Gravitational Lensing in GR = Newton x 2?

Nick Kaiser

KITP Cosmic Web - 2022.02.27

Introduction

- Early observations of lensing in cosmology: 80's. Tony Tyson group (with Jarvis, Valdes, Wenk etc).
  - deep stacking of CCD images from CFHT
  - quadrupole moments of galaxies -> $\gamma = \{M_{11} - M_{22}, 2M_{12}\}$
  - cosmic shear (null result); gal-gal lensing (low); mass-maps (wow!)
- Lensing emerges as complement to gal clustering, CMB, flows…

Early weak-lensing

- Top figure is "cosmic shear" from Valdes et al 1983.
  - null result - but constraints on large-scale $\delta\rho/\rho$
- Tyson, Jarvis, Valdes & Mills (1984) used the same data to measure <u>galaxy-galaxy lensing</u>
- The result was a <u>surprisingly weak signal</u> (lower plot)
  - barely compatible with kinematic estimates of extended flat rotation curves

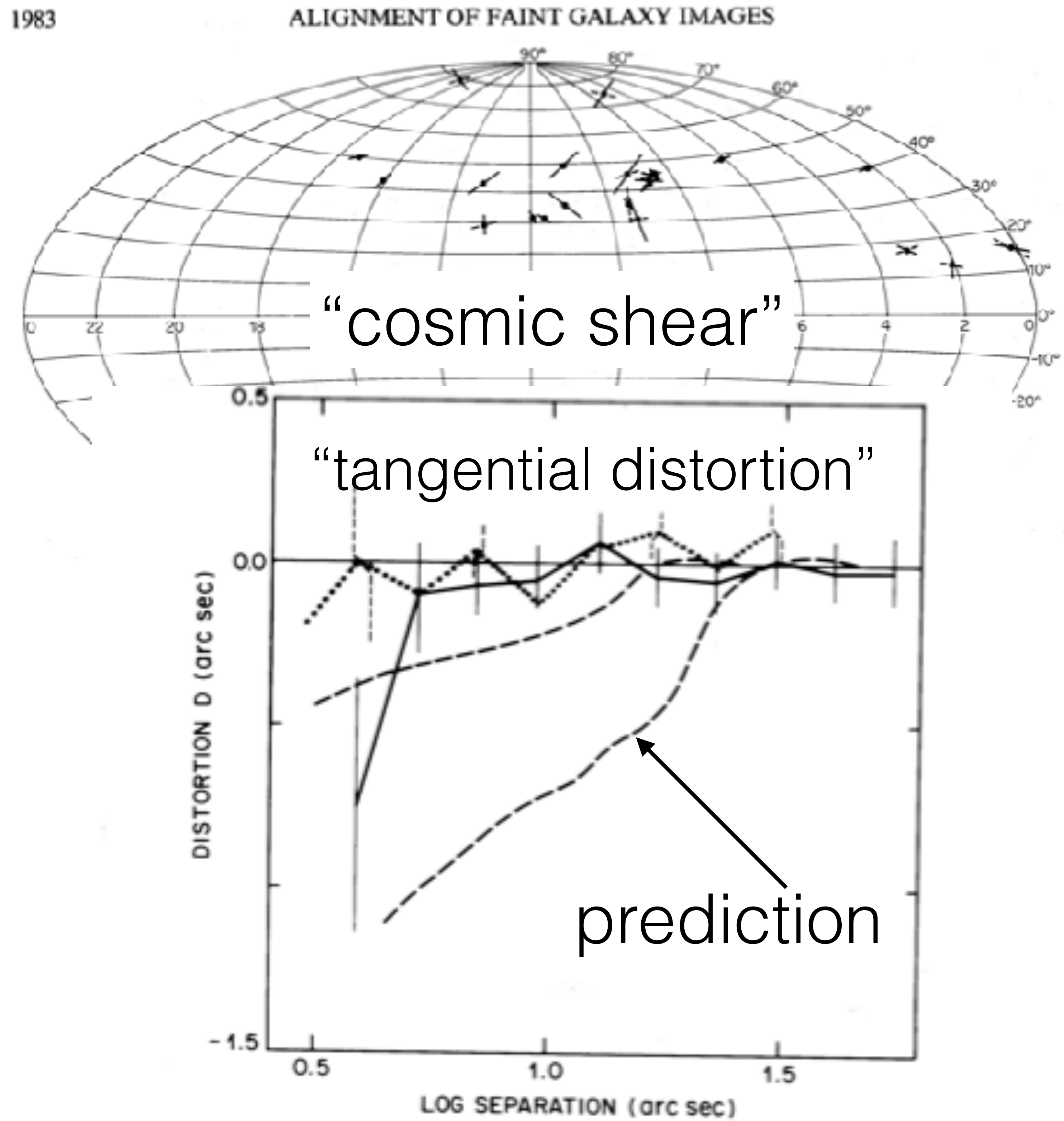

FIG. 1.—The dimensionless gravitational image distortion parameter $\mathcal{D} = \psi(M_r - M_\theta)/(M_r + M_\theta)$ in arc seconds as a function of the radial separation ψ of the foreground-background galaxy pair on the sky in arc seconds (*solid line*). Also shown (*dotted line*) is the result of a control test in which bright stars on the plate were substituted for the foreground galaxy position in the measurement of $\mathcal{D}$. The result is null (within 2 σ) in both cases, 1 σ error bars. Also shown are simulated distortions (*dashed lines*) from galaxies of mass cutoff radius 65 h^{-1} kpc and equivalent circular velocities of 200 and 300 km s^{-1}.

DETECTION OF SYSTEMATIC GRAVITATIONAL LENS GALAXY IMAGE ALIGNMENTS: MAPPING DARK MATTER IN GALAXY CLUSTERS

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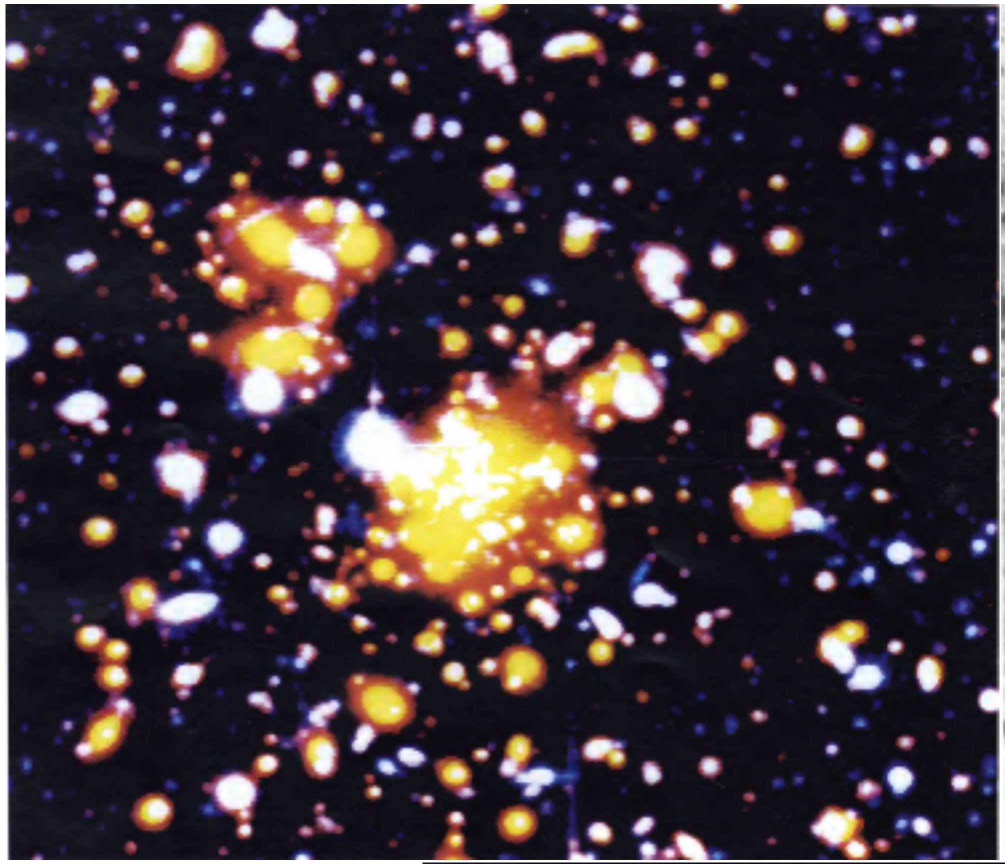

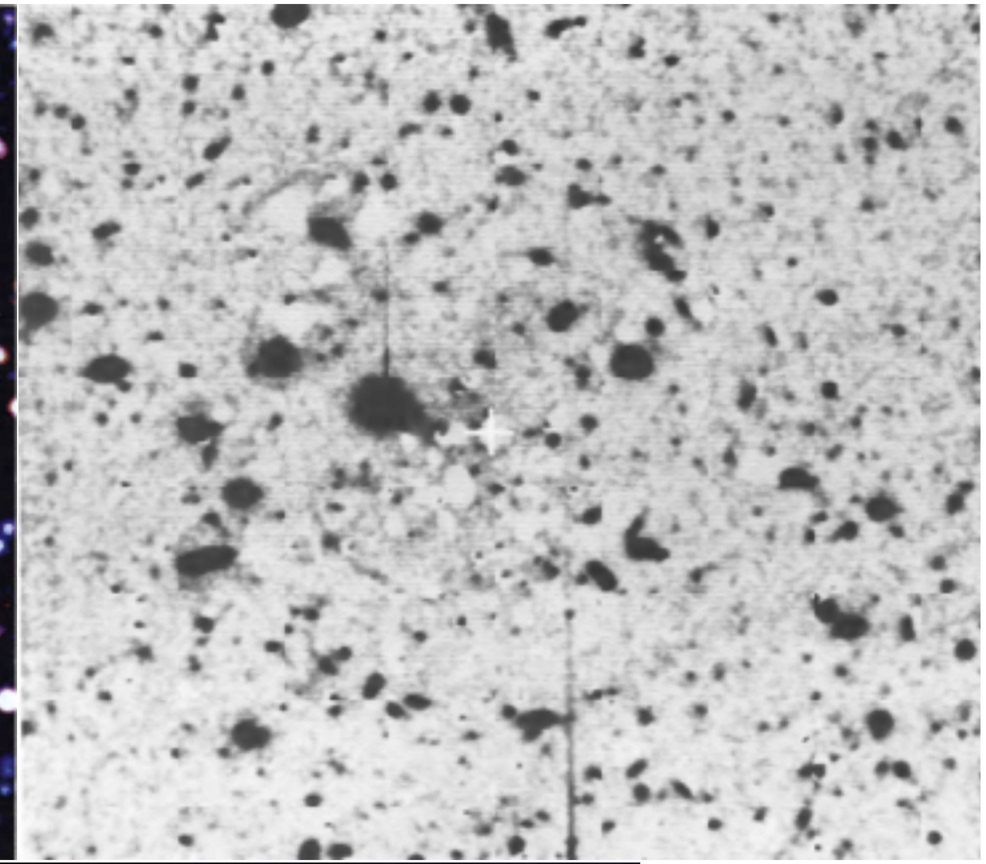

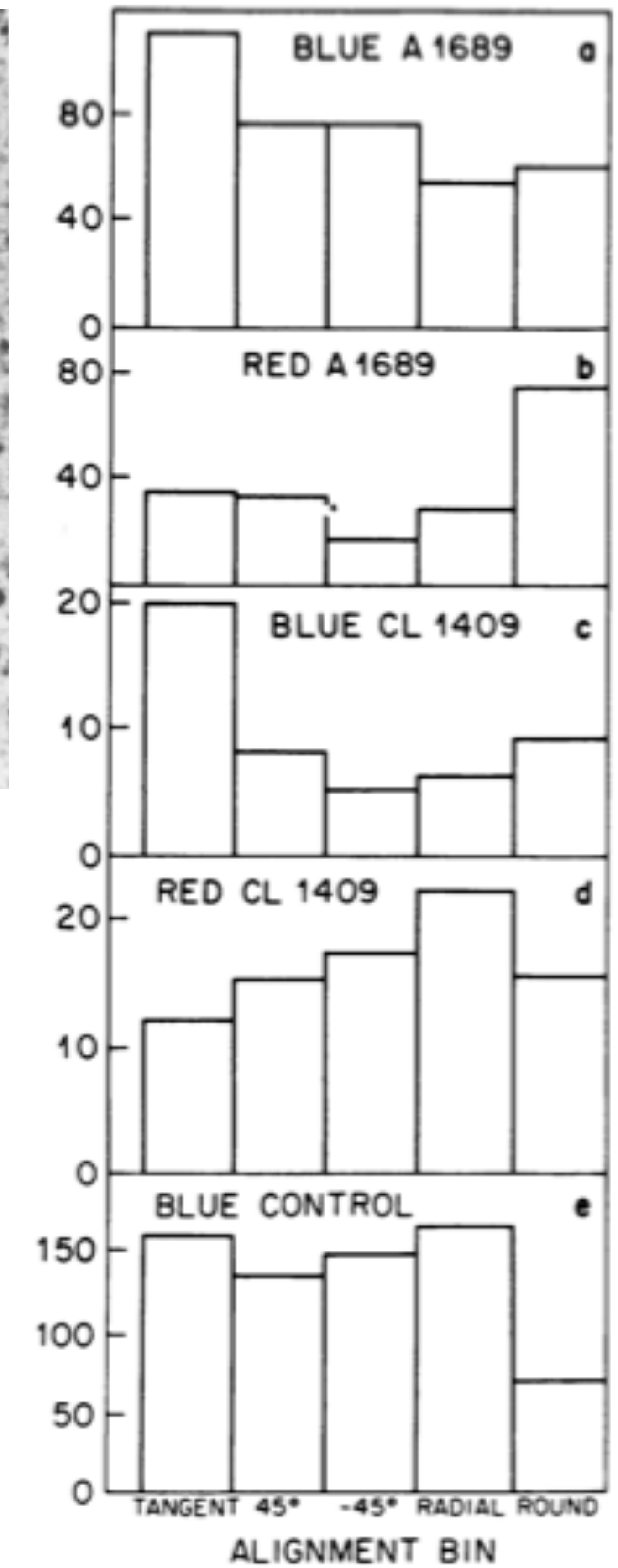

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  - deep stacking of CCD images from CFHT
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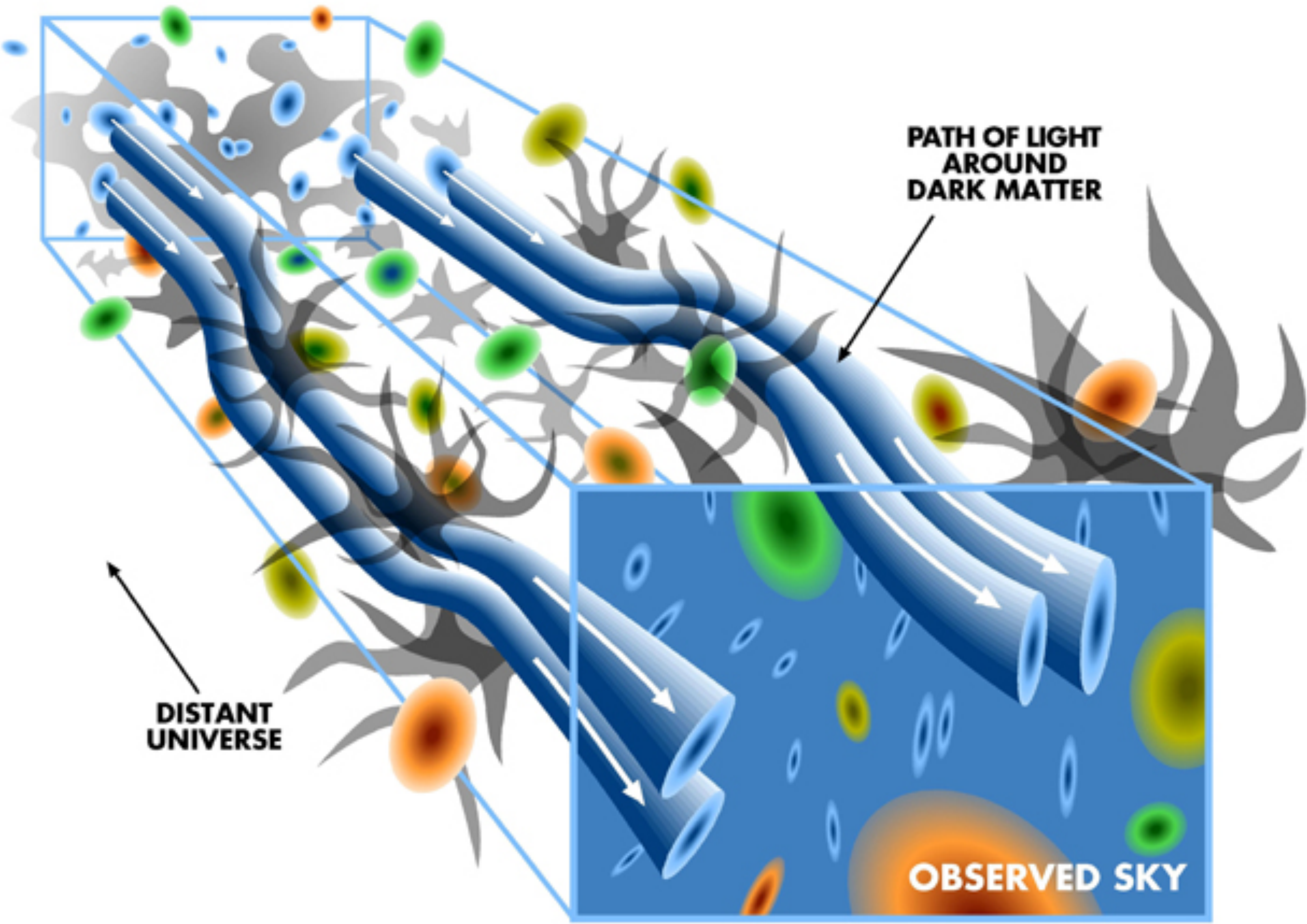
  - cosmic shear (null result); gal-gal lensing (low); mass-maps (wow!)
- Lensing emerges as complement to gal clustering, CMB, flows…
- Early theory/analysis work
  - Teresa Brainerd + Roger Blandford etc., Peter Schneider group, NK, ….
  - rapid progress on calibration - mass-maps etc.
  - model: weak-field perturbed FRW metric …
    - $ds^2 = a^2(\eta)(-(1+2\phi)d\eta^2 + (1-2\phi)(dx^2 + dy^2 + dz^2))$
    - where $\nabla^2\phi = 4\pi G\delta\rho$
  - the "lumpy glass analogy" :
    - ds = 0 -> coord. speed of light -> refractive index -> Snell's law

The "refractive index of gravity"

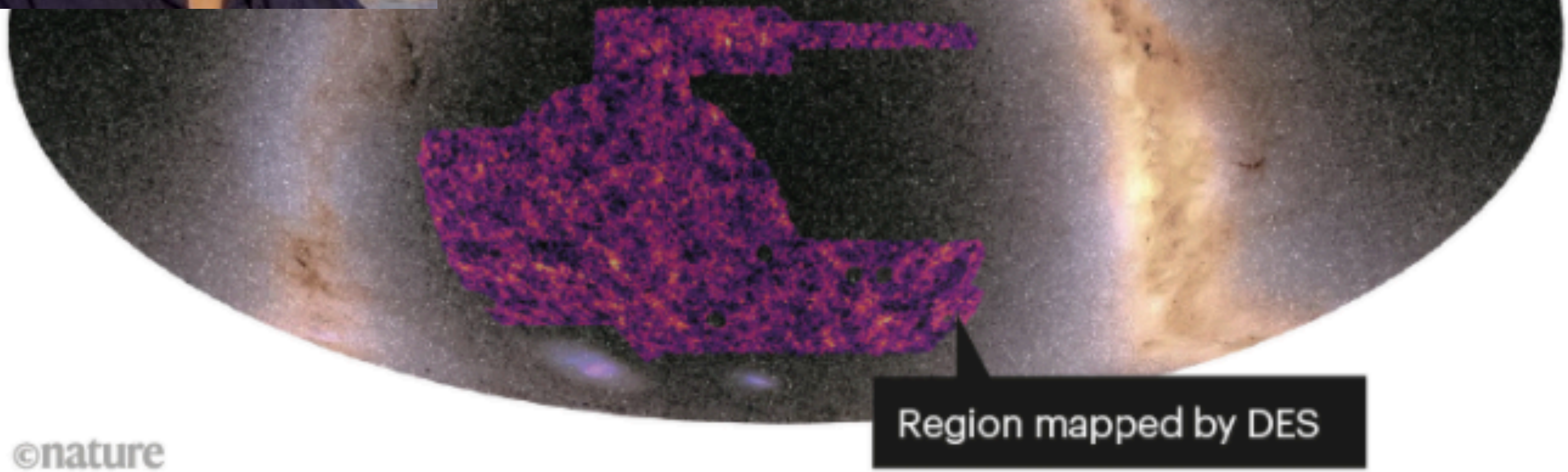
- Weak field metric:
  - $ds^2 = -(1+2\phi)dt^2 + (1-2\phi)(dx^2+dy^2+dz^2)$
- so photon trajectories ($ds = 0$) have *coordinate speed of light:*
  - $|\mathbf{dx}|/dt = \sqrt{\dfrac{1+2\Phi}{1-2\Phi}}c \simeq (1+2\Phi(\mathbf{x}))c$
- like in an inhomogeneous medium ("lumpy-glass") with *refractive index*
  - $n(\mathbf{x}) = c(|d\mathbf{x}|/dt)^{-1} \simeq 1 - 2\Phi(\mathbf{x})$
- And *Snell's law* (or *Snellius-Descartes* — or *Ibn Sahl* 984) is then
  - $d\hat{\mathbf{n}}/d\lambda = -2\nabla_\perp\Phi$
  - which is simply *twice* what Newton would have obtained
- ⚠️ in *lumpy glass*, the speed of light varies, but the frequency is fixed. In *gravity*, the frequency changes - redshift - but the physical speed is fixed.

