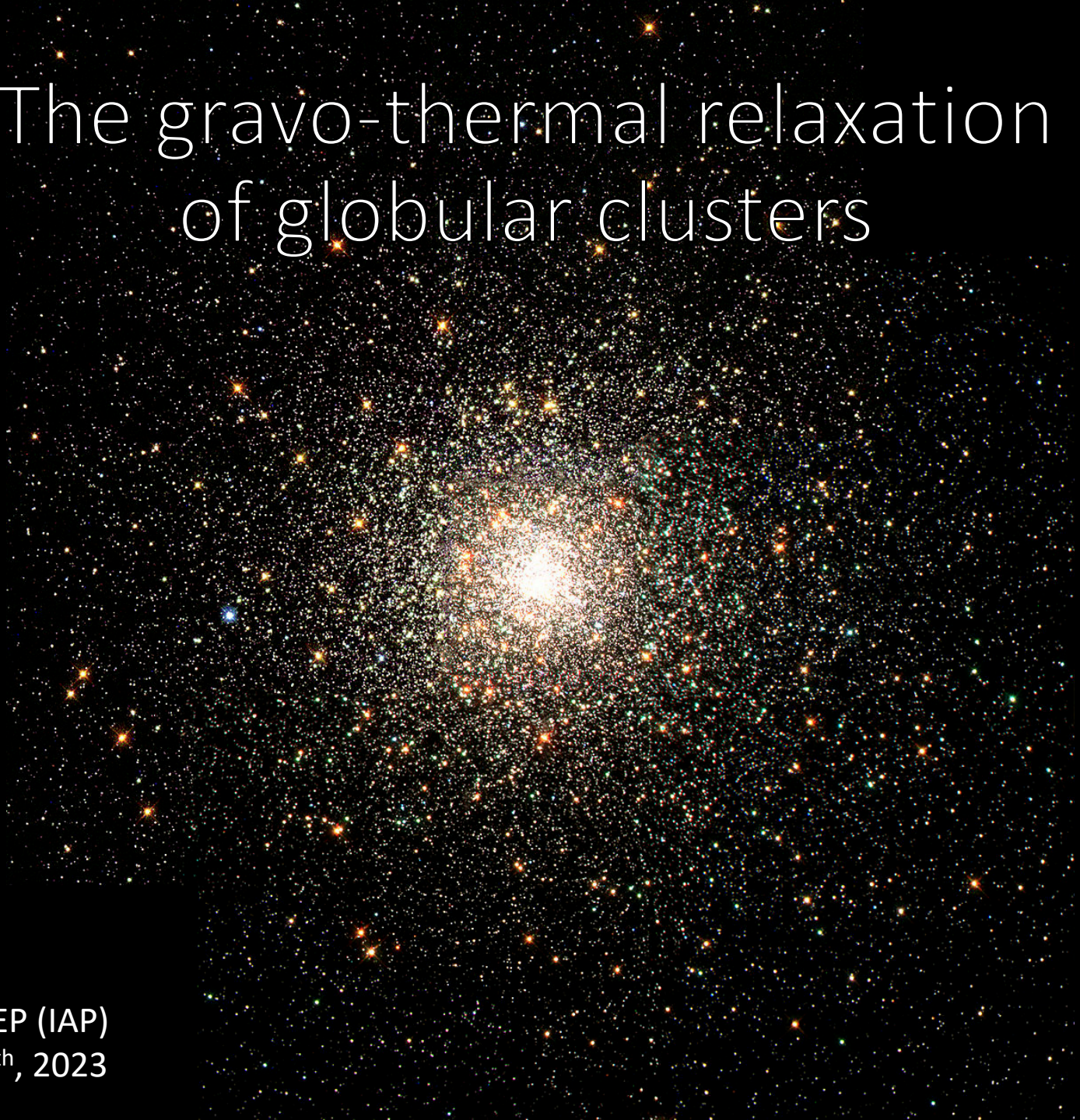
The gravo-thermal relaxation of globular clusters



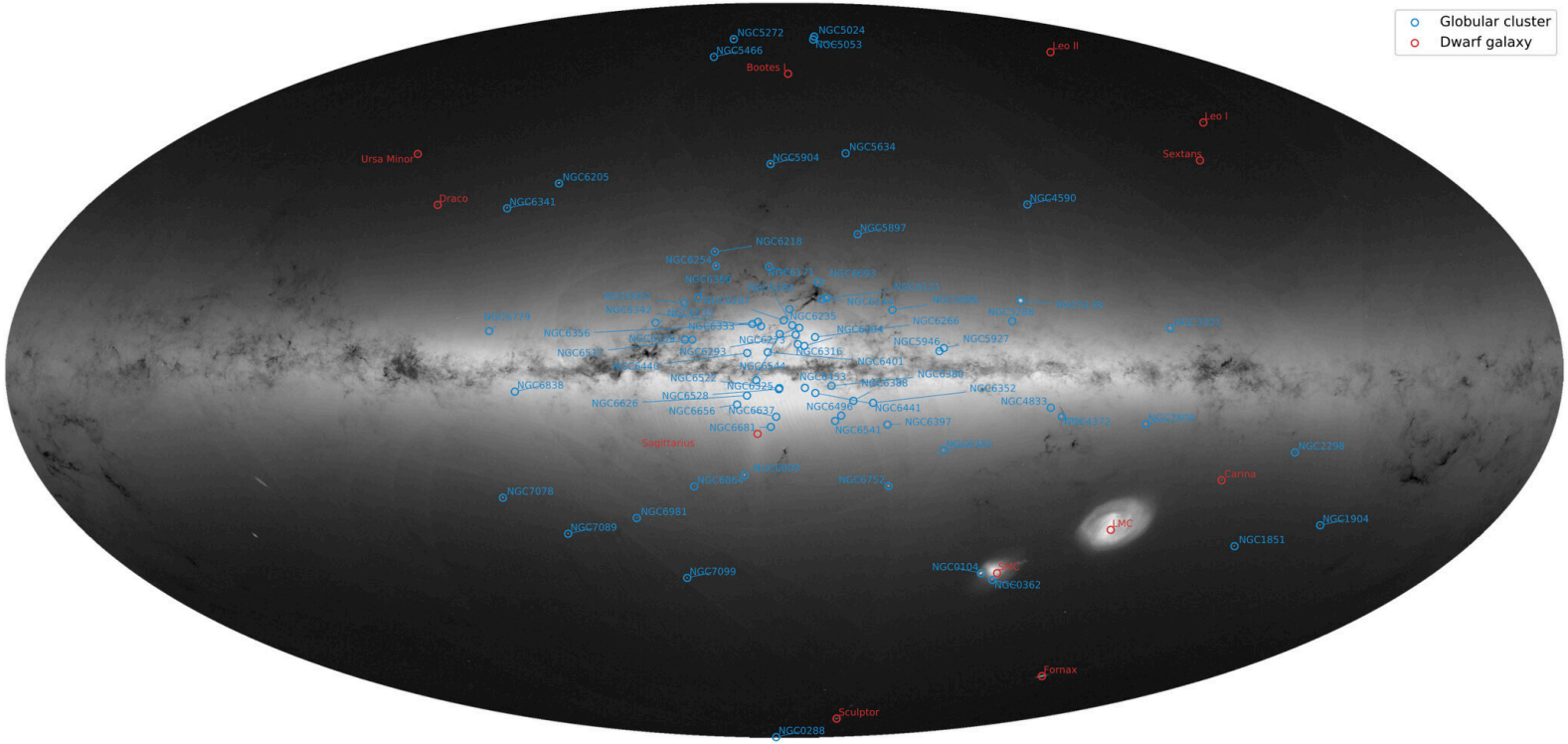
Kerwann TEP (IAP)
January 24th, 2023

Globular cluster

- Dense, spherical stellar cluster: 10^5 stars
- Size: a few pcs
- Very old: 10^{10} yrs
- Crossing time: 10^5 yrs

M80

Globular clusters near us



→ 150 GC in the Galaxy

Globular cluster

- Dense, spherical stellar cluster: 10^5 stars
- Size: a few pcs
- Very old: 10^{10} yrs
- Crossing time: 10^5 yrs
- Relaxation time: 10^{10} yrs

→ Core collapse (20%)

M80

Compare to the Galaxy:

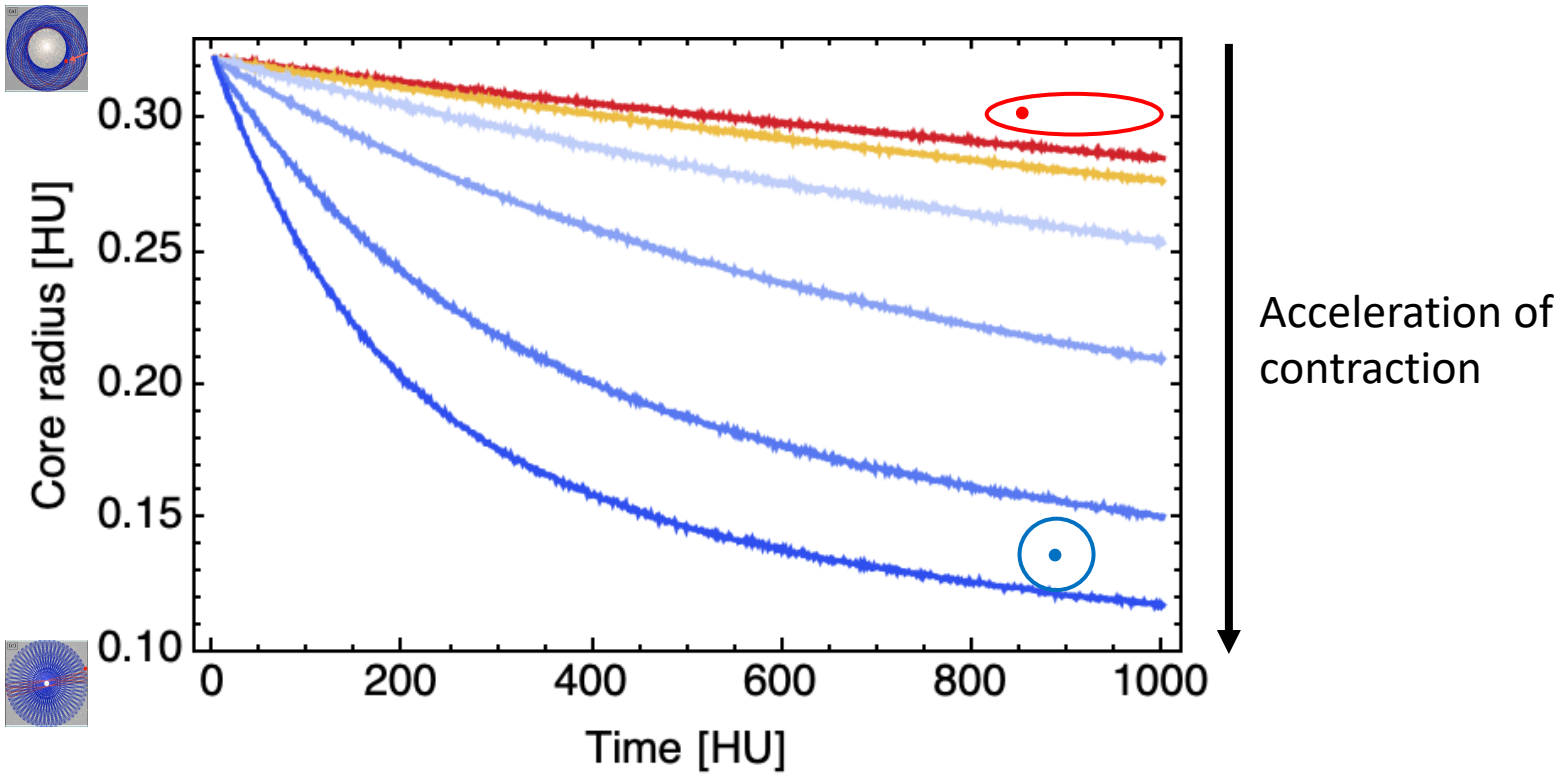
→ Number of stars: 10^{11} stars

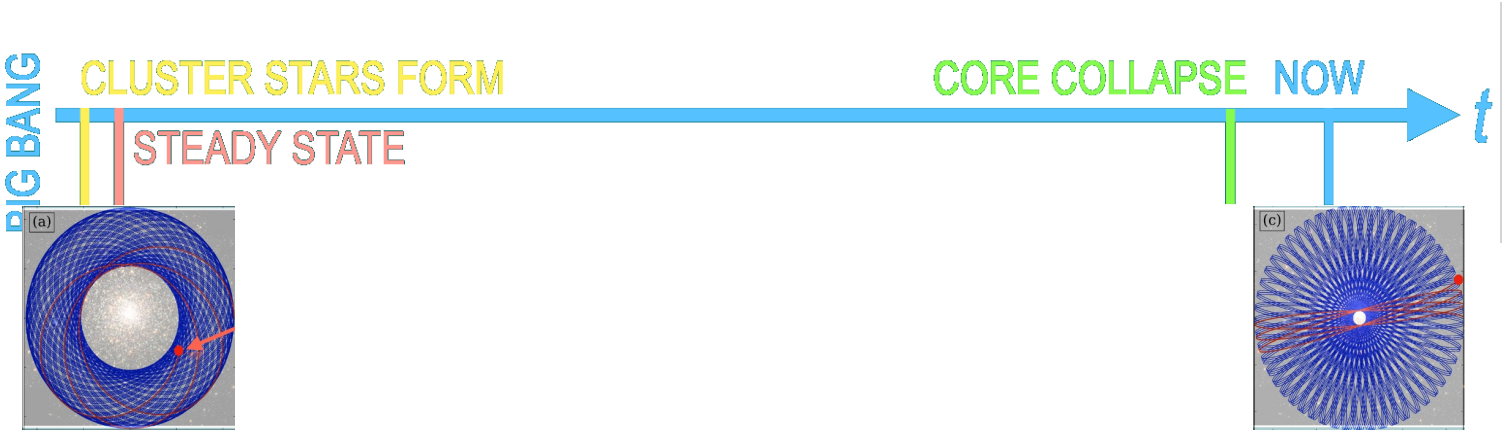
→ Diameter: ~ 30 kpc

→ Age of the MW: 10^{10} yrs

→ Relaxation time: 10^{13} yrs

Context


Core radius [HU]
0.30
0.25
0.20
0.15
0.10
0
200
400
600
800
1000
Time [HU]
Acceleration of contraction


BIG BANG
CLUSTER STARS FORM
STEADY STATE
CORE COLLAPSE
NOW
t
(a)
(c)

Objectives :

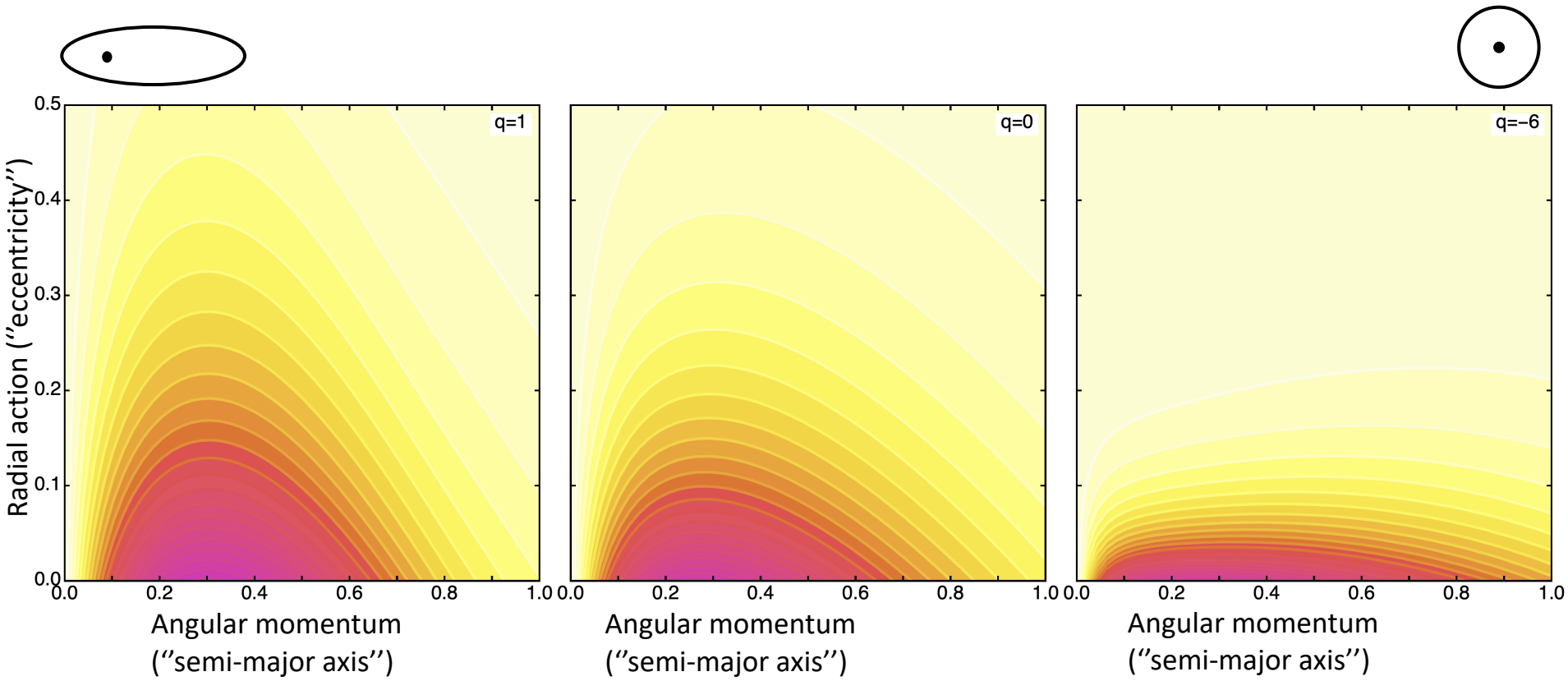
- Explain the rate of orbital diffusion

→ Using analytical considerations

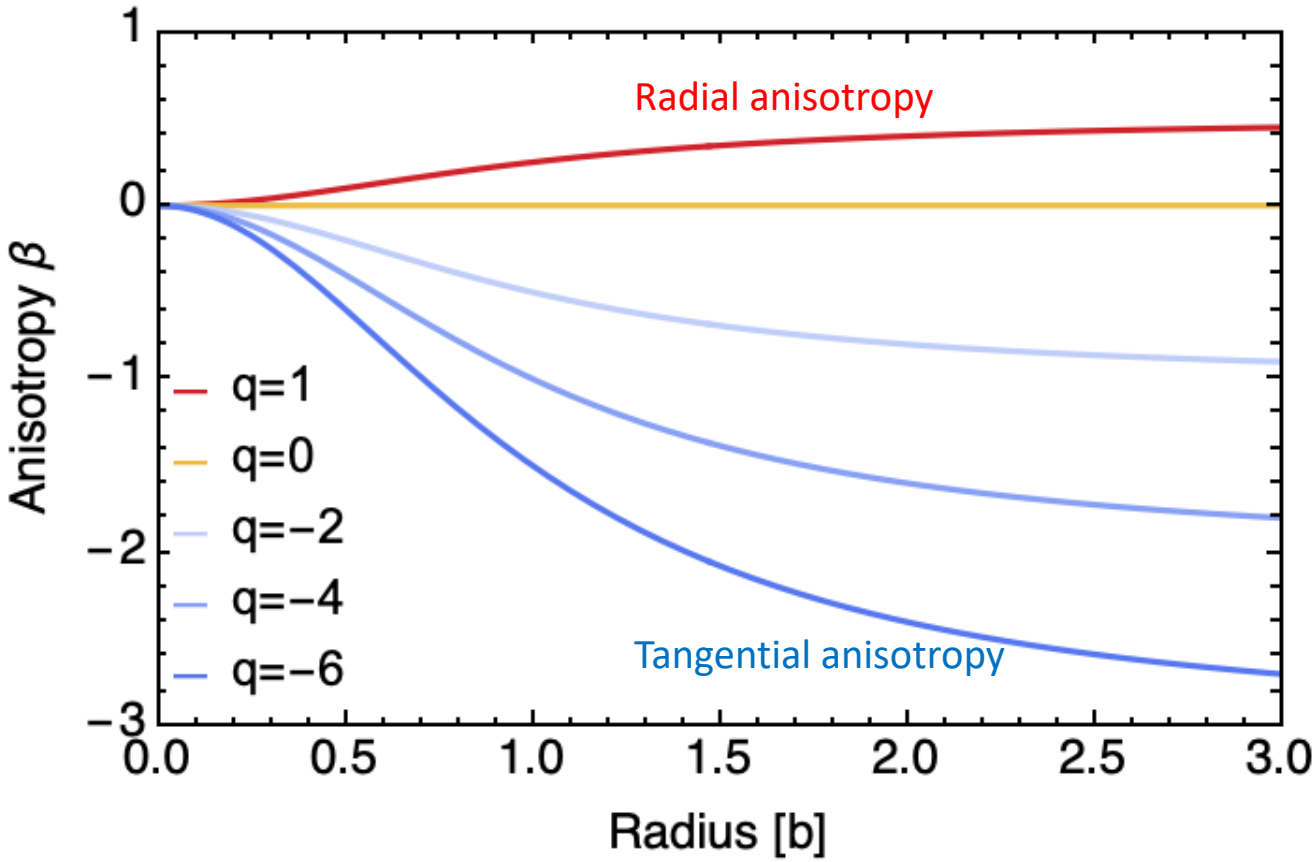
→ Comparing to NBODY simulations

The system of interest (DF)

