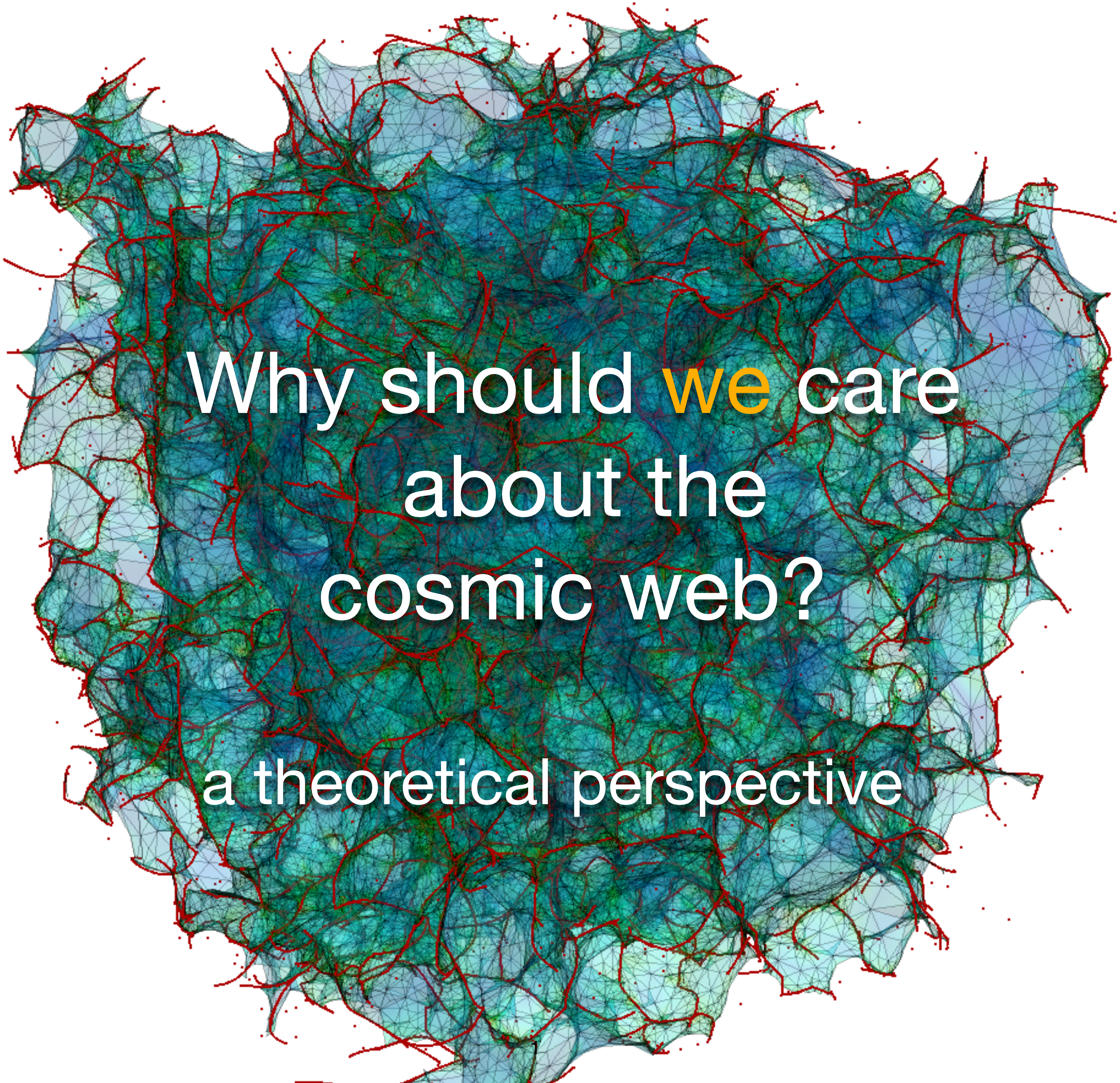
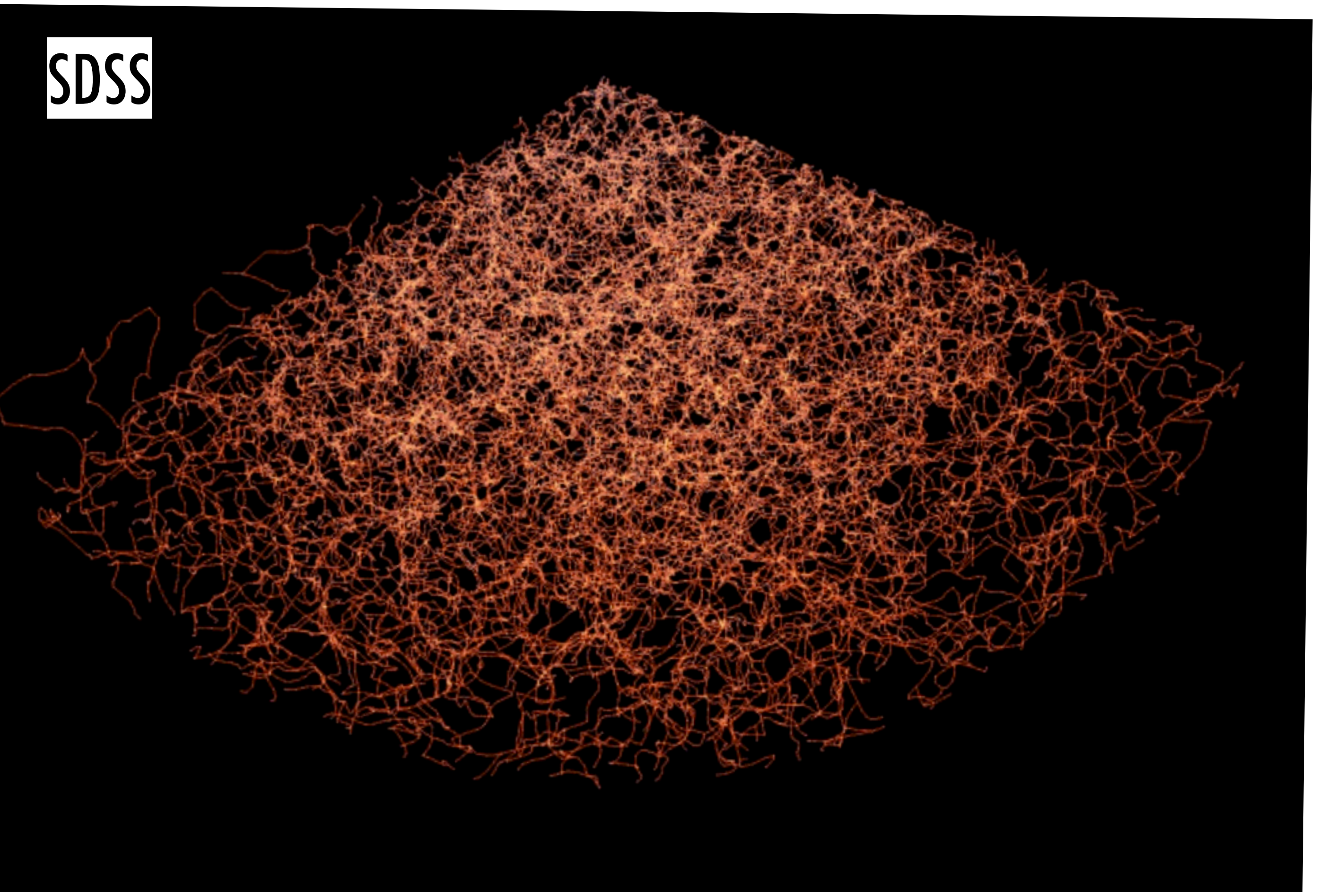
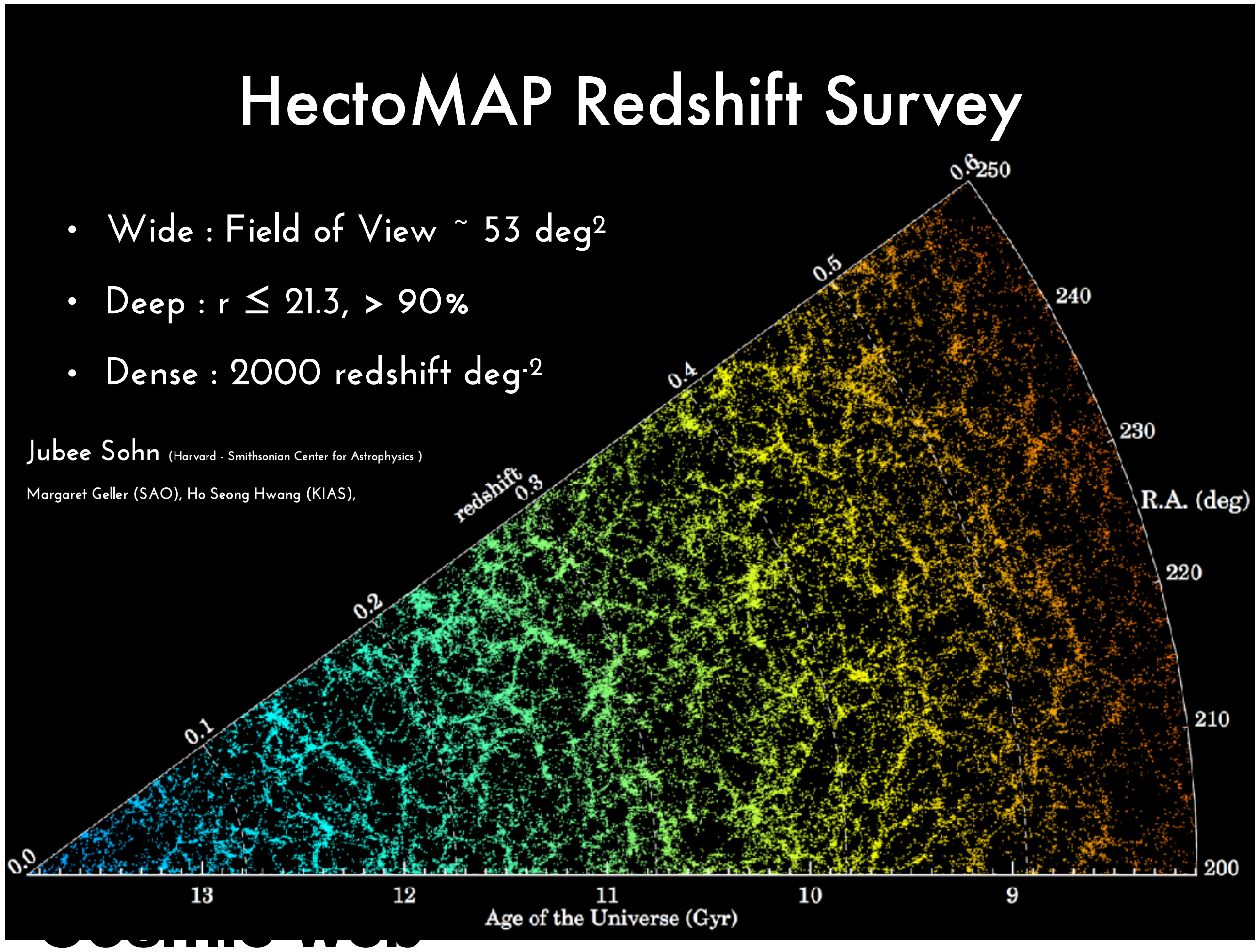
Why should we care about the cosmic web?

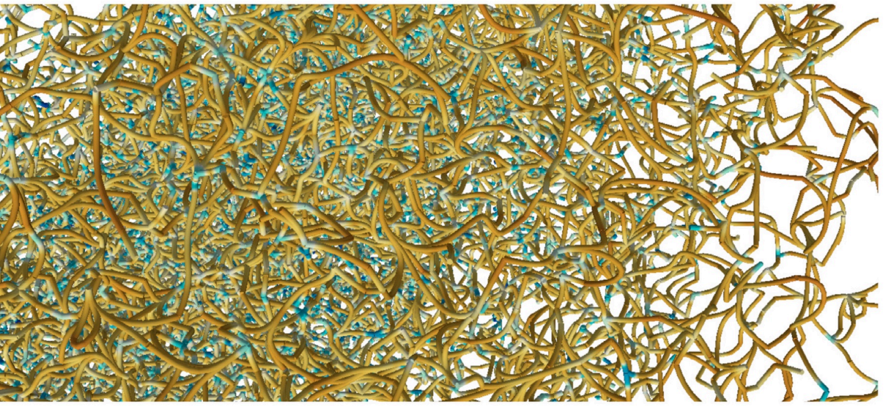
a theoretical perspective

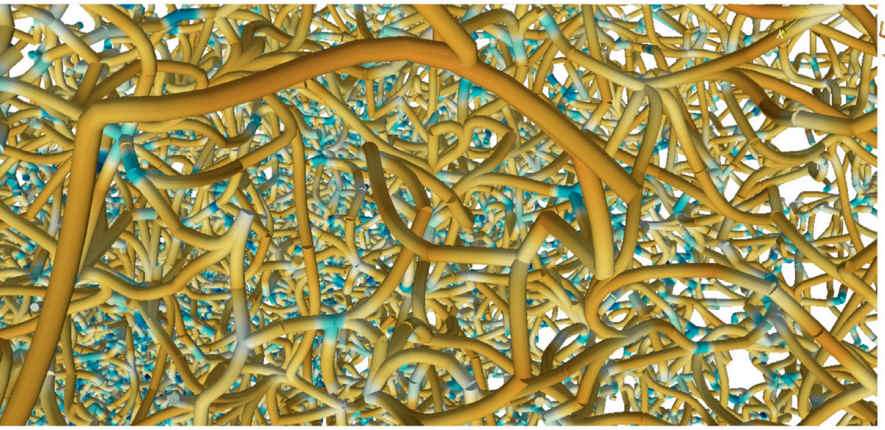


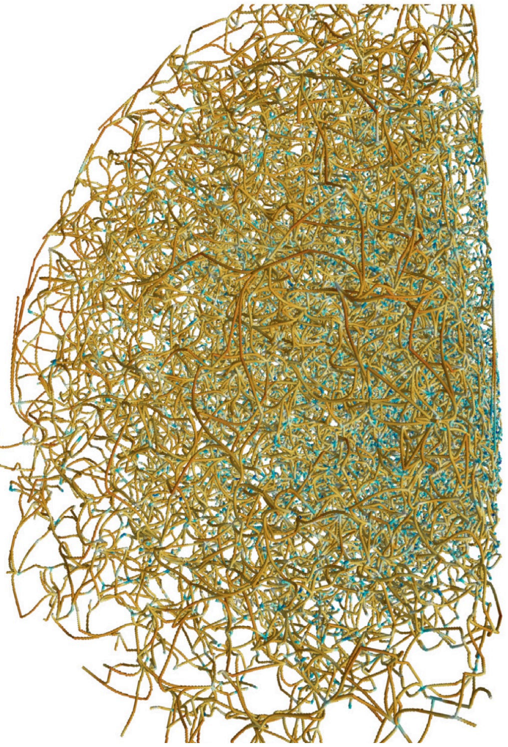
1. What is the cosmic web? according to data...

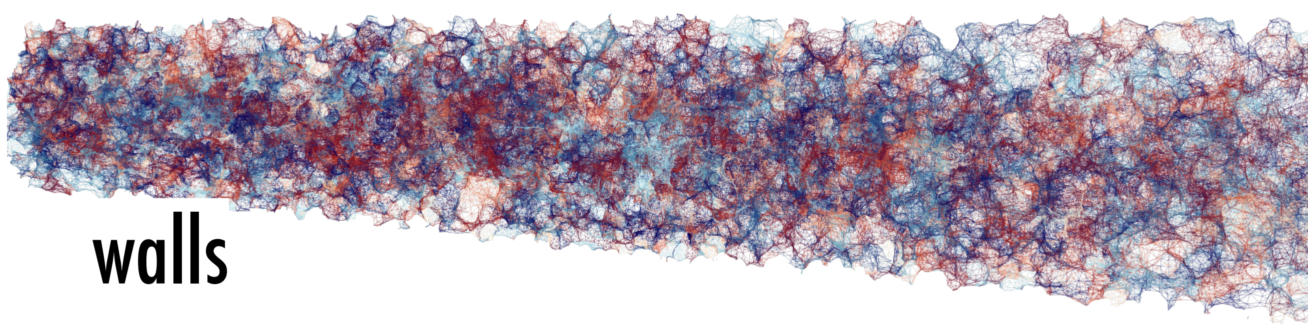


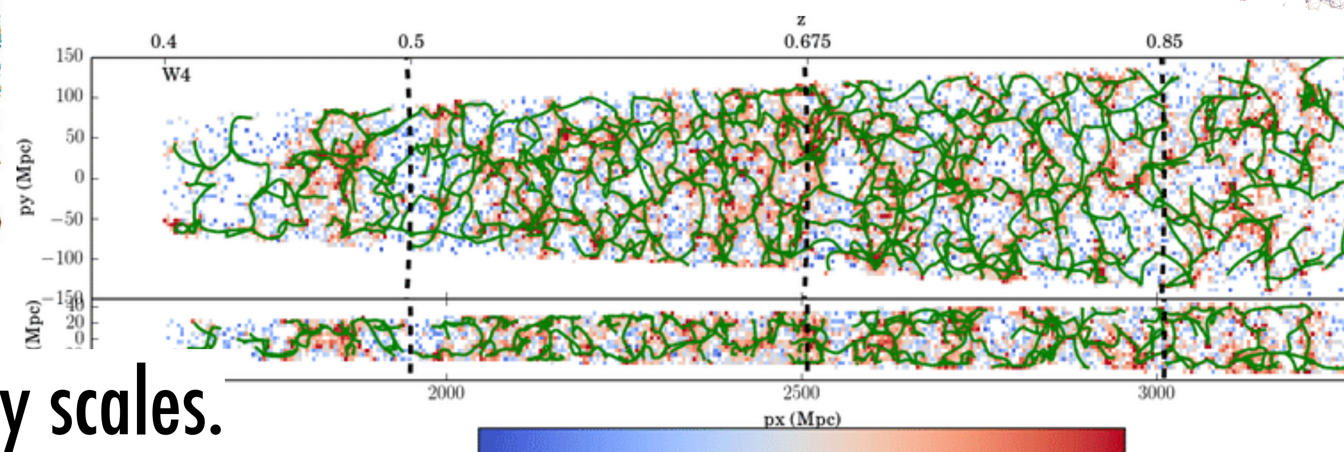












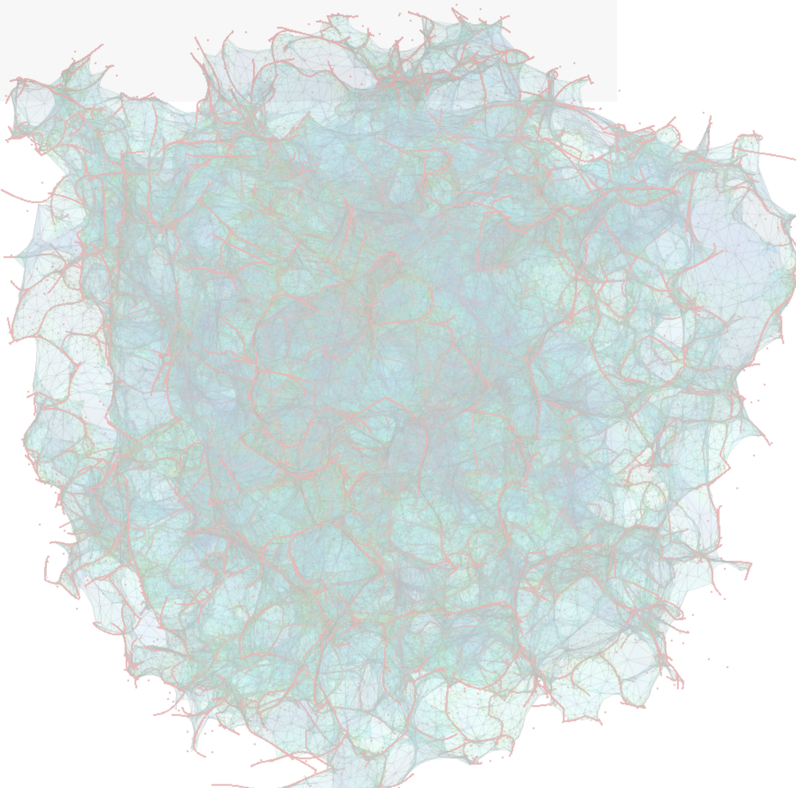
It exists on many scales.

1. What is the cosmic web? according to @chatGPT

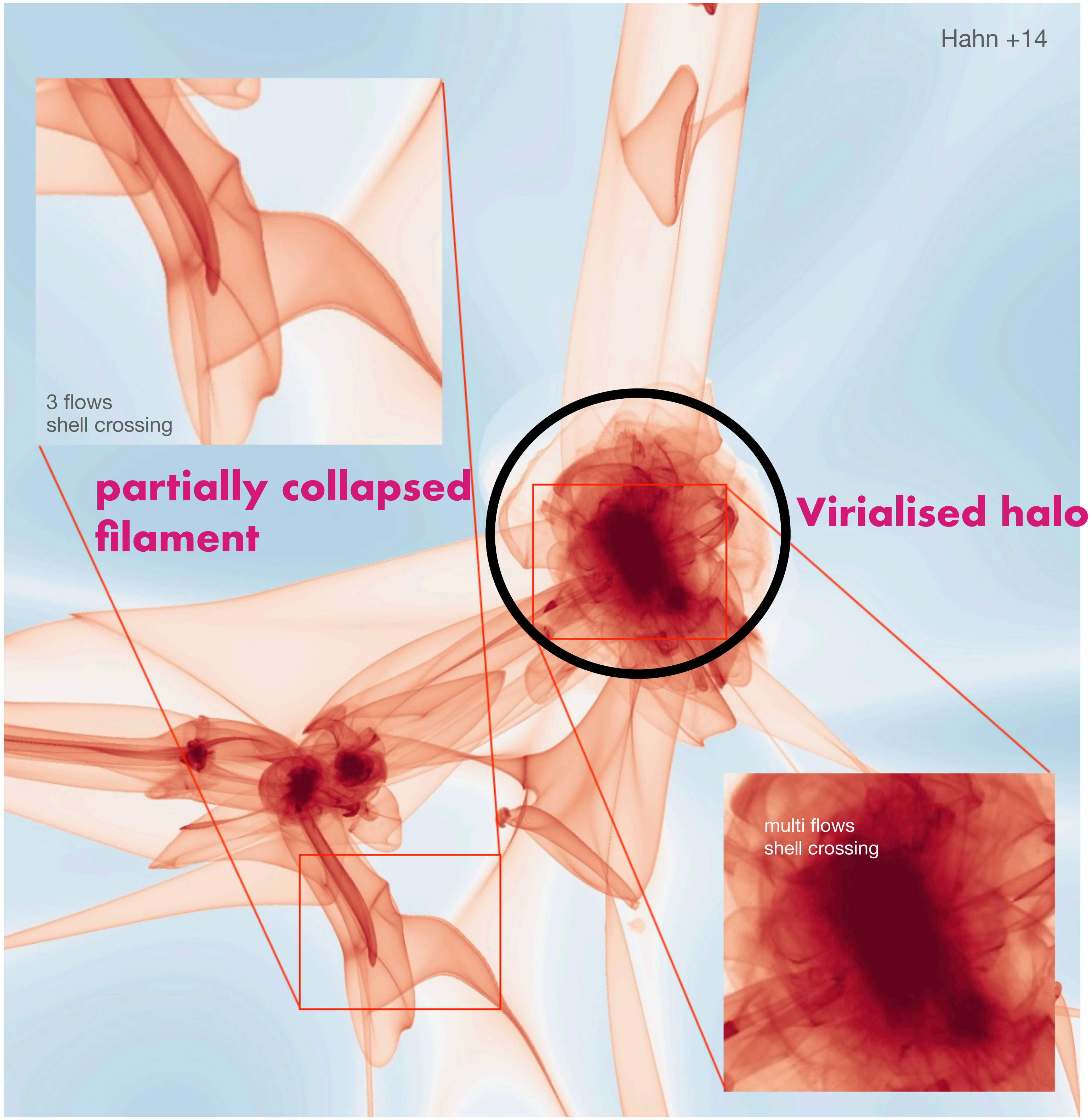
what is the cosmic web

The Cosmic Web refers to the large-scale structure of the universe, composed of galaxies and dark matter, which are interconnected by filaments of dark matter and gas. These filaments form a web-like pattern that extends throughout the observable universe, giving rise to the idea of the "Cosmic Web." The structure of the Cosmic Web is thought to play a key role in the evolution and distribution of galaxies, as well as in the formation of large-scale structures like galaxy clusters and superclusters.

Plausible but ...



The cosmic web is a dynamically relevant intermediate-density boundary between cosmology and galaxy formation.

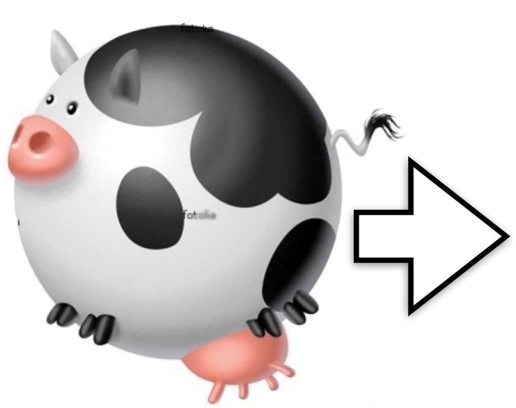


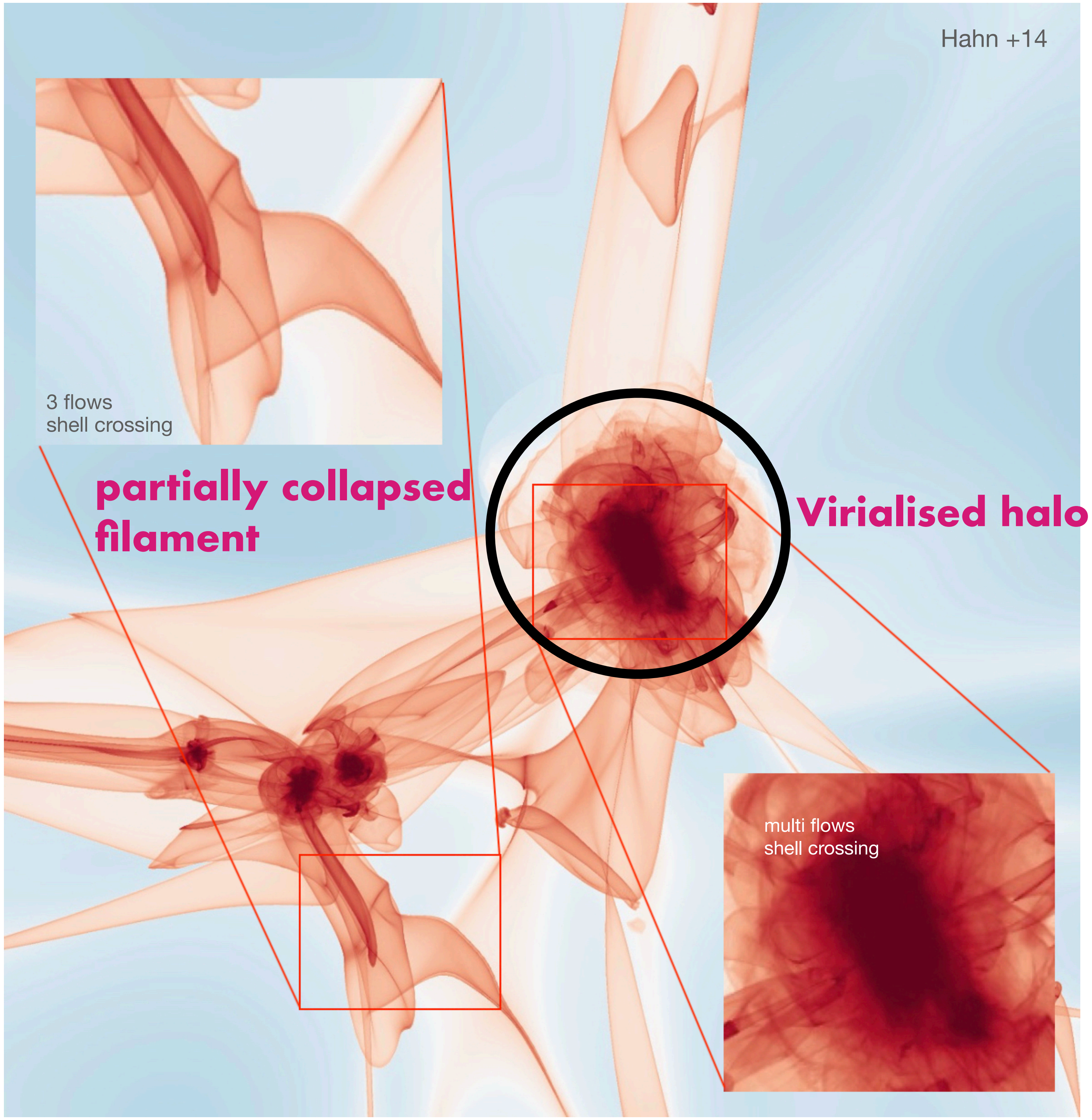
When halo collapse, neighbouring filaments+walls are in place.

~~The cosmic web is a dynamically relevant intermediate-density boundary between cosmology and galaxy formation.~~

Since it exists on many scales

The cosmic web is a dynamically relevant anisotropic (=spin 2) boundary between a given scale and a larger scale.





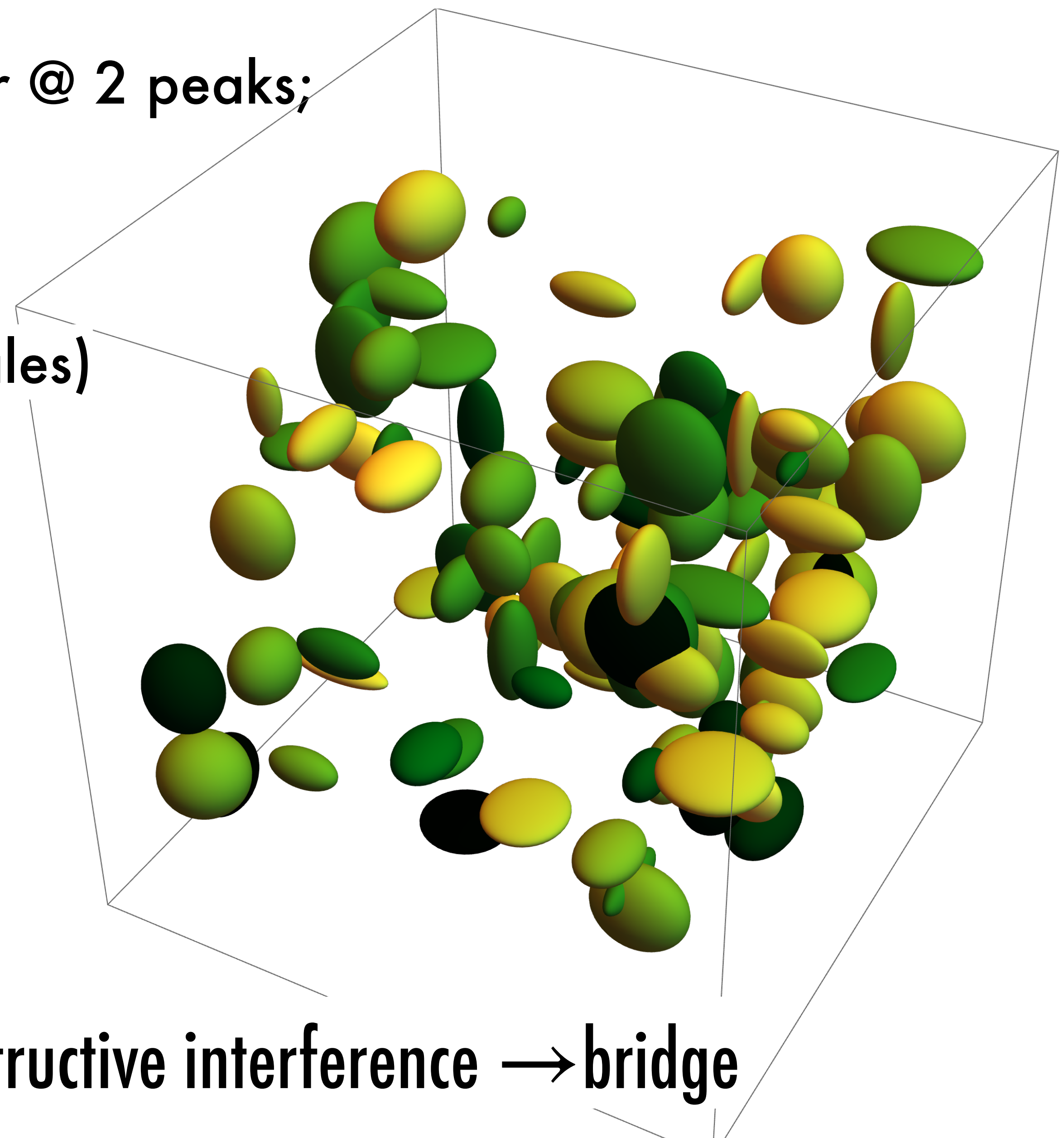
We must consider peaks rigged = dressed by their sets of (wall + filament) saddle critical pts.

1. What is the cosmic web? a spin-2 two-point process

$$\textbf{cosmic web} = \text{alignment of eigframes} \left[\frac{\partial^2 \bar{\psi}}{\partial x_i \partial x_j}\right]_{pk}$$

- tides are longer range (than density) when aligned with something

BKP96: alignment of shear tensor @ 2 peaks;

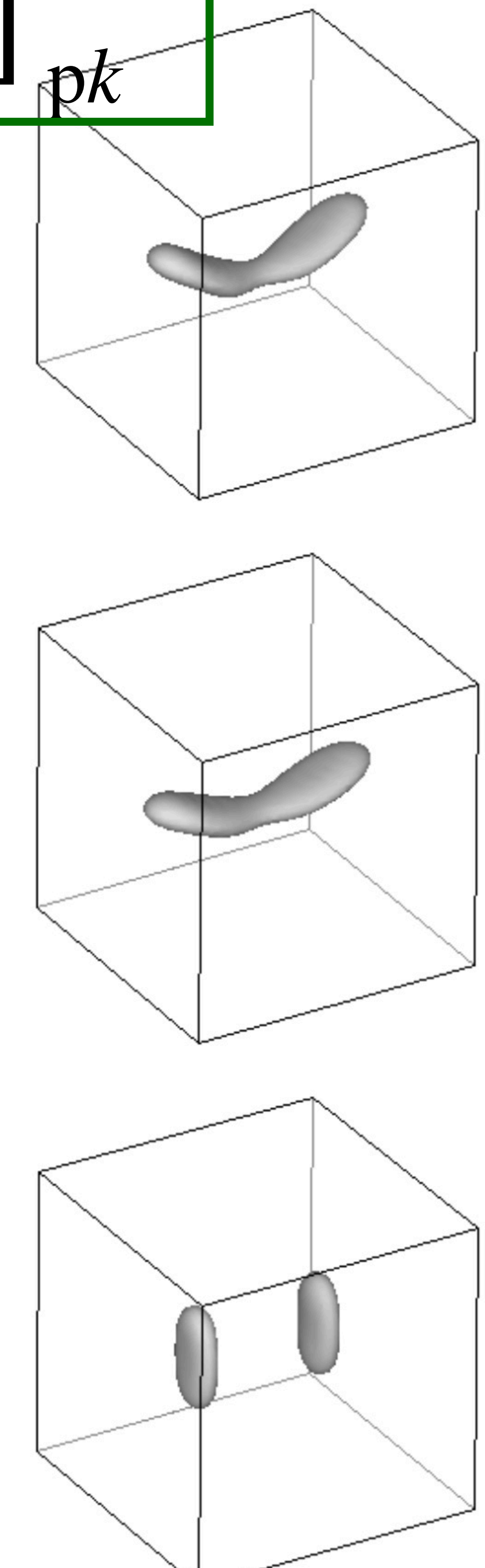


- Predicts LSS in ICs (on large scales)

  unexpected result in 96

- Applicable on any scale

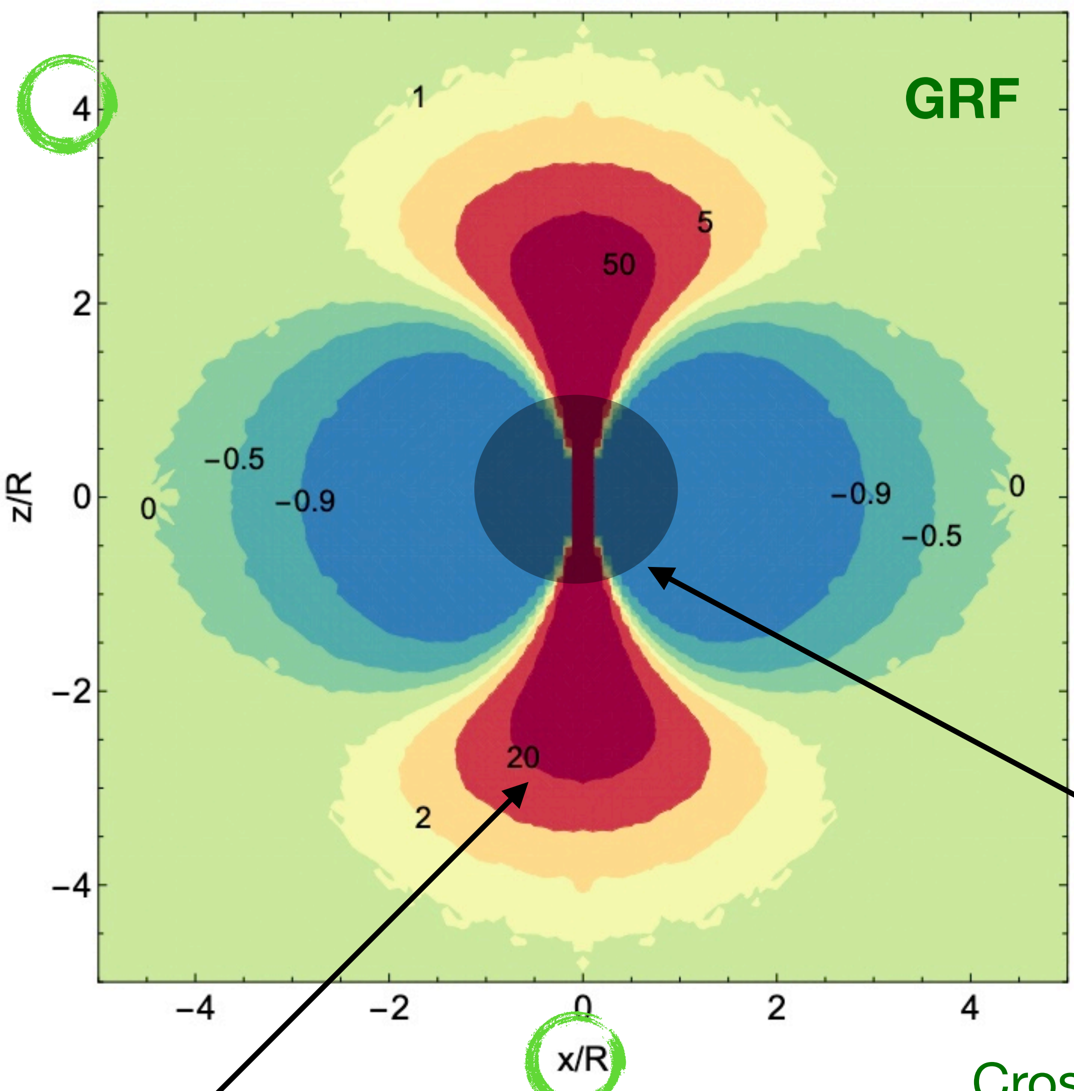
  important for this talk



alignment $\rightarrow$ high degree of constructive interference $\rightarrow$ bridge

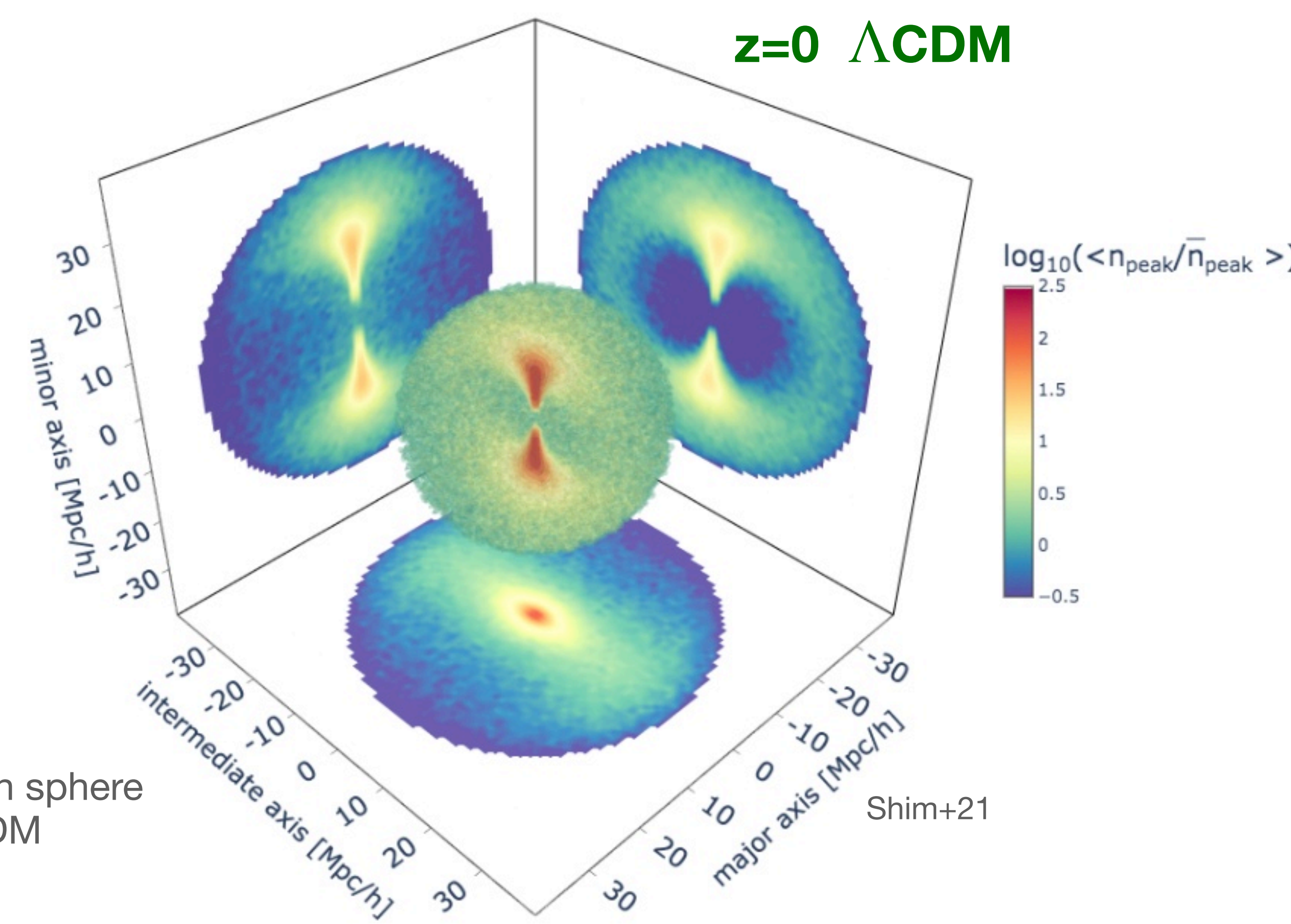
$$\textbf{cosmic web} \approx \text{metric set by eigframe} \left[\frac{\partial^2\rho}{\partial x_i \partial x_j}\right]_{\text{sad}}$$

More recently, alignment w.r.t. (filament or wall) saddle eigen-frame = spin-2 one-point process.



