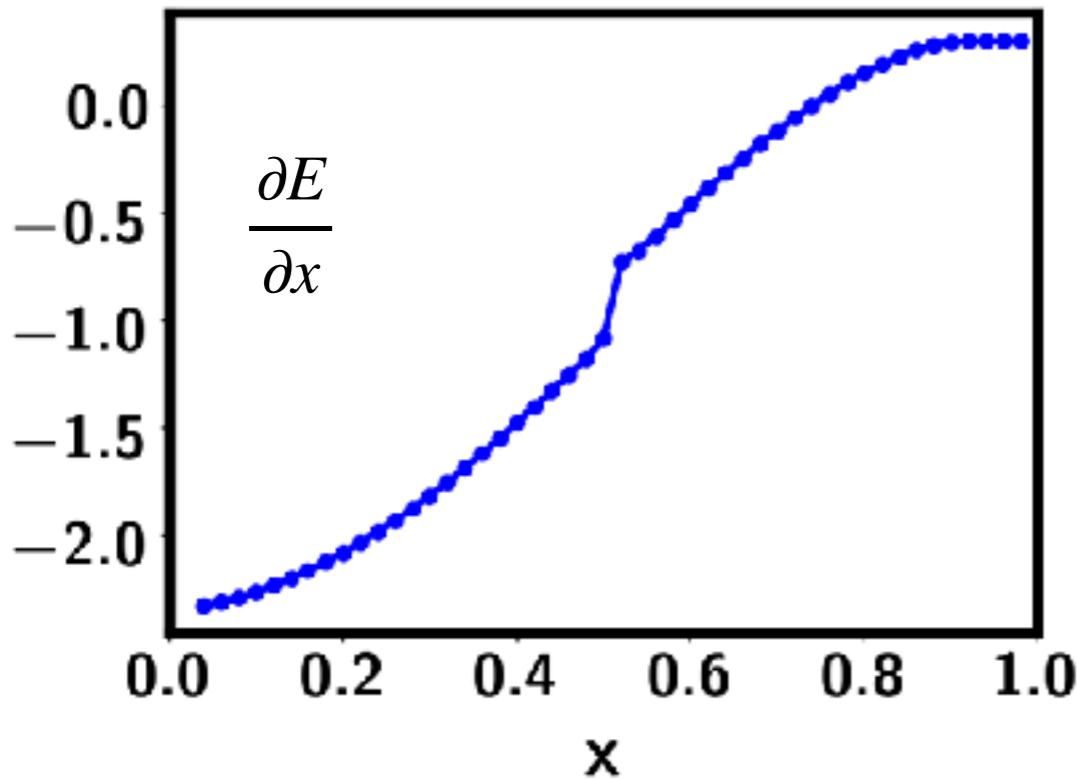
# DMRG in 1D

$x=1/3$ , infinite DMRG;

$J/t = 0.5$



A small pocket+a spin mode at  $Q=\pi$



$x=0.5$ : CDW/wigner crystal of doublon;

$J/t = 0.5$

# Luttinger Theorem

Masanori Yamanaka, Masaki Oshikawa and Ian Affleck, PRL, 1997

## 1D LSM theorem:

Consider a Hamiltonian with short-range hopping, and assume it is translationally invariant and also conserves the total particle number (i.e. has charge U(1) symmetry) and parity or time-reversal. In a chain of length L with periodic boundary conditions, there is at least one low-energy ( $O(1/L)$ ) state above the ground state, if the fermion number per unit cell  $v$  is not an integer. The low-energy state has crystal momentum  $2\pi v$  relative to the ground state

**Gapless state at**

$$\frac{Q}{2\pi} = \frac{1+x}{2}$$

**Conventional Luttinger Liquid (LL)**

$$\frac{2k_F}{2\pi} = \frac{1+x}{2}$$

**Fractional Luttinger Liquid (LL\*)**

$$\frac{Q}{2\pi} = \frac{x}{2} + \frac{1}{2}$$

small pocket + “spin liquid”

