Quantum fluids of light
first successes and many exciting perspectives

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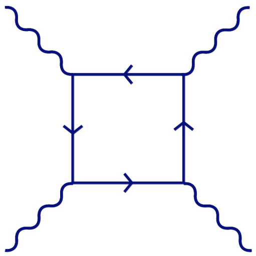
Why not hydrodynamics of light ?

Light field/beam composed by a huge number of photons

- in vacuo photons travel along straight line at c
- (practically) do not interact with each other
- in cavity, collisional thermalization slower than with walls and losses

=> **optics typically dominated by single-particle physics**

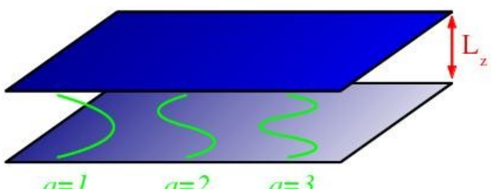


$$\sigma \sim \alpha^4 \frac{\hbar^2}{m^2 c^2} \left(\frac{\hbar \omega}{m c^2} \right)^6$$

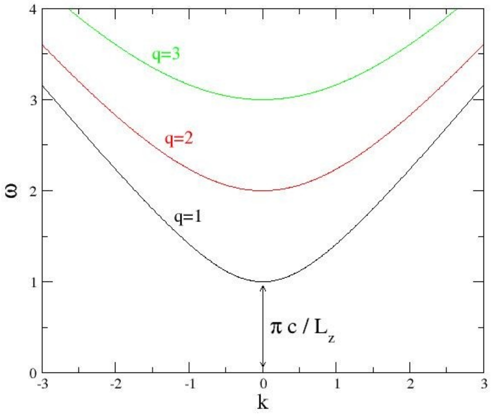
In photonic structure:

$\chi^{(3)}$ nonlinearity → photon-photon interactions

Spatial confinement → effective photon mass



=> **collective behaviour of a quantum fluid**



Many experiments so far:

BEC, superfluidity, synthetic magnetism, Chern insulators...

In this talk: a few selected topics

Review of BEC/superfluidity, quantum hydrodynamics,
IQH in 4D photonic lattices, conservative photon fluids

Standing on the shoulders of giants

Laserlight — First Example of a Second-Order Phase Transition Far Away from Thermal Equilibrium*

R. Graham and H. Haken
I. Institut für theoretische Physik der Universität Stuttgart

Received April 23, 1970

We solve the functional Fokker-Planck equation established in a previous paper in the vicinity of laser threshold. The stationary solution is obtained explicitly in the form $P = N \exp[-\varphi(\{\bar{u}, \bar{u}^*\})]$. φ has exactly the same form as the Ginzburg-Landau expression for the free energy of a superconductor, if the pair wave function is replaced by the electromagnetic field amplitude $\bar{u}$. This gives us the key for a thermodynamic reinterpretation of all laser phenomena.

In particular the laser threshold appears as a second-order phase transition in all details. It is indicated that our theory provides a new formalism also for the Ginzburg-Landau theory.

Volume 67, Number 27 PHYSICAL REVIEW LETTERS 30 December 1991

Vortices and Defect Statistics in Two-Dimensional Optical Chaos

F. T. Arecchi,[a] G. Giacomelli, P. L. Ramazza, and S. Residori
Istituto Nazionale di Ottica, Largo E. Fermi, 6, 50125 Firenze, Italy
(Received 1 April 1991)

We present the first direct experimental evidence of topological defects in nonlinear optics. For increasing Fresnel numbers F, the two-dimensional field is characterized by an increasing number of topological defects, from a single vortex, up to a large number of vortices with zero net topological charge. At variance with linear scattering from a fixed phase plate, here the defect pattern evolves in time according to the nonlinear dynamics. We assign the scaling exponents for the mean number of defects, their mean separation, and the charge unbalance as functions of F, as well as the correlation time of the defect pattern.

C. R. Acad. Sci. Paris, t. 317, Série II, p. 1287-1292, 1993 1287

Optique/*Optics*

Diffraction non linéaire

Yves Pomeau et Sergio Rica

Résumé – Une expérience classique en mécanique des fluides est la formation de structures vorticales à l'arrière d'un obstacle, comme par exemple l'écoulement de Bénard-von-Kármán. Est-il possible d'imaginer une expérience similaire en optique ? C'est-à-dire, en illuminant un obstacle pourrait-on engendrer des structures tourbillonnaires caractéristiques d'un régime pré-turbulent ? Cette Note est consacrée au problème de la génération de vorticité dans les ondes électromagnétiques.

PHYSICAL REVIEW A VOLUME 54, NUMBER 1 JULY 1996

Hydrodynamic phenomena in laser physics: Modes with flow and vortices behind an obstacle in an optical channel

M. Vaupel, K. Staliunas, and C. O. Weiss
Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany
(Received 16 February 1995; revised manuscript received 20 February 1996)

The transverse patterns of an active resonator with cylindrical optics are investigated. This resonator configuration corresponds to a "channel" form of the potential for the "photon fluid." Simultaneous emission of different transverse modes along the channel, periodic nucleation of vortices in the form of a vortex street (vortices of alternating senses of rotation appearing in a flow behind an obstacle), accelerated flow in a "tilted channel," and destabilization of the one-directional flow in the channel are demonstrated and interpreted in terms of tilted waves and beating of channel modes, as well as in fluid terms, illustrating the fluid dynamics correspondence of class-*A* lasers. [S1050-2947(96)02407-9]

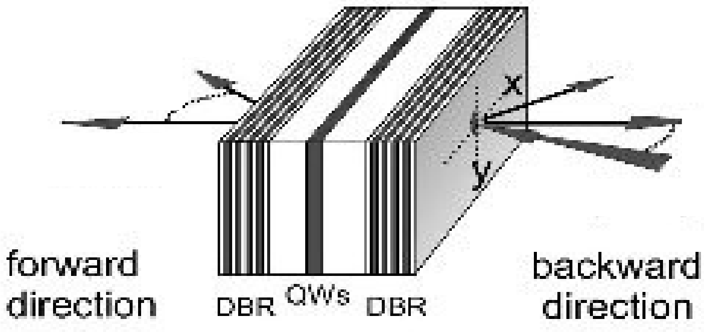
And of course many others:

Coullet, Gil, Rocca, Brambilla, Lugiato...

Part 0:

A primer to BEC and superfluidity in semiconductor microcavities

Planar DBR microcavity with QWs



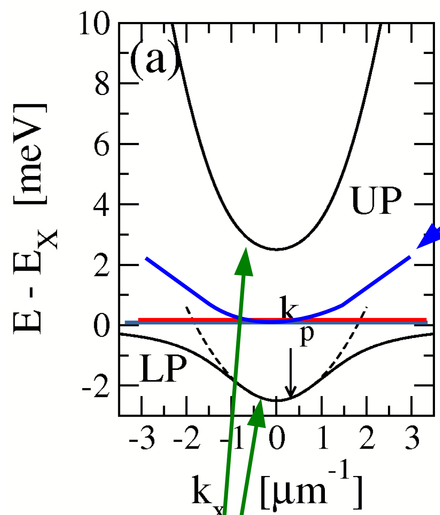
Photon

- DBR: stack λ/4 layers (e.g. GaAs/AlAs)
- Cavity layer → confined photonic mode, delocalized along 2D plane:

$$\omega_C(\boldsymbol{k}) = \omega_C^0 \sqrt{1 + \boldsymbol{k}^2 / k_z^{\,2}}$$

Exciton

- e-h pair in QW: sort of H atom. **Exciton**
- bosons for $n_{exc}\, a^2_{Bohr} \ll 1$ (verified by QMC)
- Excitons delocalized along cavity plane. Flat exciton dispersion $\omega_x(\mathbf{k}) \approx \omega_x$
- Optical $\chi^{(3)}$ from exciton collisions



Polaritons

Exciton radiatively coupled to cavity photon at same in-plane k

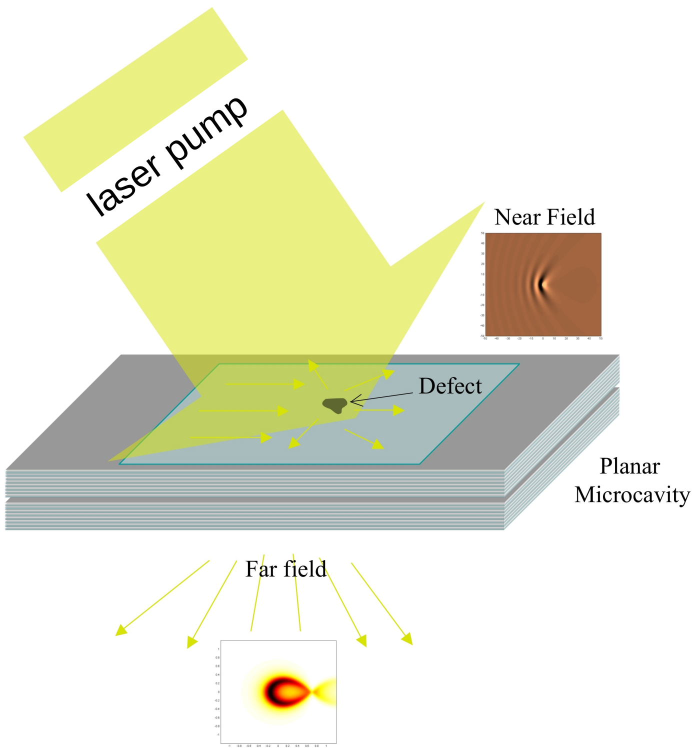
Bosonic superpositions of exciton and photon, called **polaritons**

Two-dimensional gas of polaritons

Small effective mass $m_{pol} \approx 10^{-4}\, m_e$ → originally promising for BEC studies

Exciton → interactions. Photons → radiative coupling to external world

How to create and detect the photon gas?

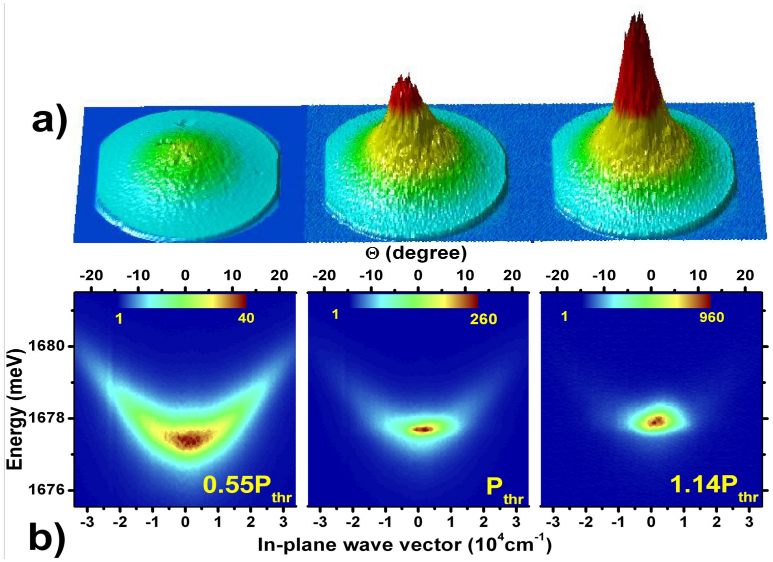


Pump needed to compensate losses: stationary state is NOT thermodynamical equilibrium

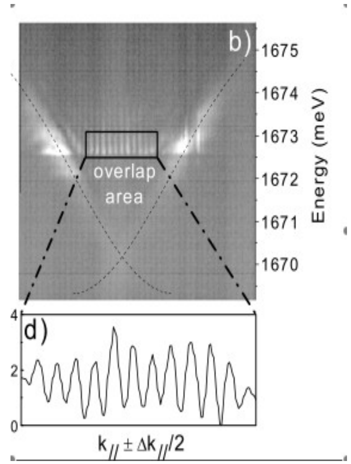
- Coherent laser pump: directly injects photon BEC in cavity, may lock BEC phase
- Incoherent (optical or electric) pump: BEC transition similar to laser threshold
  spontaneous breaking of U(1) symmetry

Classical and quantum correlations of in-plane field directly transfer to emitted radiation

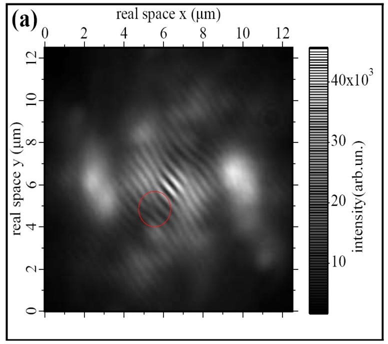
2006 - Photon/polariton Bose-Einstein condensation



Momentum distribution
Kasprzak et al., Nature **443**, 409 (2006)

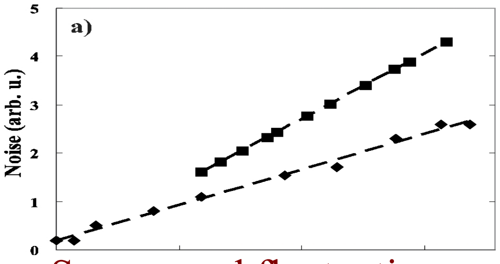


Interference
Richard et al., PRL **94**, 187401 (2005)



Quantized vortices
K. Lagoudakis *et al.*
Nature Physics 4, 706 (2008).

Many features very similar to atomic BEC

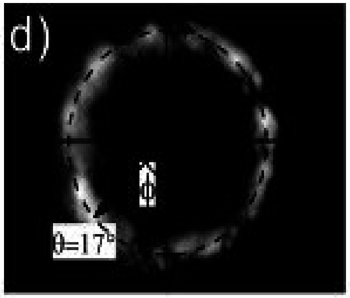


Suppressed fluctuations
A. Baas et al., PRL **96**, 176401 (2006)

But also differences due to non-equilibrium:

- BEC @ $k \neq 0$ → volcano effect
- T-reversal broken → $n(k) \neq n(-k)$
- interesting questions about thermalization

Photon/polariton BEC closely related to laser operation in VCSELs



BEC on k-space ring
M. Richard et al.,
PRL **94**, 187401 (2005)

