Large-scale structure at the interface of numerical and analytical techniques

Oliver Hahn
Institute for Astrophysics & Institute for Mathematics
University of Vienna, Austria

with Cornelius Rampf, Florian List, Cora Uhlemann, Michaël Michaux, Natalia Porqueres, Sonja Schobesberger, Raul Angulo, Mateja Gosenca

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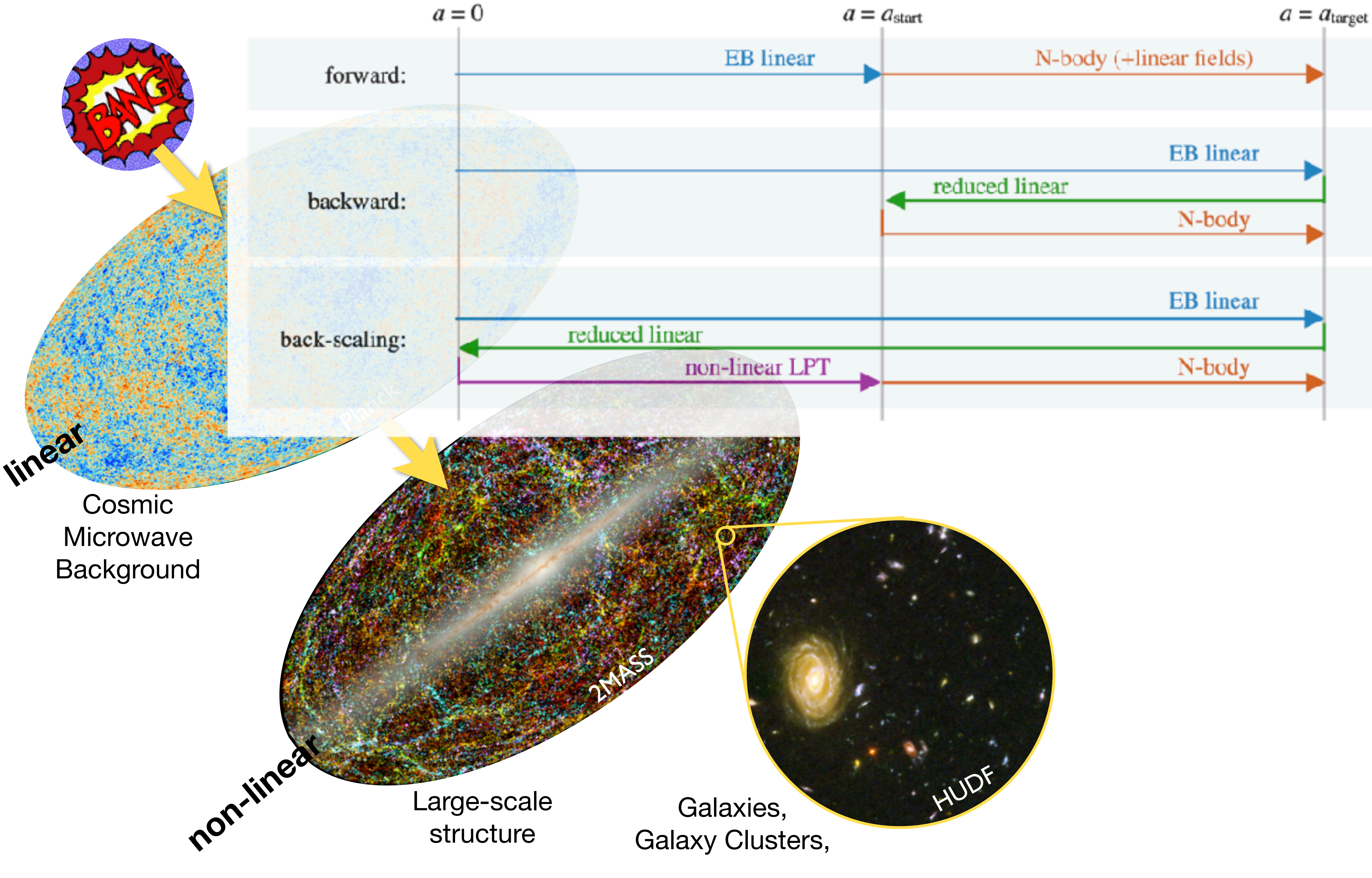
The Inhomogeneous Universe: evolution

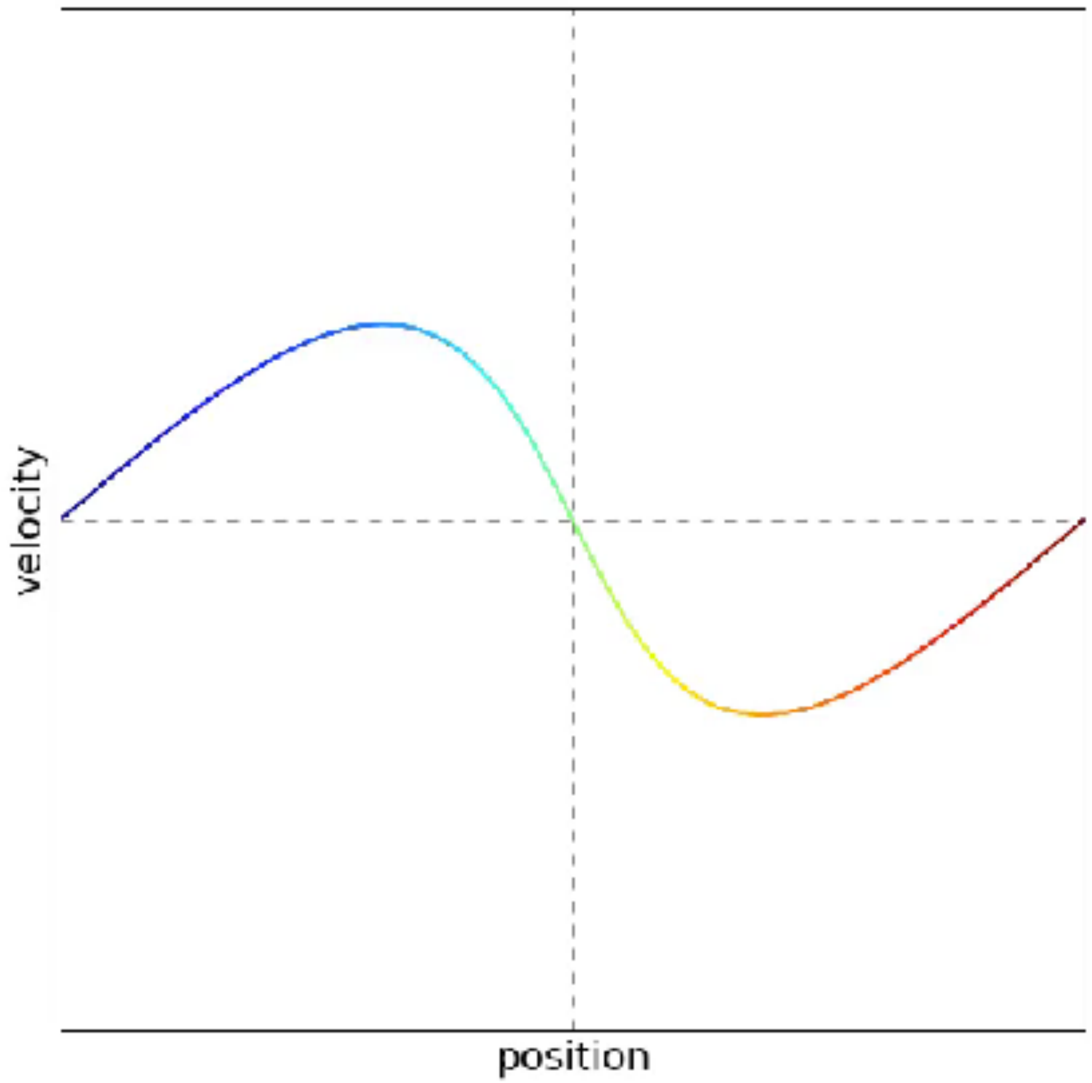


Evolution of Fluctuations

Cold Dark Matter lives on Lagrangian submanifold

Solve Vlasov-Poisson on submanifold characteristics $(\boldsymbol{q},t) \mapsto (\boldsymbol{x}(\boldsymbol{q},t), \boldsymbol{p}(\boldsymbol{q},t))$

$$\frac{\partial f_m}{\partial t} + \frac{p_i}{ma^2}\frac{\partial f_m}{\partial x^i} - m\frac{\partial \phi}{\partial x^i}\frac{\partial f_m}{\partial p_i} = 0 \quad \Leftrightarrow \quad \mathbf{x}'' + \mathcal{H}\mathbf{x}' = -\nabla\phi$$



1D singularities: Rampf, Frisch & OH (2021)

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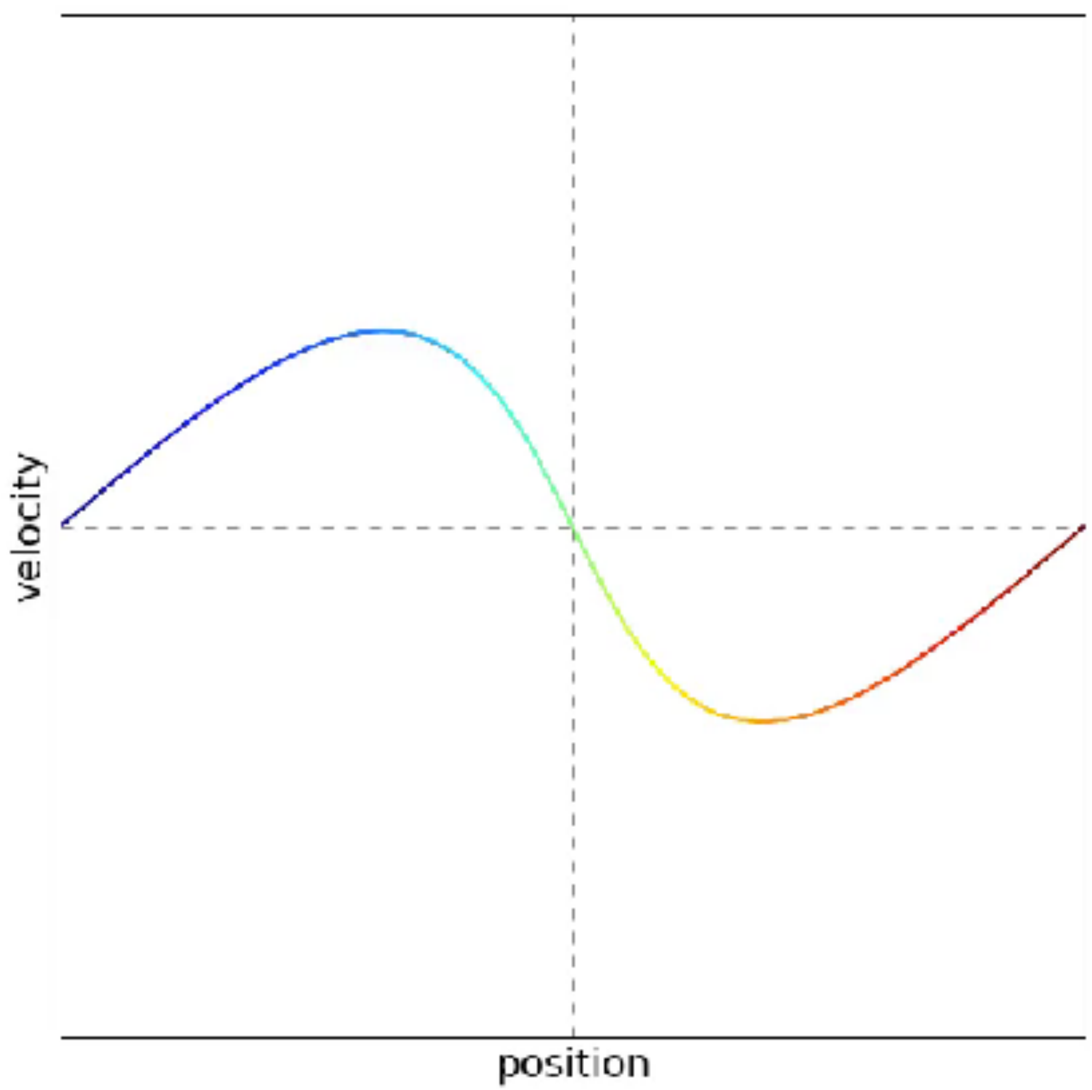
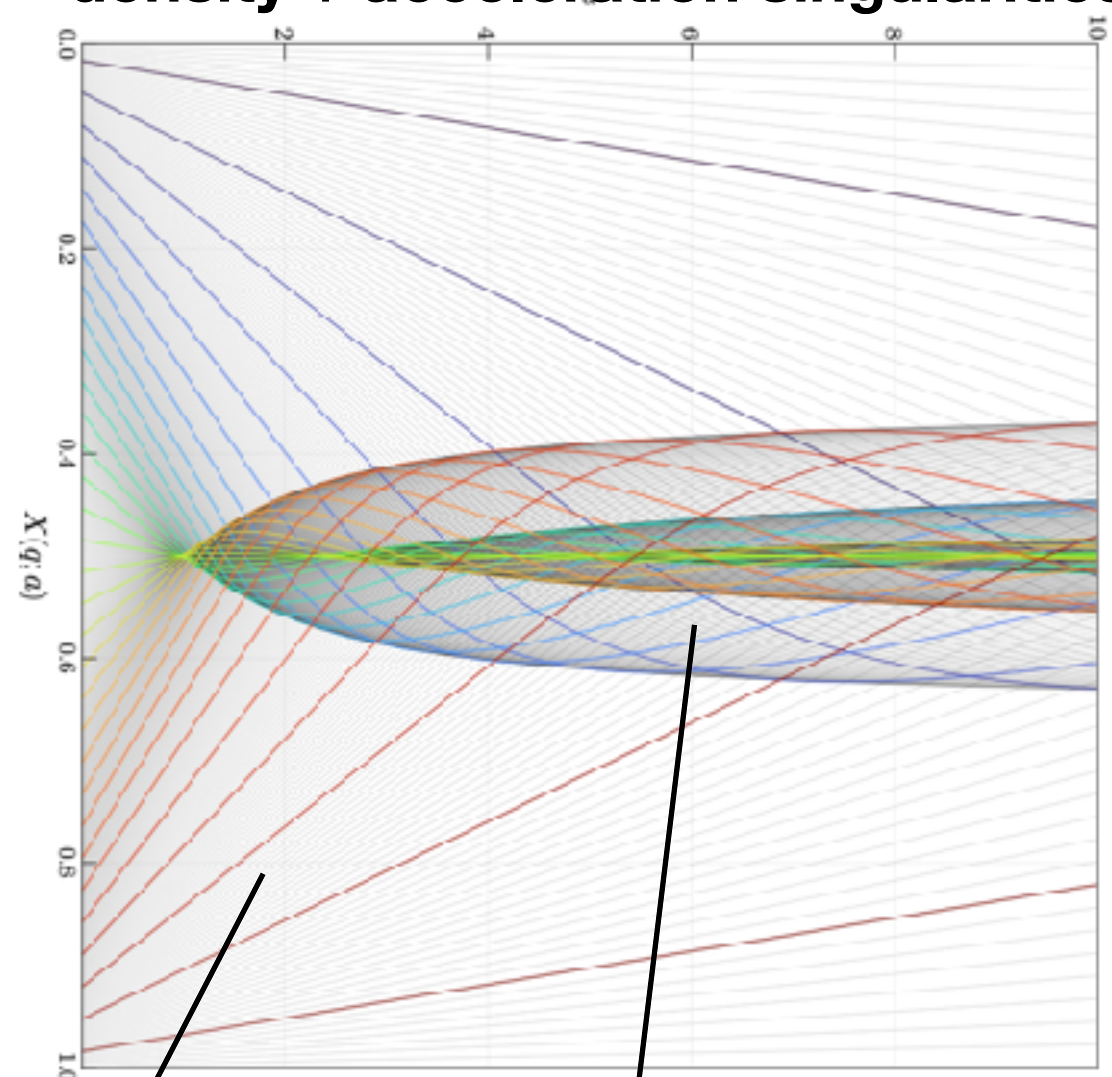