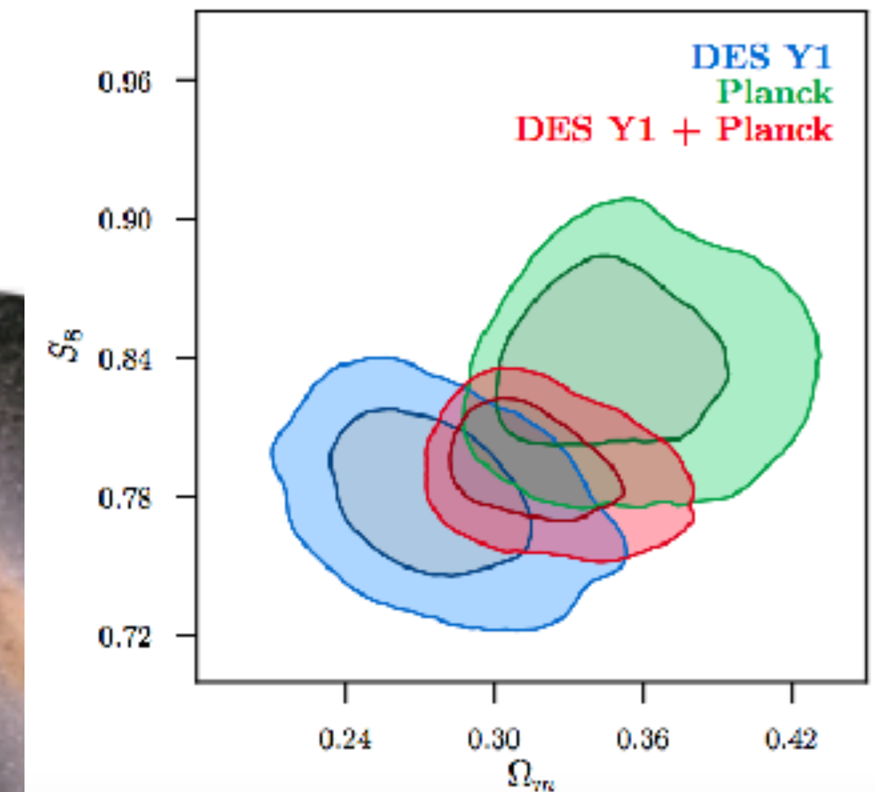
Lensing by the “cosmic-web”



3+ decades later…. state of the art:

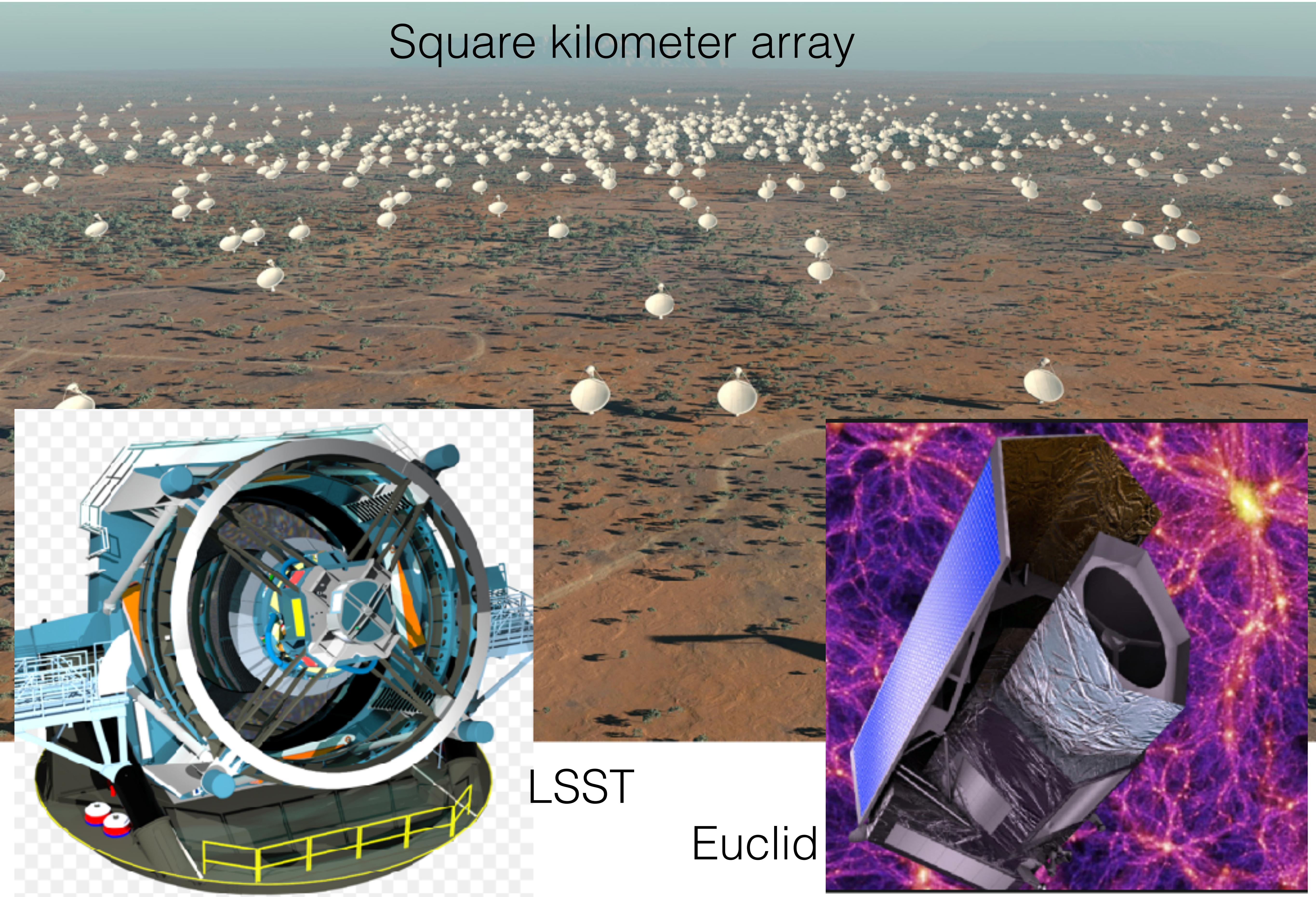
Dark Energy Survey yr 3 mass map + Niall Jeffrey (LPENS)





See also Judit Prat Marti's talk last week

The next decade - a golden age for cosmology



this talk:

- While the calculational model described above is widely used, on closer inspection it seems somewhat questionable:
  - it refers to the *coordinate* speed of light, and calculates the deflection of light rays *with respect to a coordinate system that is somewhat arbitrary*
  - it deviates from the more honest approach adopted previously, which is to use the geodesic deviation equation - or Raychaudhuri's equation - in which only physical quantities appear
- To illustrate this, I will discuss 2 situations in which the lumpy-glass model appears to give the wrong answer:
  - measurement of light bending "in the lab"
  - Zel'dovich ('63) classic "empty beam" calculation

Isaac Newton on gravitational lensing (1704)

BOOK III.

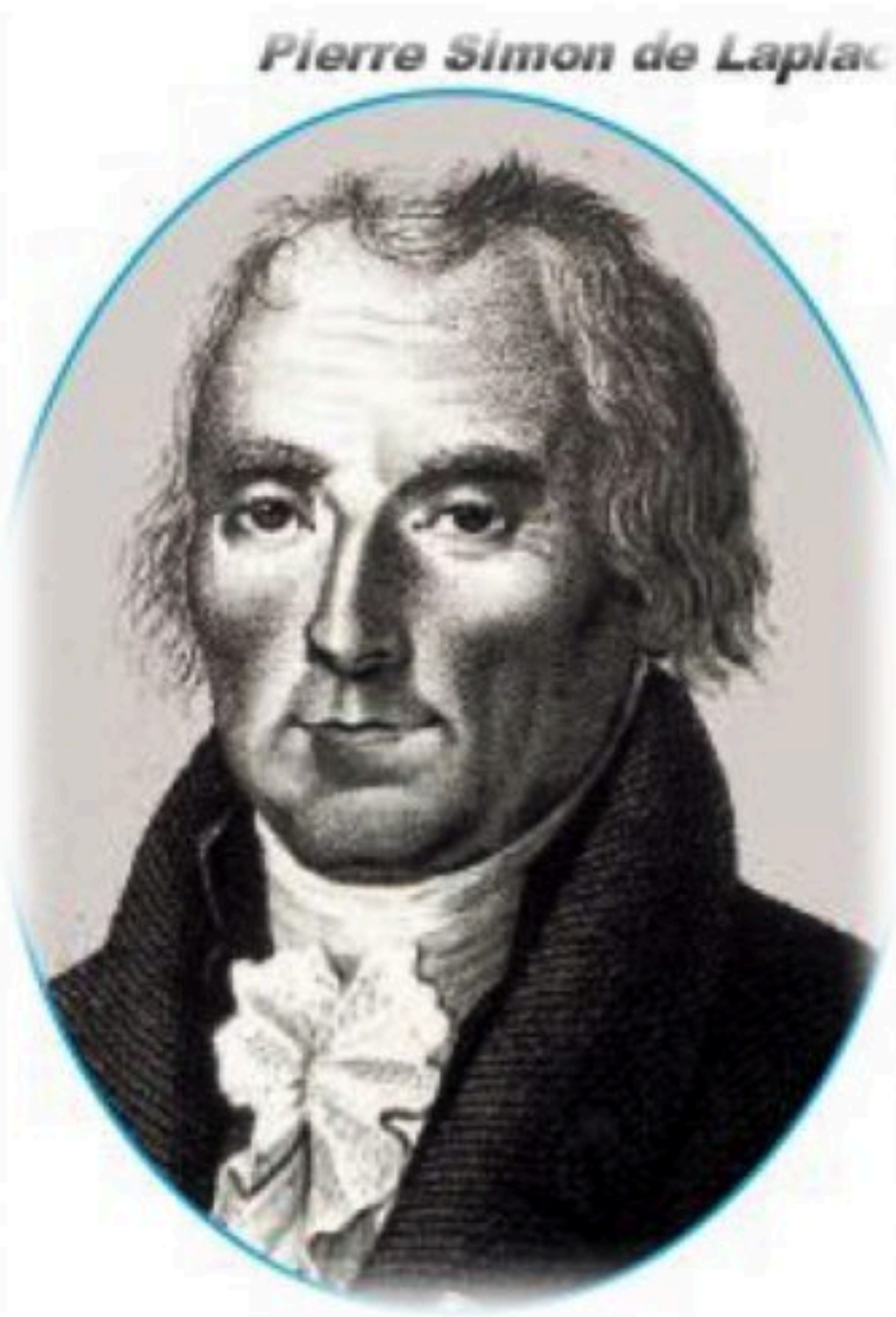
When I made the foregoing Obſervations, I deſign'd to repeat moſt of them with more care and exactneſs, and to make ſome new ones for determining the manner how the Rays of Light are bent in their paſſage by Bodies, for making the Fringes of Colours with the dark lines between them. But I was then interrupted, and cannot now think of taking theſe things into farther Conſideration. And ſince I have not finiſh'd this part of my Deſign, I ſhall conclude with propoſing only ſome Queries, in order to a farther ſearch to be made by others.

Query 1. Do not Bodies act upon Light at a diſtance, and by their action bend its Rays; and is not this action (*cæteris paribus*) ſtrongeſt at the leaſt diſtance?

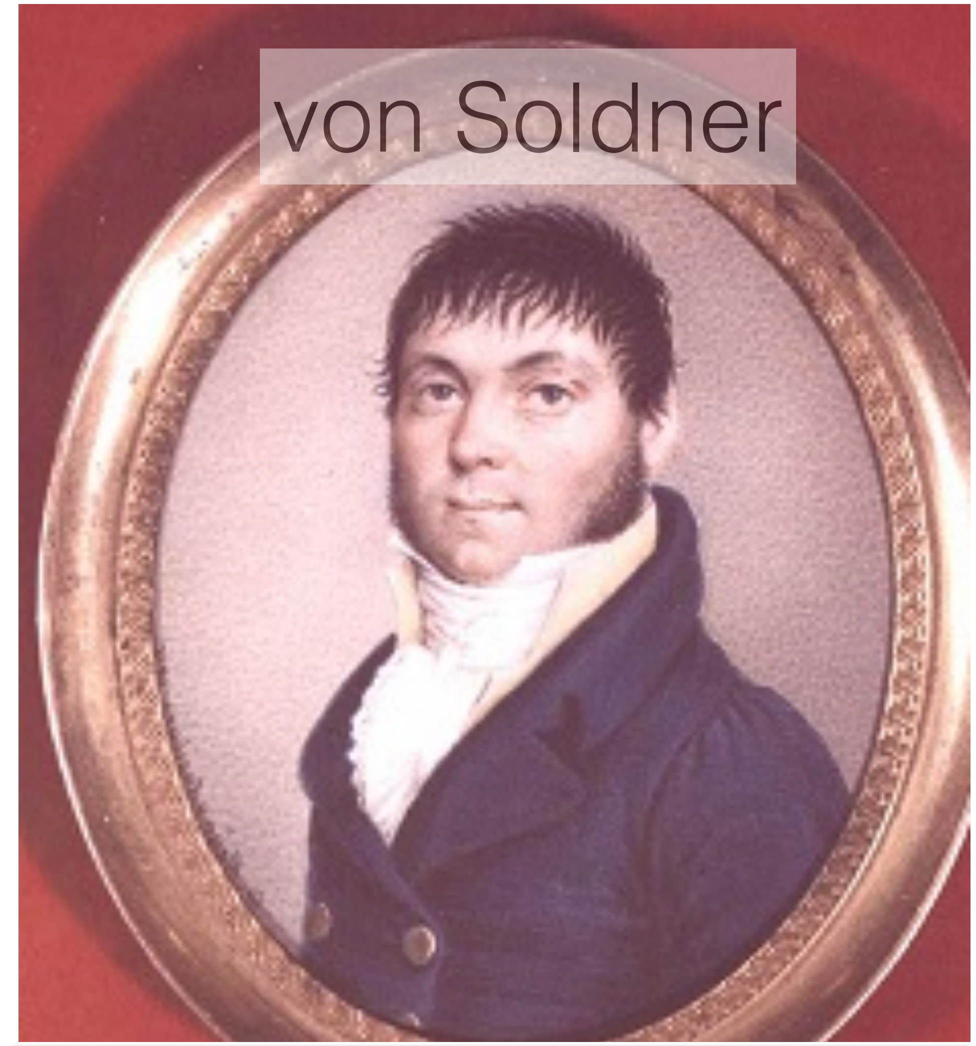
Note: Ole Romer had measured speed of light (to 20% precision) in 1676

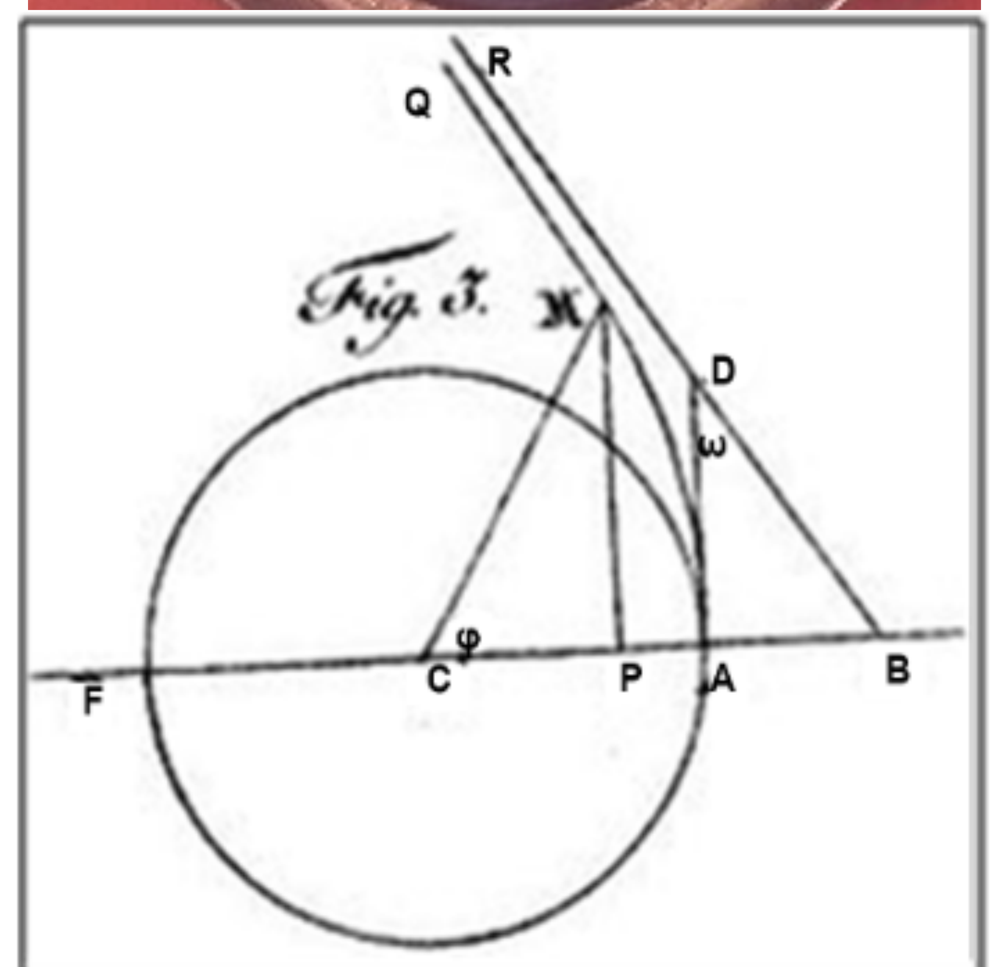
Newton in confinement (from the plague)