- Concentration of radial/circular orbits



The system of interest



- N-body system bound by gravity
- Plummer potential
- Velocity anisotropy

→ No rotation

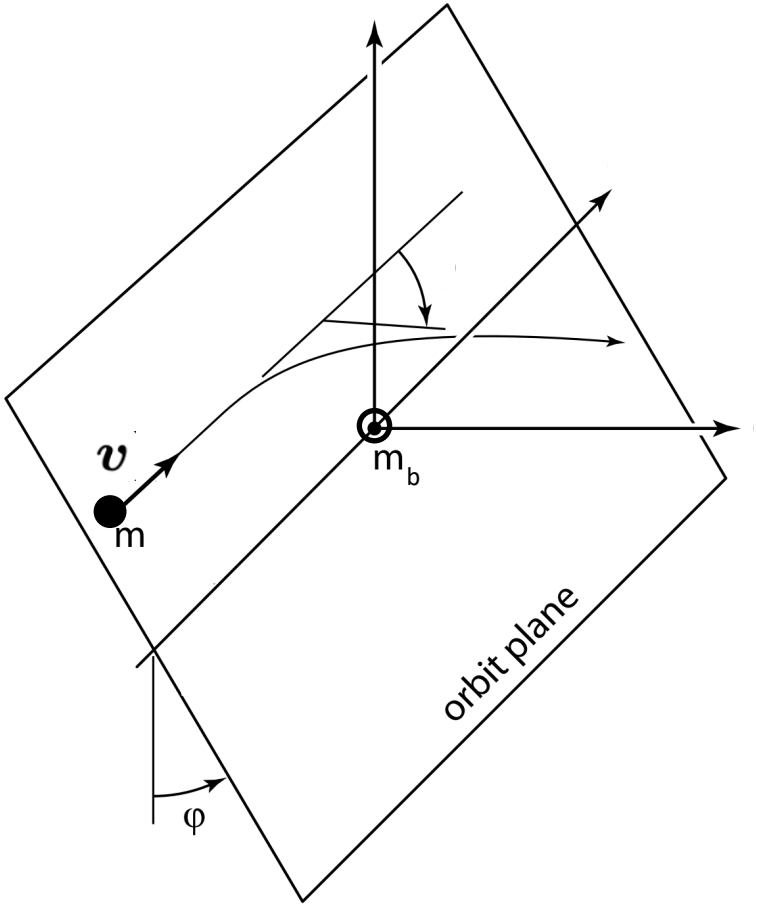
$$\beta(r) = 1 - \frac{\sigma_{\rm t}^2}{2\sigma_{\rm r}^2} = \frac{q}{2}\frac{r^2}{1+r^2}$$

Chandrasekhar theory (NR)

- Infinite, locally homogeneous
- Fixed background of field stars
- Test stars follow straight lines
- 2-body deflections along orbit

→ weak, local, uncorrelated

→ cumulative along test star's orbit



Binney & Tremaine (2008)

Chandrasekhar theory (NR)

- Infinite, locally homogeneous
- Fixed background of field stars: impacted by anisotropy
- Test stars follow straight lines
- 2-body deflections along orbit

→ weak, local, uncorrelated

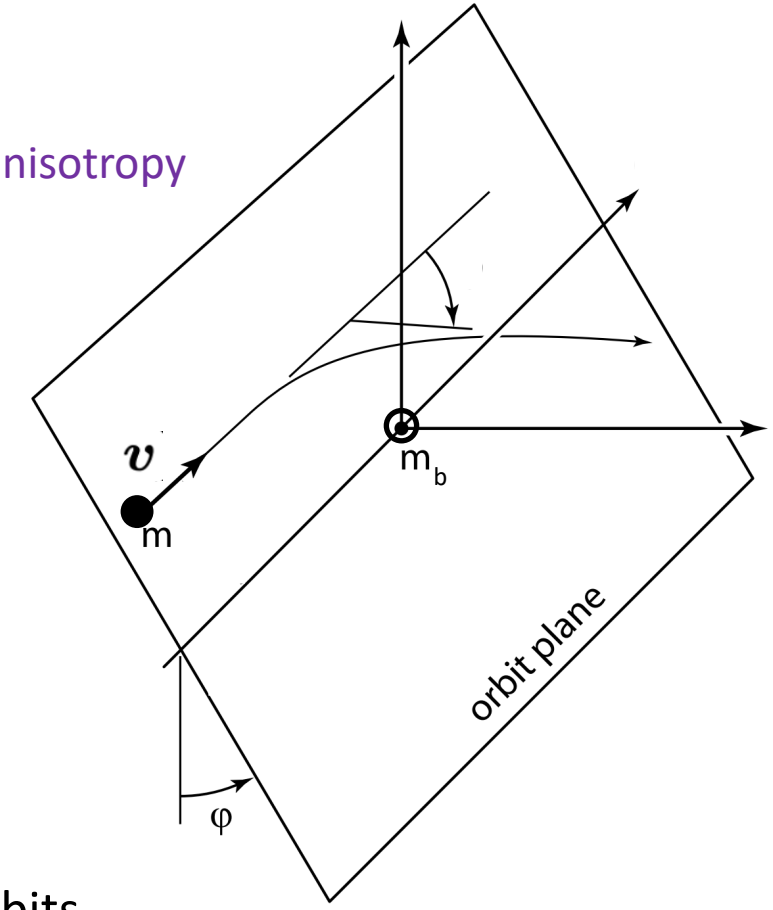
→ cumulative along test star's orbit

Average over orientations
and encounters: impacted by anisotropy

→ Drift (Δv) and diffusion (Δv^2) of test stars' orbits

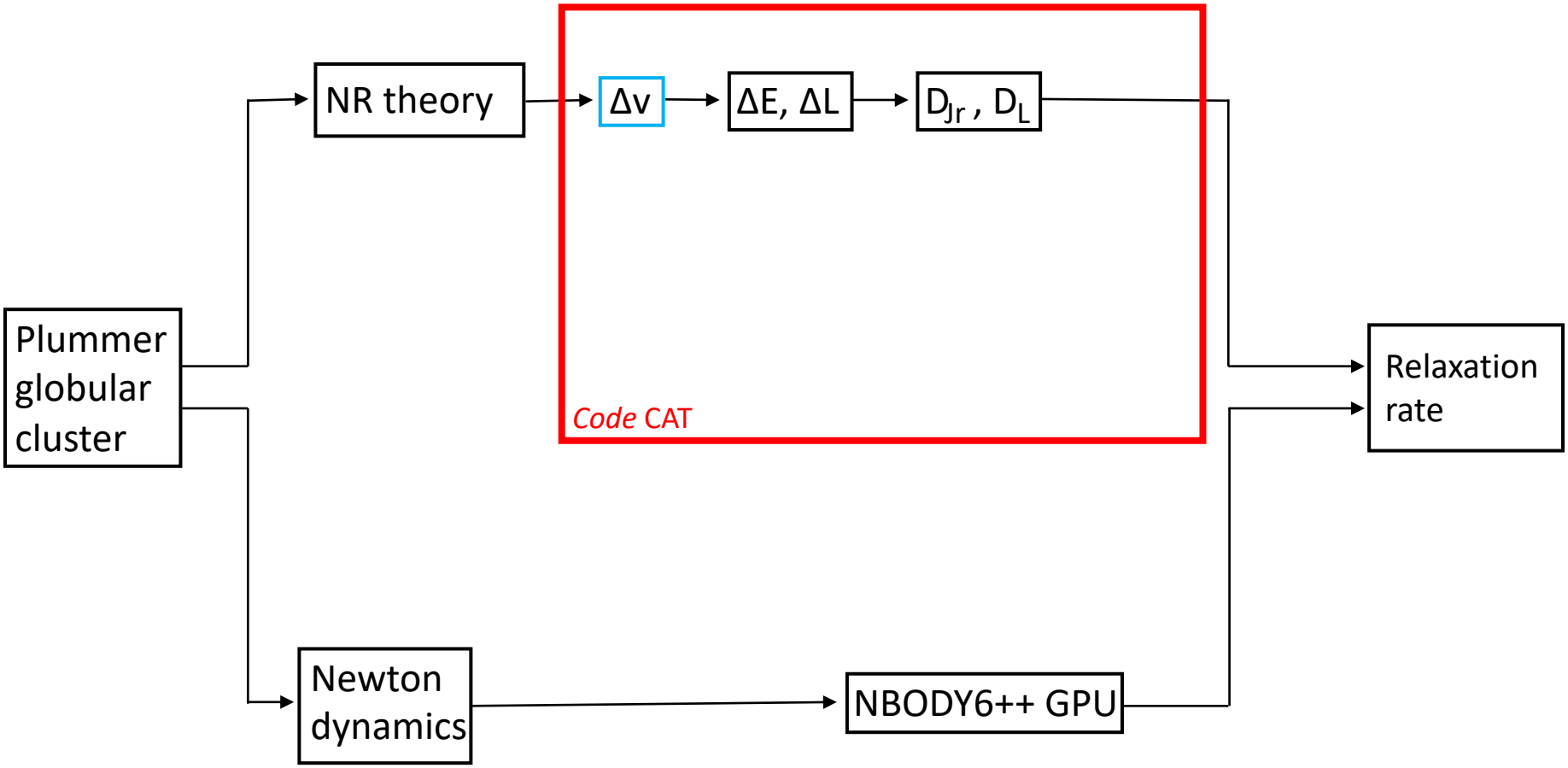
→ Fokker-Planck evolution equation

→ Free parameter: Coulomb log $\boxed{\ln \Lambda}$



Binney & Tremaine (2008)

Computation of secular response



Fokker-Planck equation

$$\frac{\partial \boxed{F(\boldsymbol{J})}}{\partial t} = -\frac{\partial}{\partial \boldsymbol{J}} \cdot \boxed{\boldsymbol{F}(\boldsymbol{J})} = -\frac{\partial}{\partial \boldsymbol{J}} \cdot \left[\boxed{\boldsymbol{D}_1(\boldsymbol{J})} \boxed{F(\boldsymbol{J})} - \frac{1}{2} \frac{\partial}{\partial \boldsymbol{J}} \cdot \left[\boxed{\boldsymbol{D}_2(\boldsymbol{J})} \boxed{F(\boldsymbol{J})} \right] \right]$$

$$\boldsymbol{D}_1(\boldsymbol{J}) = \begin{pmatrix} D_{J_\mathrm{r}} \\ D_L \end{pmatrix}, \quad \boldsymbol{D}_2(\boldsymbol{J}) = \begin{pmatrix} D_{J_\mathrm{r} J_\mathrm{r}} & D_{J_\mathrm{r} L} \\ D_{J_\mathrm{r} L} & D_{LL} \end{pmatrix}$$