1D singularities: Rampf, Frisch & OH (2021)

density + acceleration singularities



Lagrangian Perturbation Theory

(for single fluid with cold initial data)

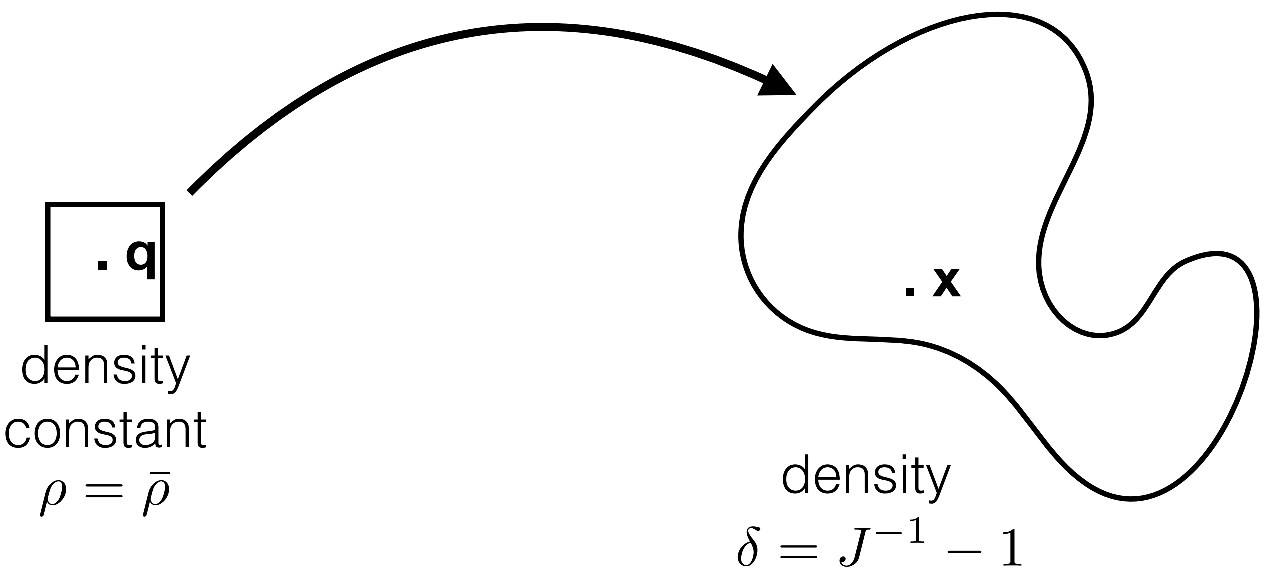
Lagrangian map

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t)$$

Overdensity given by Jacobian

$$\delta(\boldsymbol{x},t) = \frac{1}{J(\boldsymbol{q},t)} - 1$$

$$J := \det \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{q}}$$



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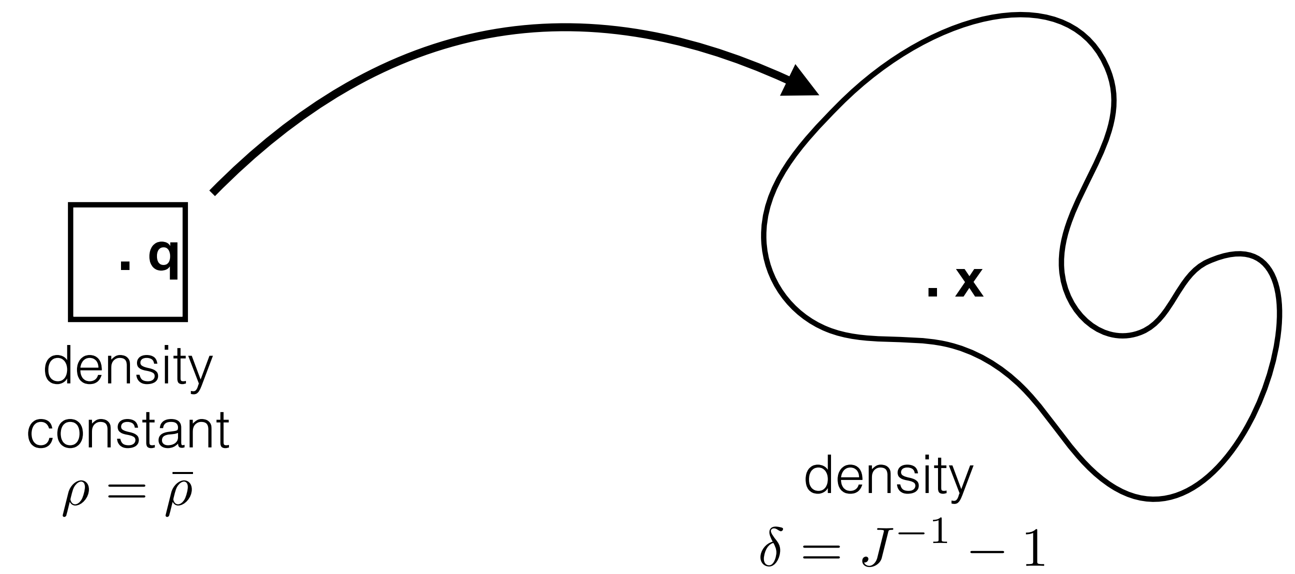


We want to solve this as a perturbative series (D is small parameter)

$$\mathbf{\Psi}(\mathbf{q},\tau) = \sum_{n=1}^{\infty} D(\tau)^n \, \mathbf{\Psi}^{(n)}(\mathbf{q})$$

Buchert (1994), Catelan (1995), Bouchet+(1995), n=3
Rampf (2012), Zheligovsky&Frisch (2014), Matsubara (2015), all order

We will go to n=3 (3LPT), Michaux et al. (2021), and n very large, Rampf & Hahn (2021)

For LCDM, actually $D^{(n)}(\tau) \neq D^n(\tau)$ **, see Rampf, Schobesberger & OH(2022)**

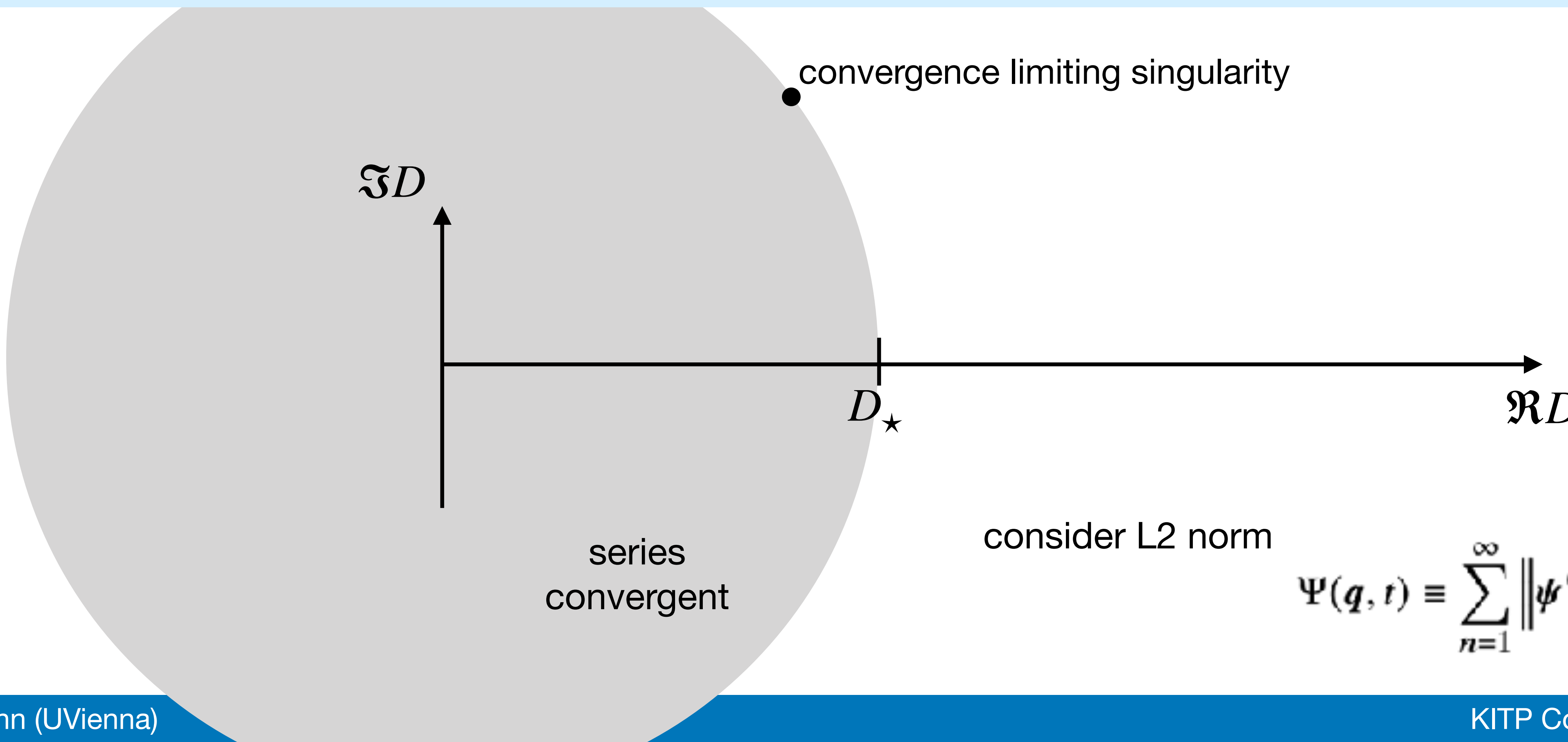
Shell-crossing — the limit of LPT?

gives rise to cosmic structure, and marks the breakdown of cold limit

BUT: Does LPT provide a convergent expansion until equations break down?

LPT expansion: $$\boldsymbol{\Psi}(\mathbf{q},\tau) = \sum_{n=1}^{\infty} D(\tau)^n \, \boldsymbol{\Psi}^{(n)}(\mathbf{q})$$

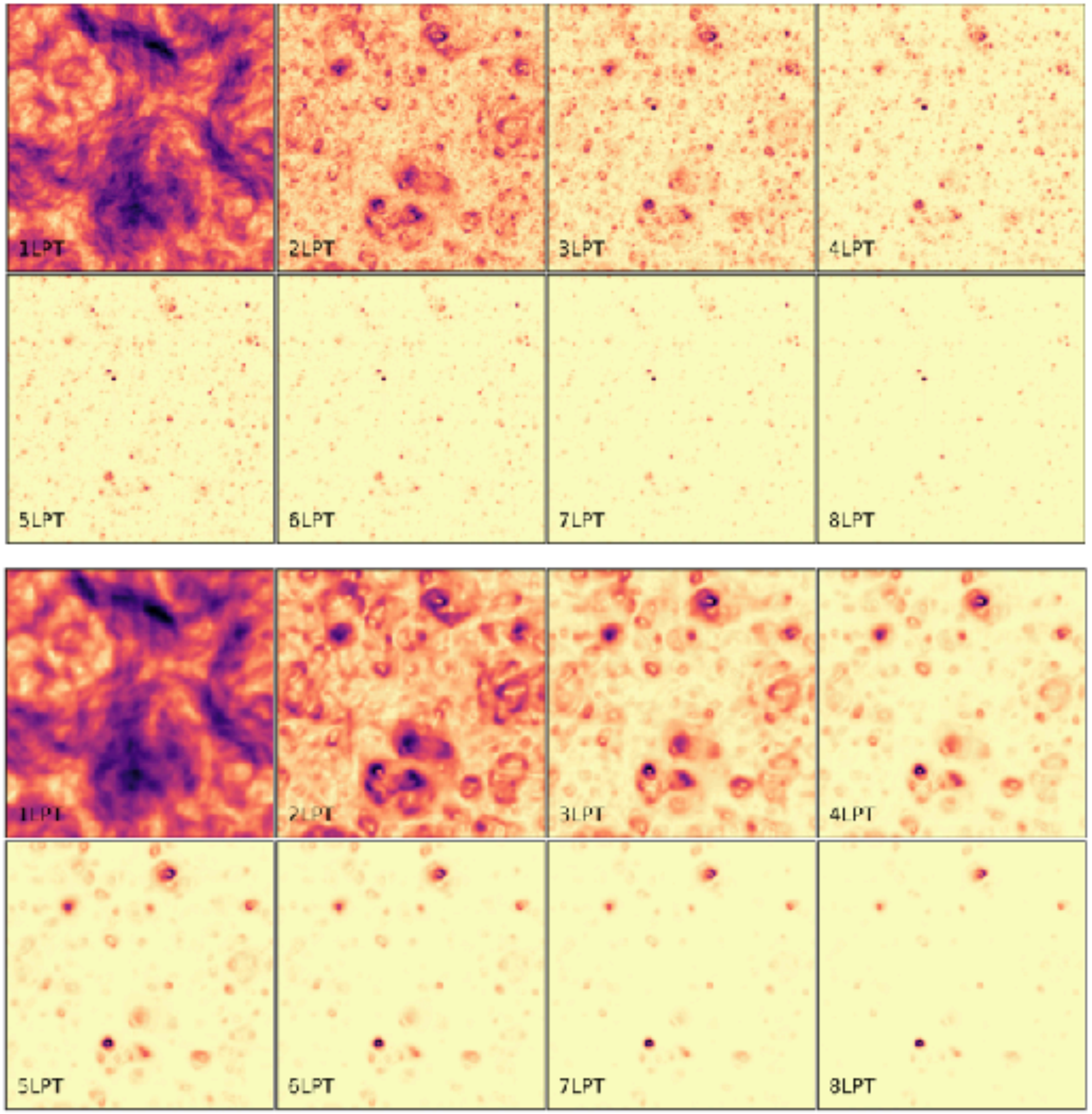
It has never been shown if LPT is convergent and if so what is its radius of convergence for realistic (random) ICs



consider L2 norm

$$\Psi(q,t) \equiv \sum_{n=1}^{\infty} \left\| \psi^{(n)}(q) \right\| D^n$$

To go where no-one has gone before…



until recently: 2LPT was state of art

Have now implementation of all-order recursion relations of LPT!