John Mitchell & Laplace -> Black holes ca. 1783



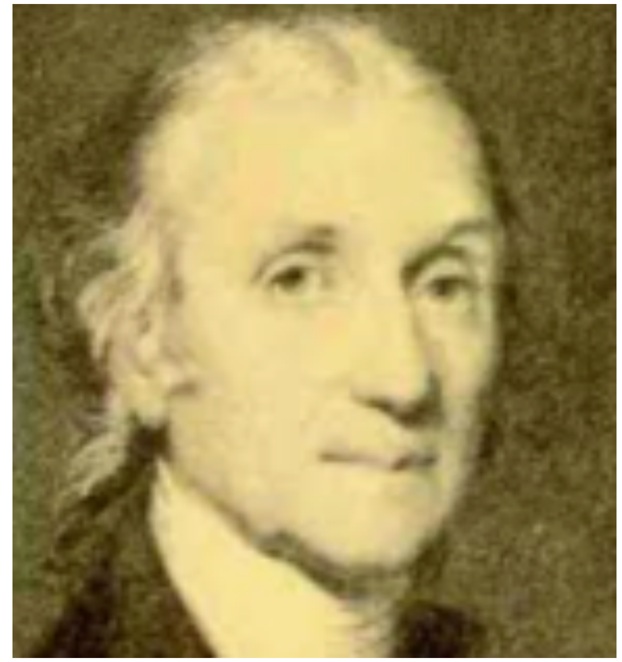
1802: Solar light deflection = 0.84"





missing the famous factor 2 from GR

Newtonian light deflection



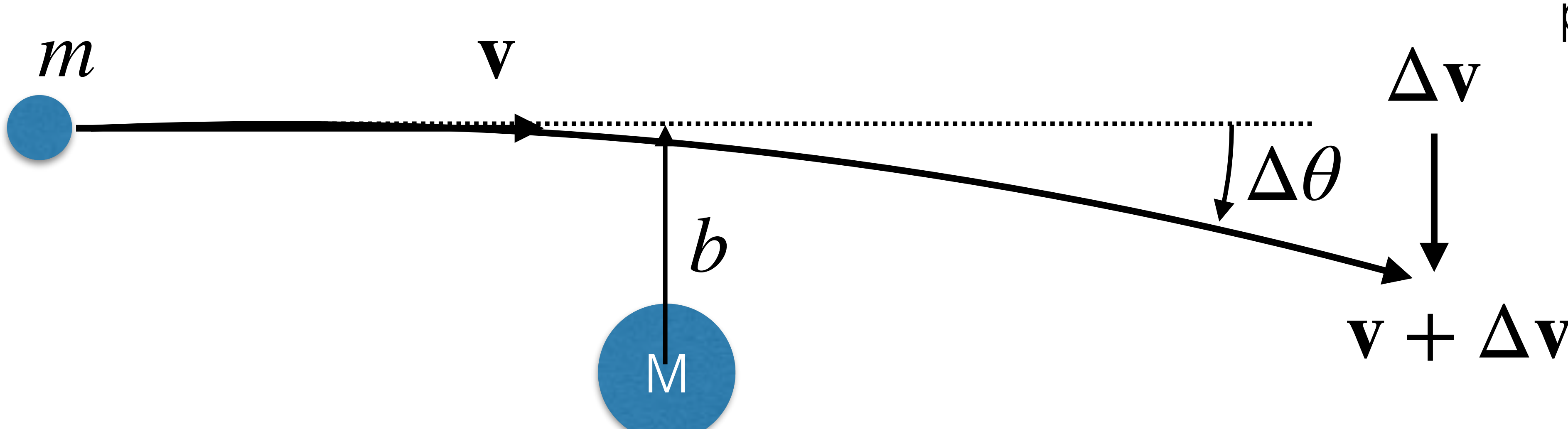
duration of collision

$$\Delta v \sim \boxed{\frac{GM}{b^2}} \times \Delta t \sim \boxed{\frac{GM}{b^2}} \times \frac{b}{v} \quad \Rightarrow \quad \Delta\theta = \frac{\Delta v}{v} = \boxed{\frac{2GM}{b}} \times \frac{1}{v^2}$$

acceleration

deflection angle

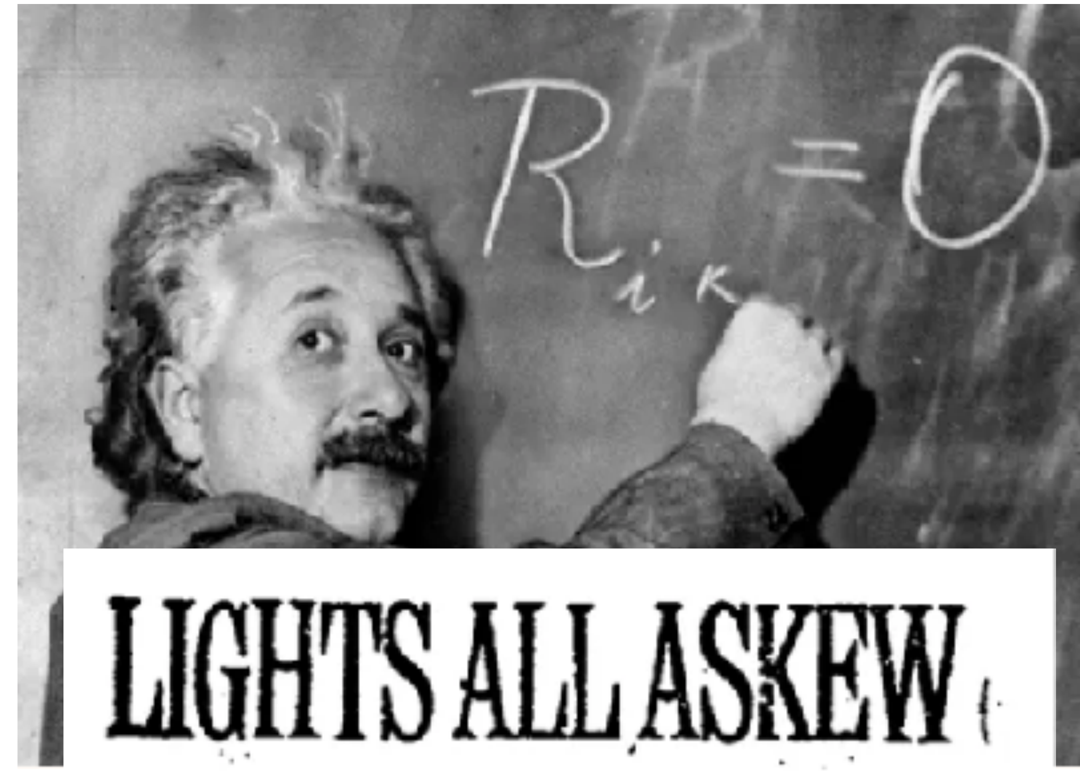
gravitational potential



for light deflection replace $v \to c$

Early 20th century: Einstein and GR

- 1911 - rocket thought experiments
  - predicts 0.84" solar bending angle
  - Lenard later accuses AE of plagiarism
- 1912 - Brazilian eclipse experiment
  - failed (to prove him wrong!)
- 1915 - GR paper published (with factor 2)
  - controversy over Hilbert paper
- 1919 - Eddington eclipse trip - success!




LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

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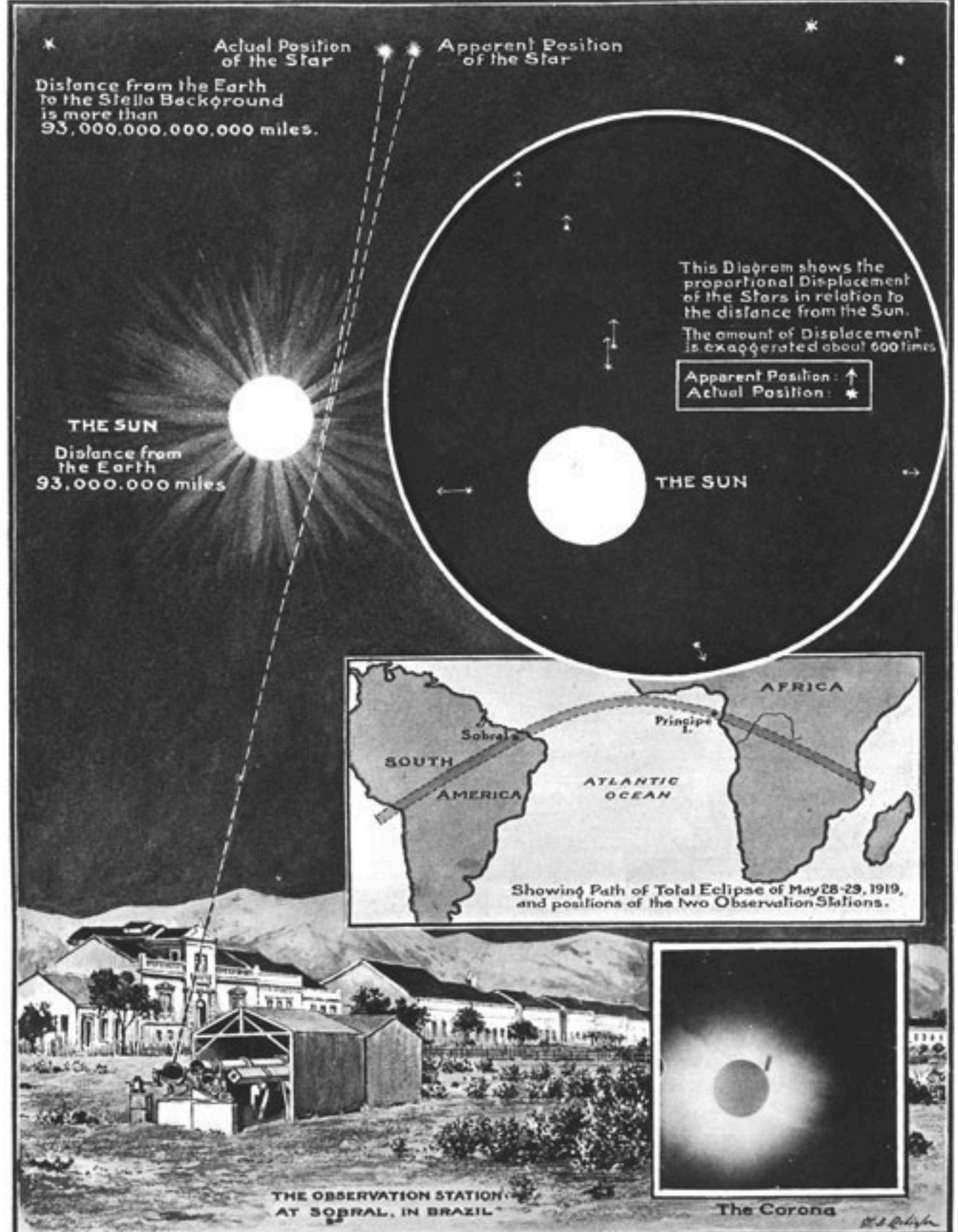
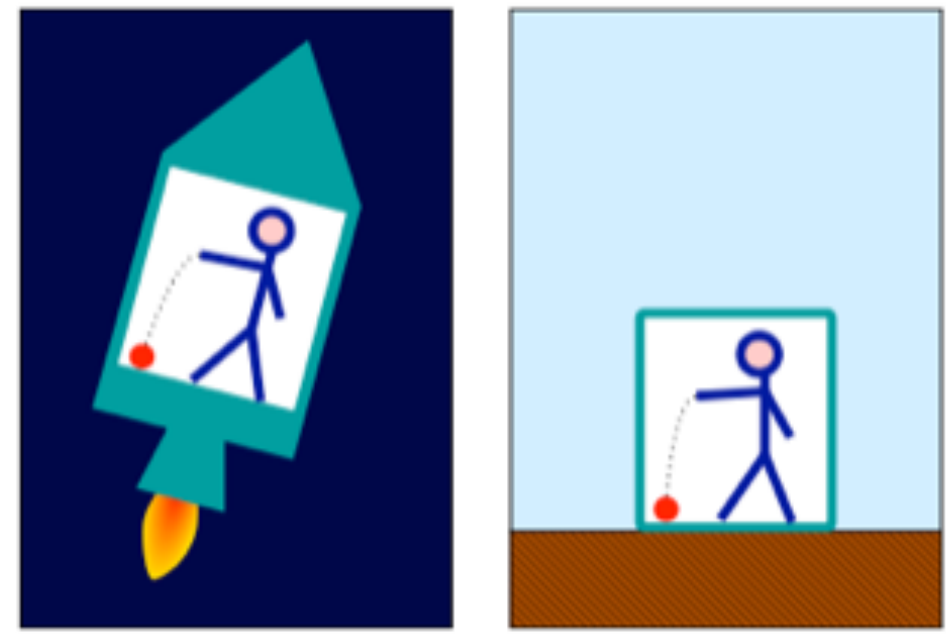
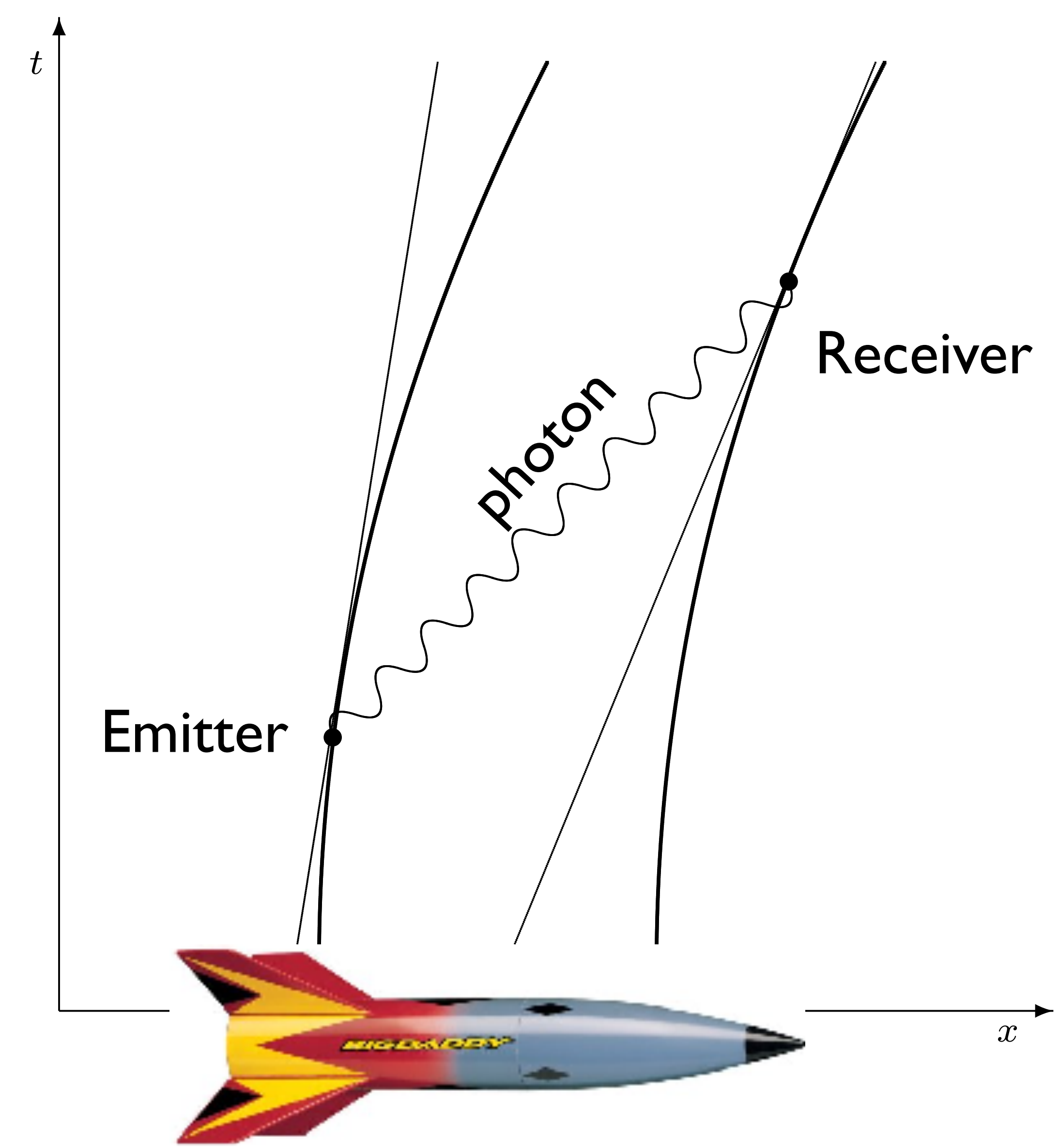


Taken from the 22 November 1919 edition of the Illustrated London News.