$\boxed{F(\boldsymbol{J})}$ Action DF

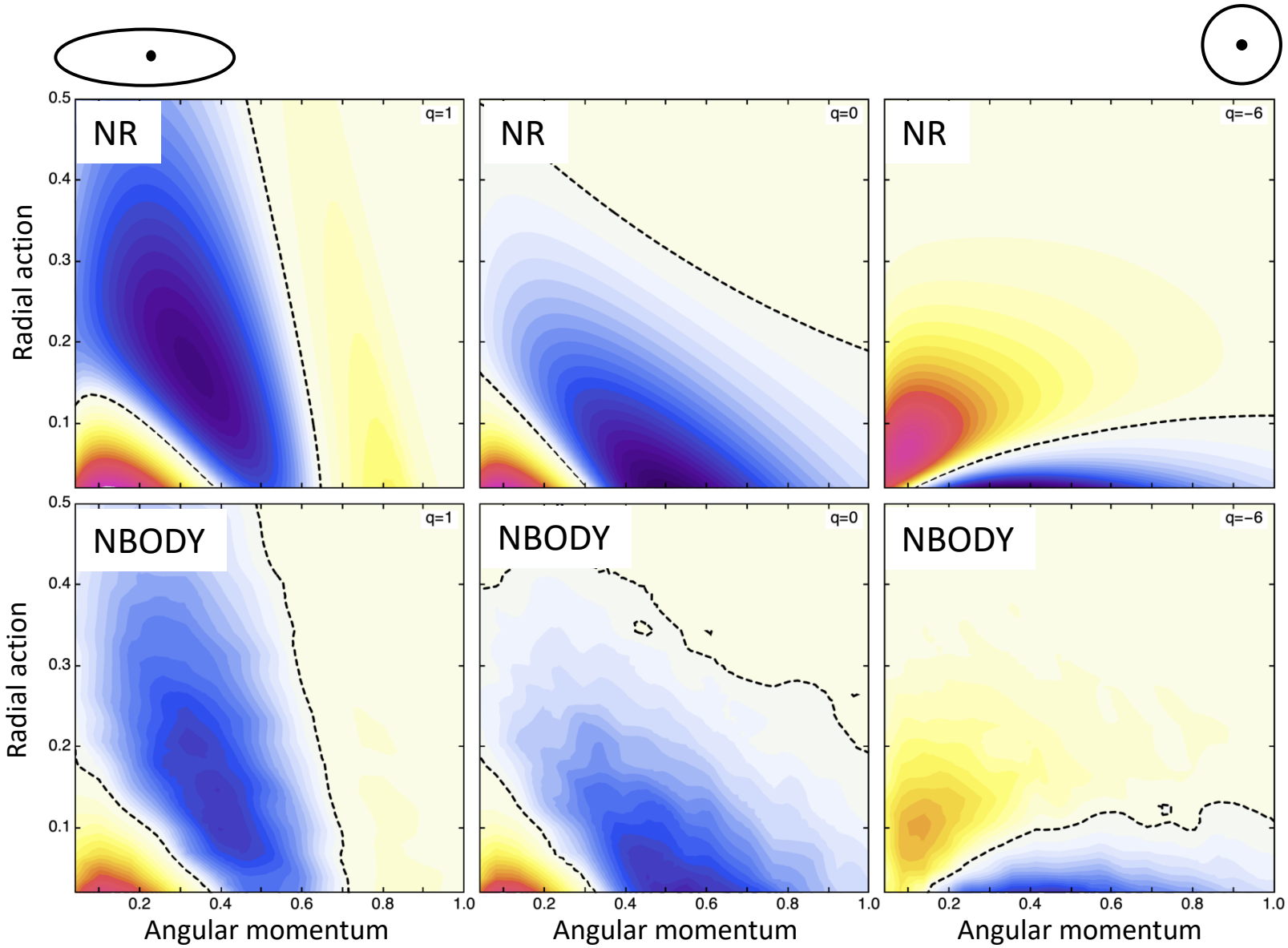
$\boxed{\boldsymbol{F}(\boldsymbol{J})}$ Flux

$\boxed{\boldsymbol{D}_1(\boldsymbol{J})}$ Dynamical friction

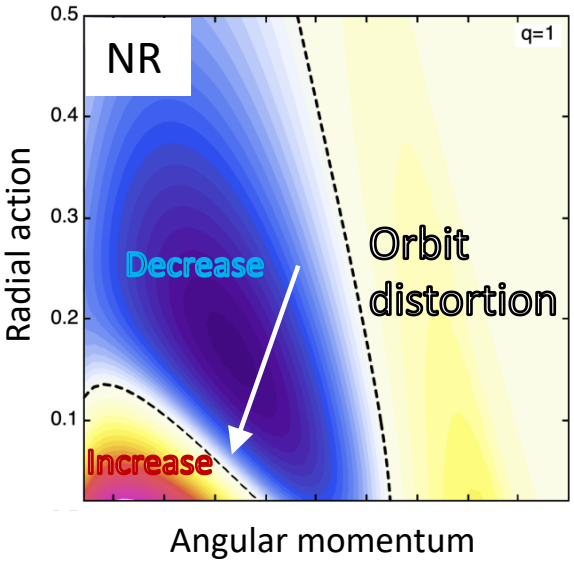
$\boxed{\boldsymbol{D}_2(\boldsymbol{J})}$ Orbital diffusion

} Orbit averaging (difficult)