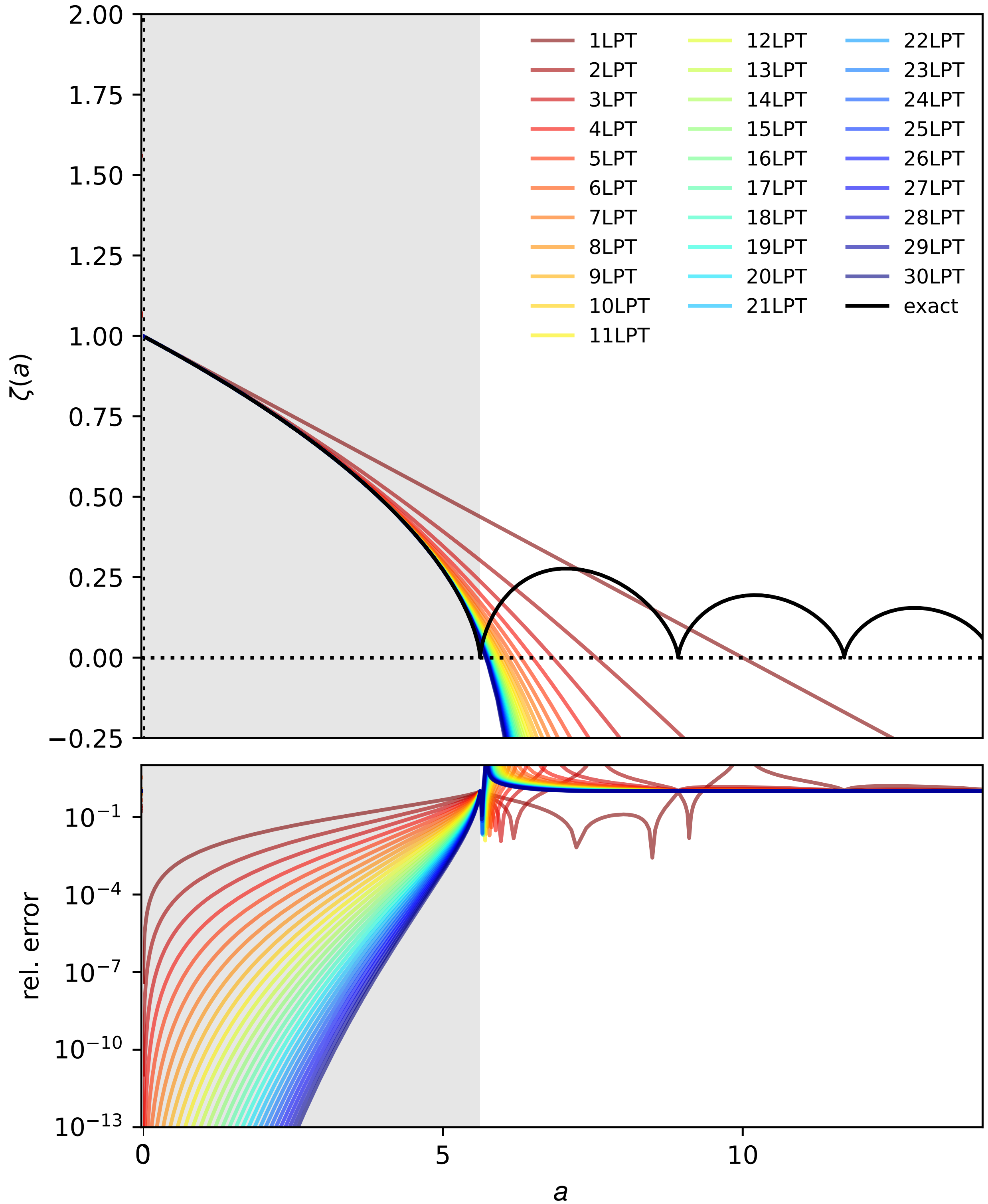
LPT converges everywhere even after first shell-crossing in field

But: For late times, converges very slowly for isolated set of points
(see also Nadkarni-Ghosh & Chernoff 2011)

Rampf+OH (2021)

on request: all order **monofonIC** code

Convergence limiting singularities



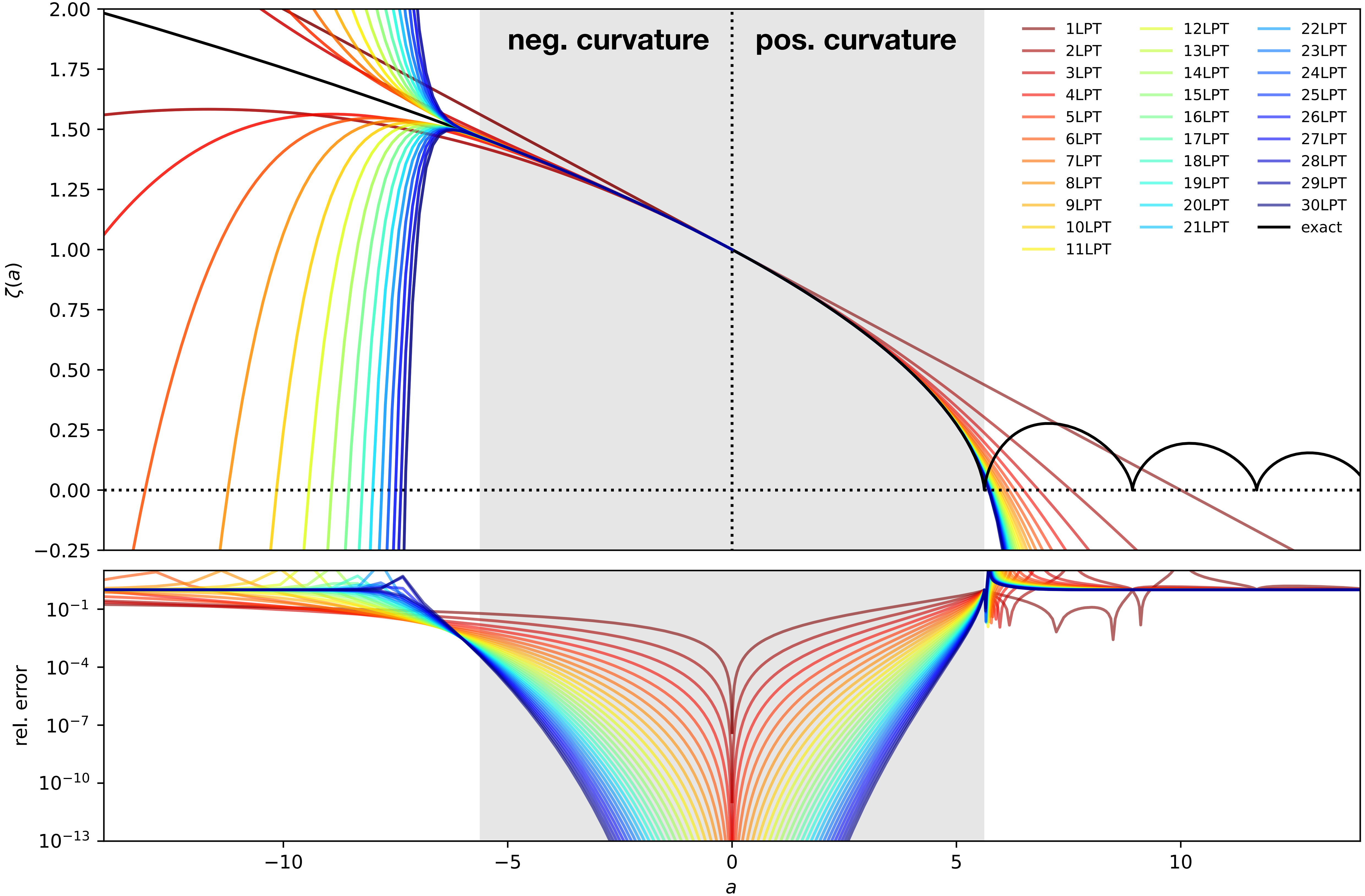
Spherical collapse has a physical singularity, acceleration is infinite

This leads to slow convergence of LPT, also no transition to bound states

numerous ad hoc UV completions in the literature
e.g. ALPT (Kitaura&Hess 2013), MUSCLE (Neyrinck 2016),…