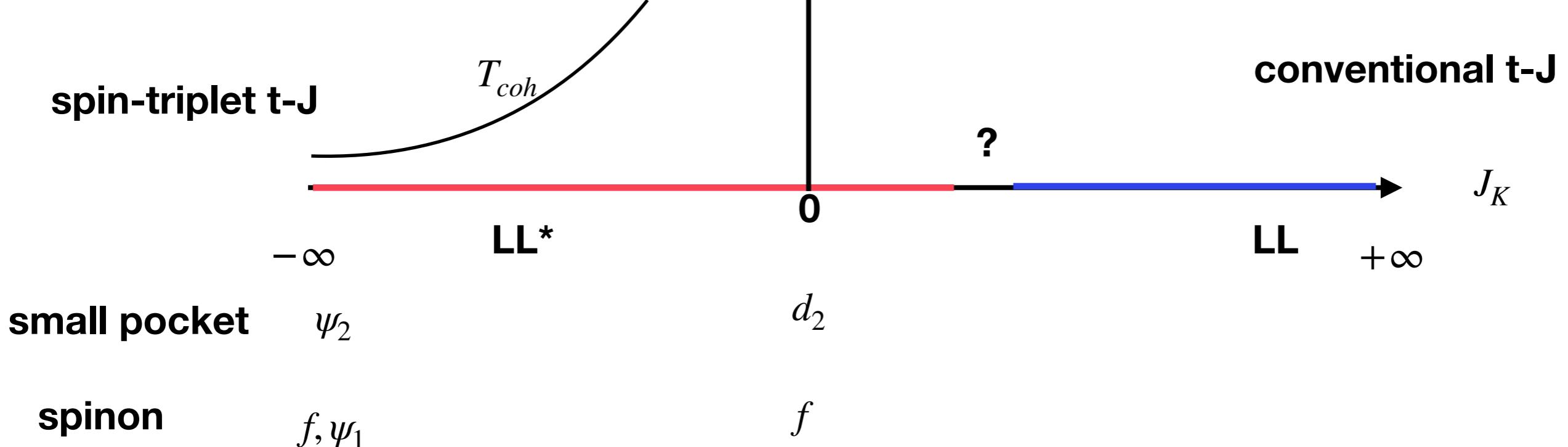
# Emergent quasi-particle

**Kondo-Heisenberg model**

$$H = -t d_{i;2}^\dagger d_{j;2} + J_K \sum_i (d_{i;2}^\dagger \vec{\sigma} d_{i;2}) \cdot \vec{S}_{i;1} + J \sum_{ij} \vec{S}_{i;1} \cdot \vec{S}_{j;1}$$

**Three-fermion parton theory:**

$$c_{i;\sigma} = \frac{1}{2} \sum_{\sigma'} \sum_{a,b=1,2} \epsilon_{ab} f_{i;\sigma'}^\dagger \psi_{i;a\sigma} \psi_{i;b\sigma'}$$