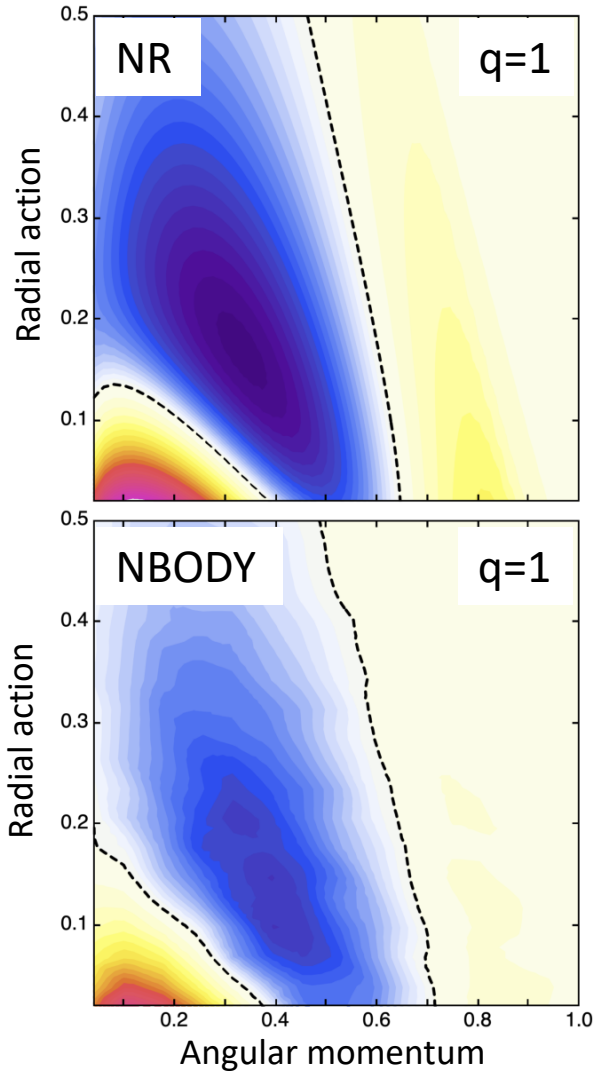
Relaxation rate



Relaxation rate



Relaxation rate

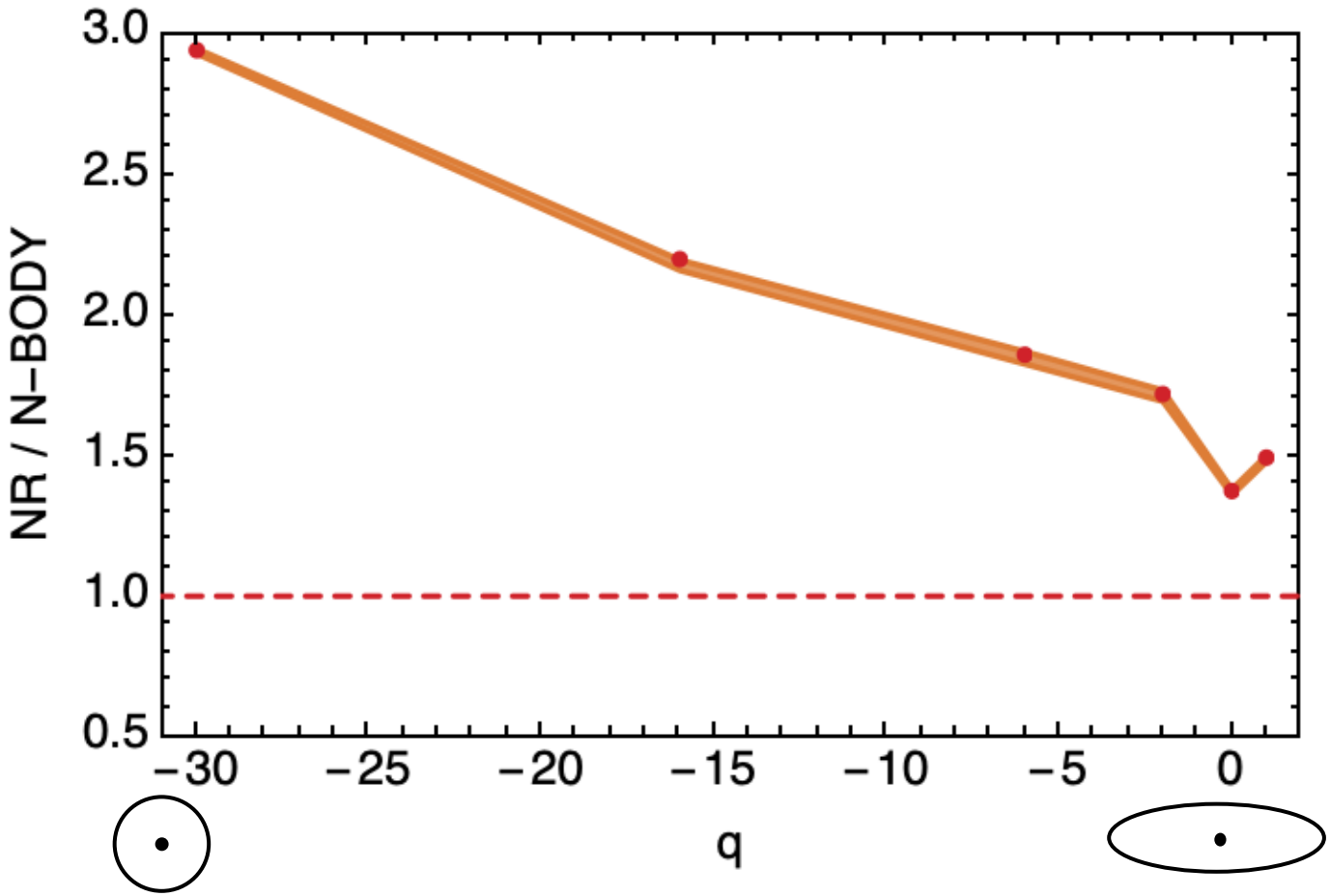


Qualitative agreement between
NR theory and NBODY simulations

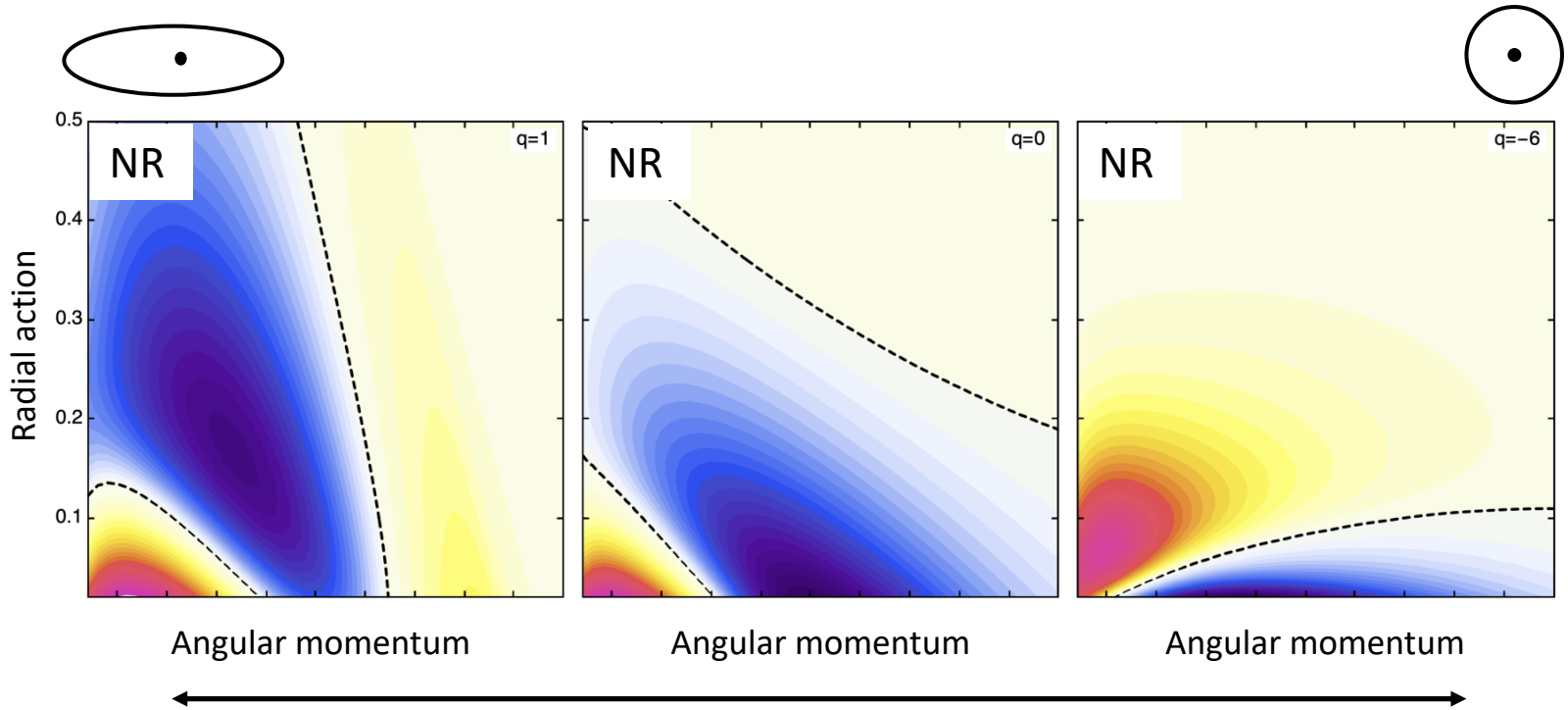
Up to overall prefactor
(Darker colors for NR theory)

Deviations

- Far away interactions via 2 bodies
- Isotropic King sphere: prefactor ~1.4, by Theuns (1996)
- Isotropic isochrone: prefactor ~1.5, by Fouvry et al. (2021)