Correlation sphere of DM

Correlation zone of saddle



Cross correlation of peaks relative to a given saddle

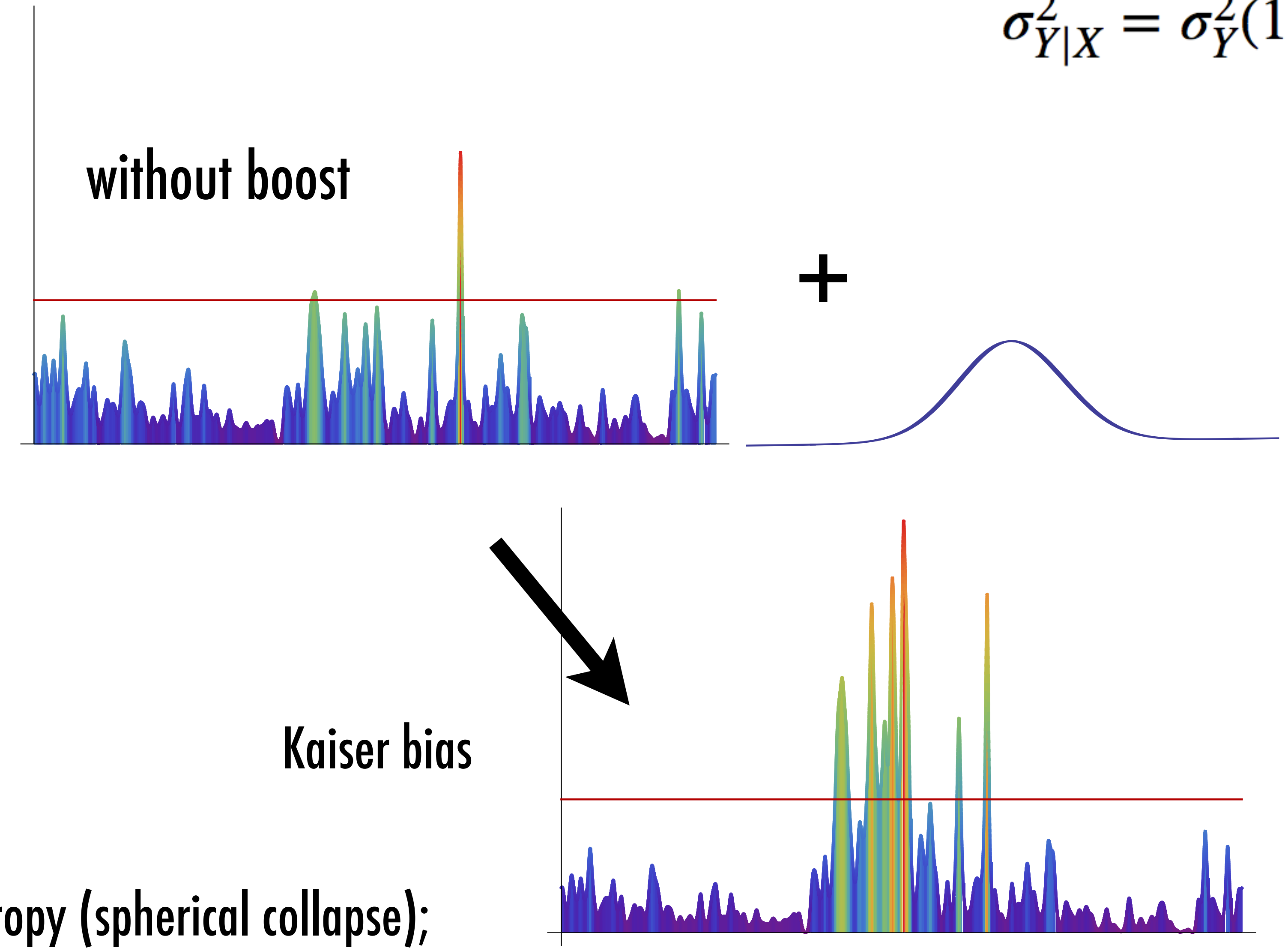
one should consider peaks dressed by neighbouring critical pts.

$$\textbf{cosmic web} \approx \text{metric set by eigframe} \left[\frac{\partial^2 \rho}{\partial x_i \partial x_j} \right]_{\text{sad}}$$

- partial alignment will change (=bias) anisotropically the mean and variance of things → specific signature of CW

$$E(Y|x) = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu_X)$$

$$\sigma^2_{Y|X} = \sigma^2_Y (1 - \rho^2)$$

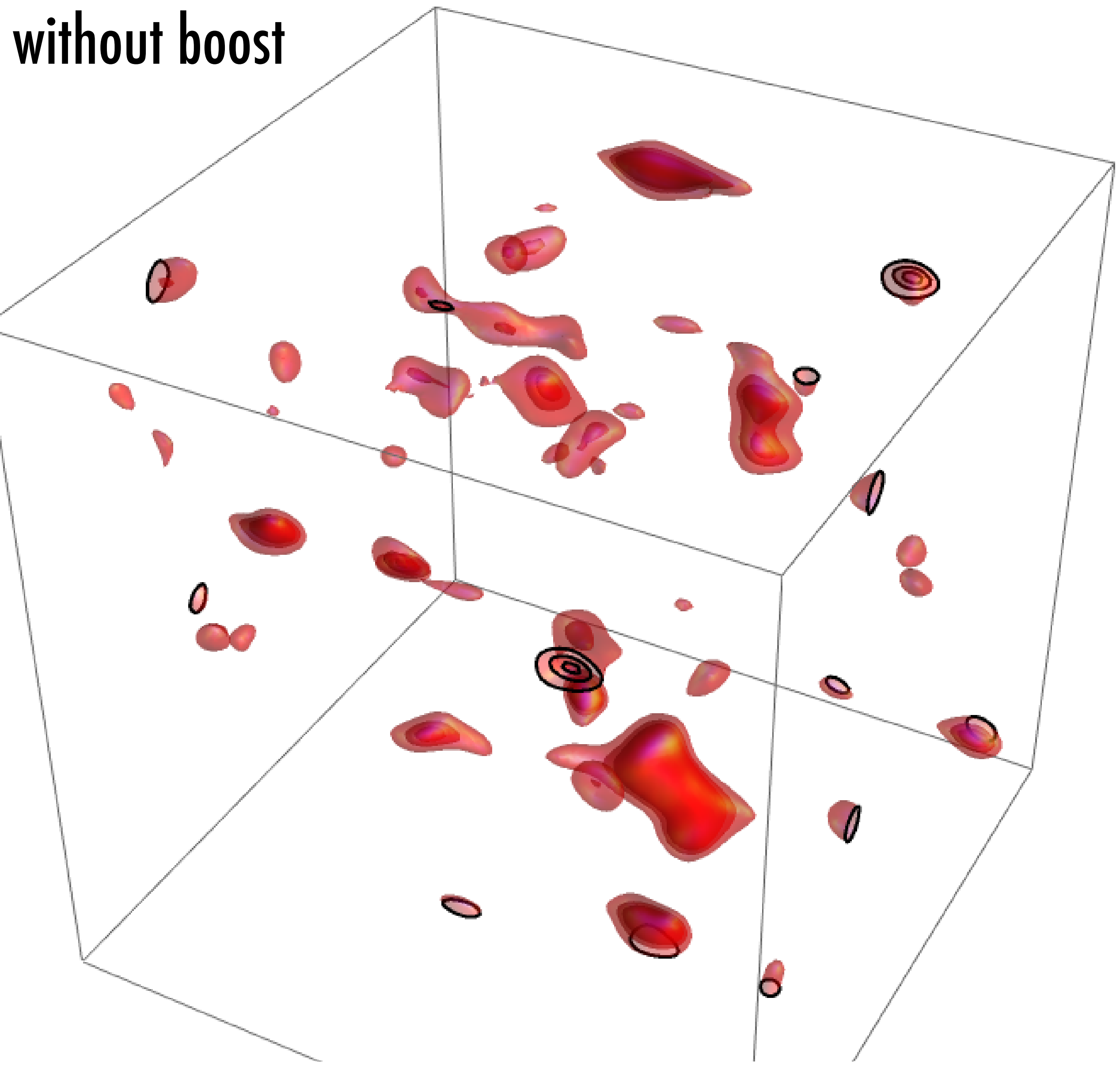


spin 0 pt constraint (=density) → isotropy (spherical collapse);

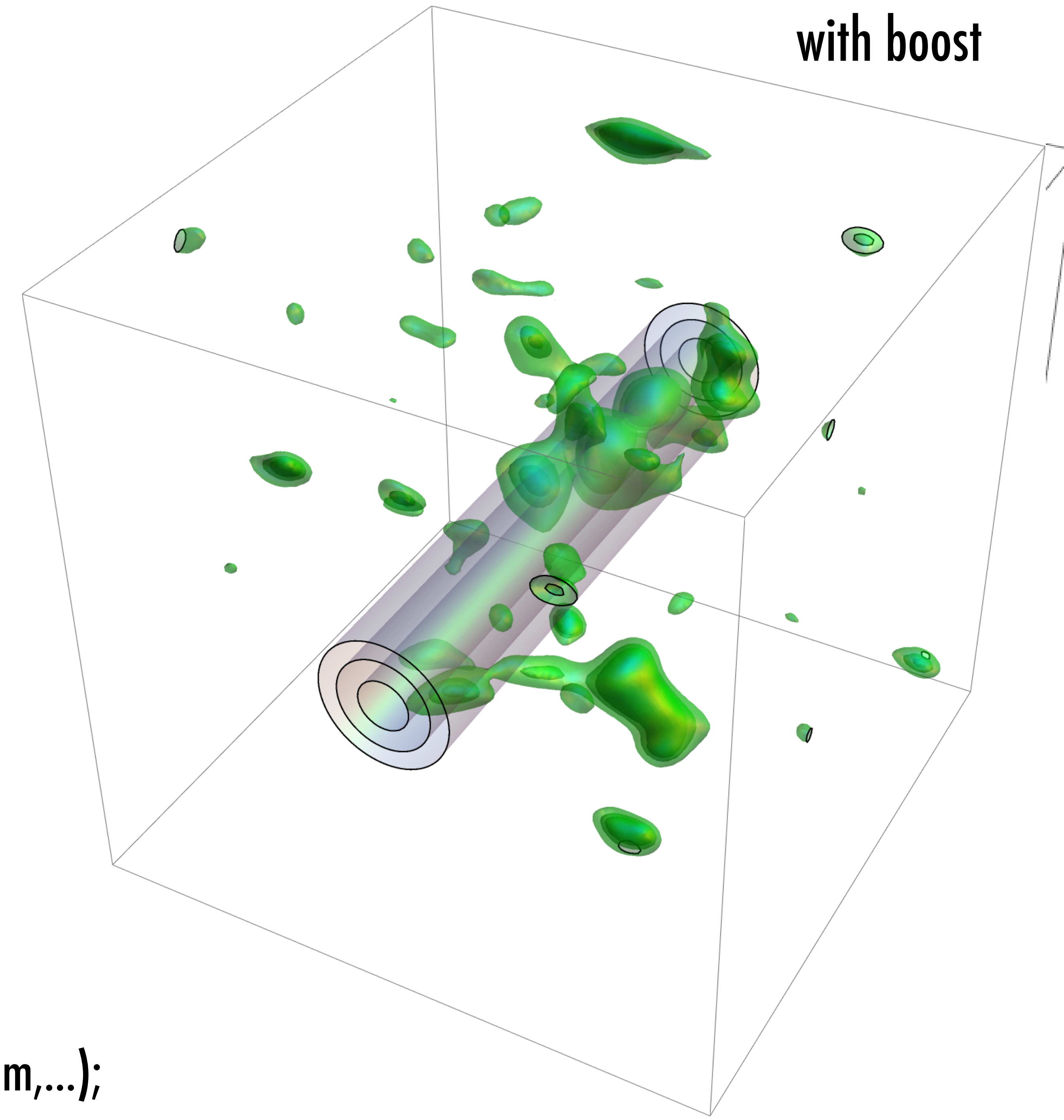
1. Kaiser bias on cosmic web

- partial alignment will change (=bias) anisotropically the mean and variance of things $\longrightarrow$ specific signature of CW

without boost



with boost



spin 0 pt constraint (=density) $\longrightarrow$ isotropy (spherical collapse);

spin 2 pt contraint (= CW) $\longrightarrow$ anisotropy (e.g. Angular momentum,...);

$$\textbf{cosmic web} \approx \text{metric set by eigframe} \left[\frac{\partial^2\rho}{\partial x_i \partial x_j}\right]_{\text{sad}}$$

- partial alignment will change (=bias) anisotropically the mean and variance of things = specific signature of CW

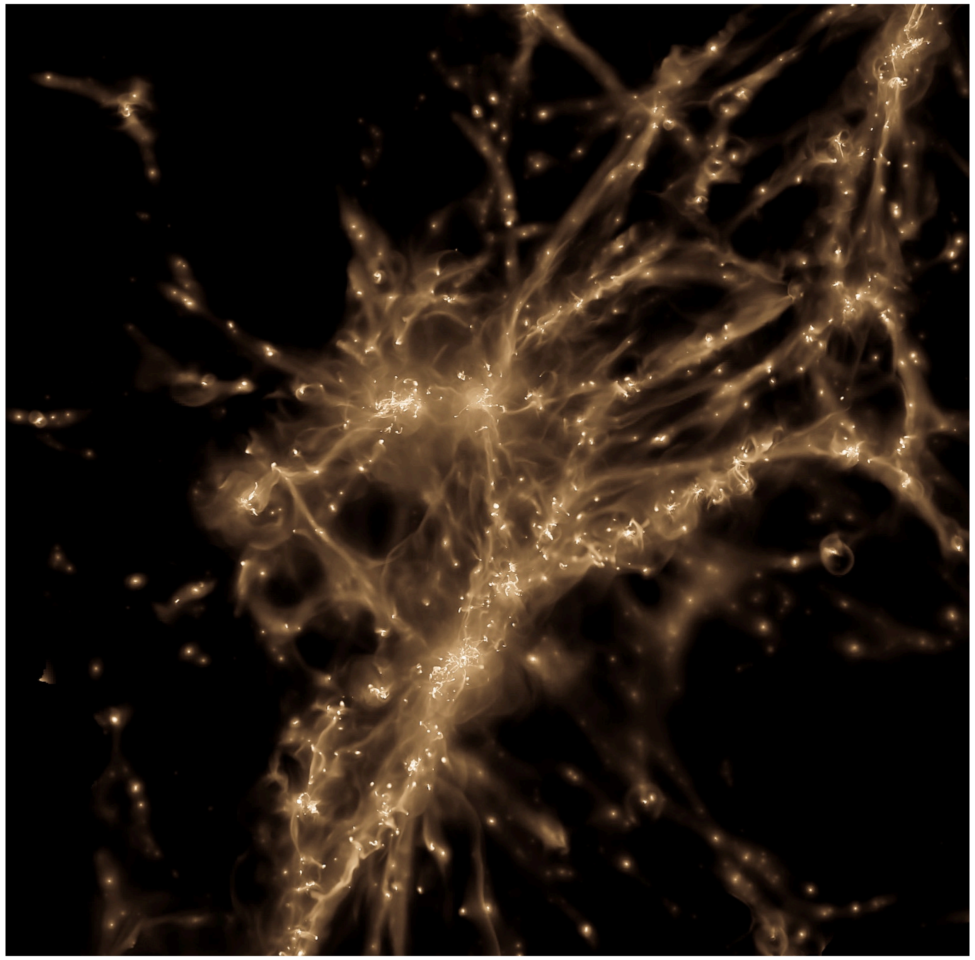
$$E(Y|x) = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x - \mu_X)$$

$$\sigma^2_{Y|X} = \sigma^2_Y(1-\rho^2)$$

revisit

- tidal torque theory
- excursion set theory
- critical event theory
- disc settling

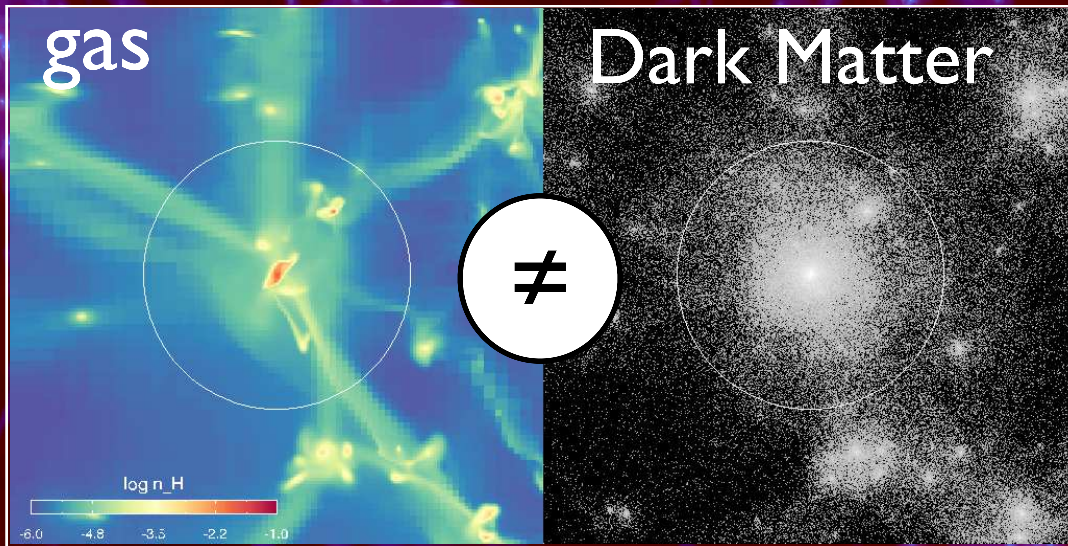
Sometimes **small** for DM



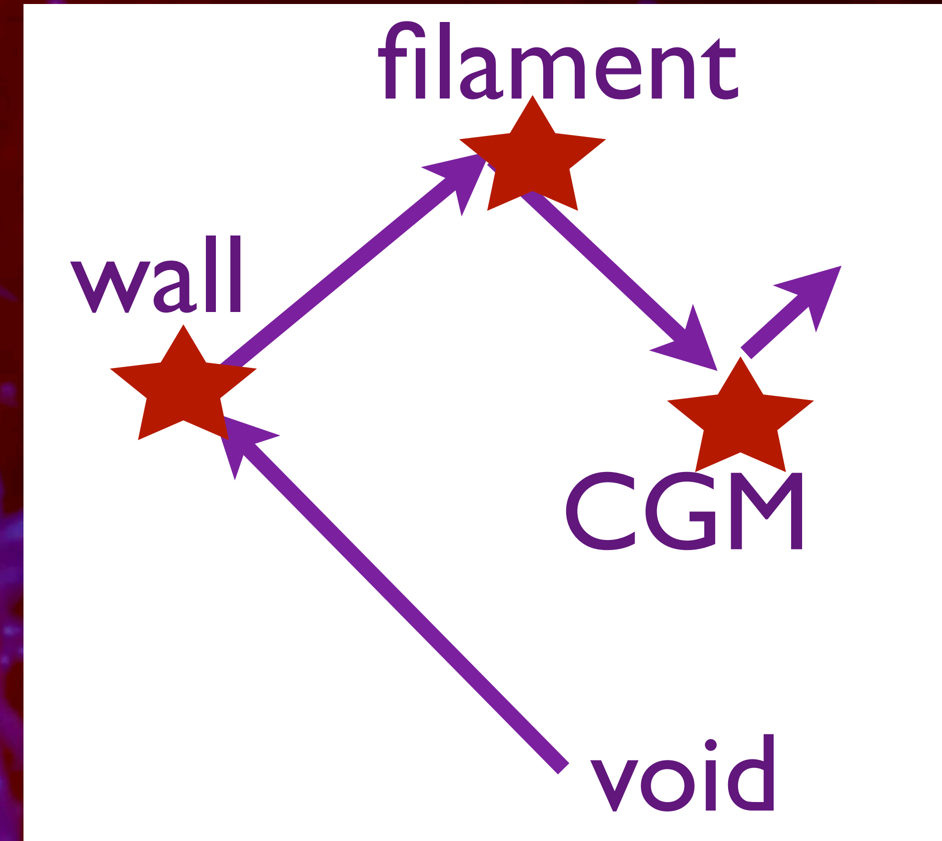
BUT It really matters for baryons

alignments funnel gas along CW : small scales inherit coherence and stability

CW drives secondary infall :



$$t_{\rm dyn} \sim 1/\sqrt{\rho}$$



MILKY WAY

IRON

STARS

GAS

DARK MATTER

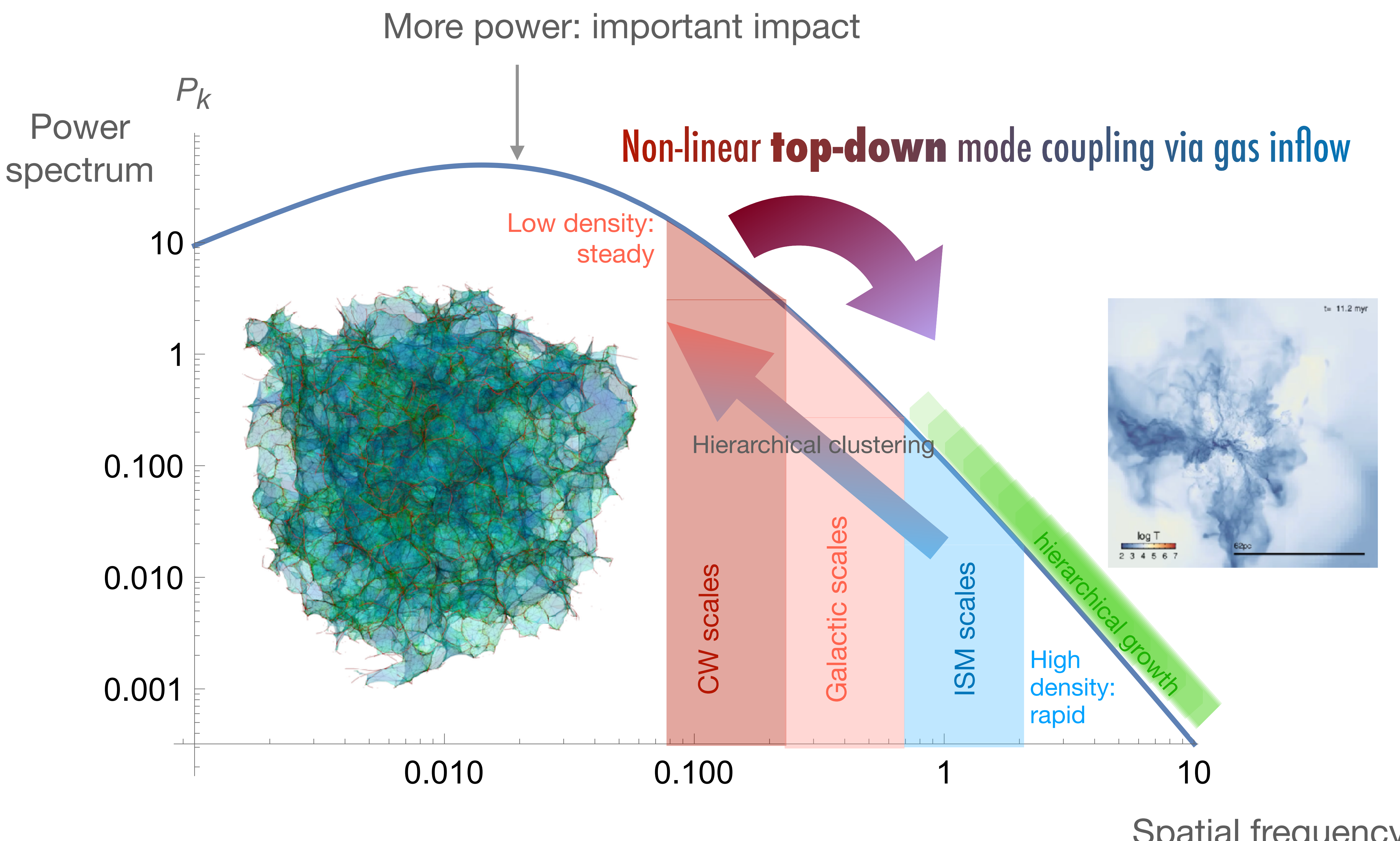
Z = 6

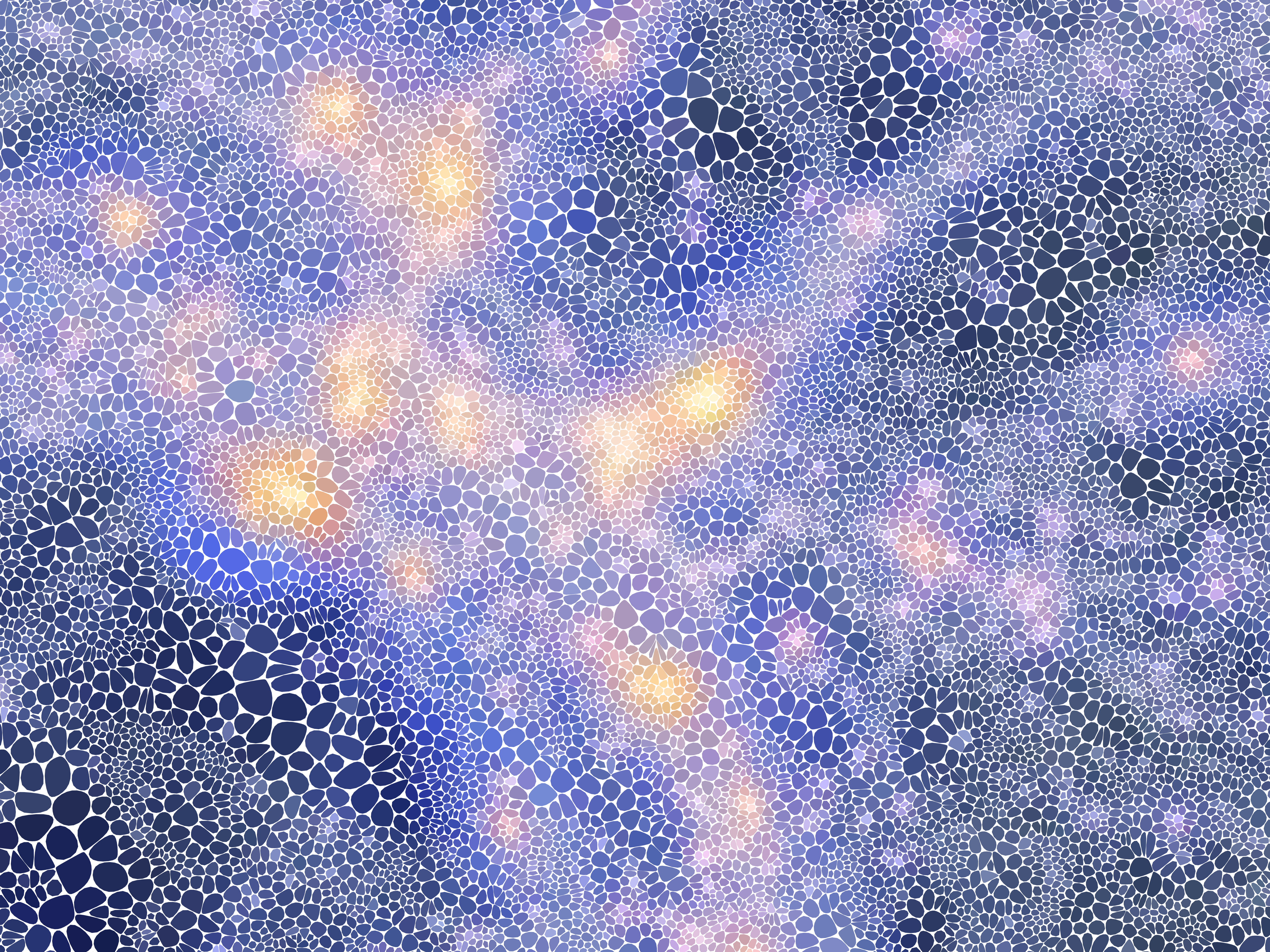
12.9 GYR AGO

Disks (re)form because LSS are large (dynamically young) and (partially) an-isotropic : they induce persistent angular momentum advection of gas along filaments which stratifies accordingly.

1. Impact of CW on non-linear dynamics is non linear & top down

On galactic scales, the Shape of initial P_k is such that galaxies inherit stability from LSS via cold flows

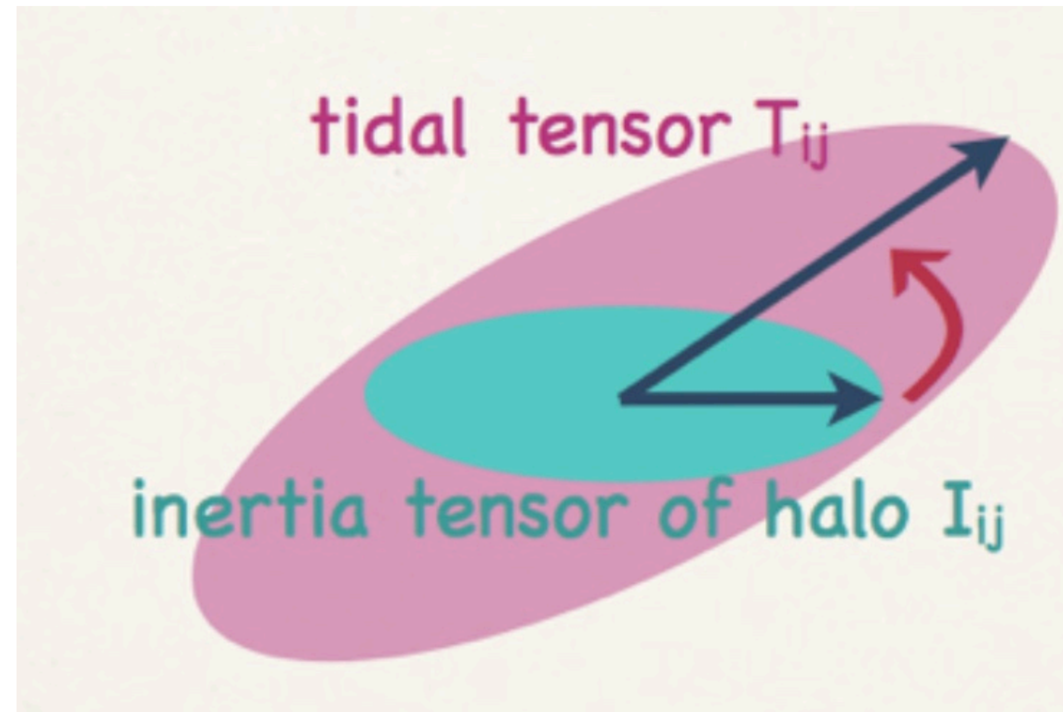




2.1 Revisiting tidal torque theory subject to CW

- saddle metric changes (=biases) anisotropically the mean and variance of things = specific signature of CW

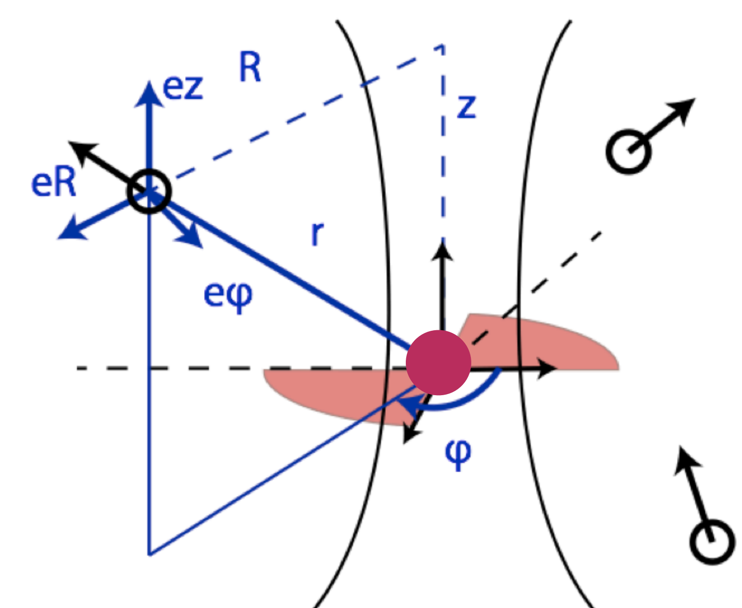
revisit
- tidal torque theory
- excursion set theory
- critical event theory
- disc settling



Tidal torque theory reflects the mis alignment of two tensors on different scales

Angular momentum = anti symmetric contraction of two tensors

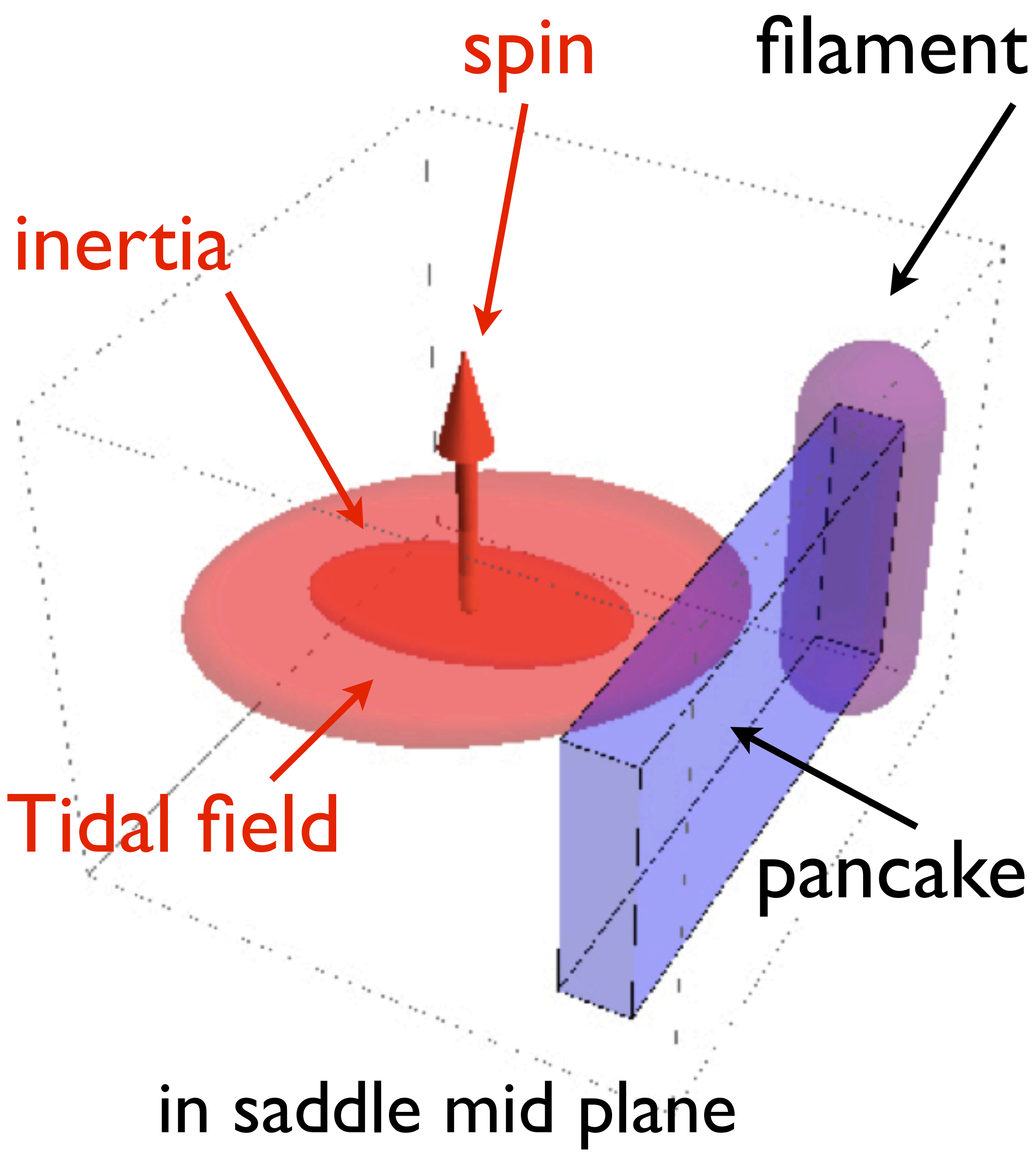
$$L_k = \epsilon_{ijk} I_{kl} \psi_{,lj}$$



aligment between frame of saddle and separation vector to halo.

2.1 Revisiting tidal torque theory subject to CW

Tidal torque theory reflects the mis-alignment of two tensors on different scales



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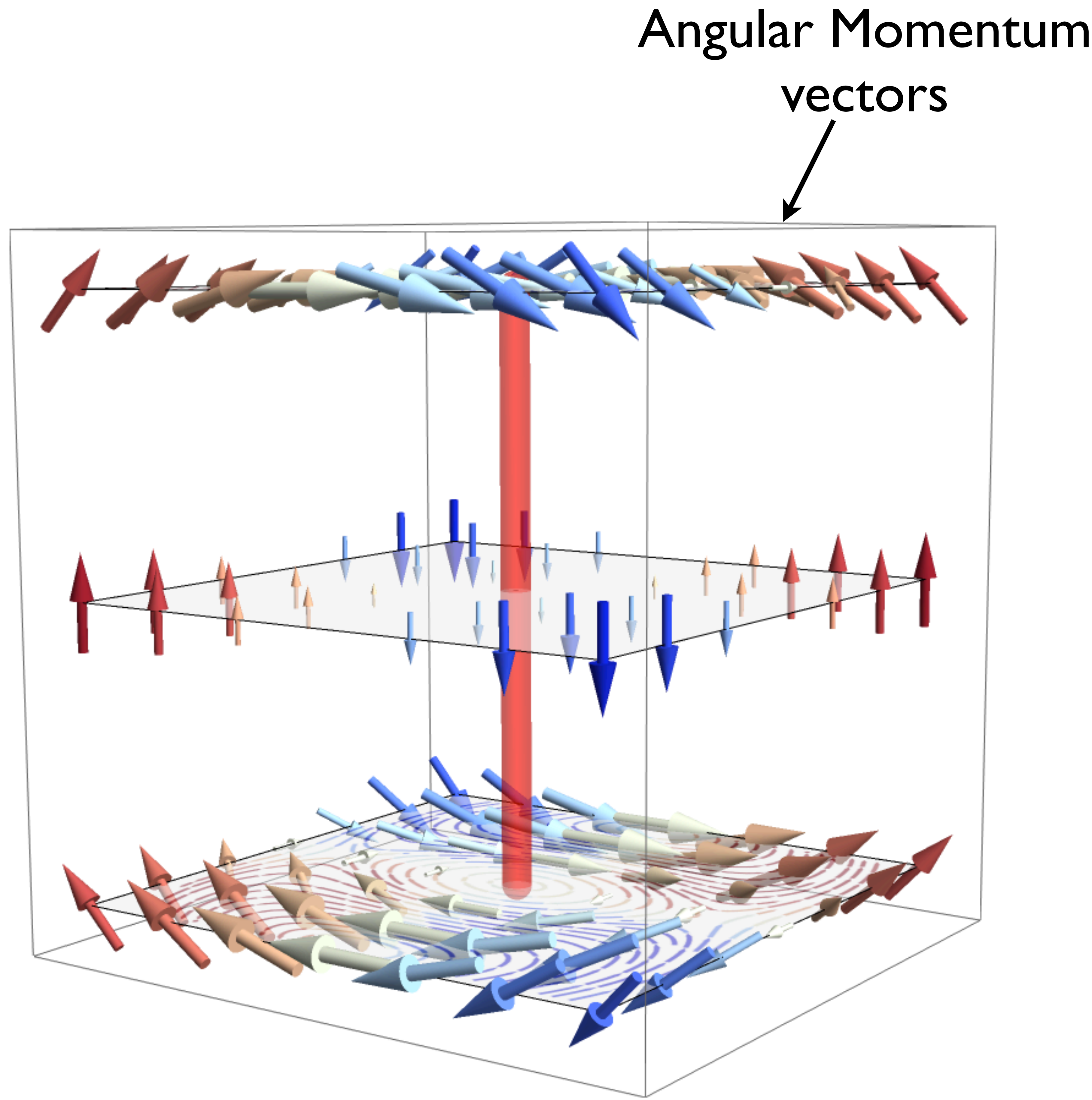


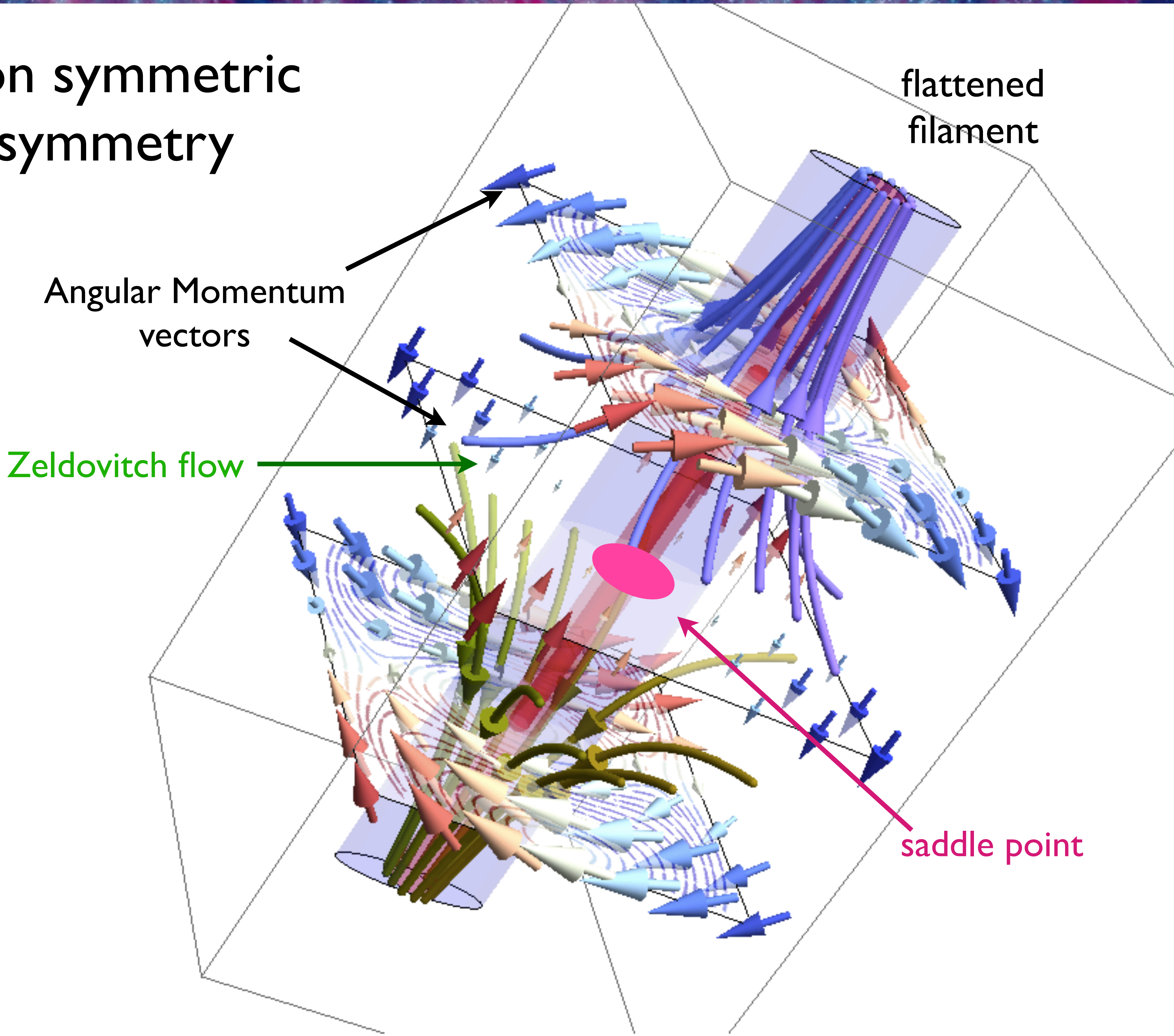
2.1 Revisiting tidal torque theory subject to CW

- point reflection symmetric
- vanish if no a-symmetry

perp. along e_ϕ

spin // to filament

perp = along e_ϕ



2.1 Revisiting tidal torque theory subject to CW

Geometry of the saddle provides **a natural 'metric'** (local frame as defined by Hessian @ saddle) relative to which **dynamical evolution** of DH is predicted.

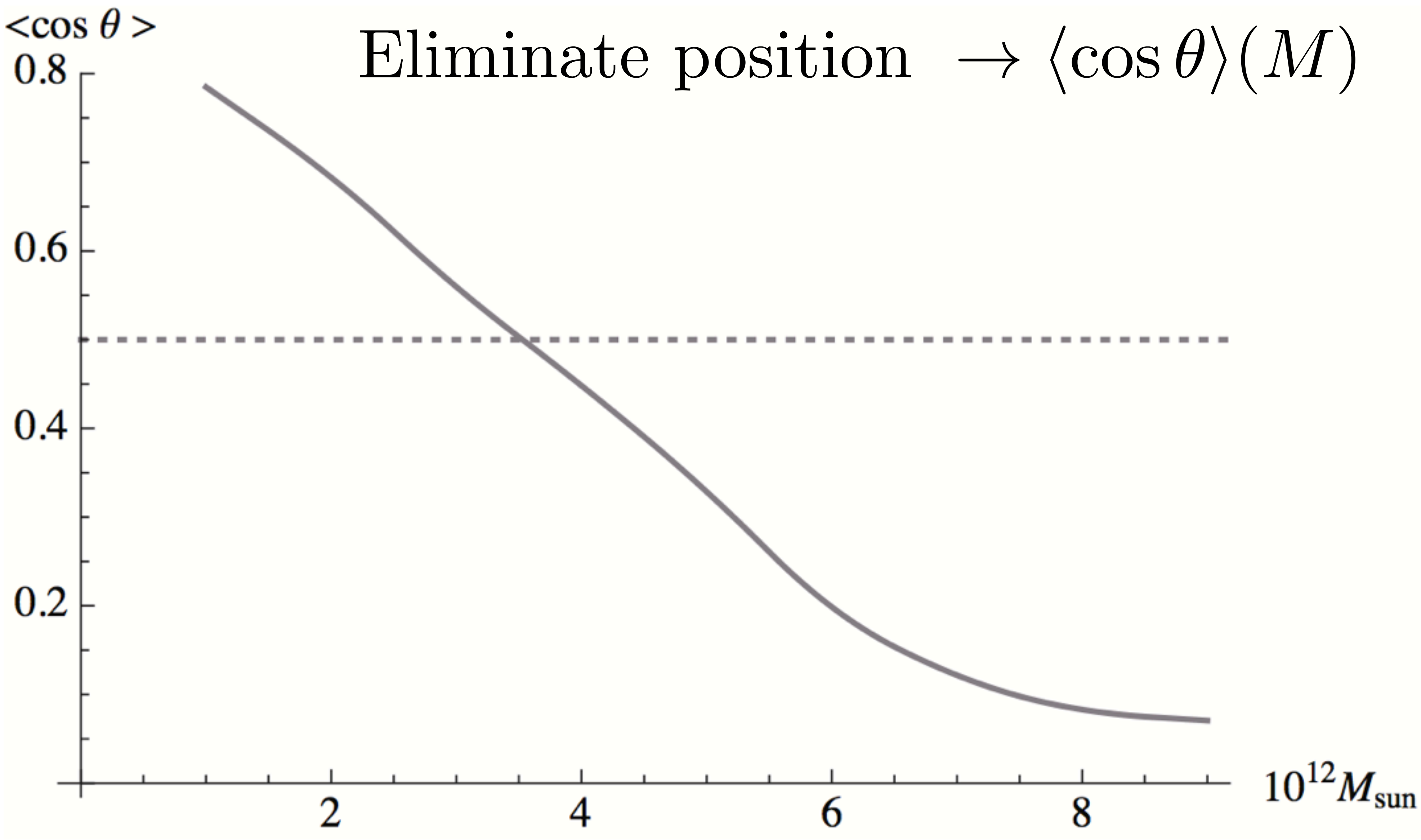
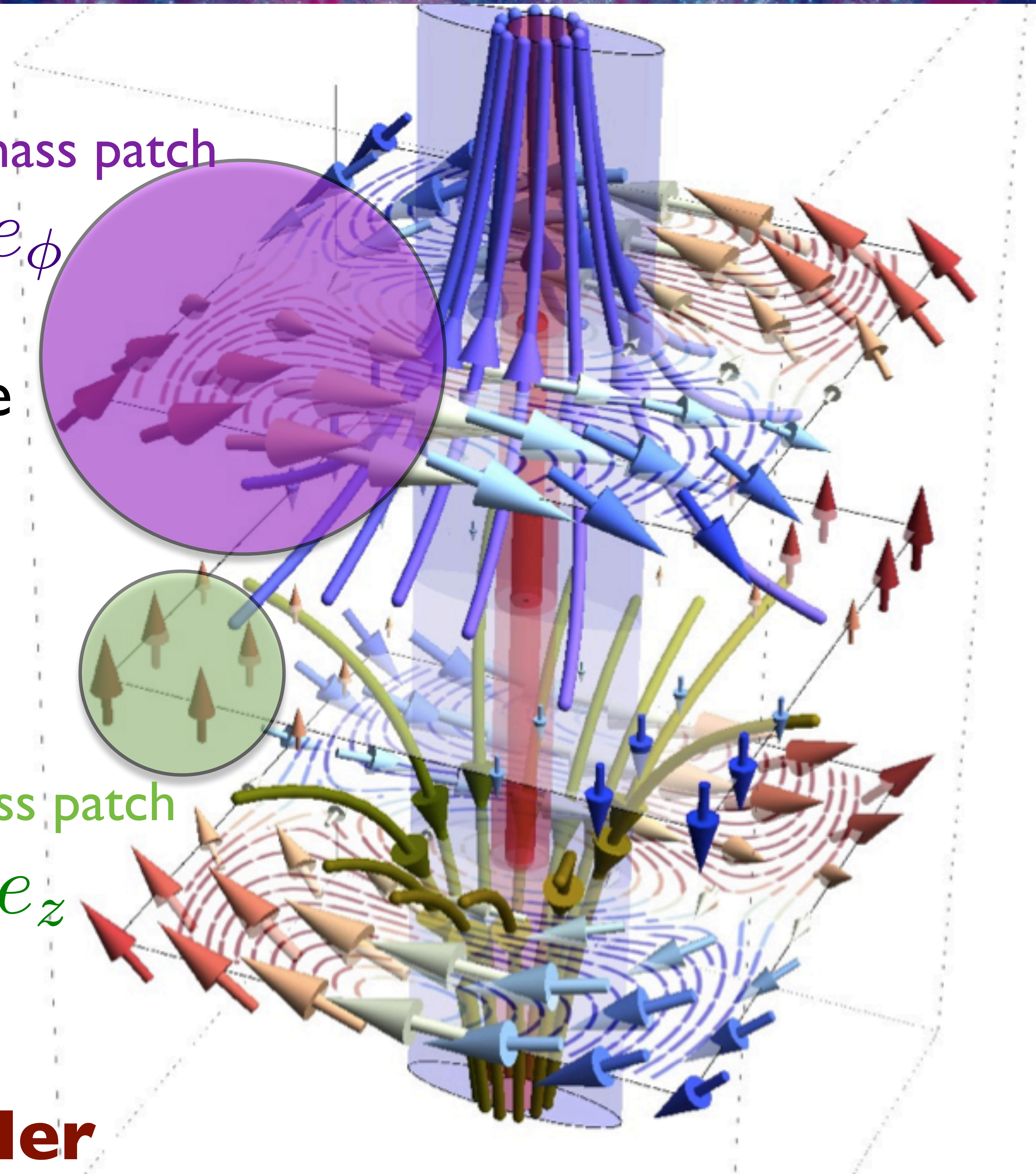


Figure 6. Mean alignment between spin and filament as a function of mass for a filament smoothing scale of 5 Mpc/h. The spin flip transition mass is around $4\,10^{12}\,M_\odot$.

geometric split ⟶ mass split

2.1 Revisiting tidal torque theory subject to CW



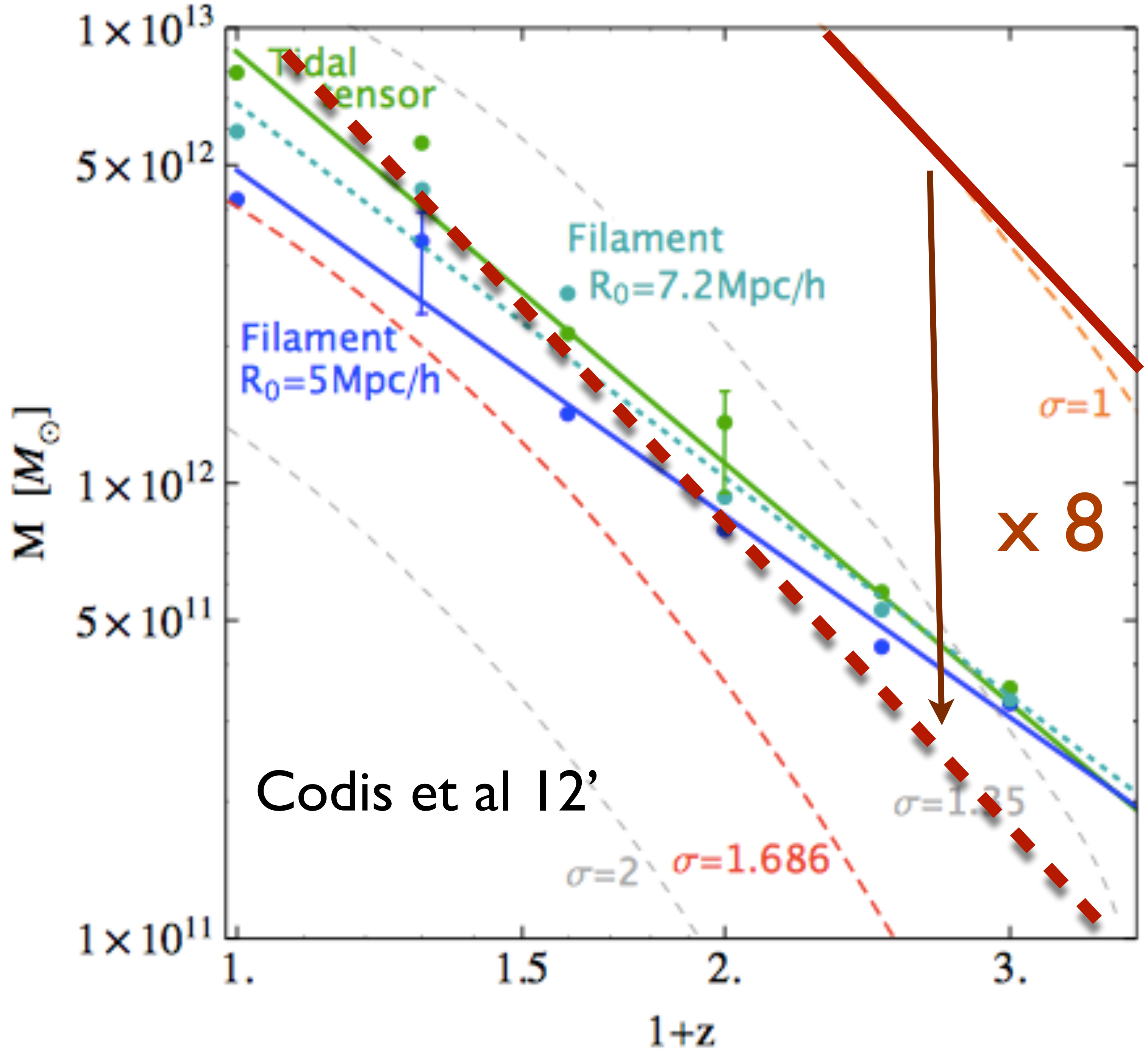
Lagrangian theory capture spin flip

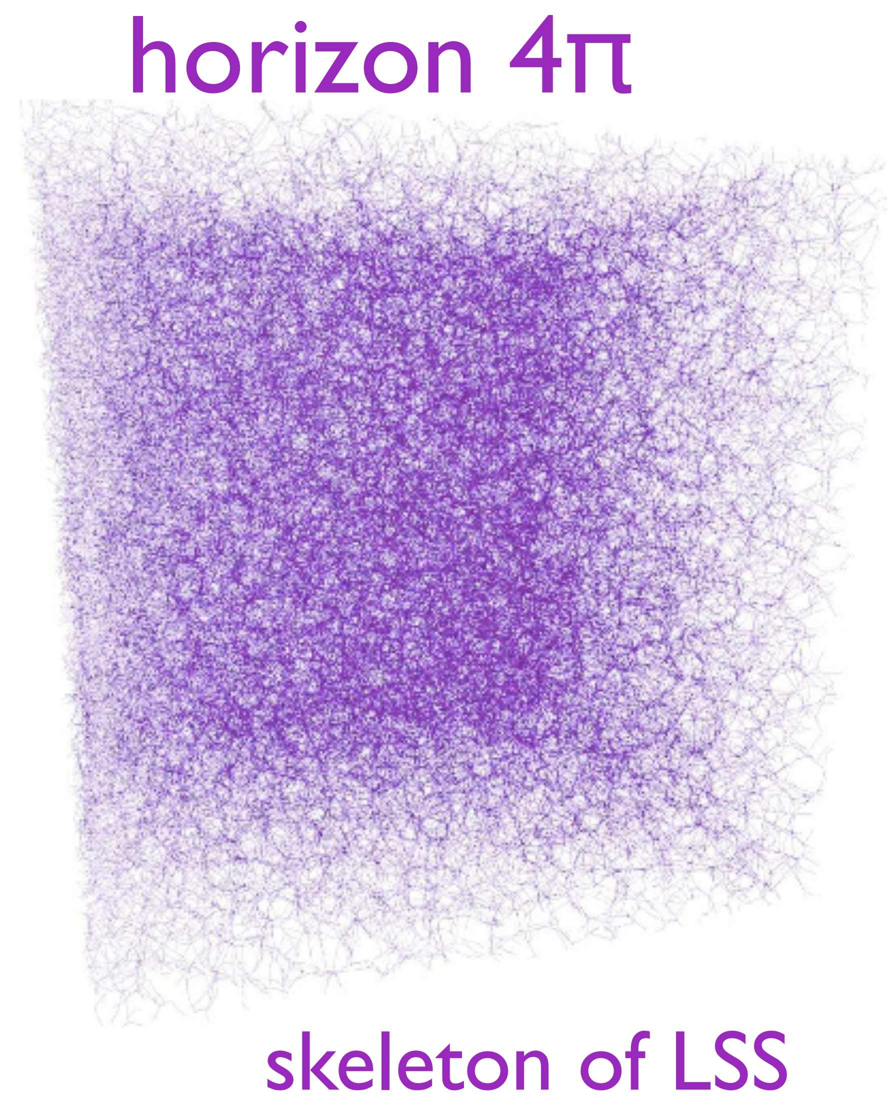
Transition mass associated with **size** of quadrant