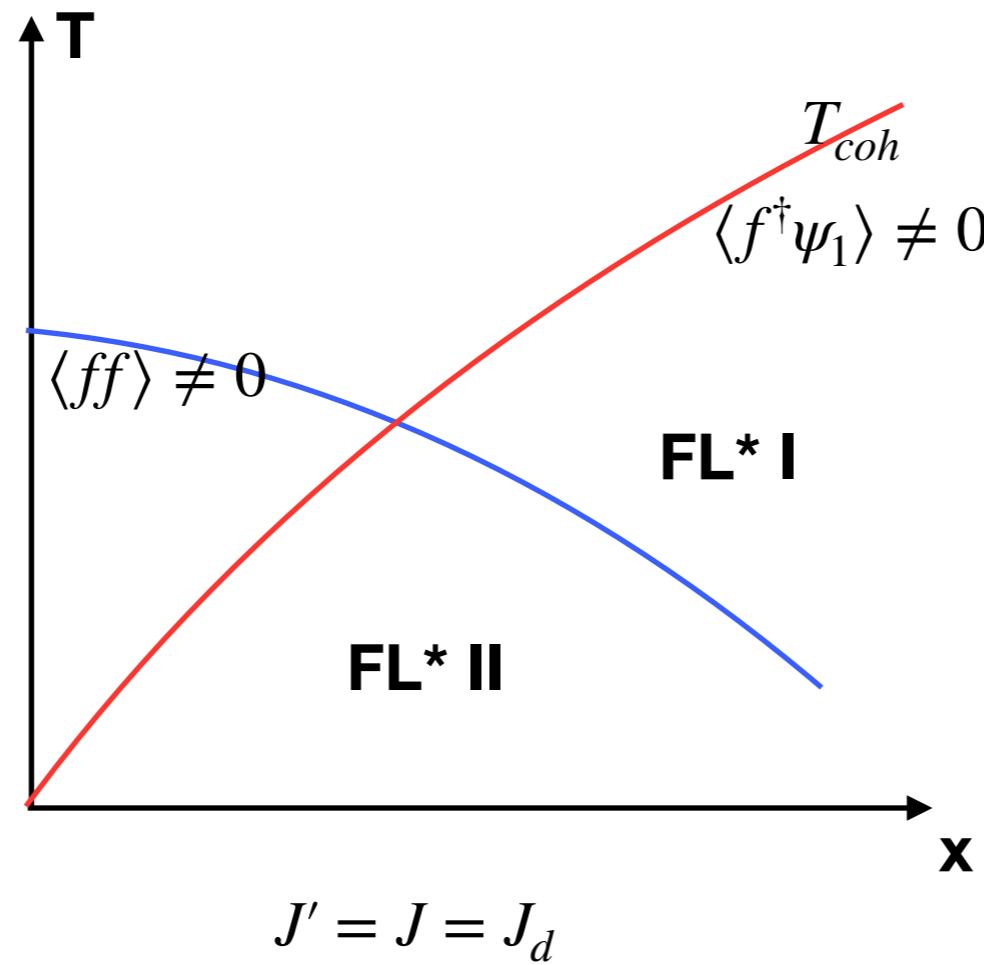


**Below a coherent temperature,**

$$\psi_2 = \sqrt{Z} d_2$$

$$Z \propto x$$

# Square Lattice



$FL^* I$ : spin liquid part is a spinon Fermi surface

$FL^* II$ : spin liquid part is a  $Z_2$  Dirac fermion SL

**Conjecture:** For reasonable value of  $J'=J$ , normal state has a small Fermi surface

Antiferromagnetic metal is also possible, but still beyond SDW mean field description.

# Small to Large Fermi surface transition



## Two-orbital Hubbard model

$$H = H_K + \frac{U_1}{2} \sum_i n_{1;i}(n_{1;i} - 1) + \frac{U_2}{2} \sum_i n_{2;i}(n_{2;i} - 1)$$