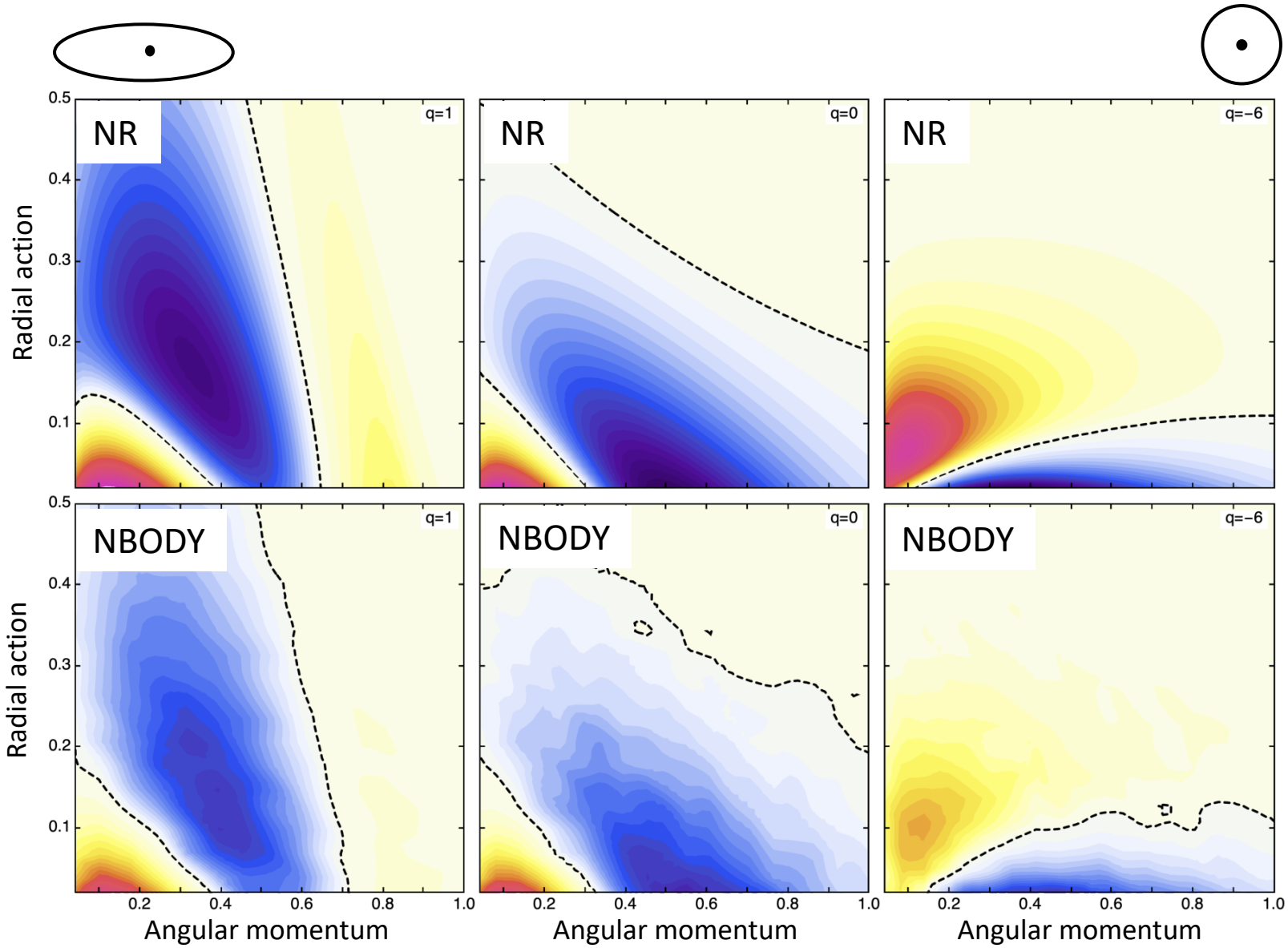


Relaxation rate

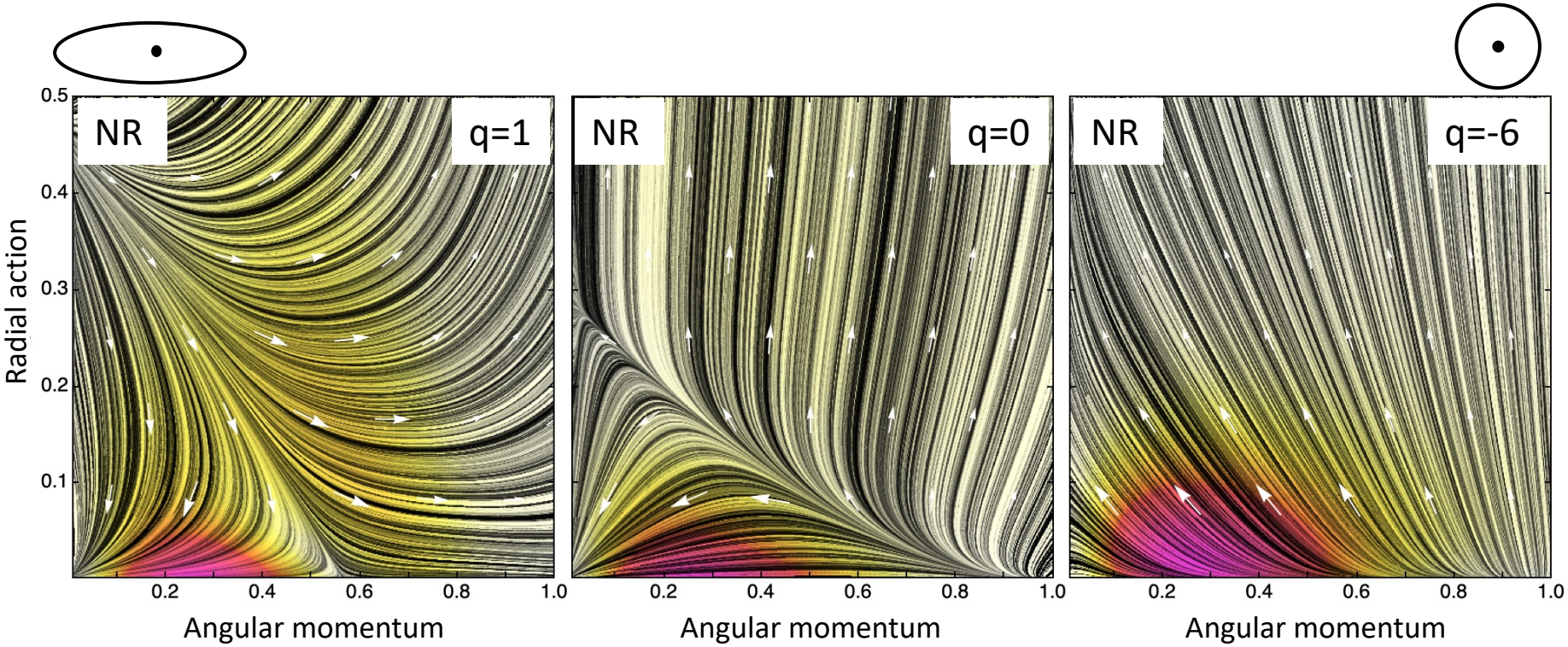


Impact of anisotropy on the rate of orbital change

Relaxation rate



Flux

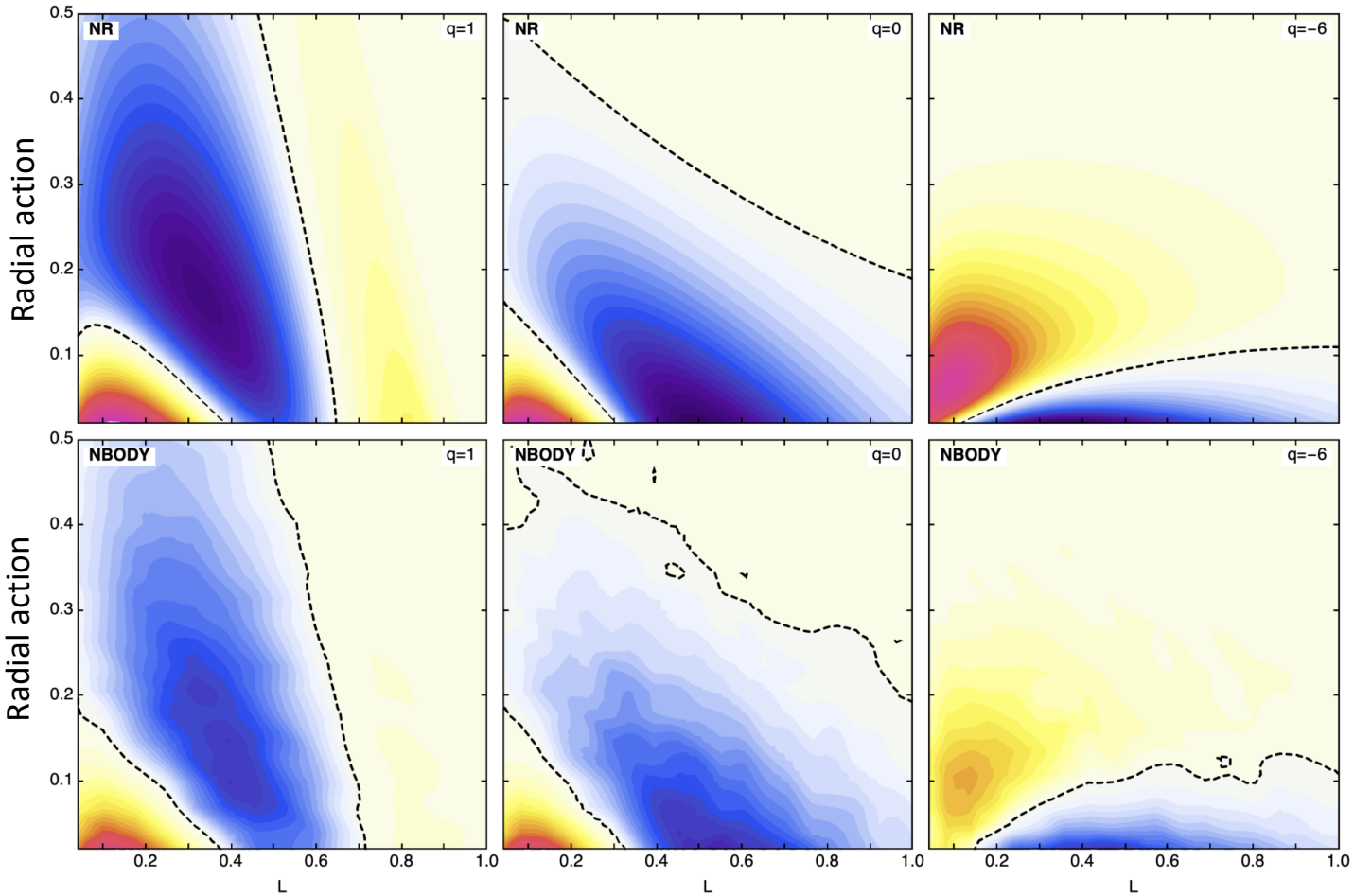


- Reshuffling of orbits towards isotropisation:

→ q=1 : orbits diffuse towards circular orbits

→ q=-6 : orbits diffuse towards radial orbits

Conclusions



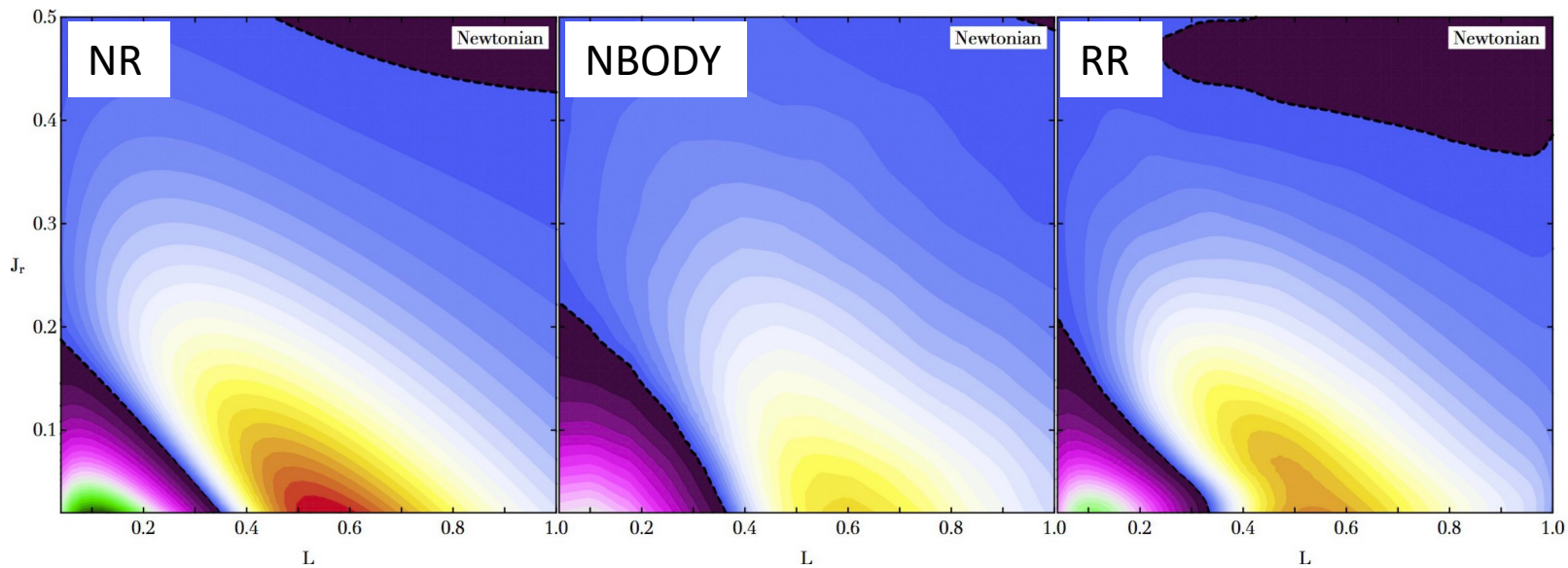
- Qualitative match between NBODY and NR theory

→ Captures faster contraction via rate of orbital change

Perspectives

- Computation of other quantities: $\beta(r)$, $R_c(t)$, $S(t)$, etc
- Self-consistent FP integration of anisotropic clusters
- Other spheres ? (truncated, cuspy, rotating)

- Anisotropic RR ? (next work: Coulomb log, prefactor, collective effects)



Isotropic isochrone cluster: Fouvry et al. (2021)