ROI x8 smaller

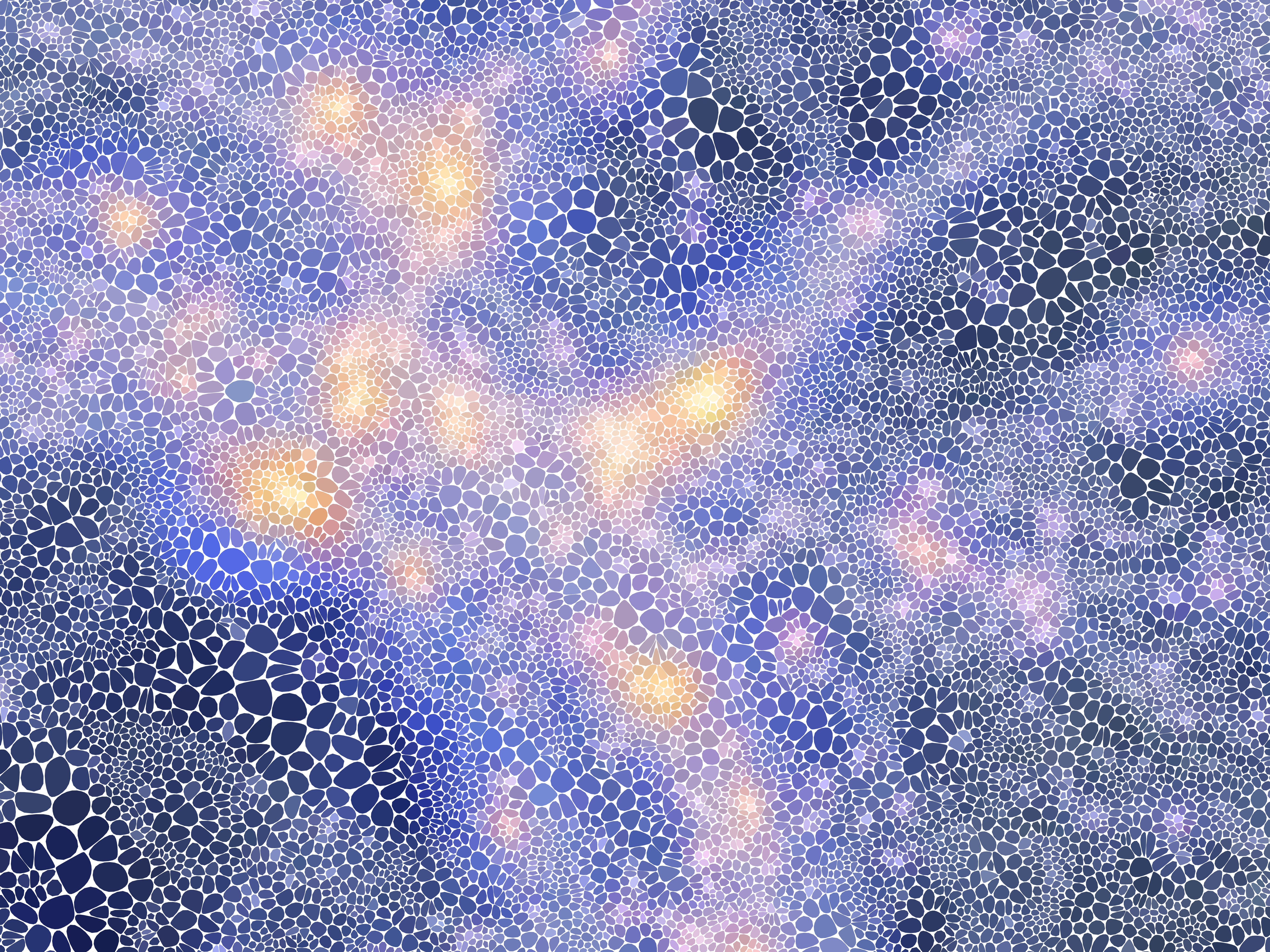
2.1 Revisiting tidal torque theory subject to CW

Only 2 ***ingredients***: a) spin is spin one b) filaments flattened





Transition mass versus redshift → intrinsic alignments

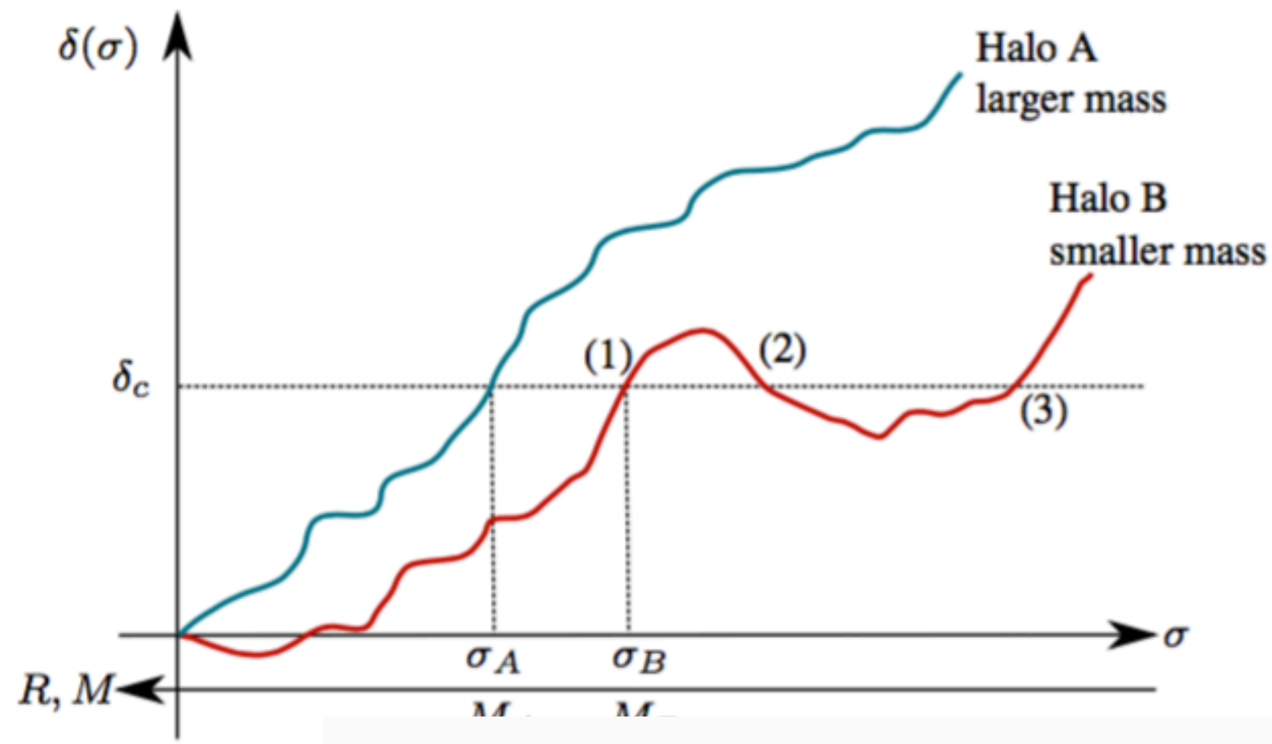


2.2 Revisiting (up-crossing) excursion set theory subject to CW

- metric changes (=biases) anisotropically the mean and variance of
  Excursion = specific signature of CW

revisit

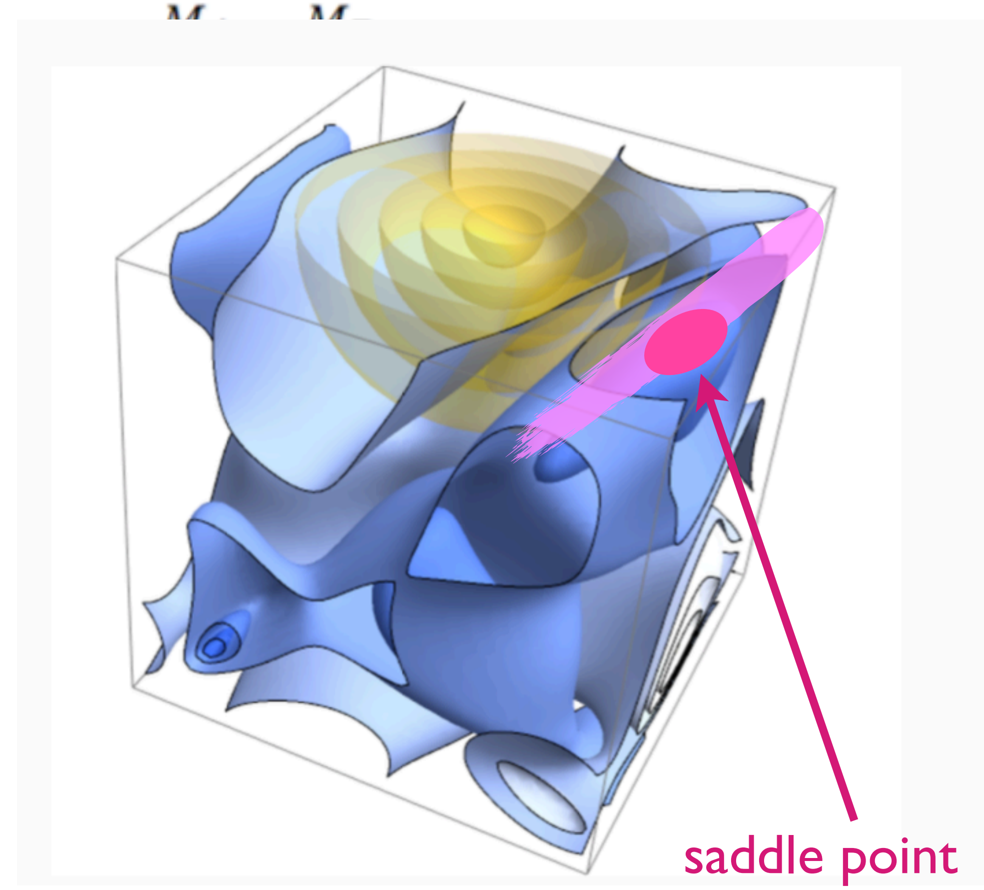
- tidal torque theory
- excursion set theory (Press Schechter)
- critical event theory
- disc settling



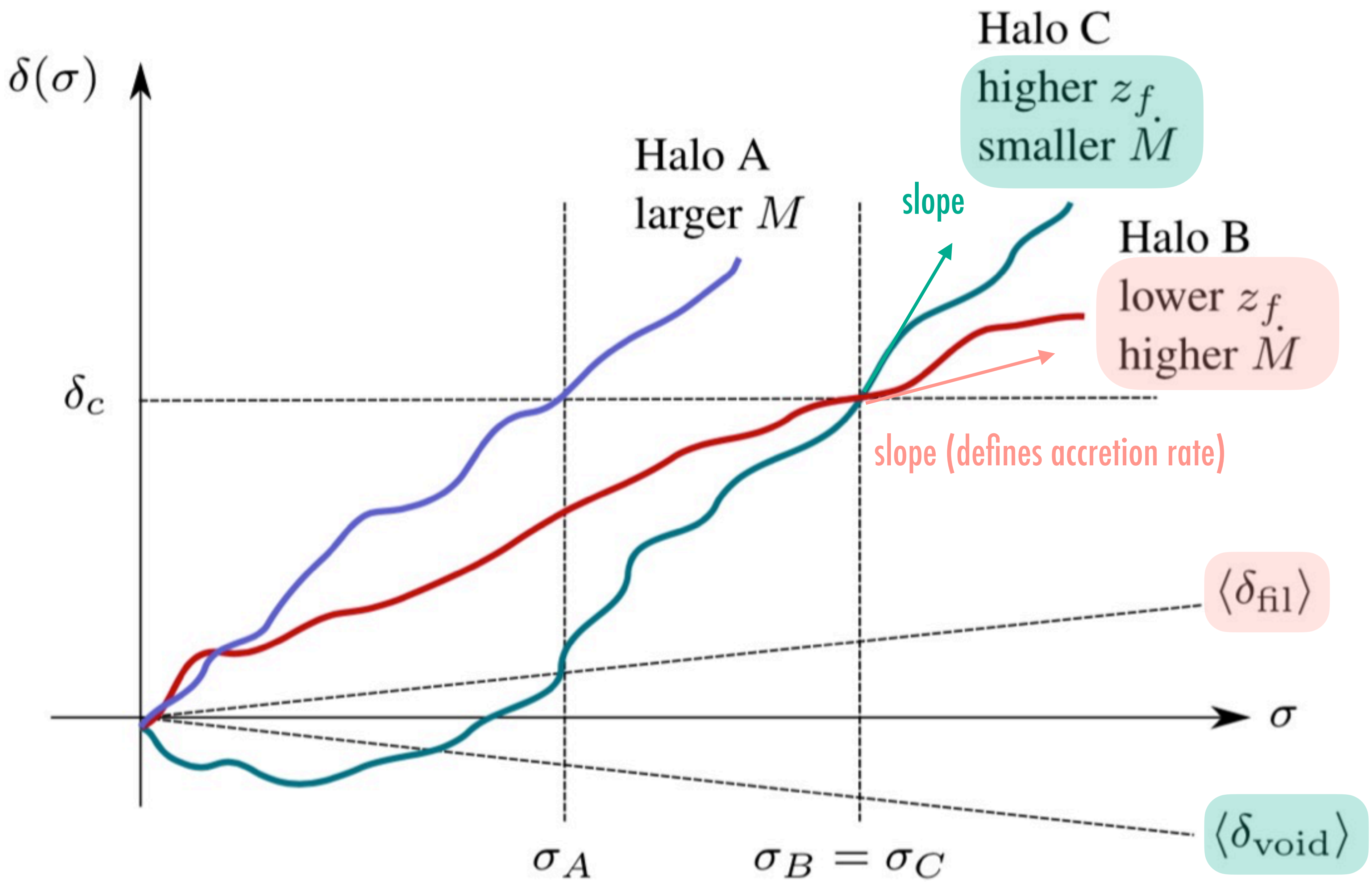
Excursion set theory quantifies barrier crossing

$$\mathcal{P}(\delta, \partial_R \delta | \mathcal{S}addle)$$

set of paths (=excursion) compatible with saddle



2.2 Revisiting (up-crossing) excursion set theory subject to CW



Halos with same mass can have different slope because of tides

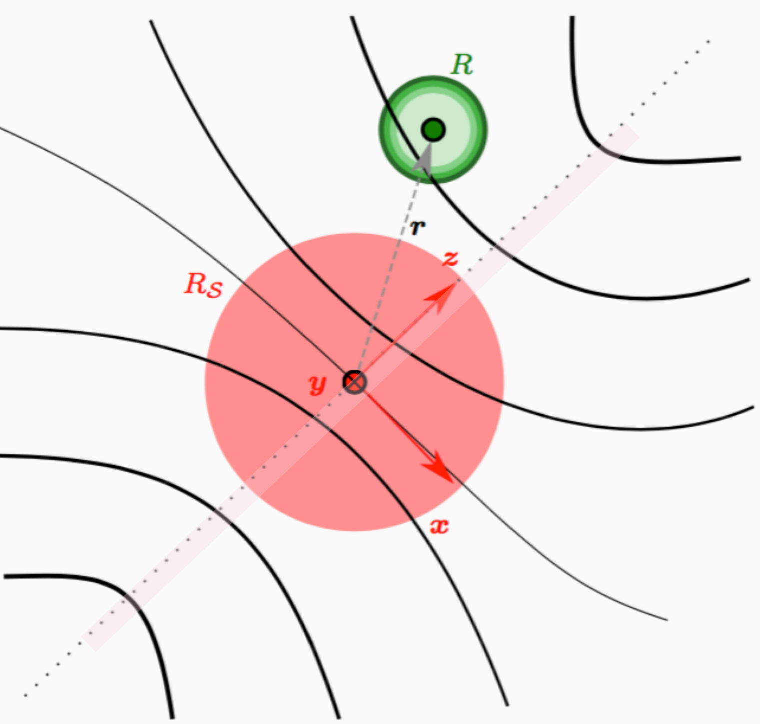
2.2 Typical mass subject to CW

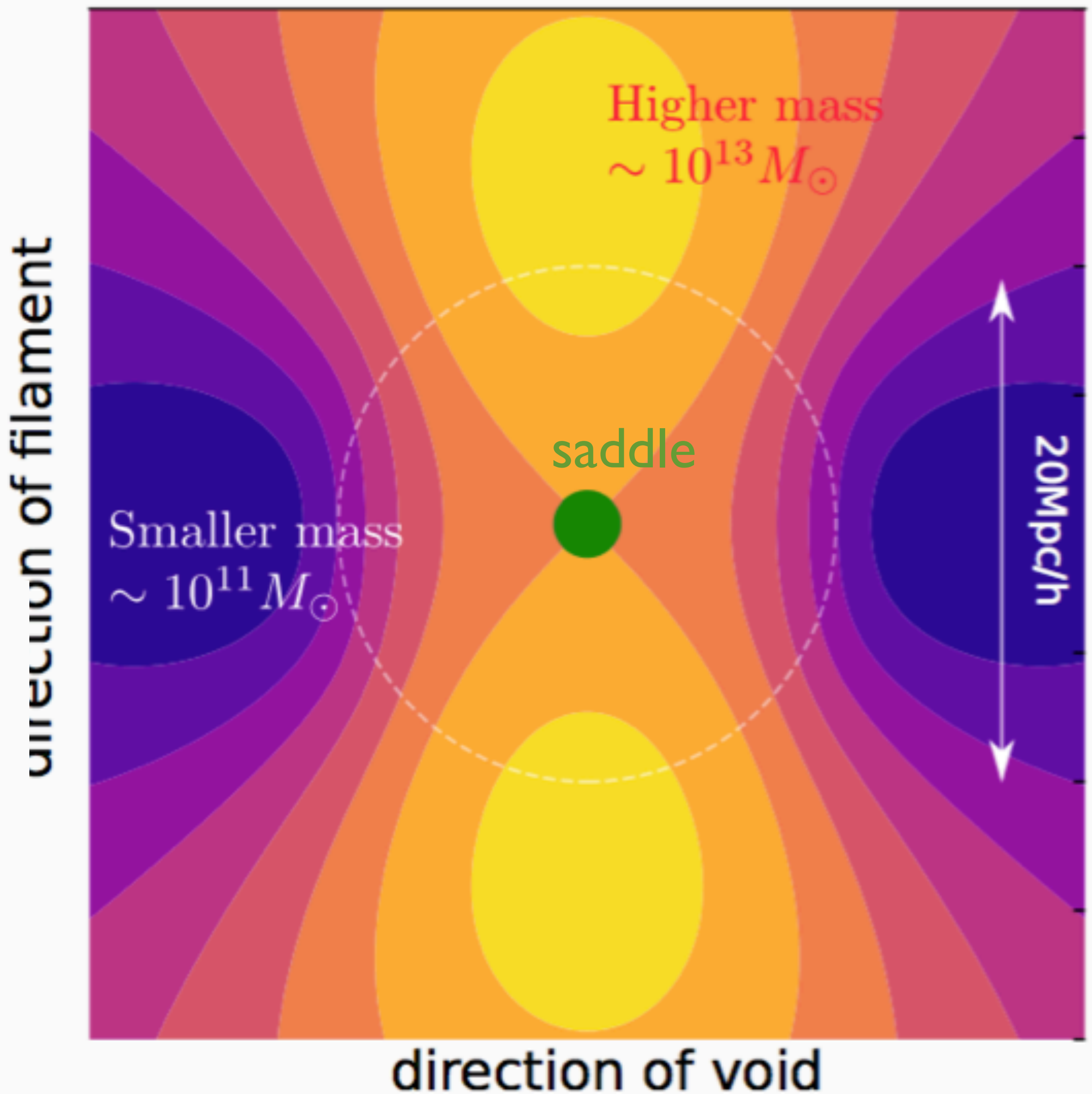
Extra degree of freedom, Q(θ,φ), provides a supplementary vector space

$$q_{ij} = \nabla_i \nabla_j \varphi$$

$$\mathcal{Q} = \sum_i \sum_j \frac{r_i \bar{q}_{ij} r_j}{\|\mathbf{r}\|^2}$$

$$\mathcal{P}(\delta, \partial_R \delta | \mathcal{S}addle)$$



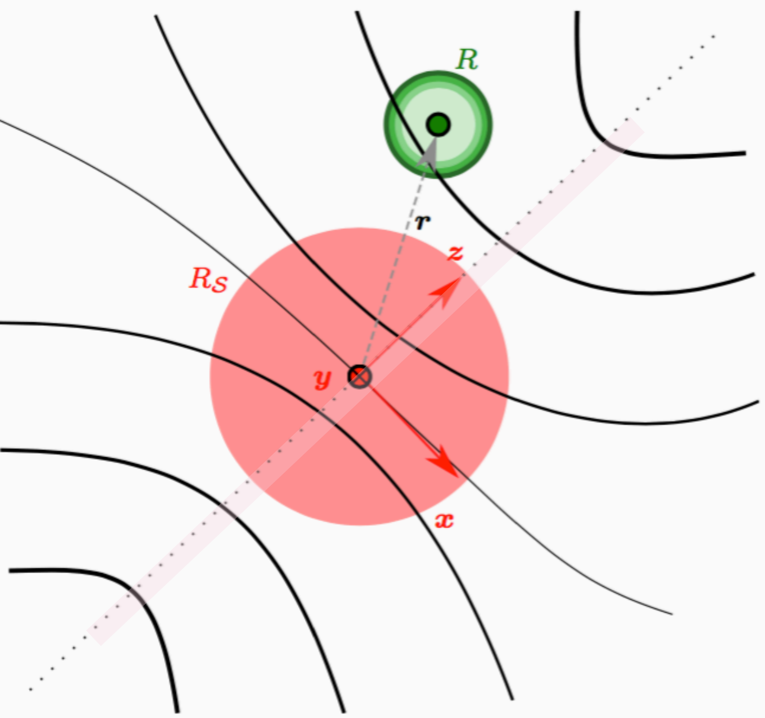


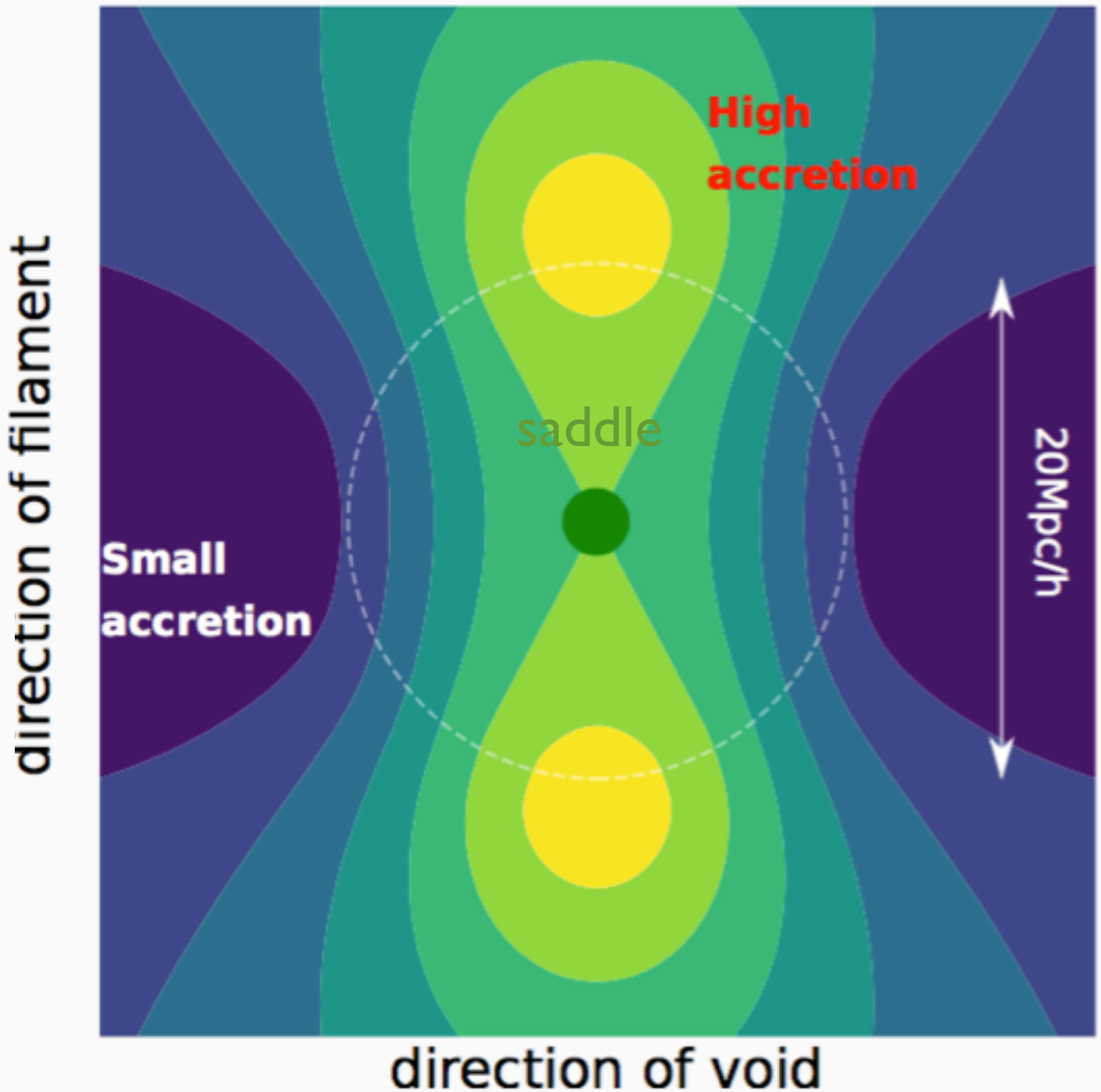
ξ_{20}: corr. density-tide + density

$$\Delta M_\star(\mathbf{r}) \propto \delta_{\mathcal{S}} \xi_{20}(r) \mathcal{Q}$$

2.2 typical accretion rate subject to CW

$$\mathcal{P}(\delta, \partial_R \delta | \mathcal{S}addle)$$

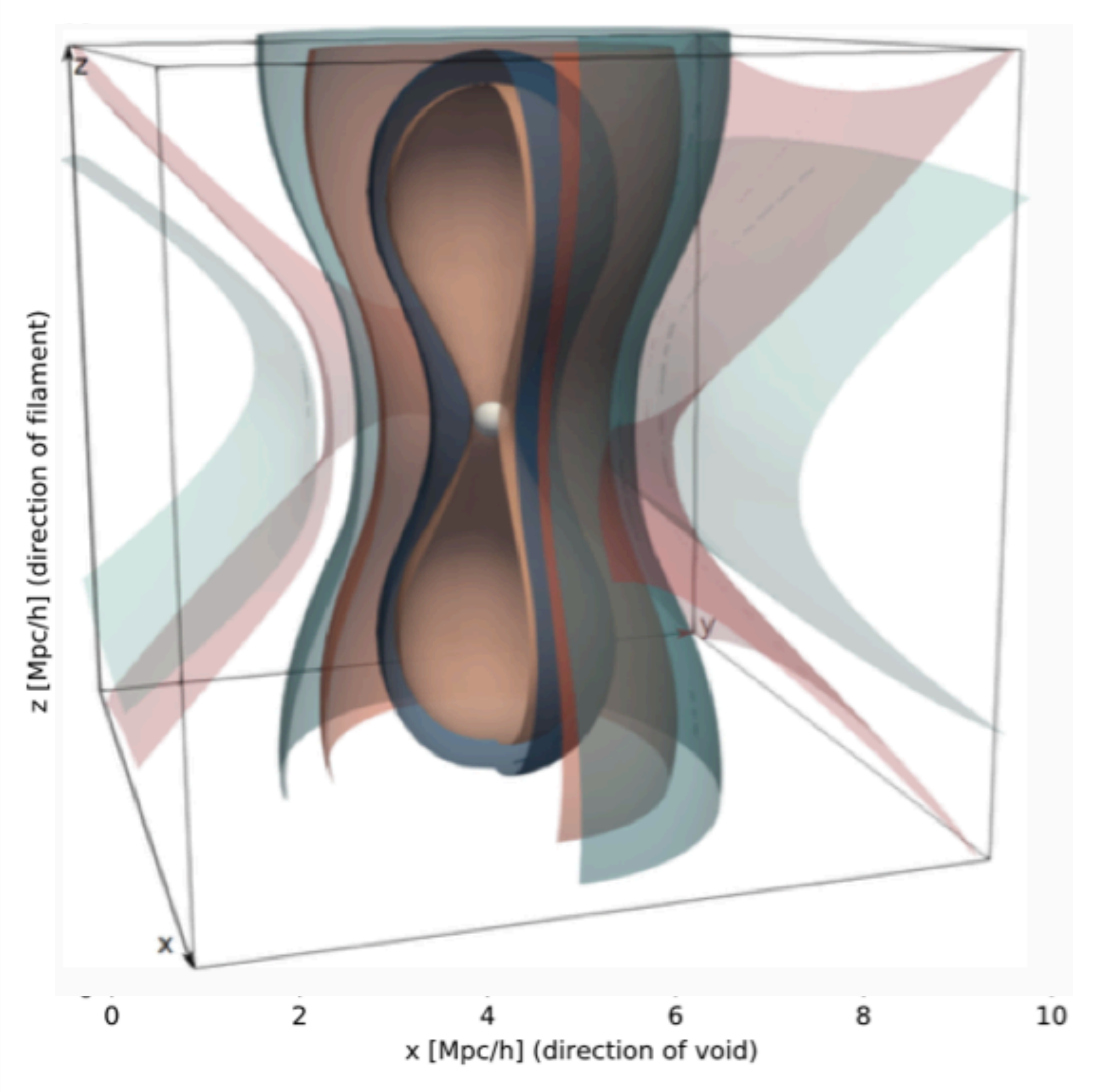




$$\Delta \dot{M}(\mathbf{r}) \propto \left[\xi'_{20} - \frac{\sigma - \xi'_l \xi_l}{\sigma^2 - \xi^2} \xi_{20} \right] \mathcal{Q}$$

ξ'_{20}: corr. slope-tide + variance of field

2.2 Revisiting (up-crossing) excursion set theory subject to CW



cross product of normals

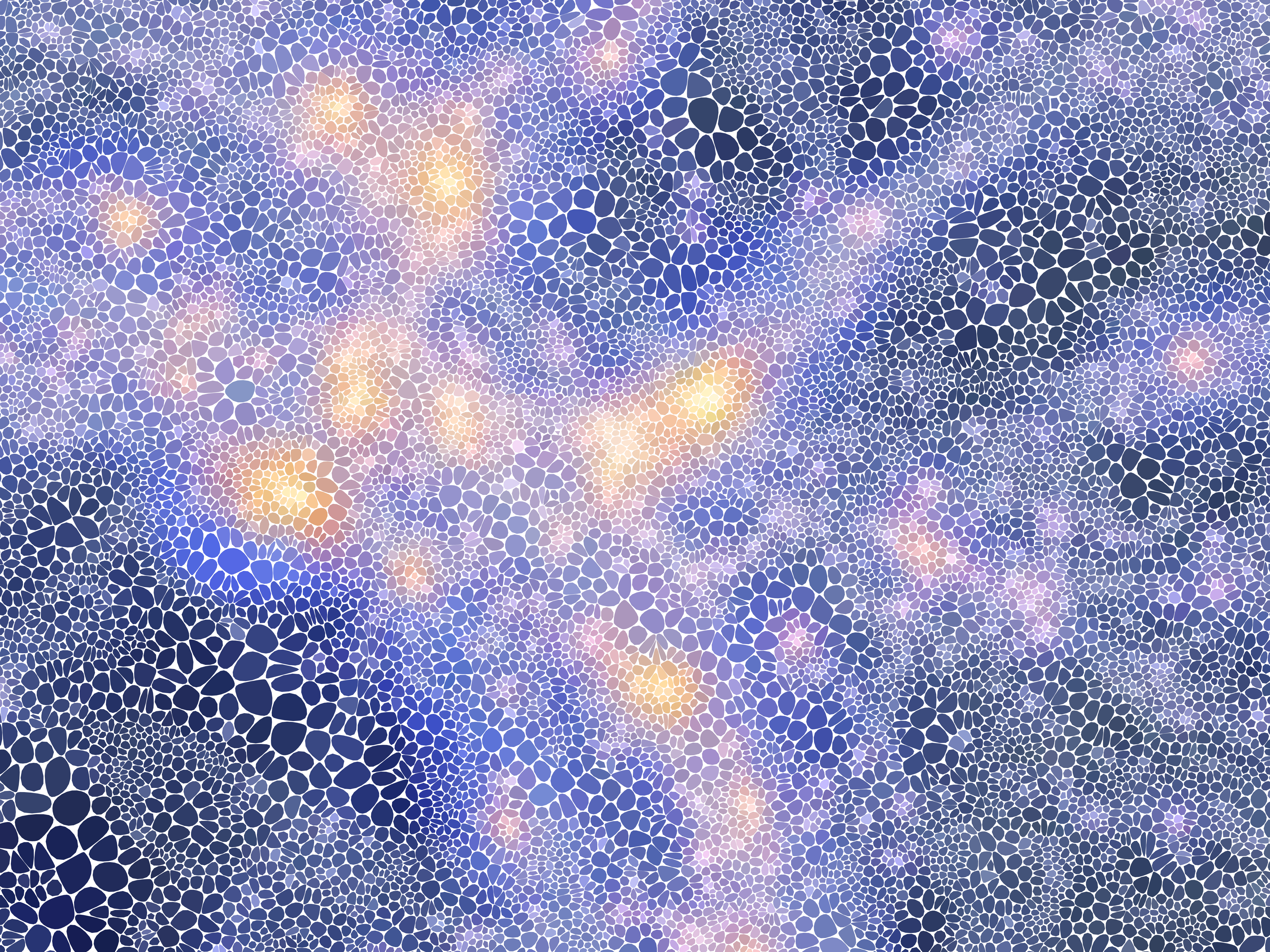
$$\left(\frac{\partial \dot{M}_\star}{\partial r}\frac{\partial M_\star}{\partial \mathcal{Q}} - \frac{\partial \dot{M}_\star}{\partial \mathcal{Q}}\frac{\partial M_\star}{\partial r}\right)\tilde{\nabla}\mathcal{Q}$$

- background: ρ
- dotted M
- dashed $\dot{M}$

$\Rightarrow$ different gradients

accretion rate is not a function of mass and density alone

applies also to formation time, concentration (?), kinetic anisotropy...



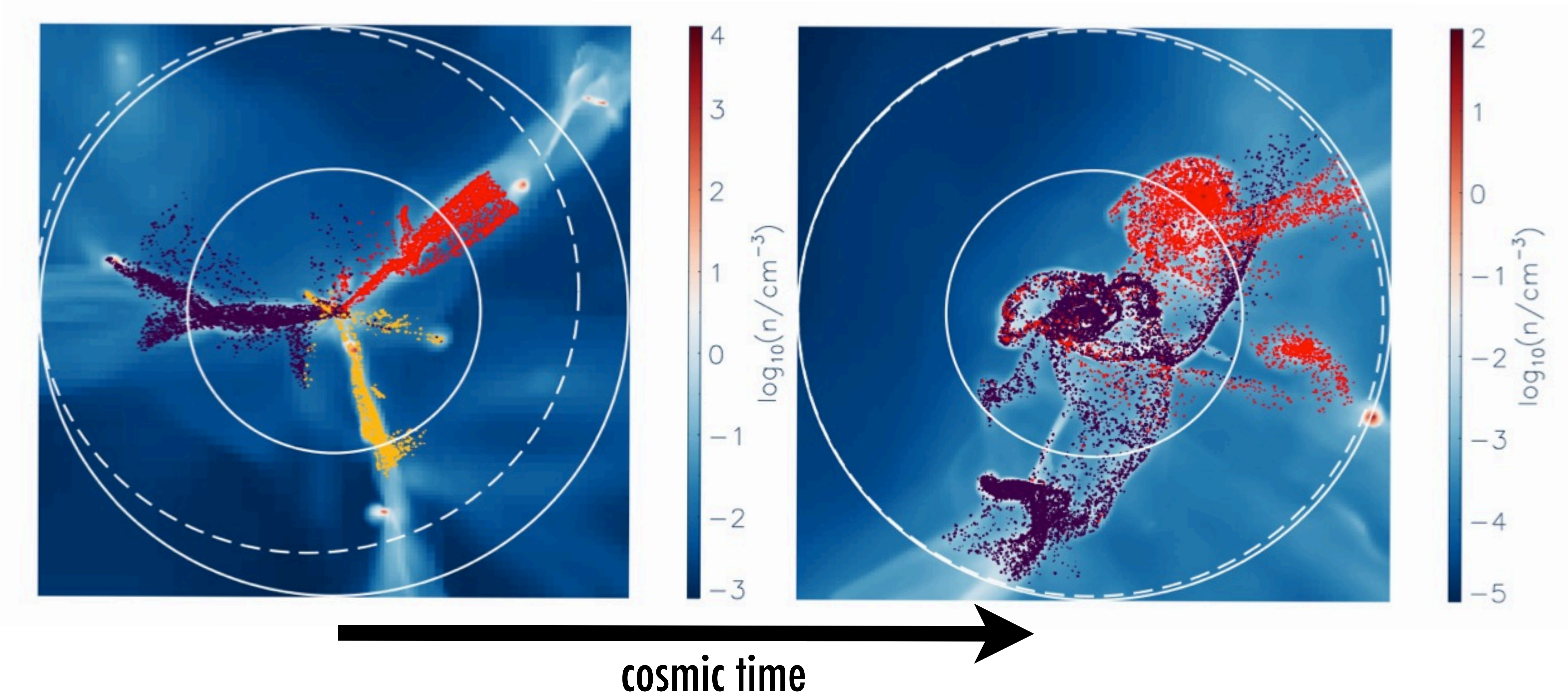
2.3 Critical events:Galactic motivation

revisit

- tidal torque theory
- excursion set theory
- critical event theory
- disc settling

- metric changes (=biases) anisotropically the mean and variance of mergers = specific signature of CW

filament disconnect = cold gas inflow truncation



2.3 Synopsis of merger events

What happens to neighbouring critical pts?

Peak merger

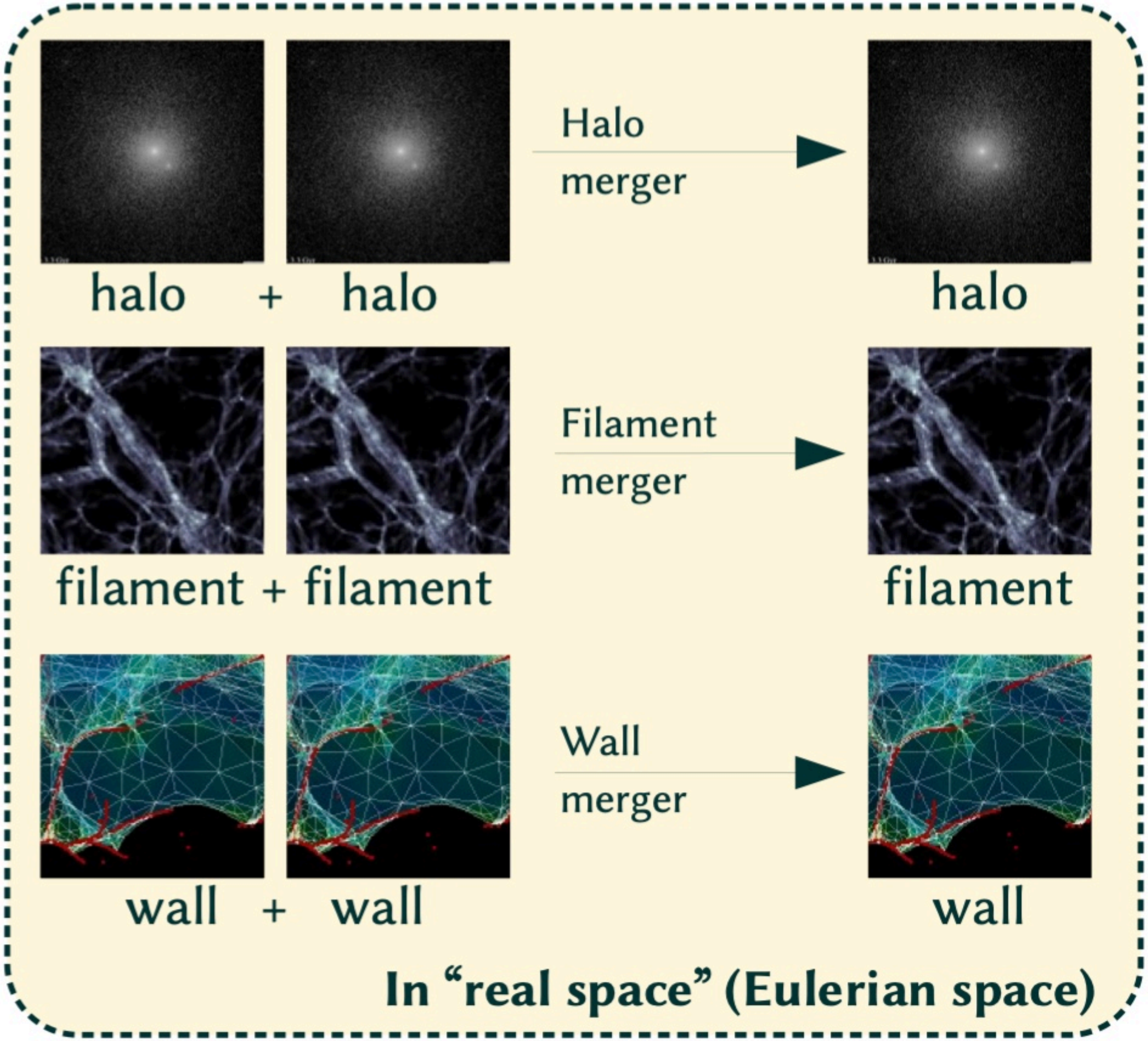
Filament vanishing

Filament merger

Wall vanishing

Wall merger

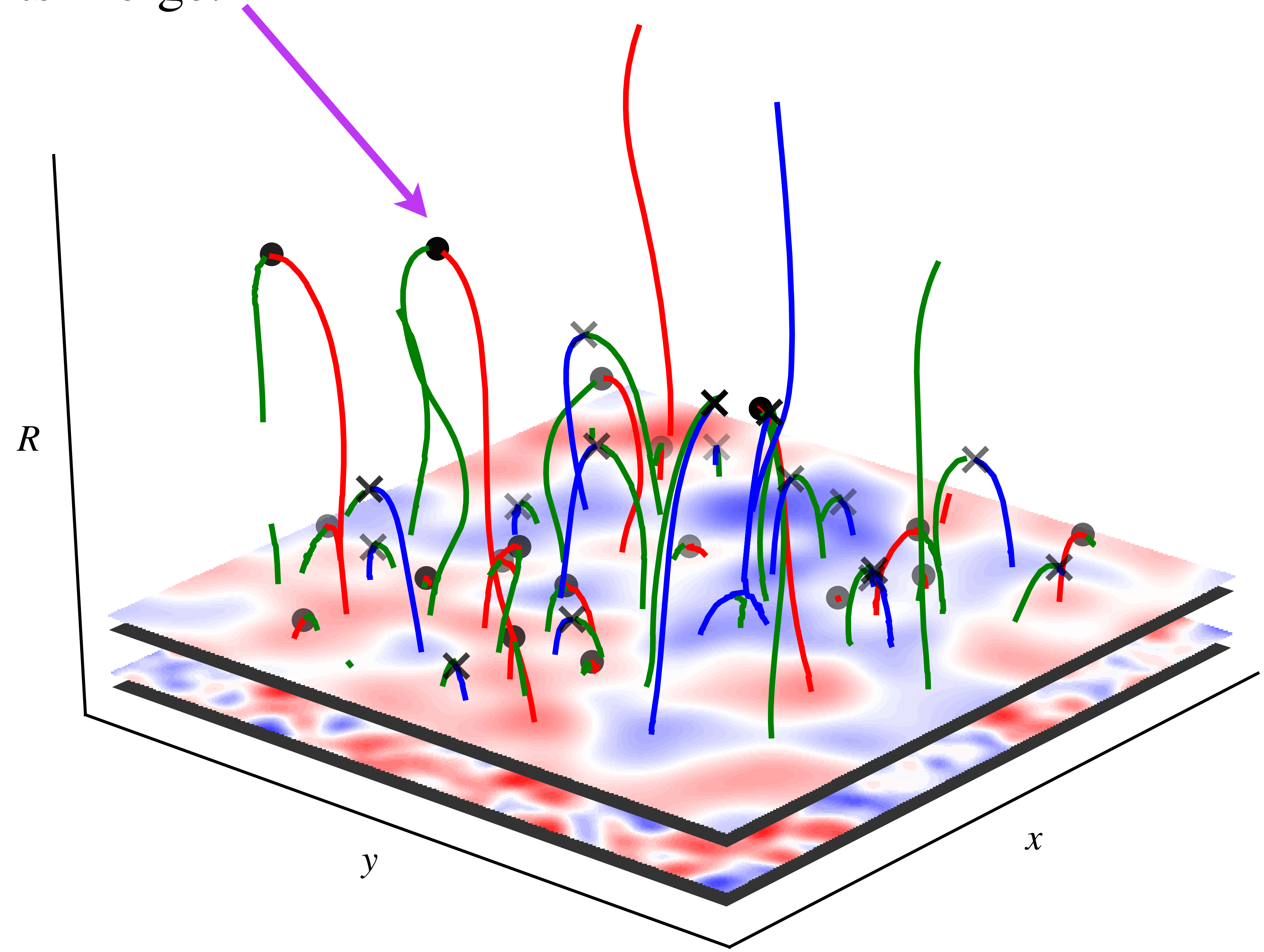
Void vanishing



2.3 Critical event PDF: formal definition

$$\frac{\partial^2 \mathcal{N}}{\partial r^3 \partial R} \equiv \langle \delta_{\rm D}^{(3)}(\mathbf{r}-\mathbf{r}_0)\delta_{\rm D}(R-R_0)\rangle \,,$$

where $\mathbf{r}_0$ is a (double) critical point in real space and R_0 the scale at which the two critical points merge.