2008 - Superfluid light

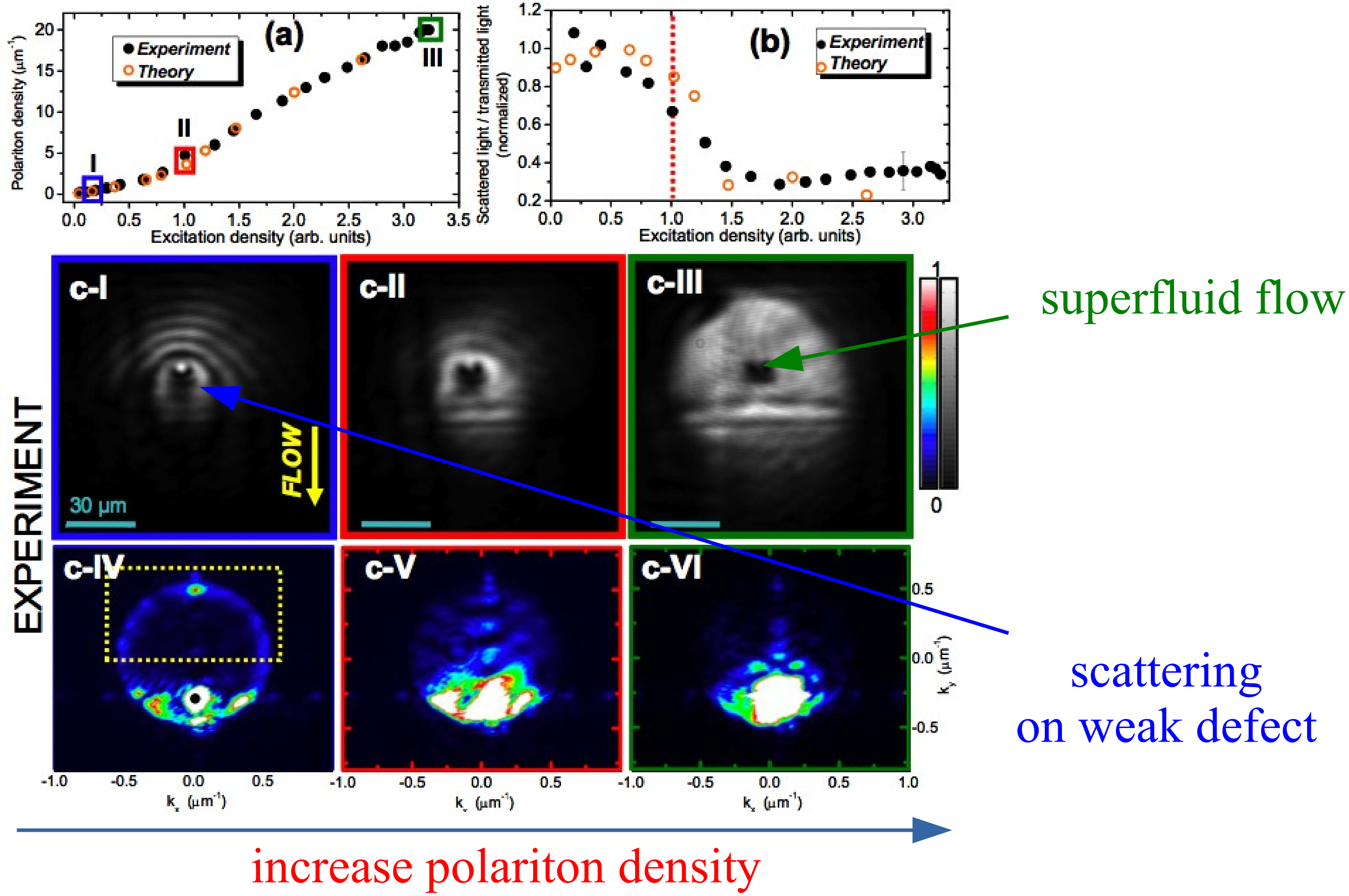
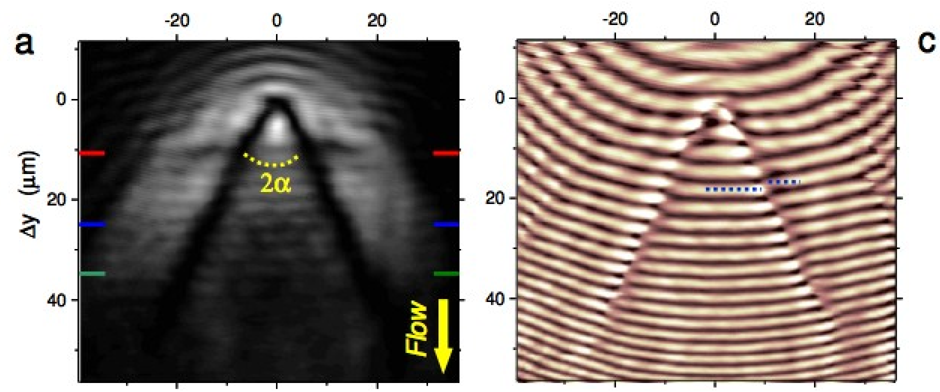


Figure from LKB-P6 group:
J.Lefrère, A.Amo, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, Nature Phys. **5**, 805 (2009)

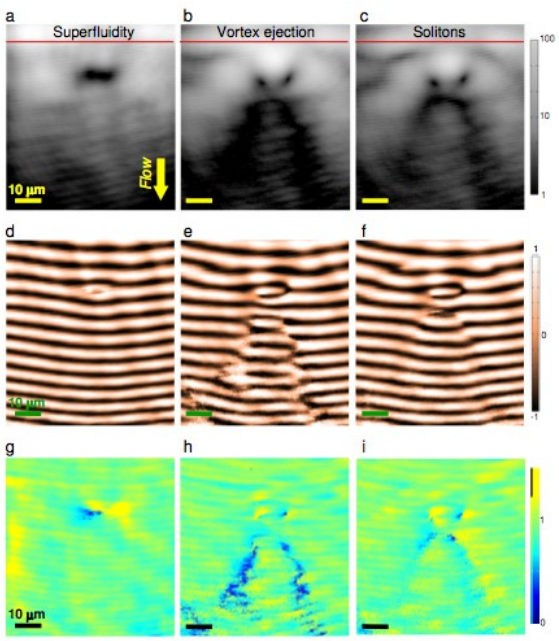
Theory: IC and C. Ciuti, PRL **93**, 166401 (2004).

2009-10 - Superfluid hydrodynamics

Oblique dark solitons →



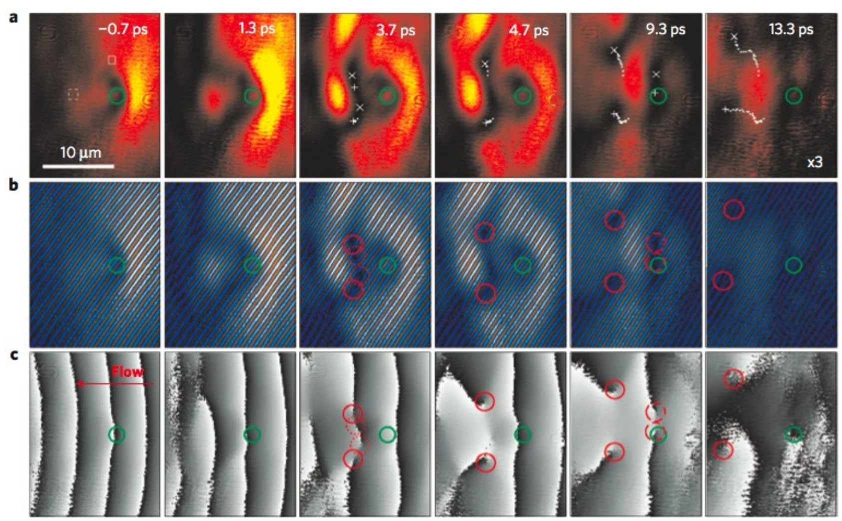
A. Amo, et al., Science **332**, 1167 (2011)



← Turbulent behaviours

A. Amo, et al., Science **332**, 1167 (2011)

Hydrodynamic nucleation → of vortices



Nardin et al., Nat. Phys. 7, 635 (2011)

Role of interactions crucial in determining regimes as a function of v/c_s

Part I:

Quantum hydrodynamics

Artificial black holes in atom and photon fluids

The (hopefully forthcoming) tale of Navier and Stokes meeting Heisenberg at Hawking's place

Acoustic (and fishic) black hole horizon

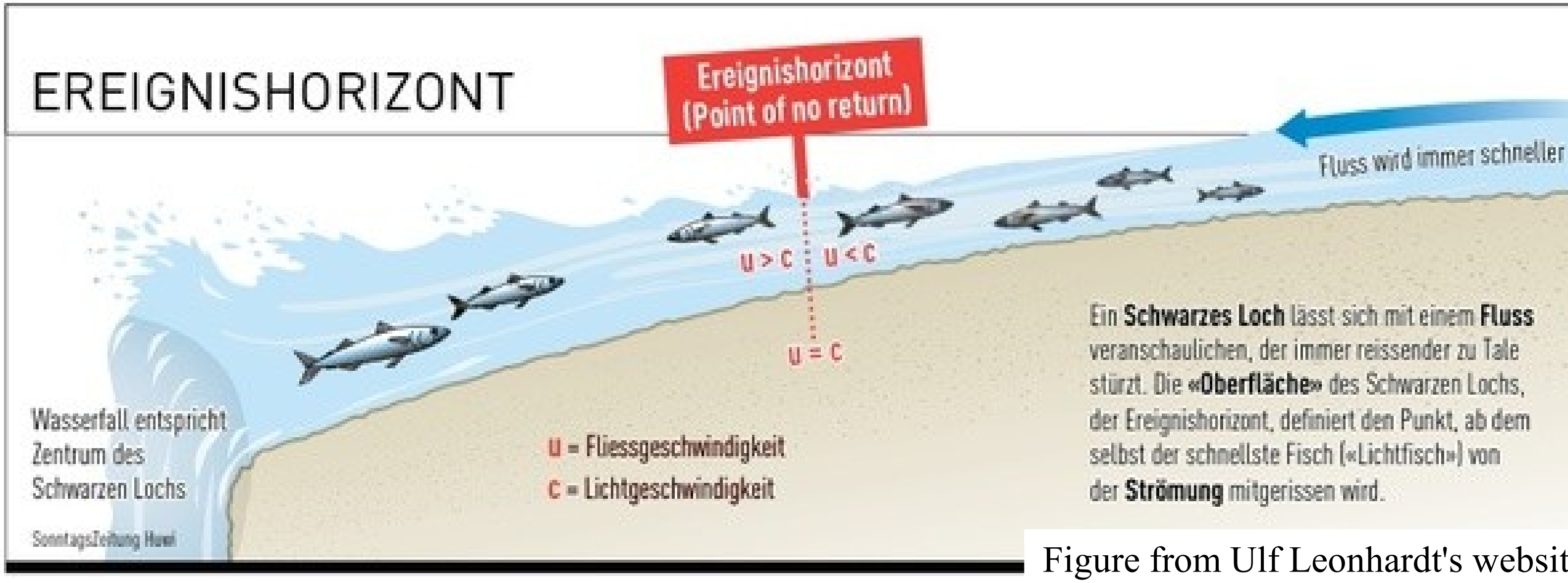


Figure from Ulf Leonhardt's website

- Horizon region separating:

  sub-sonic (i.e. sub-fishic) flow (upstream)

  from super-sonic (i.e. sub-fishic) flow (downstream)

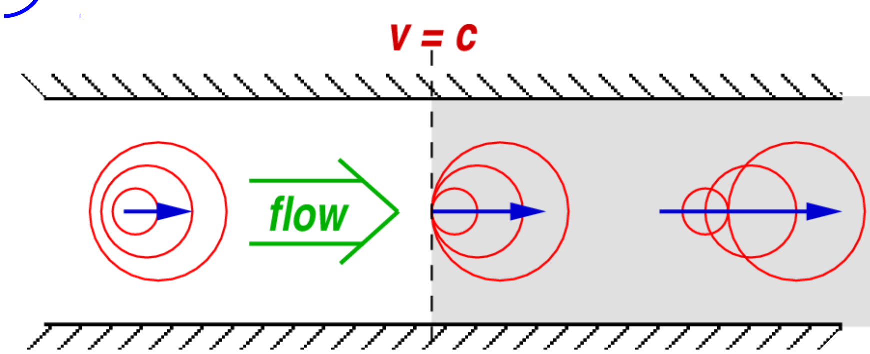
- Excitations (i.e. fish) in super-sonic (i.e. super-fishic) region

  can not travel (i.e. swim) back through horizon

Behavior analogous to
Astrophysical black hole horizon

What happens with quantum sound ?

Hawking radiation of acoustic phonons ?



Unruh, PRL 1981
Barceló, Liberati, Visser, Liv. Rev. Relativity **14**, 3 (2011)

Analog Hawking radiation

Unruh PRL '81:

Sound propagation on superfluid ↔ light propagation on space-time with curved metric determined by density n(x) and velocity v(x) fields

$$ds^2 = G_{\mu\nu}\, dx^\mu dx^\nu = \frac{n(x)}{c_s(x)} \left[-c_s(x)^2 dt^2 + (d\vec{x} - \vec{v}(x)\, dt)(d\vec{x} - \vec{v}(x)\, dt) \right]$$

Wave equation for superfluid phase $\frac{1}{\sqrt{-G}}\, \partial_\mu \left[\sqrt{-G}\, G^{\mu\nu} \partial_\nu \right] \varphi(x,t) = 0$

Once quantized → quantum field theory in a curved space time

Astrophysical black holes: Hawking emission from horizon at $T_H = \frac{\hbar c^3}{8\pi k_B G M}$

Acoustic black holes:

- emit Hawking radiation of phonons at $T_H = \frac{\hbar}{4\pi k_B c_s} \left[\frac{d}{dx} (c_s^2 - v^2) \right]_H$
- in nK range for μm-sized ultracold atomic BECs (not so bad...)
- much higher for superfluids of light thanks to small photon mass

as first proposed by F. Marino, PRA **78**, 063804 (2008)

What about acoustic horizons in fluids of light?

Polariton-polariton interactions

- Bogoliubov phonon dispersion on top of polariton condensate

Pump at an angle

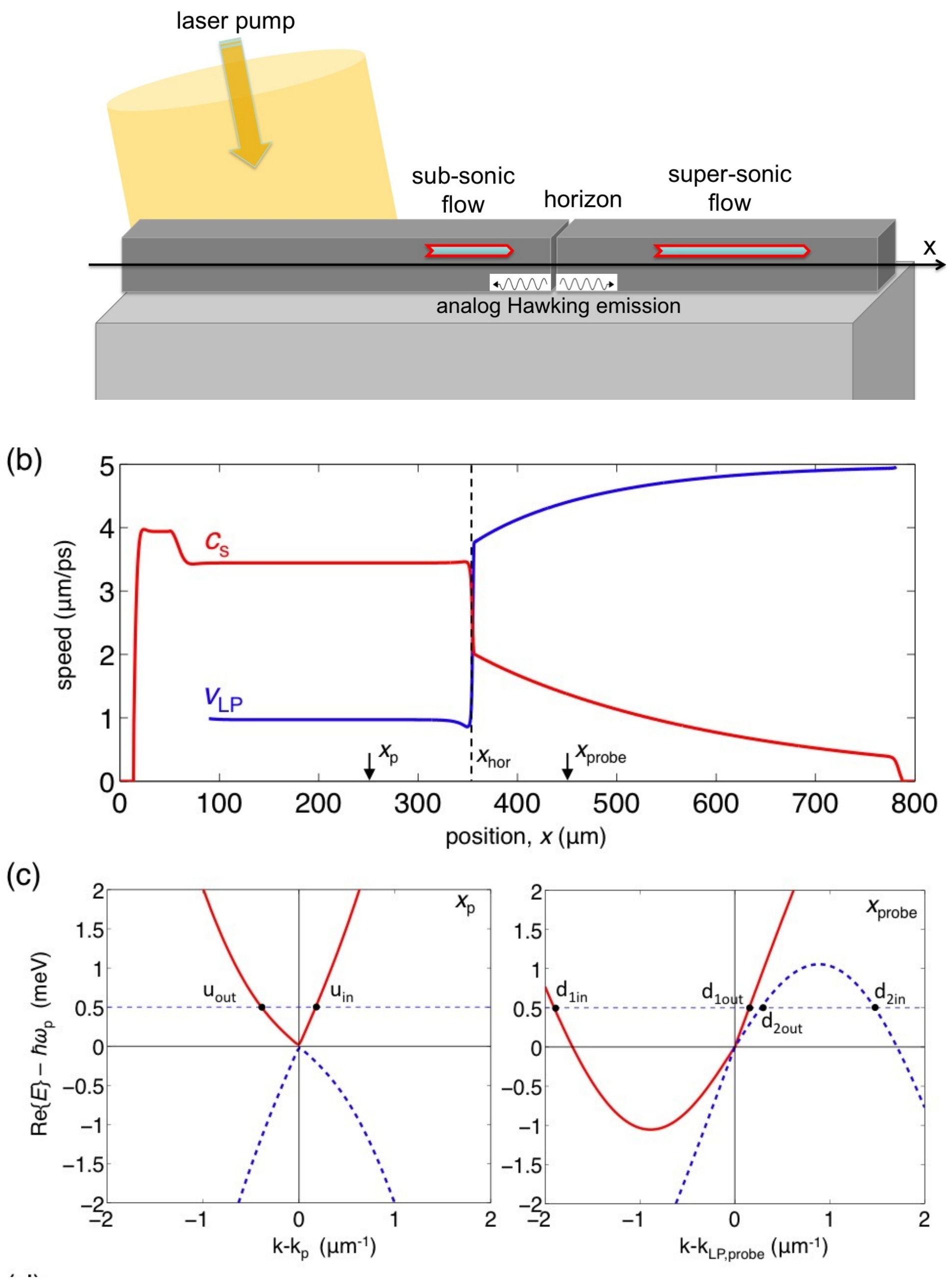
- finite in-plane wavevector, so condensate is flowing