Rotating clusters

$$F(\mathbf{J}) = F_0(\widetilde{\mathbf{J}})(1 + \alpha\,\mathrm{Sign}(L_z))$$

Rotating clusters

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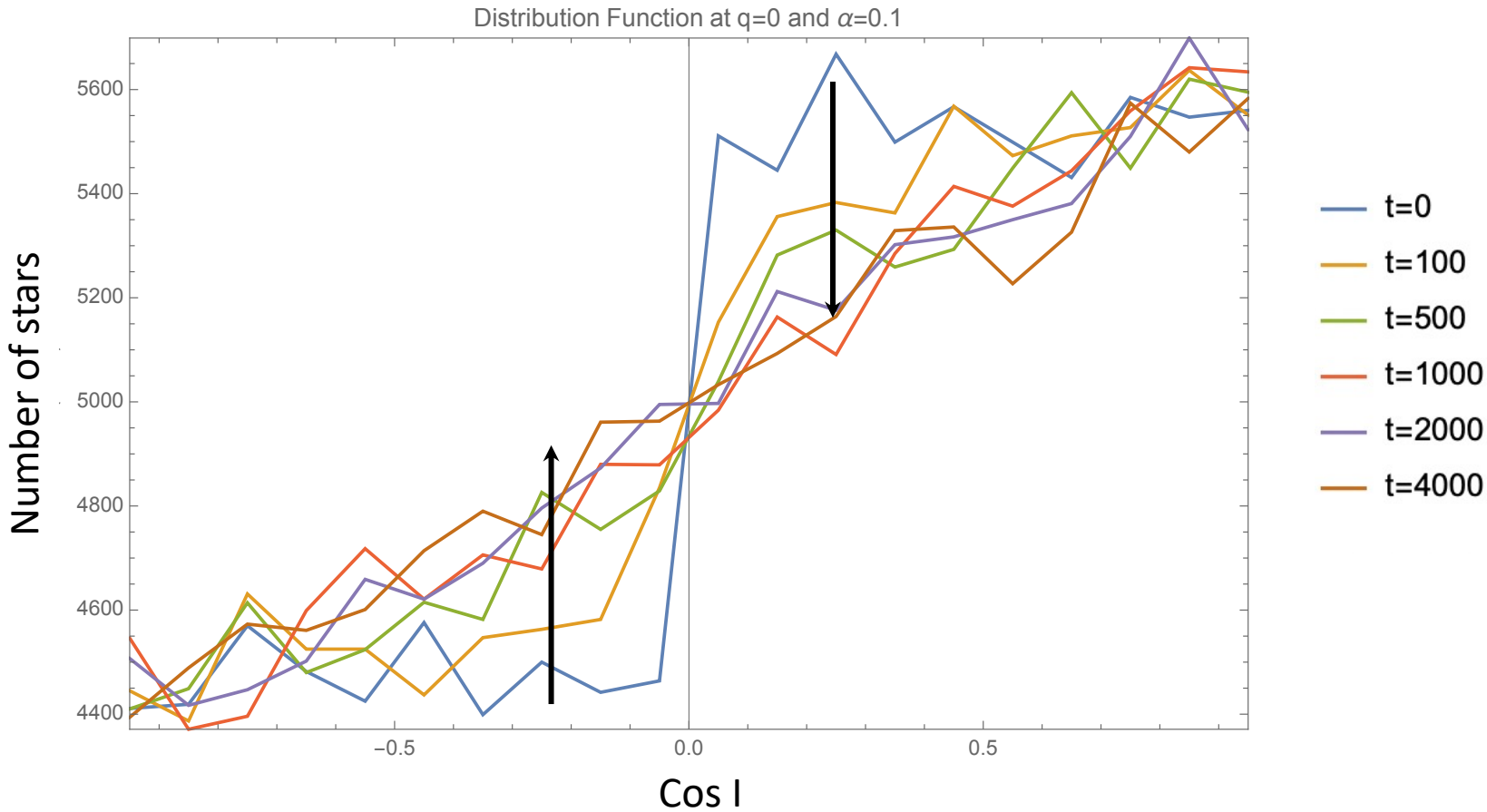
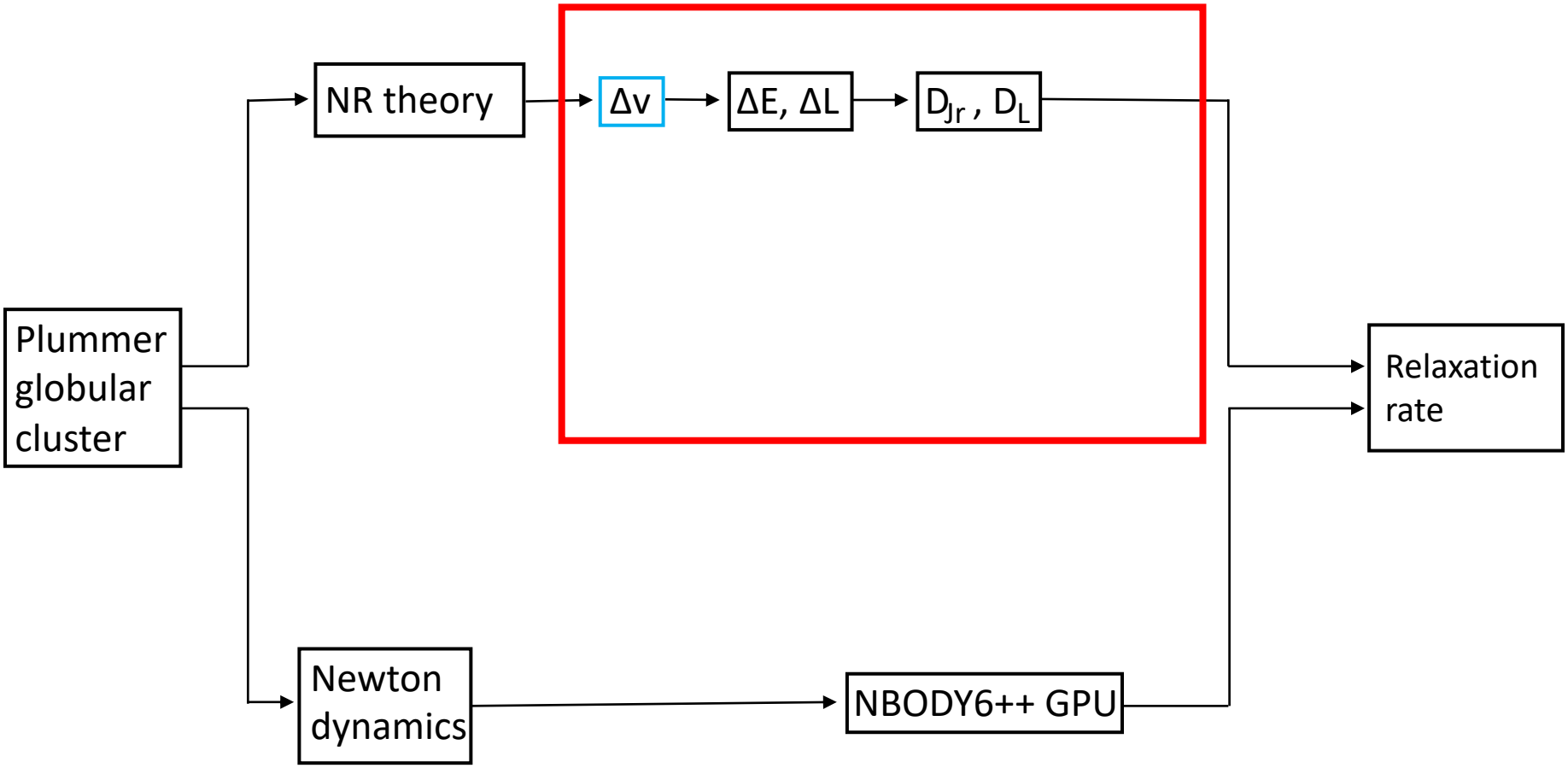


Computation of secular response



Fokker-Planck equation (no rotation)

$$\frac{\partial \boxed{F(\boldsymbol{J})}}{\partial t} = -\frac{\partial}{\partial \boldsymbol{J}} \cdot \boxed{\boldsymbol{F}(\boldsymbol{J})} = -\frac{\partial}{\partial \boldsymbol{J}} \cdot \left[\boxed{\boldsymbol{D}_1(\boldsymbol{J})} \boxed{F(\boldsymbol{J})} - \frac{1}{2} \frac{\partial}{\partial \boldsymbol{J}} \cdot \left[\boxed{\boldsymbol{D}_2(\boldsymbol{J})} \boxed{F(\boldsymbol{J})} \right] \right]$$

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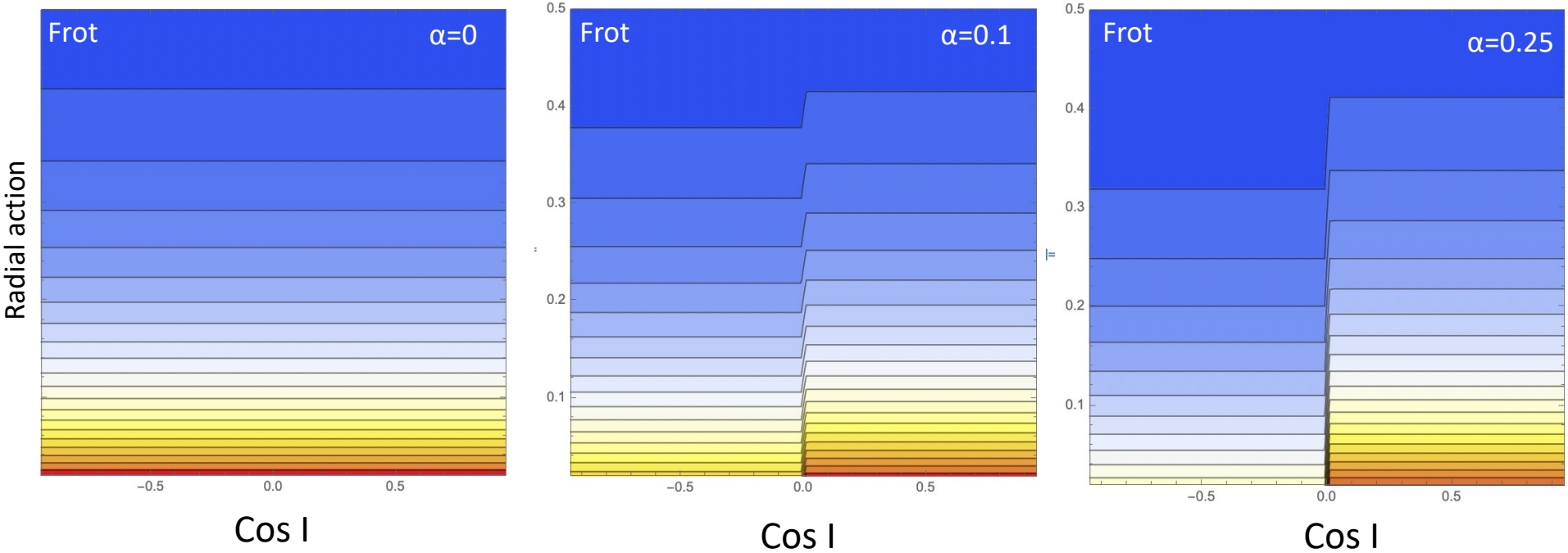
Fokker-Planck equation (rotation)

$$\frac{\partial \boxed{F(\boldsymbol{J})}}{\partial t} = -\frac{\partial}{\partial \boldsymbol{J}} \cdot \boxed{\boldsymbol{F}(\boldsymbol{J})} = -\frac{\partial}{\partial \boldsymbol{J}} \cdot \left[\boxed{\boldsymbol{D}_1(\boldsymbol{J})} \boxed{F(\boldsymbol{J})} - \frac{1}{2} \frac{\partial}{\partial \boldsymbol{J}} \cdot \left[\boxed{\boldsymbol{D}_2(\boldsymbol{J})} \boxed{F(\boldsymbol{J})} \right] \right]$$