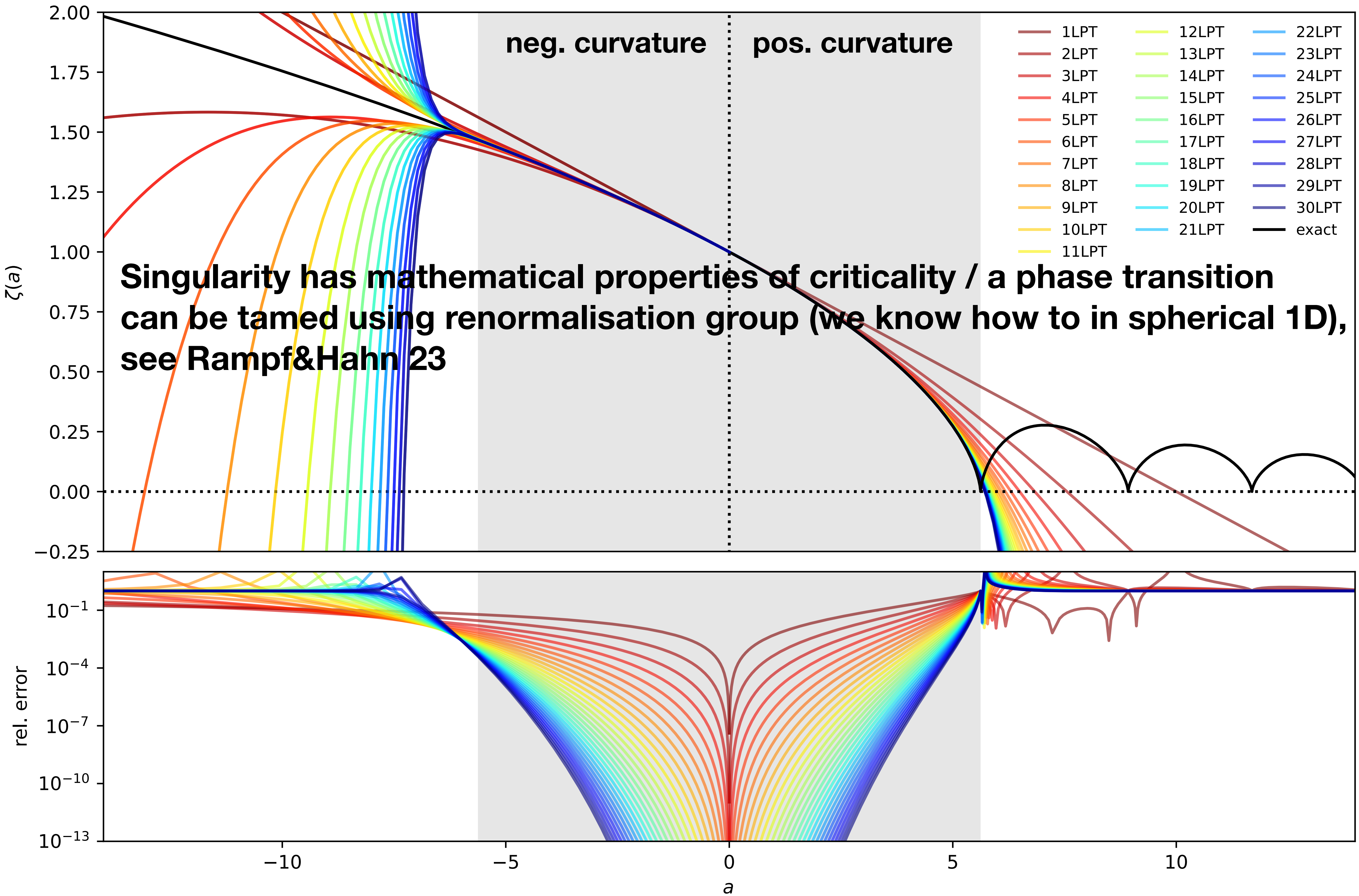
Rampf & Hahn 2023

Convergence limiting singularities



Rampf & Hahn 2023

Convergence limiting singularities



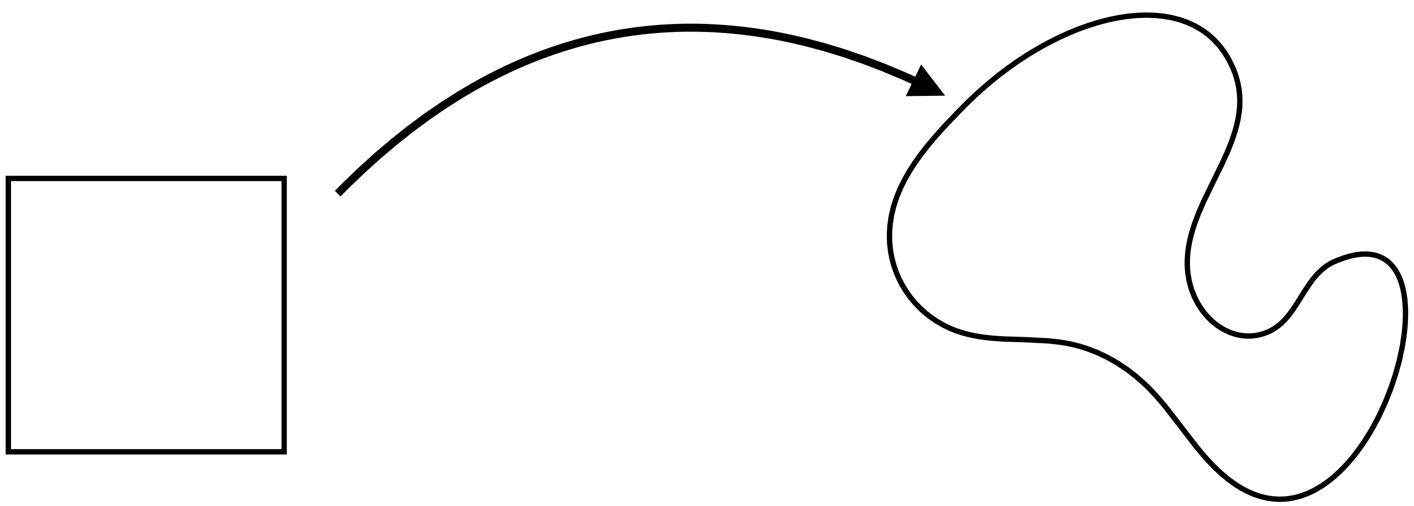
Singularity has mathematical properties of criticality / a phase transition can be tamed using renormalisation group (we know how to in spherical 1D), see Rampf&Hahn 23

Rampf & Hahn 2023

Discrete evolution vs. fluid evolution

Lagrangian description, evolution of fluid element

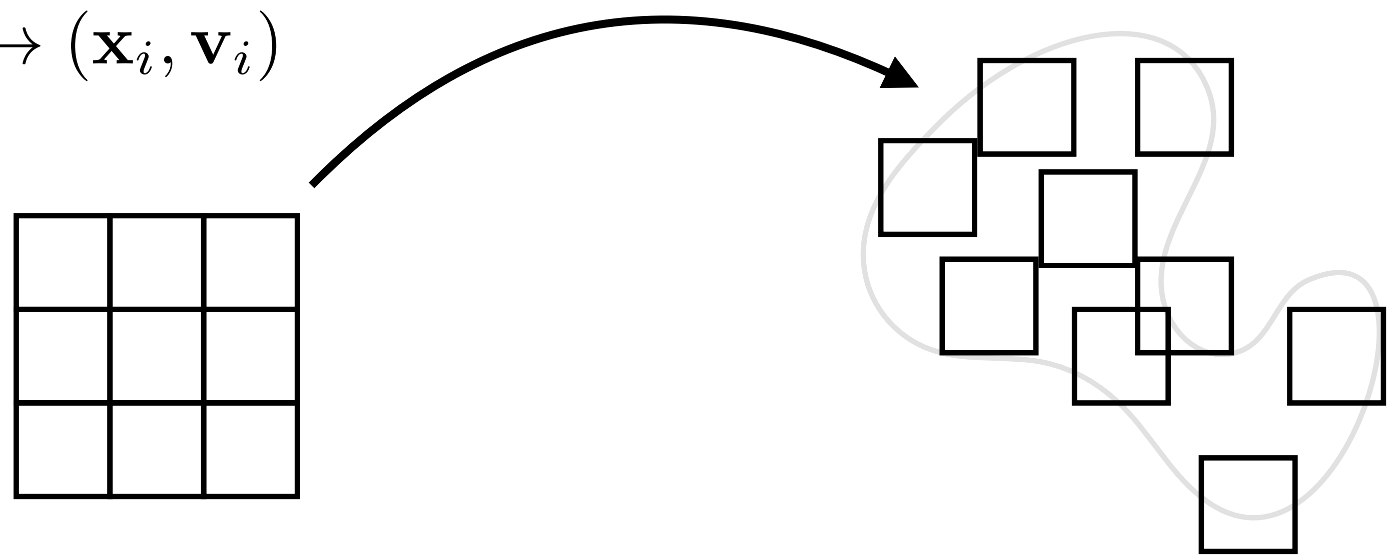
$$\mathbb{Q} \subset \mathbb{R}^3 \to \mathbb{R}^6 \;:\; \mathbf{q} \mapsto (\mathbf{x}_\mathbf{q}(t), \mathbf{v}_\mathbf{q}(t))$$



$$\frac{\mathrm{D} f_m}{\mathrm{D}t} = 0$$

The N-body approximation:
cover distribution function with N characteristics, estimate f_m from them

$$i \in \{1 \ldots N\} \mapsto (\mathbf{x}_i, \mathbf{v}_i)$$

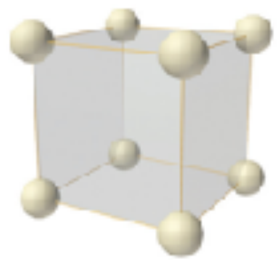


This can re-introduce short-range interactions -> softening…

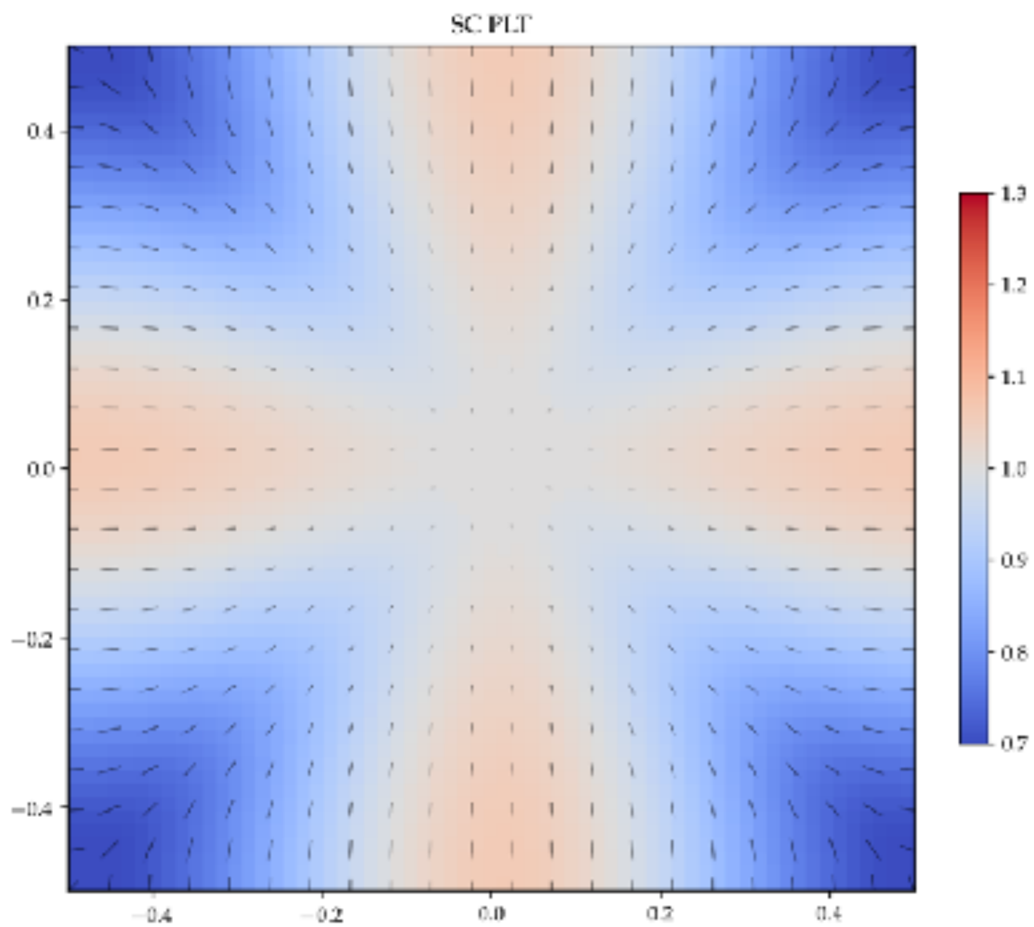
Convergence of LPT and N-body…

I've talked a lot about discreteness and Vlasov in the past. Here is a new take:

SC
(simple cubic)

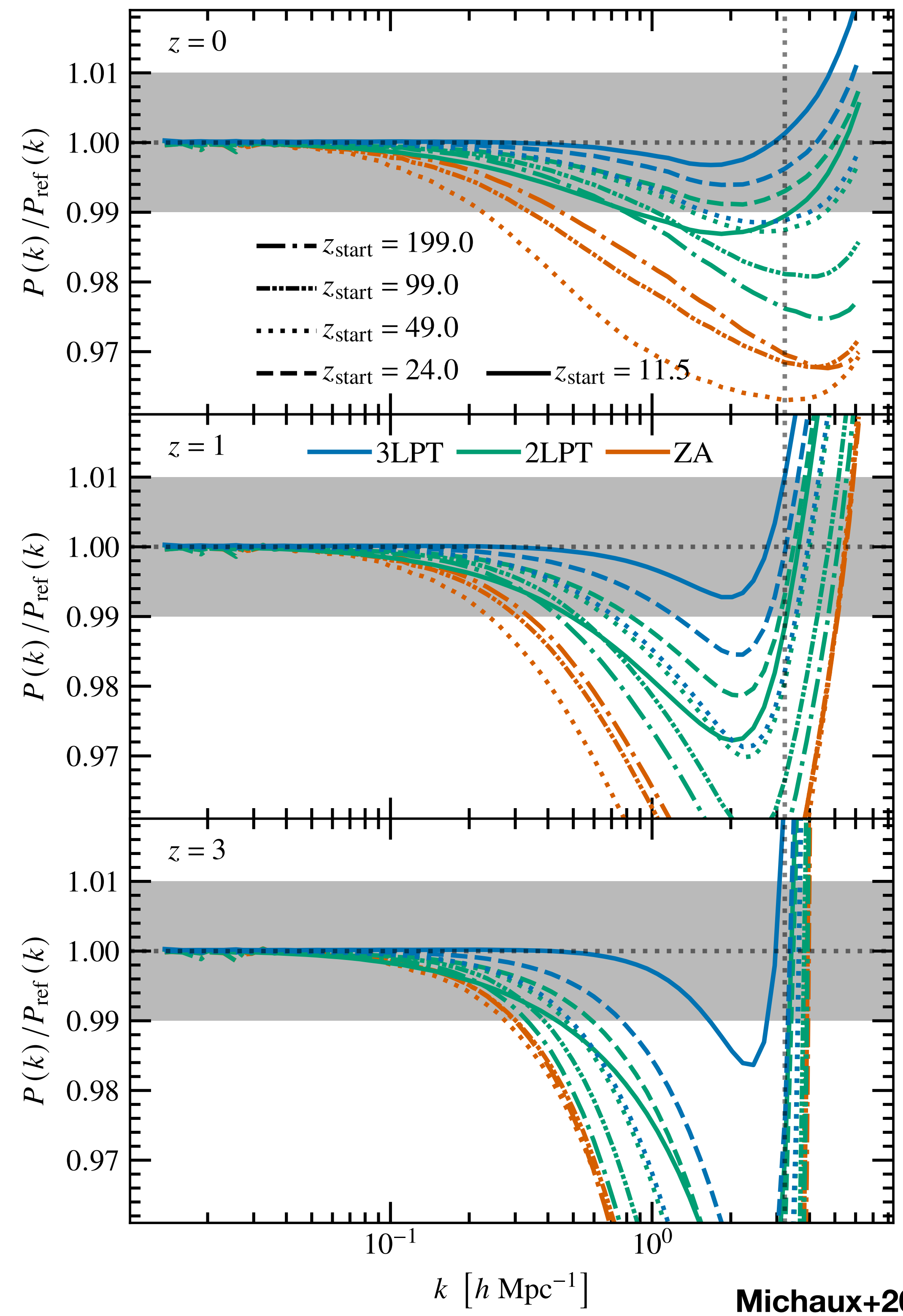


Particle motion on small scales not isotropic.
Can be calculated for Bravais lattices:



cf. Joyce+2005, Joyce&Marcos 2007, Marcos 2008,
but also Garrison+2016

same line style = same z_start, same color = same order LPT



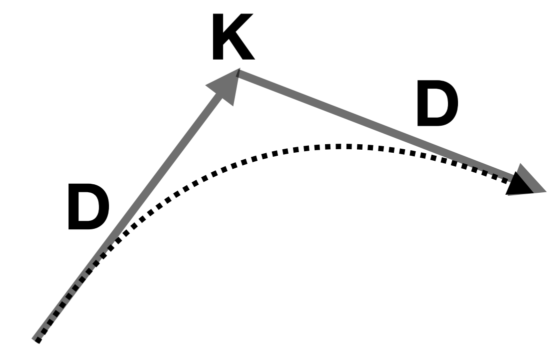
Michaux+2020

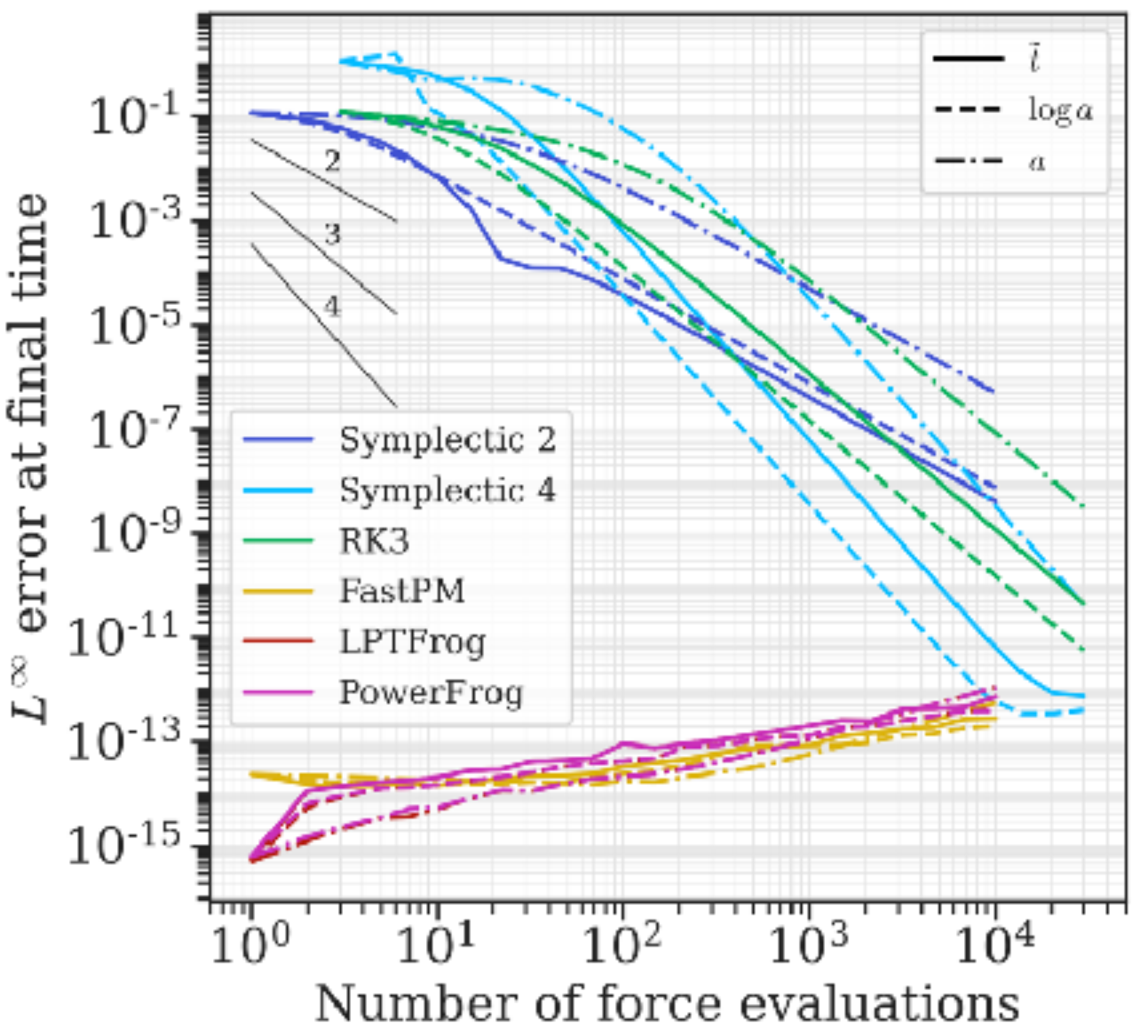
… discreteness effects large at early times. better to start as late as possible…

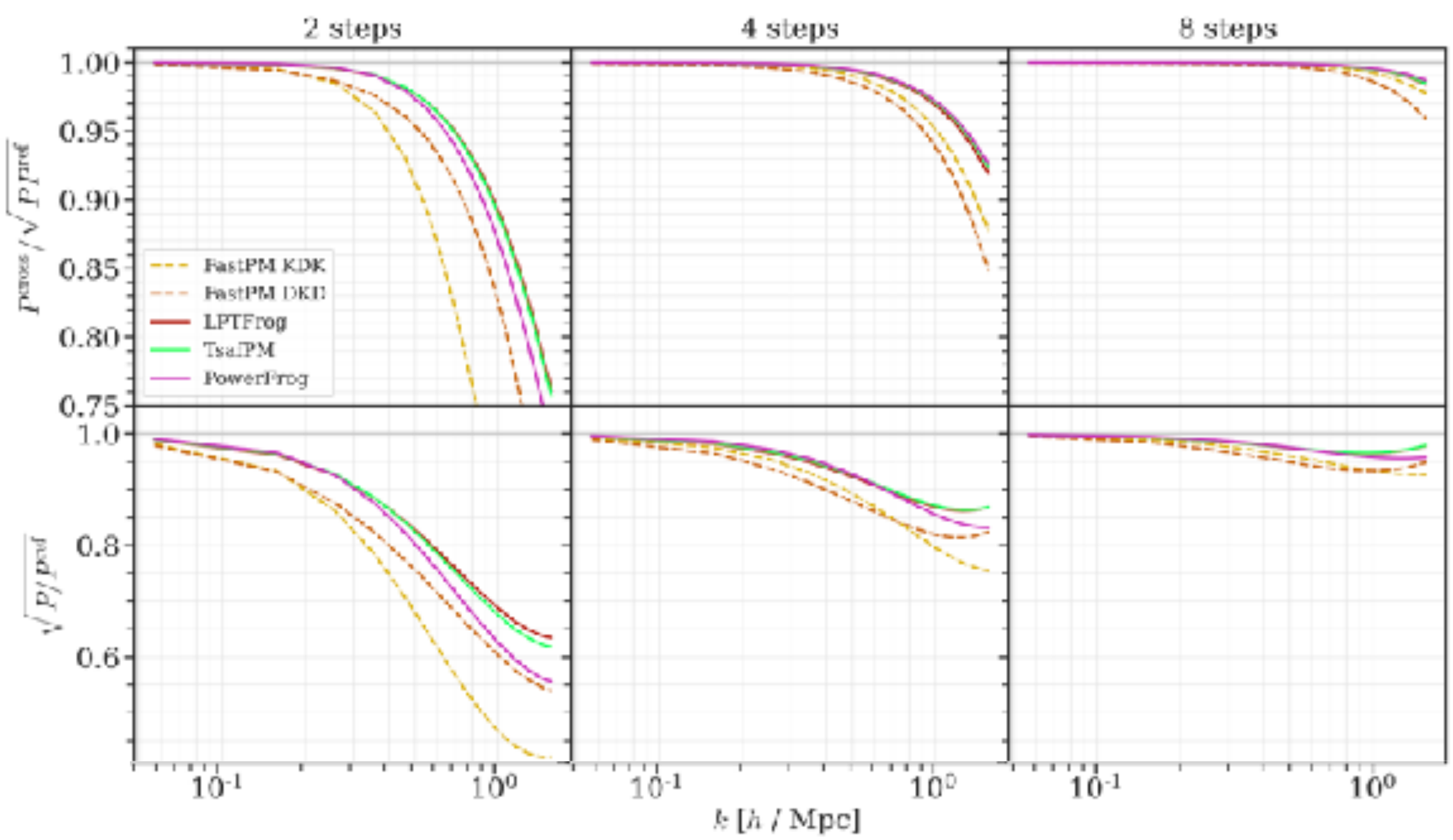
Perturbation-theory informed integrators

Integrators for N-body simulations are agnostic about perturbation theory (derived naively from Hamiltonian)

Exception: FastPM (Feng+16 - match Zel'dovich)
BUT: Can make them agree to 2nd order!