Tailored pump spot + Defect

→ Horizon with large surface gravity

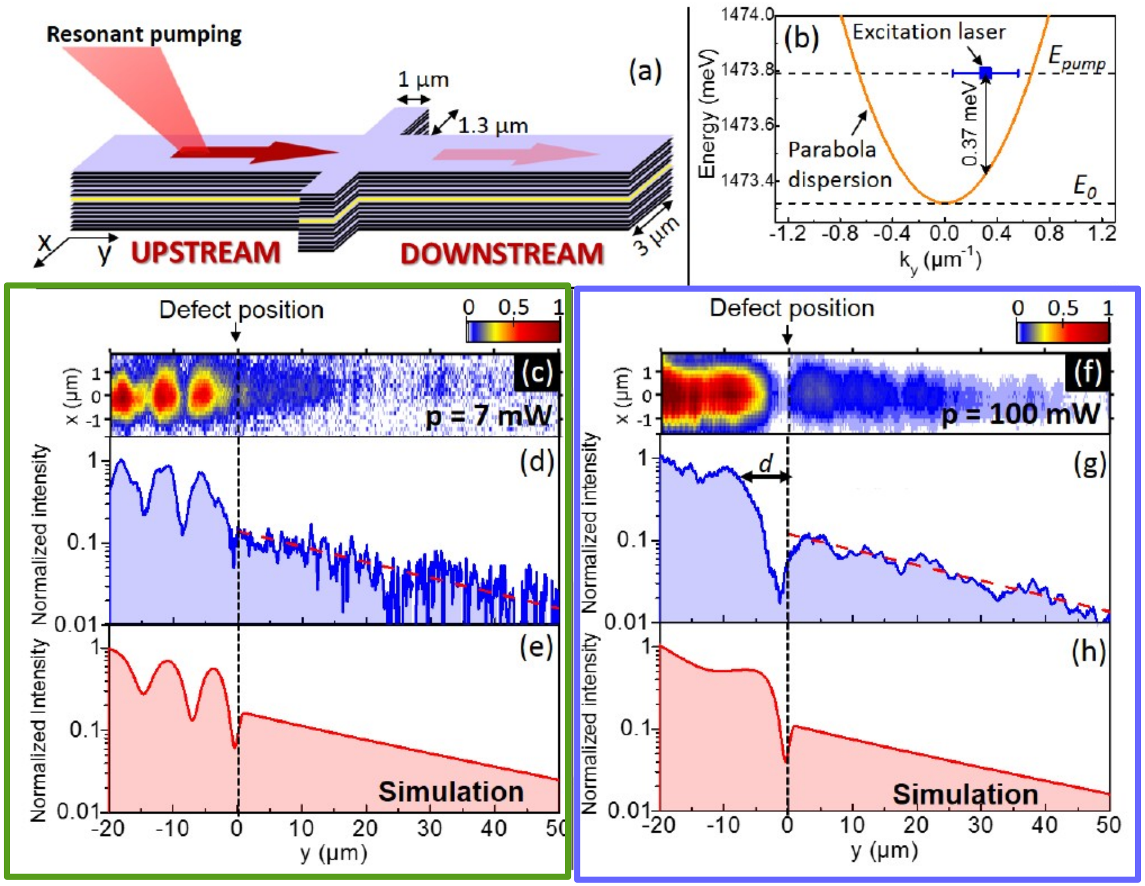
Hawking emission

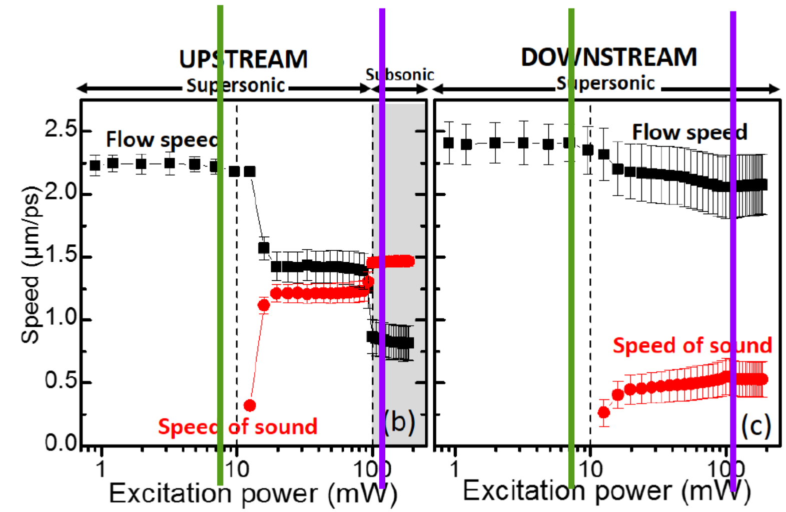
- phonons on photon fluid
- correlations of emitted light



D. Gerace and IC, PRB **86**, 144505 (2012); similar results in Solnyshkov et. al., PRB 84, 233405 (2011)

Very recent experimental results @ LPN





BH created!

The hunt for
Hawking radiation
is now open!!

H.-S. Nguyen, Gerace, IC, *et al.,* PRL **114**, 036402 (2015)

How to detect Hawking radiation?

Density-density correlation function $G^{(2)}(x,x') = \frac{\langle : n(x)\, n(x') : \rangle}{\langle n(x) \rangle \langle n(x') \rangle}$

Prediction of gravitational analogy:

→ entanglement in Hawking pairs gives long-range in/out correlations

$$G_2(x,x') = 1 - \frac{\xi_1 \xi_2}{16\pi\, c_1 c_2} \frac{k^2}{\sqrt{n^2 \xi_1 \xi_2}} \frac{c_1 c_2}{(c_1 - v)(v - c_2)} \cosh^{-2}\left[\frac{k}{2}\left(\frac{x}{c_1 - v} + \frac{x'}{v - c_2}\right)\right]$$

→ allows to isolate Hawking phonons from background of incoherent thermal phonons

→ the *"Balbinot-Fabbri" moustache*

R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, IC, PRA 78, 021603 (2008).

Ab initio numerics: Wigner-Monte Carlo

At t=0, homogeneous system:

- Condensate wavefunction in plane-wave state
- Quantum + thermal fluctuations in plane wave Bogoliubov modes
- Gaussian α_k, variance $<|\alpha_k|^2> = [2 \tanh(E_k / 2k_B T)]^{-1} \to ½$ for $T \to 0$.

$$\psi(x,t=0)=e^{i k_0 x}\left[\sqrt{n_0}+\sum_k \left(u_k e^{i k x}\alpha_k + v_k e^{-i k x}\alpha_k^\star\right)\right]$$

At later times: conservative (for atoms!) evolution under GPE

$$i\hbar\ \partial_t \psi(x) = -\frac{\hbar^2}{2\text{m}}\ \partial_x^2 \psi(x) + V(x)\psi(x) + g(x)\,|\psi(x)|^2 \psi(x)$$

Expectation values of observables:

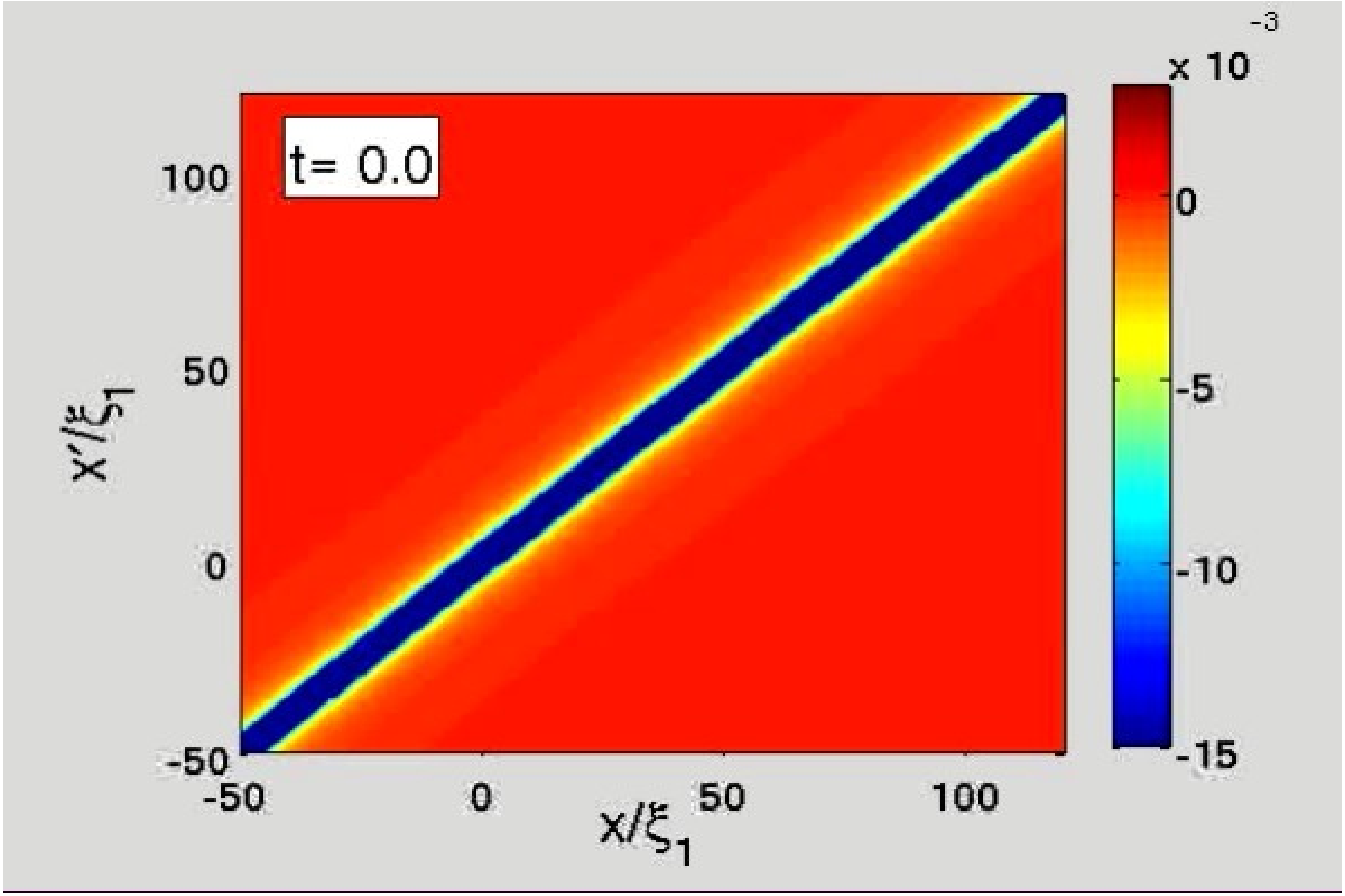
- Average over noise provides symmetrically-ordered observables

$$\langle \psi^\star(x)\ \psi(x')\rangle_W = \frac{1}{2}\langle \hat{\psi}^\dagger(x)\ \hat{\psi}(x') + \hat{\psi}(x')\ \hat{\psi}^\dagger(x)\rangle_Q$$

Equivalent to Bogoliubov, but can explore longer-time dynamics

A. Sinatra, C. Lobo, Y. Castin, J. Phys. B 35, 3599 (2002)

Density correlations in atomic gas: the movie



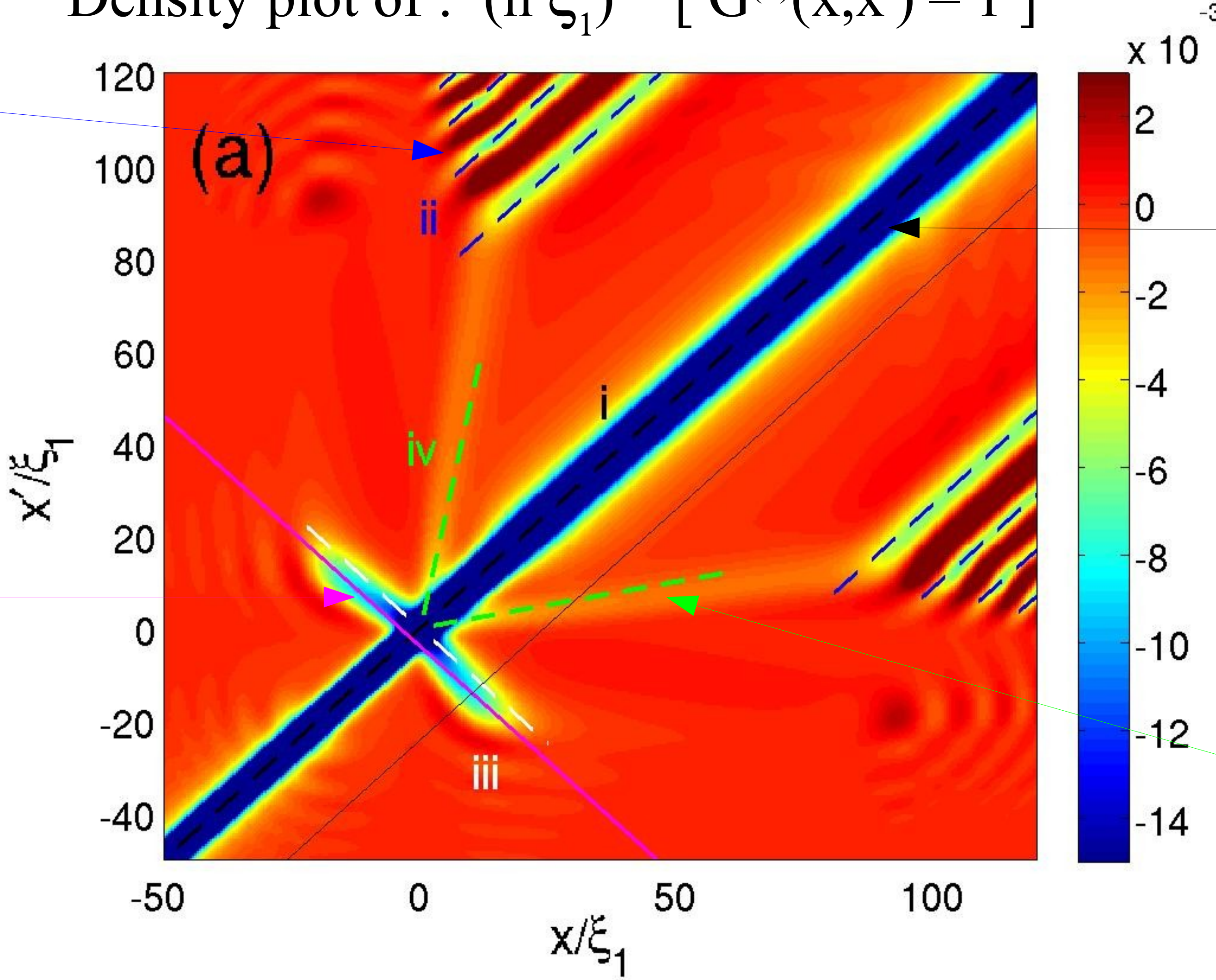
IC, S.Fagnocchi, A.Recati, R.Balbinot, A.Fabbri, New J. Phys. 10, 103001 (2008)

A snapshot of density correlations

Density plot of : $(n\ \xi_1) * [\ G^{(2)}(x,x') - 1\ ]$

(ii)
Dynamical
Casimir
emission

(i)
Many-body
antibunching



(iii)
Hawking
in / out
"Balbinot-
-Fabbri
moustache"

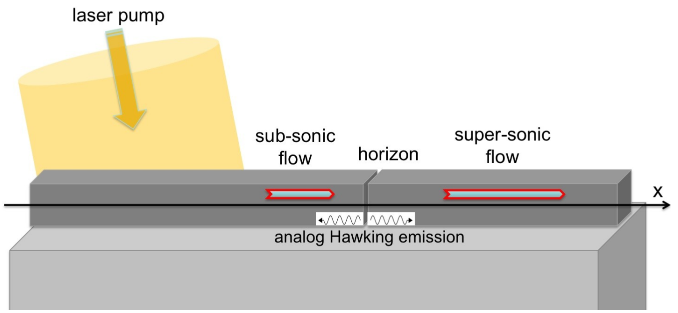
(iv)
Hawking
in / in

IC, S.Fagnocchi, A.Recati, R.Balbinot, A.Fabbri, New J. Phys. 10, 103001 (2008)