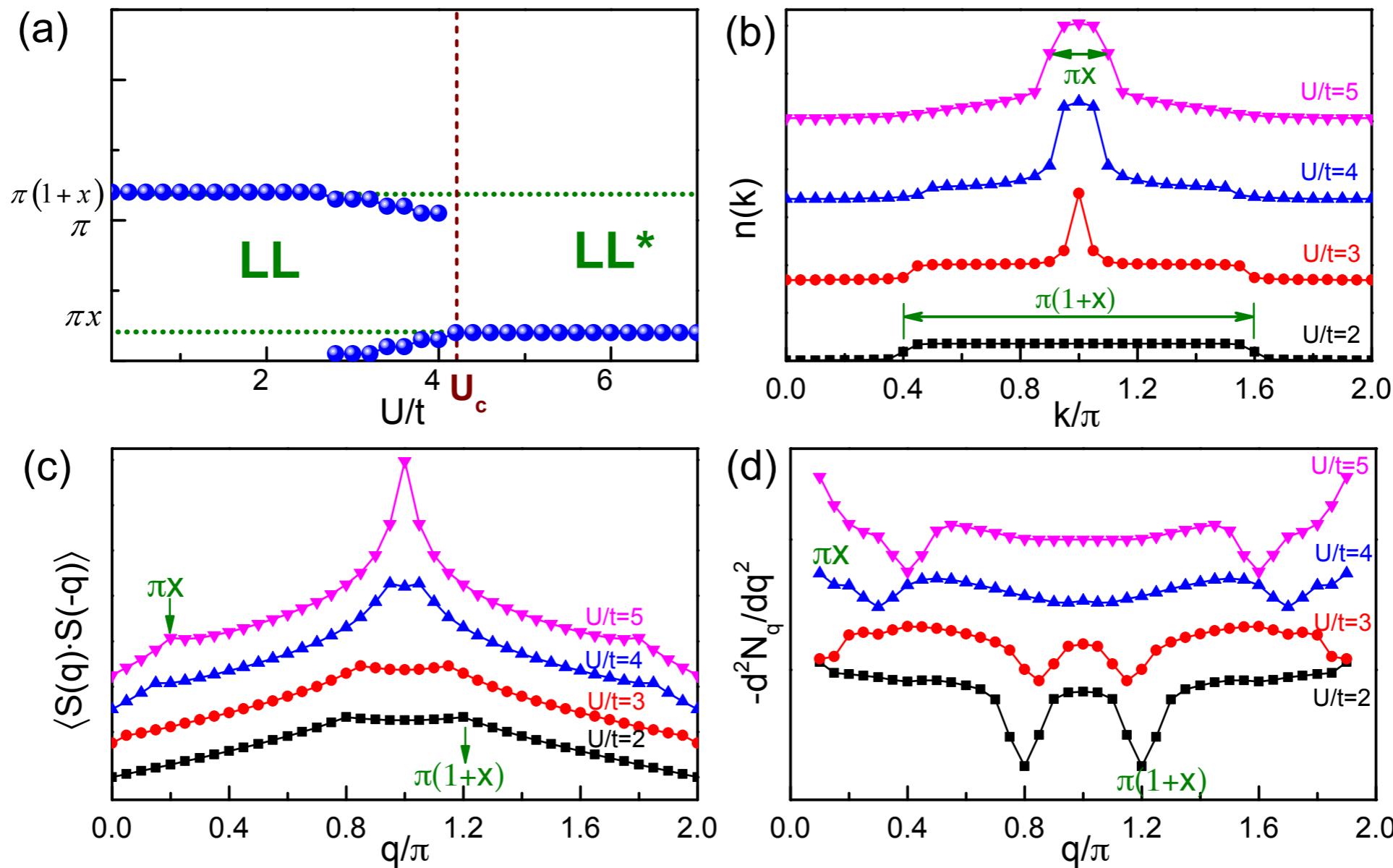
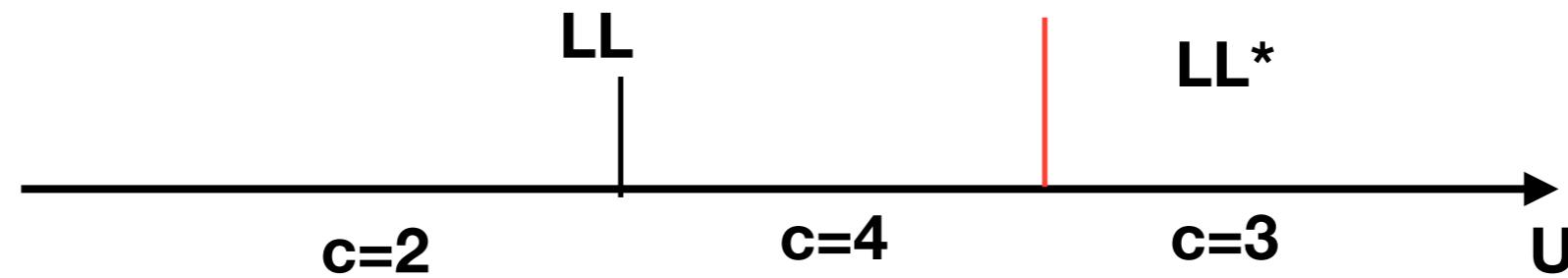
$$+ U' \sum_i n_{1;i} n_{2;i} - 2J_H \sum_i (\mathbf{S}_{1;i} \cdot \mathbf{S}_{2;i} + \frac{1}{4} n_{i;1} n_{i;2})$$

$$H_K = \sum_i \epsilon_{dd} n_{2;i} + V \sum_i (d_{i;1}^\dagger d_{i;2} + h.c.) + \sum_{\langle ij \rangle} t_{1;ij} d_{1;i}^\dagger d_{1;j} + \sum_{\langle ij \rangle} t_{2;ij} d_{2;i}^\dagger d_{2;j}$$
$$+ \sum_{\langle ij \rangle} t_{12;ij} d_{1;i}^\dagger d_{2;j} + h.c.$$