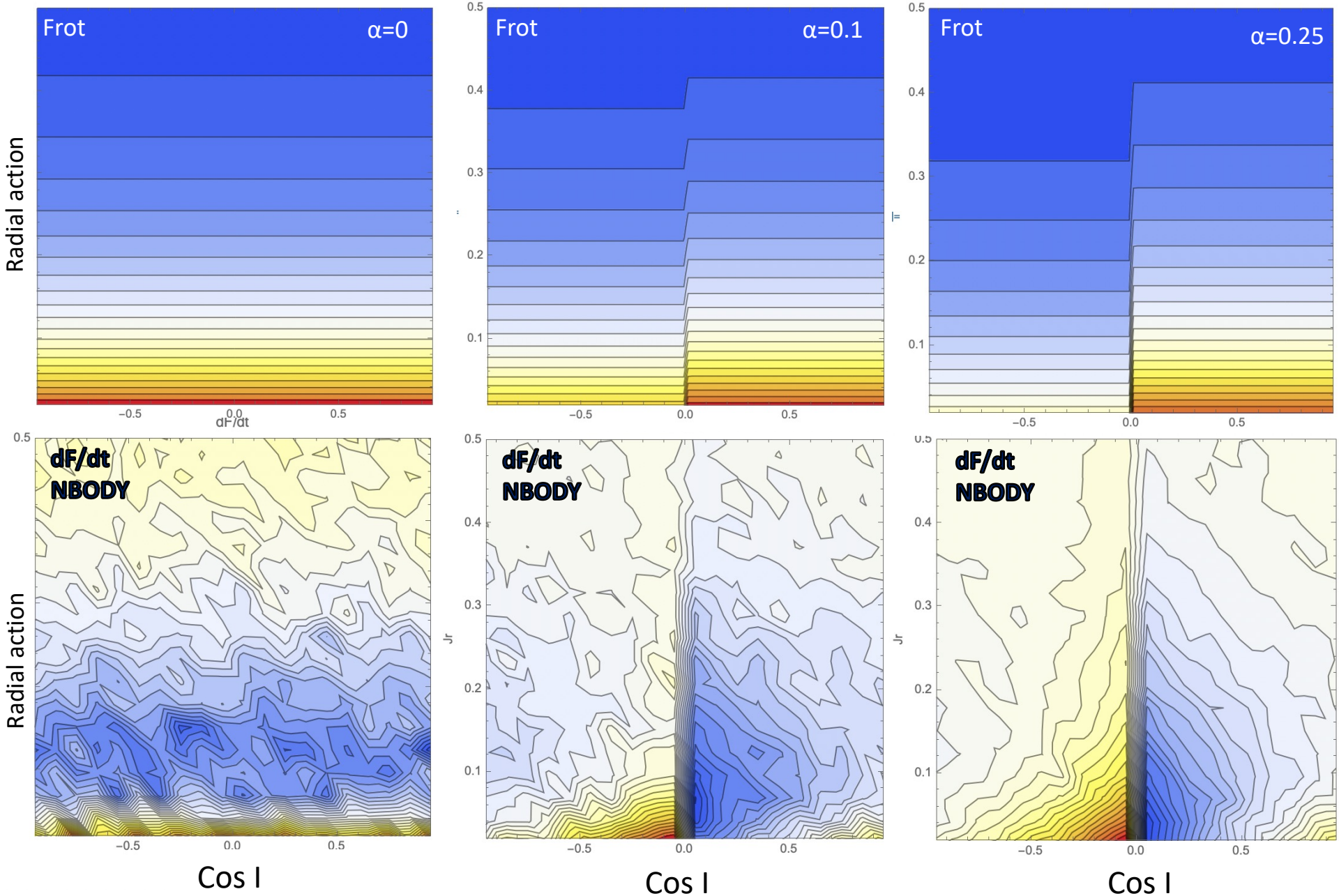
$$\boldsymbol{D}_1(\boldsymbol{J}) = \begin{pmatrix} D_{J_r} \\ D_L \\ D_{L_z} \end{pmatrix} \quad \boldsymbol{D}_2(\boldsymbol{J}) = \begin{pmatrix} D_{J_r J_r} & D_{J_r L} & D_{J_r L_z} \\ D_{L J_r} & D_{LL} & D_{L L_z} \\ D_{L_z J_r} & D_{L_z L} & D_{L_z L_z} \end{pmatrix}$$

2D => 3D !

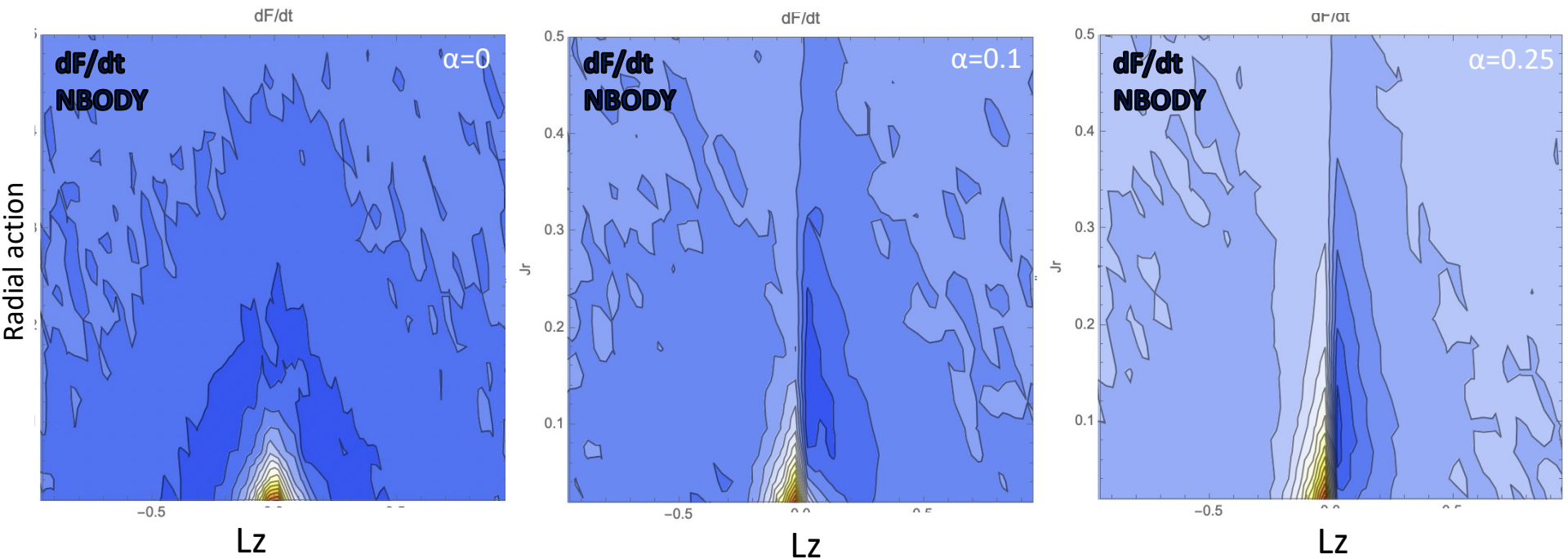
Rotating clusters (q=0)



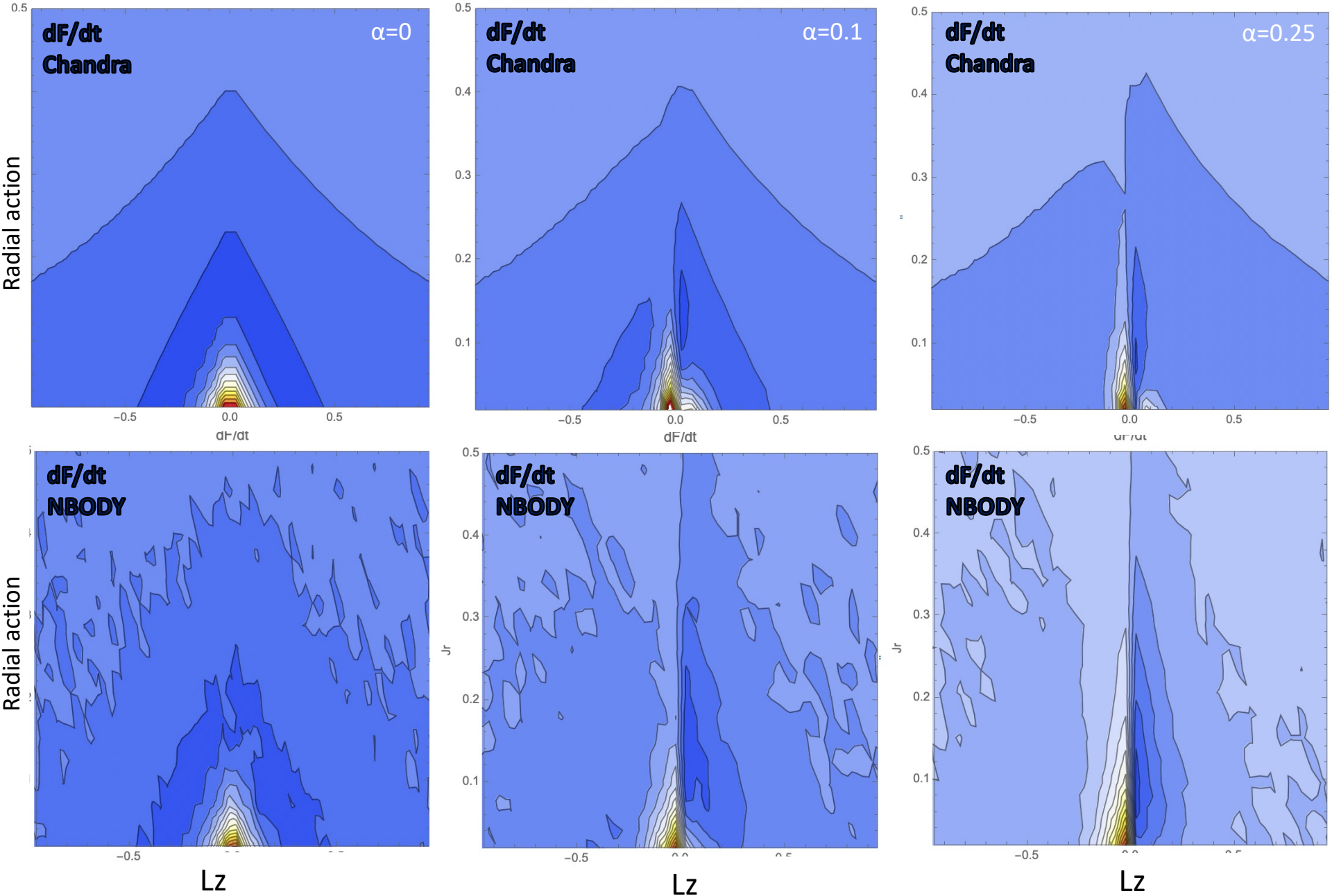
Rotating clusters (q=0)



Rotating clusters (q=0)



Rotating clusters (q=0)



Rotating clusters (q=1)