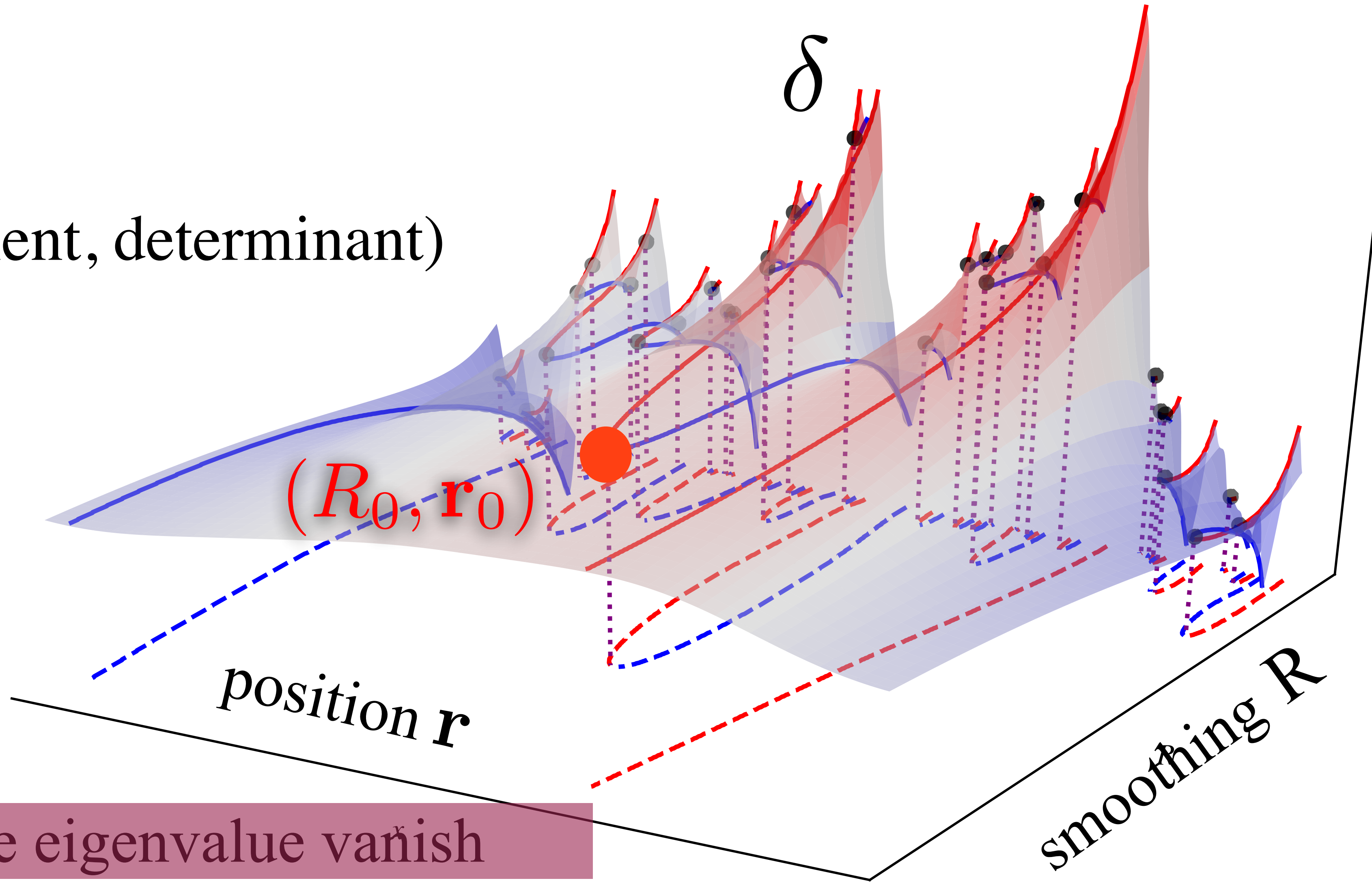


2.4 Critical events PDF: Derivation

$$\frac{\partial^2 \mathcal{N}}{\partial r^3 \partial R} \equiv \langle \delta_{\rm D}^{(3)}(\mathbf{r}-\mathbf{r}_0)\delta_{\rm D}(R-R_0)\rangle \,,$$

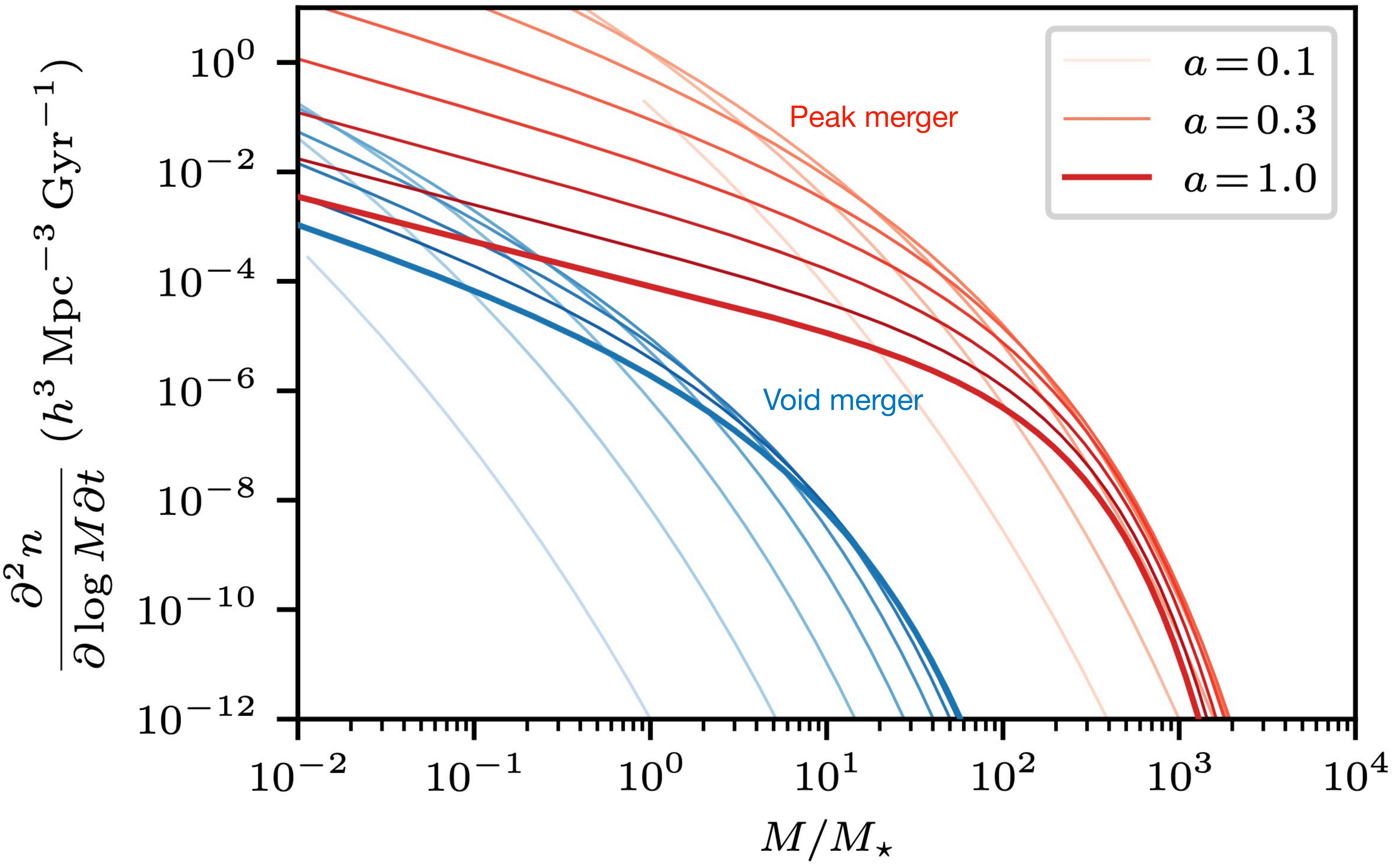
where $\mathbf{r}_0$ is a (double) critical point in real space and R_0 the scale at which the two critical points merge.

- ✓ Invoque ergodicity
- ✓ Change variable to (gradient, determinant)

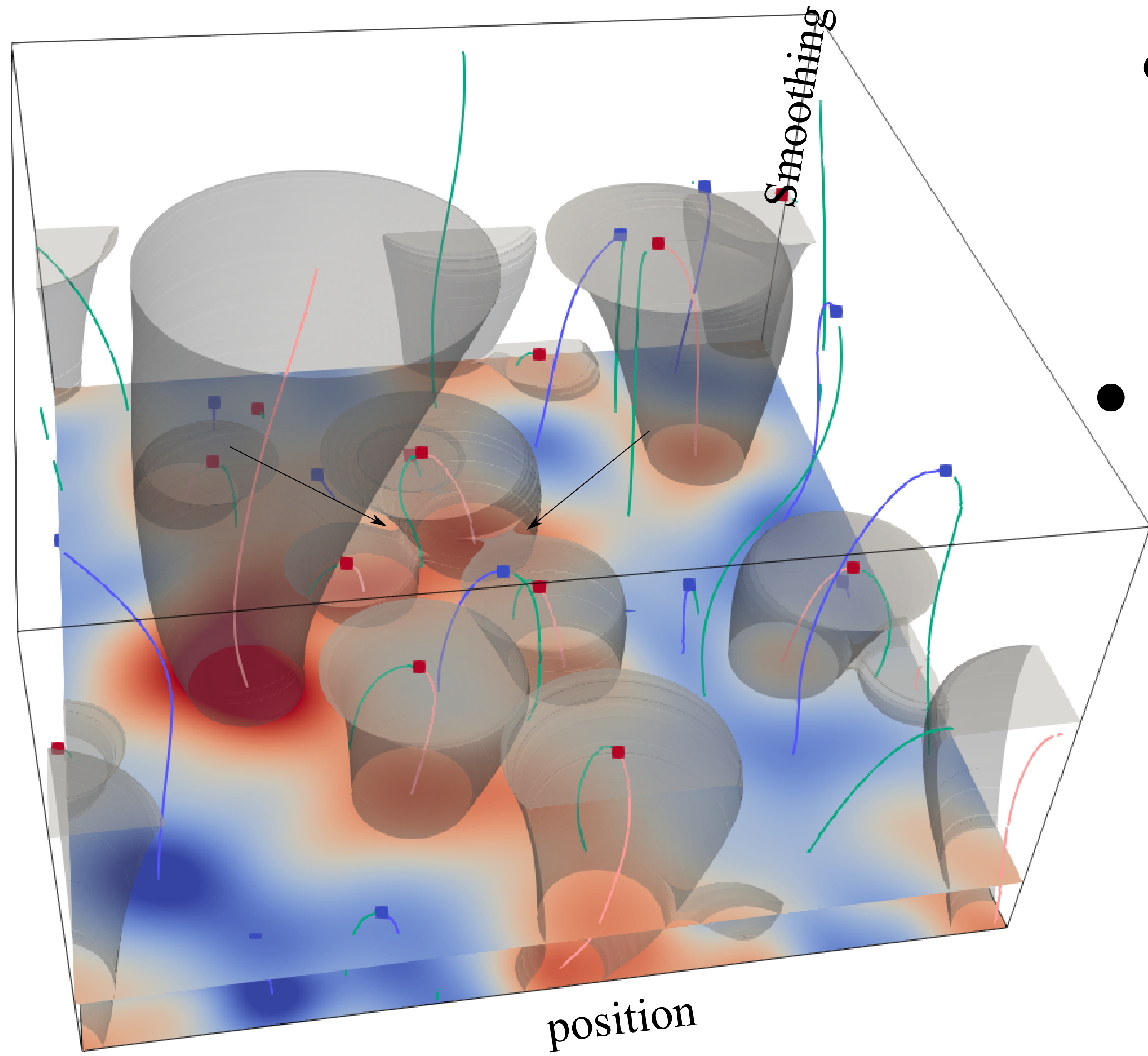


At $(R_0, \mathbf{r}_0)$ gradient and one eigenvalue vanish

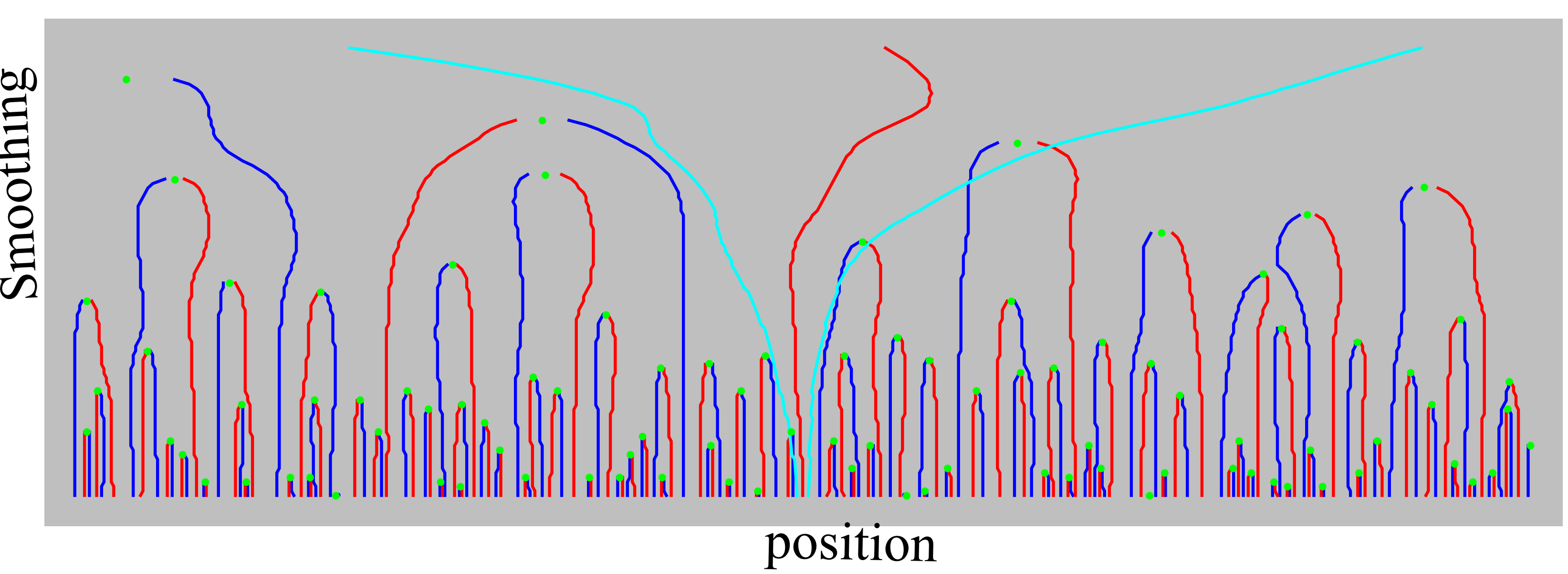
2.3 merger event function



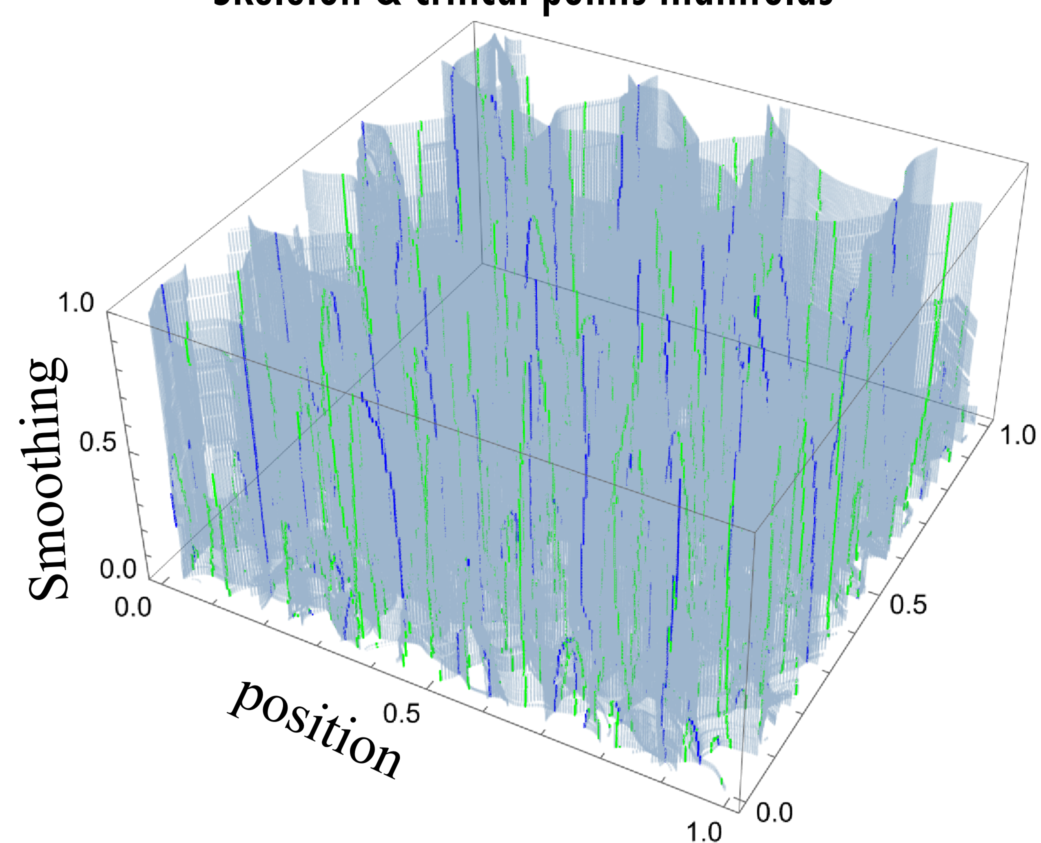
2.4 Critical events within lightcone: AM & number count

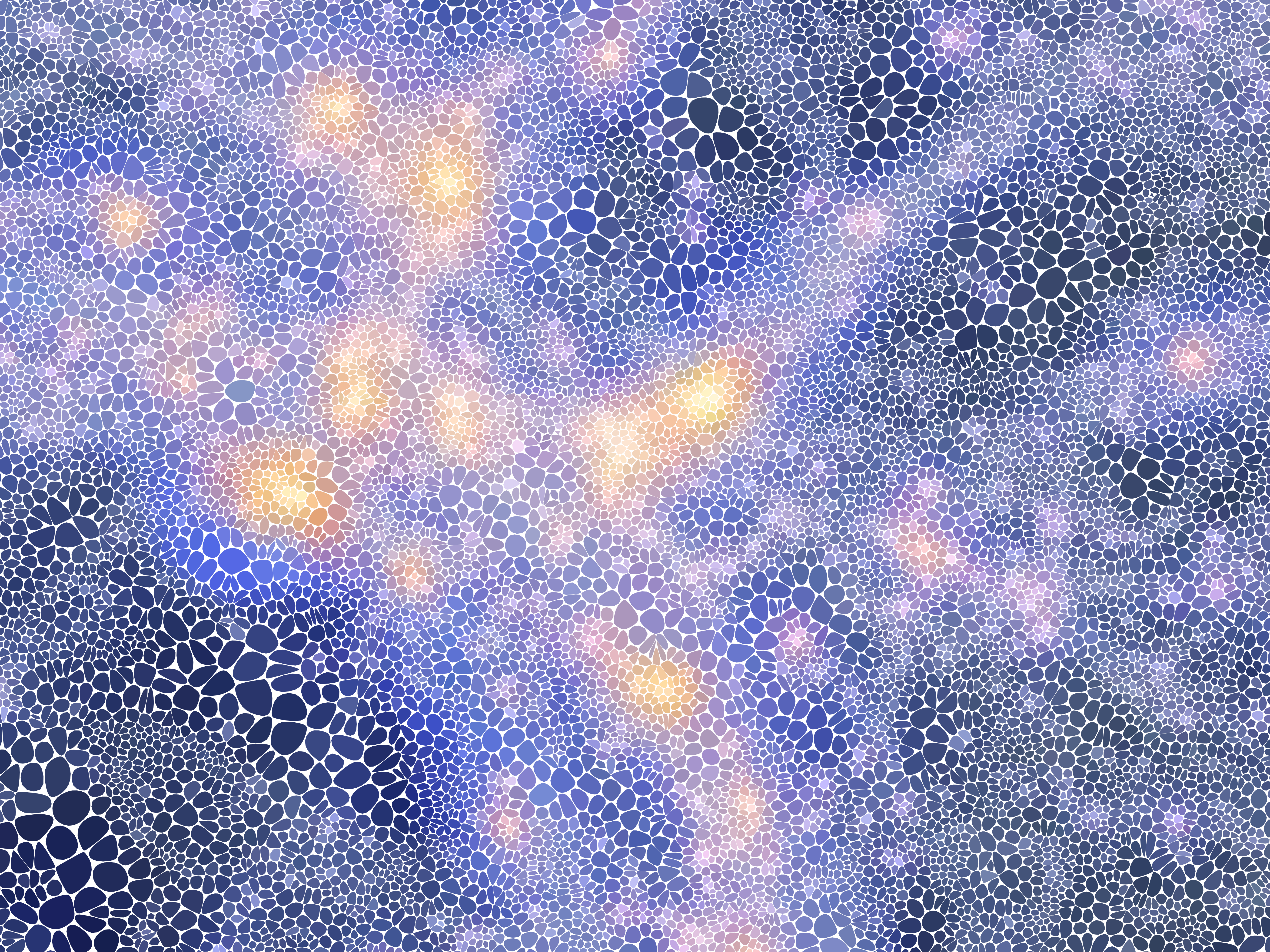


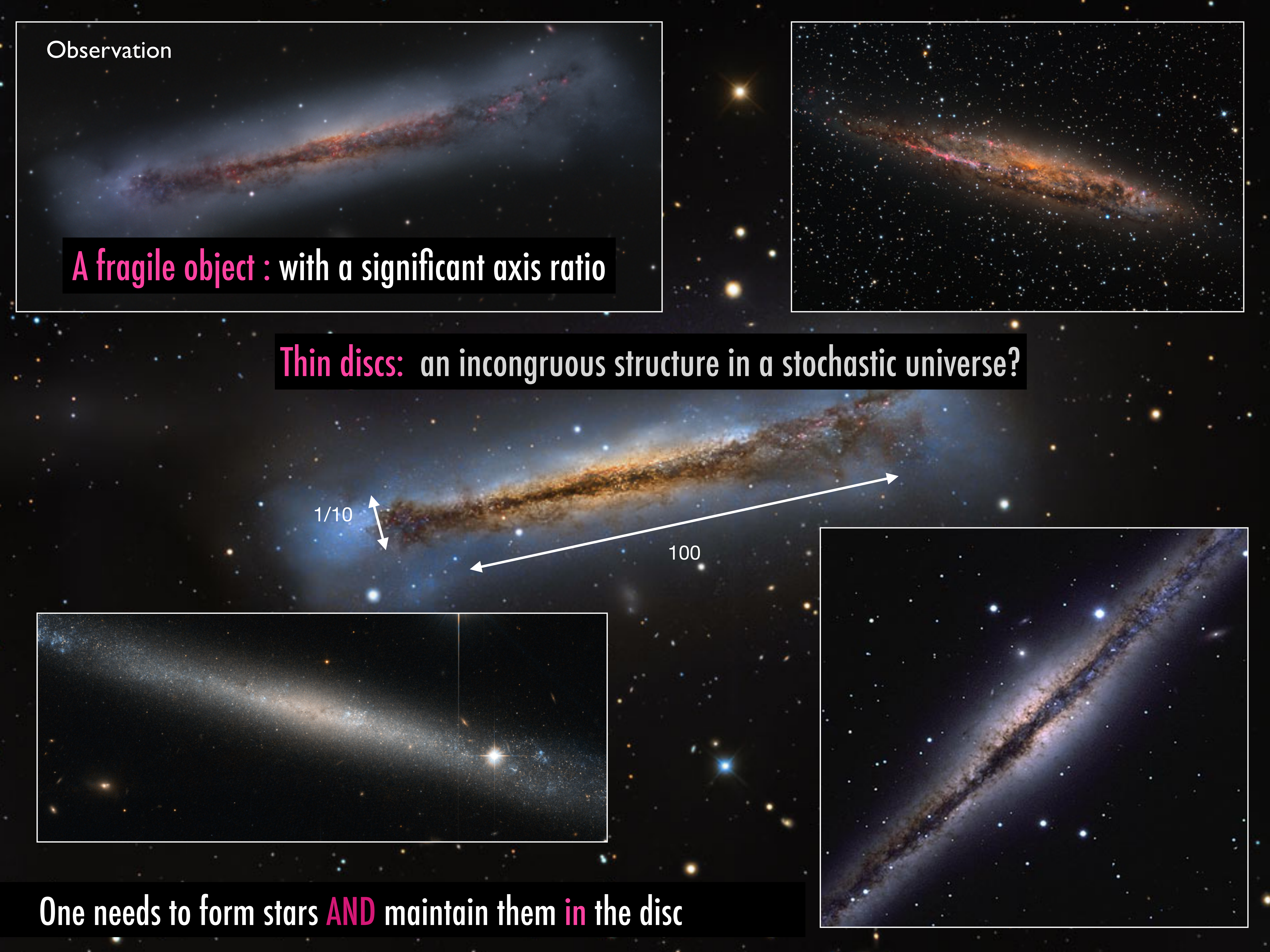
- number of critical event within "past lightcone" of peak defines typical number of mergers ~ 4
- Orientation of saddle frame at surrounding event defines a proxy for the angular momentum of mergers.



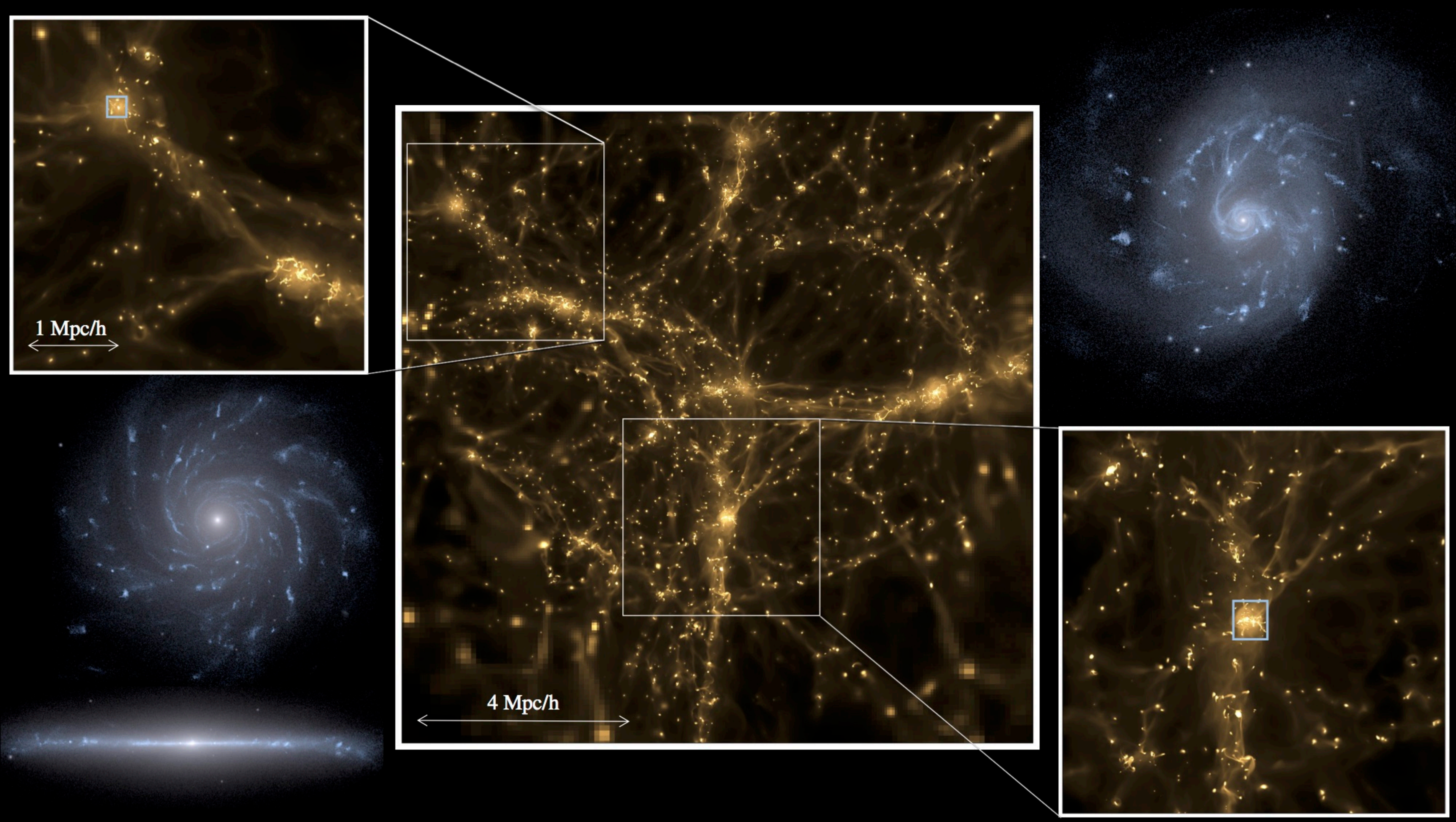
Skeleton & critical points manifolds



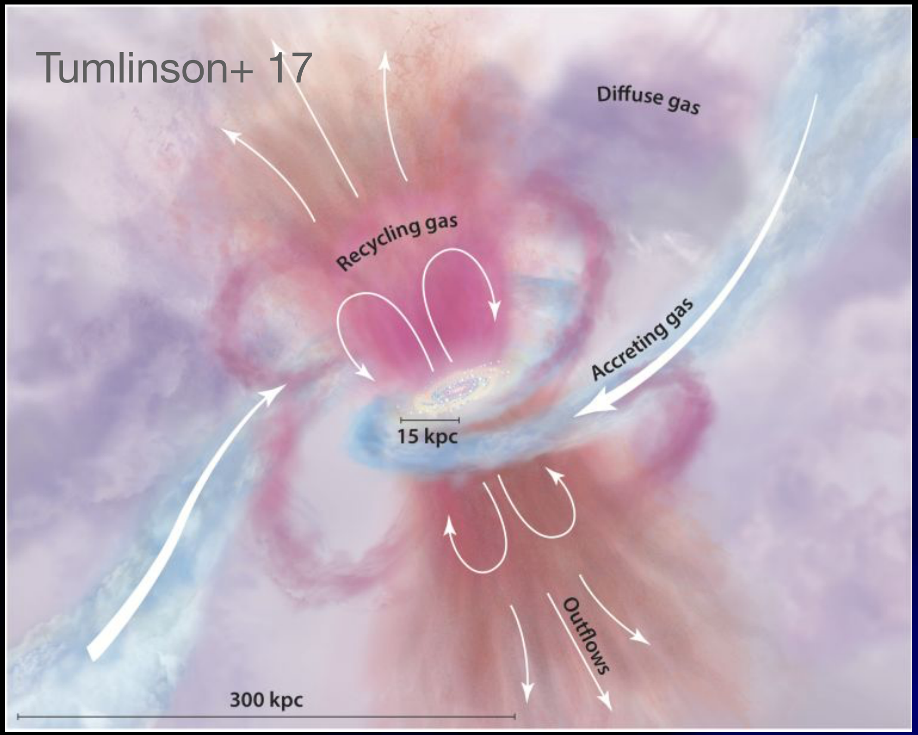



Observation
A fragile object : with a significant axis ratio
Thin discs: an incongruous structure in a stochastic universe?
1/10
100
One needs to form stars AND maintain them in the disc

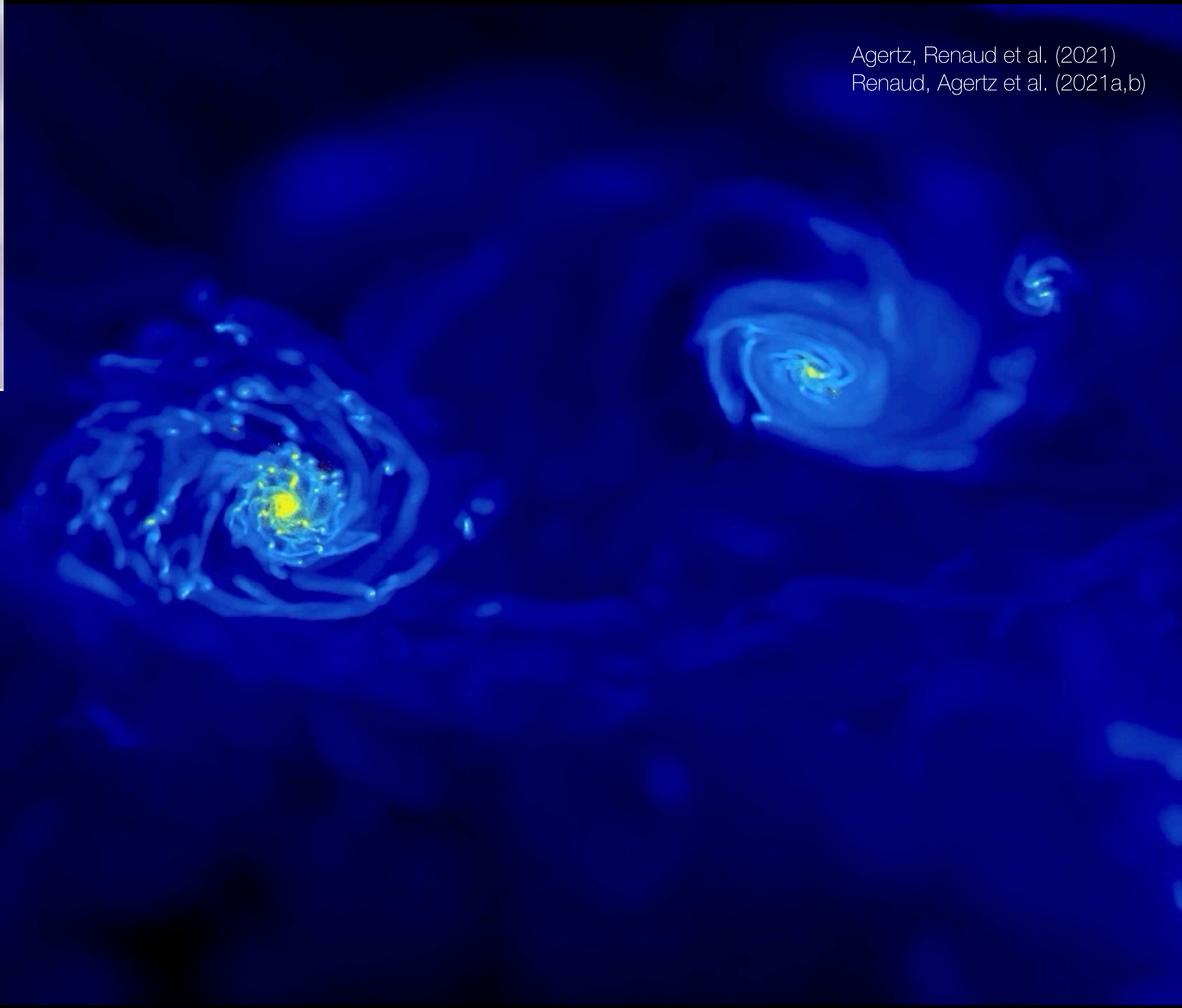
Cosmological simulations produce thin discs



New Horizon Simulation



Disc torqued by GCM

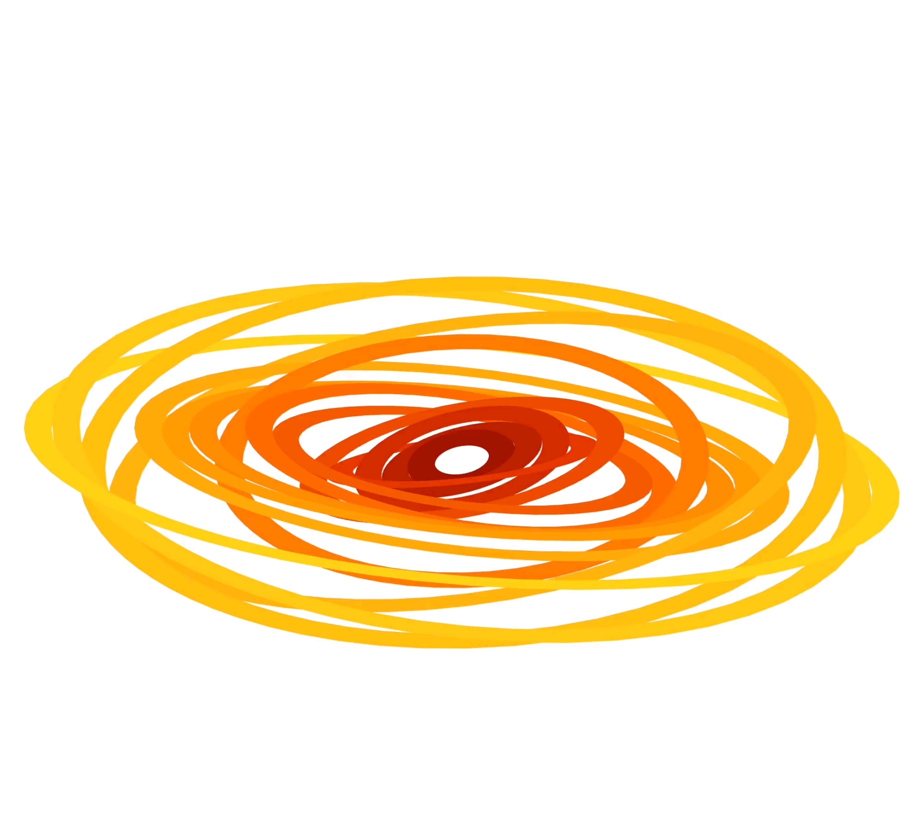


Cosmic web sets up
reservoir of free energy in CGM = the fuel for thin disc emergence

Synopsis of thin disc emergence:

- Why do disc settle ? Because Q→1
- But Why does Q→1? Because tighter control loop ($t_{\rm dyn} \ll 1$) via wake
- But how does it impact settling? Because wake also stiffens coupling

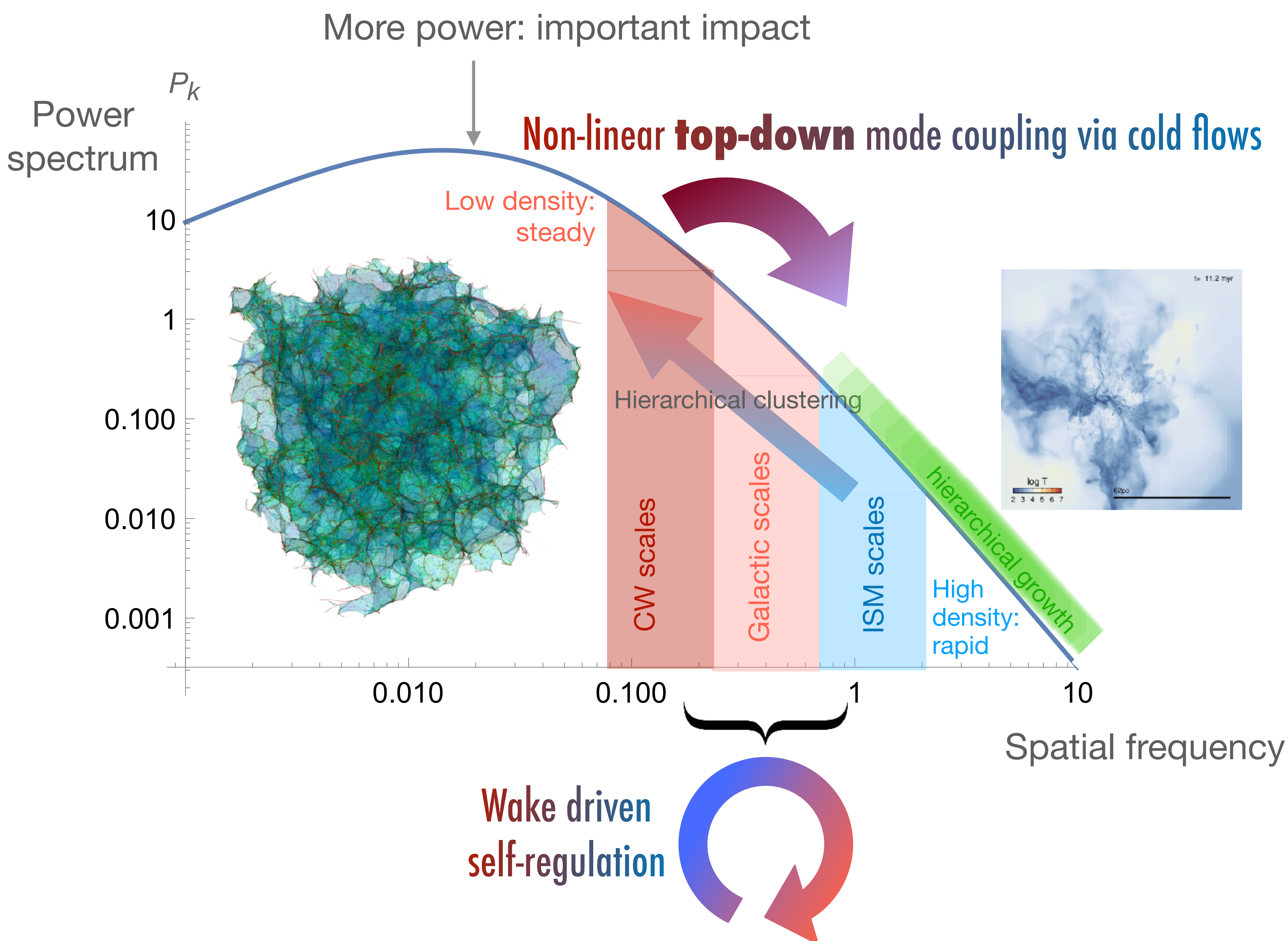
New Horizon



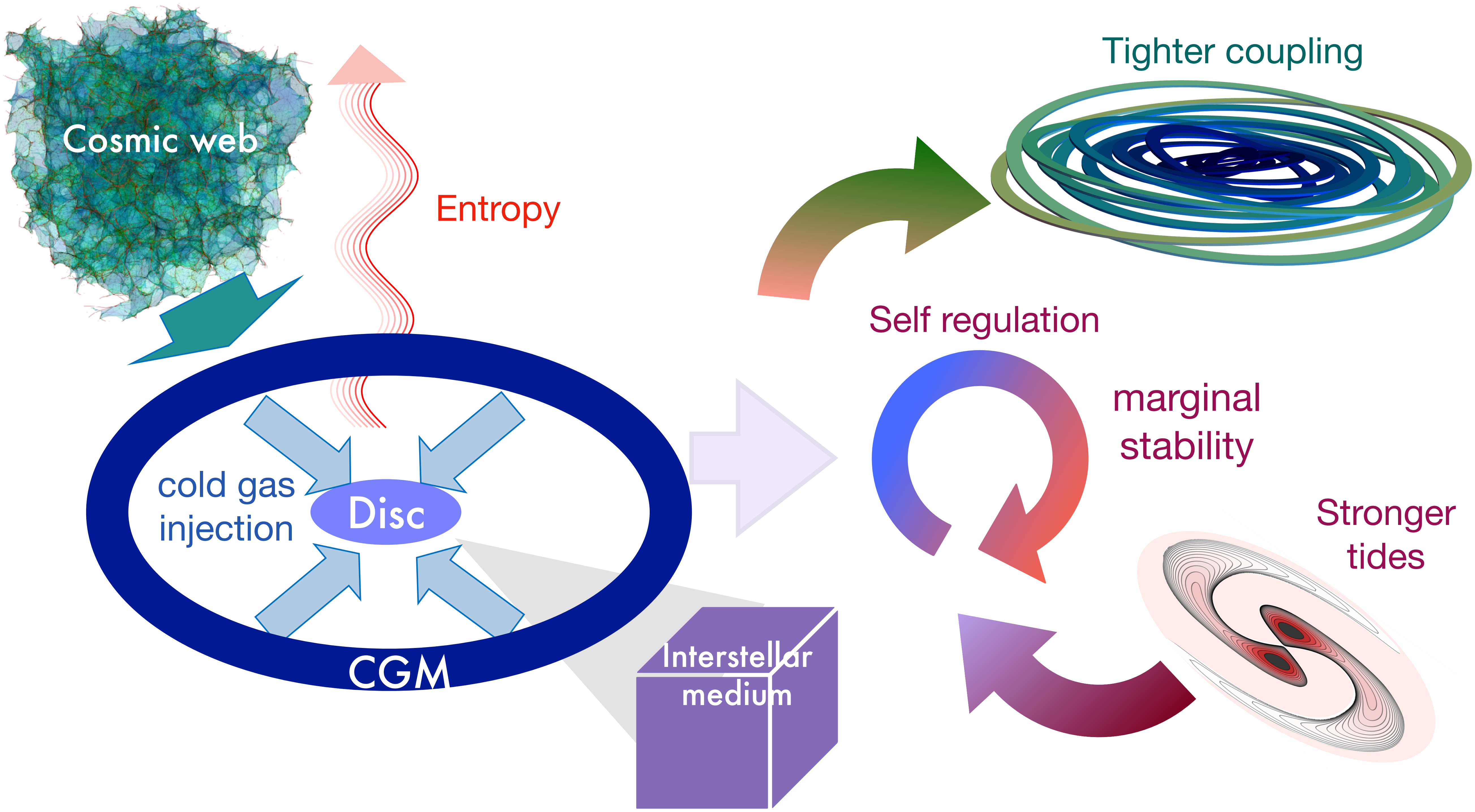
Ring toy model

1. Impact of CW on homeostatic thin disc

On galactic scales, the Shape of initial P_k is such that galaxies inherit stability from LSS via gas inflow, which, in turn, sets up CGM engine/reservoir required to self-regulate thin discs

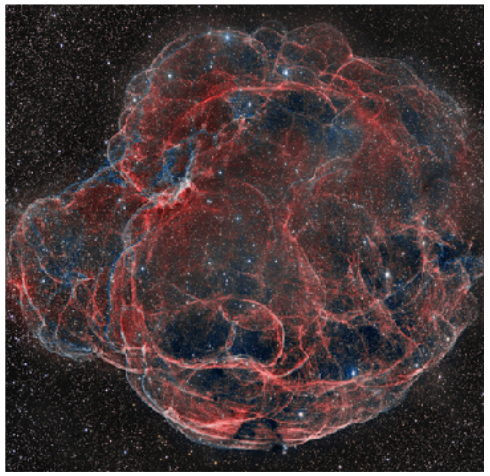


2.4 Synopsis of thin disc emergence induced by CW



- Three components system coupled by gravitation.
- A CGM reservoir fed by the CW (top down *causation*)
- Convergence towards marginal stability : acceleration of dynamical control-loop by wakes
- Tightening of stellar disc by boosting of torques, & increased dissipation.