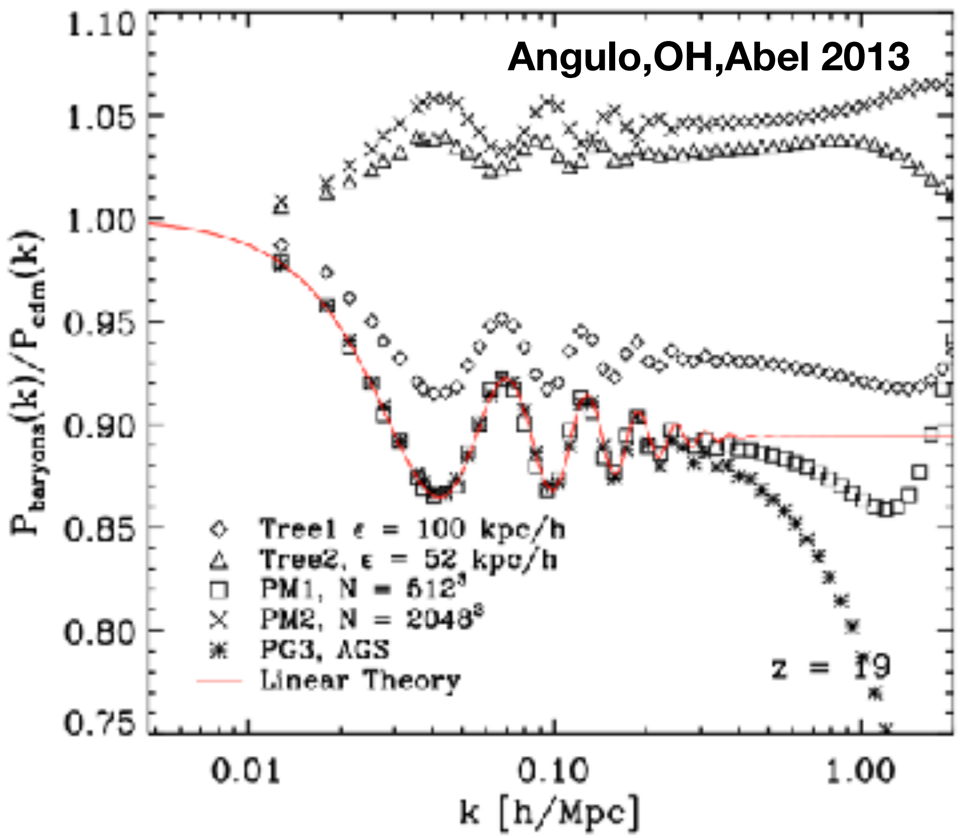




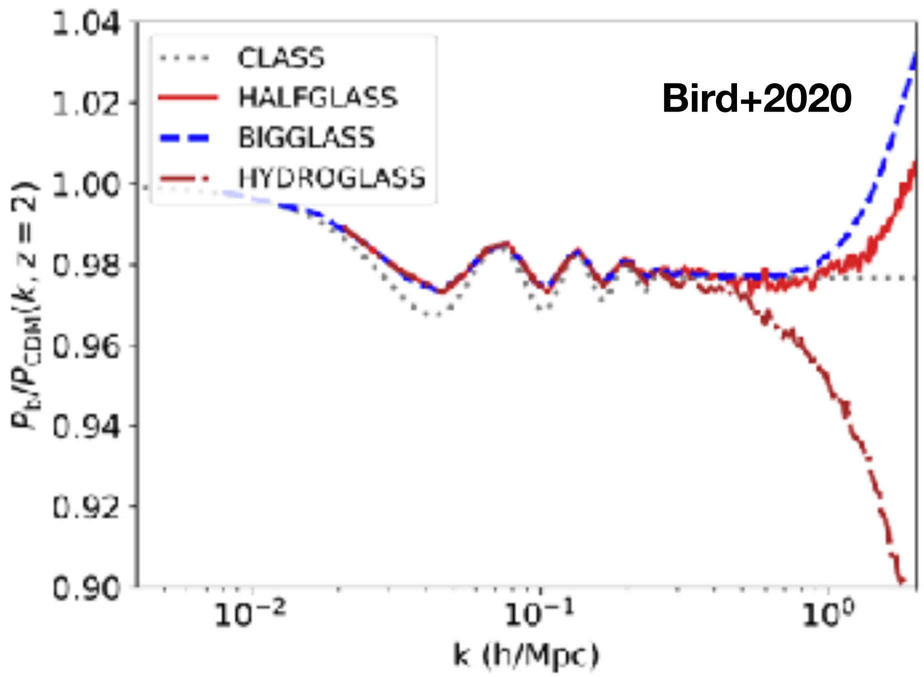
List&Hahn 23

Adding baryons: two fluid baryon+CDM simulations

N-body two-fluid sims have completely dominant discreteness errors



we thought it requires large softening,
or decorrelated particle arrangements…



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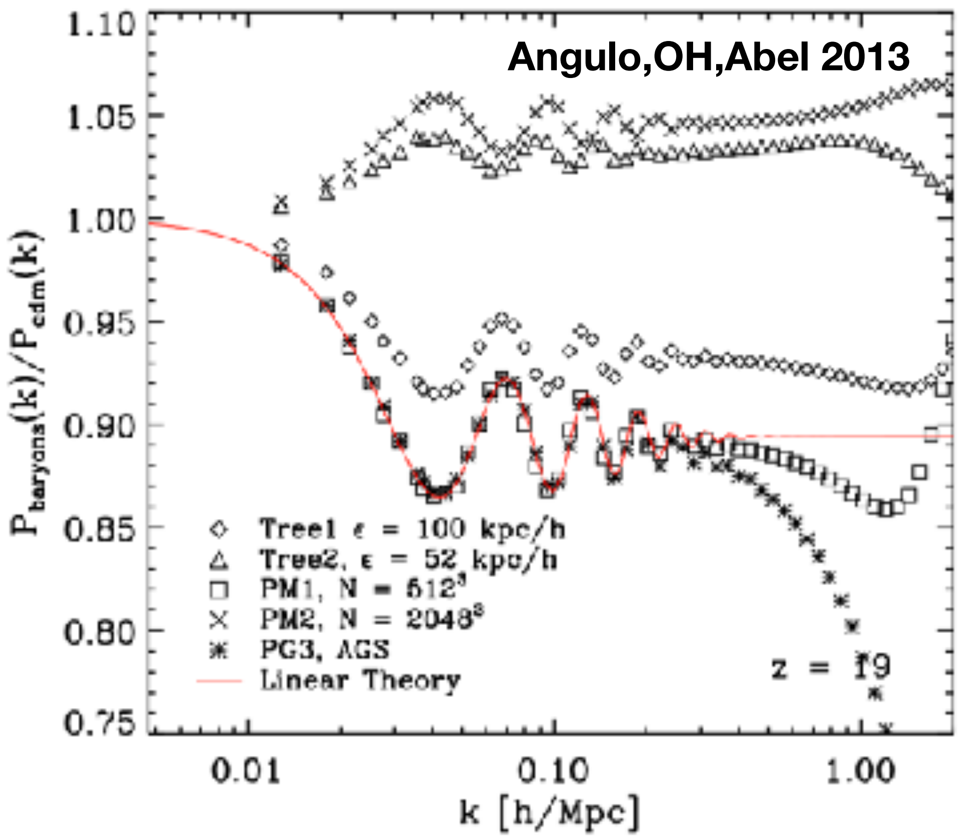
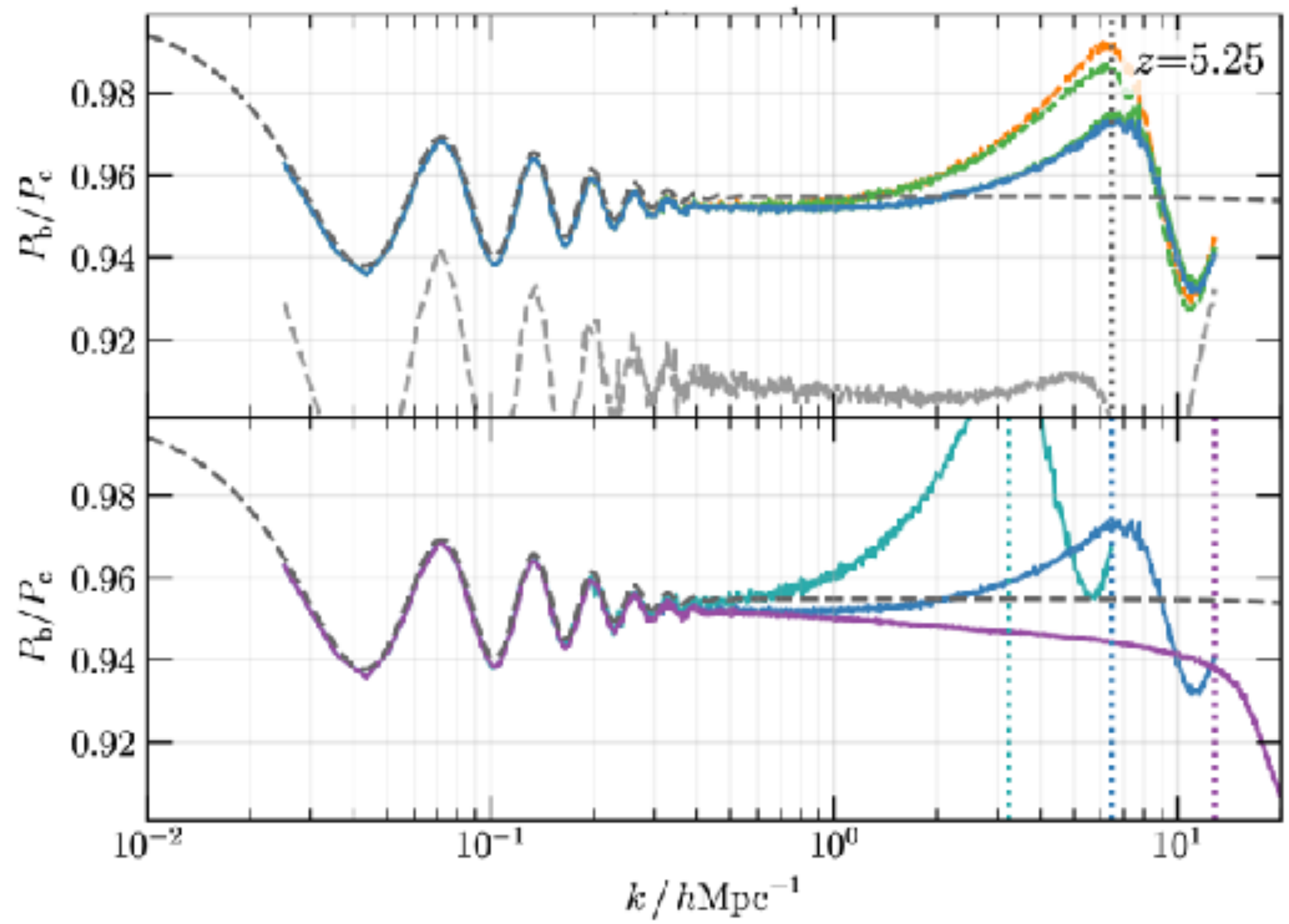
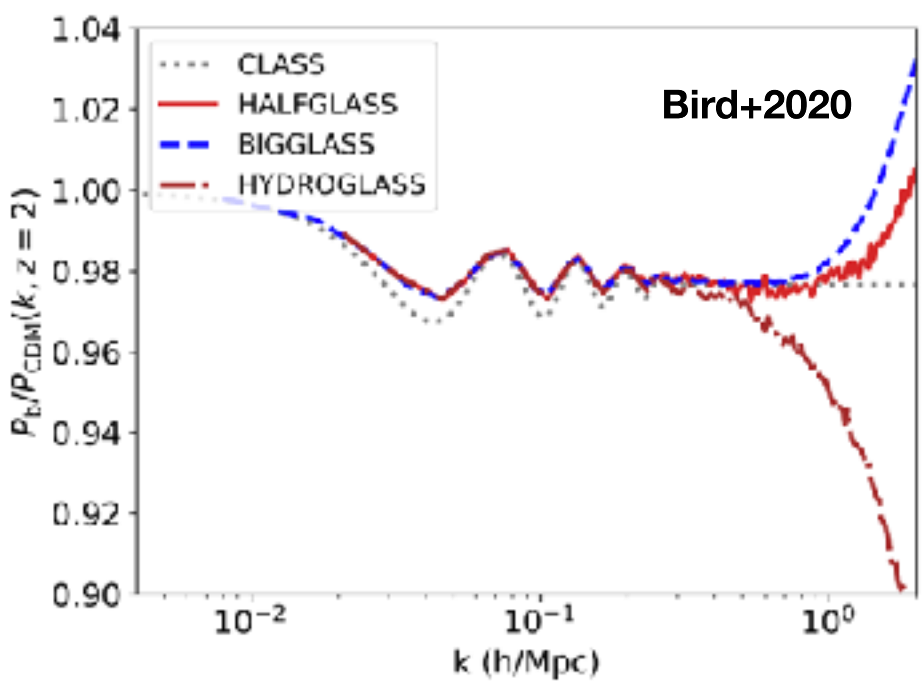


correct solution is to incorporate isocurvature perturbations correctly:



we thought it requires large softening, or decorrelated particle arrangements.



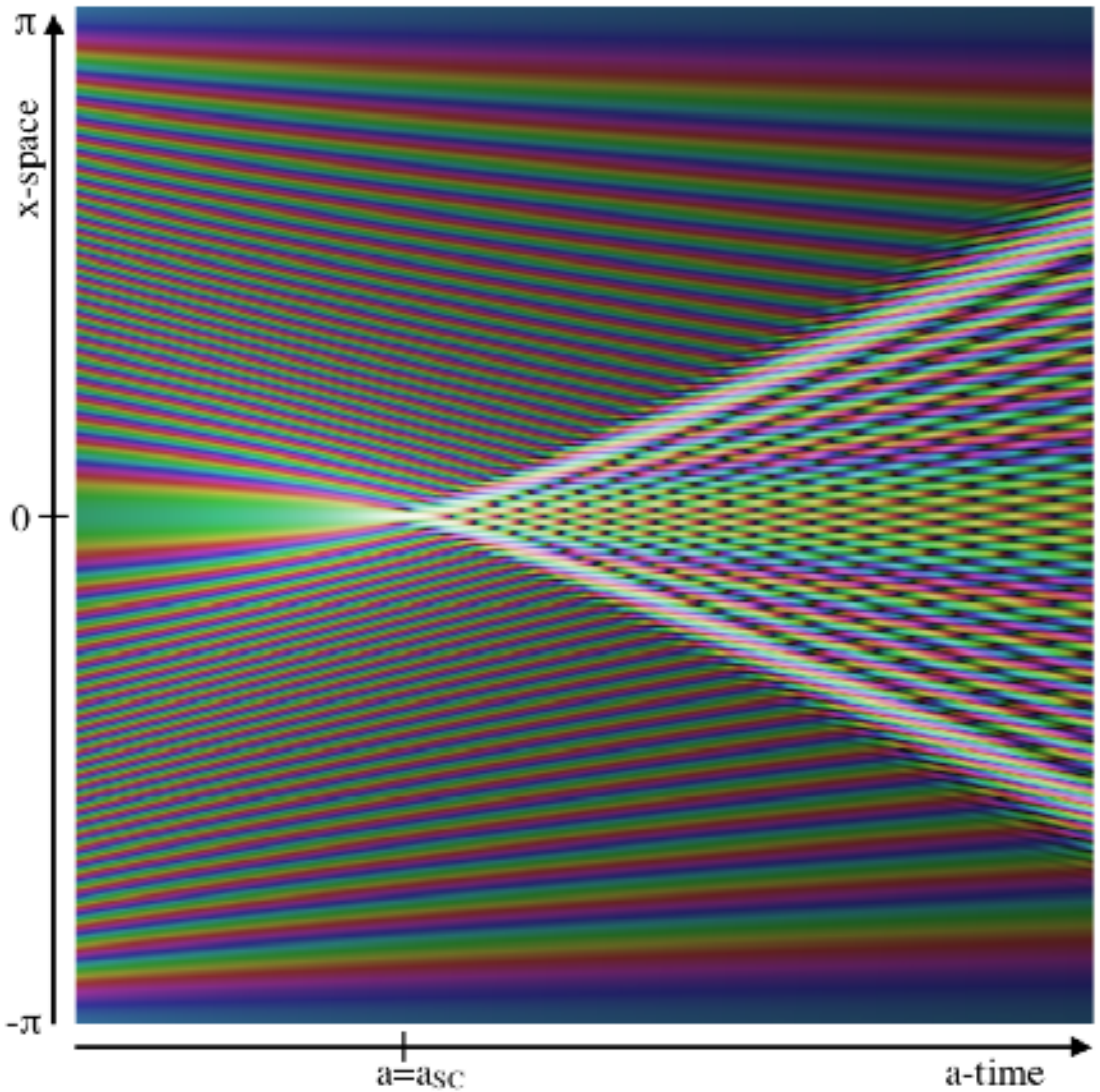
by perturbing not only particle pos+vel but also particle masses

(see Hahn,Rampf,Uhlemann 2021, Rampf,Uhlemann,Hahn 2021)

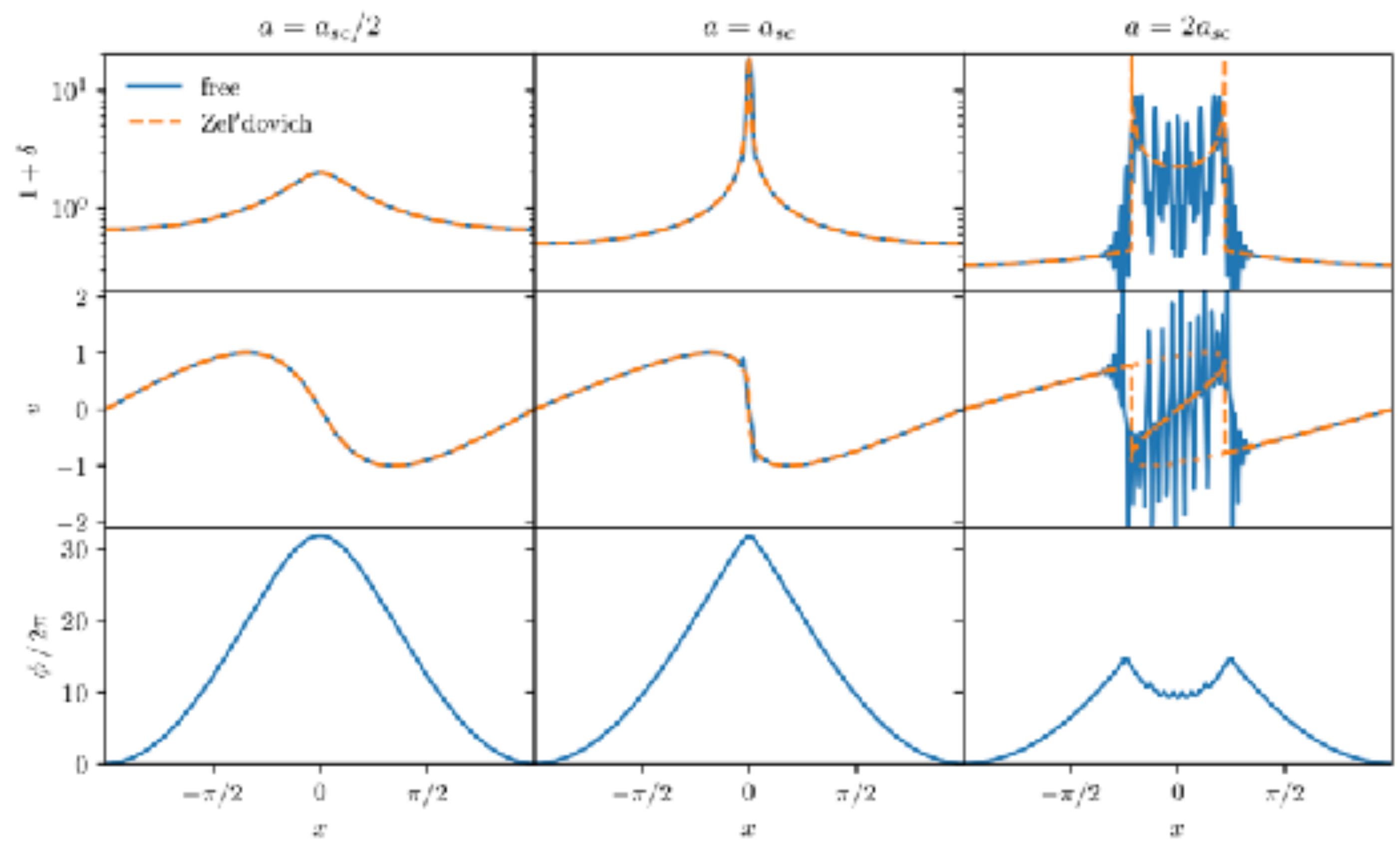
Lagrangian Perturbative Dynamics in a Field Framework I

Use QM inspired transition matrix q->x to predict transition probabilities to go from Lagrangian to Eulerian space