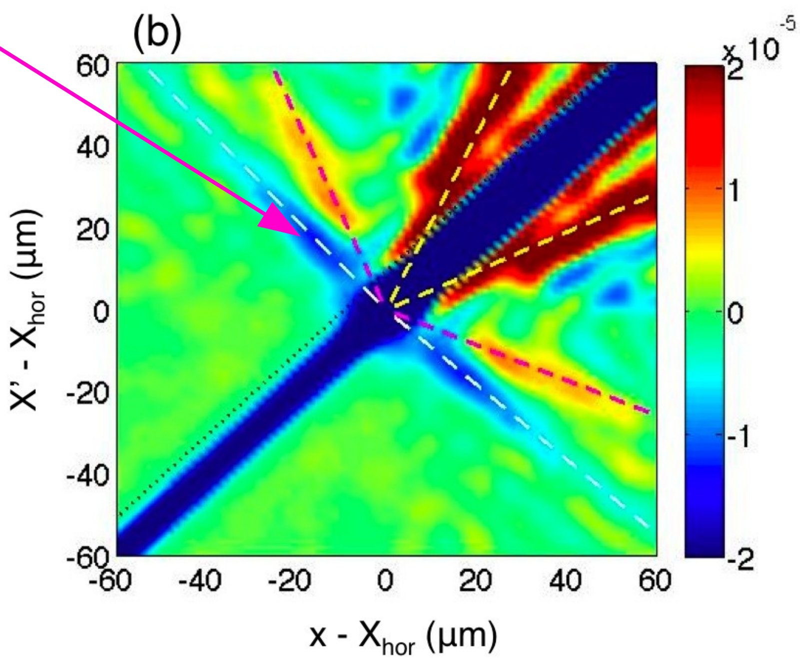
Hawking emission in driven-dissipative photon fluids

- Wigner-MC simulation with driving/losses: $$i\,dE=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|E|^2-\frac{i}{2}\gamma\right\}E\;dt+F_{ext}(x,t)\;dt+dW$$

- Near-field emission pattern from wire :
  Correlation function of intensity noise
  at different positions (x,x')



- Signature of Hawking radiation processes:
  "Balbinot-Fabbri" correlation tongues
  Conversion of zero-point fluctuations
  into correlated pairs of Bogoliubov phonons
  propagating away from horizon



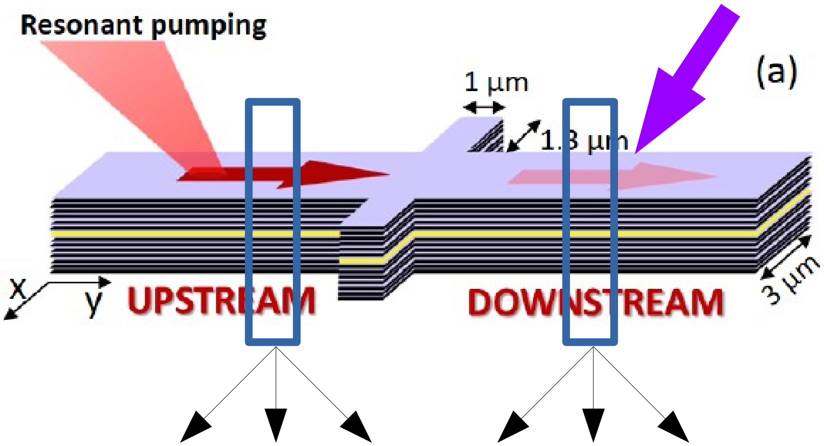
- In optics language:
  parametric emission of entangled photons
  flow+horizon play role of pump
  photons dressed by fluid into phonons

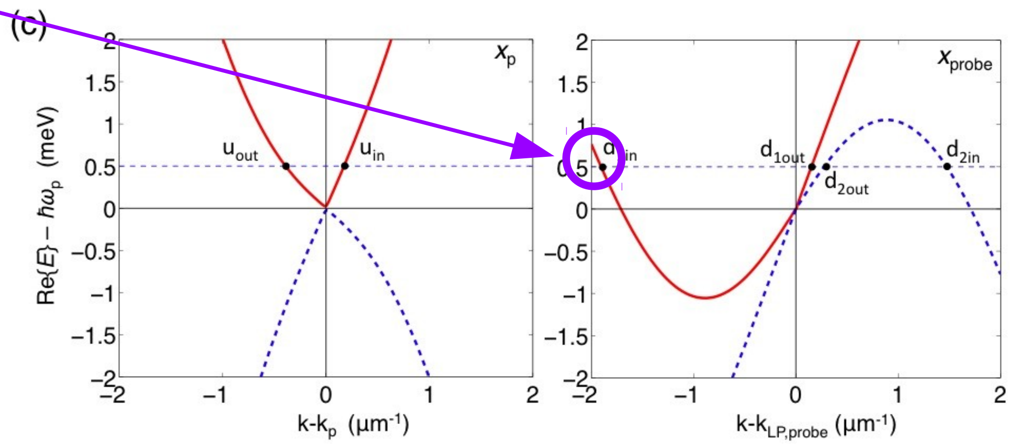
- Proposed experiment:
  - steady state under cw pumping
  - collect near-field emission
  - measure intensity noise
  - integrate over long time to extract signal out of shot noise

D. Gerace and IC, PRB **86**, 144505 (2012)

Pump-probe detection of (classical) HR

CW probe beam





- CW probe at ω_{probe} , frequency resolved detect

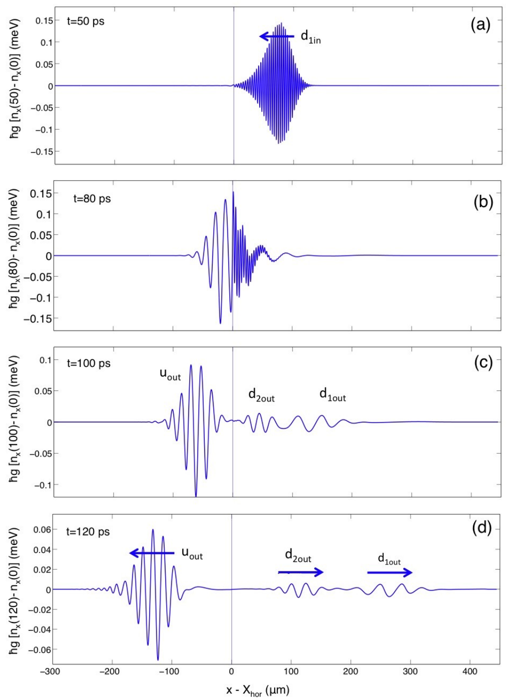
  at ω_{probe} and FWM signal @ $2\omega_{pump}$-ω_{probe}

- Stimulated Hawking on mode d_{2out} → peak in angular distribution

- Scattering matrix S(ω):

  T_H/ω scaling @ low ω → signature of thermal Hawking emiss.

- In contrast to pulse expt, no need for temporal resolution

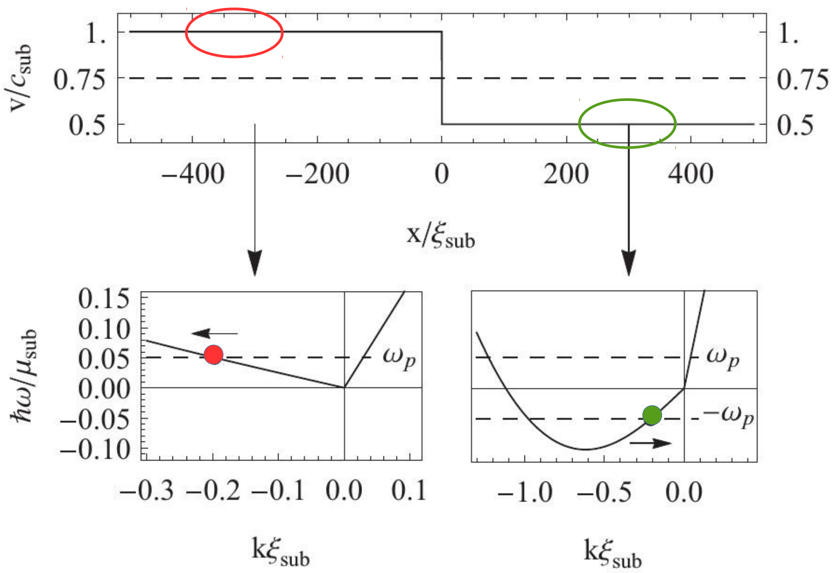


Gerace, IC, PRB 2012

Expt with surface waves on water (Weinfurtner, Unruh, PRL 2010) appears not conclusive as no horizon present, new expt in progress (Rousseaux)

How to assess quantum nature of HR?

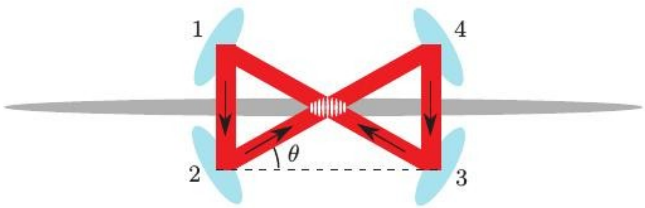
- Signal in density/intensity correlations reinforced at finite T by stimulated Hawking emission.
  → Not a signature of quantum origin of emission

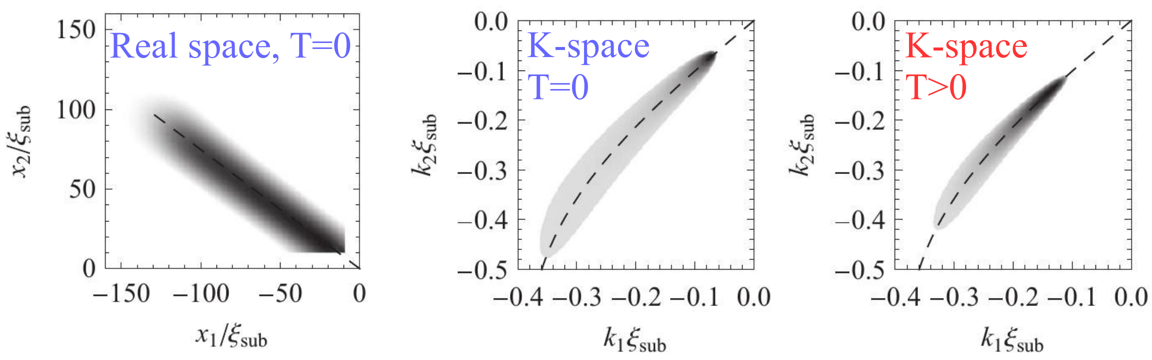


- Peres-Horodecki criterion for entanglement in bipartite systems
  → correlations of quadratures of phonon operators on either side of the horizon

- Phonon wavepacket operators localized in real- and momentum spaces

- To measure phonon quadratures with spatial and spectral selectivity:
  → optomechanical coupling, measure cavity frequency shift





Finazzi, IC, *Entangled phonons in atomic Bose-Einstein condensates,* PRA 2014

Is HR in microcavities a quantum process?

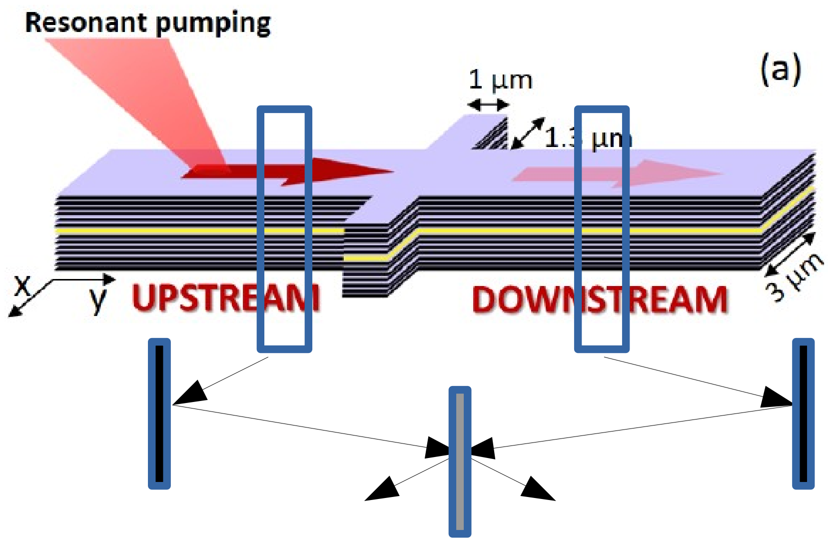
Unavoidable losses of microcavity device generate excitations in the fluid:
lost photon → creates hole → Bogoliubov excitation

Spurious excitations up to $\omega \sim gn$, comparable to T_H

X.Busch, R.Parentani, IC, *Spectrum and entanglement of phonons in quantum fluids of light*, PRA 2014

Stimulate Hawking processes giving rise to "thermal" Hawking signal:

- Density correlation signal reinforced
- Quantum entanglement still present ?
  - How to detect it ?
  - Hong-Ou-Mandel on emitted light spatially + wavevector-selected ?



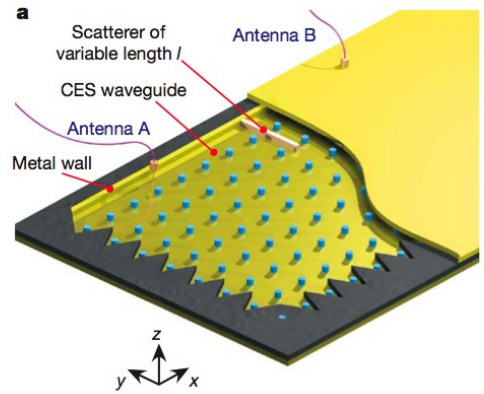
Part II:

Synthetic gauge fields and quantum magnetism with light

Photonic (Chern) topological insulator

MIT '09, Soljacic group

Original proposal Haldane-Raghu, PRL 2008



Wang et al., Nature 461, 772 (2009)

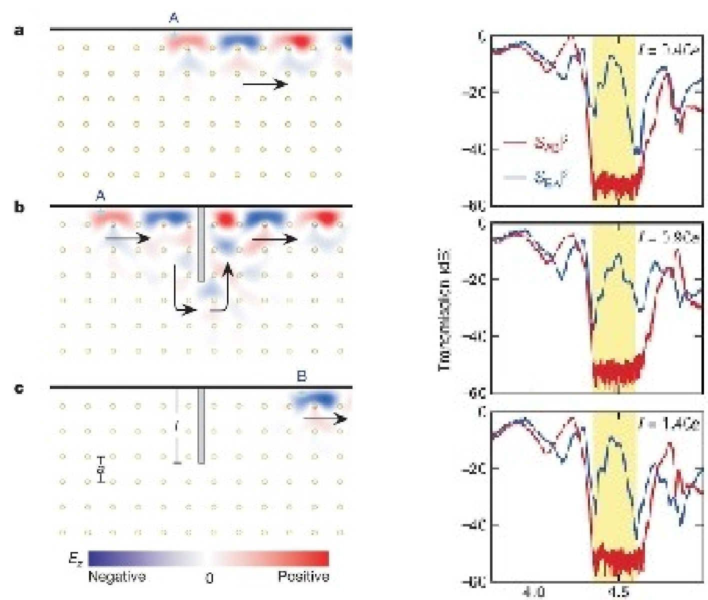
Magneto-optical photonic crystals for μ-waves

T-reversal broken by magnetic elements

Band wih non-trivial Chern number:

→ chiral edge states within gaps

- unidirectional propagation
- immune to back-scattering by defects



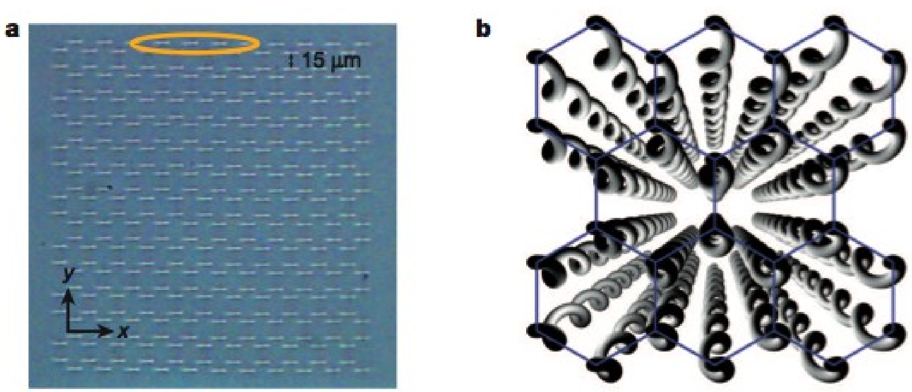
Wang et al., Nature 461, 772 (2009)

Synthetic gauge fields for photons

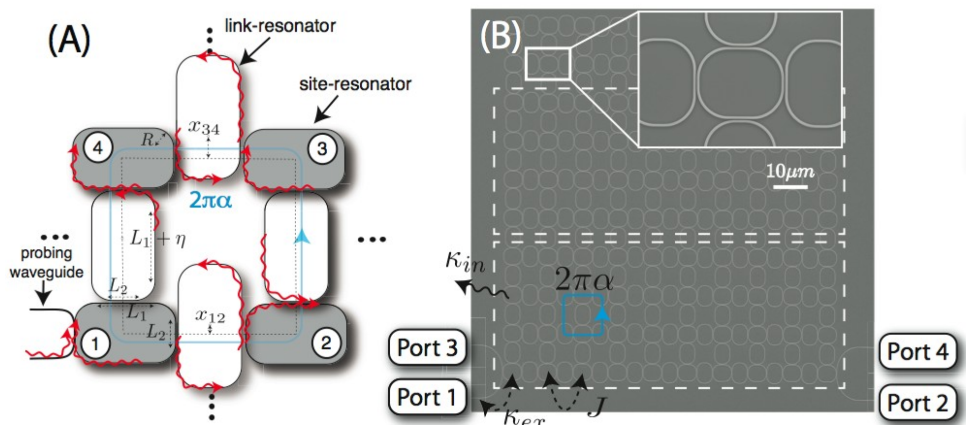
2D lattice of coupled cavities with tunneling phase

$$H = \sum_i \hbar\omega_\circ \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}} + \sum_i \left[\hbar F_i(t)\, \hat{a}_i^\dagger + \mathrm{h.c.}\right]$$

- Floquet bands in helically deformed waveguide lattices → Rechtsman/Segev
- silicon ring cavities → Hafezi/Taylor (JQI)
- electronic circuits with lumped elements → J. Simon (Chicago)
- related: honeycomb potential for polaritons → A. Amo/J.Bloch (LPN)



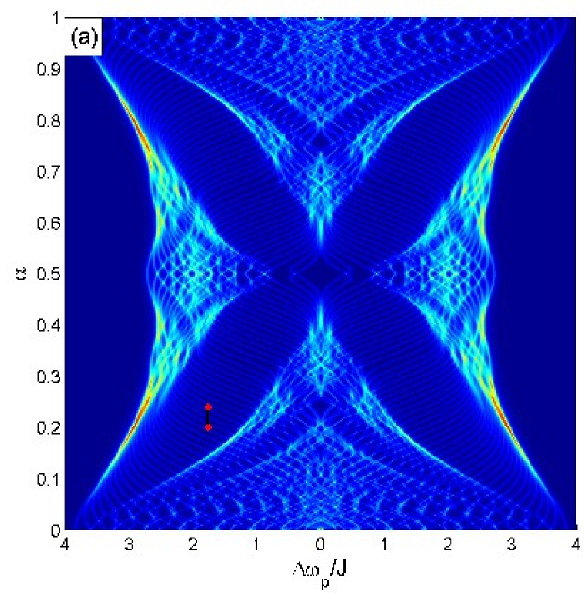
Rechtsman, Plotnik, et al., Nature 496, 196 (2013)



Hafezi et al.,Nat. Phot. **7**, 1001 (2013)

Hofstadter butterfly and chiral edge states

2D square lattice of coupled cavities
at large magnetic flux

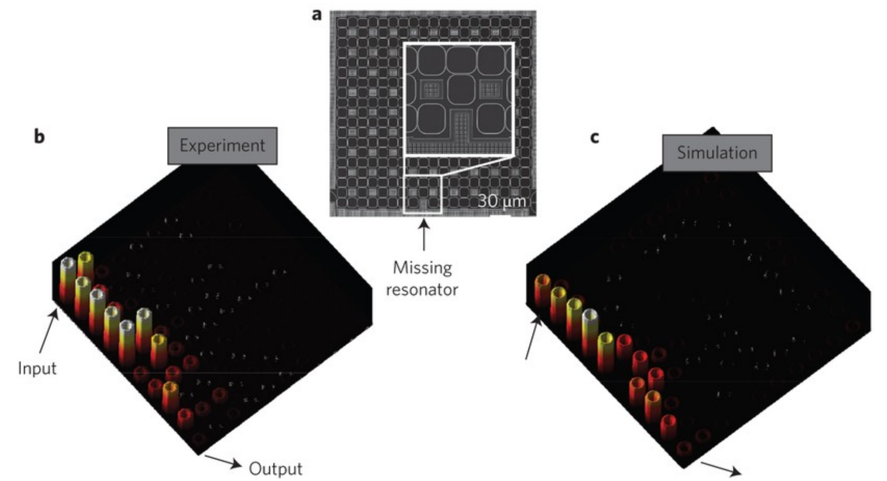


- eigenstates organize in bulk Hofstadter bands
- Berry connection in k-space: $A_{n,k} = i\langle u_{n,k} | \nabla_k u_{n,k}\rangle$

Bulk-edge correspondance:

$A_{n,k}$ has non-trivial Chern number
→ chiral edge states within gaps

- unidirectional propagation
- (almost) immune to scattering by defects
- T-reversal not broken, 2x pseudo-spin bands with opposite Chern

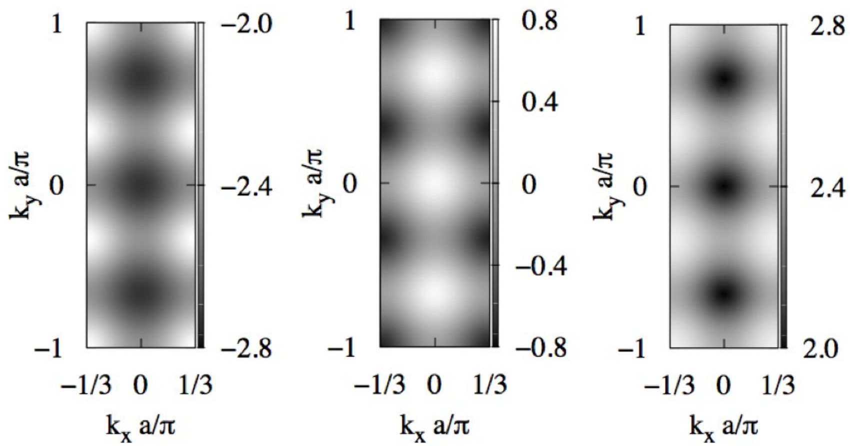


Hafezi et al.,Nat. Phot. **7**, 1001 (2013)

How to observe topological properties of bulk?

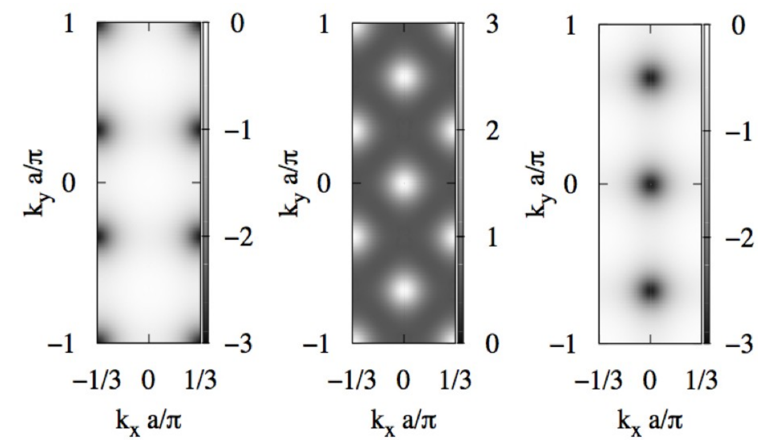
Lattice at strong magnetic flux, e.g. $\alpha = 1/3$

Band dispersion



Berry curvature

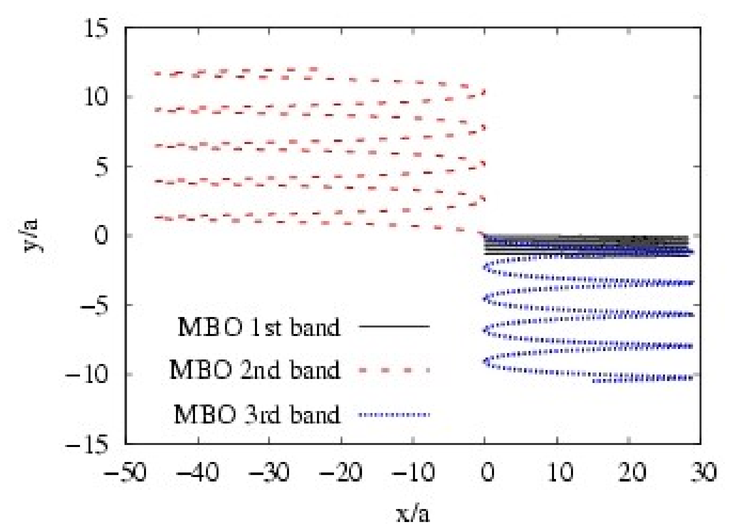
$$\boldsymbol{\Omega}_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathcal{A}_{n,\mathbf{k}} = \nabla_{\mathbf{k}} \times [i\langle u_{n,\mathbf{k}} | \nabla_{\mathbf{k}} u_{n,\mathbf{k}} \rangle]$$



Semiclassical eqs. of motion:

$$\hbar \dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$

$$\hbar \dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}} \mathcal{E}_{n,\mathbf{k}} - e\mathbf{E} \times \boldsymbol{\Omega}_n(\mathbf{k})$$

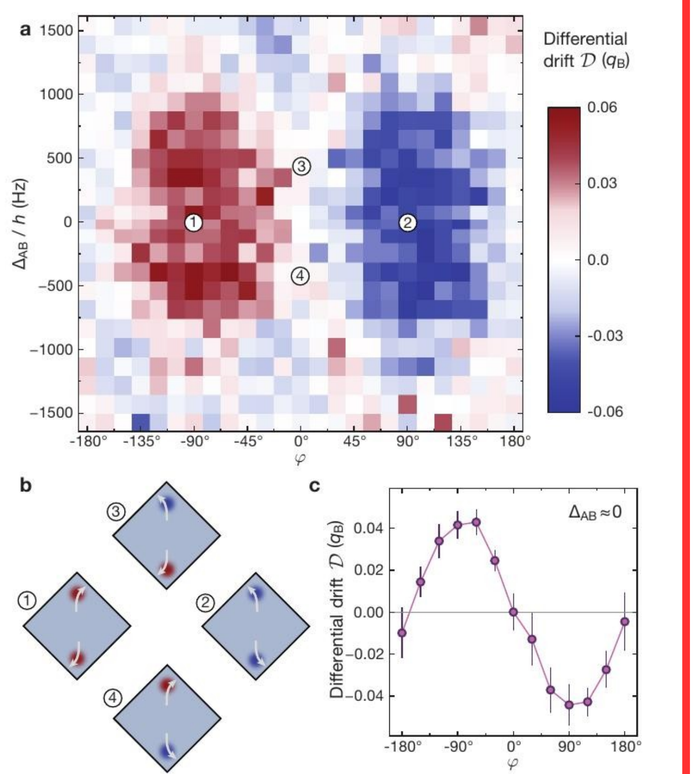


Magnetic Bloch oscillations display a net lateral drift

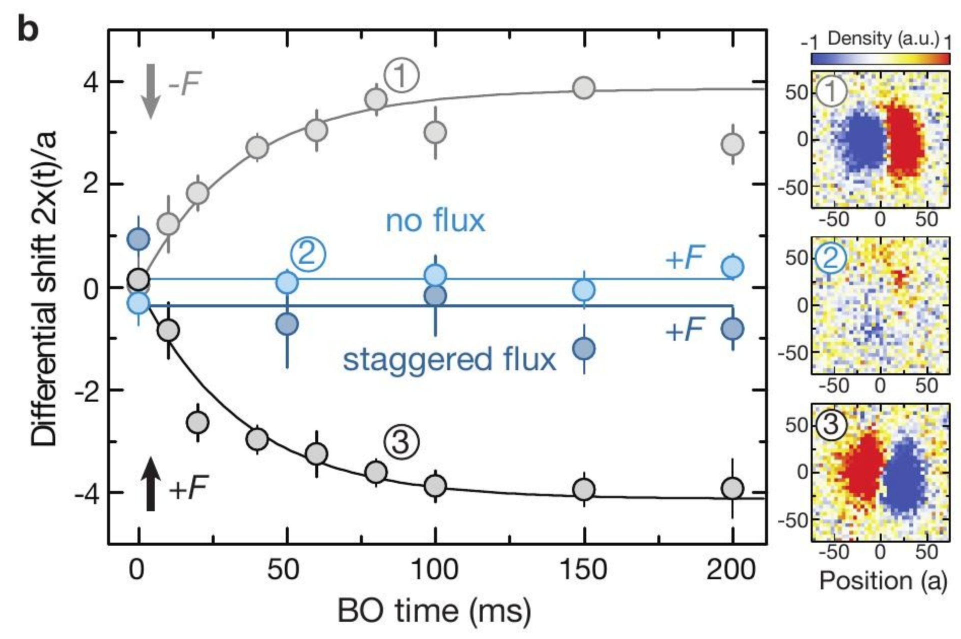
- Initial photon wavepacket injected with laser pulse
- spatial gradient of cavity frequency → uniform force
- Berry curvature → sort of **k**-space magnetic field

Figures from Cominotti-IC, EPL **103**, 10001 (2013).
First proposal in Dudarev, IC et al. PRL 92, 153005 (2004). See also Price-Cooper, PRA 83, 033620 (2012).

Experiments with atoms



Jotzu et al., Nature 515, 237-240 (2014)



Aidelsburger et al. Nat. Phys. 11, 162 (2015)

Semiclassical equations of motion

$$\hbar\dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar\dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}}\mathcal{E}_{n,\mathbf{k}} + e\mathbf{E}\times\mathbf{\Omega}_n(\mathbf{k})$$

Slow momentum shift under uniform force eE (as in Bloch oscillations)

Berry curvature of band Ω_n → lateral displacement in space of atomic cloud

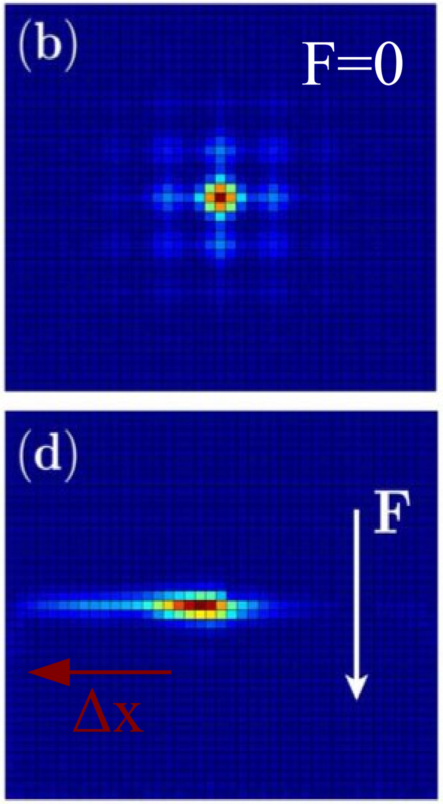
Anomalous Hall effect vs. Integer Quantum Hall effect

Photonic system

Cavity lattice geometry → promising in view of interacting photon gases, but radiative losses.