2.4 Upshot of the various processes in the intra galactic medium



Destabilising effects

- supernovae
- Turbulence

————

- Minor merger
- accretion
- flybys

Stabilising effects

- Stellar formation
- Cooling
- Shocks

—————

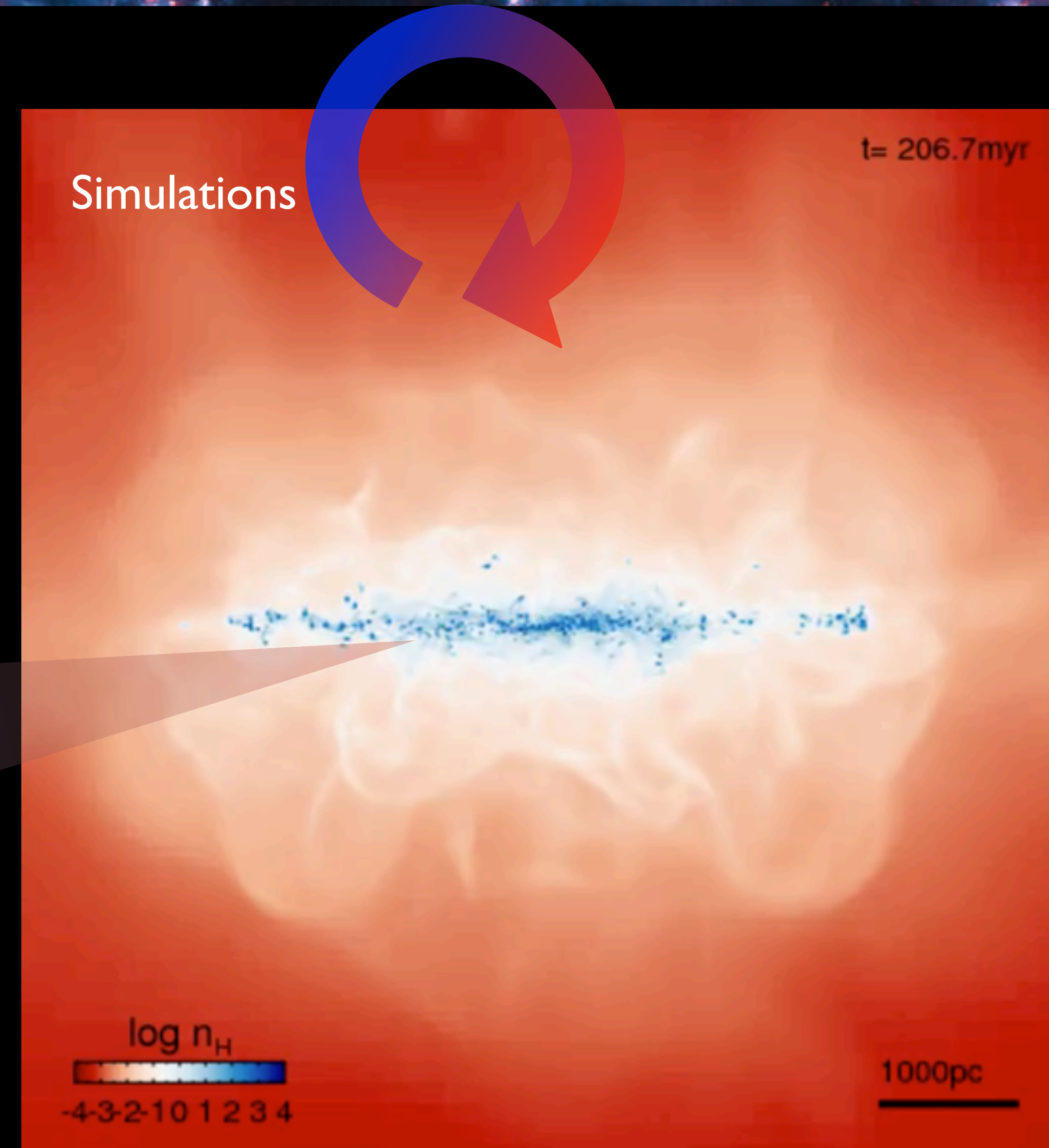
- aligned accretion

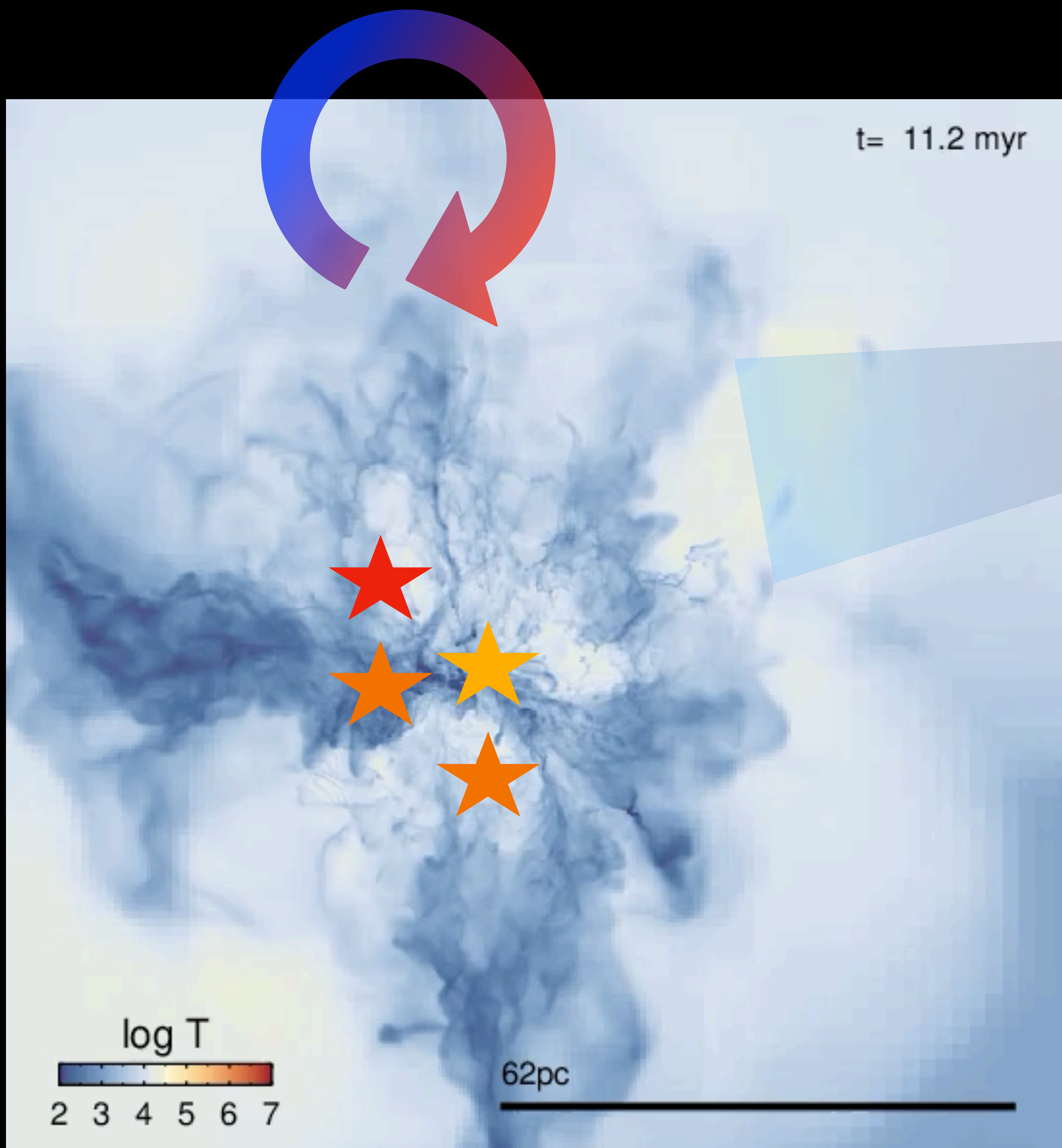
Cosmic perturbation

Free energy reservoir in CGM

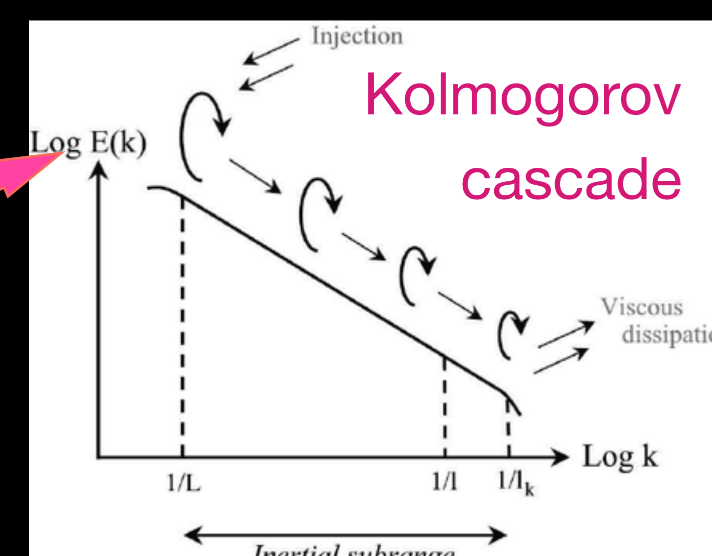
Internal Structure @ small scales: simulation & theory

State-of-the-art simulations illustrates the level of perturbation
on smaller (molecular cloud) scales



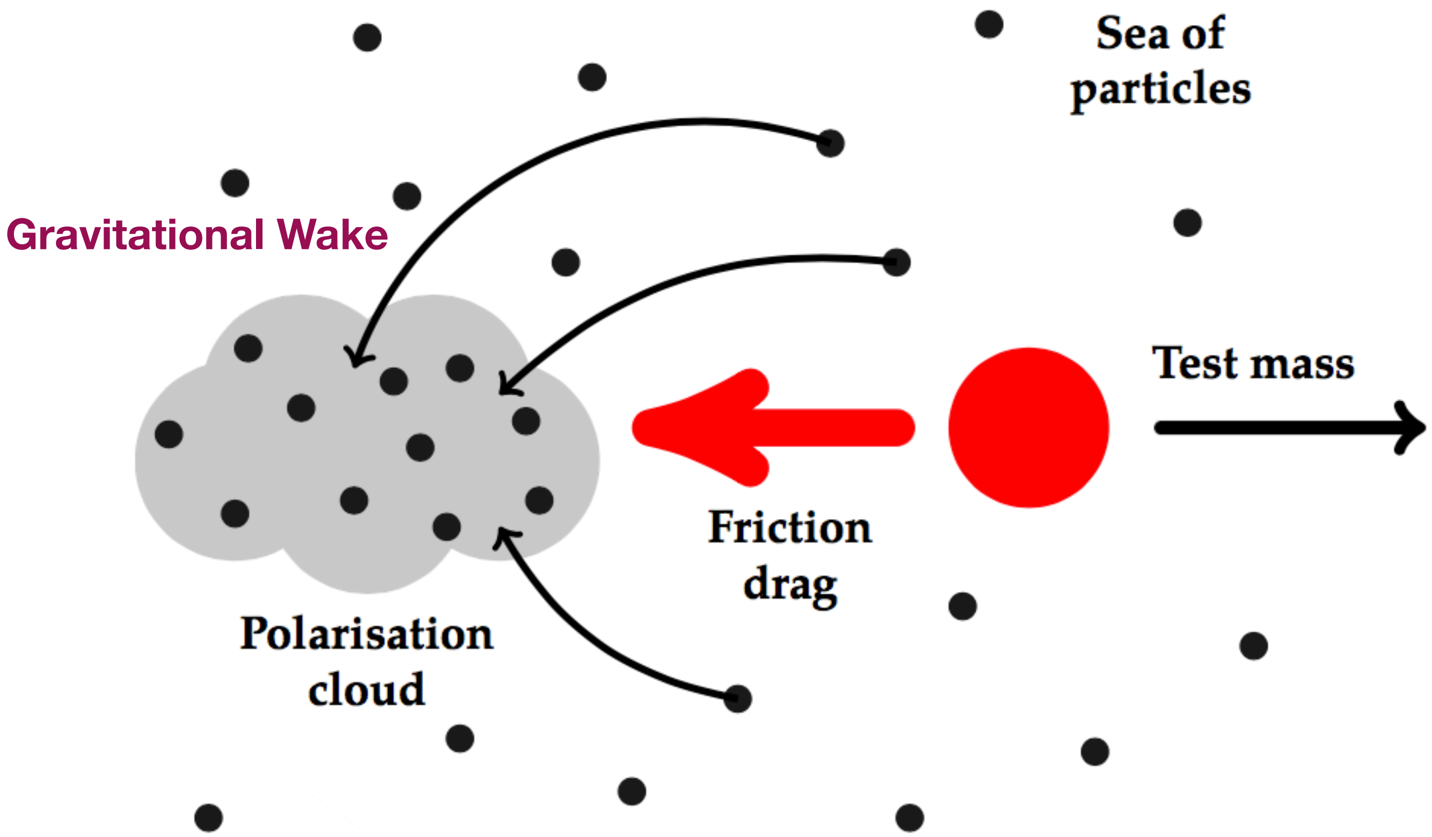


Turbulent cascade
controlled by
energy **injection** scale

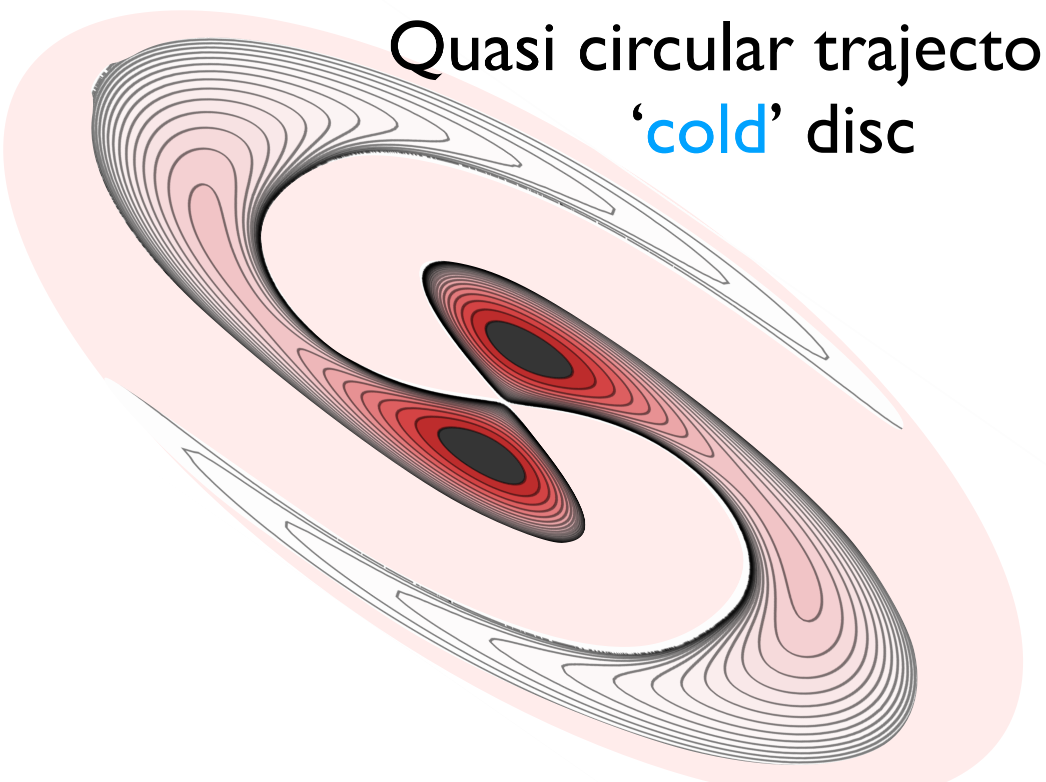


Quid of the effect of wakes on injection scale?

Chandrasekhar polarisation



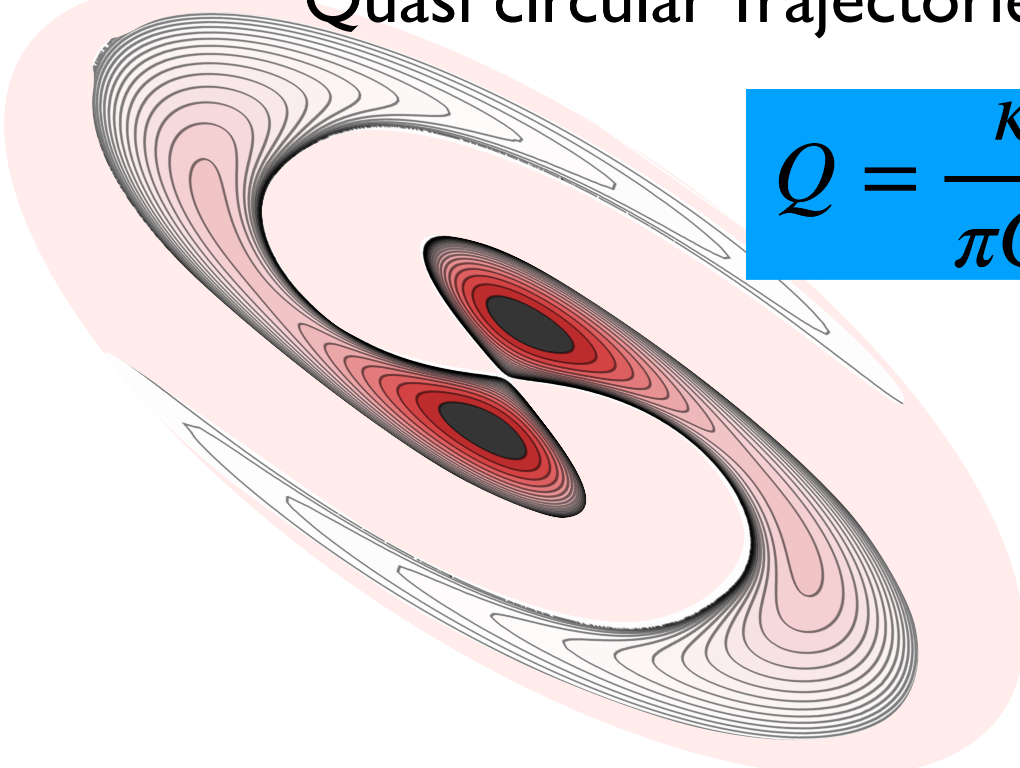
Quasi circular trajectories:
'cold' disc



→ No significant relative motion to oppose gravitation

Gravitational wake/polarisation/dressing

Quasi circular Trajectories: 'cold' disc

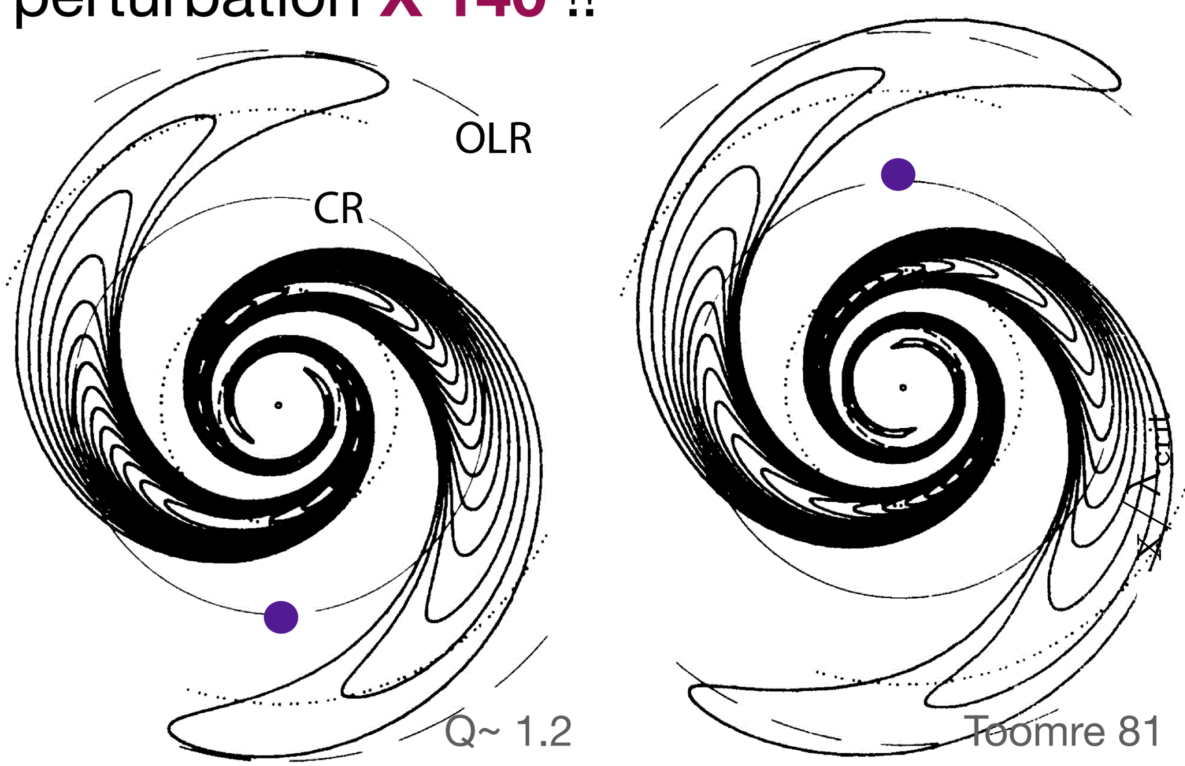


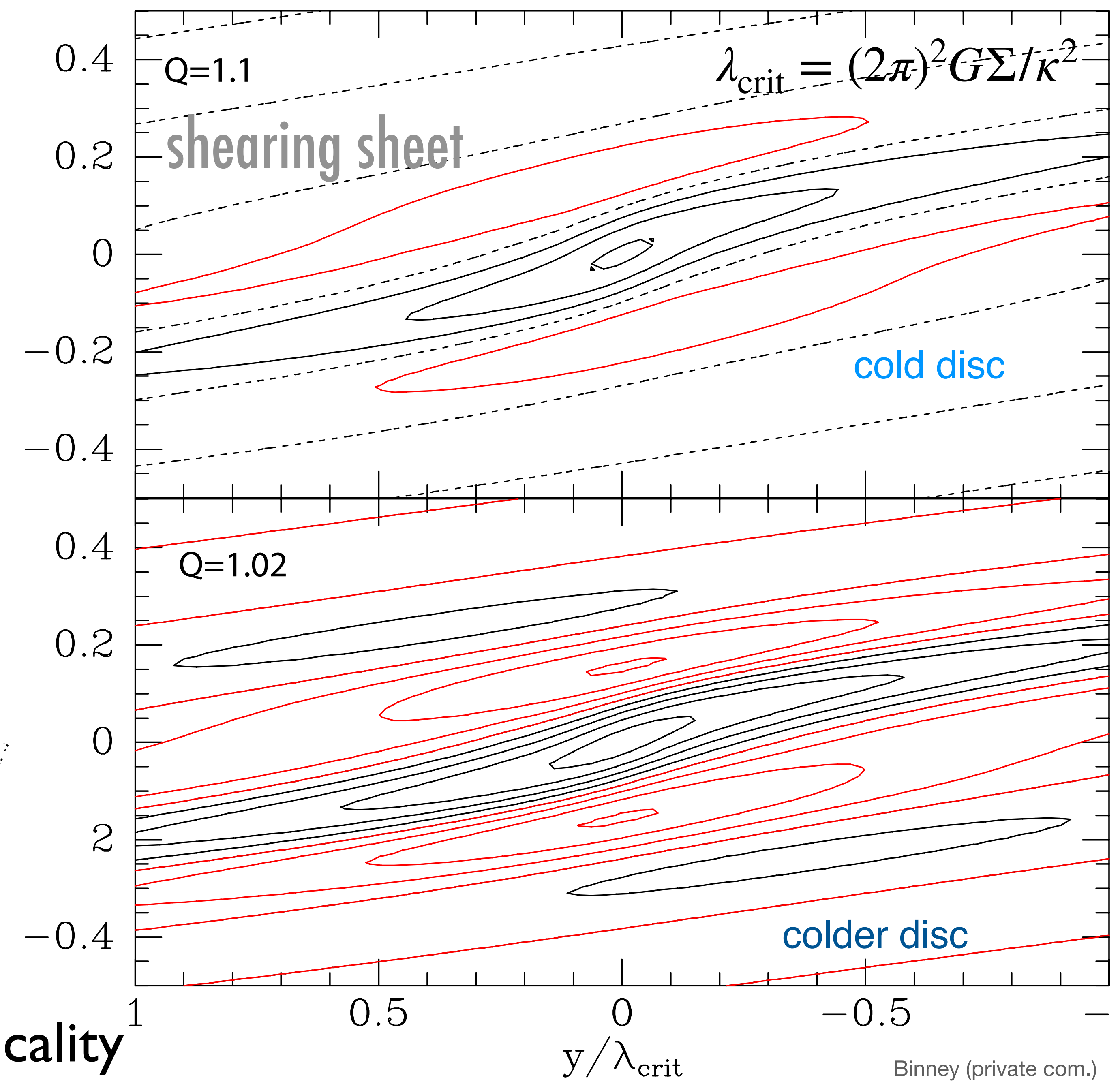
$$Q = \frac{\kappa\sigma}{\pi G\Sigma} \to 1$$

- colder disc means **larger** wake
- colder disc means **stronger** wake
- colder disc means **shorter** dynamical time

Mass in **wake** = mass in perturbation **X 140** !!

Kalnajs





Binney (private com.)

→ long range correlation: self organised criticality

On the importance of gravitational dressing

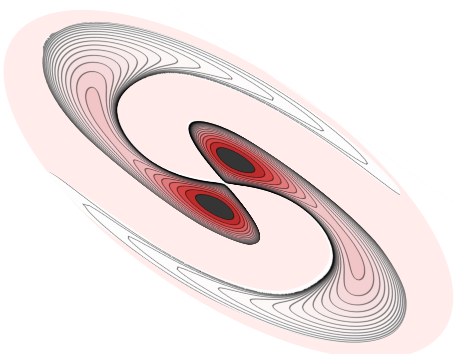
For cold discs…

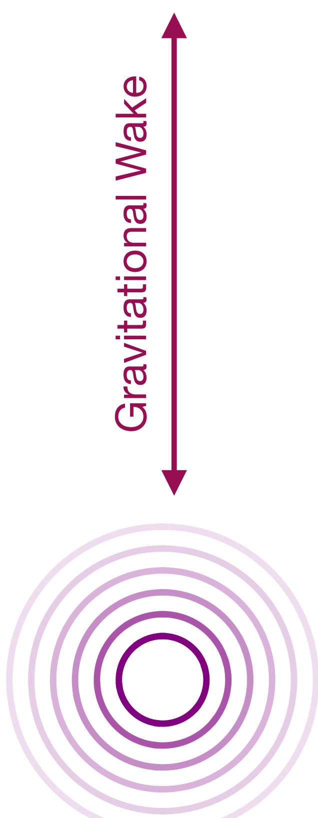
$$Q = \frac{\kappa\sigma}{\pi\Sigma} \to 1$$

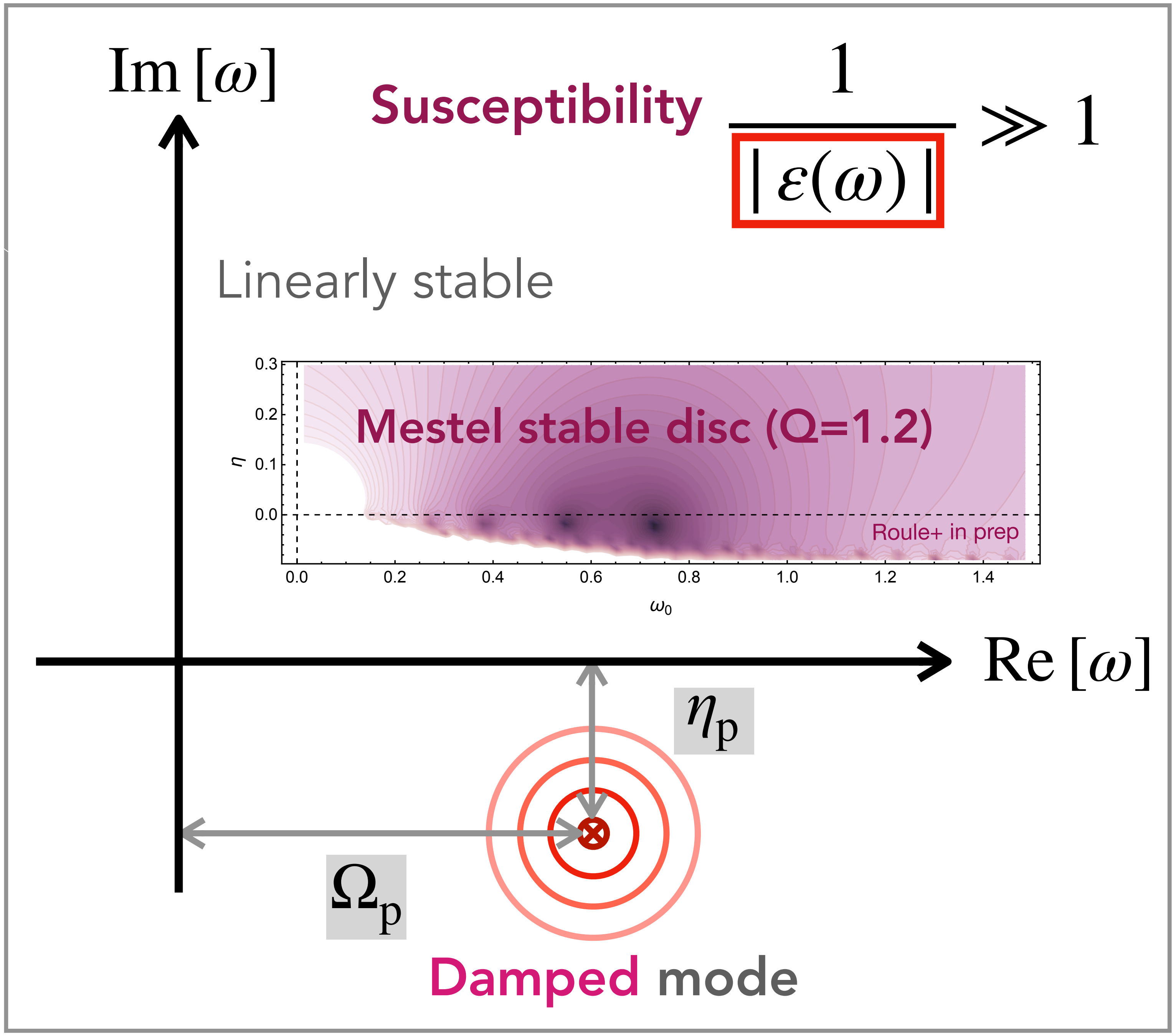
Gravitational "*Dielectric*" function ϵ

$$\epsilon(Q) \equiv \mathcal{D}(\omega, k) = \det\big(1 - \mathbf{M}(\omega)\big)$$

Dispersion relation — Response matrix







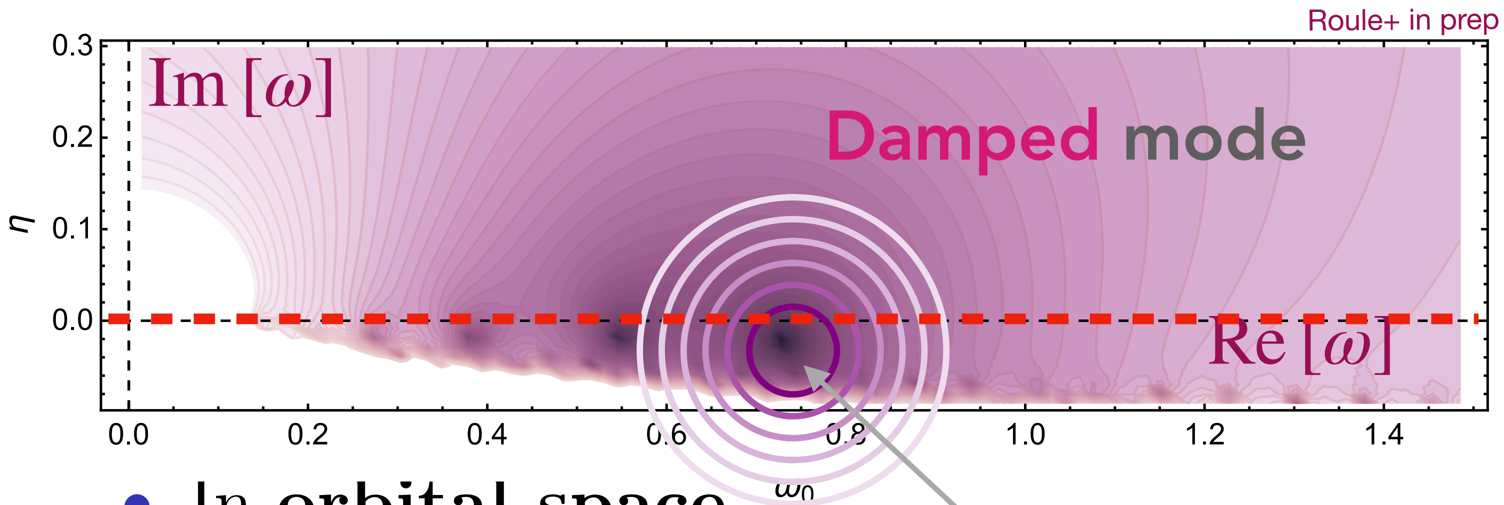
$$[\delta\psi]_{\rm dressed} = \frac{[\delta\psi]_{\rm bare}}{|\varepsilon(\omega)|}$$

$$T_{\rm dressed} \simeq |\varepsilon|\, T_{\rm bare}$$

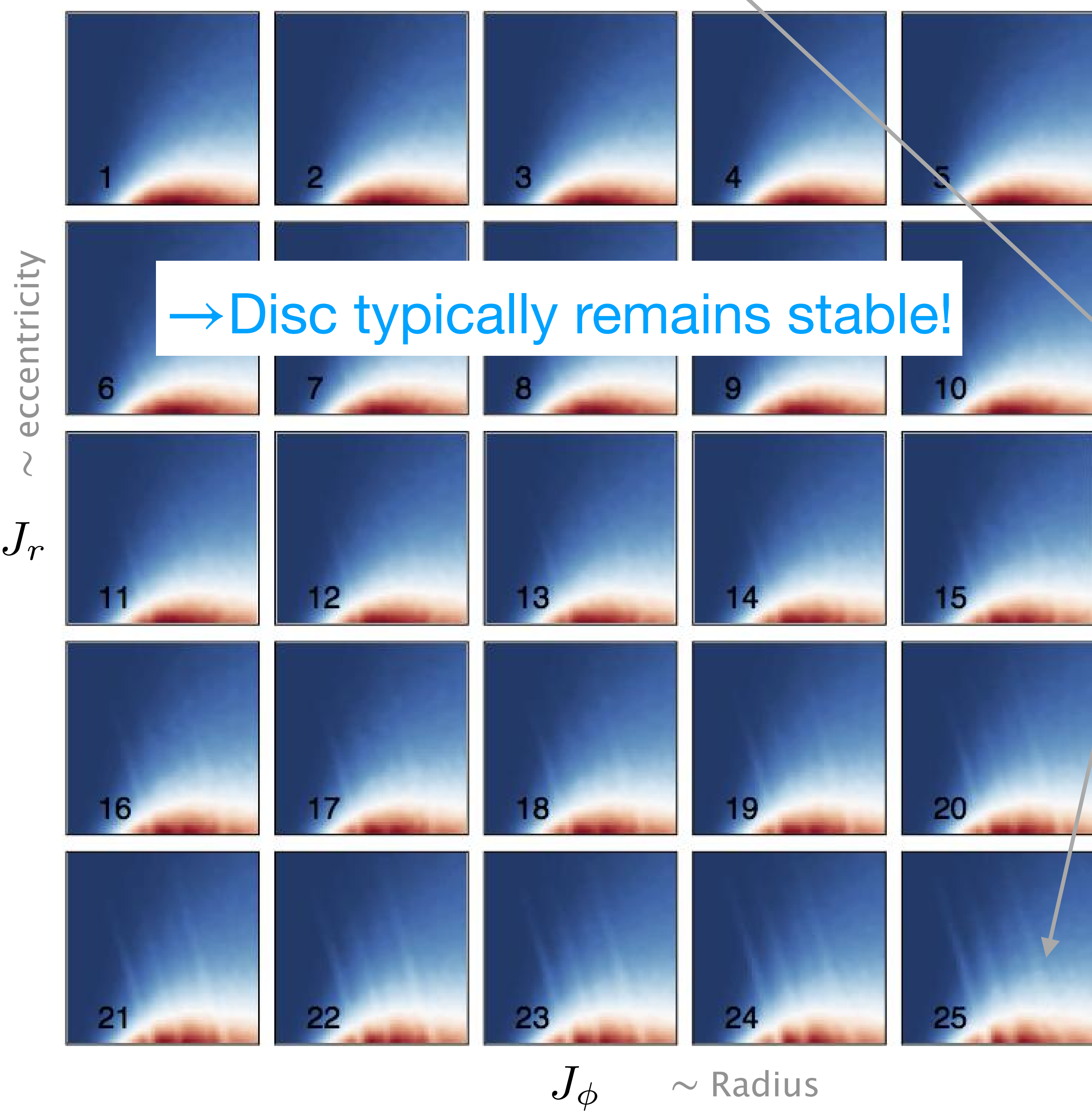
$$\Omega_{\rm dressed} \simeq \frac{1}{|\varepsilon|}\, \Omega_{\rm bare}$$

thanks to **cosmic web** which sets up cold disc

Wake drastically boost orbital frequencies, stiffening coupling/tightening control loops

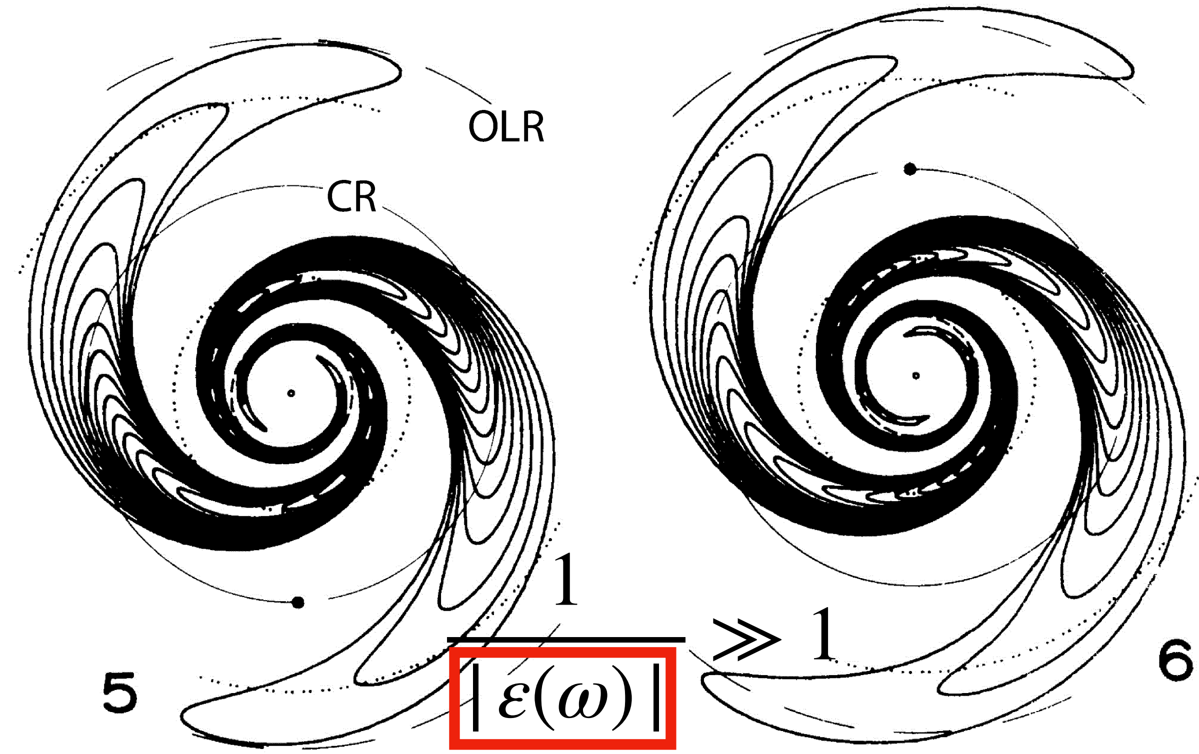


- **In orbital space**



→Disc typically remains stable!

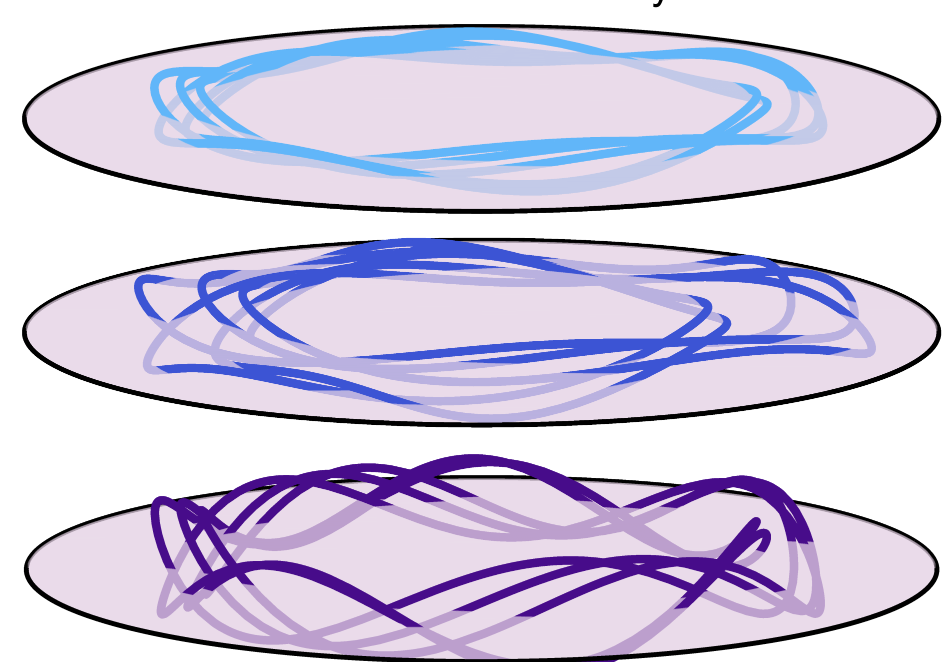
Mass in **wake** = mass in perturbation **X 140**



$$\frac{1}{|\varepsilon(\omega)|} \gg 1$$

One-to-one correspondance between **position** of 4 **weakly damped** mode and position of **ridges**

Orbital diffusion time = t_{dyn} **/ 140x 140**



Self regulating loop boosted by wake

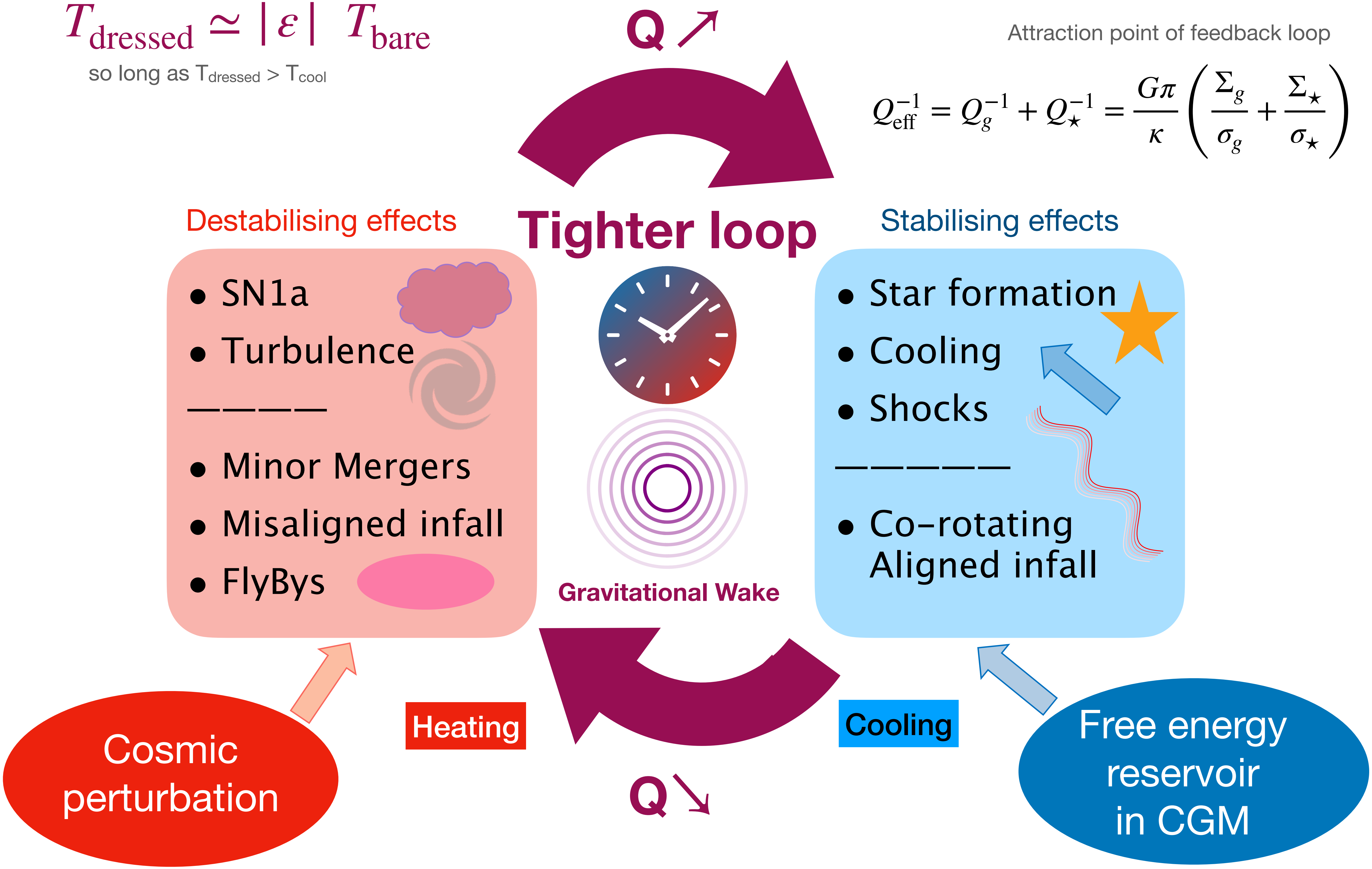
Transition to secularly-driven morphology promoting self-regulation around an effective Toomre $Q \sim 1$.

$$T_{\rm dressed} \simeq |\varepsilon|\ T_{\rm bare}$$

so long as $T_{\rm dressed} > T_{\rm cool}$

Attraction point of feedback loop

$$Q_{\rm eff}^{-1} = Q_g^{-1} + Q_\star^{-1} = \frac{G\pi}{\kappa}\left(\frac{\Sigma_g}{\sigma_g} + \frac{\Sigma_\star}{\sigma_\star}\right)$$

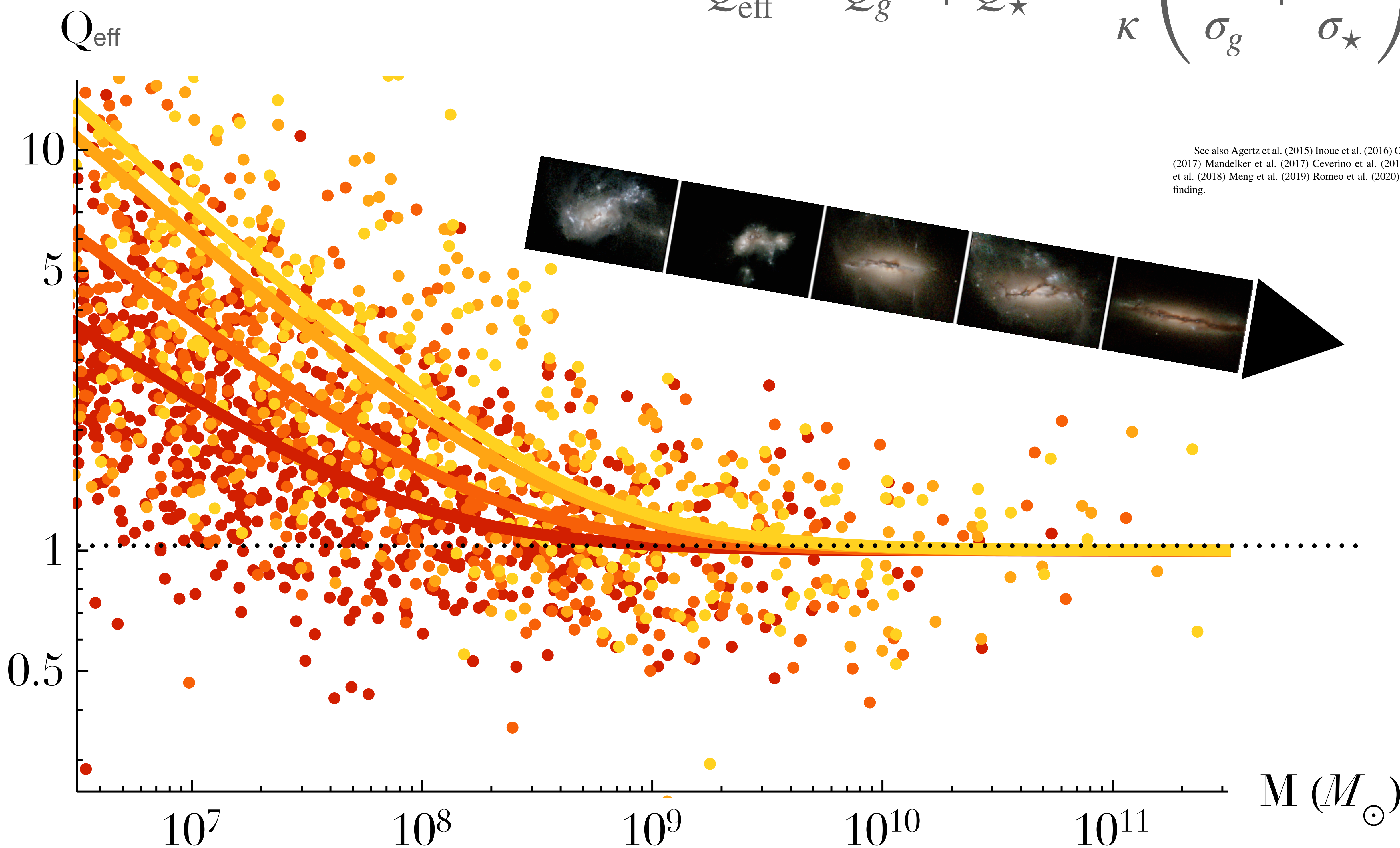


Open system with control loop generates complexity through self-organisation

Toomre Q convergence with mass and z

Toomre Q (★+gas) parameter convergence as a function of *both* mass and redshift

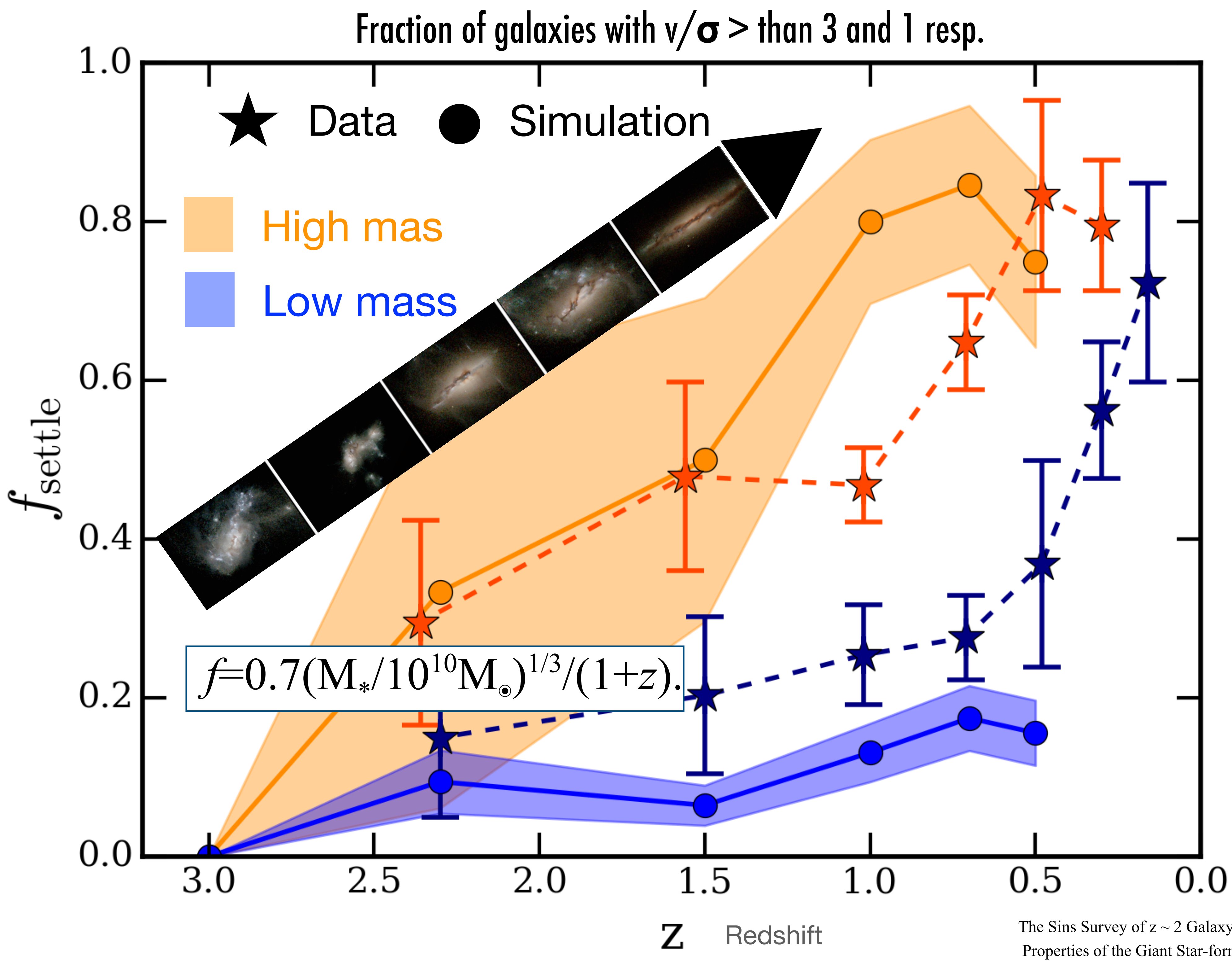
$$Q_{\rm eff}^{-1} = Q_g^{-1} + Q_\star^{-1} = \frac{\pi}{\kappa}\left(\frac{\Sigma_g}{\sigma_g} + \frac{\Sigma_\star}{\sigma_\star}\right)$$



See also Agertz et al. (2015) Inoue et al. (2016) Oklopčić et al. (2017) Mandelker et al. (2017) Ceverino et al. (2017) Krumholz et al. (2018) Meng et al. (2019) Romeo et al. (2020) for a similar finding.