Einstein ca 1910: happiest thought - the equivalence principle (EP)

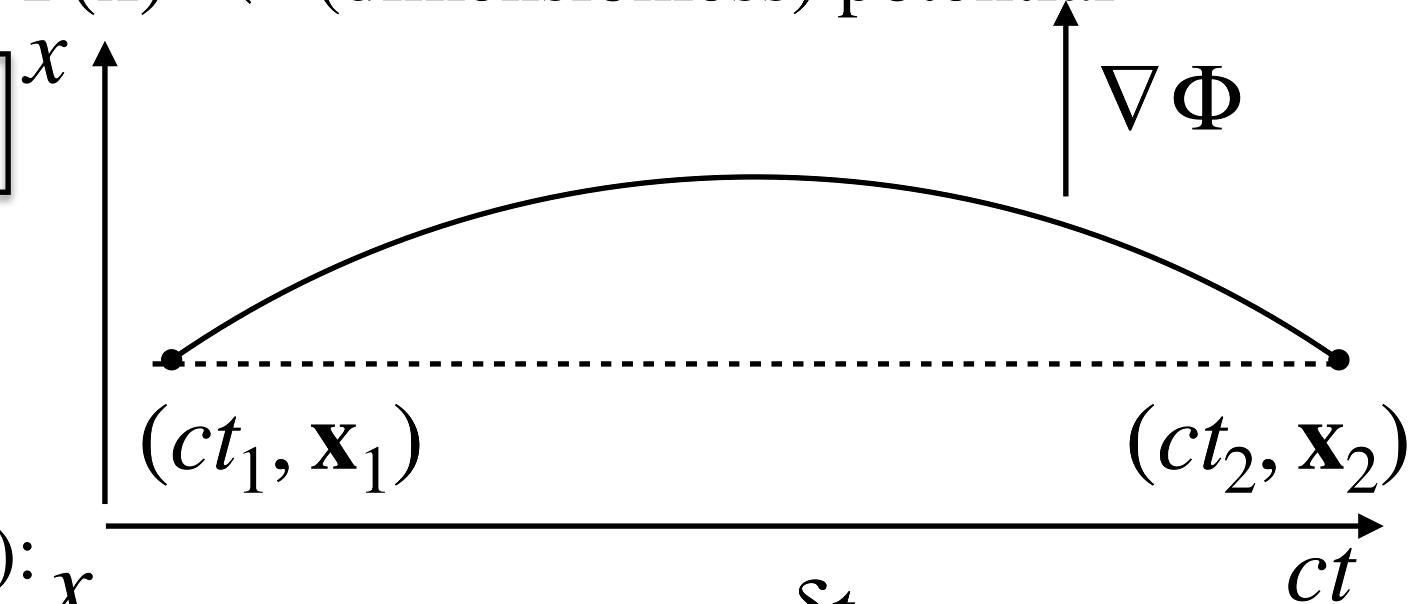


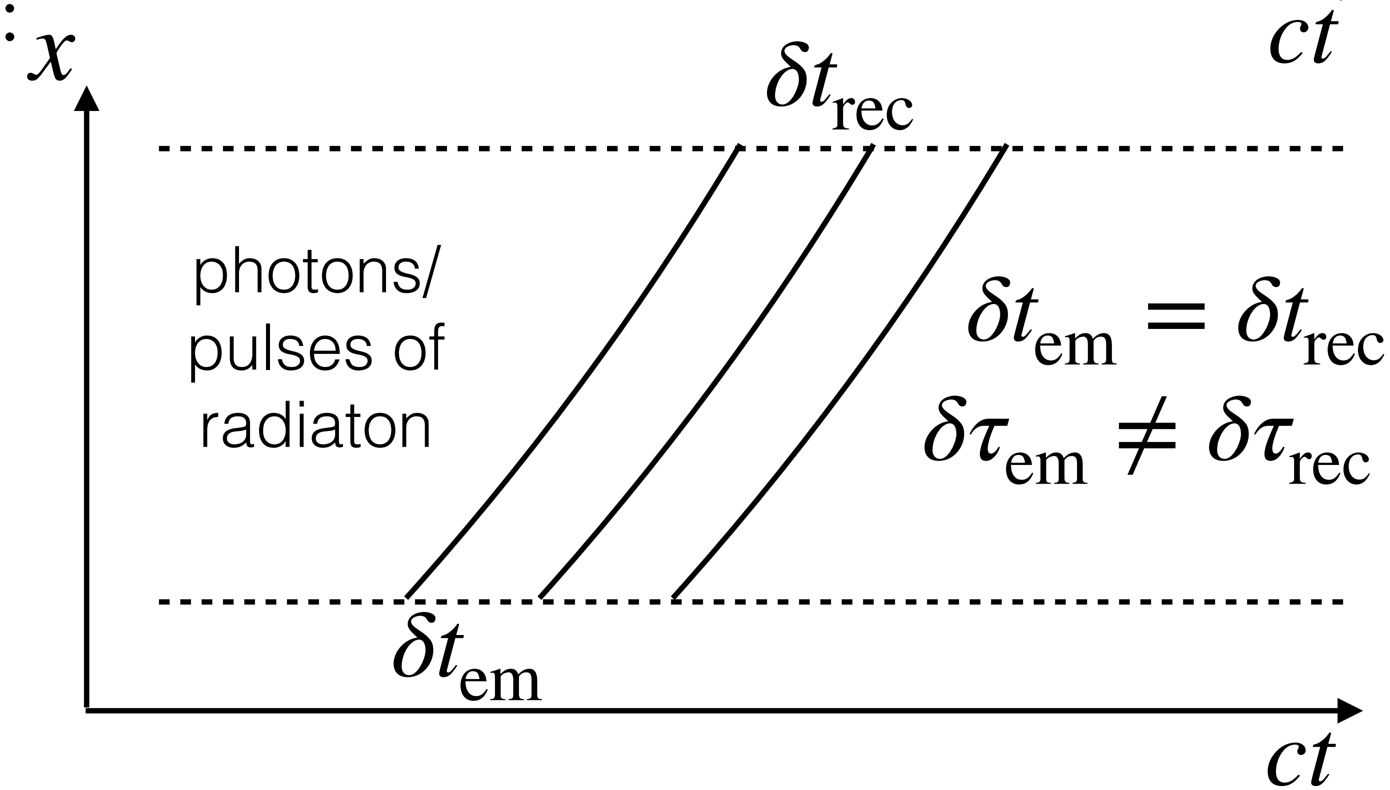
- Reads newspaper article about a tiler falling to his death from a roof
  - going to free-fall "*switches off*" gravity (locally)
- physics we see sitting on the Earth is the same as if we were in a rocket in empty space being accelerated
  - EP: *gravity and acceleration are equivalent*
- Q: What is the metric of space-time in an accelerating rocket?
  - i.e. SR, but with spatial coordinates tied to the body of the rocket? Rindler coordinates.
- A: $ds^2 = -(1 + 2\mathbf{a} \cdot \mathbf{x}/c^2)c^2 dT^2 + |\mathbf{dx}|^2$
  - so *time is warped in an accelerating frame*
  - time runs faster (slower) at the nose (tail) of the rocket!
  - clocks drift out of synchrony



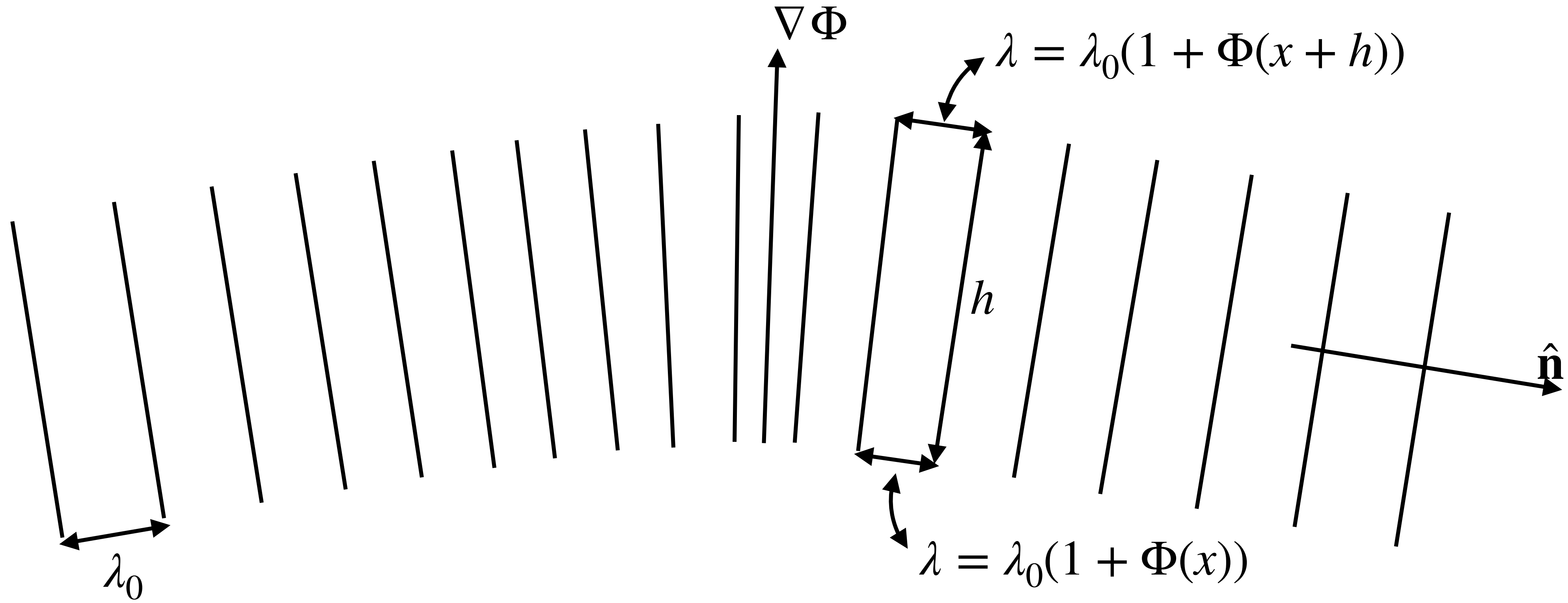
Equivalence -> metric in a "gravitational" field

- Equivalence: $\mathbf{x} \cdot \mathbf{a}/c^2 \;\Rightarrow\; \mathbf{x} \cdot \nabla\Phi = \Phi(\mathbf{x}) \;\Leftarrow$ (dimensionless) potential
- $$ds^2 = -(1 + 2\Phi(\mathbf{x}))c^2dt^2 + |\mathbf{dx}|^2$$
- Explains:
  - parabolic ballistic trajectories
  - "*geodesics*" (maximal proper time):
  - $$\int d\tau = \int d\lambda \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$$
- Predicts:
  - gravitational redshift
  - and hence light deflection
    - the *same* as Newtonian prediction for a particle moving with speed $v = c$
- Several attempts to measure the light bending by the Sun were unsuccessful (and so failed to prove him wrong!)





Einstein 1910: Light deflection from the equivalence principle



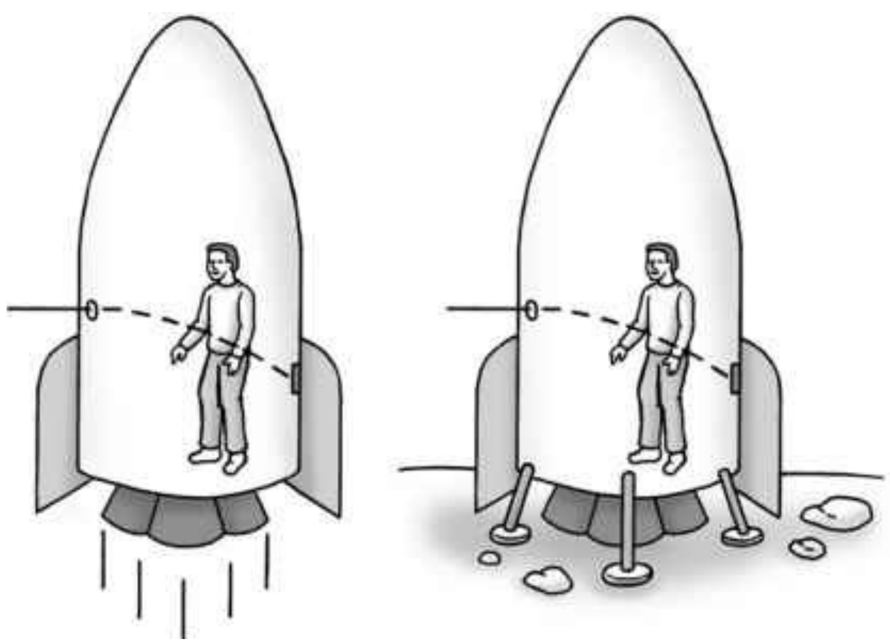
deflection in propagating 1 wavelength

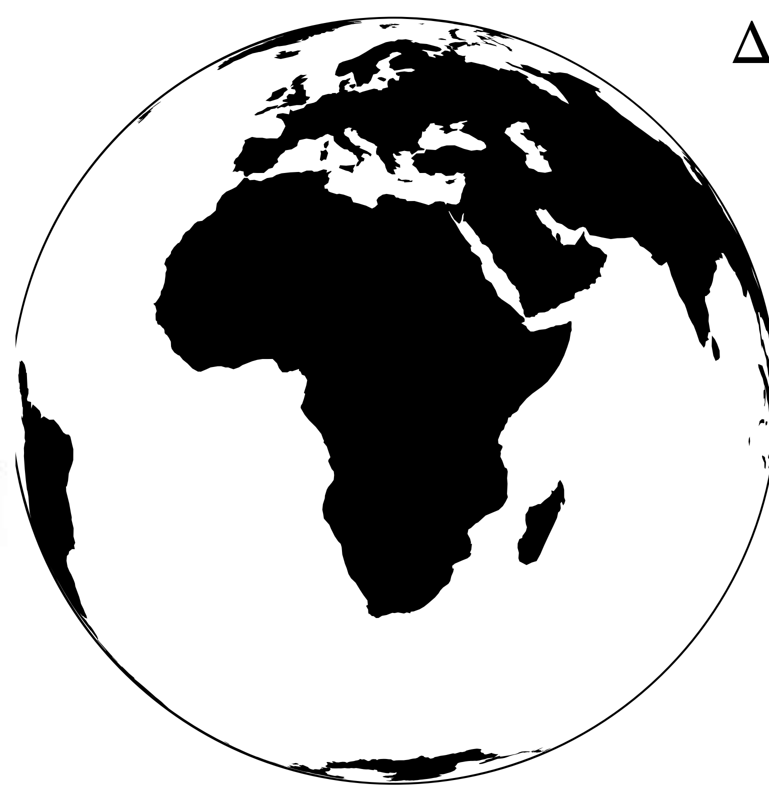
$$\Delta\theta_{\rm def} = \Delta\lambda / h = \lambda_0 \nabla_\perp \Phi$$

gradient perpendicular
to the path

Snell's law:

$$d\hat{\mathbf{n}}/dl = -\nabla_\perp\Phi$$





same as Newtonian theory - no extra factor 2

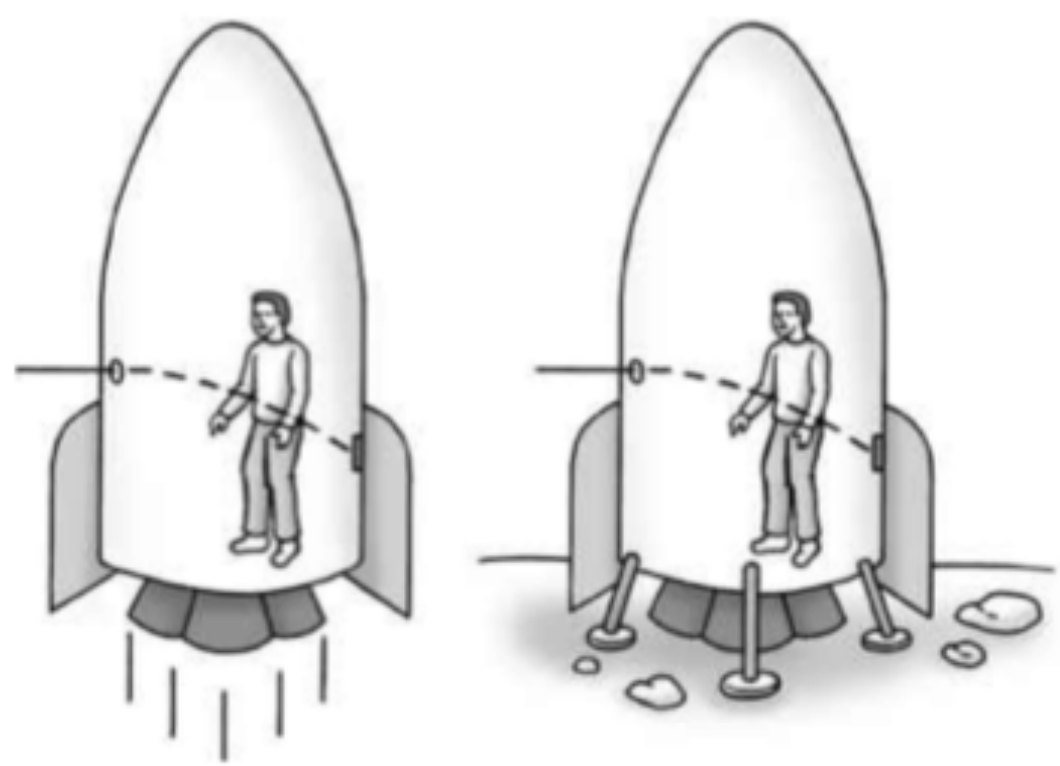
Einstein 1910-1915: The gravitational field equations

Newton	Einstein
potential : ϕ	metric : $g_{\mu\nu}(\overrightarrow{x})$
gravity : $\mathbf{g} = -\nabla\phi$	connection : $\Gamma^{\mu}{}_{\nu\beta}$
tide : $\nabla\nabla\phi = \partial^2\phi/\partial x_i \partial x_i$	curvature : $R^{\mu}{}_{\alpha\nu\beta}$
Poisson : $\nabla^2\phi = 4\pi G\rho$	Einstein : $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi\kappa T_{\mu\nu}$
tidal acceleration : $\frac{d^2\Delta\mathbf{x}}{dt^2} = -\Delta\mathbf{x}\cdot\nabla\nabla\phi$	geodesic deviation : $\frac{d^2\Delta x^\alpha}{d\lambda^2} = R^{\alpha}{}_{\beta\mu\nu}\frac{dx^\beta}{d\lambda}\Delta x^\mu \frac{dx^\nu}{d\lambda}$

- metric -> connection > curvature generally non-linear. In addition we have coordinate freedom -> complex to solve; hard to interpret solutions
- but for weak fields we can choose coordinates (Lorenz gauge) such that
- $$\boxed{ds^2 = -(1 + 2\Phi)c^2dt^2 + (1 - 2\Phi)|\mathbf{dx}|^2}$$
- where $\nabla^2\Phi = 4\pi G\rho c^2$
- "*weak-field*" or "*Newtonian-limit*" metric has warping of *space* as well as of time

Q1: light deflection "in the lab"

- Shine a laser-pointer horizontally in a terrestrial laboratory and measure the deflection of light
- Do we see the GR prediction (including the factor 2)?
    - If *not*, why not? There *is* a gravitational field present, right?
        - and then why should we trust this in other applications?
    - But if *so*, what is wrong with AE's ca. 1911 argument?

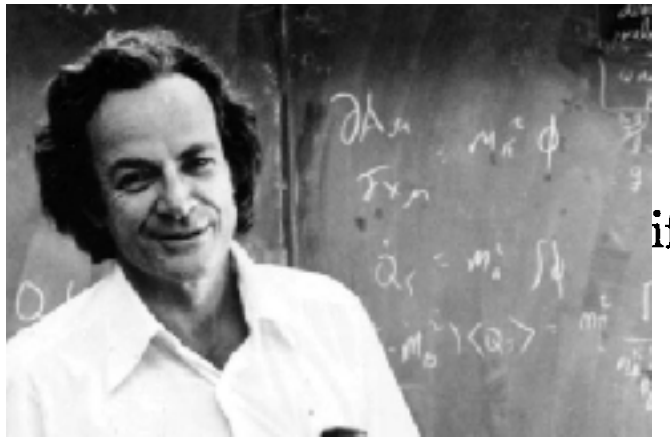


- We'll come back to this presently, but before that, you might want to think about how you would go about doing this experiment ….

Problem 2: Light focussing *à la* Raychaudhuri

- Long prior to the 80's people were thinking about how matter in the universe would affect sizes (and therefore flux densities) and shapes of distant galaxies
  - Feynman gave a seminar in 1964 at Caltech arguing that this would introduce extra scatter in Sandage's Hubble diagram (expanded on in 1967 by Jim Gunn)
  - Zel'dovich wrote a classic paper in 1963 on the subject
  - These were followed by numerous papers by Ron Kantowski and by Charles Dyer and others (and the subject has re-surfaced with some controversial claims for how lumpiness affects the Hubble diagram)
- These papers were relatively honest, in that they used <u>Raychaudhuri's equation</u> for the transport of the expansion, vorticity and shear of a bundle of light rays
- Derived from the <u>geodesic deviation equation</u> in which one calculates the evolution of the *physical* - rather than *coordinate* - ray separation

ON THE PROPAGATION OF LIGHT IN INHOMOGENEOUS COSMOLOGIES. I. MEAN EFFECTS



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ABSTRACT

The statistical effects of local inhomogeneities on the propagation of light are investigated, and deviations (including rms fluctuations) from the idealized behavior in homogeneous universes are investigated by a perturbation-theoretic approach. The effect discussed by Feynman and recently by Bertotti of the density of the intergalactic medium being systematically lower than the mean mass density is examined, and expressions for the effect valid at all redshifts are derived.