Obtain a field version of Zeldovich trajectories:



Interference = multi-streaming



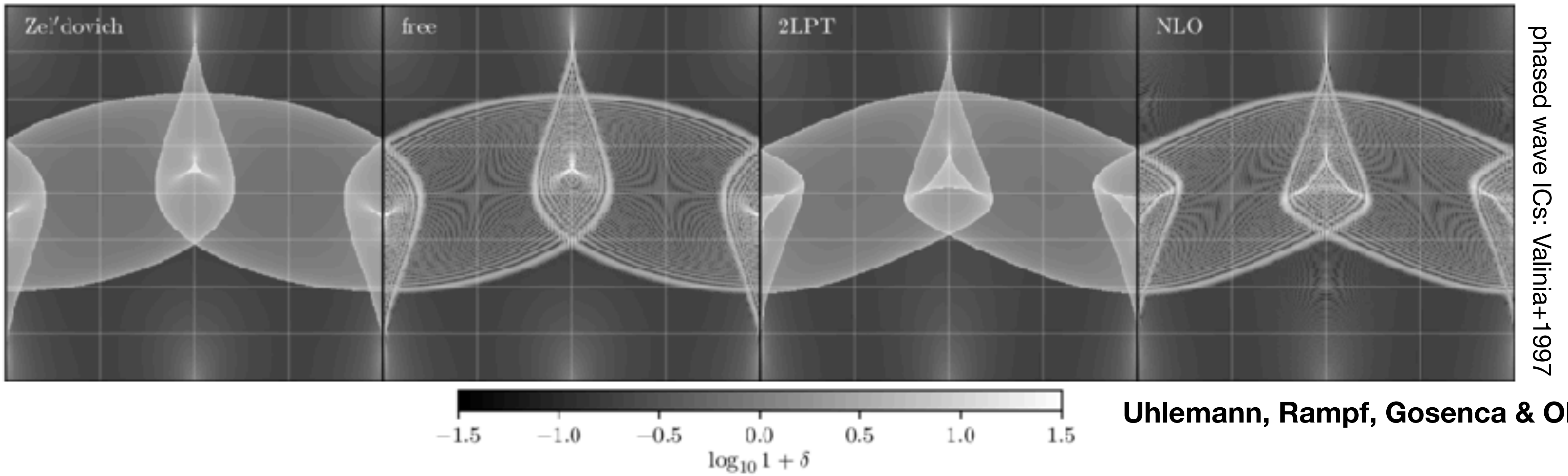
dynamics 'smoothed' by hbar scale

Uhlemann, Rampf, Gosenca & OH (2019)

Lagrangian Perturbative Dynamics in a Field Framework II

This can be expanded to n-th order LPT

Comparison 1LPT, 2LPT with 1PPT, 2PPT for phased wave ICs



Uhlemann, Rampf, Gosenca & OH (2018)

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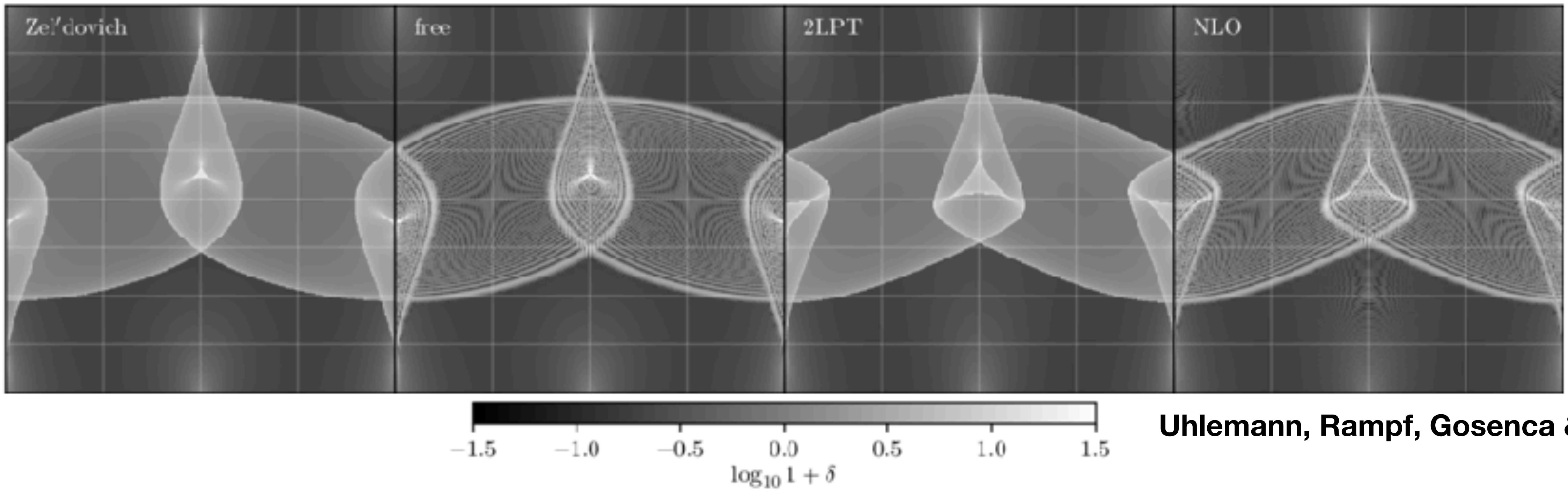
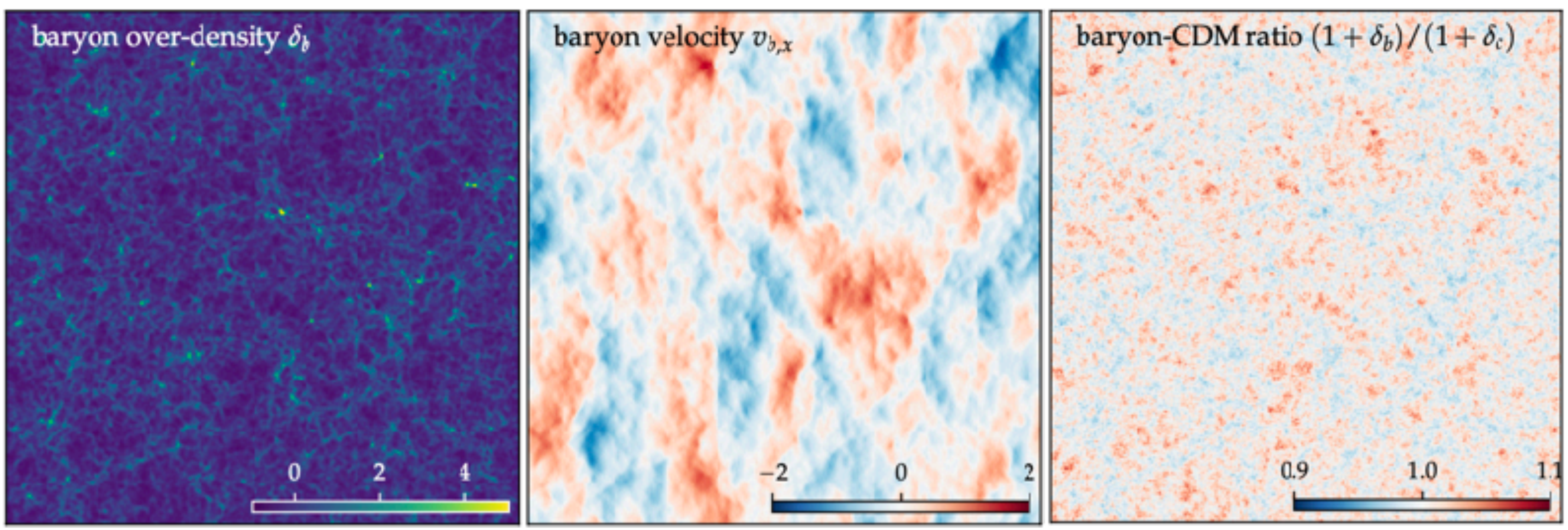


phased wave ICs: Valinia+1997

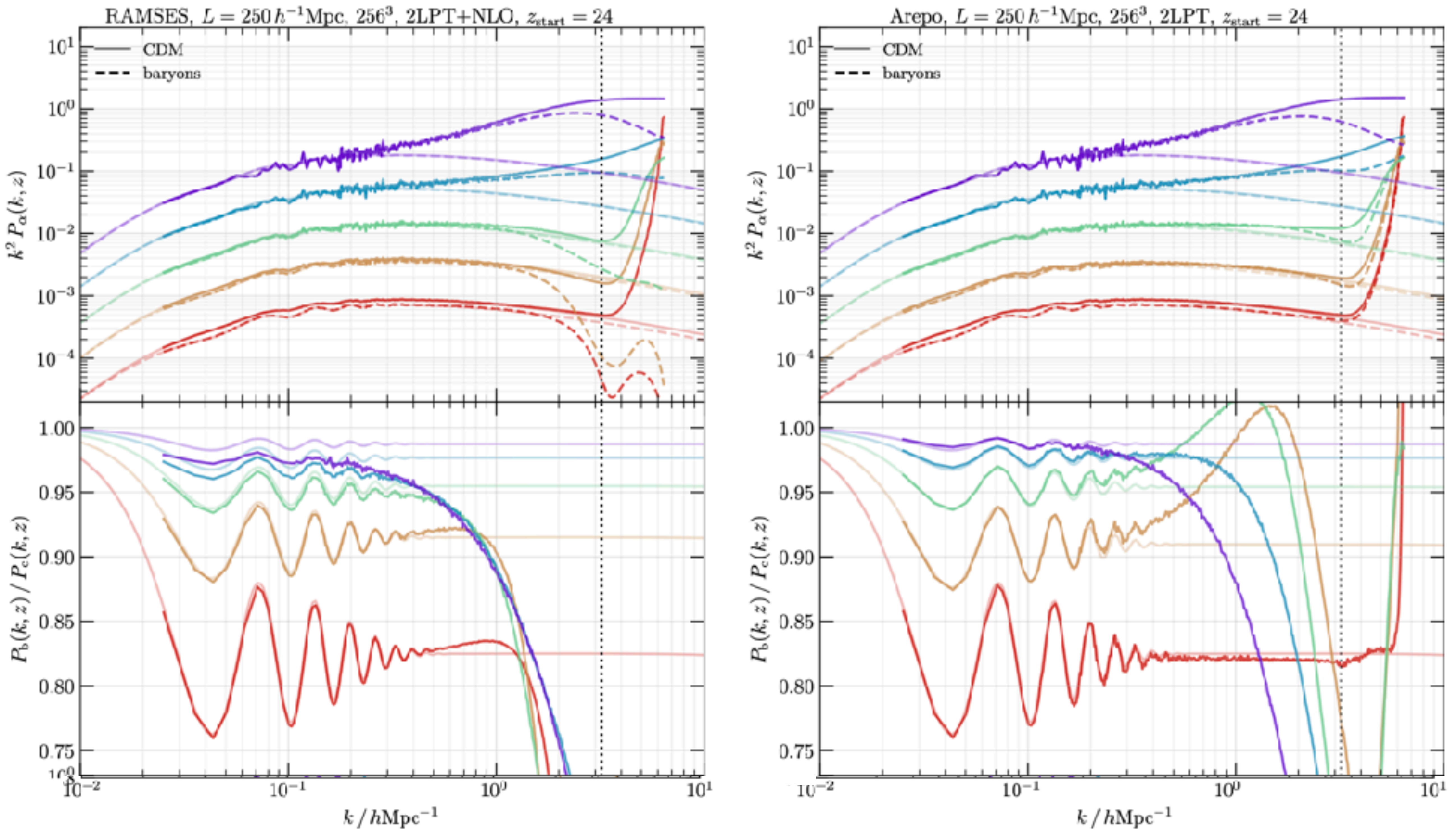
Uhlemann, Rampf, Gosenca & OH (2018)