Disc settling : fraction of settled discs

Match between simulation and observation as a function of *both* mass and redshift



The Sins Survey of z ~ 2 Galaxy Kinematics: Properties of the Giant Star-forming Clumps. Astrophys. J., 733, 101-130 (2011)

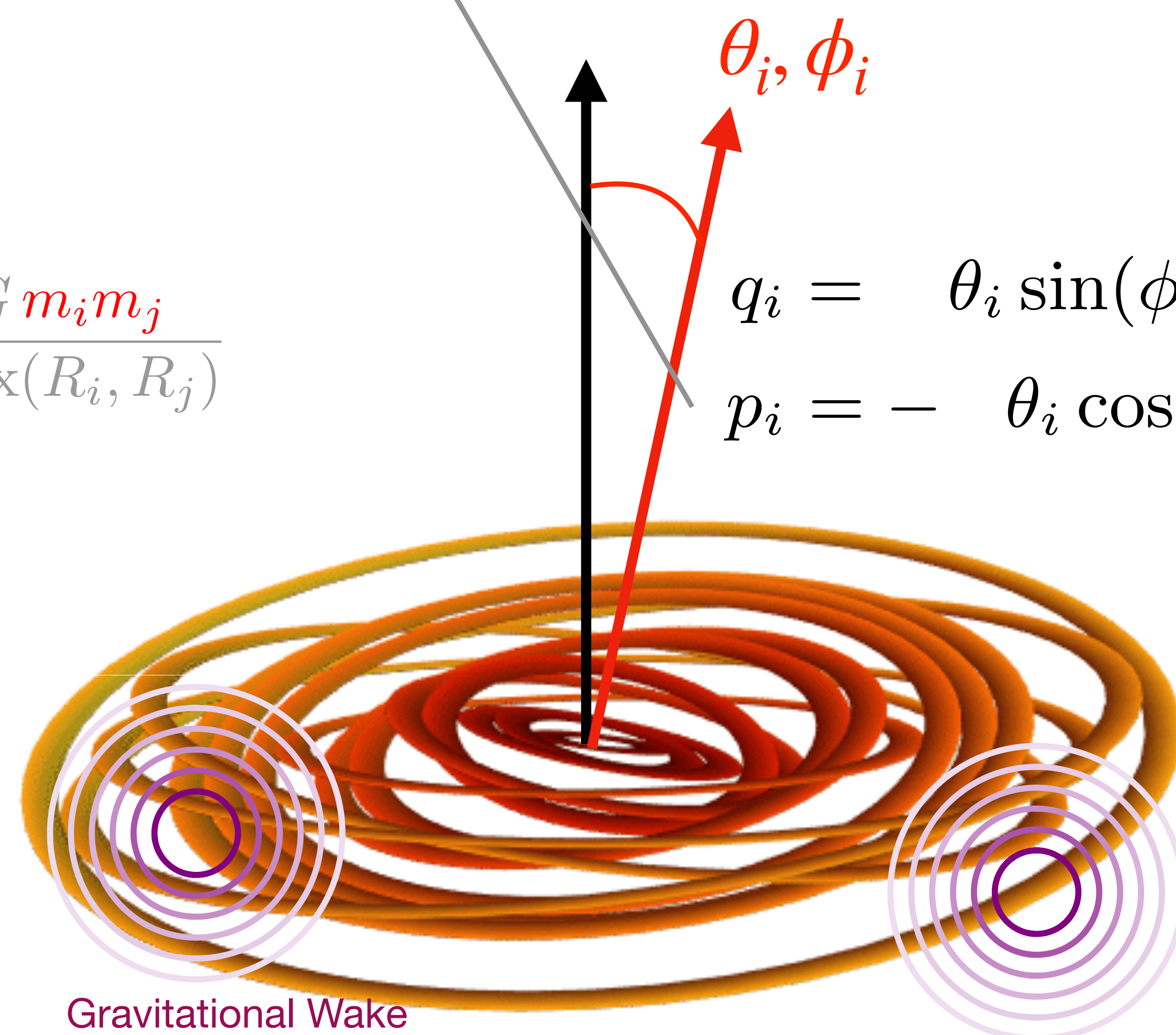
Ring Toy model: secular damping by wake growth

Lagrange Laplace theory of rings *(small eccentricity small inclinaison)*

x and y components of **angular momentum**

$$H(\mathbf{p},\mathbf{q}) = \frac{1}{2}\mathbf{p}^{\mathrm{T}}\cdot\mathbf{A}\cdot\mathbf{p} + \frac{1}{2}\mathbf{q}^{\mathrm{T}}\cdot\mathbf{A}\cdot\mathbf{q}\,,$$

$$A_{ij} \propto -\frac{G\, m_i m_j}{\max(R_i, R_j)}$$



$$q_i = \theta_i \sin(\phi_i)$$

$$p_i = -\theta_i \cos(\phi_i)$$

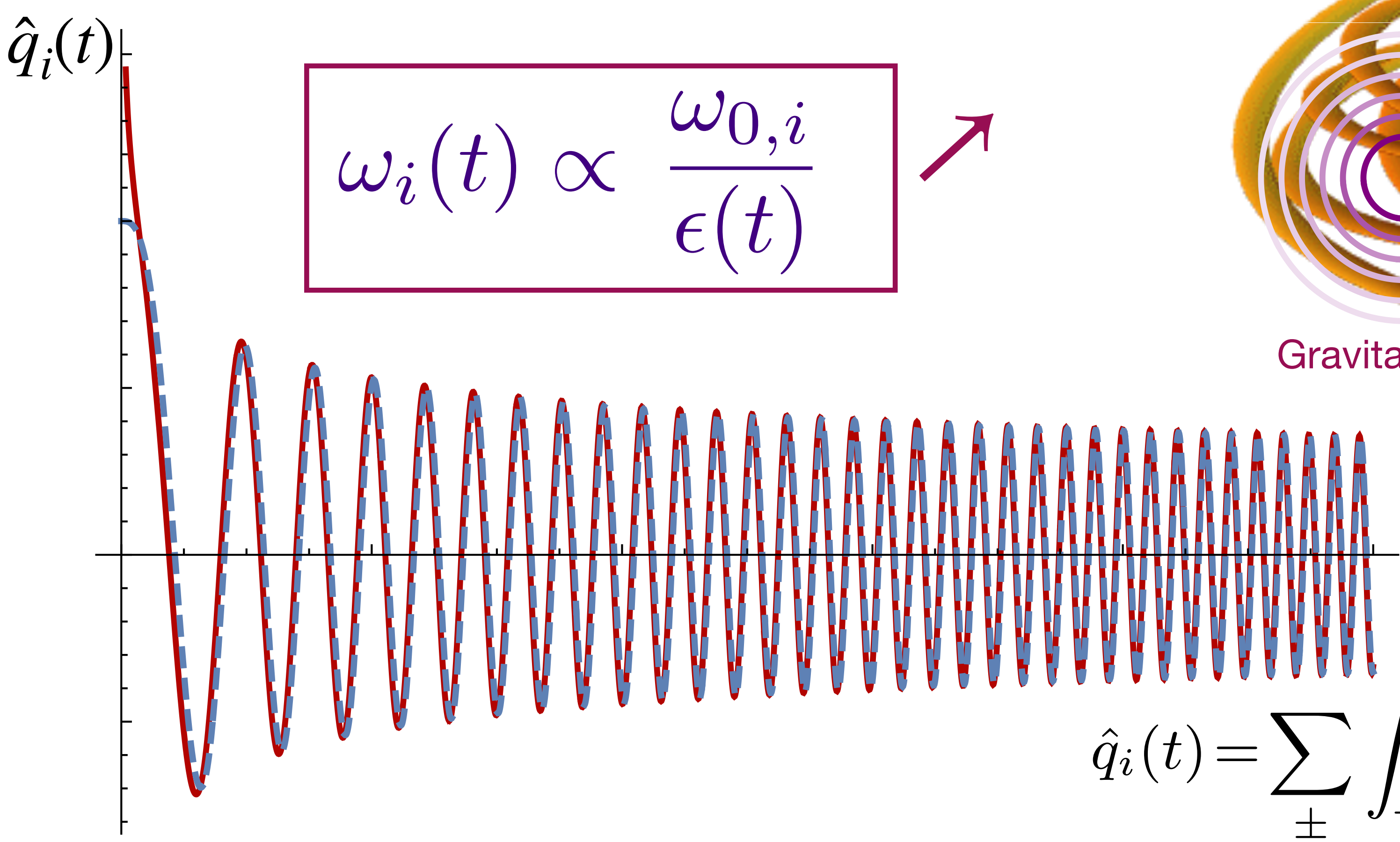
In eigenframe of A

Eigen frequency

$$\ddot{\hat{q}}_i + \omega_i^2(t)\hat{q}_i = \xi_i$$

forcing

$$\omega_i(t) \propto \frac{\omega_{0,i}}{\epsilon(t)} \nearrow$$



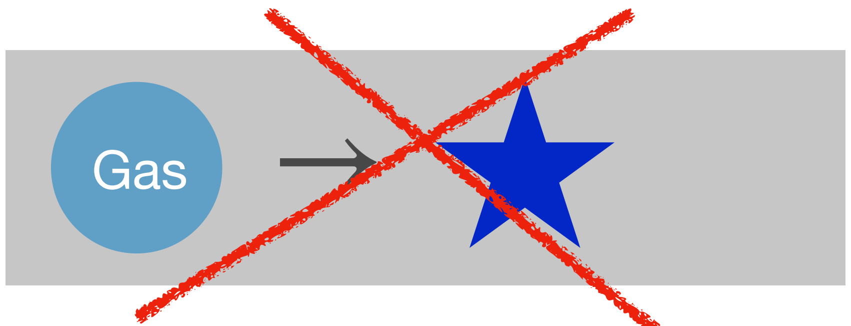
Secular WKB solution

$$\hat{q}_i(t) = \sum_{\pm}\int_{-\infty}^{\infty}\frac{\hat{\xi}_i(t')}{\sqrt{\omega_i(t)\omega_i(t')}}\exp\left(\pm\imath\int_{t'}^{t}\omega_i(\tau)d\tau\right)dt'$$

Why finite thickness? Chemistry of emergence

Let us write down effective (closed loop) production rate for cold stellar component

Auto-catalysis of the cold component (via wakes) converts kinetic evolution into a logistic differential equation.



Gas + ★ → ★ + ★

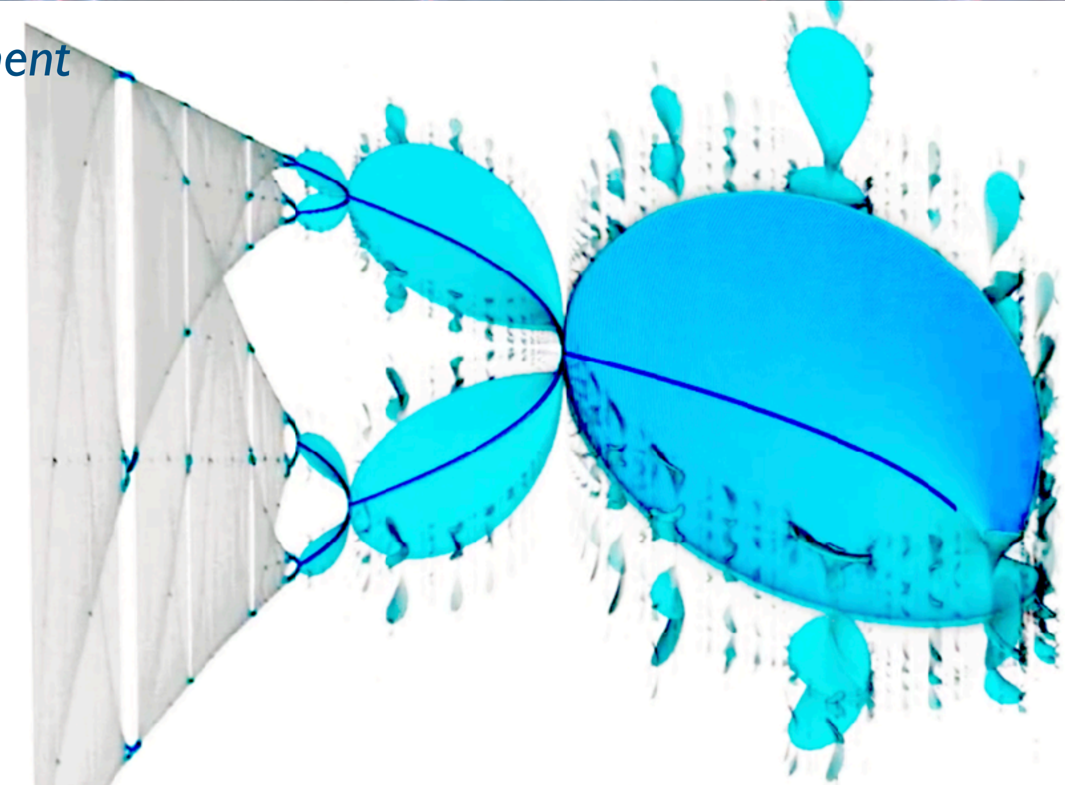
$$\frac{d}{dt}\bigstar = r\bigstar(1-\bigstar)$$

r: control parameter

Logistic *ODE (cf Ecology, Chaos, Covid, Innovation etc..)*

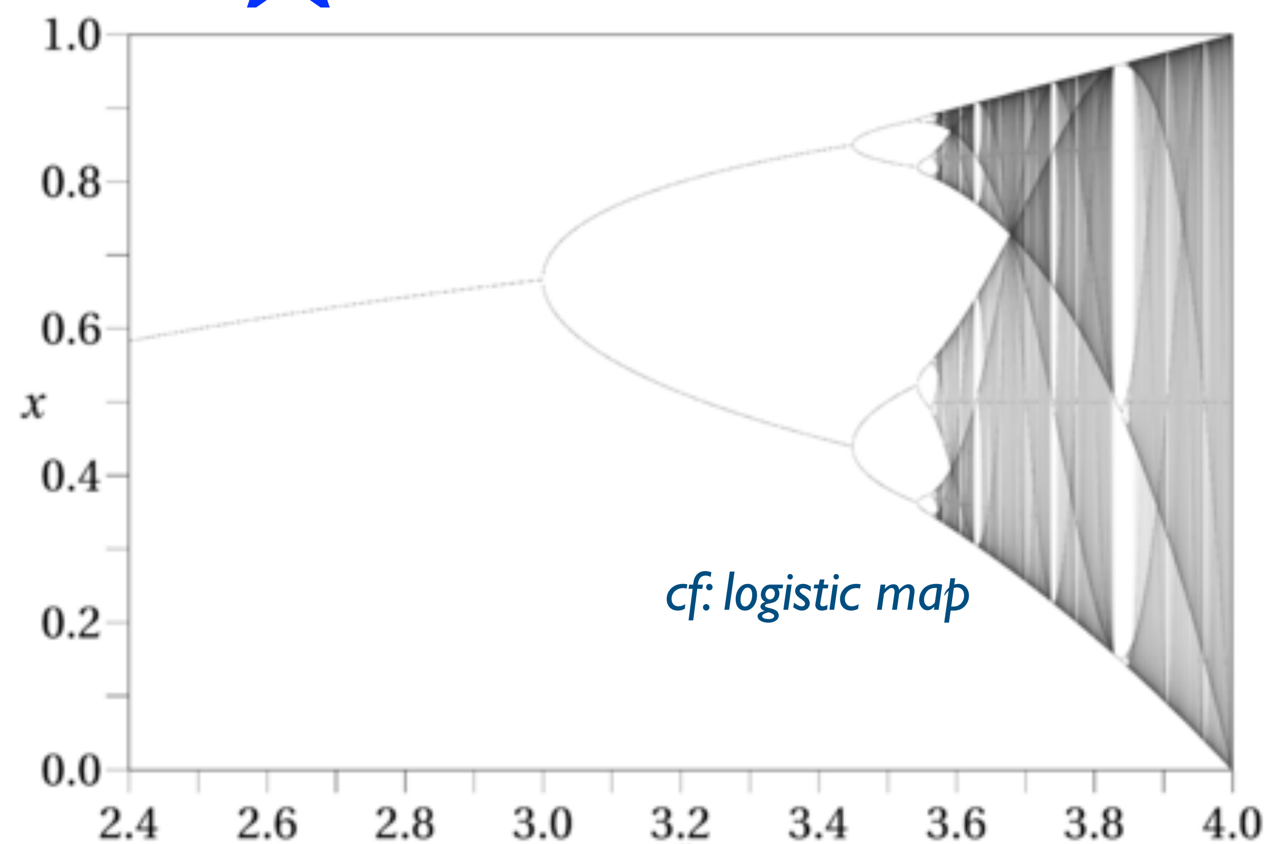
= Simplest **quadratic** model for self -regulation

= Taylor expansion of effective production rate



Link to Mandelbrot Set (Veritassium 2021)

★ = cold stellar component



r control parameter

Chemistry of emergence... introduce heating

*Now let us take into account for the **vertical** secular diffusion of the cold component*

Dissipation converts kinetic instability point into an attractor.

Reaction-Diffusion equation (cf morphogenesis)

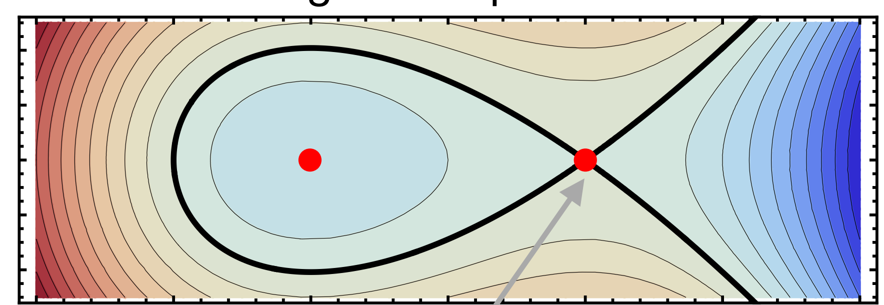
Fokker Planck orbital diffusion

$$\frac{d}{dt}\bigstar = \delta_D \bigstar (1 - \bigstar) + \Delta\bigstar$$

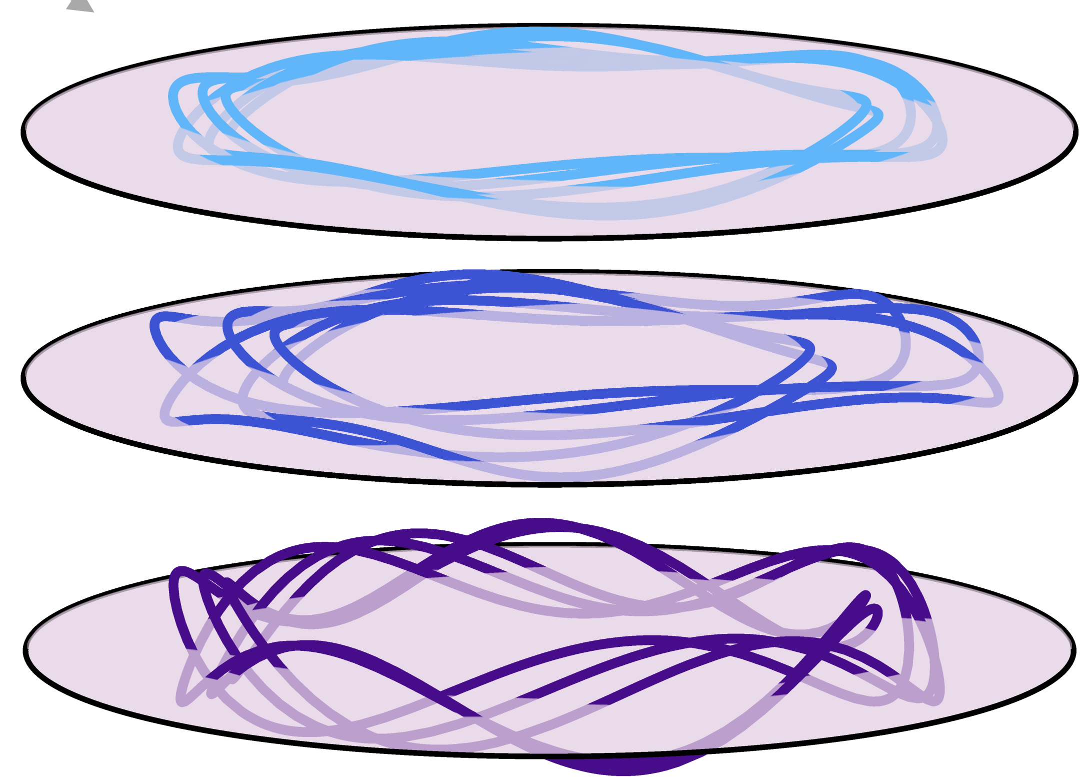
Cooling

Heating

Logistic map Hamiltonian



New point of equilibrium with finite disc thickness



Chemistry of emergence… introduce heating

*Now let us take into account for the **vertical** secular diffusion of the cold component*

Dissipation converts kinetic instability point into an attractor.

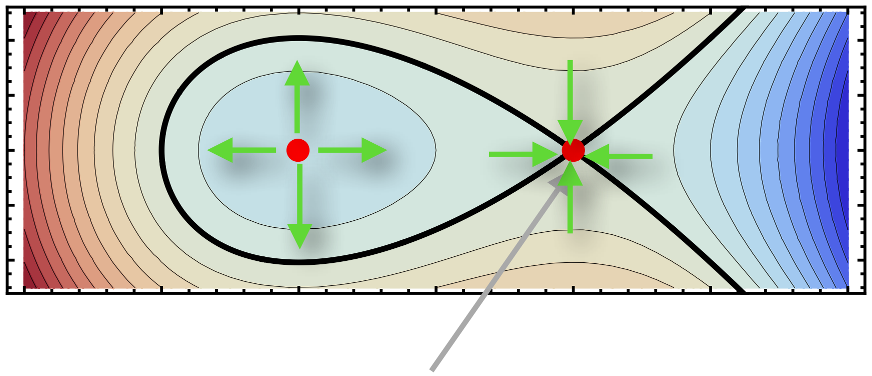
Reaction-Diffusion equation (cf morphogenesis)

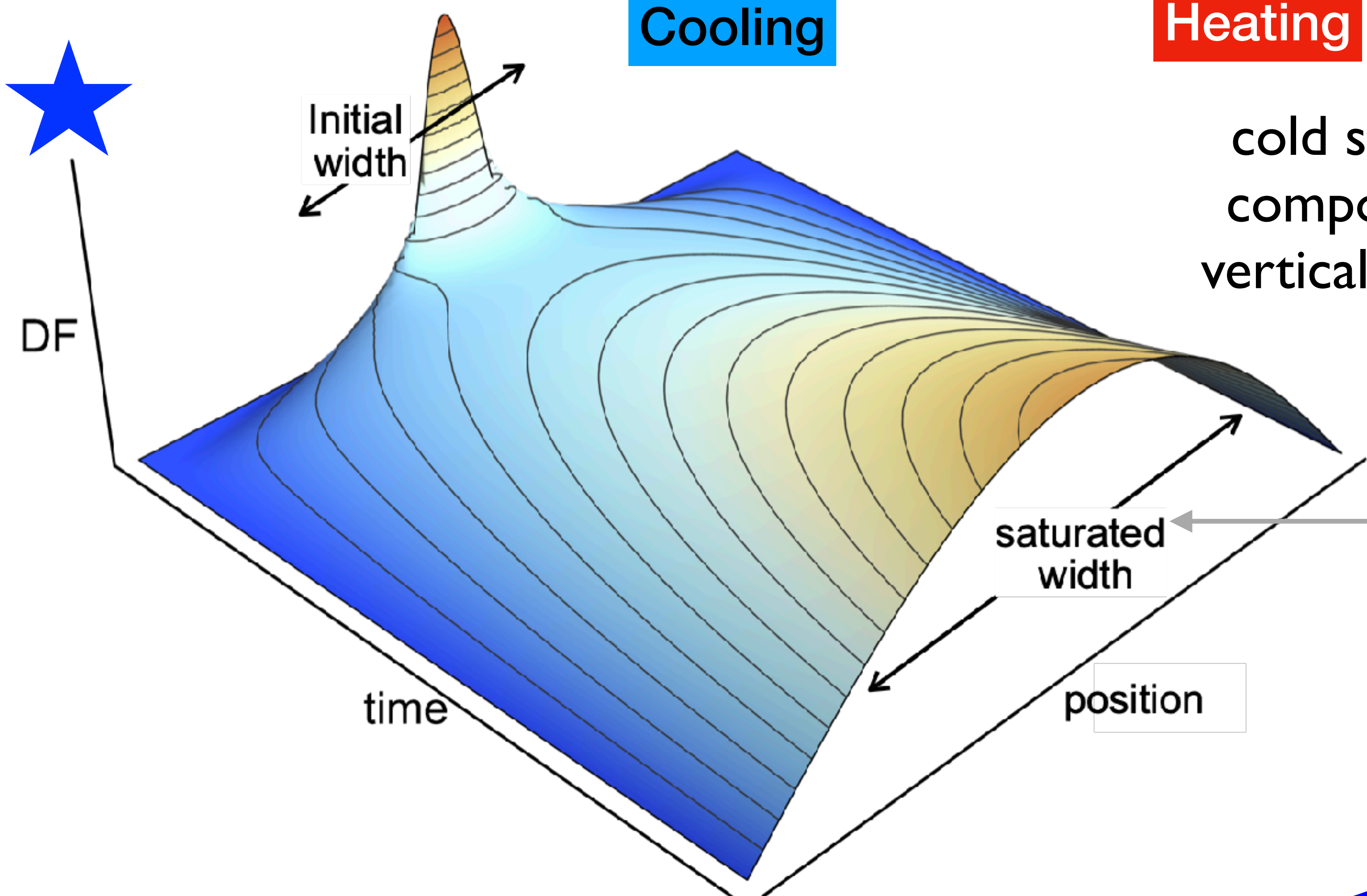
$$\frac{d}{dt}\bigstar = \delta_D \bigstar (1 - \bigstar) + \Delta \bigstar$$

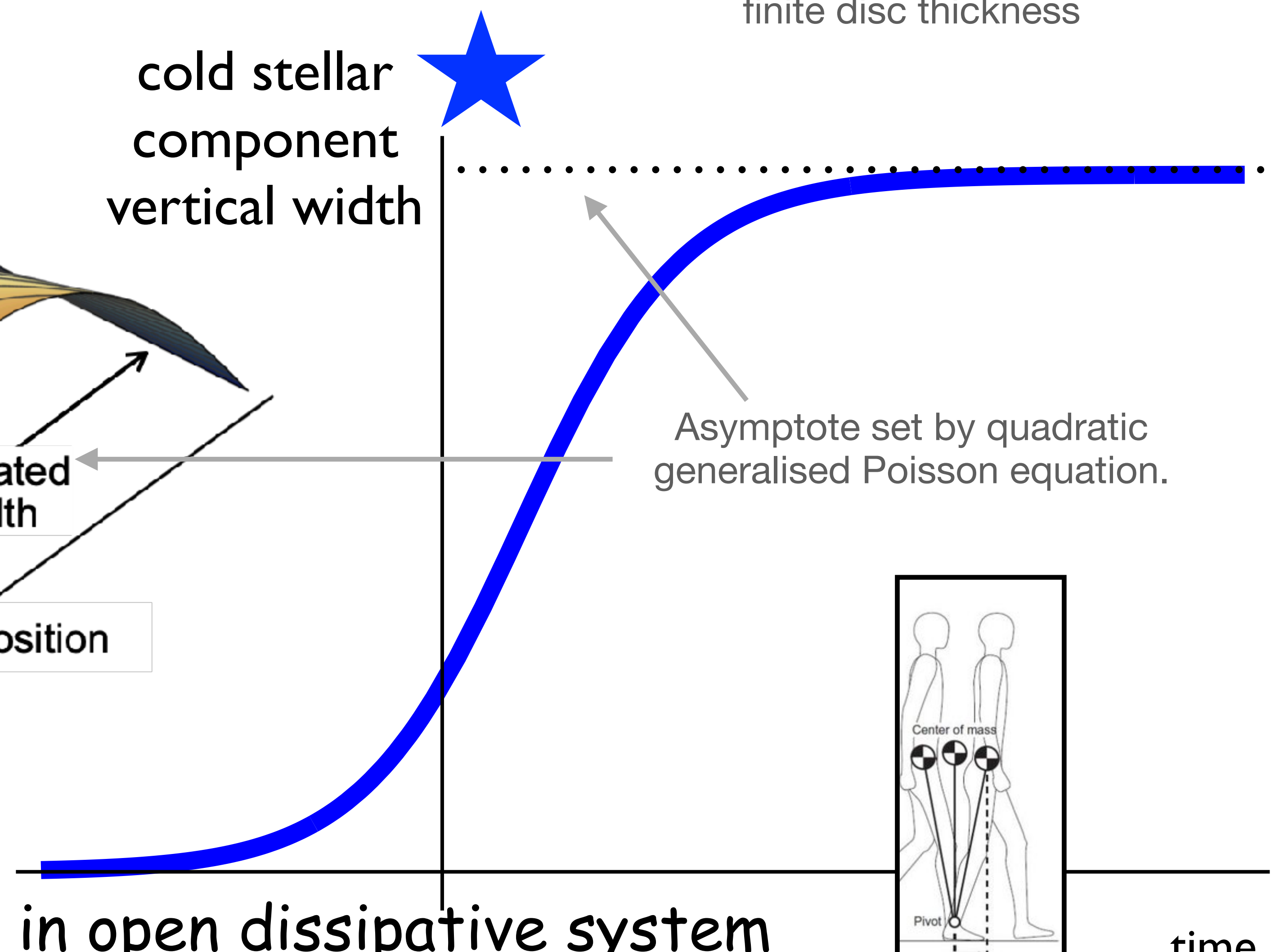
Cooling: $\delta_D \bigstar (1 - \bigstar)$

Heating: $\Delta \bigstar$ — Fokker Planck orbital diffusion

Logistic map Hamiltonian







→Emergence of thin **fixed width** disc in open dissipative system

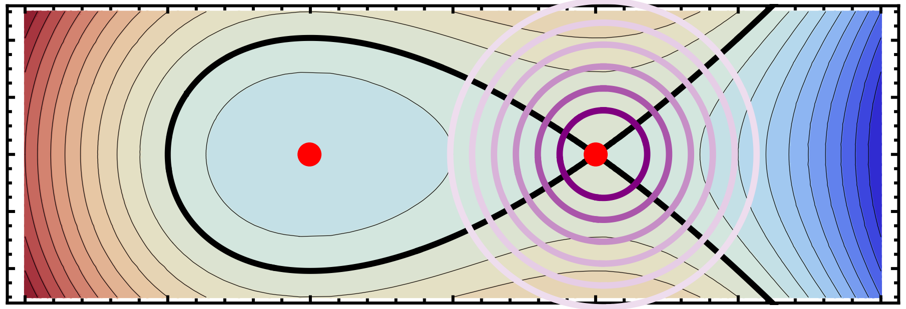
Chemistry of emergence... introduce tides

Now let us take into account for the ***vertical*** *secular diffusion of the cold component*

Dissipation converts kinetic instability point into an attractor.

Gravitational Wake

Logistic map Hamiltonian



Dressed Reaction-Diffusion equation (cf morphogenesis)

$$\frac{d}{dt}\bigstar = \frac{\delta_D}{\epsilon^2}\bigstar(1-\bigstar) + \frac{1}{\epsilon^2}\Delta\bigstar$$

wake driven $\varepsilon(z) \to 0$ as $Q \to 1$

SF efficiency

$$\eta_{\rm dressed} \propto \eta_{\rm raw}/\epsilon^2(Q)$$

$\sim$ quadratic in ε

$$D_{\rm dressed} \propto D_{\rm raw}/\epsilon^2(Q)$$

Diffusion

$$\Longrightarrow \quad dt \to \frac{dt}{\epsilon^2}$$

Rapid correction

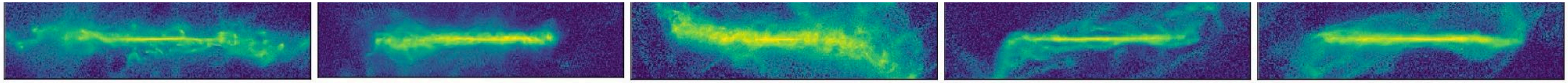
→ Cosmic resilience of thin disc driven by CW

→ Operates swiftly near self-organised Criticality

→ Robustness / feedback details

No fine tuning !

all discs are fairly thin whatever the feedback



Emergence cf: self-steering Bike on slope of increasing steepness

Disc resilience is direct analog of self-steering bike on slope of increasing steepness.

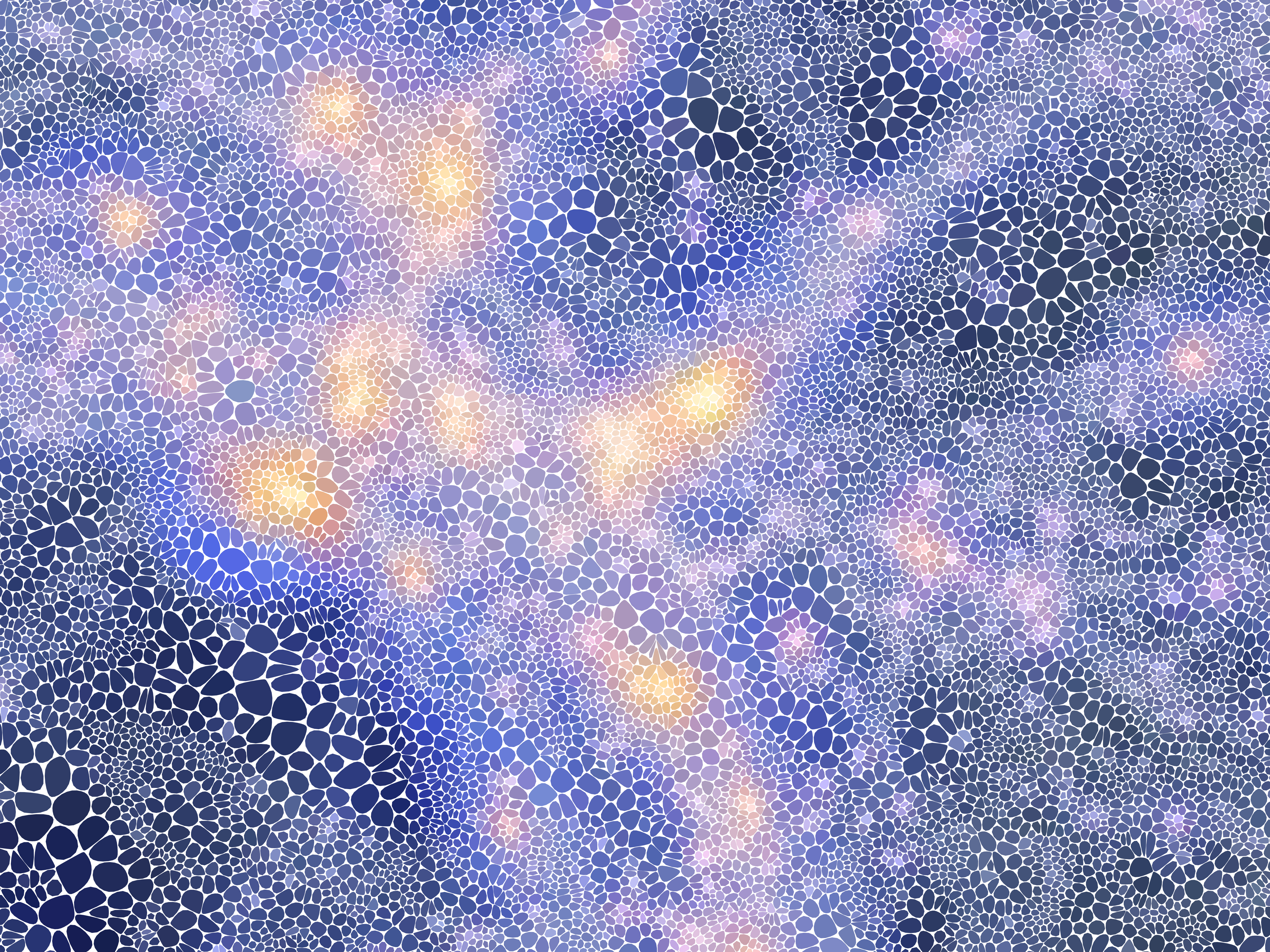
remarkably,
the bike's analog
spontaneously emerges
thanks to the CW!

Conclusion:

We should care about the cosmic web!

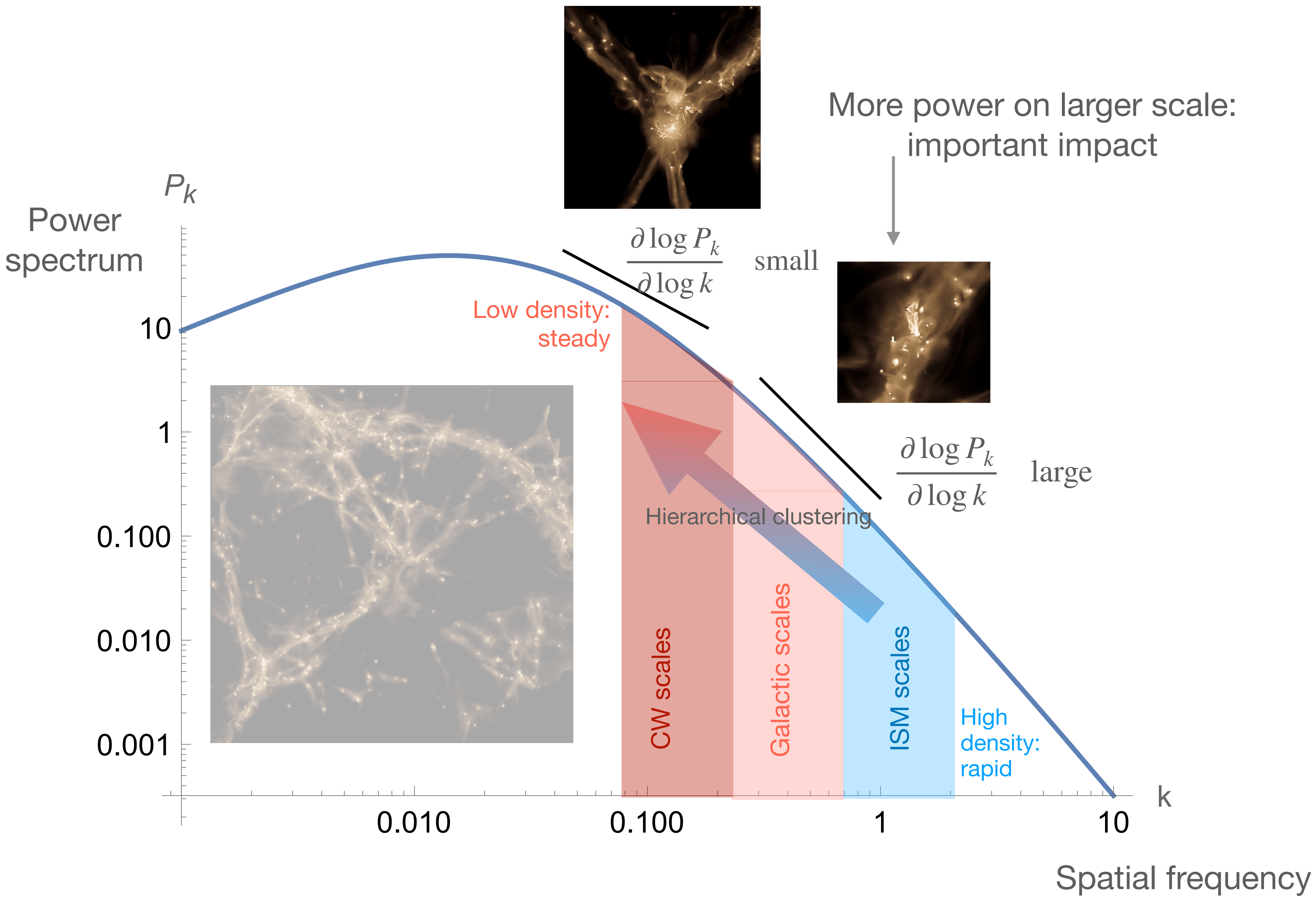
$$\mathbf{cosmic\ \ web} = \text{metric set by}\ \ \text{eigframe}\left[\frac{\partial^2 \rho}{\partial x_i \partial x_j}\right]_{\text{sad}}$$

Merci !



1. Impact of CW on non-linear dynamics is top down

Change in Pk shape reflects dark halos larger or smaller than filament cross-section



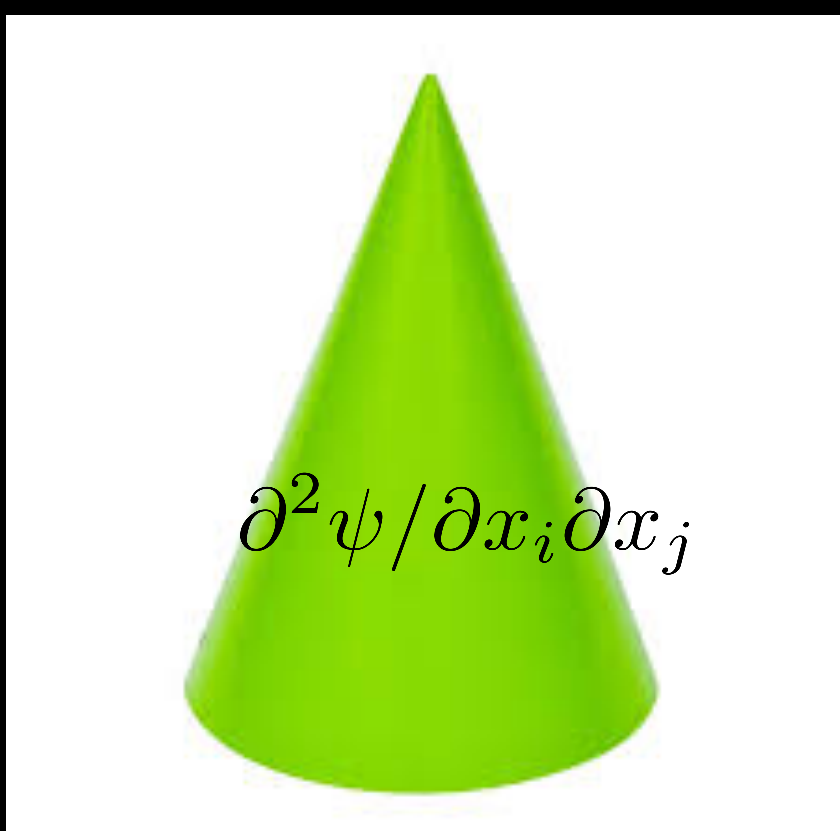
1. What is the cosmic web? a fruitful theoretical spin

- Galaxy property driven by the past lightcone of tidal tensor $\partial^2\psi/\partial x_i \partial x_j$'s non-linear evolution impacted by *scale-coupling*/differential *time delays*

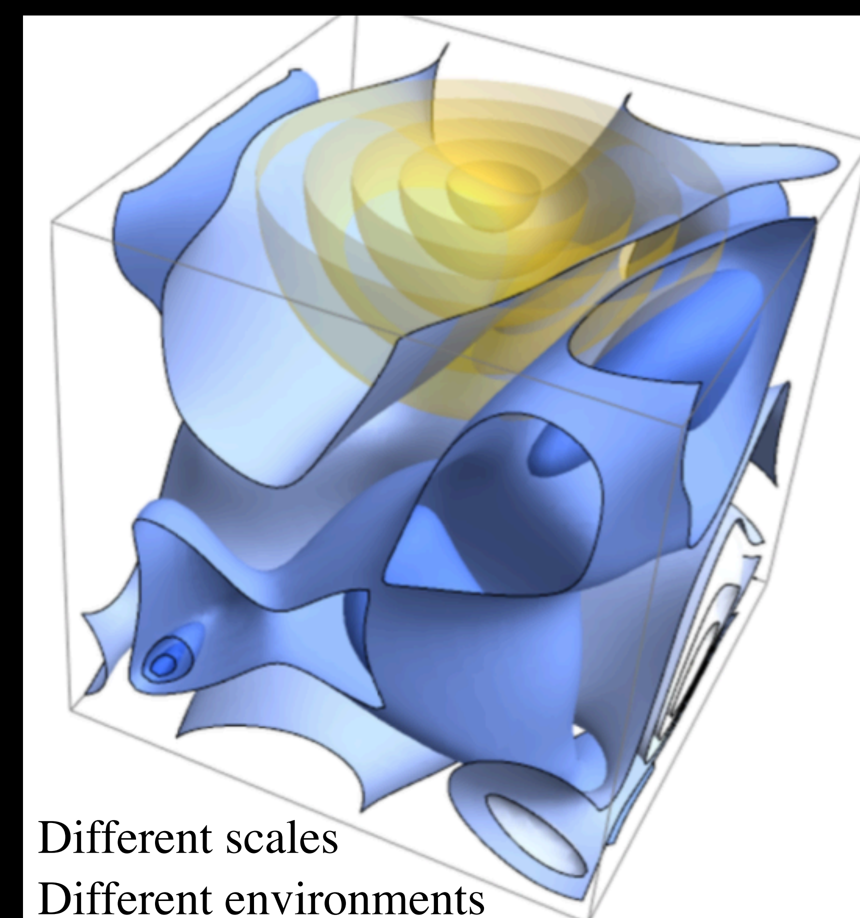
$$\langle f_{\rm NL}(IC)\rangle \neq f_{\rm NL}(\langle IC\rangle)$$

$$\langle f_{\rm NL}(IC)\rangle_{\theta,\phi} \neq f_{\rm NL}(\langle IC\rangle_{\theta,\phi})$$

Spherical collapse does not capture filamentary/wall tides...