I. INTRODUCTION

In an unpublished colloquium given at the California Institute of Technology in 1964, Feynman discussed the effect on observed angular diameters of distant objects if the intergalactic medium has lower density than the mean mass density, as would be the case if a significant fraction of the total mass were contained in galaxies. It is an obvious extension of the existence of this effect that luminosities will also be affected, though this was apparently not realized at the time. This realization prompted the conviction that the effect of known kinds of deviations of the real Universe from the homogeneous isotropic models (upon which predictions had been based in the past) upon observable quantities like luminosity and angular diameter should be investigated. The author (1967) has recently made such a study for angular diameters; the present work deals primarily with mean statistical effects upon luminosity. A third paper will deal with possible extreme effects one may expect to encounter more rarely. Some of the results discussed here have been discussed independently by Bertotti (1966) and Zel'dovich (1965).

OBSERVATIONS IN A UNIVERSE HOMOGENEOUS IN THE MEAN

Ya. B. Zel'dovich

Translated from Astronomicheskii Zhurnal, Vol. 41, No. 1,
pp. 19-24, January-February, 1964
Original article submitted June 12, 1963

- considers light propagation in *inhomogeneous* cosmologies
- the first known "*cone diagram*"
- angular diameter $D_a(z)$ plots
  - uses $\Delta = z/(1+z)$
- bias in D_a for galaxies seen along underdense lines of sight
- shape distortion from external mass
- FLRW curvature from local light-beam focussing - Raychaudhuri...
  - not $\mathbf{G} = 8\pi G\mathbf{T}$ + symmetry

Fig. 1.

Fig. 2.

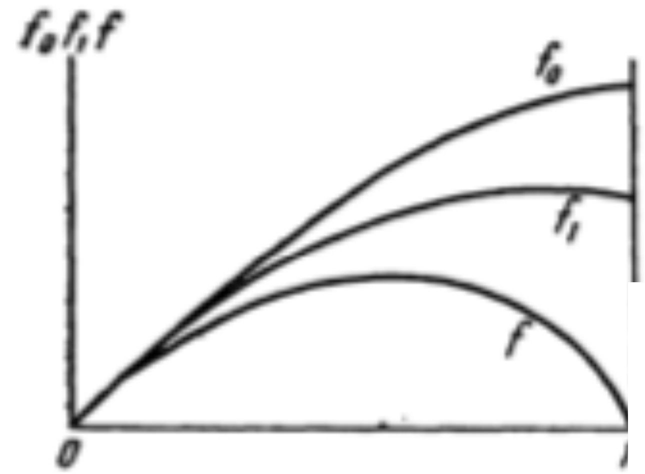
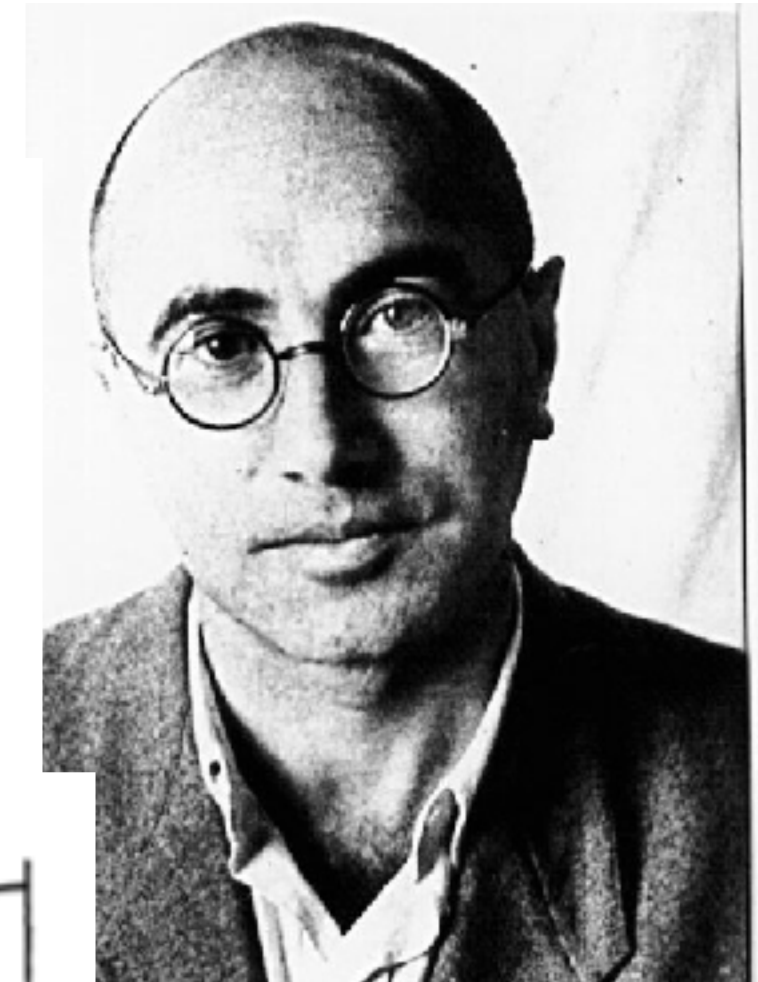
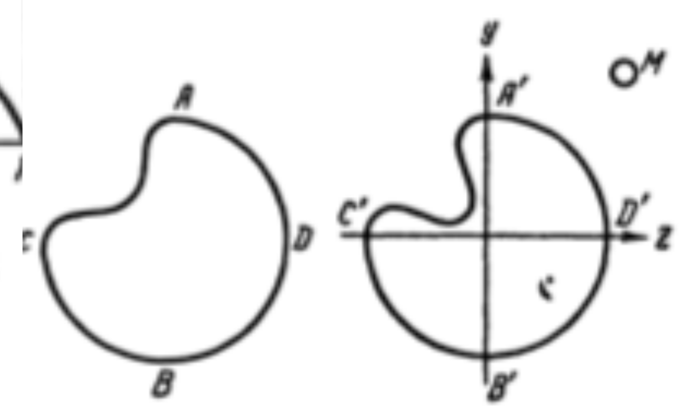


Fig. 6. $f_0 = \Delta\left(1 - \frac{\Delta}{2}\right)$; $f_1 = \frac{2}{5}\ [1 - (1-\Delta)^{5/2}]$; $f = 2(1-\Delta)(1-\sqrt{1-\Delta})$





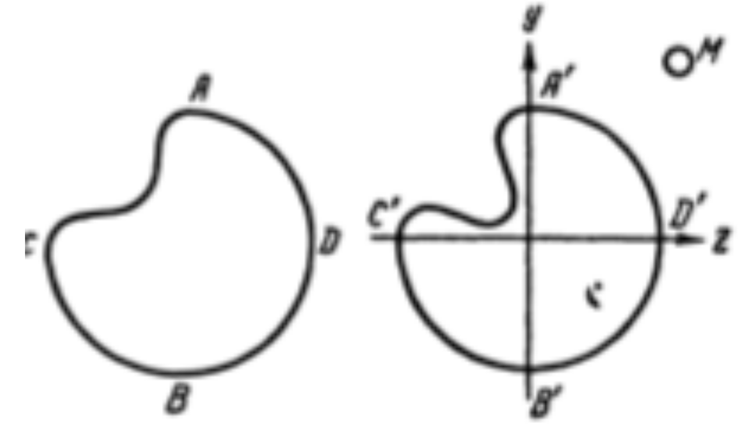
The mass M deforms the observed shape of the object, so that the latter becomes contracted along the axis joining it to M and elongated in the perpendicular direction.

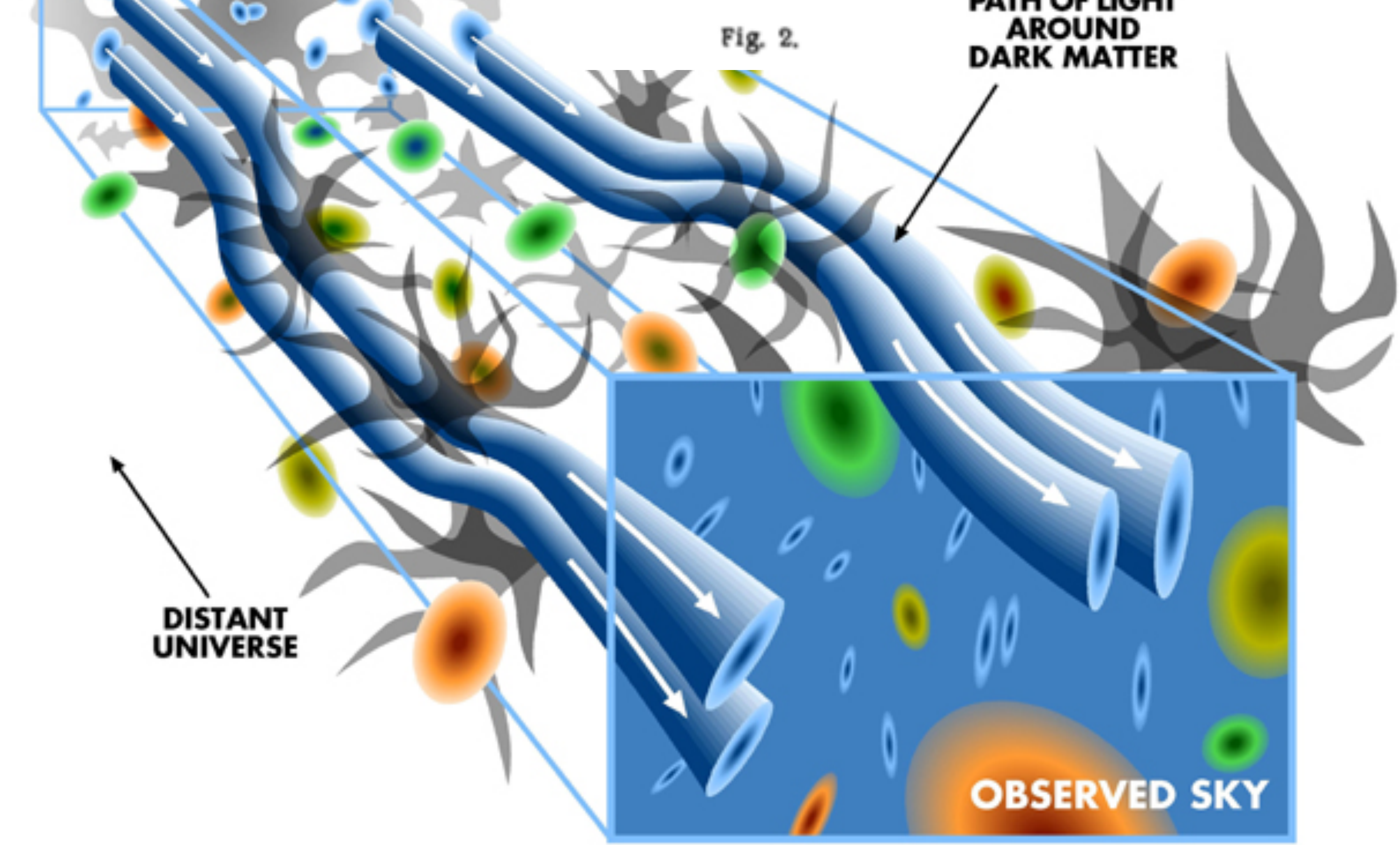

O
θ
A
B

Fig. 1.

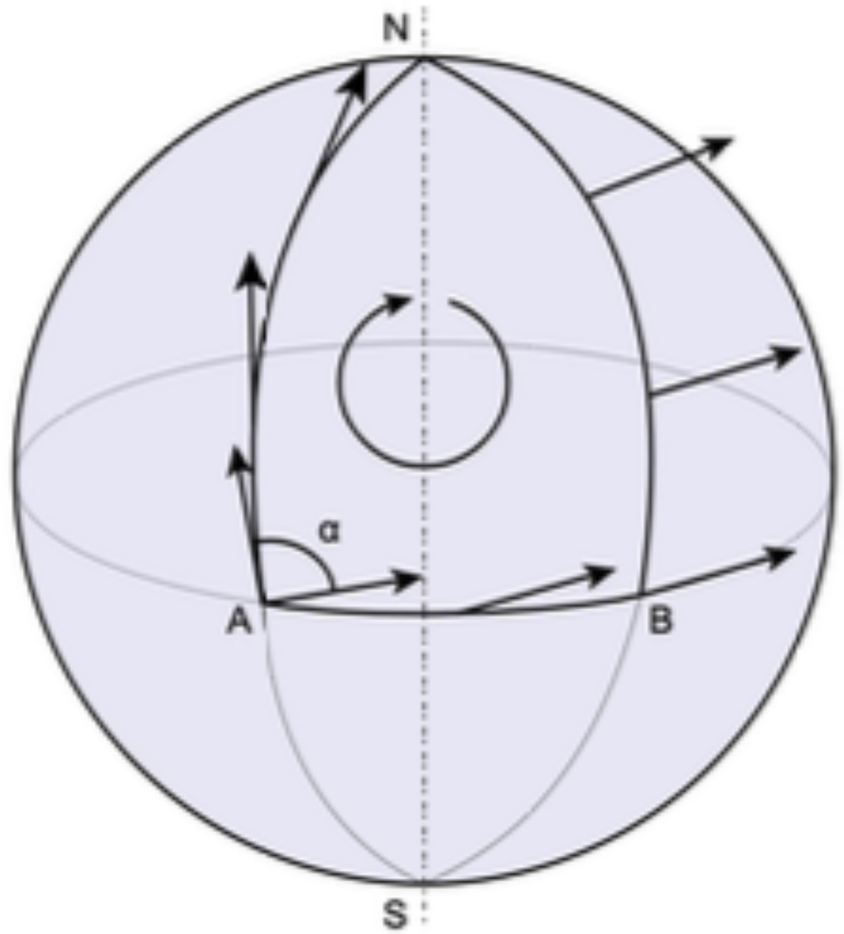

O
θ
A
B

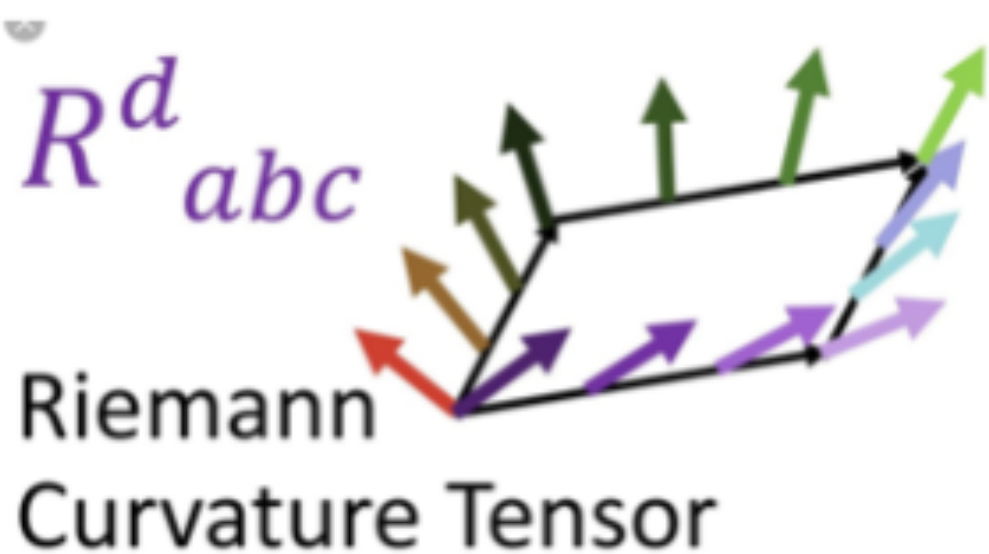
Fig. 2.

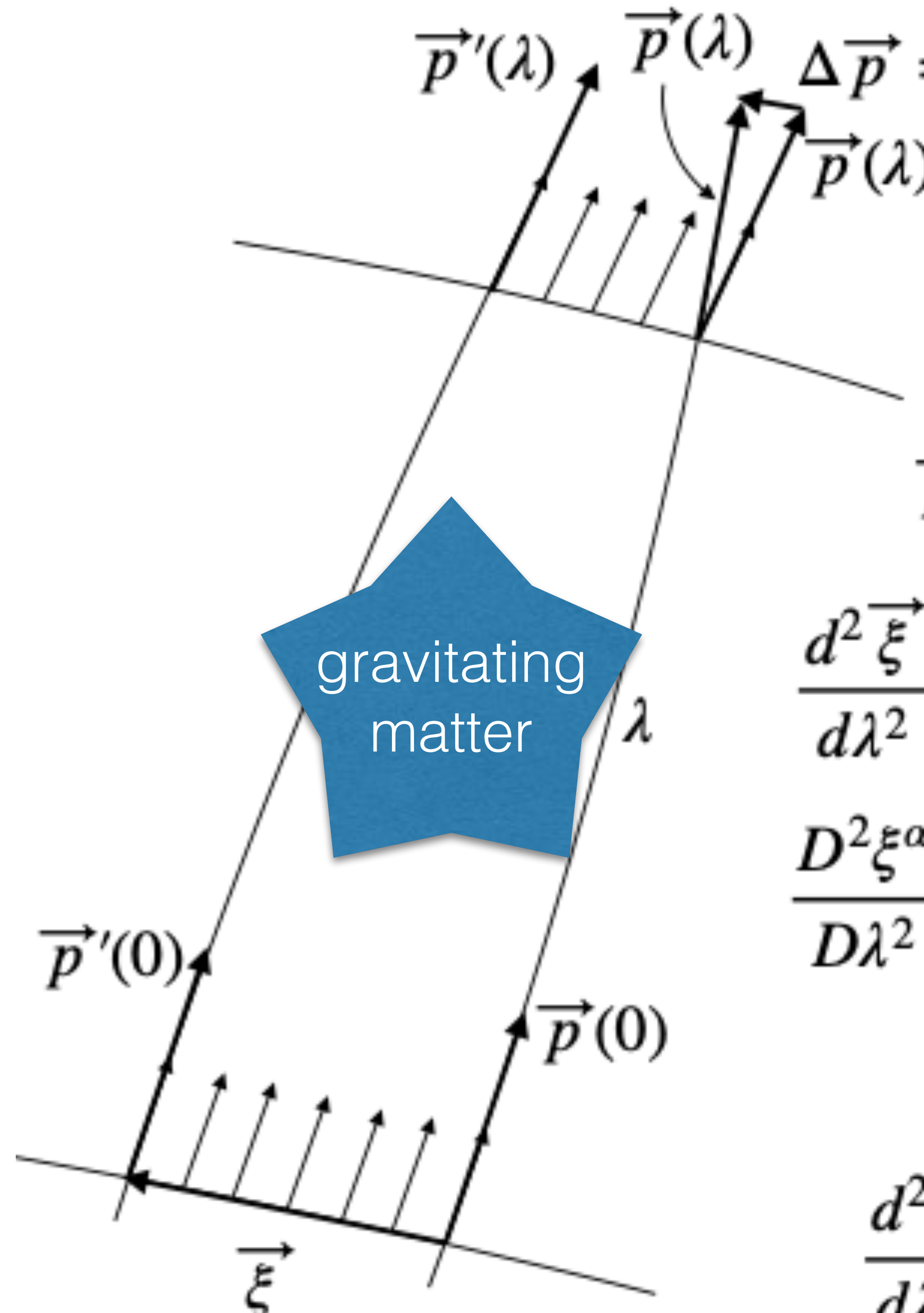

A
C
D
B
y
A'
M
C'
D'
z
B'