Spin-triplet t-J model: U,J\_H large limit

# Small to Large Fermi surface transition

$$U = 4J_H, U' = 2J_H$$

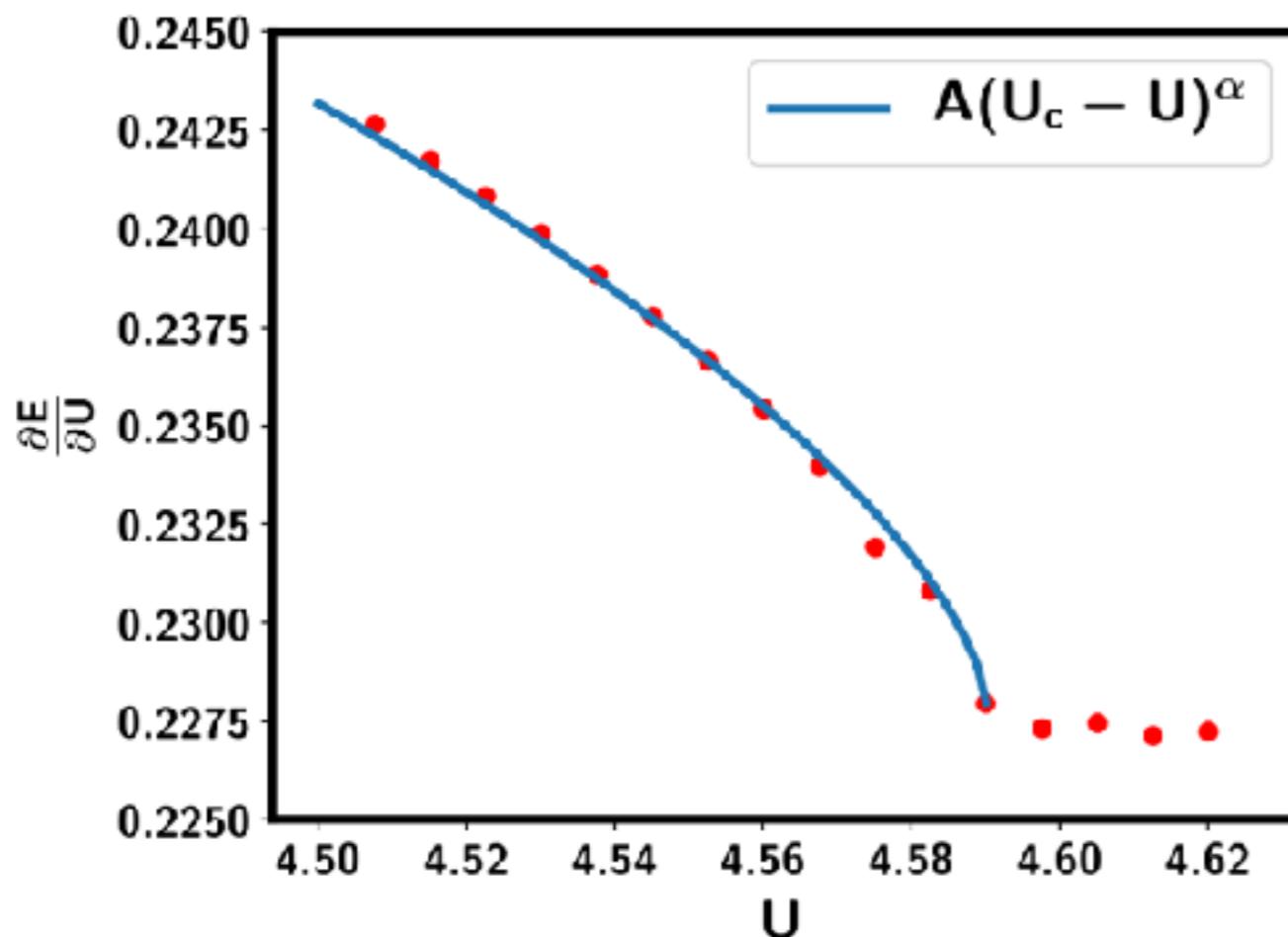


# Small to Large Fermi surface transition

A simple picture: chemical potential tuned Mott transition for one Fermi surface

$$U - U_c \propto -(\mu - \mu_c)$$

3D version: T.Senthil, M.Vojta and S.Sachdev (2003)



**chemical potential tuned Mott transition**

$$\langle n \rangle = -\frac{\partial E}{\partial \mu} = \begin{cases} A\sqrt{\mu - \mu_c} + 1, & \mu > \mu_c, \\ 1 & \mu < \mu_c \end{cases}$$

**Our result:**

$$\frac{\partial E}{\partial U} = \begin{cases} A(U_c - U)^\alpha + C, & U < U_c, \\ C & U > U_c \end{cases}$$

$$\alpha \approx 0.64$$

**Two possibilities:**

Coupling to the small pocket is relevant and modify the exponent.