Proto halo will be impacted by ***all*** *components of Tidal tensor (not just trace, also eigenvectors+other minors) over past light cone*

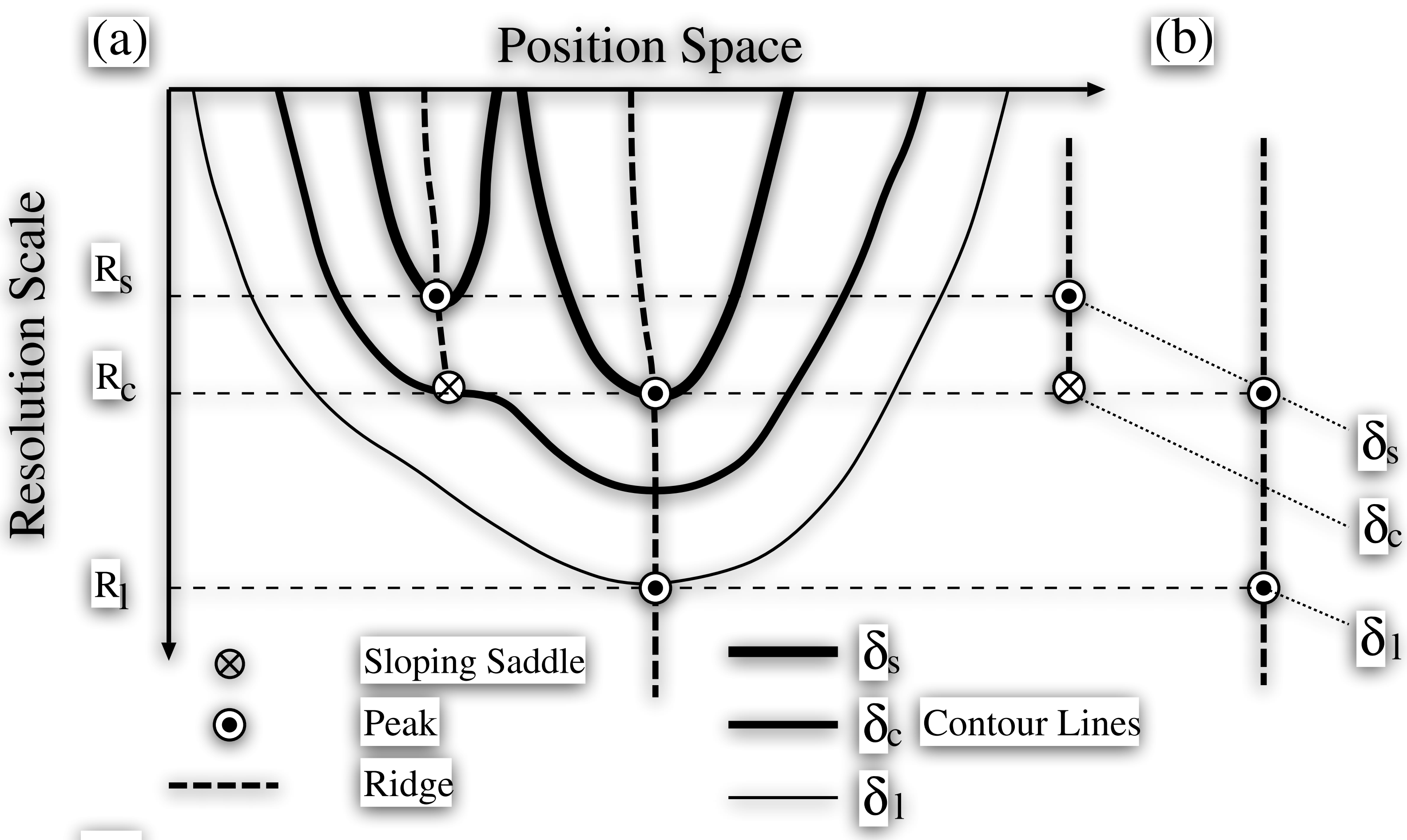


Context: skeleton tree

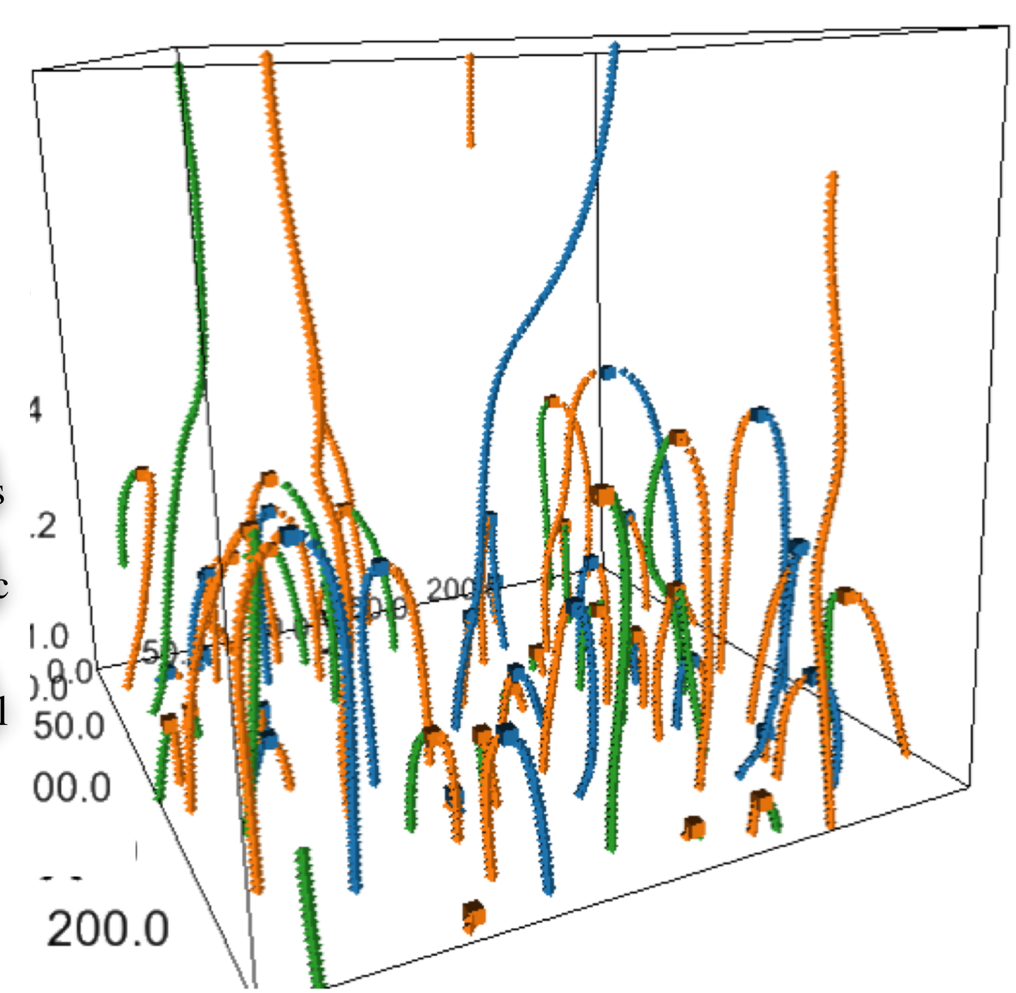
Statistics of Merging Peaks of Random Gaussian Fluctuations: Skeleton Tree Formalism

Hitoshi HANAMI **2001**

Physics Section, Faculty of Humanities and Social Sciences, Iwate University, Morioka 020 JAPAN



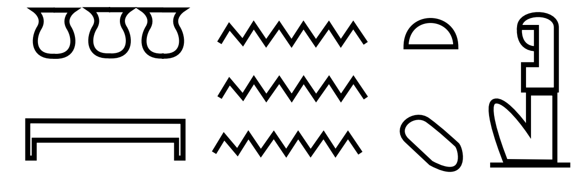
Marulli+2009



Cadiou+ in prep

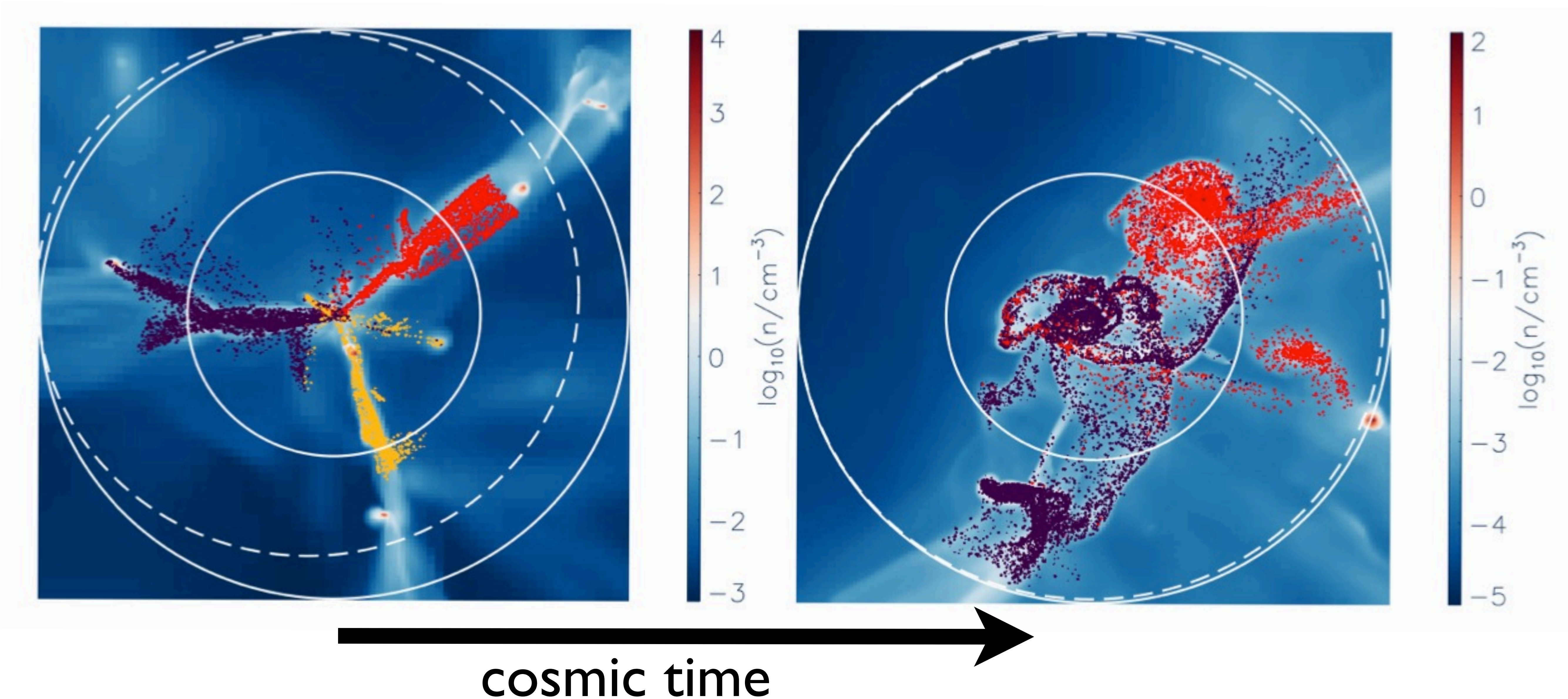
Extend Hanami '01 to *other* critical events

Galactic motivation

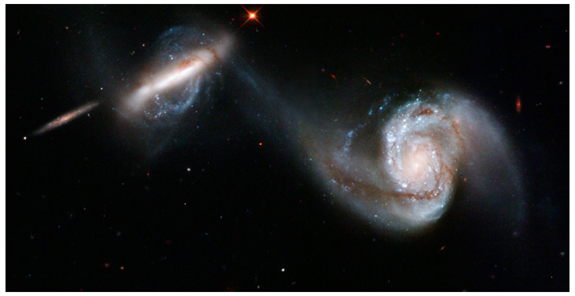


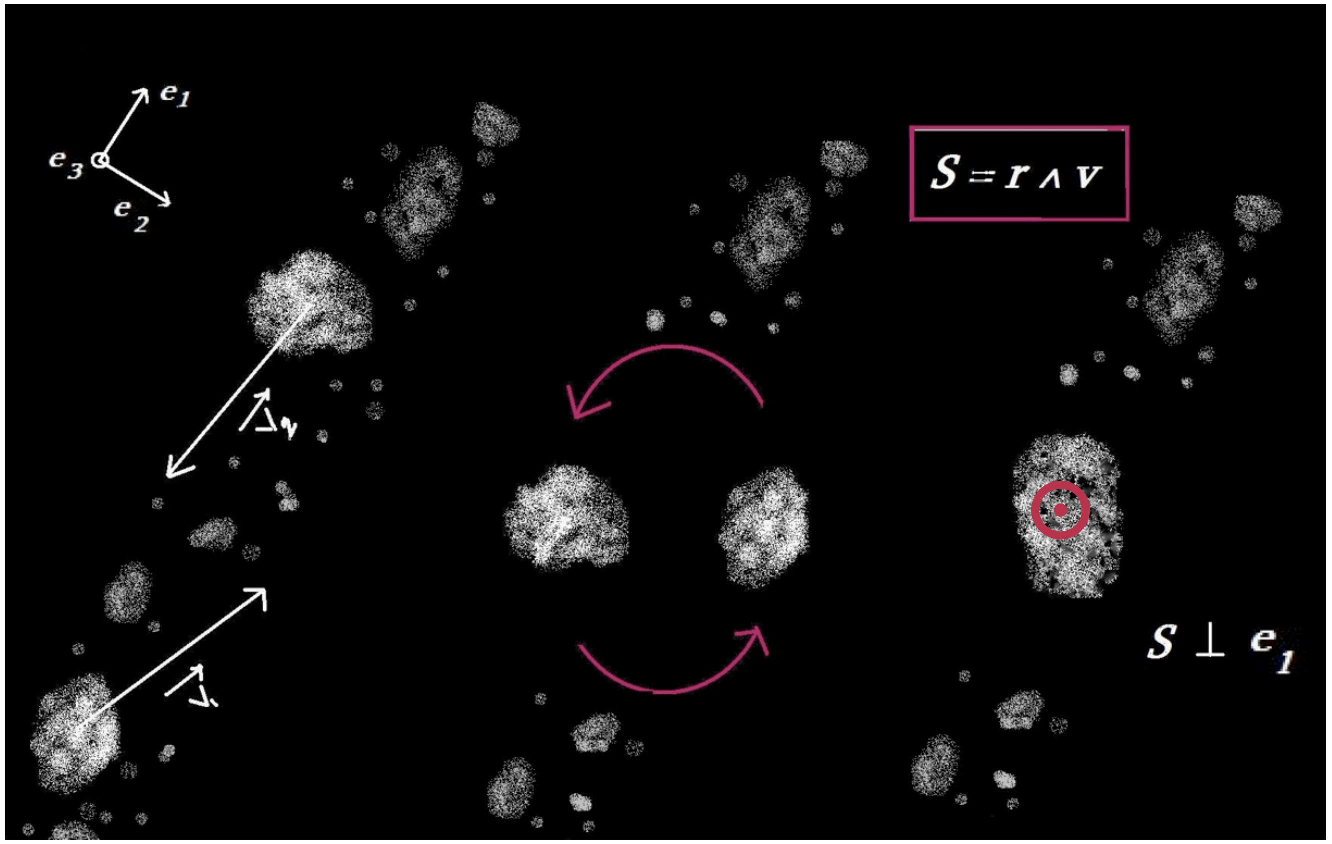
filament disconnect

= cold gas inflow truncation

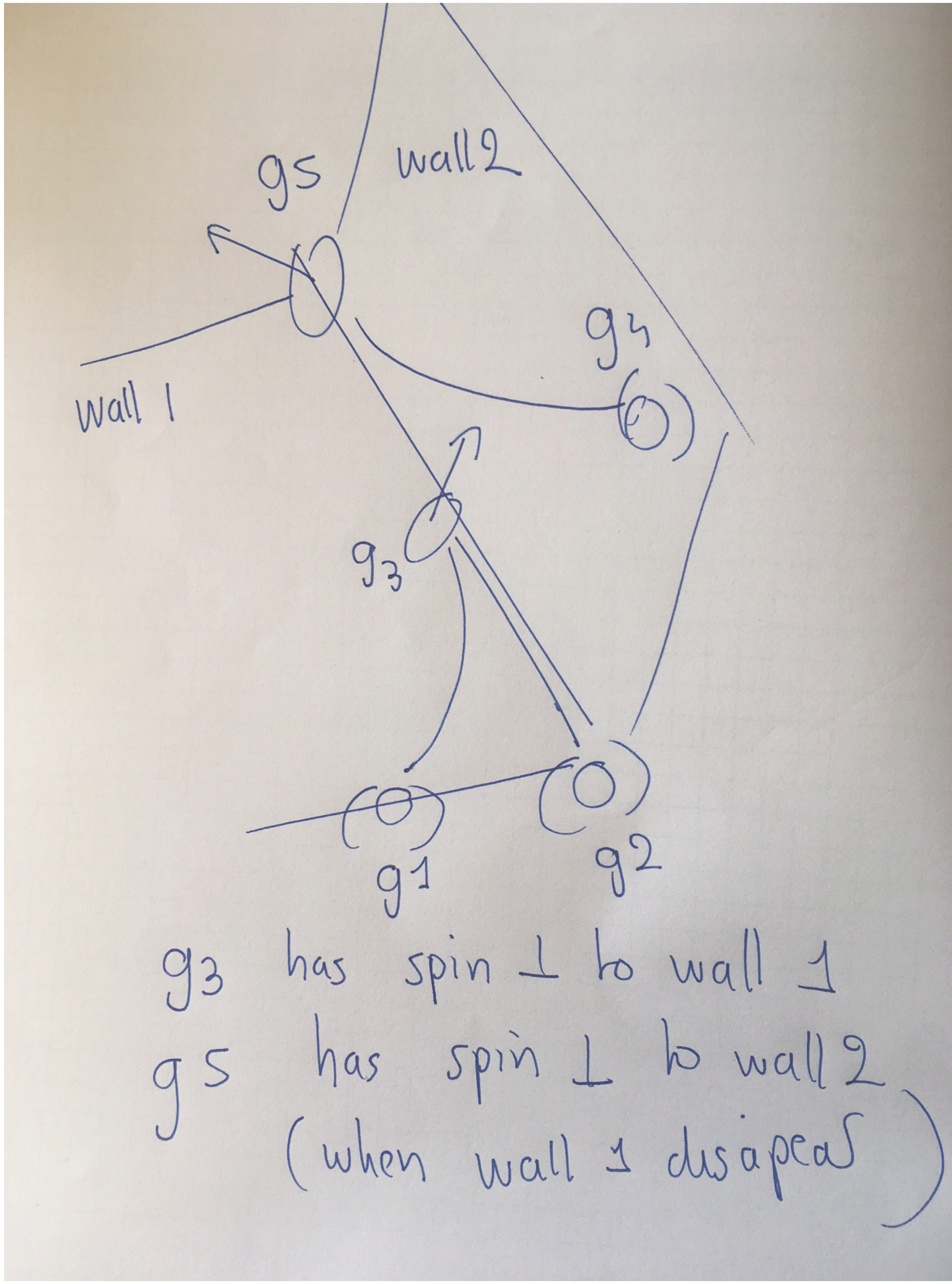


Galactic motivation 2





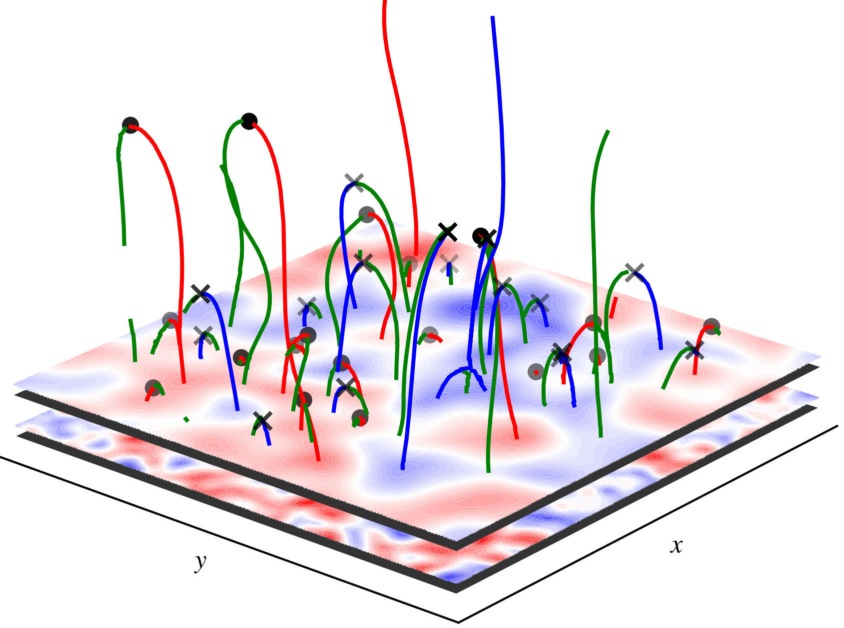
codis et al 2012

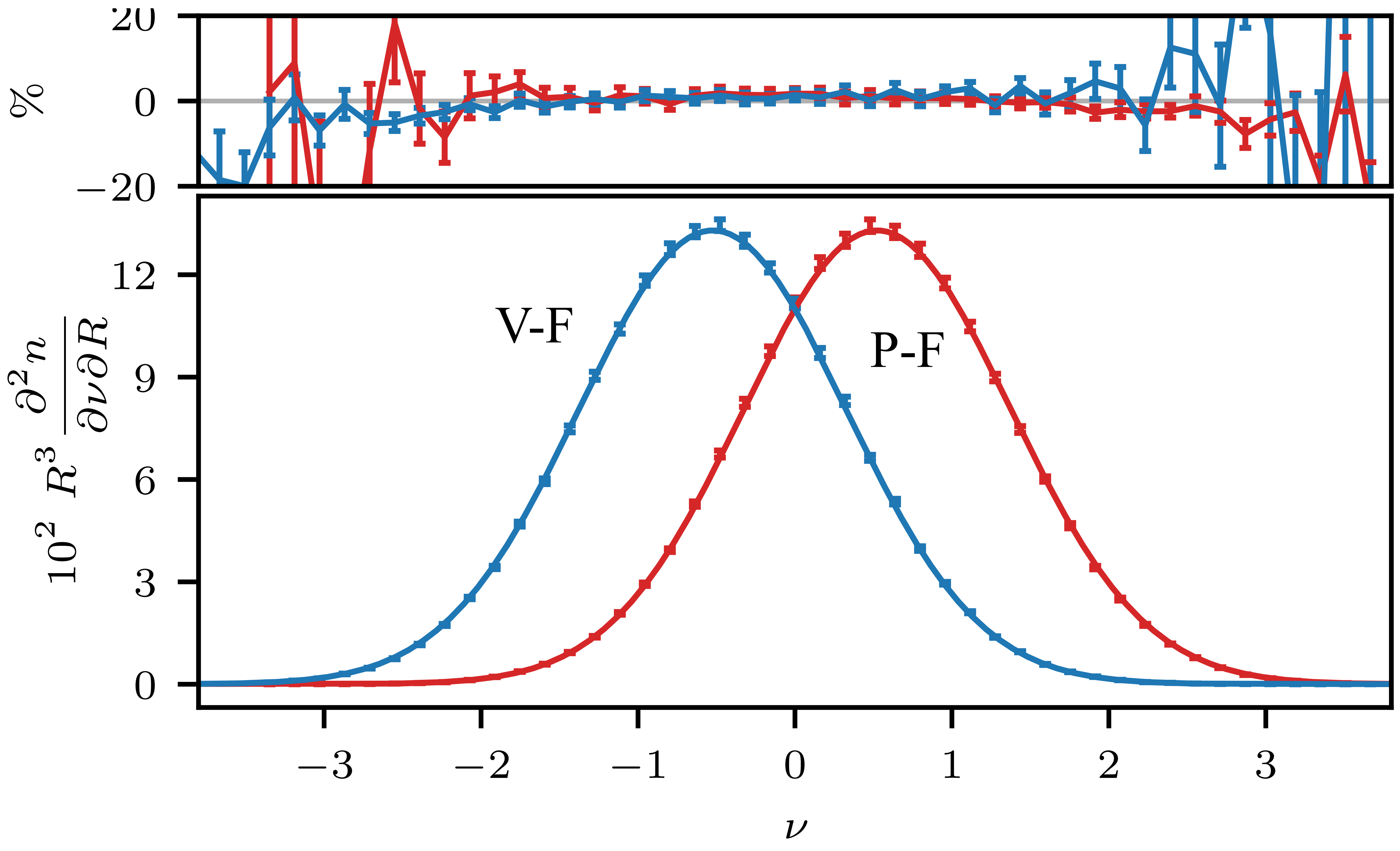


wall disappearance

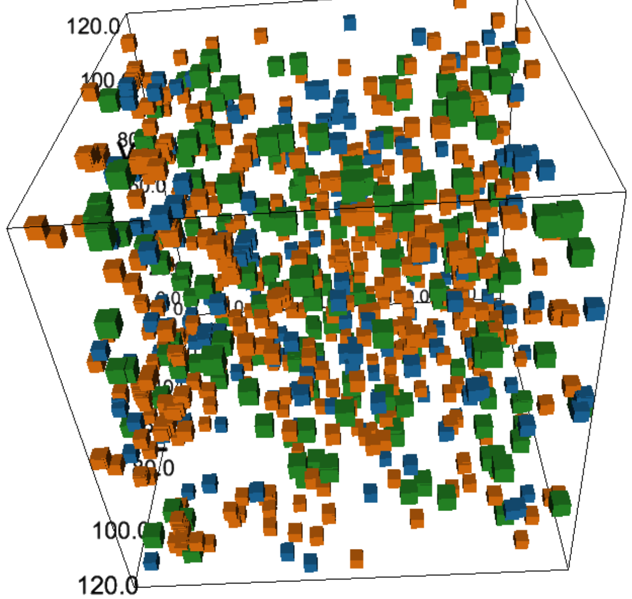
= spin flip

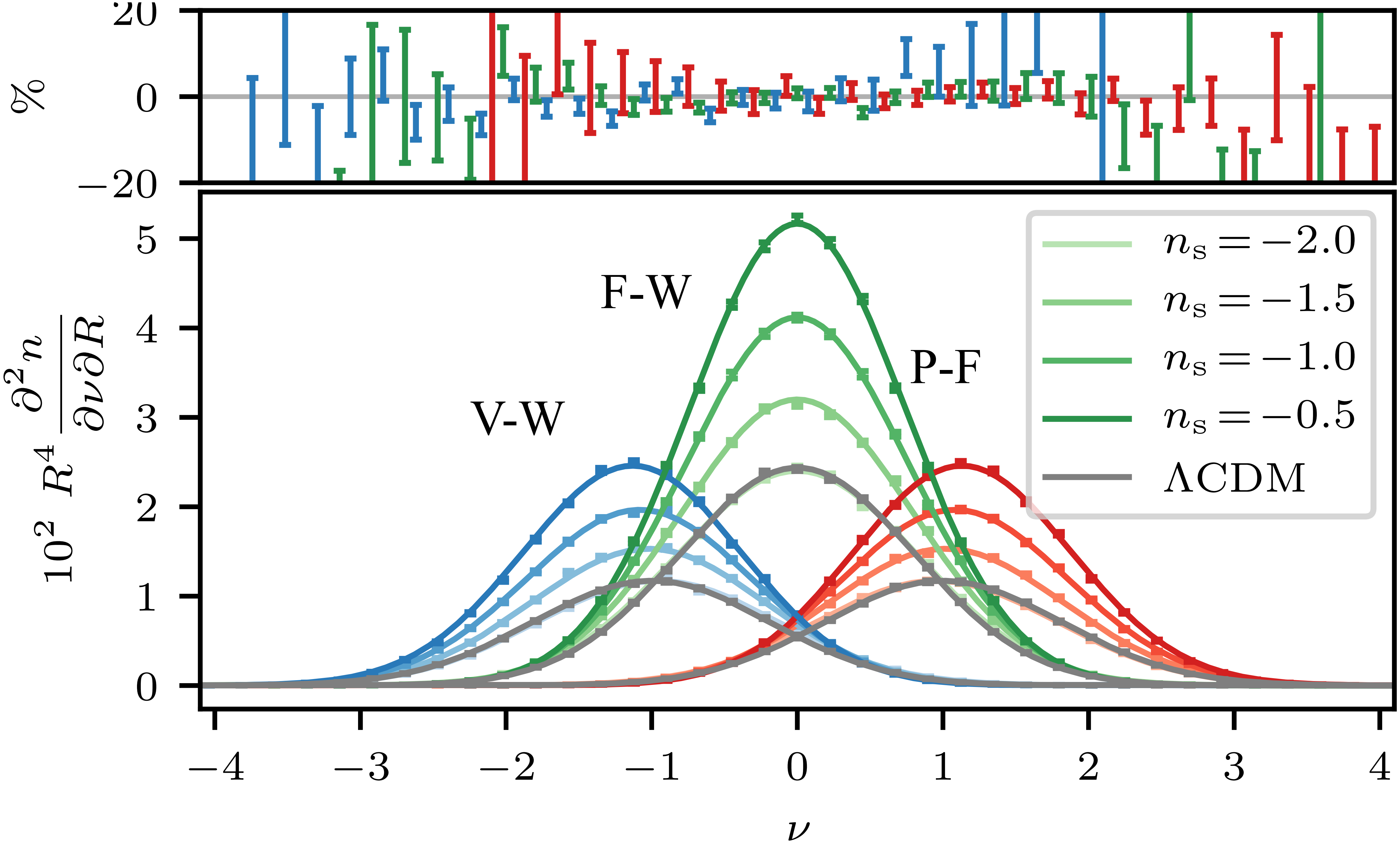
Validation: 2D event counts





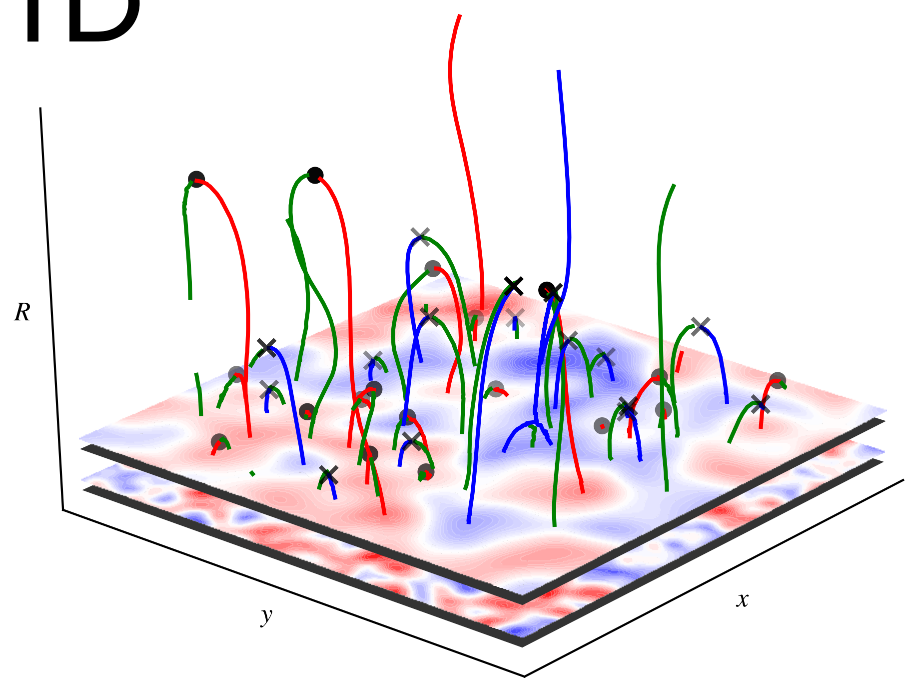
Validation: 3D event counts

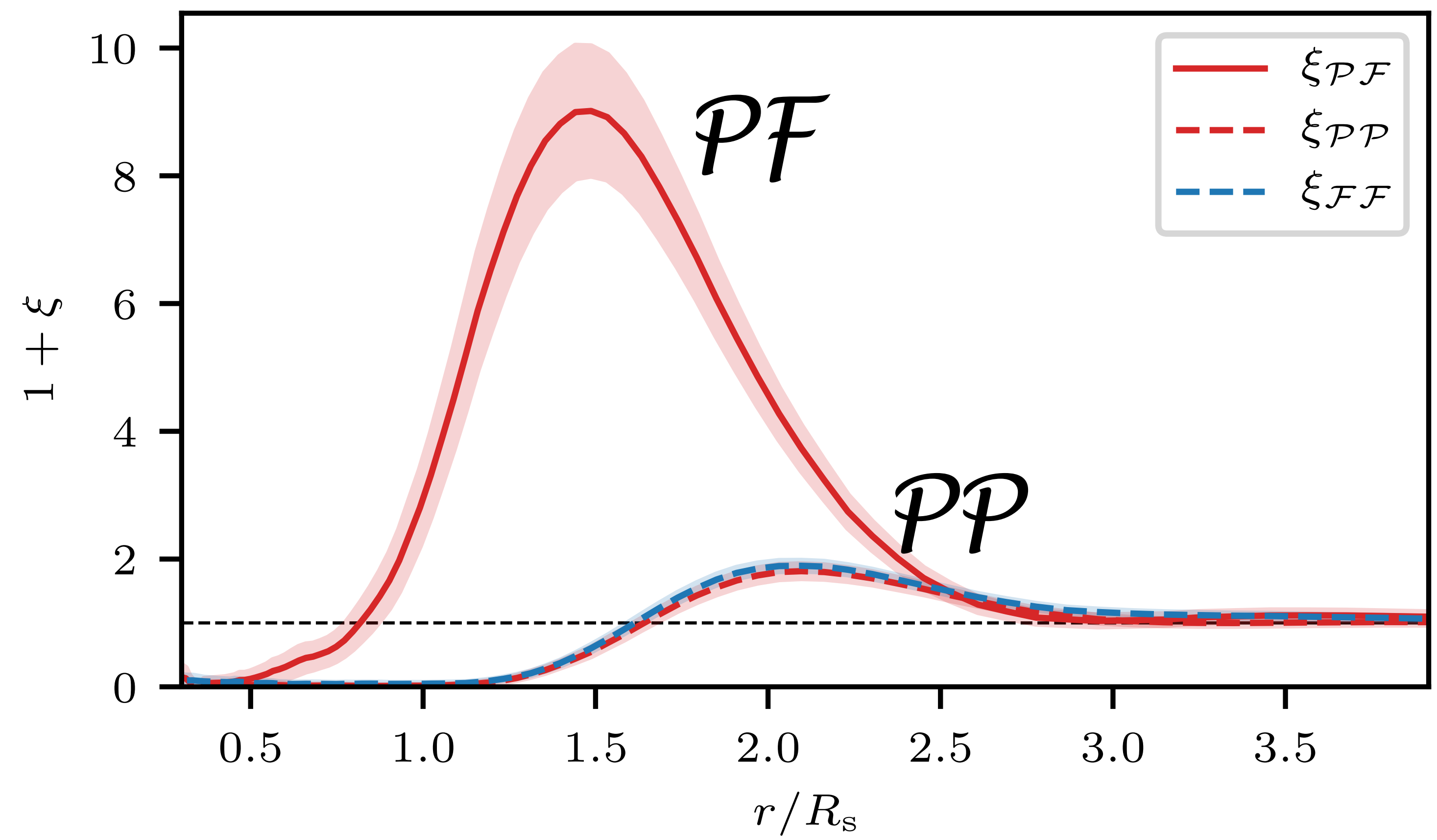




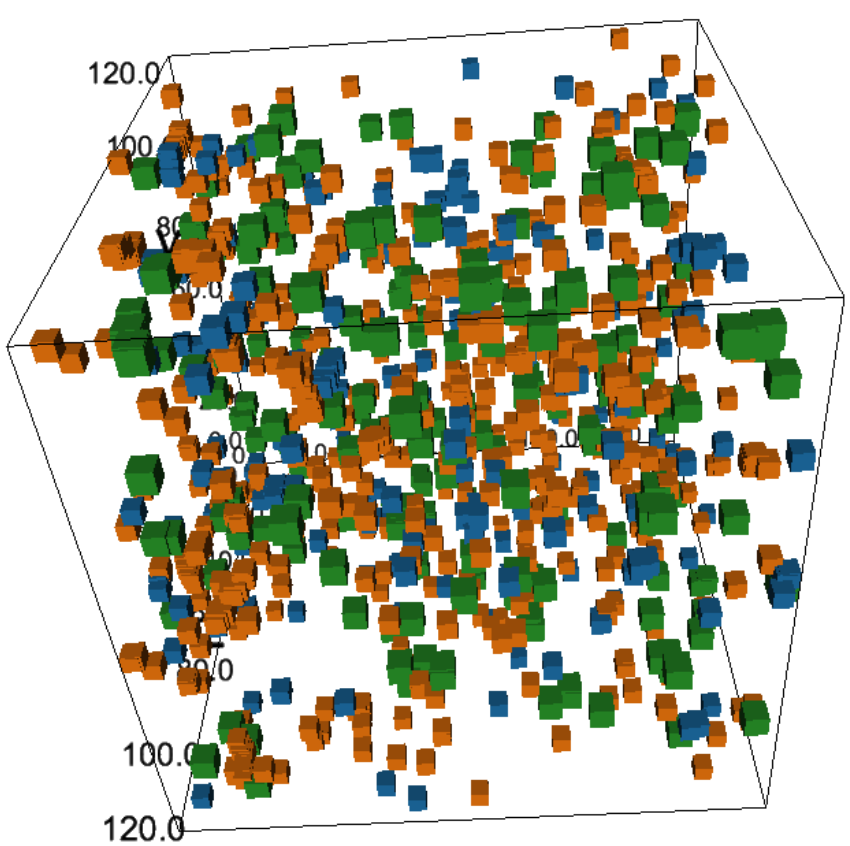
Two-point clustering of events

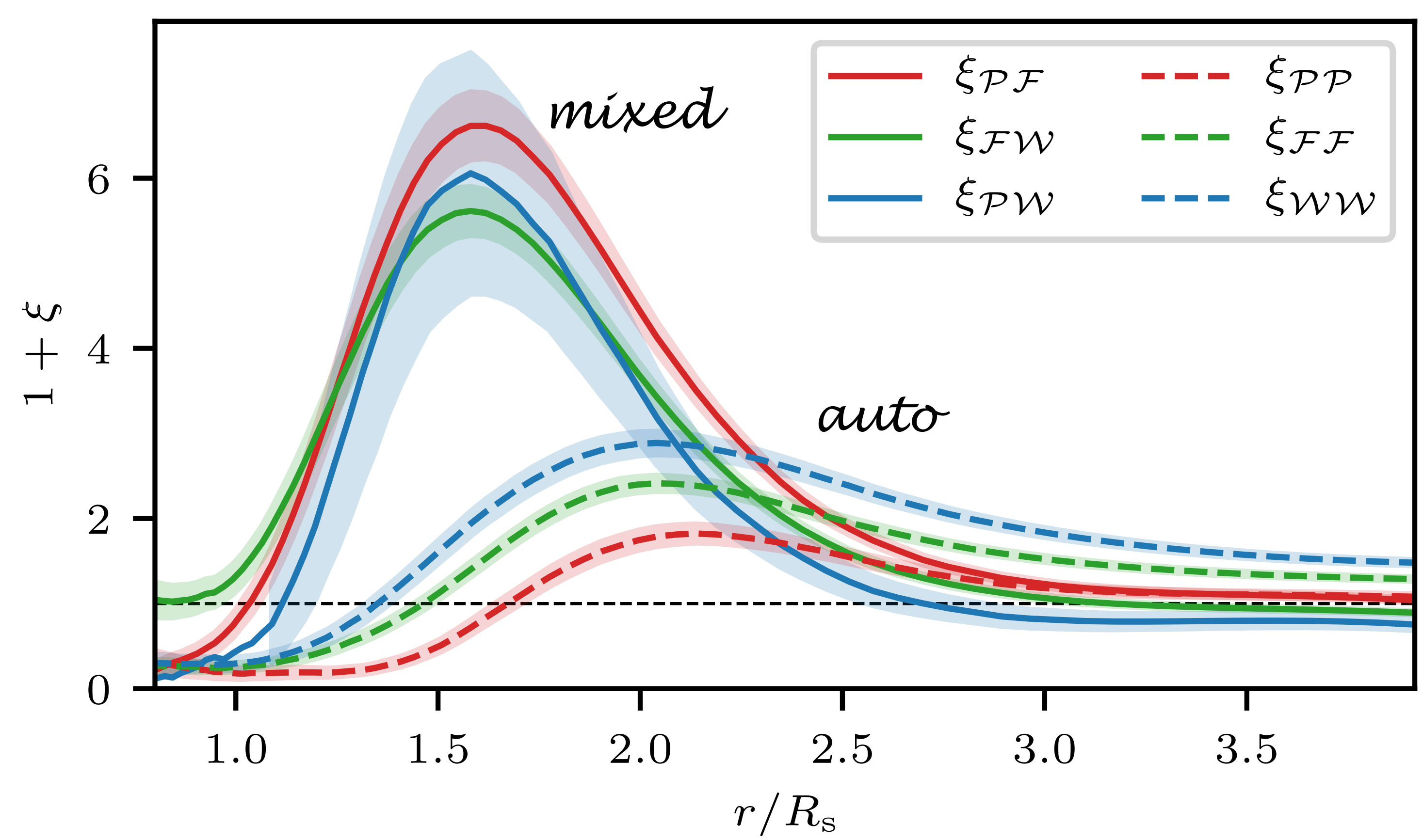
2+1D





3+1D

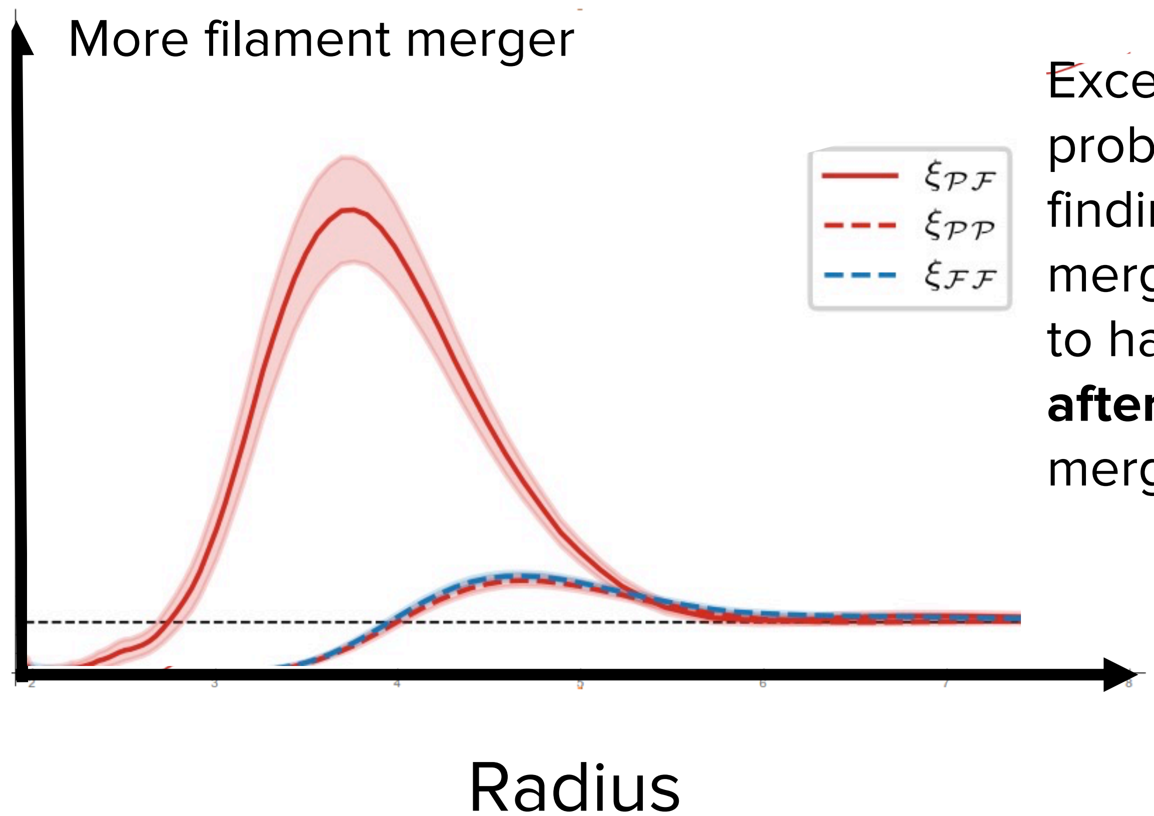




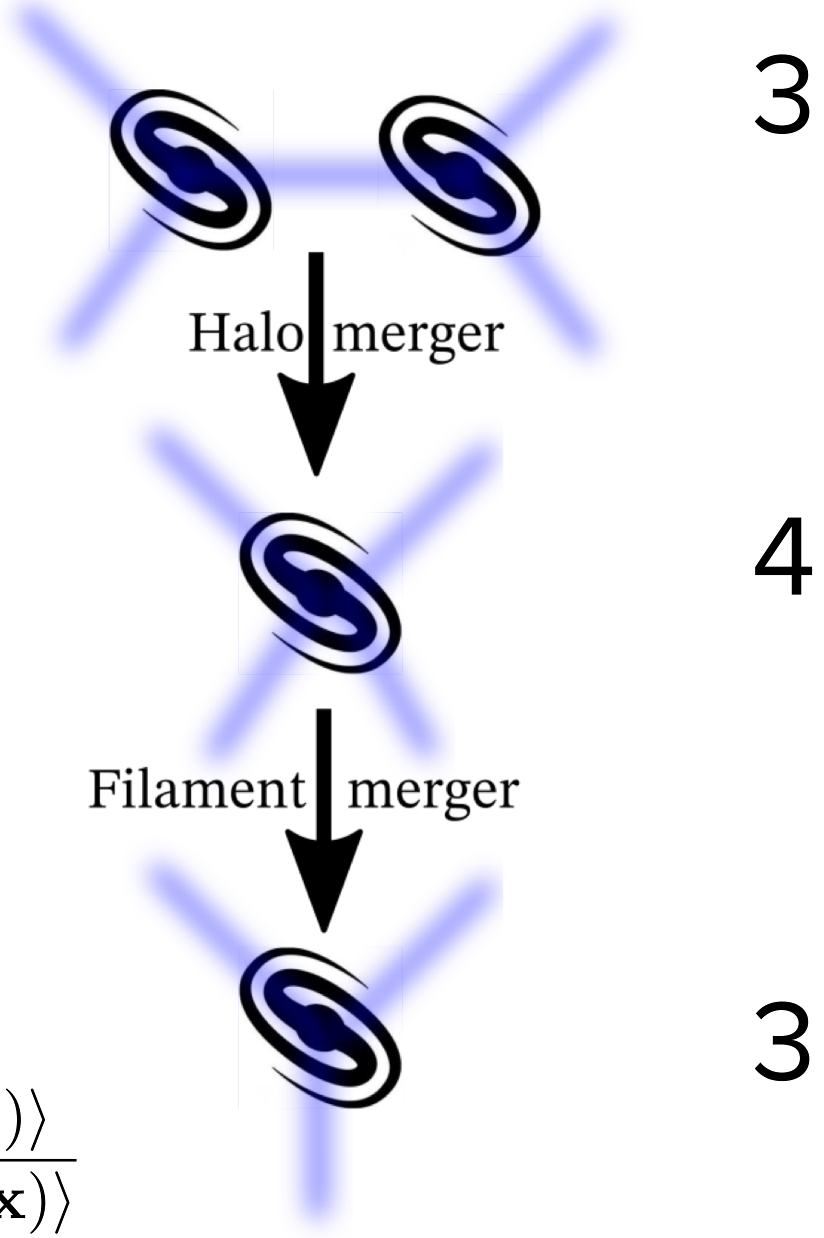
Application: preserving cosmic connectivity

On the connectivity of halos

Compute frequency of filament merger compared to halo merger in the vicinity of a halo merger event $\xi_{\rm hf}(r)$ $\xi_{\rm hh}(r)$.