Can upgrade also to 2-fluid baryon+CDM version, use as ICs for Eulerian codes



built into monofonIC, see OH et al. 2021, Rampf et al. 2021

A fairer comparison of Eulerian and Lagrangian codes for precision cosmology…

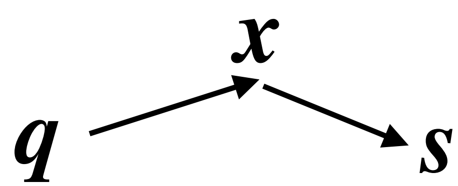


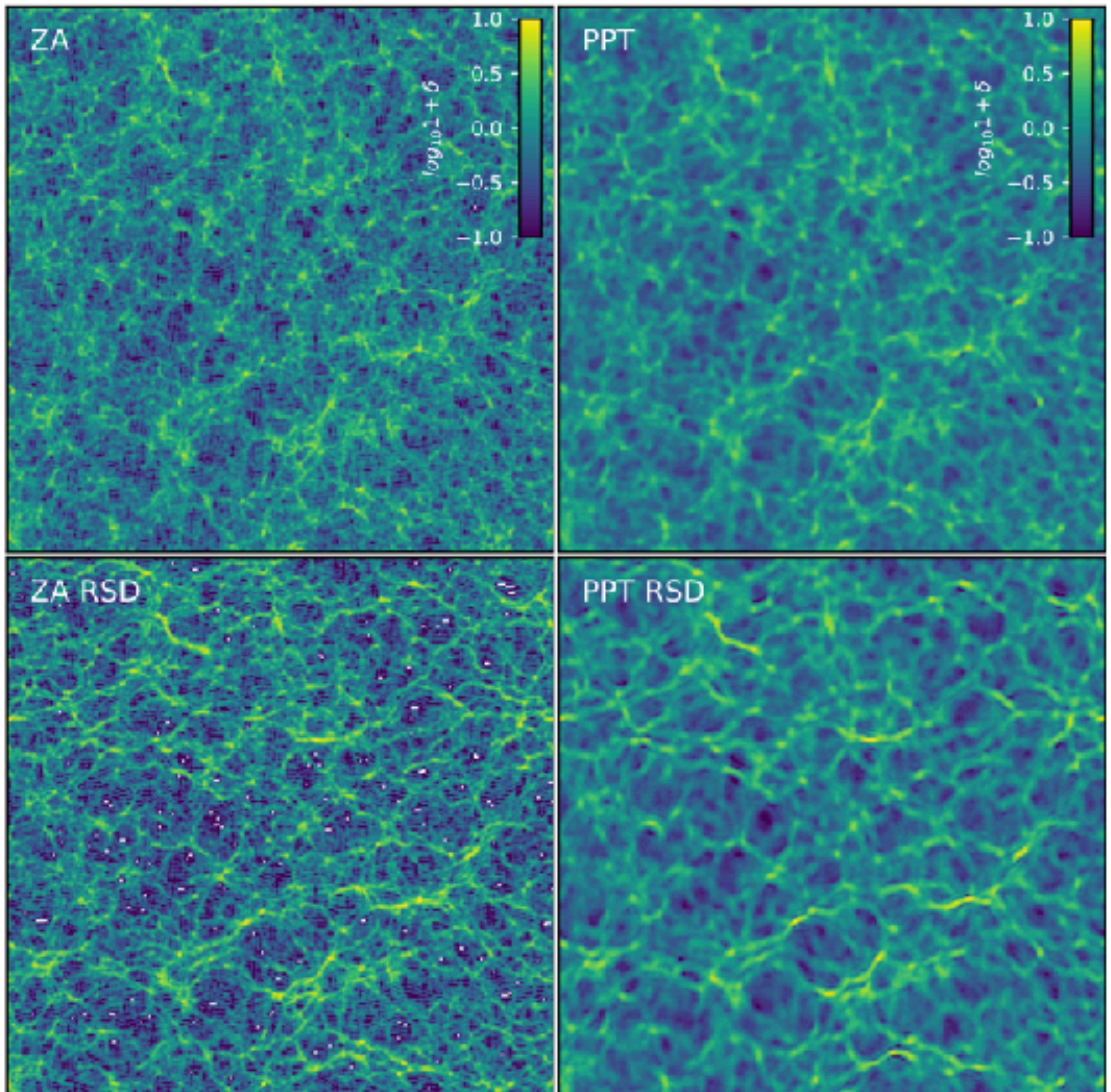
OH et al. 2021, Rampf et al. 2021

PPT as forward model for IGM

In LPT: $\boldsymbol{s} := \boldsymbol{x} + f(a)\,(\boldsymbol{\Psi}\cdot\hat{\boldsymbol{e}}_{\mathrm{LOS}})\,\hat{\boldsymbol{e}}_{\mathrm{LOS}}$

As propagator:





Porqueres+2020

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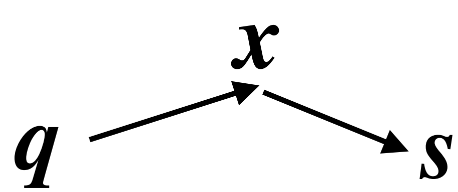
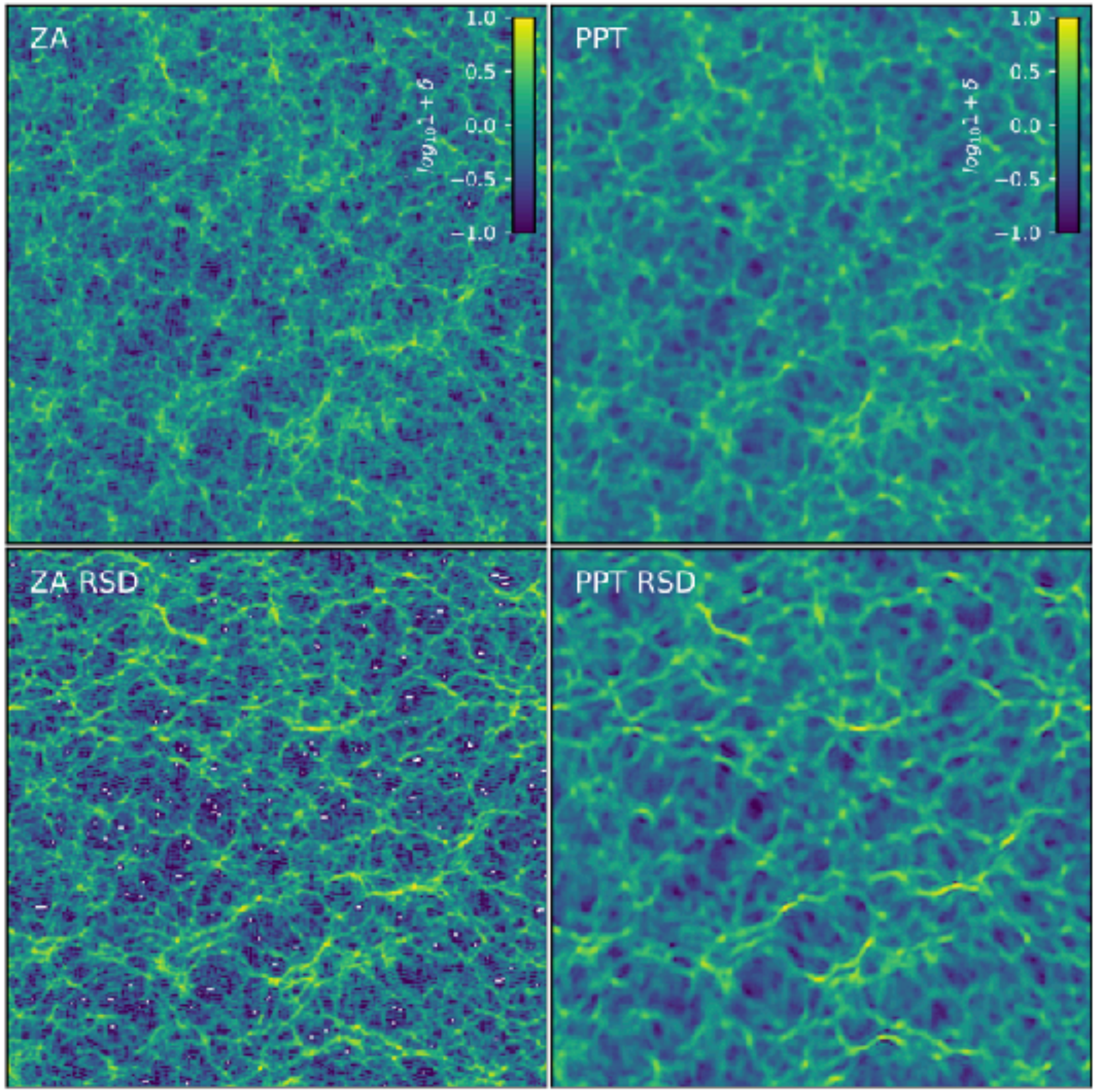
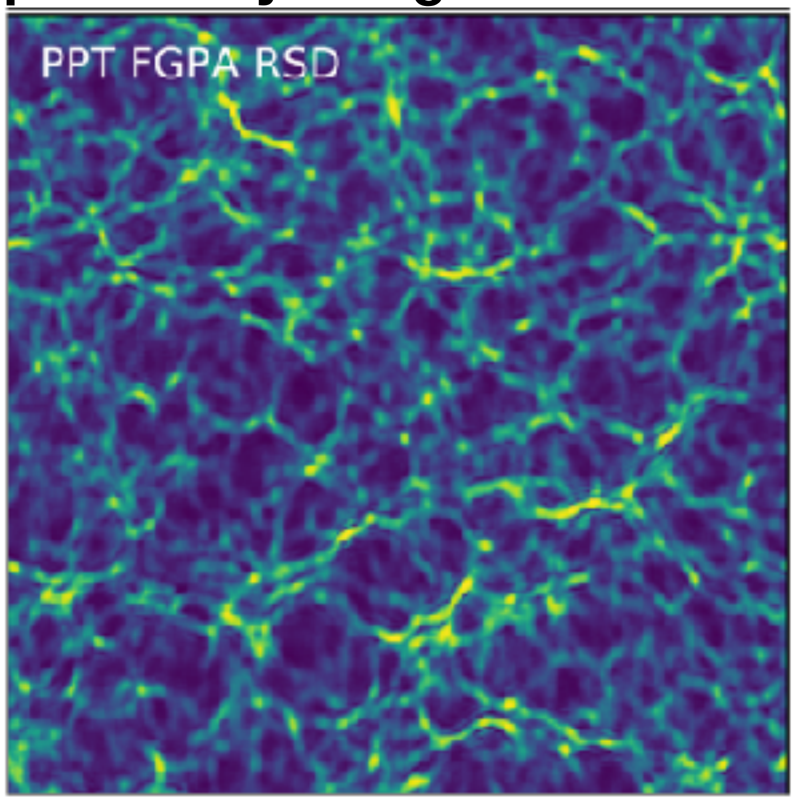




Upgrade to photon absorption probability using FGPA



redshift-space transmitted flux

-> fast field-level forward model

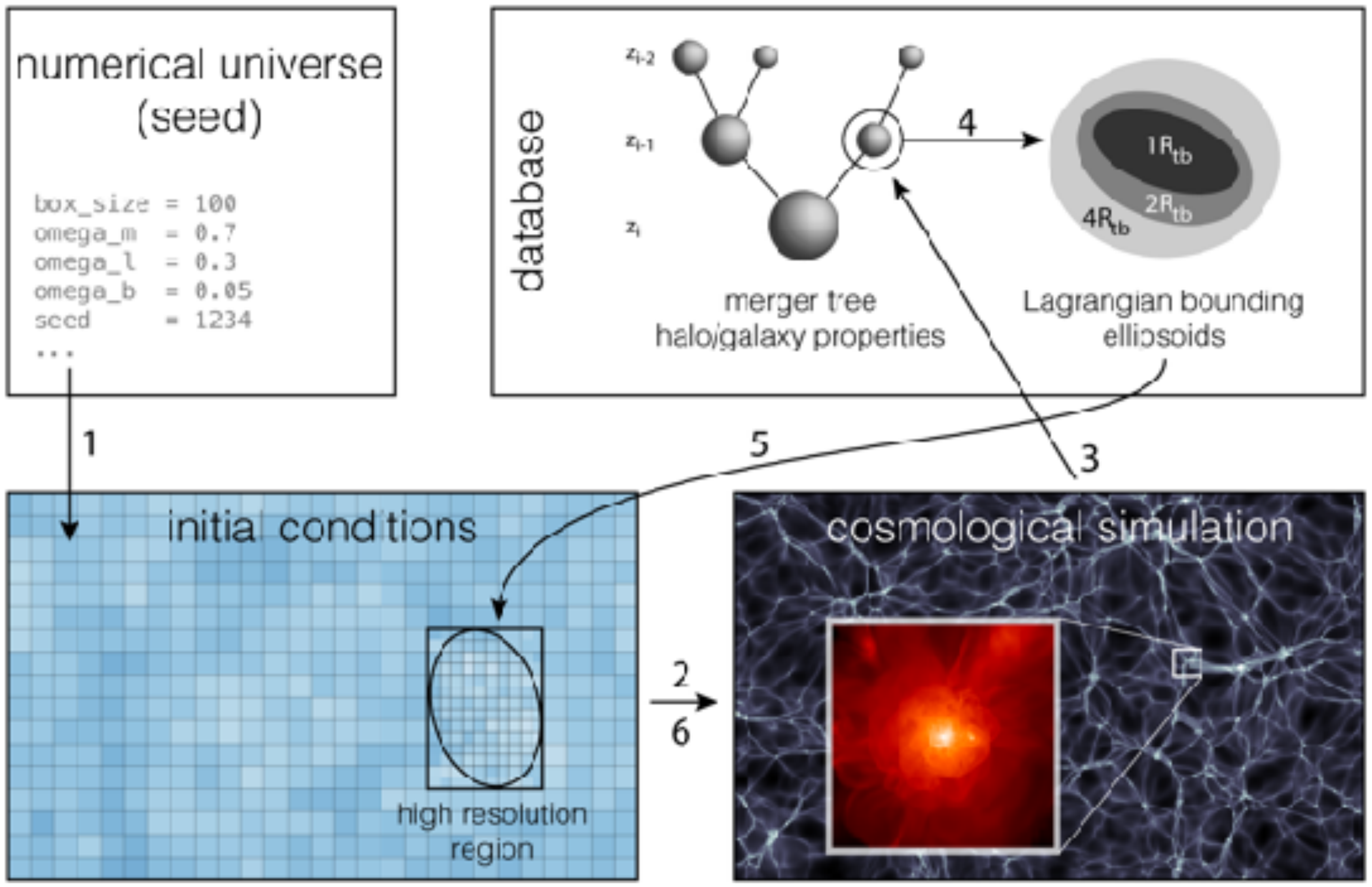
Porqueres+2020

The cosmICweb platform - a protohalo database for Zooms

Enhancing the MUSIC1 ecosystem for the next decade

w/ Michael Buehlmann

http://cosmicweb.astro.univie.ac.at



1: create ICs from cosmo parameters and random seed

2: running simulation, storing snapshots

3: structure finding and linking across time: merger trees

4: for each halo, find Lagrangian patch (origin)

5: for chosen halo, refine that patch in ICs

6: run zoom simulation with additional physics, etc.

cosmICweb:
A database and web interface for

1. **Finding** the right objects to re-simulate
2. **Obtaining** initial conditions for these objects
3. **Referencing** objects in articles / papers

If interested, do get in touch!

Buehlmann, OH, et al. 2023, in prep.

slide courtesy M. Buehlmann

SUMMARY

- LPT has key role in ICs for cosmological simulations
- Demonstrated convergence of LPT beyond shell-crossing
- 3LPT needed for precision era N-body simulations,
  push to late starts to reduce errors
- new propagator approach for LPT on the grid
  - can be used for Eulerian grid code ICs
  - and for IGM/field-level forward modelling
- new LPT inspired integrators (beyond 'FastPM')

MUSIC2 monofonIC **https://bitbucket.org/ohahn/monofonic**

single resolution (=only full cosmological volume) version

- direct integration of CLASS
- up to 3LPT, (nLPT exists already, not public just yet)
- PLT corrections
- more accurate treatment of baryons
- new propagator approach for Eulerian baryons
- modular architecture: multi code, easily extensible
- MPI+threads (no more memory limits)

MUSIC2 cosmICweb **https://cosmicweb.univie.ac.at**

- proto-halo patches for full merger tree (incl. Agora, Eagle, …)
- numerical observatory (unified framework for zoom resimulations)