PATH OF LIGHT
AROUND
DARK MATTER
DISTANT
UNIVERSE
OBSERVED SKY

parallel transport -> curvature -> focussing -> Raychaudhuri







$$\Delta\vec{p} = \mathbf{R}(\ ,\vec{p},\vec{\xi},\lambda\vec{p})$$

$$\vec{p} = \frac{d\vec{x}}{d\lambda}$$

$$\frac{d^2\vec{\xi}}{d\lambda^2} = \mathbf{R}(\ ,\vec{p},\vec{\xi},\vec{p})$$

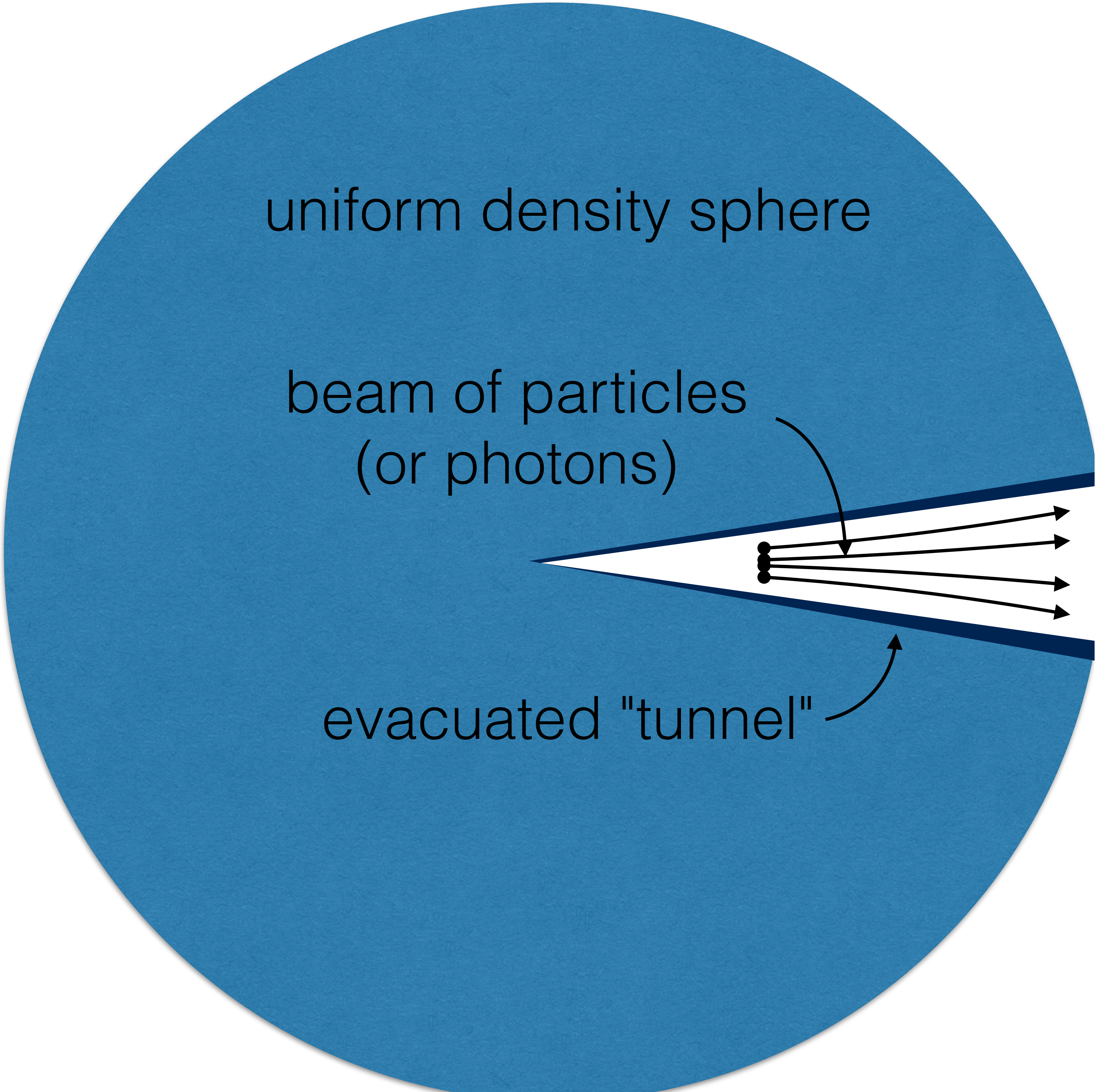
$$\frac{D^2\xi^\alpha}{D\lambda^2} = (R^\alpha{}_{\mu\beta\nu}p^\mu p^\nu)\xi^\beta$$

tidal field

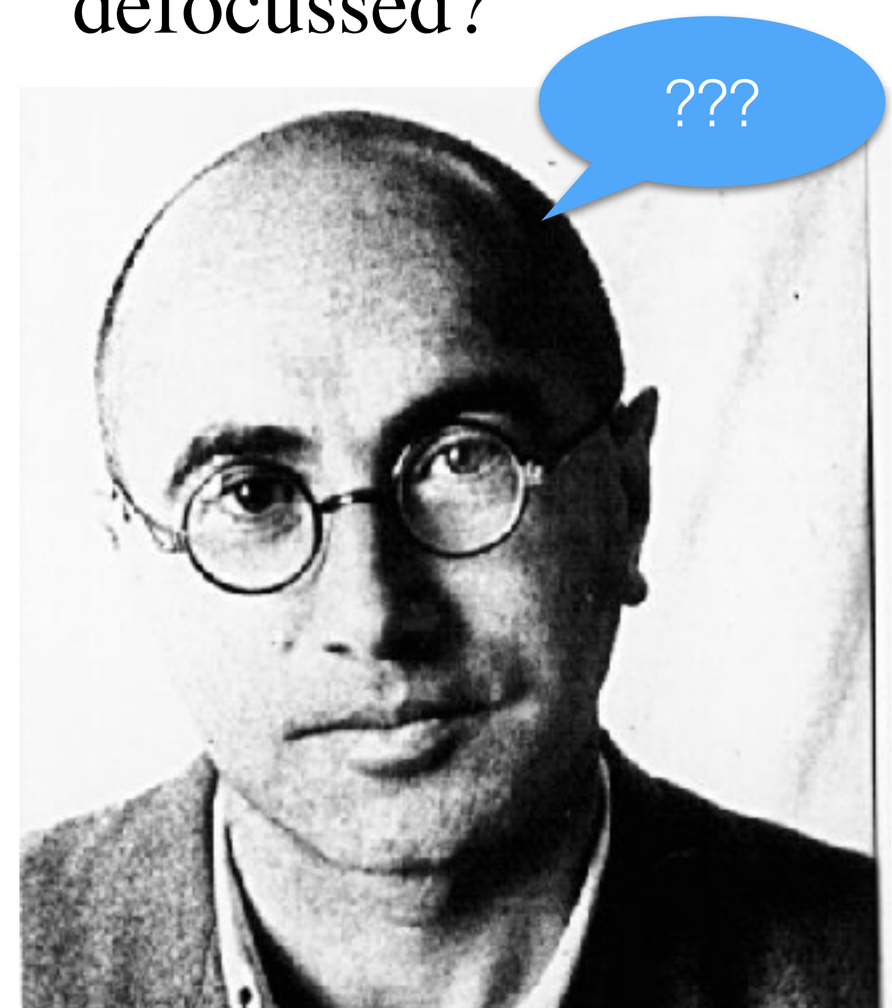
$$\frac{d^2\xi_i}{d\lambda^2} = -\phi_{ij}\xi_j$$

Raychaudhuri: directly relates focussing and curvature = matter
the metric never enters!

Zel'dovich's 1963 "empty beam" calculation



- Is there a gravitational field in the "tunnel"?
- Would Newton say that a beam of test particles would be defocussed?
- What about a beam of light? Would that get defocussed?



Geraint's tunnel

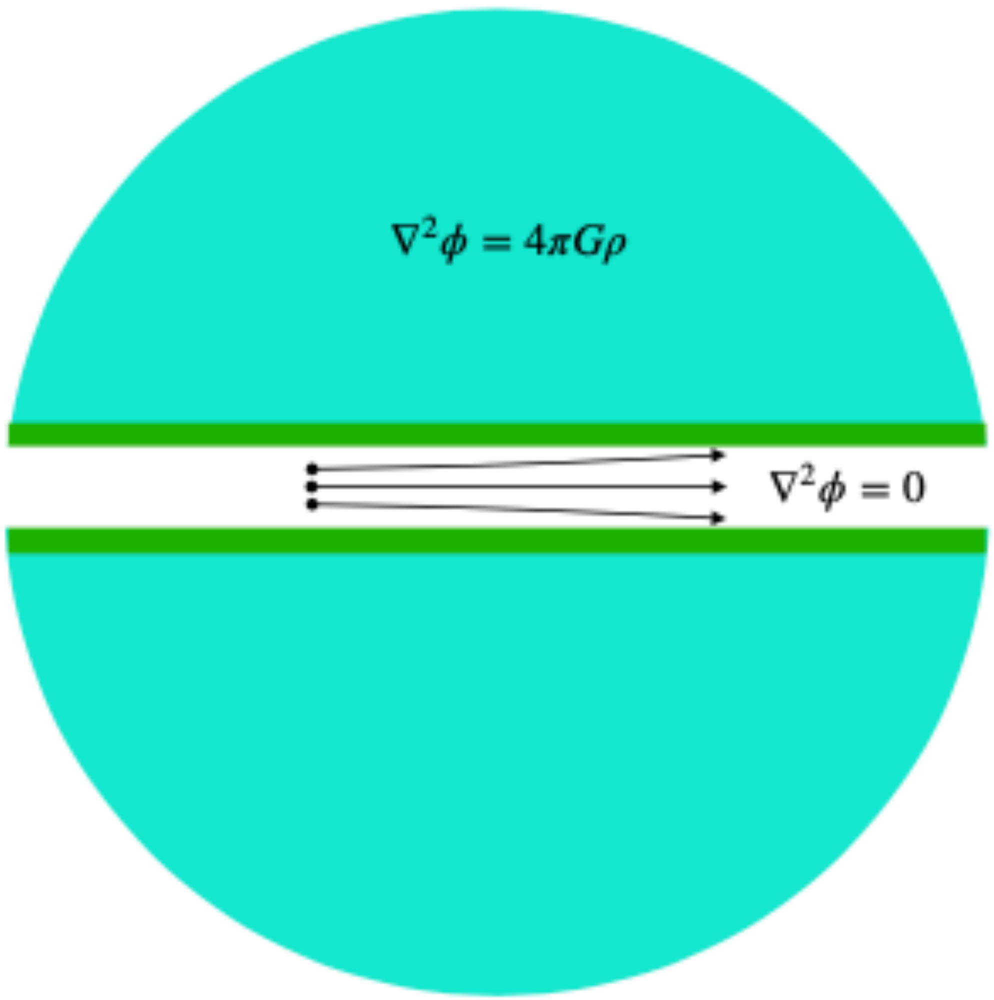
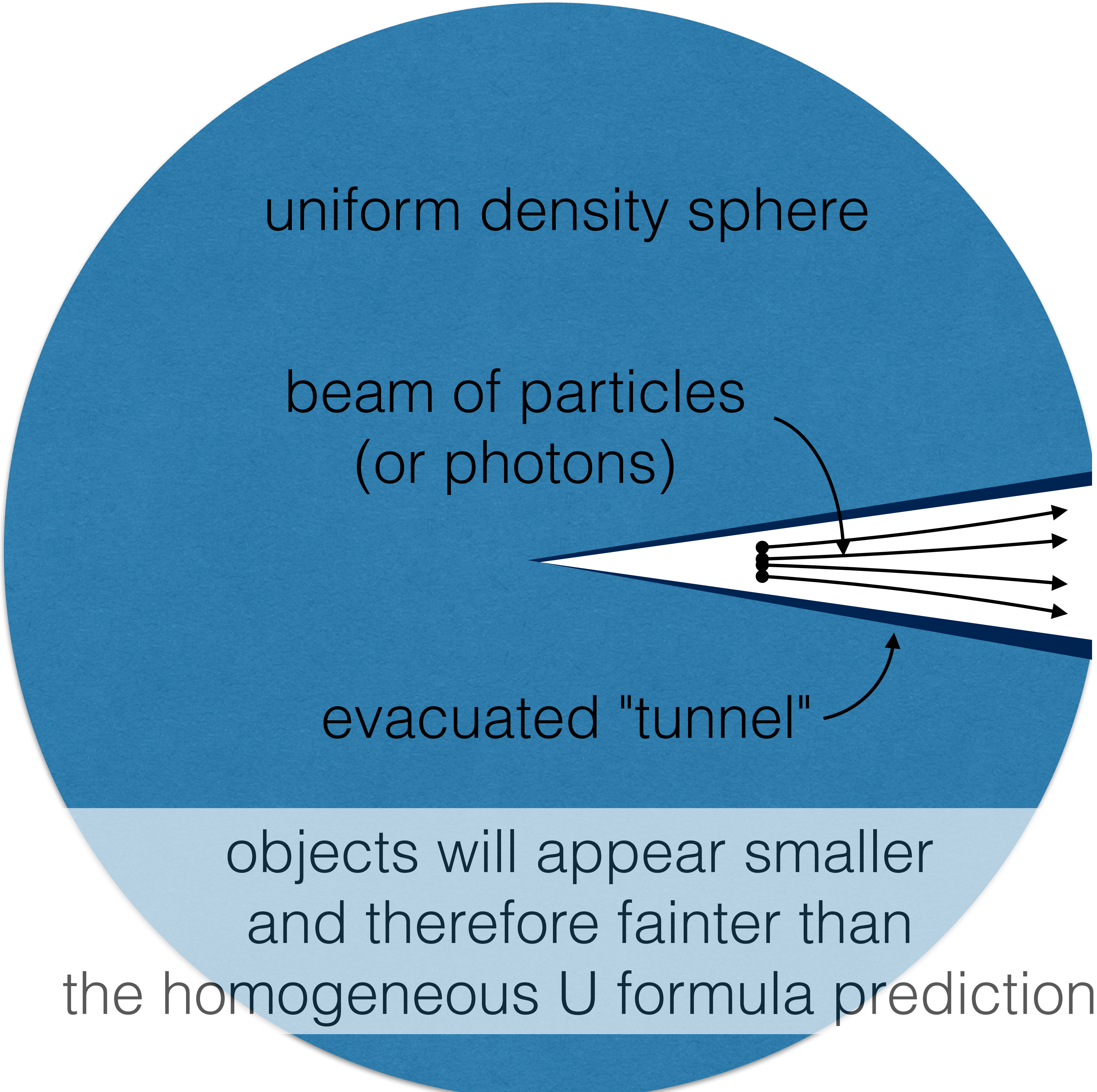
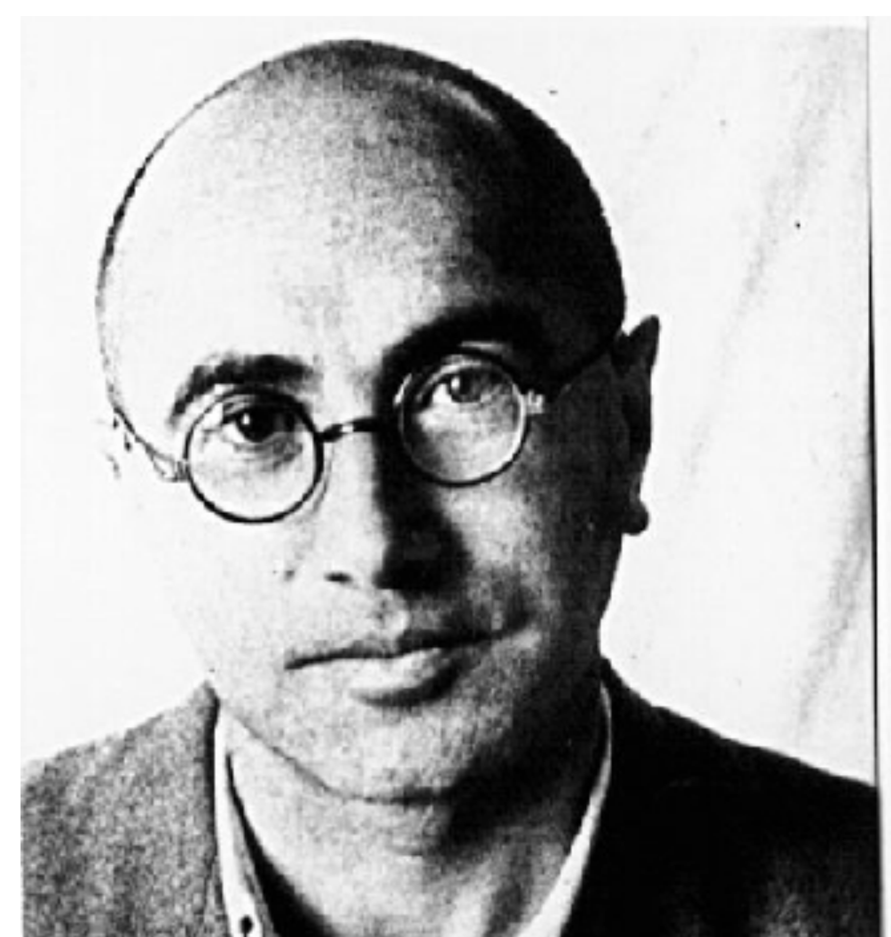


Figure 10.11: We consider a uniform density sphere through which we have drilled a tunnel. The tunnel is assumed to be very narrow compared to the radius of the sphere and we will assume that the material excavated has been compressed to make the walls of the tunnel (though this is a detail that doesn't have any significant impact). The first question is: if we fire an initially parallel beam of non-relativistic test particles down the tunnel, do they get defocussed? The second question is: what would happen if we were to fire a beam of photons – or ultra-relativistic test-particles – down the tunnel.

Zel'dovich's 1963 "empty beam" calculation



- Is there a gravitational field in the "tunnel"? **yes**
- Would Newton say that a beam of test particles would be defocussed? **yes**
- What about a beam of light? Would that get defocussed? **no!**



Geraint's tunnel

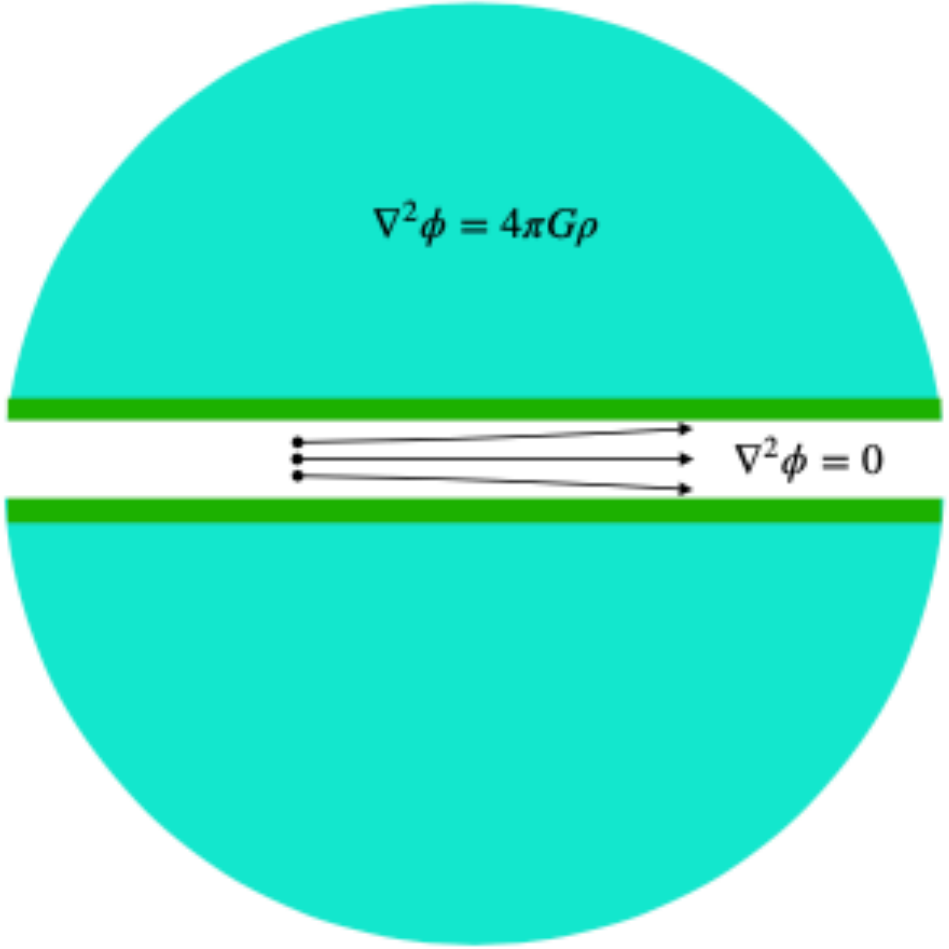


Figure 10.11: We consider a uniform density sphere through which we have drilled a tunnel. The tunnel is assumed to be very narrow compared to the radius of the sphere and we will assume that the material excavated has been compressed to make the walls of the tunnel (though this is a detail that doesn't have any significant impact). The first question is: if we fire an initially parallel beam of non-relativistic test particles down the tunnel, do they get defocussed? The second question is: what would happen if we were to fire a beam of photons – or ultra-relativistic test-particles – down the tunnel.