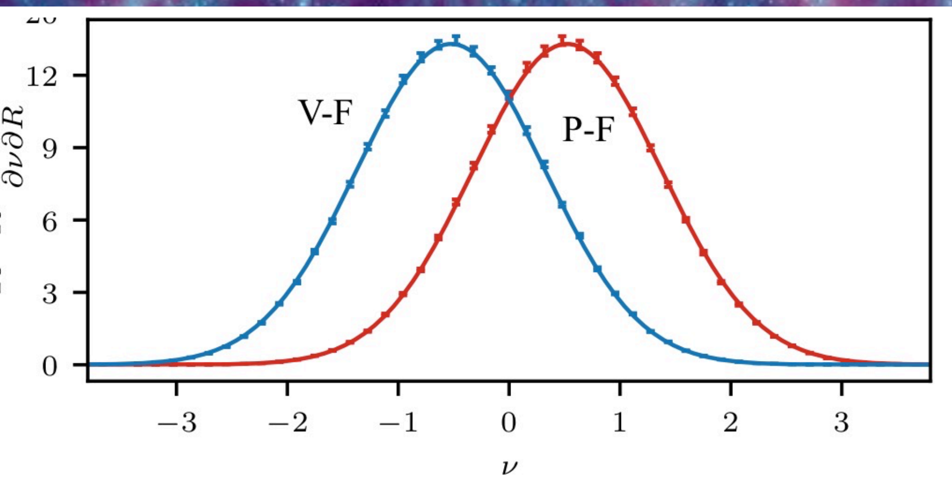


Excess of probability of finding filament merger compared to halo merger **after** a halo merger

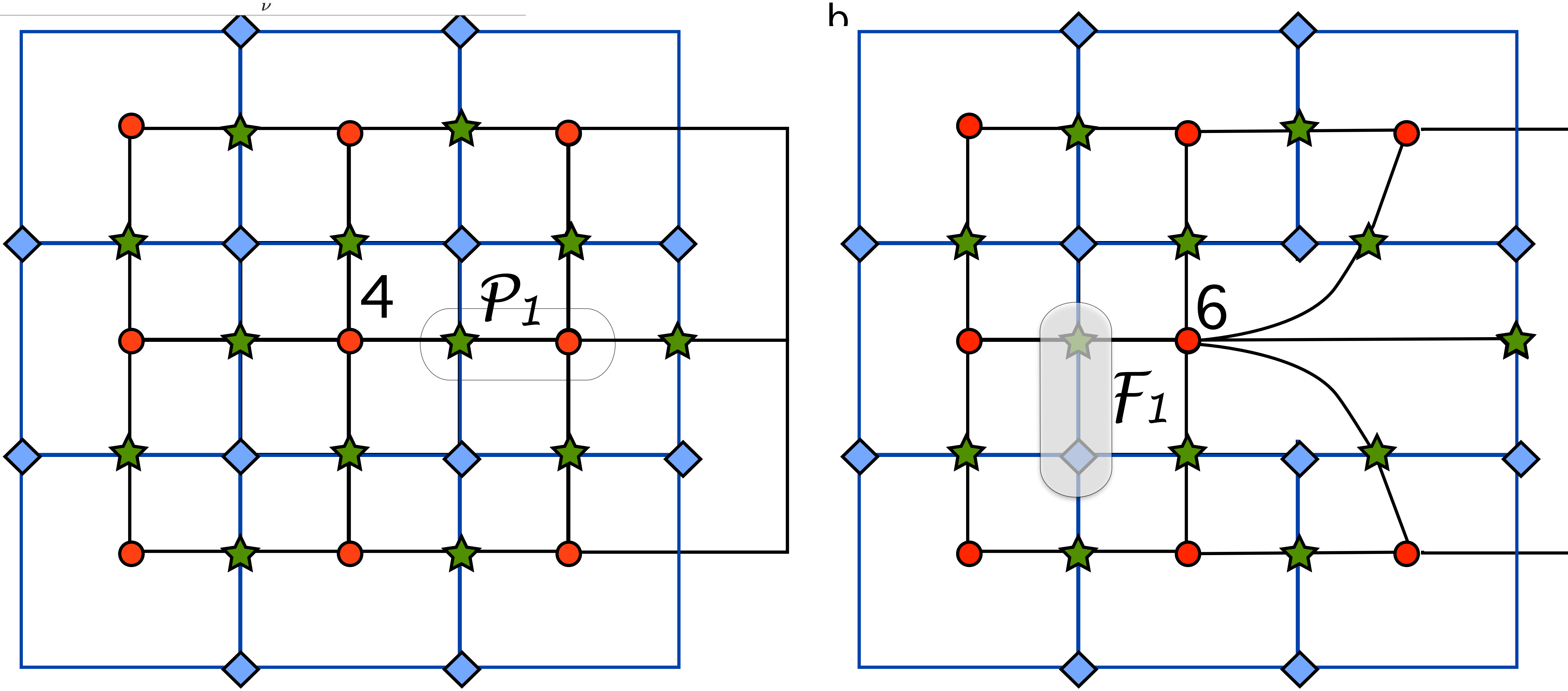


$$1+\xi_p = \frac{\langle \mathrm{cond}_p(\mathbf{x})\mathrm{cond}_p(\mathbf{y})\rangle}{\langle \mathrm{cond}_p(\mathbf{x})\rangle^2}, \quad 1+\xi_f = \frac{\langle \mathrm{cond}_f(\mathbf{x})\mathrm{cond}_p(\mathbf{y})\rangle}{\langle \mathrm{cond}_f(\mathbf{x})\rangle\langle \mathrm{cond}_p(\mathbf{x})\rangle}$$

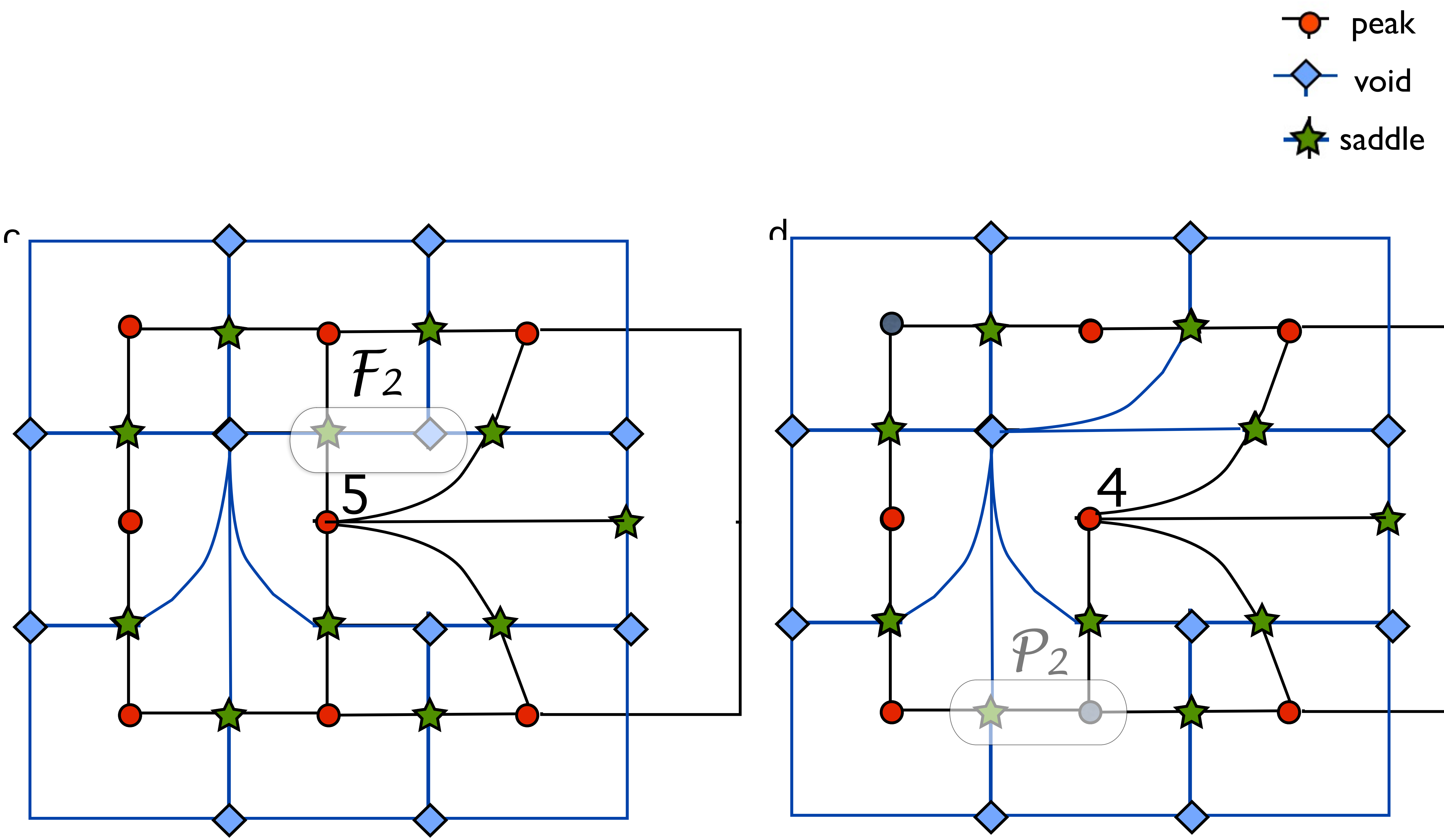
Application: preserving 2D connectivity



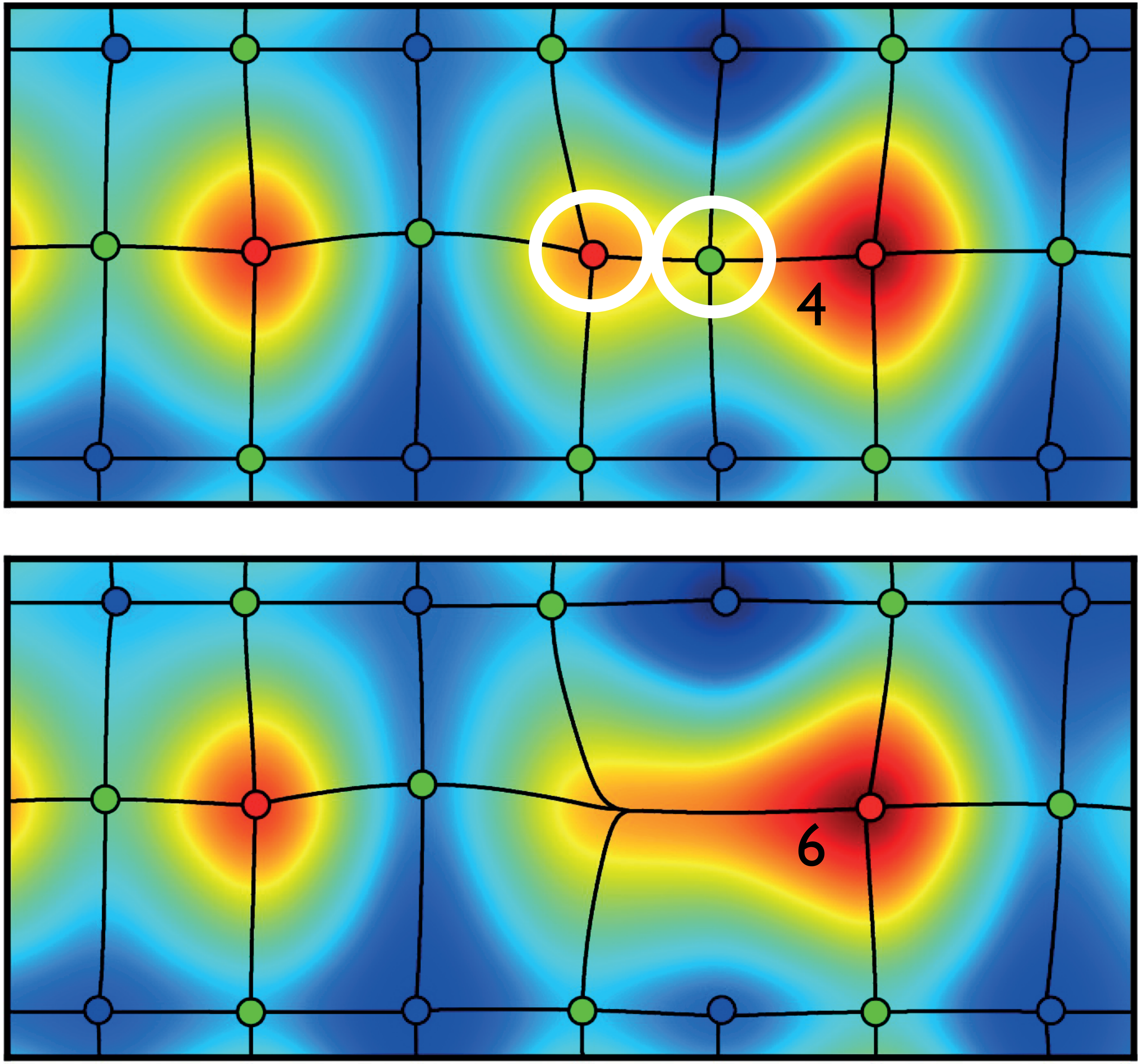
$\mathcal{P}$-$\mathcal{F}$-$\mathcal{F}$-$\mathcal{P}$



Application: preserving 2D connectivity



smoothing cancels low persistence pairs



Critical event PDF: derivation

$$\frac{\partial^2 \mathcal{N}}{\partial r^3 \partial R} \equiv \langle \delta_{\rm D}^{(3)}(\mathbf{r}-\mathbf{r}_0)\delta_{\rm D}(R-R_0)\rangle \,,$$

where $\mathbf{r}_0$ is a (double) critical point in real space and R_0 the scale at which the two critical points merge.

$$d(\delta) \equiv \det(\nabla\nabla\delta) = \lambda_1\lambda_2\lambda_3$$

$$\frac{\partial^2 \mathcal{N}}{\partial r^3 \partial R} = \left\langle J\, \delta_{\rm D}^{(3)}(\nabla\delta)\delta_{\rm D}(d)\right\rangle$$

$$J(d,\delta) = \left| \begin{matrix} \partial_R d & \vec{\nabla} d \\ \partial_R \vec{\nabla}\delta^T & \vec{\nabla}\vec{\nabla}\delta \end{matrix} \right|$$

Critical event PDF: derivation

$$\frac{\partial^2 \mathcal{N}}{\partial r^3 \partial R} = \left\langle J\, \delta_{\rm D}^{(3)}(\nabla\delta)\delta_{\rm D}(d) \right\rangle$$

$$J(d,\delta) = \begin{vmatrix} \partial_R d & \vec{\nabla} d \\ \partial_R \vec{\nabla}\delta^T & \vec{\nabla}\vec{\nabla}\delta \end{vmatrix} = \begin{vmatrix} \partial_R d & \vec{\nabla} d \\ -R\vec{\nabla}\Delta\delta^T & \vec{\nabla}\vec{\nabla}\delta \end{vmatrix},$$

$$\frac{J(d,\delta)}{\sigma_1\sigma_2^4\sigma_3} = |x_{11}x_{22}| \begin{vmatrix} \partial_R x_{33} & x_{33i} \\ \partial_R x_i & x_{ij} \end{vmatrix},$$

$$= |x_{11}x_{22}| \begin{vmatrix} \partial_R x_{33} & x_{133} & x_{233} & x_{333} \\ \partial_R x_1 & x_{11} & 0 & 0 \\ \partial_R x_2 & 0 & x_{22} & 0 \\ \partial_R x_3 & 0 & 0 & 0 \end{vmatrix}$$

$$= |x_{11}x_{22}|^2 |\partial_R x_3| |x_{333}|,$$

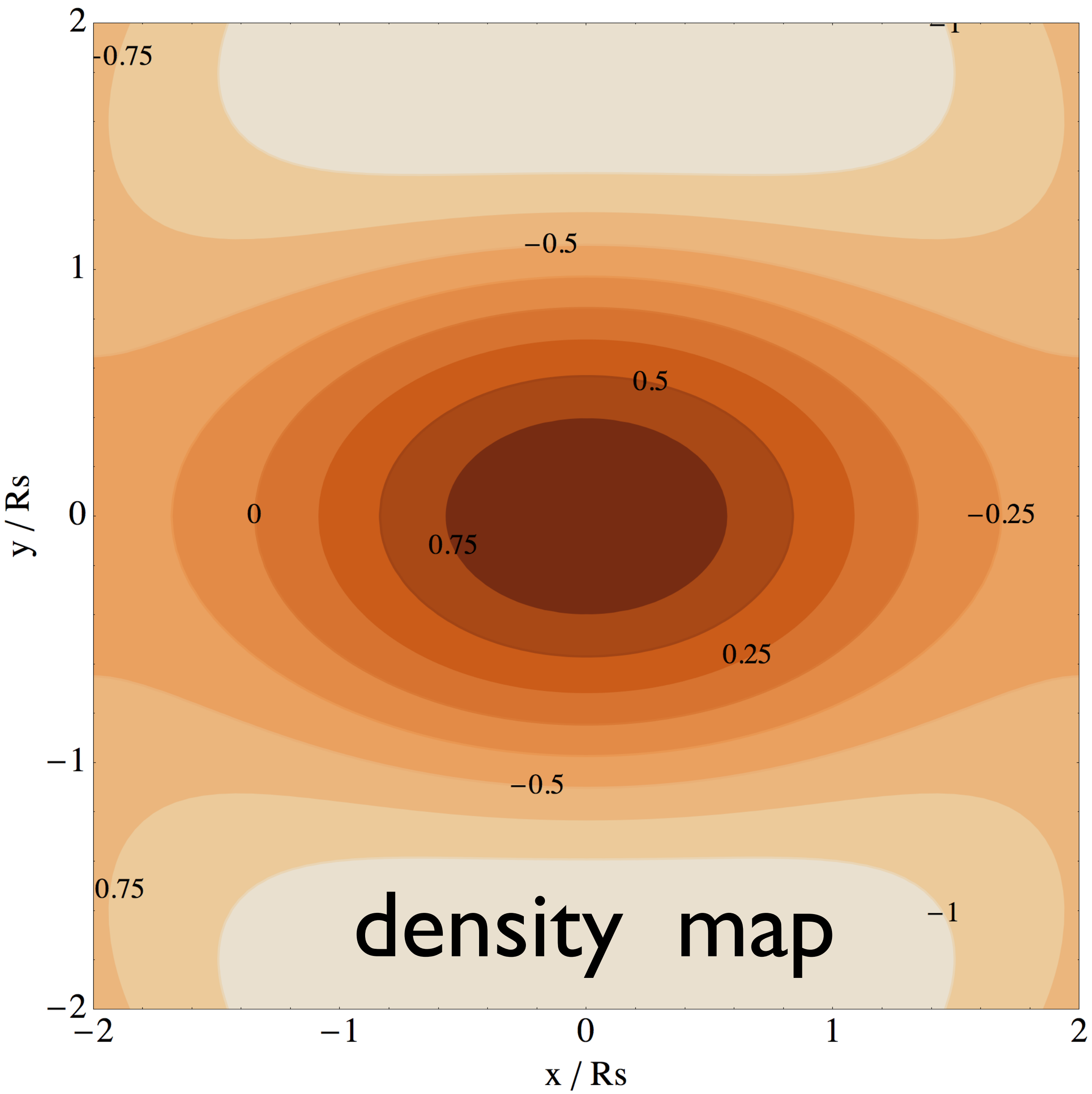
$$x \equiv \frac{\delta}{\sigma_0},\; x_k \equiv \frac{\nabla_k\delta}{\sigma_1},\; x_{kl} \equiv \frac{\nabla_k\nabla_l\delta}{\sigma_2},\; x_{klm} \equiv \frac{\nabla_m\nabla_l\nabla_k\delta}{\sigma_3}$$

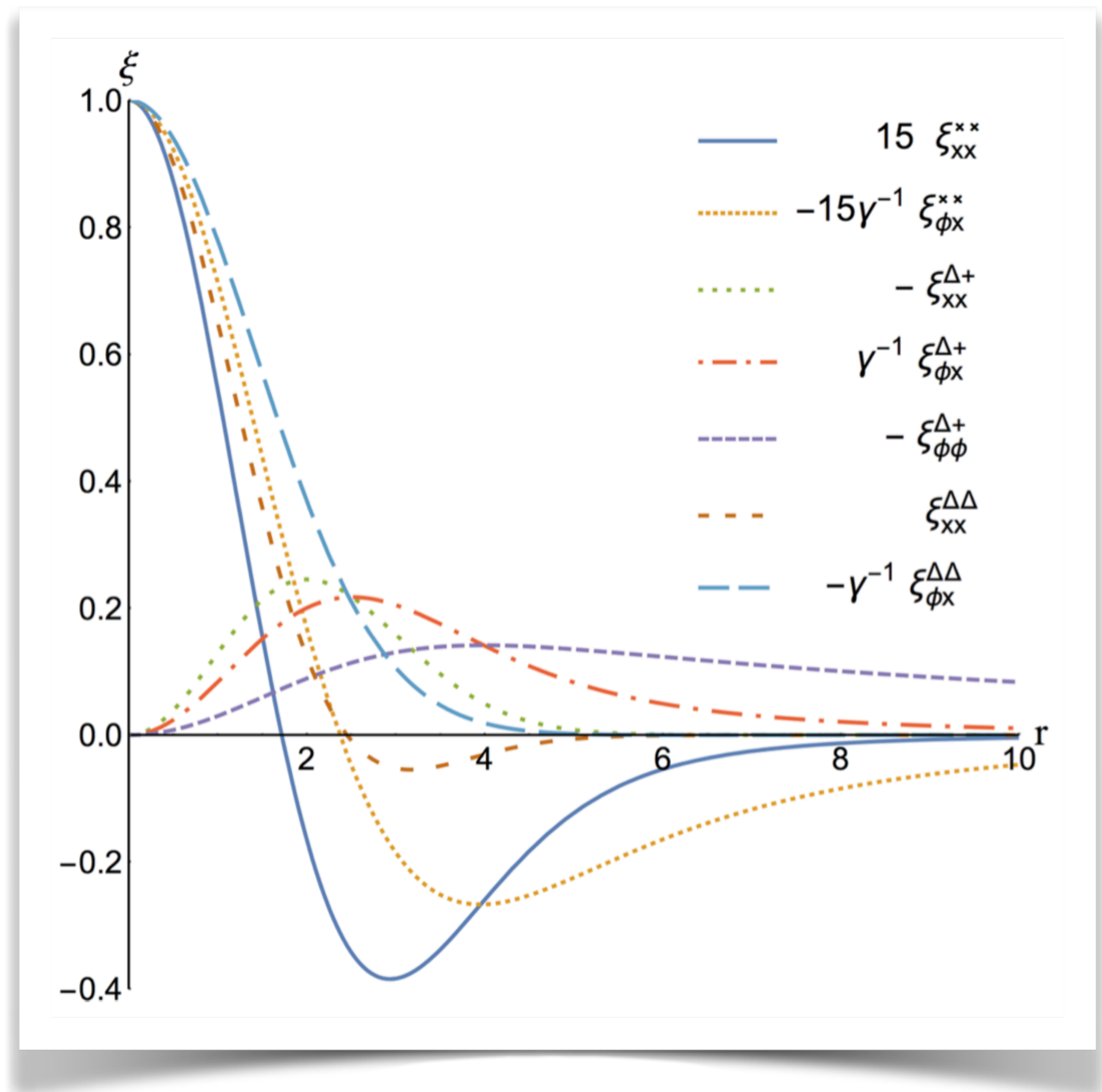
2D Theory of Tidal Torque @ saddle?

$$\delta(\mathbf{r},\kappa,I_1,\nu|\mathrm{ext}) = \frac{I_1(\xi^{\Delta\Delta}_{\phi\delta}+\gamma\xi^{\Delta\Delta}_{\phi\phi})+\nu(\xi^{\Delta\Delta}_{\phi\phi}+\gamma\xi^{\Delta\Delta}_{\phi\delta})}{1-\gamma^2} + 4\left(\hat{\mathbf{r}}^{\mathrm{T}}\cdot\overline{\mathbf{H}}\cdot\hat{\mathbf{r}}\right)\xi^{\Delta+}_{\phi\delta}\,,$$

Hessian

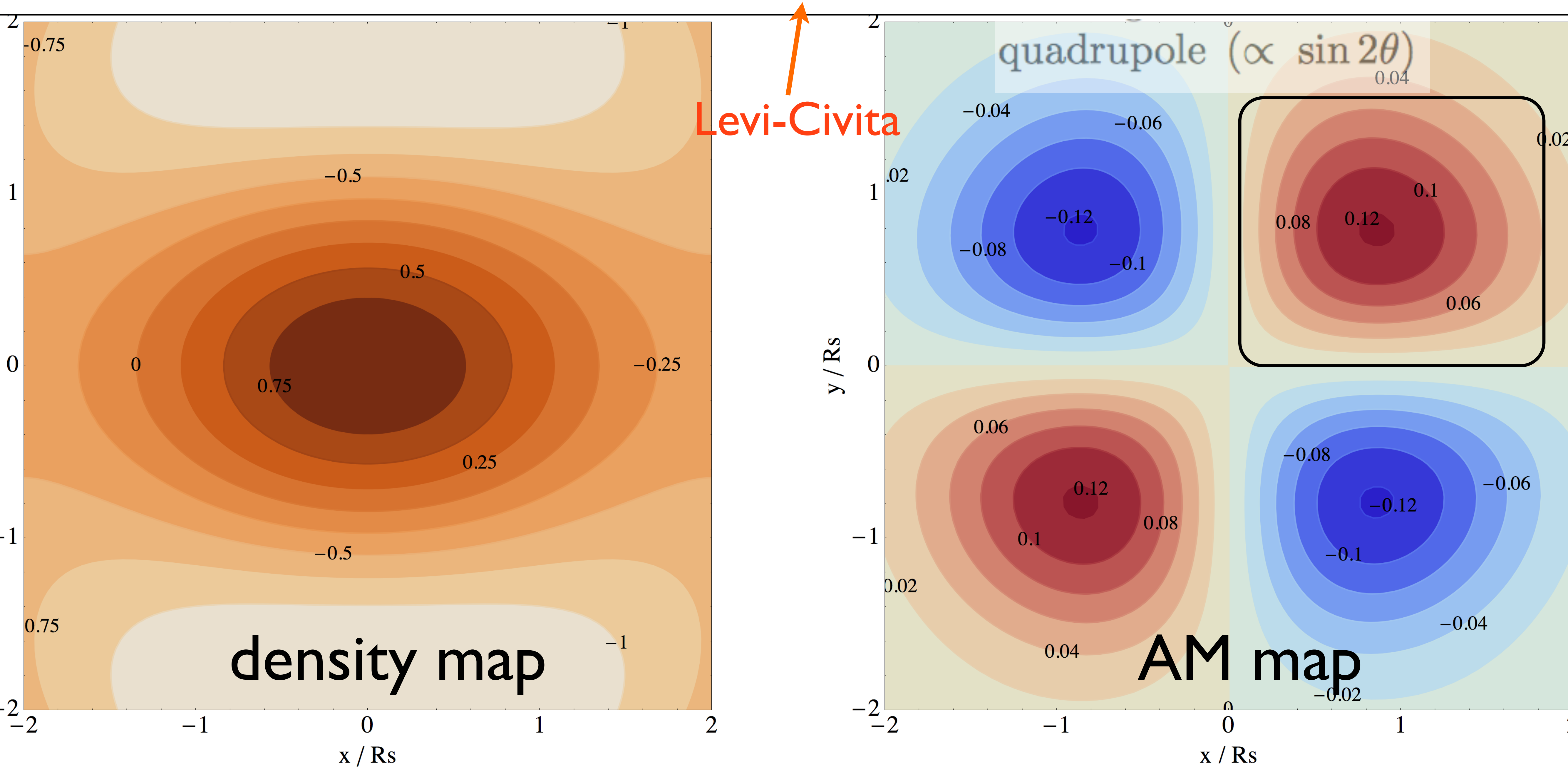
$$f^{+} = (f_{11}-f_{22})/2 \text{ and } f^{\times} = f_{12}.$$





2D Theory of Tidal Torque @ saddle?

$$\langle L_z|\mathrm{ext}\rangle = L_z(\mathbf{r},\kappa,I_1,\nu|\mathrm{ext}) = -16(\hat{\mathbf{r}}^{\mathrm{T}}\cdot\epsilon\cdot\overline{\mathbf{H}}\cdot\hat{\mathbf{r}})\left(L_z^{(1)}(r)+2(\hat{\mathbf{r}}^{\mathrm{T}}\cdot\overline{\mathbf{H}}\cdot\hat{\mathbf{r}})L_z^{(2)}(r)\right)$$

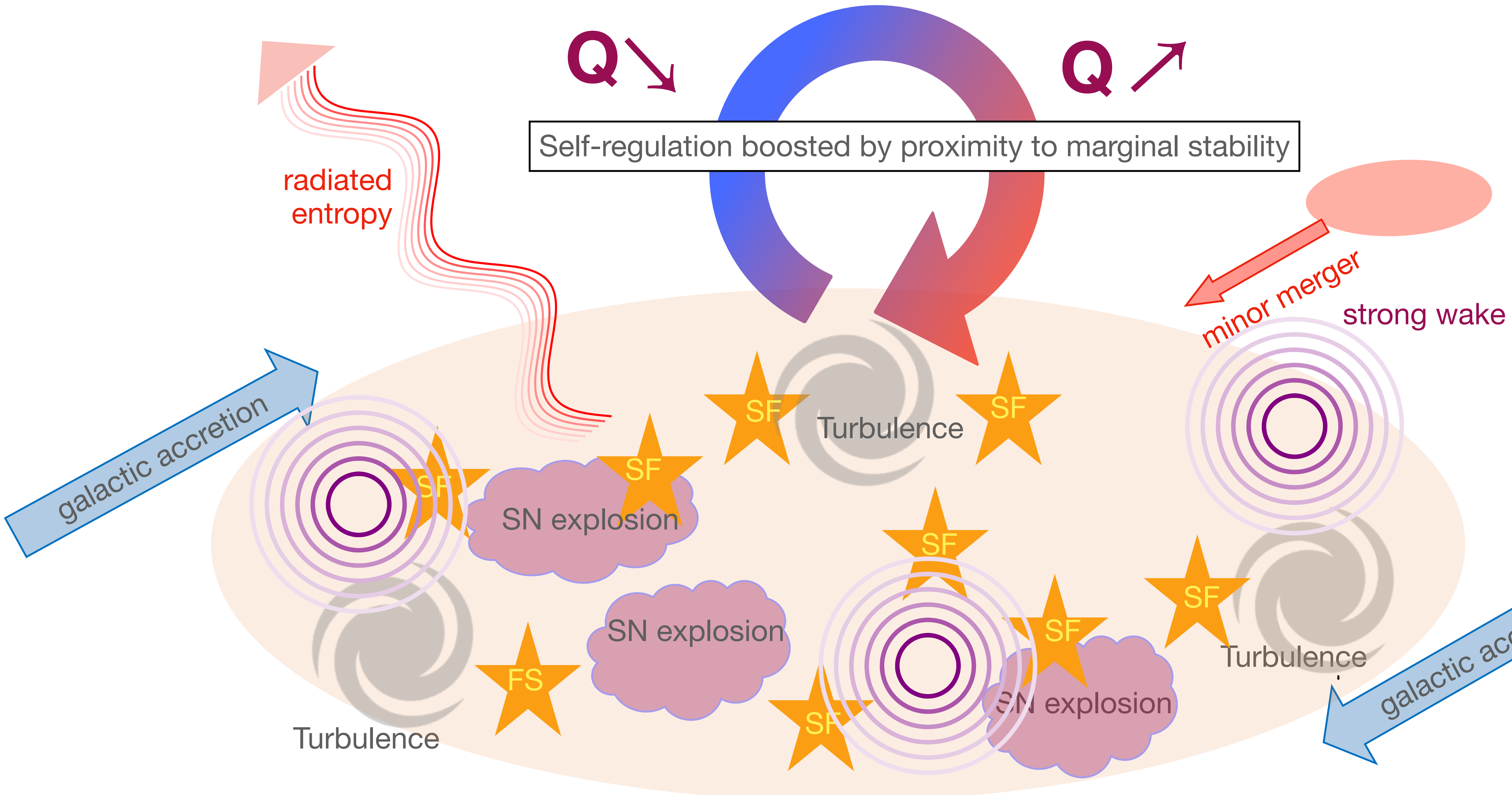


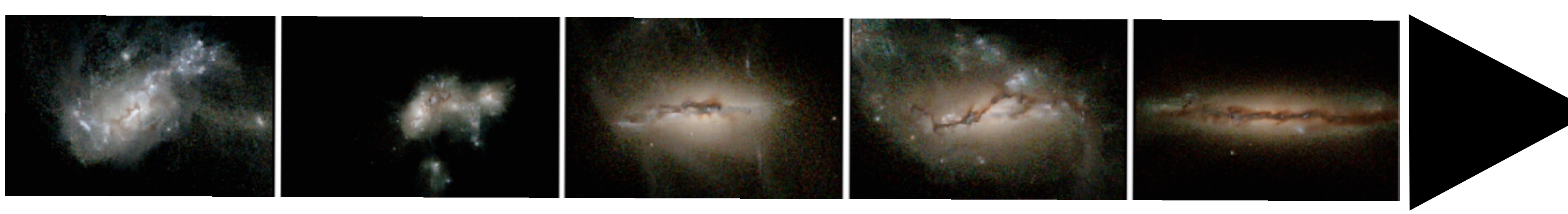
$$L_z^{(1)}(r) = \frac{\nu}{1-\gamma^2}\left[(\xi^{\Delta+}_{\phi\phi}+\gamma\xi^{\Delta+}_{\phi\delta})\xi^{\times\times}_{\delta\delta} - (\xi^{\Delta+}_{\phi\delta}+\gamma\xi^{\Delta+}_{\delta\delta})\xi^{\times\times}_{\phi\delta}\right]$$

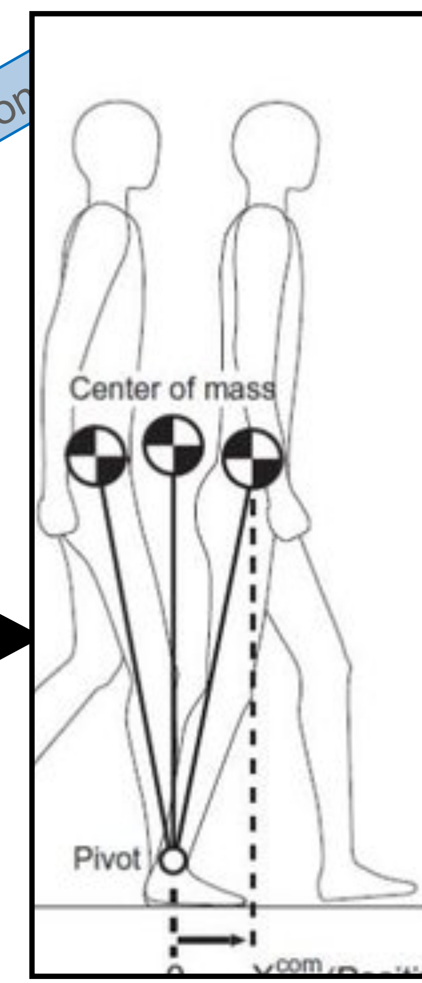
$$L_z^{(2)}(r) = (\xi^{\Delta\Delta}_{\phi x}\xi^{\times\times}_{\delta\delta} - \xi^{\times\times}_{\phi\delta}\xi^{\Delta\Delta}_{\delta\delta}) + \frac{I_1}{1-\gamma^2}\left[(\xi^{\Delta+}_{\phi\delta}+\gamma\xi^{\Delta+}_{\phi\phi})\xi^{\times\times}_{\delta\delta} - (\xi^{\Delta+}_{\delta\delta}+\gamma\xi^{\Delta+}_{\phi\delta})\xi^{\times\times}_{\phi\delta}\right]$$

Bring home message: dressing redefine clocks

New dynamical equilibrium







Wake drastically boost orbital frequencies,
tightening control loops

Impact of CGM growth

Lagrange Laplace theory of rings *(small eccentricity small inclinaison)*

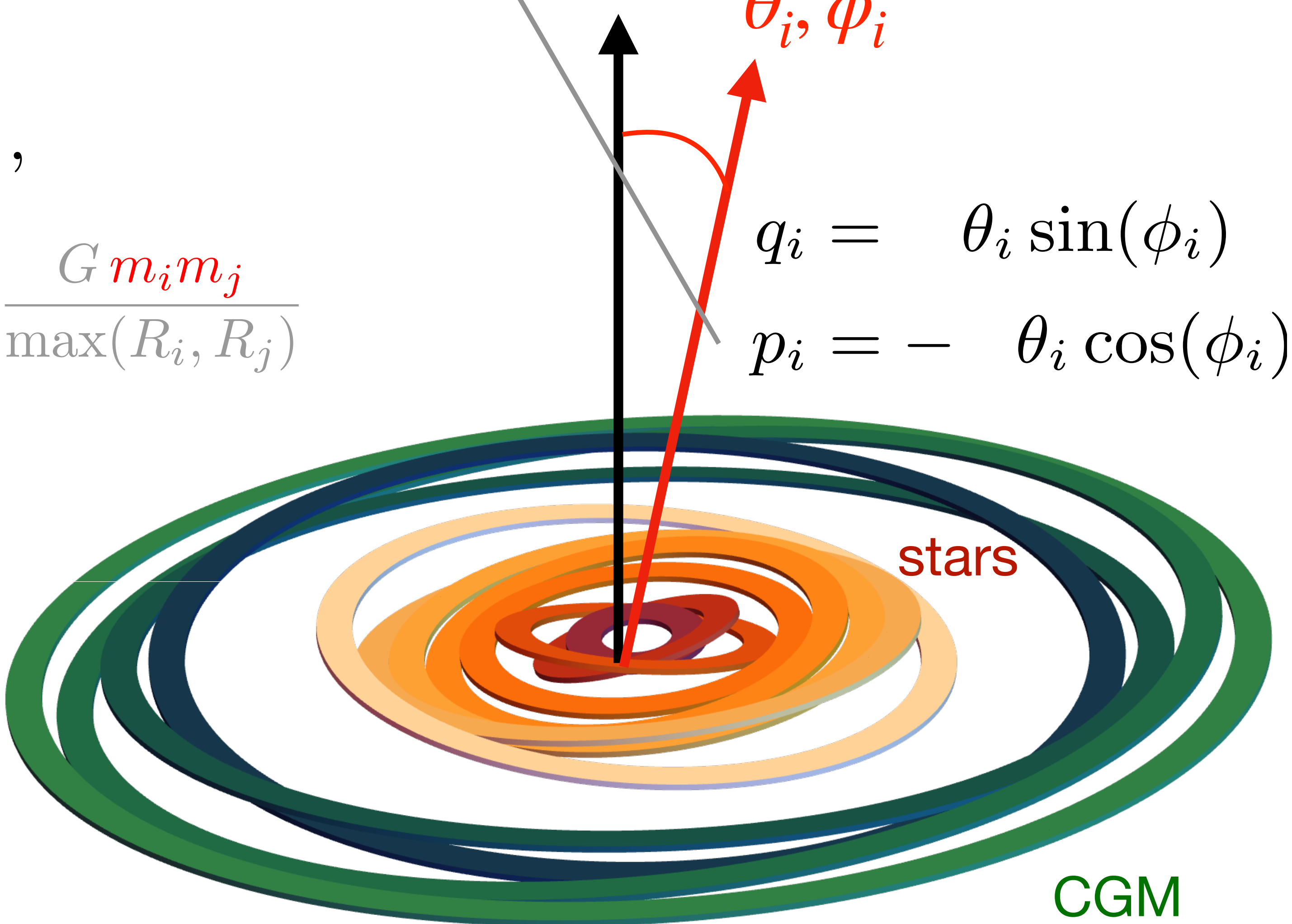
x and y components of **angular momentum**

$$H(\mathbf{p},\mathbf{q}) = \frac{1}{2}\mathbf{p}^{\mathrm{T}}\cdot\mathbf{A}\cdot\mathbf{p} + \frac{1}{2}\mathbf{q}^{\mathrm{T}}\cdot\mathbf{A}\cdot\mathbf{q}\,,$$

$$A_{ij} \propto -\frac{G\, m_i m_j}{\max(R_i, R_j)}$$

$$q_i = \theta_i \sin(\phi_i)$$

$$p_i = -\theta_i \cos(\phi_i)$$



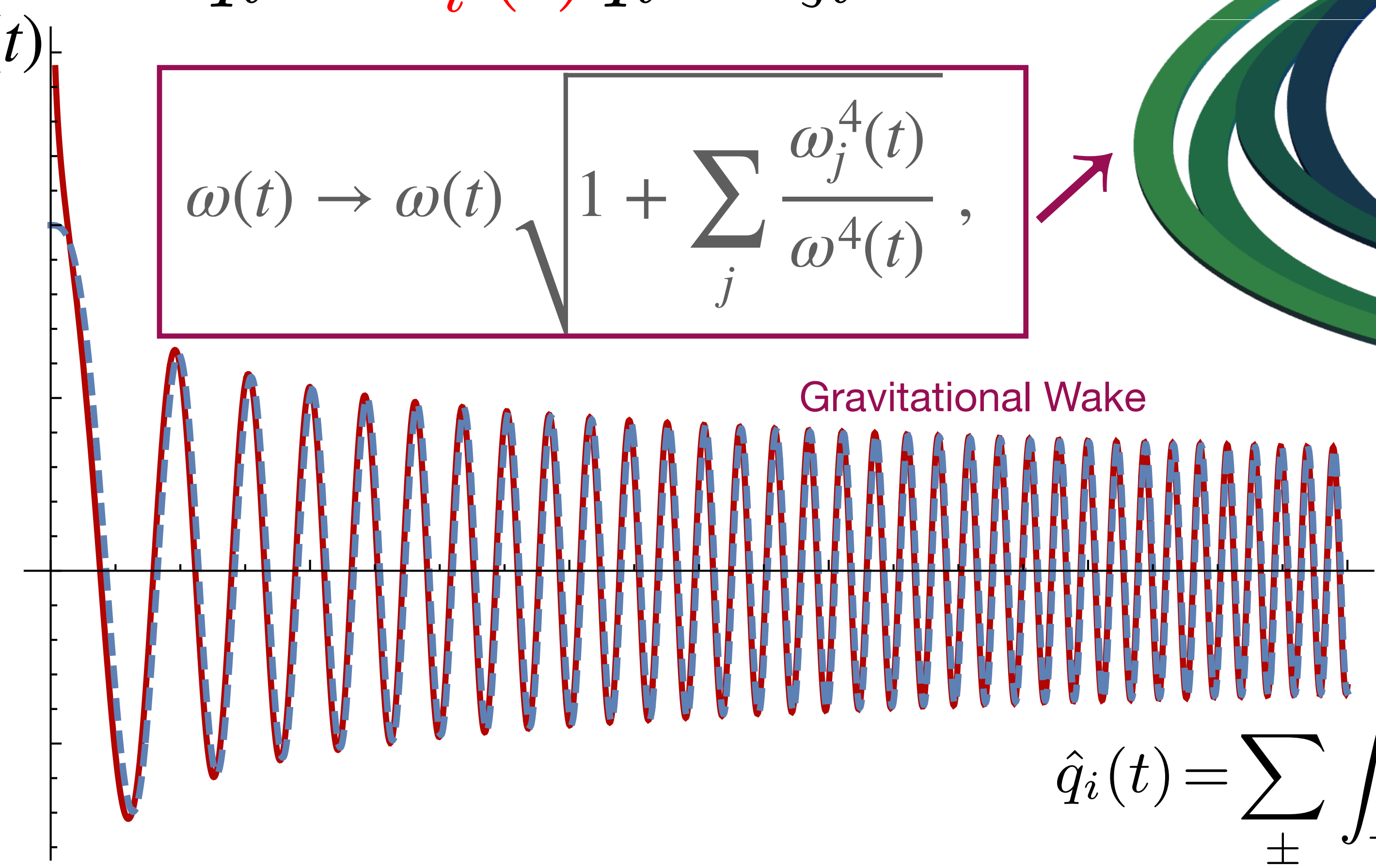
In eigenframe of A

Eigen frequency

$$\ddot{\hat{q}}_i + \omega_i^2(t)\hat{q}_i = \xi_i$$

forcing

$$\omega(t) \to \omega(t)\sqrt{1 + \sum_j \frac{\omega_j^4(t)}{\omega^4(t)}}\,,$$



Secular WKB solution

$$\hat{q}_i(t) = \sum_{\pm}\int_{-\infty}^{\infty}\frac{\hat{\xi}_i(t')}{\sqrt{\omega_i(t)\omega_i(t')}}\exp\left(\pm\imath\int_{t'}^{t}\omega_i(\tau)d\tau\right)dt'$$

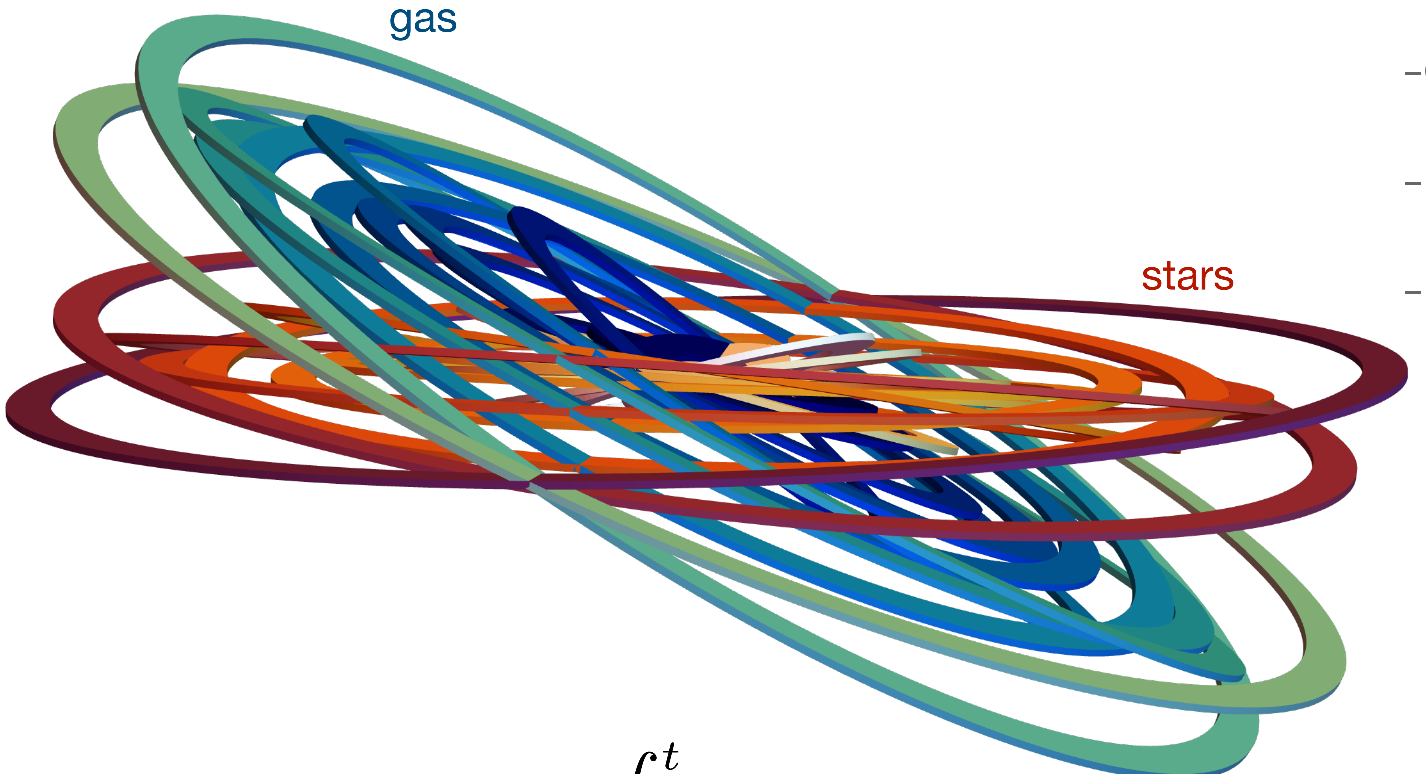
*Growth of CGM component **also** brings down the ★ modes*

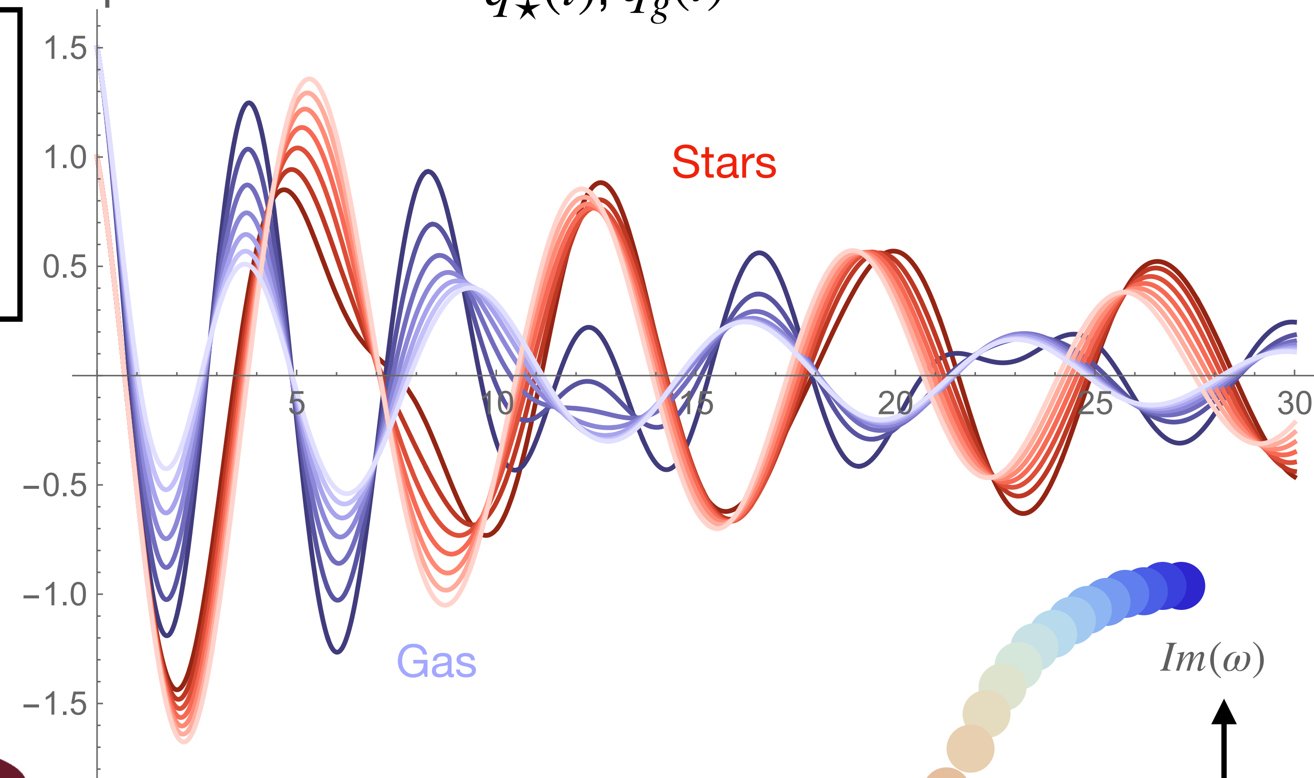
Ring Toy model: gas + star coupling

gravitational coupling — damping

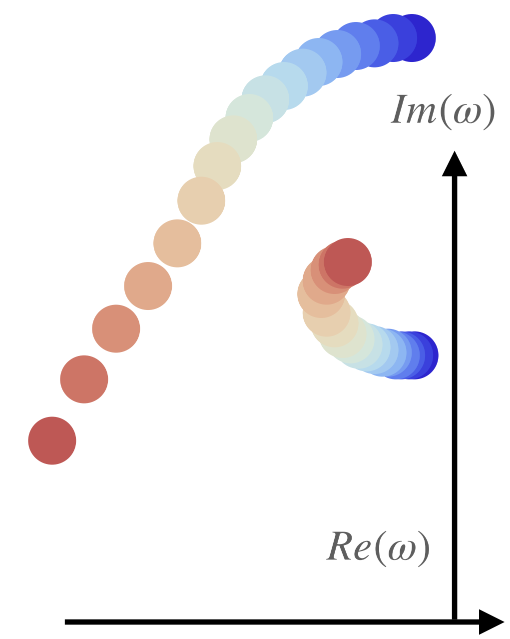
$$\ddot{q}_\star + \omega_\star^2 q_\star + \omega_{\star g}^2 q_g = 0\,,$$
$$\ddot{q}_g + \omega_g^2 \hat{q}_g + \omega_{\star g}^2 q_\star + \eta \dot{q}_g = \xi\,,$$

forcing



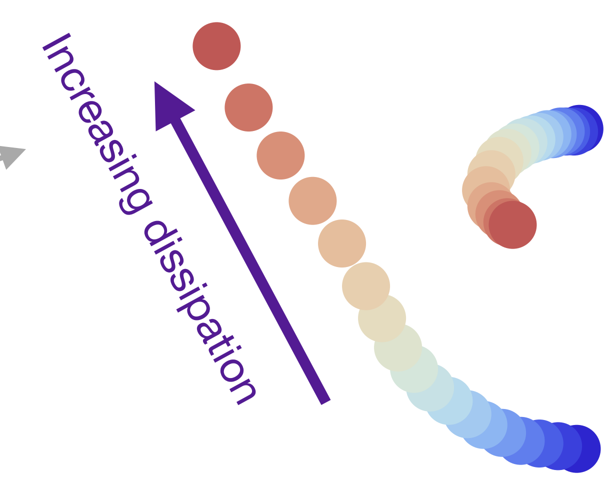


Nyquist diagram



$$q_\star(t) = -\sum_{\omega \in S_4} \frac{\omega_{g\star}^2 \int_{-\infty}^{t} \exp\left((t-\tau)\omega\right) \xi(\tau)\, d\tau}{\eta\left(3\omega^2 + \omega_\star^2\right) + 2\omega\left(2\omega^2 + \omega_g^2 + \omega_\star^2\right)}\,,$$

$$S_4 = \{\omega |\, \left(\omega^2 + \omega_\star^2\right)\left(\omega\left(\eta + \omega\right) + \omega_g^2\right) = \omega_{g\star}^4\},$$



*Dissipation in gas **also** brings down the ★ modes*

See also Bertin Romeo (1988) 195, 105-113