Short time to observe BO's, but experiment @ non-eq steady state even better

Coherent pumping $H_d = \sum_i F_i(t)\, \hat{b}_i + F_i^*(t)\, \hat{b}_i^\dagger$ + losses at rate γ



Pump spatially localized on central site only:

- couples to all k's within Brillouin zone
- resonance condition selects specific states

In the presence of force F:

motion in BZ → lateral drift in real space by Berry curvature

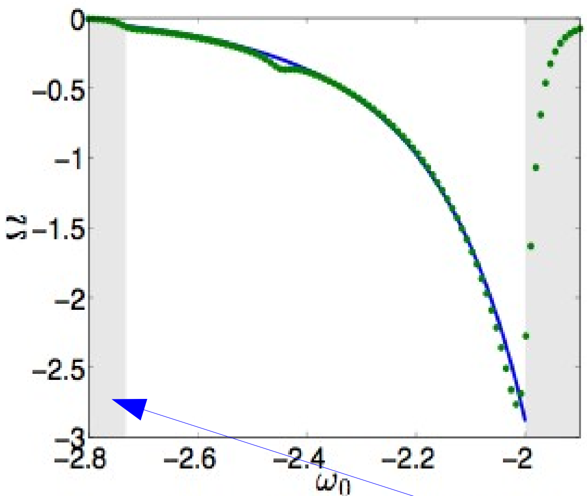
$$\hbar \dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar \dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}} \mathcal{E}_{n,\mathbf{k}} - e\mathbf{E} \times \mathbf{\Omega}_n(\mathbf{k})$$

Detectable as lateral shift of intensity distribution by Δx perpendicular to F

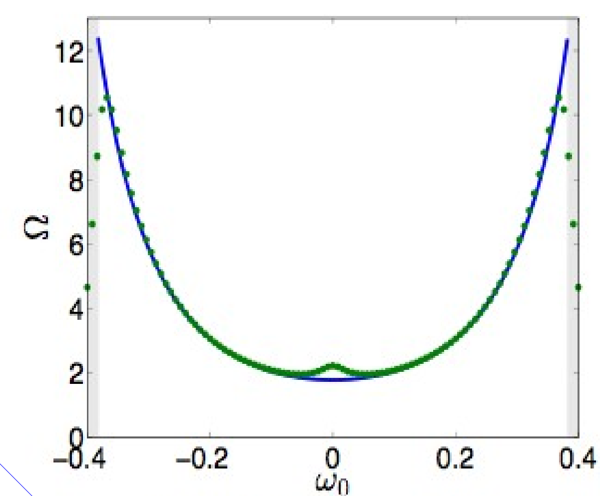
T. Ozawa and IC, *Anomalous and Quantum Hall Effects in Lossy Photonic Lattices*, PRL (2014)

More quantitatively

		1st	2nd	3rd	4th	5th	6th
$\alpha = \frac{1}{3}$	$\mathcal{C}$	-1	$+2$	-1			
	$\mathcal{C}_n$	-0.91	-	-0.91			
$\alpha = \frac{1}{5}$	$\mathcal{C}$	-1	-1	$+4$	-1	-1	
	$\mathcal{C}_n$	-0.97	-0.66^*	-	-0.66^*	-0.97	
$\alpha = \frac{1}{6}$	$\mathcal{C}$	-1	-1	$+2$	$+2$	-1	-1
	$\mathcal{C}_n$	-0.96	-1.06	-	-	-1.06	-0.96
$\alpha = \frac{3}{7}$	$\mathcal{C}$	$+2$	-5	$+2$	$+2$	$+2$	-5
	$\mathcal{C}_n$	2.05	-	-	2.01	-	-
$\alpha = \frac{4}{9}$	$\mathcal{C}$	$+2$	$+2$	-7	$+2$	$+2$	$+2$
	$\mathcal{C}_n$	1.96	-	-	2.02	1.92	2.02
$\alpha = \frac{5}{11}$	$\mathcal{C}$	$+2$	$+2$	-9	$+2$	$+2$	$+2$
	$\mathcal{C}_n$	1.92	1.88	-	-	2.06	1.91



(a) Lowest band of $\alpha = 1/3$



(b) Middle band of $\alpha = 1/5$

band gap

Low loss (γ < bandwidth) → $\Delta x = F\,\Omega(k_0)/2\gamma$ (anomalous Hall eff.)

Large loss (bandwidth < γ < bandgap) → $\Delta x = q\,\text{Chern} / 2\pi\gamma$ (integer-QH)

Integer quantum Hall effect for photons (in spite of no Fermi level)

Photon phase observable => expts sensitive to gauge-variant quantities!!

T. Ozawa and IC, *Anomalous and Quantum Hall Effects in Lossy Photonic Lattices*, PRL (2014)

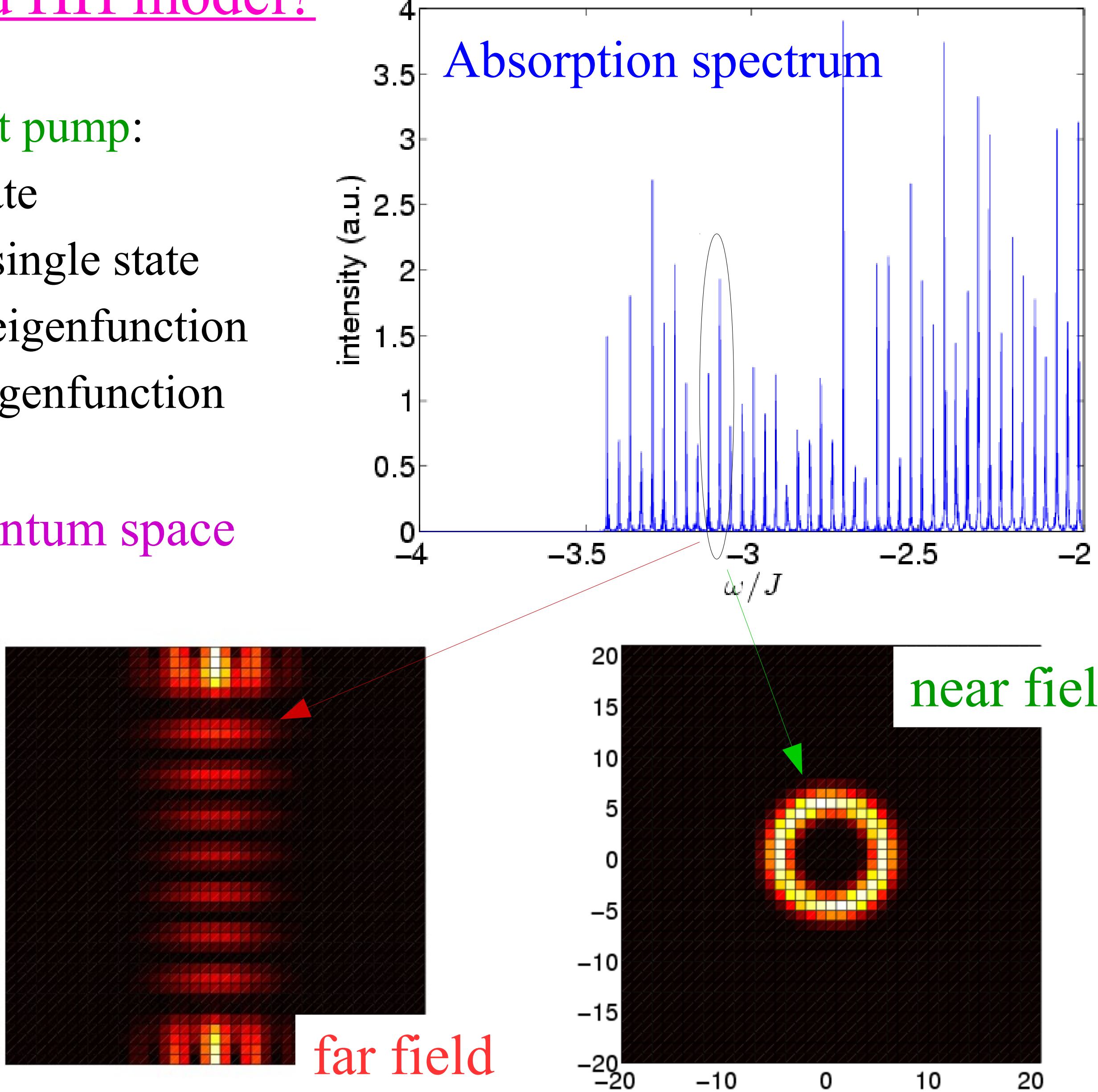
What happens with harmonic trap?

What are the quantum mechanical eigenstates of a harmonically trapped HH model?

Straightforward in optics under coherent pump:

- each absorption peak → an eigenstate
- coherent pump frequency selects a single state
  - near-field image → real-space eigenfunction
  - far-field emission → k-space eigenfunction

Toroidal Landau levels in momentum space



Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , PRL 2014

A. C. Berceanu, T. Ozawa, H. M. Price, IC, in preparation (2015)

Part III:

Towards higher dimensions

What about higher dimensions?

Generalize of semiclassical equations to 4D:

$$\begin{cases} \dot{r}^{\mu}(\boldsymbol{k}) = \dfrac{\partial \mathcal{E}(\boldsymbol{k})}{\partial k_{\mu}} - \dot{k}_{\nu}\Omega^{\mu\nu}(\boldsymbol{k}) \\ \dot{k}_{\mu} = -E_{\mu} - \dot{r}^{\nu} B_{\mu\nu}, \end{cases}$$

Integrate current over filled bands:

- 2D quantized Hall current depends on 1st Chern number

$$j^y = \frac{E_x}{(2\pi)^2} \int_{T^2} \Omega \, d^2 k = \frac{\nu_1}{2\pi} E_x \quad \text{analogous to} \quad j^y = \nu \frac{e^2}{h} \quad \text{well known in IQHE}$$

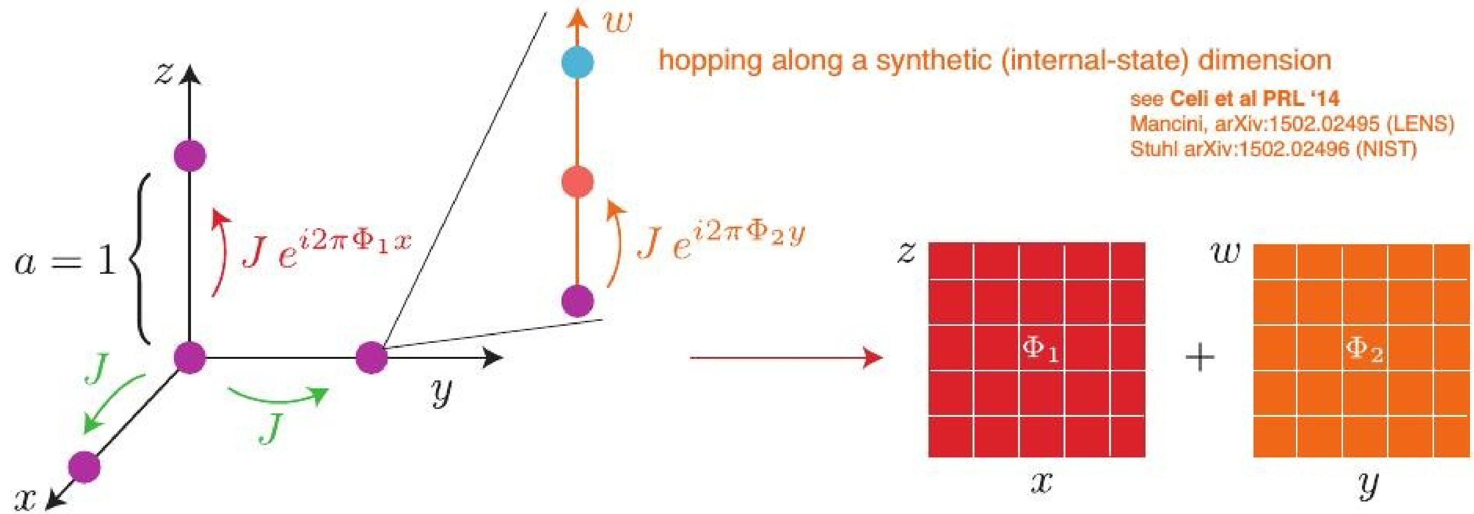
- 4D magneto-electric response depends on 2nd Chern number (non-zero in d≥4)

$$j^{\mu} = E_{\nu} \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \Omega^{\mu\nu} \mathrm{d}^4 k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_{\nu} B_{\alpha\beta}$$

$$\nu_2 = \frac{1}{4\pi^2} \int_{\mathbb{T}^4} \Omega^{xy}\Omega^{zw} + \Omega^{wx}\Omega^{yz} + \Omega^{zx}\Omega^{yw} \mathrm{d}^4 k$$

H. M. Price, O. Zilberberg, T. Ozawa, IC, N. Goldman, *Four-Dimensional Quantum Hall Effect with Ultracold Atoms*, ArXiv:1505.04387, to appear on PRL

How to create 4D system with atoms?



Internal state → Synthetic dimension w

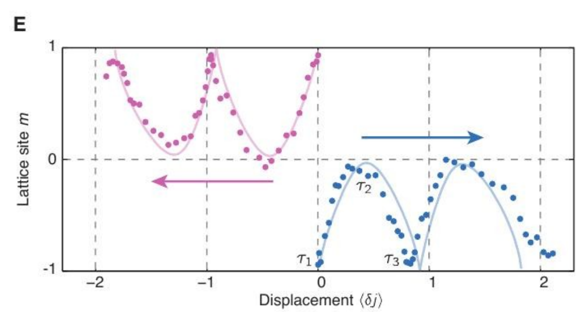
Raman processes give tunneling along w

Spatial phase of Raman beams give Peierls phase

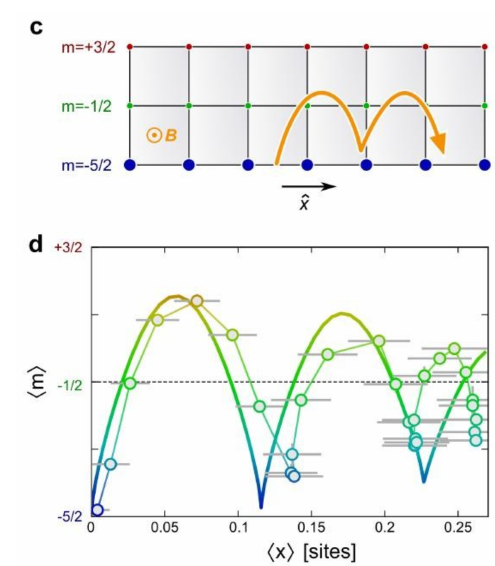
Synthetic magnetic field in xw,yw,zw planes

First experimental realization:

- 1+1 dimens. using 3 spin states
- Cyclotron + Reflection on edges

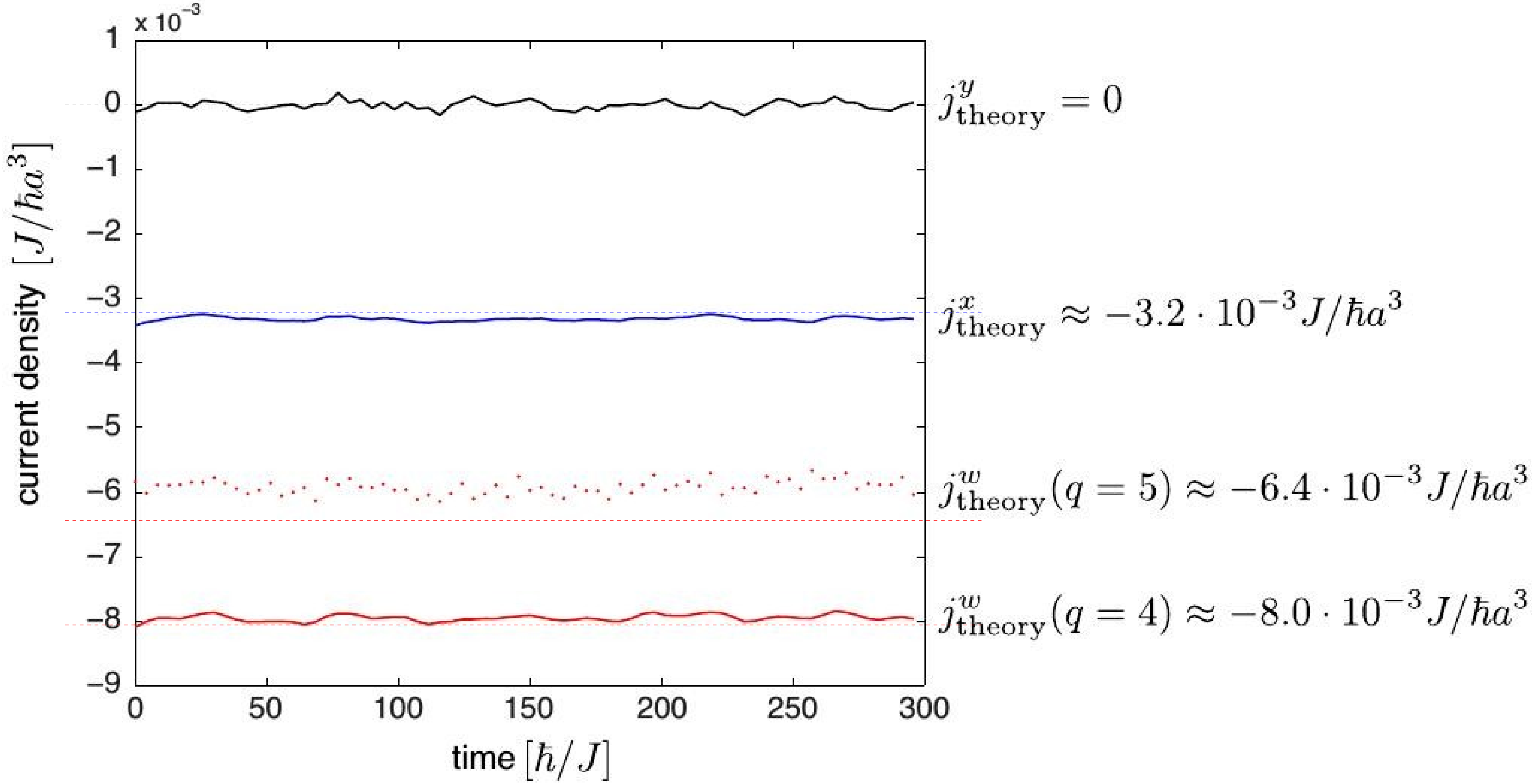


Stuhl et al., arXiv:1502.02496



Mancini et al. ArXiv:1502.02495

Numerical validation of 4D QH effect



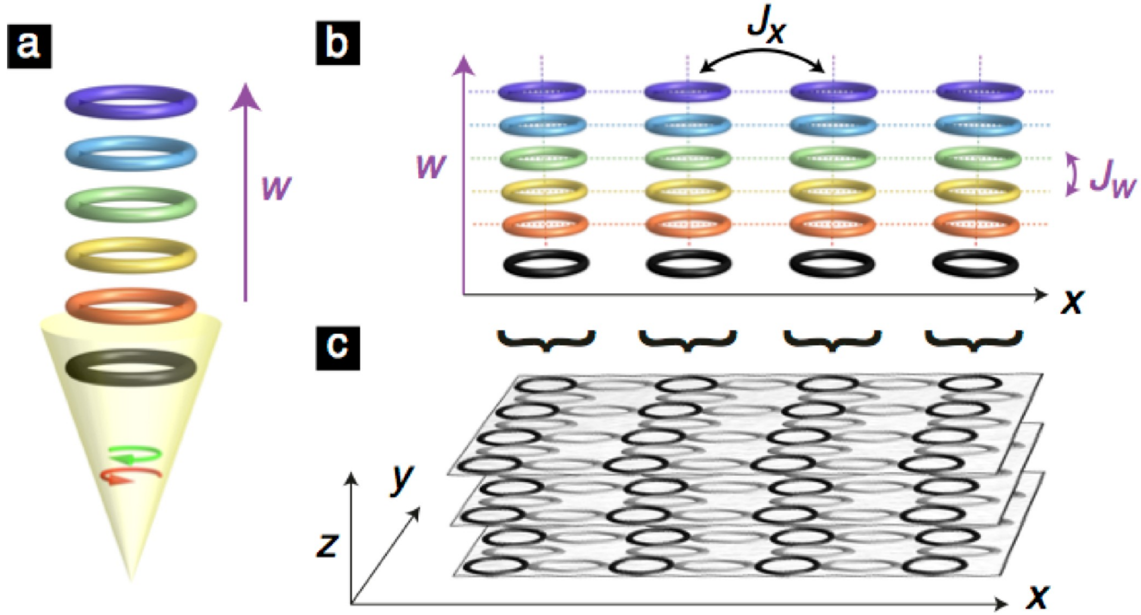
Numerical simulation of full wave equation
Add external E and B fields
Results in good agreement with semiclassics
Allowed E,B enough to see the effect

$$j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \Omega^{\mu\nu} \mathrm{d}^4 k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}$$

$$\nu_2 = \frac{1}{4\pi^2} \int_{\mathbb{T}^4} \Omega^{xy}\Omega^{zw} + \Omega^{wx}\Omega^{yz} + \Omega^{zx}\Omega^{yw} \mathrm{d}^4 k$$

H. M. Price, O. Zilberberg, T. Ozawa, IC, N. Goldman, *Four-Dimensional Quantum Hall Effect with Ultracold Atoms*, ArXiv:1505.04387, to appear on PRL

How to create synthetic dimensions for photons?



Different modes of ring resonators → synthetic dimension *w*

Tunneling along synthetic *w*:

- strong beam modulates resonator ε_{ij} at ω_{FSR} via optical $\chi^{(3)}$
- neighboring modes get linearly coupled
- phase of modulation → Peierls phase along synthetic *w*

Extends Fan's idea of synthetic gauge field via time-dependent modulation (Nat. Phys. 2008)

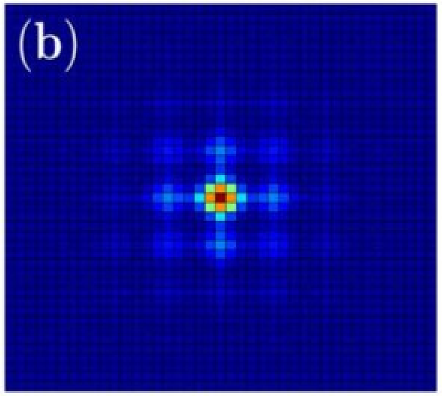
Peierls phase along *x,y,z* → Hafezi's ancilla resonators

T. Ozawa, N. Goldman, O. Zylberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, in preparation (2015)

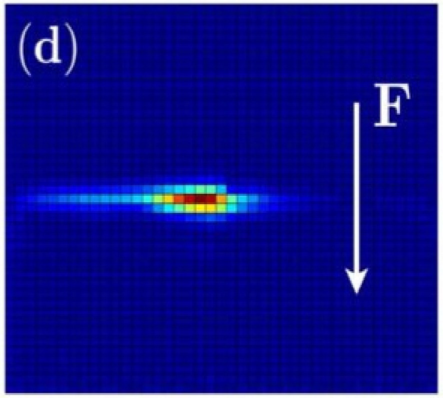
1+1 array: chiral edge states & optical isolation

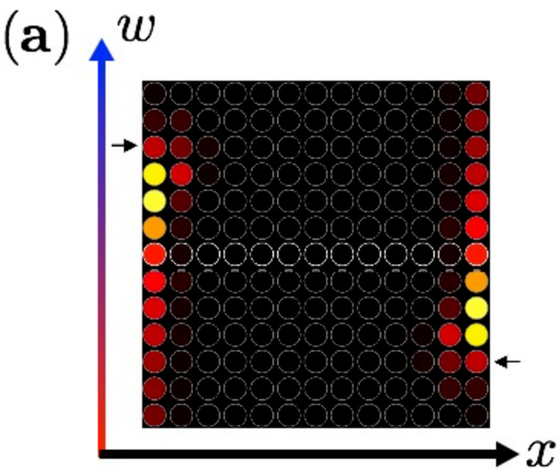
1 (physical) + 1 (synthetic) dimensions: Hofstadter model

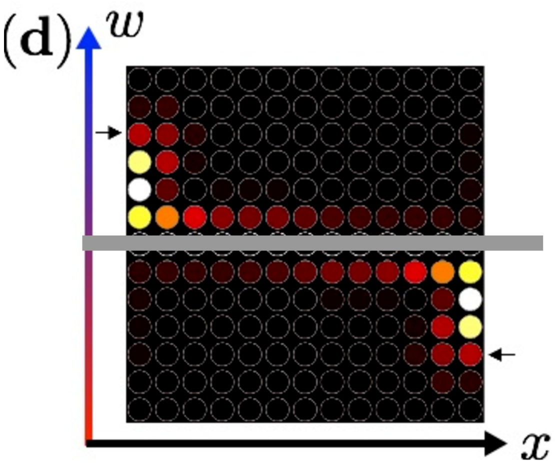
- Bulk topological invariant → Chern number
  - measured via Integer Quantum Hall effect



- Chiral states on edges:
  - Physical edges along x
  - Synthetic edges via design of ε(ω)
    (e.g. inserting absorbing impurities in chosen sites)
    → topologically protected optical isolator







T. Ozawa, N. Goldman, O. Zylberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, in preparation (2015)

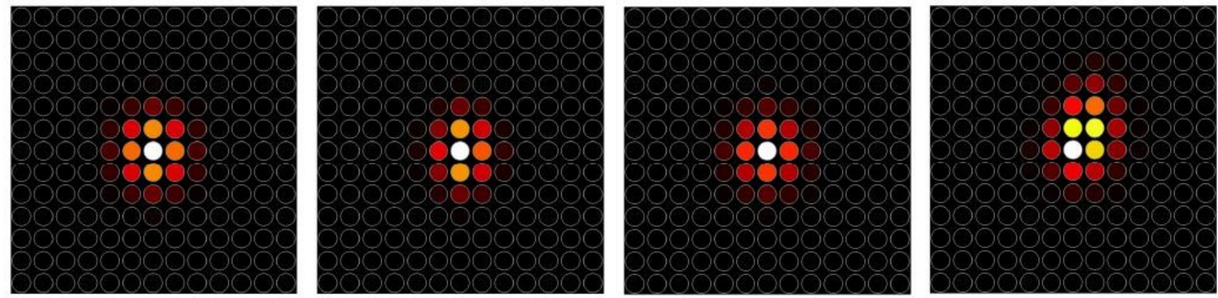
3+1 array: 4D Quantum Hall physics

4D magneto-electric response

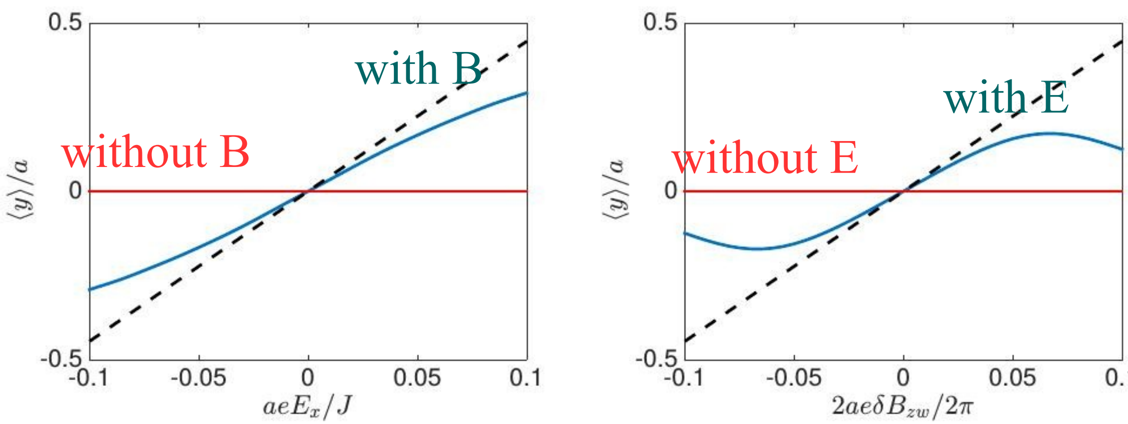
Nonlinear integer QH effect

Lateral shift of photon intensity distribution in response to external synth-E and synth-B:

- only present with both E & B
  - proportional to 2nd Chern



(a) $eE_x a, = 0$, $ea^2\delta B_{zw} = 0$ (b) $eE_x a = \frac{J}{10}$, $ea^2\delta B_{zw} = 0$ (c) $eE_x a = 0$, $ea^2\delta B_{zw} = \frac{2\pi}{10}$ (d) $eE_x a = \frac{J}{10}$, $ea^2\delta B_{zw} = \frac{2\pi}{10}$



$$j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \Omega^{\mu\nu} \mathrm{d}^4 k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}$$

$$\nu_2 = \frac{1}{4\pi^2} \int_{\mathbb{T}^4} \Omega^{xy}\Omega^{zw} + \Omega^{wx}\Omega^{yz} + \Omega^{zx}\Omega^{yw} \mathrm{d}^4 k$$

T. Ozawa, N. Goldman, O. Zylberberg, H. M. Price, and IC, *Synthetic Dimensions in Photonic Lattices: From Optical Isolation to 4D Quantum Hall Physics*, in preparation (2015)

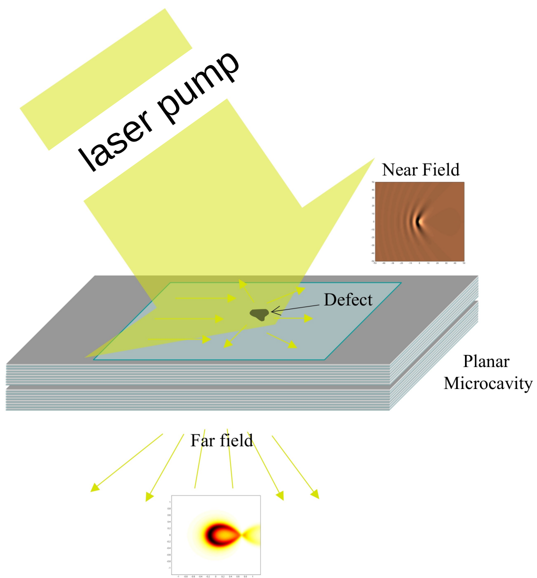
Part IV:

Quantum fluids of light

with a *conservative* dynamics

Field equation of motion

Planar microcavities & cavity arrays



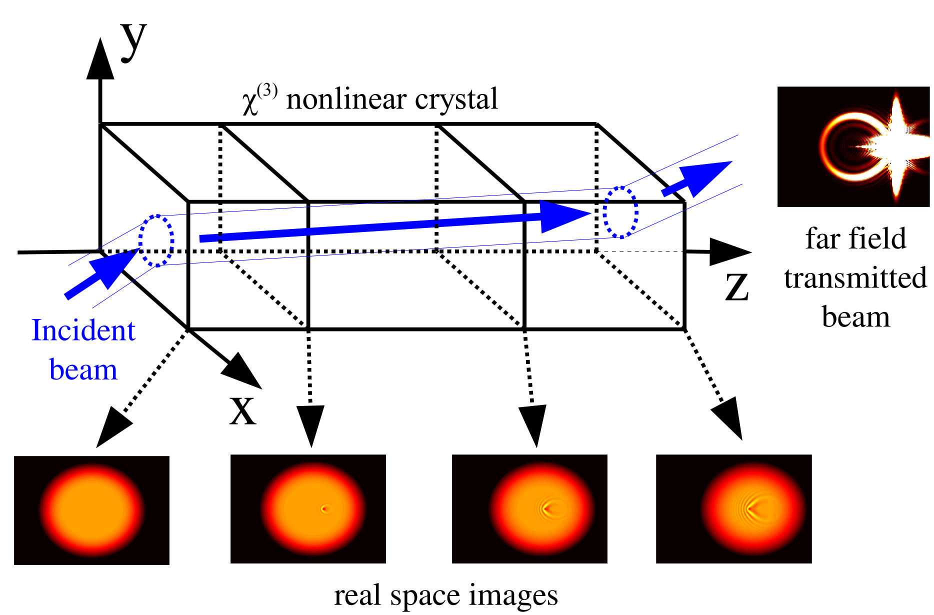
Pump needed to compensate losses:
driven-dissipative dynamics in real time
stationary state ≠ thermodyn. equilibrium