- The tube is empty so the trace of the tidal field tensor vanishes: $\nabla^2\phi = 0$
- But radial component of the tide $\partial^2\phi/\partial z^2 \neq 0$
    - it's the same as in the bulk of the sphere (or the "background")
- So there *is* a transverse tidal field: $\partial^2\phi/\partial x^2 = \partial^2\phi/\partial y^2 = -\frac{1}{2}\partial^2\phi/\partial z^2$
    - and so non-relativistic particles would be defocused
- But photons would not be defocused - no matter -> Ricci tensor = 0
    - so the focussing in GR is not 2 x Newton

What is going on?

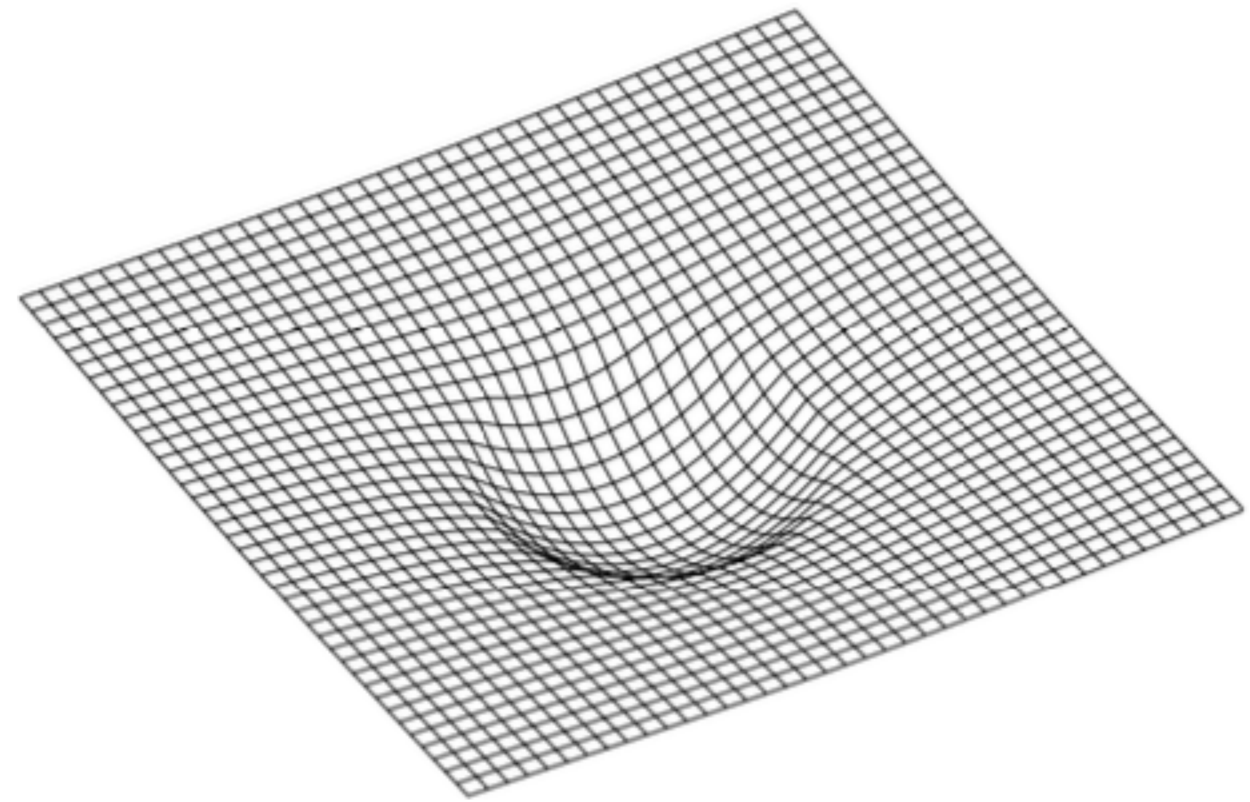
- If we assume that Einstein was right about “equivalence” (spoiler: he was!) then there is no “relativistic factor 2” involved in light bending “in the lab”
- And it seems that the defocussing of a bundle of light rays in Zel’dovich’s “empty beam” calculation (non-existent) is qualitatively different from the defocussing of non-relativistic particles in the same gravitational field.
- So the idea that light deflection and lensing is like Newton with $v \to c$ (and with a factor 2 enhancement) seems to be an utter failure
- So should we be “agog”?

Resolution: 1) light bending in the lab

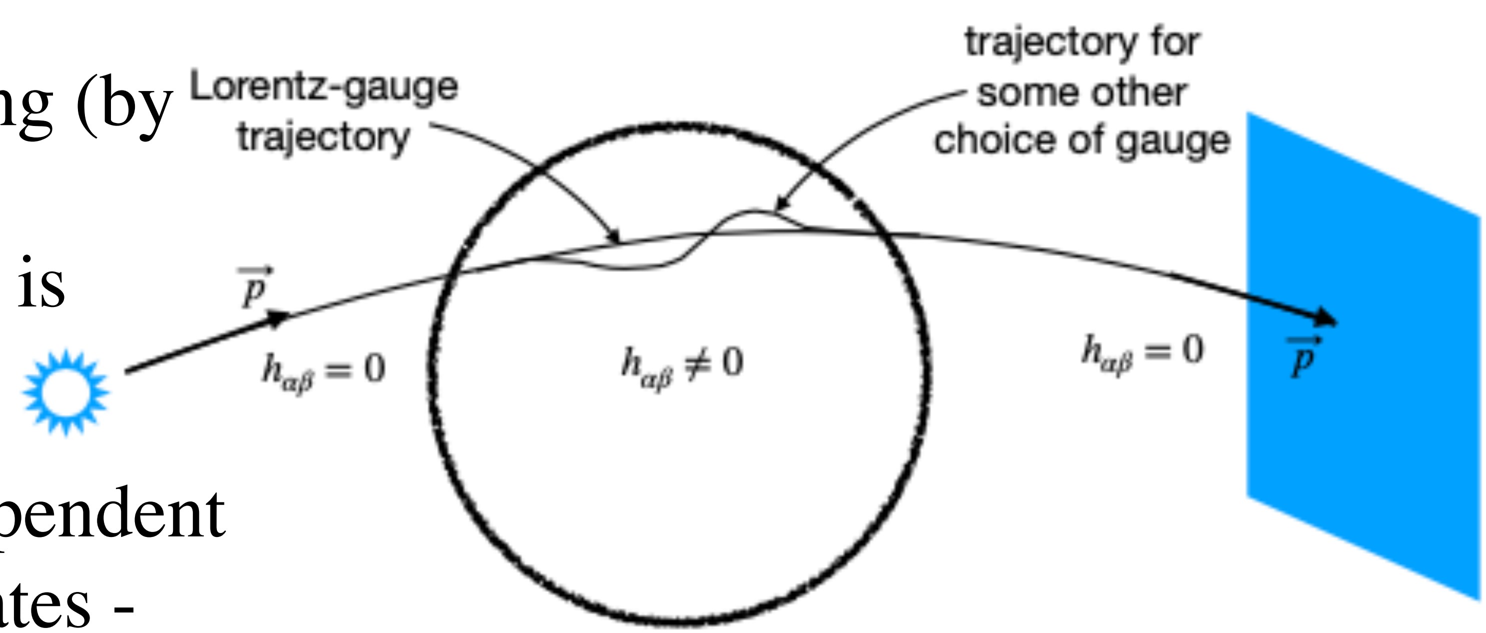
- There is *nothing* wrong with the equivalence principle argument, which tells us in-lab light deflection is the same as the Newtonian case prediction for $v = c$
- That's because it is fundamentally NOT a gravitational effect.
- We see light bending in the lab because we are *accelerated*
  - that acceleration happens to come about because we are on a massive gravitating planet, but that is irrelevant
- Bending of a single ray tells *nothing* about the gravitational field *per se*. It tells us only about the *connection* $\Gamma^\alpha{}_{\beta\gamma}$. To measure the *gravity* - i.e. the curvature - we would have to look at a pair of rays.
- Aside: There is something (mis-)named the *"local flatness theorem"* that says we can always find a coordinate system where the connection vanishes
  - e.g. the coordinate system that a freely falling (inertial) observer would construct
- Such an observer would see no light deflection. People often say that going to free-fall *"transforms away gravity"*, but that's wrong; the curvature is a *tensor*; if it vanishes in one coordinate system it vanishes in all coordinate systems.
- But what astronomers see (looking at multiple stars) does sense curvature

How to understand the extra factor 2 in GR light bending

- The equatorial plane through the Sun - a 2D surface - is curved in the same way as the 2-space *embedded* in 3-dimensions shown at the right



- The EP says that physical wavelengths are diminished (gravitational redshift) and that causes local bending as seen by fixed observers
- But there is an <u>extra</u> increase of path length for rays that pass close to the Sun because the surface is curved
- And that enhances the *global* bending (by the famous factor 2) - relative to the coordinate system at $r \to \infty$, which is spatially flat
- And while the <u>coordinate</u> path is dependent on the (arbitrary) choice of coordinates - what Eddington measured isn't



Resolution 2: Zel'dovich's empty beam calculation

- Raychaudhuri's equation is clear: *if the beam is empty there is no focussing (or defocussing)*
  - yet there *is* a non-vanishing transverse tidal field,
  - which *would* act to defocus non-relativistic particles
- Q: So is this in conflict with the weak-field gravity prediction?
- A: Yes & no.
  - Lorenz gauge WF gravity says that the *coordinate width* of an initially parallel beam of light would undergo defocussing
  - But the spatial parts of the *metric perturbation* are also varying along the beam
  - and combining these you find that the *physical width* is unchanging
  - in agreement with Raychaudhuri - but qualitatively non-Newtonian

summary/conclusions

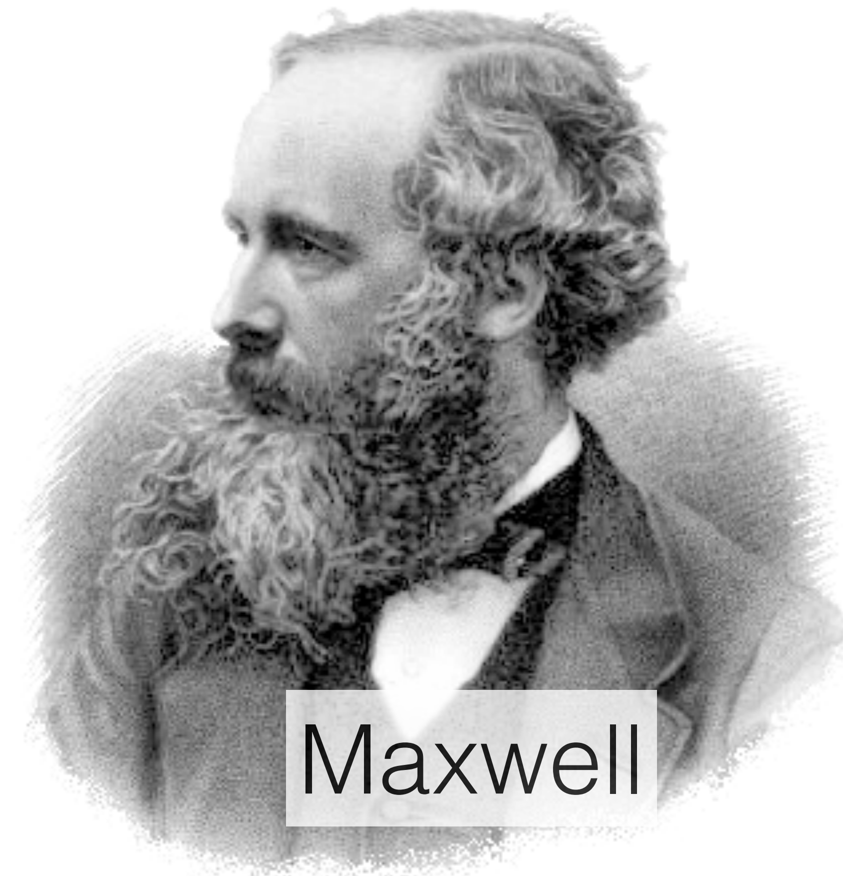
- We reviewed the “lumpy glass” analogy for lensing (in which the metric gives the effective refractive index) according to which light is deflected like Newtonian test particles with $v = c$, but twice as strongly
- We highlighted the problem that this refers to the *coordinate* speed of light, and calculates the deflection of light rays with respect to a coordinate system that - in GR - is somewhat arbitrary (gauge freedom)
    - unlike the more honest approach adopted previously, which is to use the geodesic deviation equation - or Raychaudhuri’s equation - in which only physical quantities appear
- We considered 2 situations in which the lumpy-glass model appears to give the wrong or misleading answer:
    - measurement of light bending “in the lab”
    - Zel’dovich (’63) classic “empty beam” calculation
- But were reassured that for most purposes (compact lenses) this is not a serious problem.

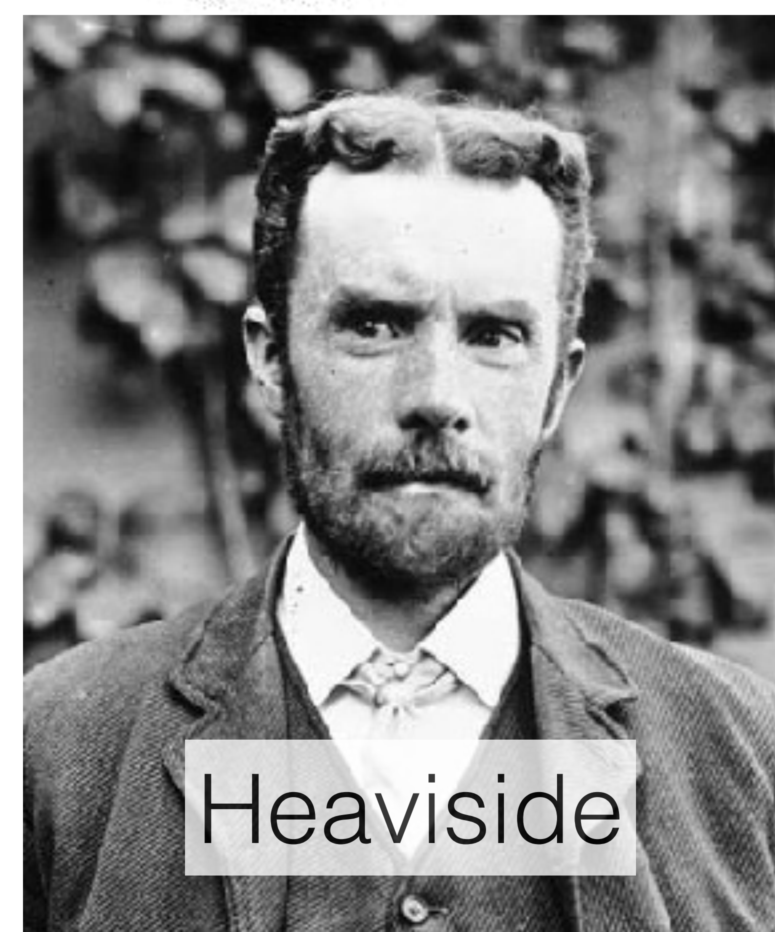
Why didn’t Newton do "Newtonian Cosmology”

- *"All of the phenomena observable at the present could have been predicted by the founders of mathematical hydrodynamics in the 18th century, or even by Newton himself*" E.A. Milne and Bill McCrea (1934)
- Newtonian could have analysed the dynamics of a uniform density expanding sphere of dust of radius R.
  - obeys Friedmann's equation -- observers can't tell where the centre is -- -- local dynamics are independent of R; the sphere can be arbitrarily large -- has a “horizon” + other features of Friedmann’s 1922 GR model
- But he had problems letting $R \to \infty$ (*“Bentley's paradox”*) - however…
- *"But if the matter was evenly disposed throughout an infinite space, it could never convene into one mass; but some of it would convene into one mass and some into another, so as to make an infinite number of great masses, scattered at great distances from one to another throughout all that infinite space. And thus might the sun and fixt stars be formed…*"
- So he was invoking what we now call *the Jeans swindle* to do gravitational instability

Aside: Mid-19th century: Why didn't Maxwell do gravity?