It is a numerical error.

**Bond dimension: D=5000**

**Need a better computer to improve the precision.**

# **Summary**

- I. A new t-J model when doped doublon is a spin-triplet**
  
- II. This t-J model is interesting**

# A three-fermion Parton theory

**Parton theory:**  =  $f_\sigma^\dagger |0\rangle$        =  $b_a^\dagger |0\rangle \quad a = x, y, z$       or =  $\psi_{1;\sigma}^\dagger \psi_{2;\sigma'}^\dagger |0\rangle$

**SO(3) slave boson theory:**  $c_i = f_i^\dagger b_i$       symmetric FL is not possible

**Three-fermion theory:**  $c_{i;\sigma} = \frac{1}{2} \sum_{\sigma'} \sum_{a,b=1,2} \epsilon_{ab} f_{i;\sigma'}^\dagger \psi_{i;a\sigma} \psi_{i;b\sigma'}$       U(2) gauge structure

**Constraint:**  $f_i^\dagger f_i + \frac{1}{2} \Psi_i^\dagger \Psi_i = 1$        $\Psi_i^\dagger \vec{\tau} \Psi_i = 0$

$U(1) : f_i \rightarrow f_i e^{2i\alpha_c(i)}, \Psi_i \rightarrow \Psi_i e^{i\alpha_c(i)}$        $SU(2) : f_i \rightarrow f_i, \Psi_i \rightarrow U_i \Psi_i, \quad U_i \in SU(2)$

**Coupling:**

$$f : 2a_c, \quad \psi_1 : a_c + a_s + \frac{1}{2}A, \quad \psi_2 : a_c - a_s + \frac{1}{2}A$$

**Density**  $n_f = 1 - x \quad n_{\psi_1} = n_{\psi_2} = x$

# FL\* below coherent temperature

$$H = \frac{1}{4}t \sum_{\langle ij \rangle} \epsilon_{ab} \epsilon_{a'b'} \psi_{i;b\beta}^\dagger \psi_{i;a\alpha}^\dagger \psi_{j;a'\alpha} \psi_{j;b'\beta'} f_{j;\beta'}^\dagger f_{i;\beta} + h.c.$$

**a,b=1,2**

$$- \frac{1}{2}J \sum_{\langle ij \rangle} f_{i;\alpha}^\dagger f_{j;\alpha} f_{j;\beta}^\dagger f_{i;\beta}$$

$\alpha, \beta = \uparrow, \downarrow$

$$- \frac{1}{2}J_d \sum_{\langle ij \rangle} \psi_{i;a\alpha}^\dagger \psi_{j;b\alpha} \psi_{j;b\beta}^\dagger \psi_{i;a\beta}$$

$$- \frac{1}{4}J' \sum_{\langle ij \rangle} (f_{i;\alpha}^\dagger \psi_{j;a\alpha} \psi_{j;a\beta}^\dagger f_{i;\beta} + \psi_{i;a\alpha}^\dagger f_{j;\alpha} f_{j;\beta}^\dagger \psi_{i;a\beta})$$

**Mean field:**  $\Phi_{i;a} f_i^\dagger \psi_{i;a}$        $\Phi = (\Phi_1, \Phi_2)^T$       transforms as fundamental rep of SU(2)

**Fix gauge:**  $\Phi_{i;1} \neq 0, \Phi_{i;2} = 0$        $f: 2a_c, \psi_1: a_c + a_s + \frac{1}{2}A, \psi_2: a_c - a_s + \frac{1}{2}A$

**Higgs**       $a_c = a_s + \frac{1}{2}A$        $f: \alpha, \psi_1: \alpha, \psi_2: A; \alpha = 2a_s + A$

**Neutral spinon at total filling 1:**  $f, \psi_1$       **Electron:**  $c \sim \langle f^\dagger \psi_1 \rangle \psi_2$

**FL\* phase if  $f, \psi_1$  forms a spin liquid**       $n_f = 1 - x \quad n_{\psi_1} = n_{\psi_2} = x$