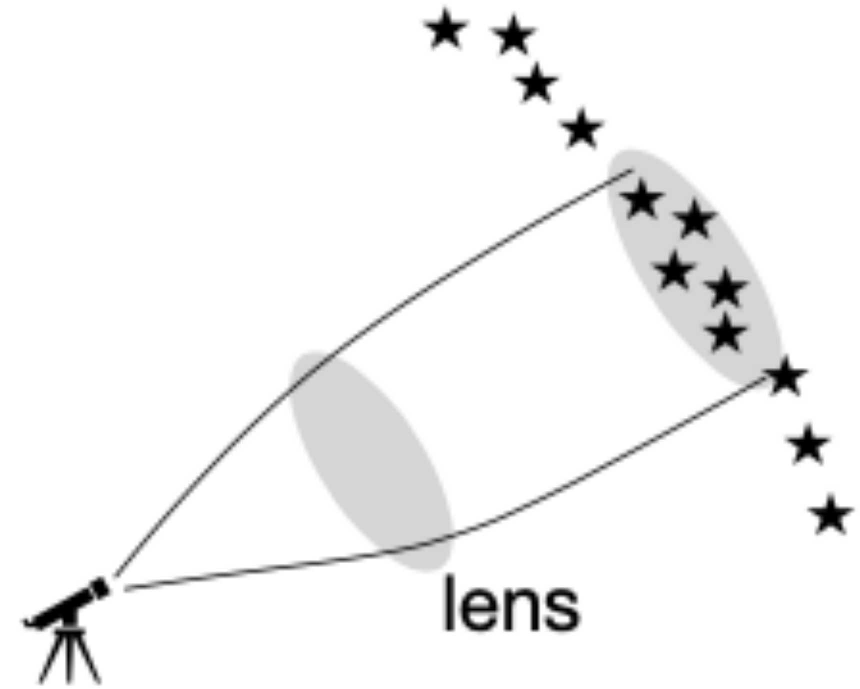
- Maxwell unified electricity and magnetism
  - a causal, relativistic, gauge, field theory...
- why didn't he follow up with gravity?
- he did get the stress tensor for gravity:
  - $T_{ij} = (g_i g_j - \frac{1}{2}\delta_{ij} |\mathbf{g}|^2)/4\pi G$
  - just like Maxwell EM stress tensor
  - but =32,000 tons per square inch here!
  - *"I do not think space is strong enough to withstand such a stress"*
- a sadly missed opportunity:
  - $\Box A^\mu = j^\mu/\mu_0 \quad \Rightarrow \quad \Box \overline{h}_{\mu\nu} = -T_{\mu\nu}/16\pi G$
  - $dp^\mu/d\tau = qF^{\mu\nu}U_\nu \Rightarrow dp_\mu/d\lambda = -\frac{1}{2}h_{\alpha\beta,\mu}p^\alpha p^\beta$

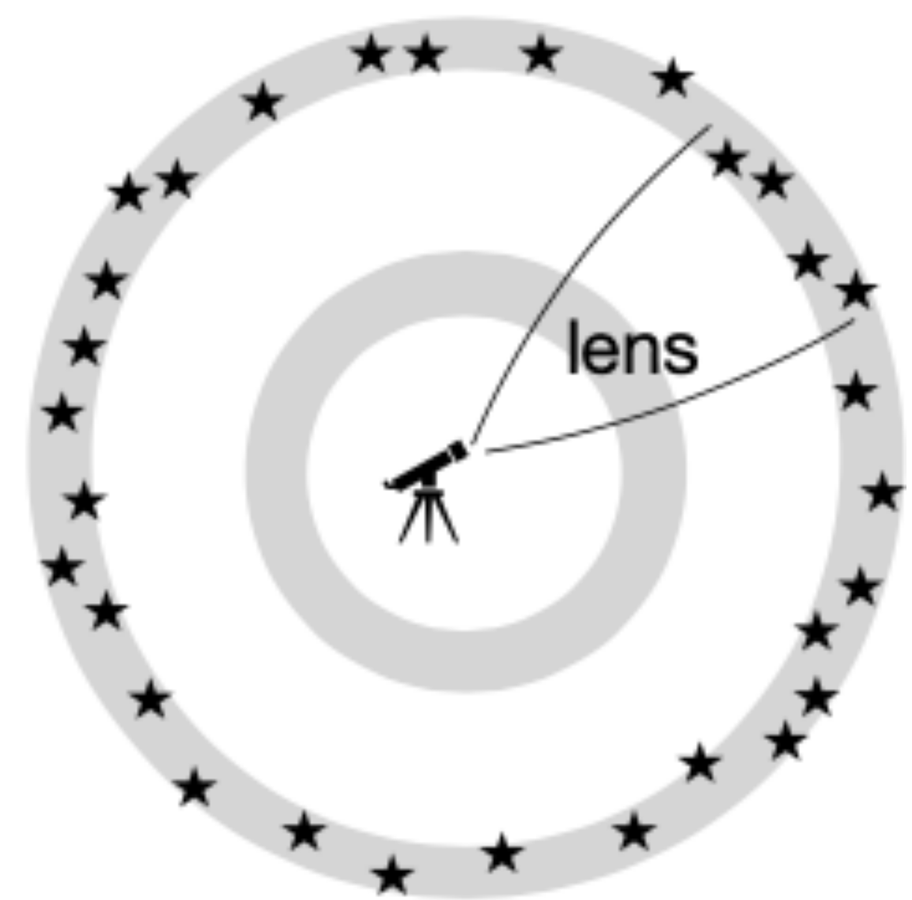




Zel'dovich '63: From Rauchaudhuri to the FLRW metric

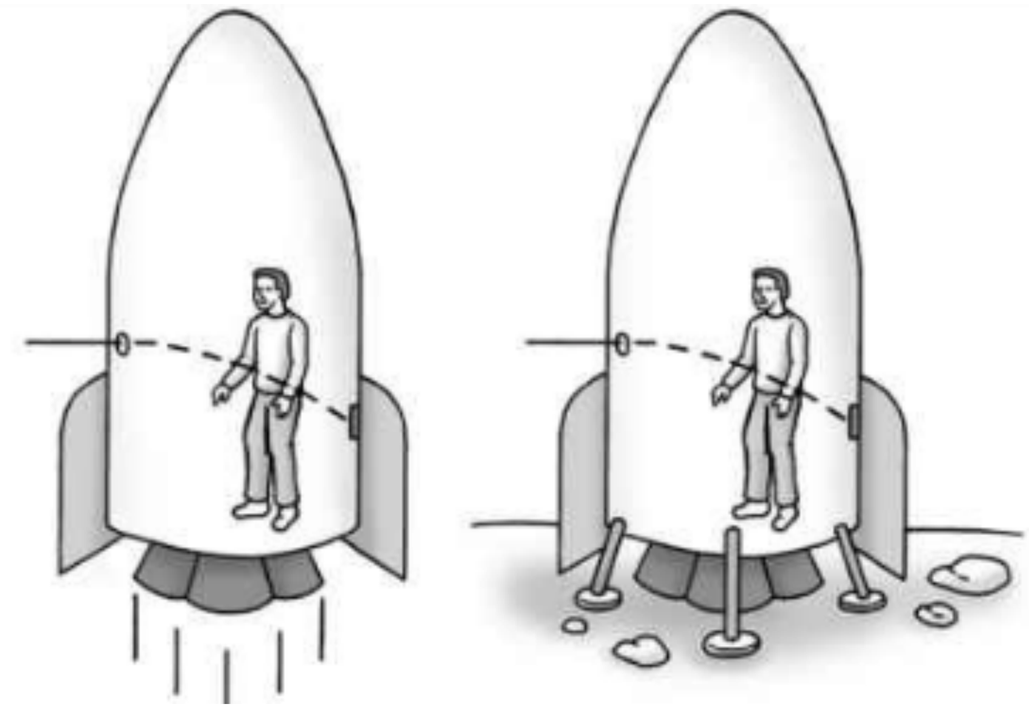
- With $r = \sqrt{A}$ and affine $d\lambda = -\,a d\tau = a^2 d\chi$ Raychaudhuri's focussing equation is
  - $$r'' \equiv \frac{d^2 r}{d\lambda^2} = -\frac{4\pi G(\rho + P/c^2)}{c^2 a^2} r$$
  - comes from GR, but mostly Newton (x2)
- and has solution, for bundle of angle θ at observer,
  - $r = \theta a \sin(\chi)$
  - proof: with $r'/\theta = -\cos(\chi)/a + \sin(\chi) a'$ etc
  - and with Friedmann equation + continuity
    - $$a'' = -\frac{4\pi G(\rho + P/c^2)}{ac^2} + \frac{1}{a^3}$$
  - $$r''/\theta = \cancel{\frac{a'}{a^2}\cos\chi} - \frac{1}{a^3}\sin\chi + a''\sin\chi - \cancel{\frac{a'}{a^2}\cos\chi}$$
- but $ds^2 = -\,c^2 dt^2 + a^2(t)(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2))$ implies $r = a\theta\sin\chi$ and hence $D_{\mathrm{A}} = a\sin\chi$
- so D_{A} + F-metric can be "derived" from local focussing





Light bending “in the lab”

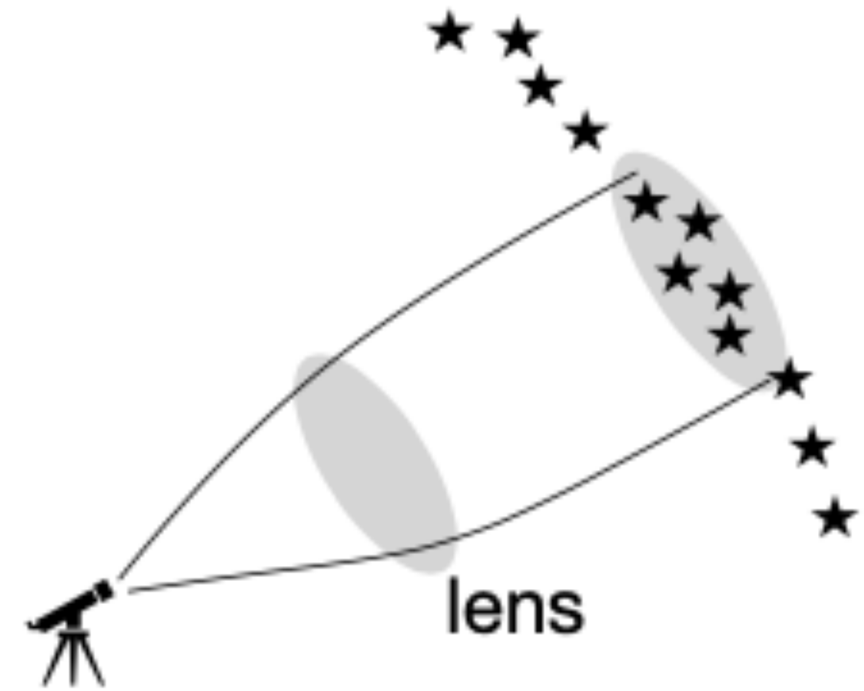
- Weak-field (Lorenz-gauge) gravity says the deflection of light by <u>gravity</u> - from the geodesic equation - is 2x Newtonian pred^{n}
- But an <u>accelerated</u> observer would see a deflection equal to the Newtonian value
- As argued by Einstein ca. 1910 using the *principle of equivalence* (EP)

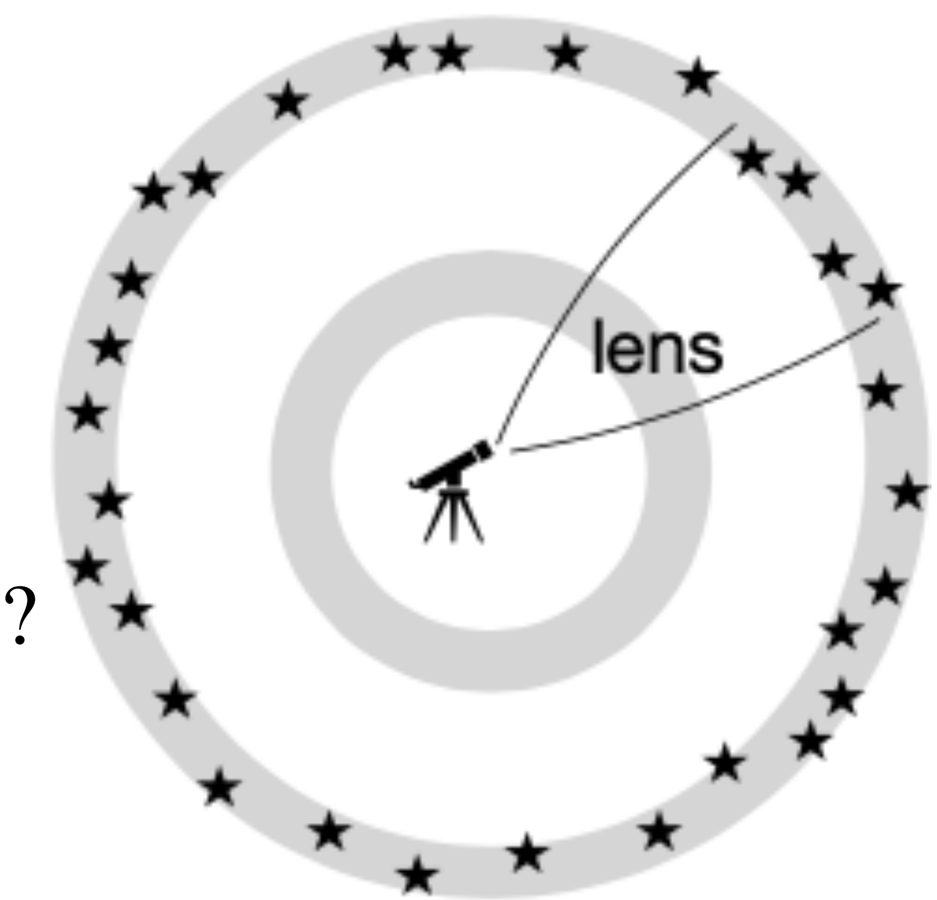


- Q: what's wrong with the EP argument?
  - A: nothing - it gives the light bending *relative to locally straight lines*
- Q: what’s wrong with the Lorenz-gauge result?
  - A: spatial part of metric is *curved* - so straight lines do not have $\mathbf{x}'' = 0$
    - their *coordinate curvature* is just equal to the EP bending
  - the extra factor 2 is a *coordinate (or gauge) artefact*
- but EP *doesn't* predict what astronomers see for images of stars near the Sun — that involves *spatial curvature* also

What if Newton *had* done Newtonian cosmology?

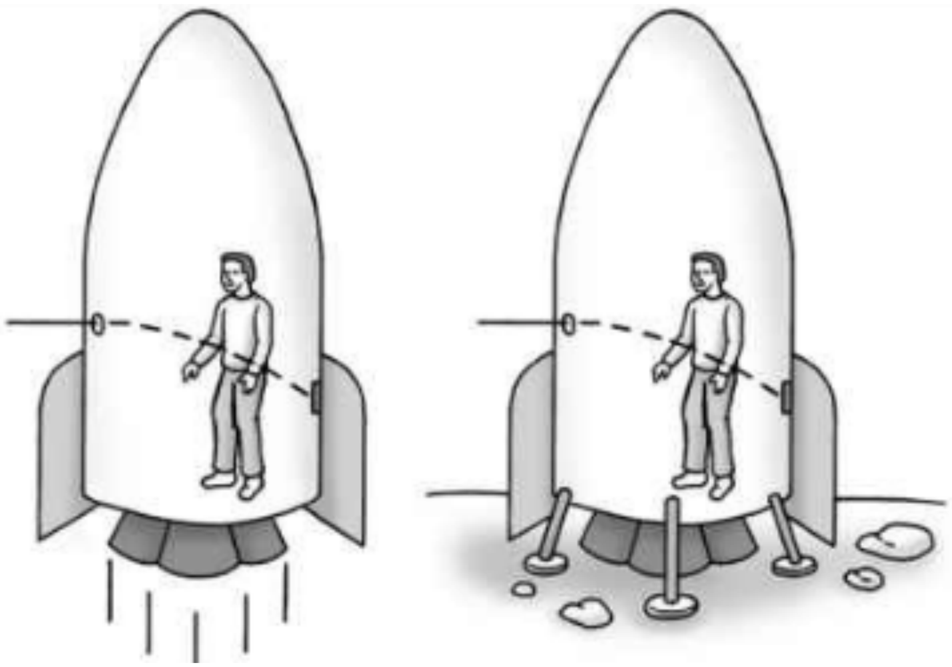
- He'd have found Friedmann's eq^{n}, energy eq^{n}, continuity eq^{n}, for an expanding universe
  - You might ask "*what would have prompted him to do so?*". Good question: but it didn't stop Alexander Friedmann in 1922.
  - 7 years before the expansion was discovered by Hubble
- But he would probably have had difficulty with the angular diameter distance
  - for the same reason he had trouble with "Bentley's paradox"
- What'd he have said about "Hoekstra's paradox"?
  - does a spherical shell lens deflect light?



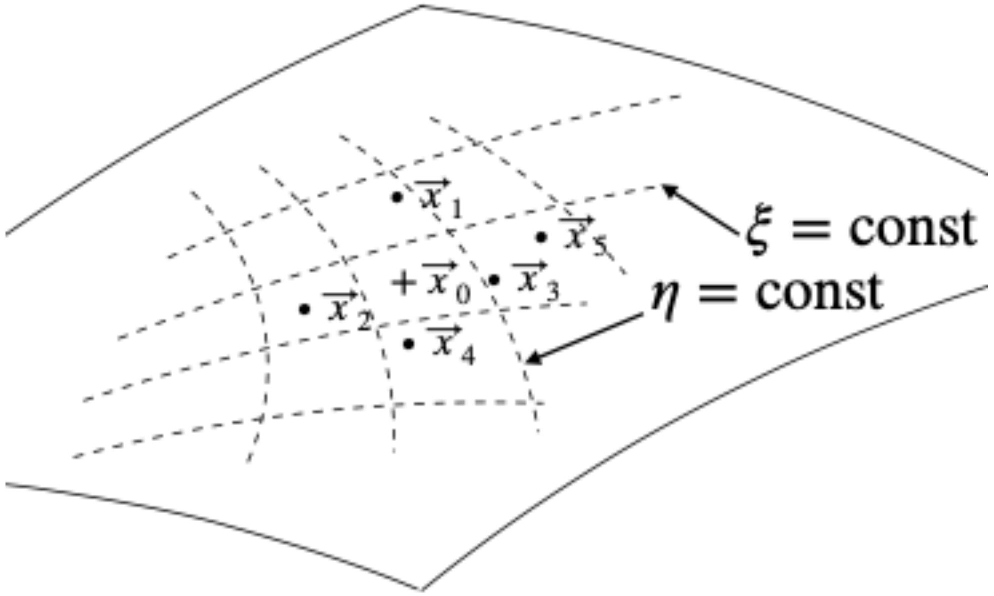


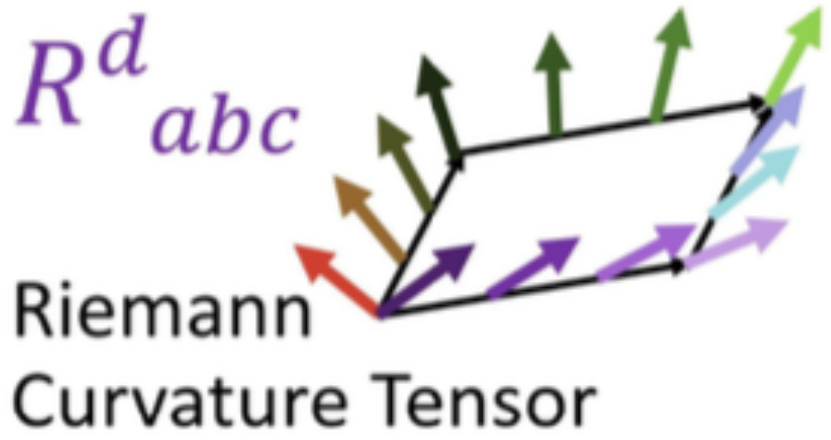
Where does the "extra factor 2" come from?

- Q: what's wrong with the EP argument?
- A: nothing - if used for *local light bending*
  - a "flat-space-time" phenomenon
- but it doesn't predict what astronomers see for images of stars seen near the Sun
  - that involves *spatial curvature* also



- The arena of GR is a smooth "*curved manifold*" on which you can lay down curvi-linear coords
- smoothness means you can always find "*locally flat*" coordinates in terms of which *local* physics is just as in SR
- but things like focussing of particles or light depend on the *curvature* of the manifold
- the curvature - a tensor - is encoded in 2nd derivatives of the metric
- and is determined by the matter stress-tensor $T_{\mu\nu}$





Development of weak-lensing

- Pioneering study of Tyson, Valdes & Wenk led the way
- Theory:
  - understanding of what was measured (shear = shape polarisation γ)
  - inversion techniques: $\gamma(\mathbf{r}) \Rightarrow \kappa(\mathbf{r}) = \mathbf{\Sigma}(\mathbf{r})/\mathbf{\Sigma}_{\text{crit}} \Leftarrow$ "*mass-maps*"
    - Σ = surface mass density, $\Sigma_{\text{crit}} \sim c\rho/H \sim 1\text{gm}/\text{cm}^2$
  - Estimation of power spectrum

- Observation:
  - (Total = dark + luminous) mass maps
  - Power-spectrum
  - Constraints on nature of DM (e.g. the "bullet cluster")
- Emphasis shifted from mapping DM to constraining "dark energy" and testing "modified gravity": DETF need for dedicated facilities => Euclid + LSST