Driven-dissipative CGLE evolution

$$i\frac{dE}{dt}=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|E|^2+\frac{i}{2}\left(\frac{P_0}{1+\alpha|E|^2}-\gamma\right)\right\}E+F_{ext}$$

Quantum correl. sensitive to dissipation

Propagating geometry



Monochromatic beam
Incident beam sets initial condition @ z=0
Conservative GPE paraxial propagation

$$i\frac{d\hat{E}}{dz}=\left\{-\frac{\hbar\nabla^2_{xy}}{2\beta}+V_{ext}+g\,\hat{E}^\dagger\hat{E}\right\}\hat{E}$$

- V_{ext}, g proportional to -(ε(r)-1) and $\chi^{(3)}$
- Mass → diffraction (xy)

How to include quantum fluctuations beyond MF

Requires going beyond monochromatic beam and explicitly including physical time

Gross-Pitaevskii-like eq. for propagation of quasi-monochromatic field

$$i\frac{\partial E}{\partial z}=-\frac{1}{2\beta_0}\left(\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}\right)-\frac{1}{2D_0}\frac{\partial^2 E}{\partial t^2}+V(r)E+g|E|^2E$$

Propagation coordinate z → time

Physical time → extra spatial variable, dispersion D_0 → temporal mass

Upon quantization → conservative many-body evolution in z: $i\frac{d}{dz}|\psi> = H|\psi>$

with $$H=N\iiint dx\, dy\, dt\left[\frac{1}{2\beta_0}\nabla\hat{E}^\dagger\nabla\hat{E}-\frac{D_0}{2}\frac{\partial\hat{E}^\dagger}{\partial t}\frac{\partial\hat{E}}{\partial t}+V\hat{E}^\dagger\hat{E}+\hat{E}^\dagger\hat{E}^\dagger\hat{E}\hat{E}\right]$$

Same z commutator $$\left[\hat{E}(x.y,t,z),\hat{E}^+(x',y',t',z)\right]=\frac{c\hbar\omega_0 v_0}{\epsilon}\delta(x-x')\,\delta(y-y')\,\delta(t-t')$$

P.-E. Larré and IC, arXiv:1412.5405, to appear on PRA. See also old work by Lai and Haus, PRA 1989

Dynamical Casimir emission at quantum quench (I)

Air / nonlinear medium interface

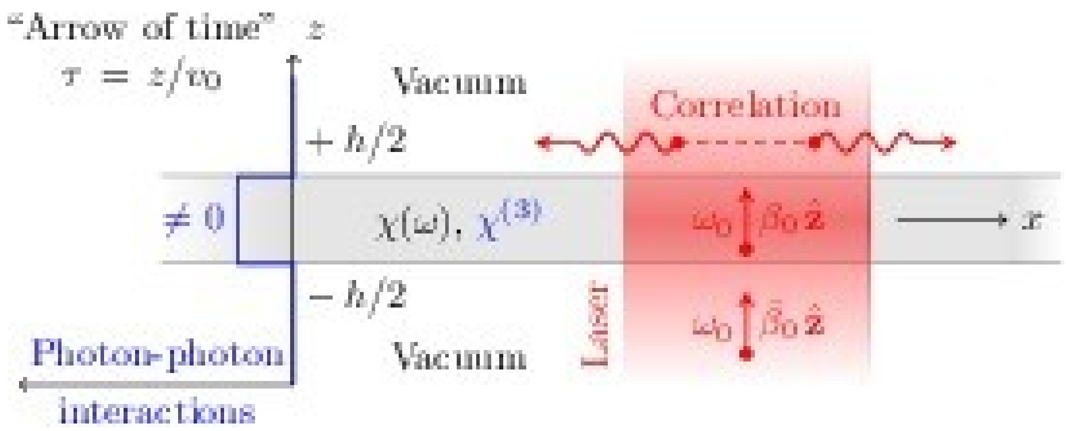
→ sudden jump in interaction constant when moving along z

Monochromatic wave

Normal incidence

Weakly nonlinear medium

→ Weakly interacting Bose gas at rest



Propagation along z

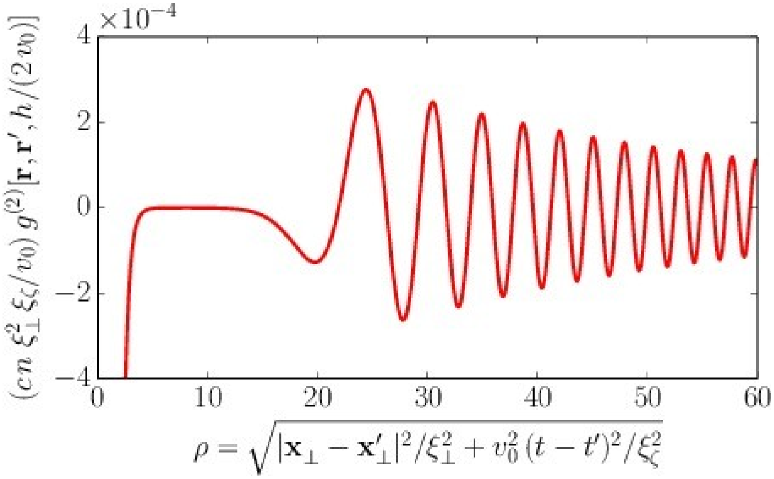
→ conservative quantum dynamics

Mismatch of Bogoliubov ground state in air and in nonlinear medium

→ emission of phonon pairs at opposite k on top of fluid of light

P.-E. Larré and IC, arXiv:1412.5405, to appear on PRA

Dynamical Casimir emission at quantum quench (II)



Observables:

- Far-field → correlated pairs of photons at opposite angles
- Near-field → peculiar pattern of intensity noise correl.

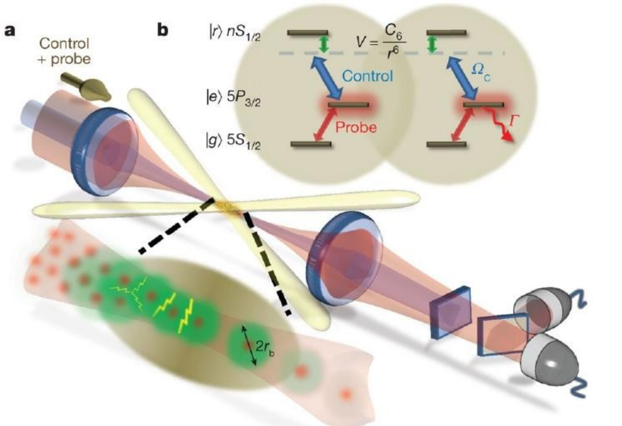
First peak propagates at the speed of sound c_s

May simulate dynamical Casimir effect & fluctuations in early universe

:

Pimp & probe expt for speed of sound c_s in spatial (xy) and temporal (t) directions:

- c_s^{xy} (Heriot-Watt – Vocke et al. Optica '15)
- c_s^{t} (Trento, in progress)



Quantum dynamics most interesting in strongly nonlinear media, e.g. Harvard expts with Rydberg polaritons

P.-E. Larré and IC, arXiv:1412.5405, to appear on PRA

Conclusions and perspectives

Dilute photon gas
GP-like equation

- 2000-6 → BEC in exciton-polaritons gas in semiconductor microcav.
- 2008-10 → superfluid hydrodynamics effects observed
- 2009-13 → synthetic gauge field for photons and topologically protected edge states observed.

- Optical microcavity systems are unavoidably lossy → driven-dissipative, non-equilibrium dynamics
  not always a hindrance for many-body physics, but can be turned into great advantage!
- Bulk cavityless geometries: paraxial propagation → conservative dynamics
  time plays role of third dimension; useful to study quantum quenches, thermalization, etc.

Many questions still open:

- quantum hydrodynamic states of photon fluid, e.g. analog Hawking emission of phonon pairs
- Integer QH effects in high-dimensional photonics
- critical properties of transition in 2D → BKT or not to BKT

Challenging perspectives on a longer run:

- strongly correlated photon gases → Tonks-Girardeau gas in 1D necklace of cavities
- with synthetic gauge field → Laughlin states, quantum Hall physics of light
- Theoretical challenge → how to create and control strongly correlated many-photon states?
- more complex quantum Hall states: non-Abelian statistical phases.
  Integrated platform for topologically protected states to store and process quantum information?

If you wish to know more...

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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(published 21 February 2013)

I. Carusotto and C. Ciuti, Reviews of Modern Physics **85**, 299 (2013)

I. Carusotto, Il Nuovo Saggiatore – SIF magazine (2013)

Save the date: May 8th-12th, 2017
2nd Workshop on Strongly Correlated Fluids of Light and Matter
Cargese, Corse

Or, even better, visit us in Trento!!
carusott@science.unitn.it

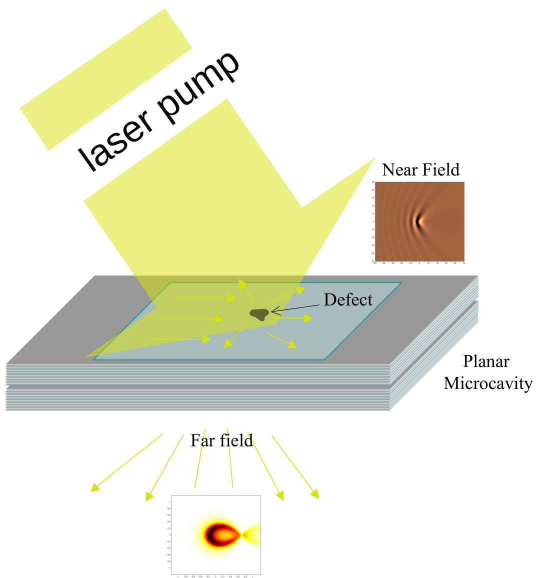
Part II:

Quantum fluids of light

with a *conservative* dynamics

Field equation of motion

Planar microcavities & cavity arrays



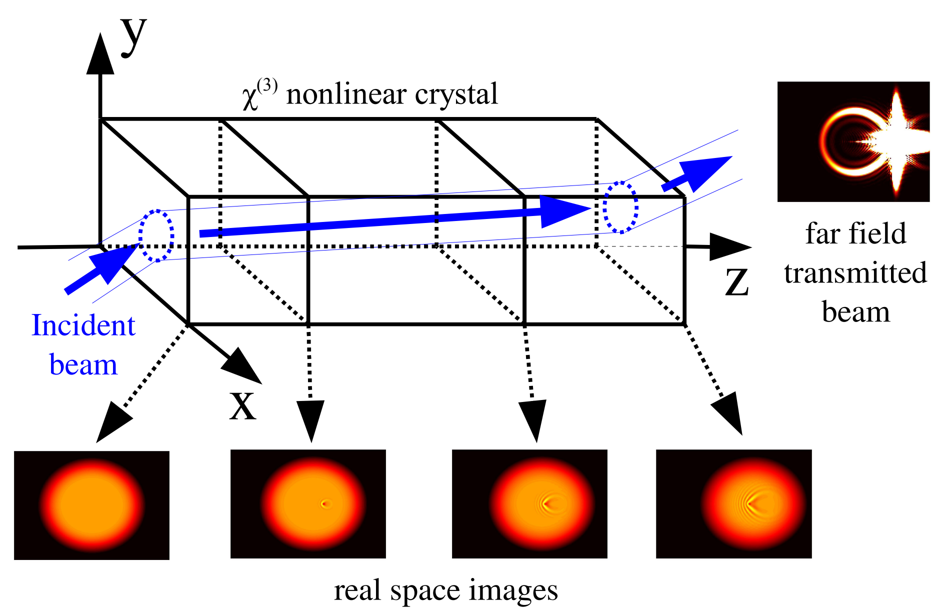
Pump needed to compensate losses:
driven-dissipative dynamics in real time
stationary state ≠ thermodyn. equilibrium

Driven-dissipative CGLE evolution

$$i\frac{dE}{dt}=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|E|^2+\frac{i}{2}\left(\frac{P_0}{1+\alpha|E|^2}-\gamma\right)\right\}E+F_{ext}$$

Quantum correl. sensitive to dissipation

Propagating geometry



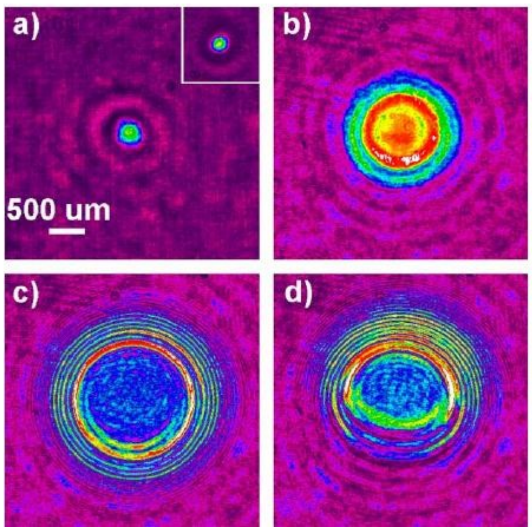
Monochromatic beam
Incident beam sets initial condition @ z=0
Conservative GPE paraxial propagation

$$i\frac{d\hat{E}}{dz}=\left\{-\frac{\hbar\nabla_{xy}^2}{2\beta}+V_{ext}+g\,\hat{E}^\dagger\hat{E}\right\}\hat{E}$$

- V_{ext} , g proportional to -(ε(r)-1) and $\chi^{(3)}$
- Mass → diffraction (xy)

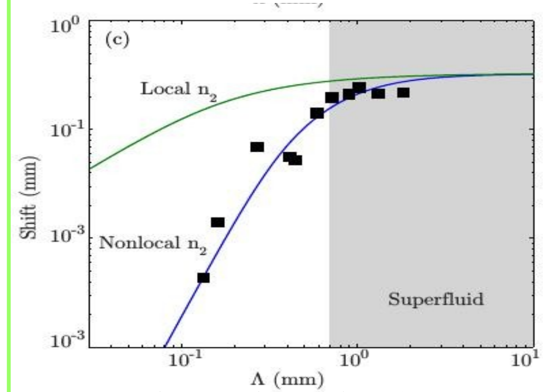
A few remarkable recent experiments

Dispersive superfluid-like shock waves



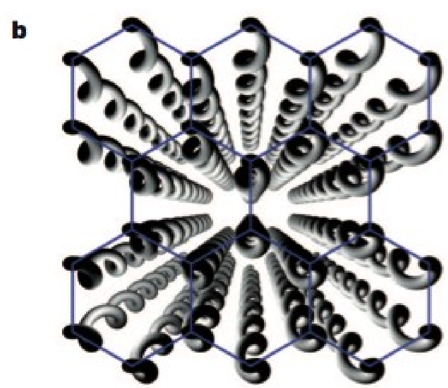
Wan et al., Nat. Phys. 3, 46 (2007)

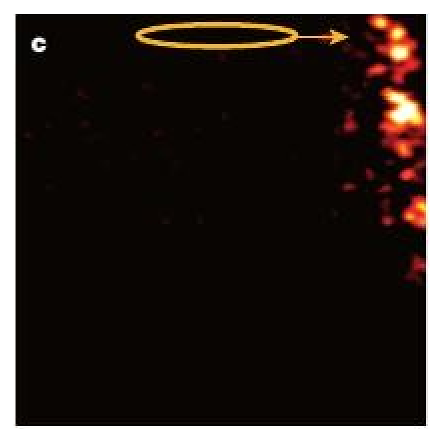
Bogoliubov dispersion of collective excitations



D. Vocke et al. Optica (2015)

Chiral edge states in (photonic) Floquet topological insulator





Rechtsman, et al.,
Nature 496, 196 (2013)

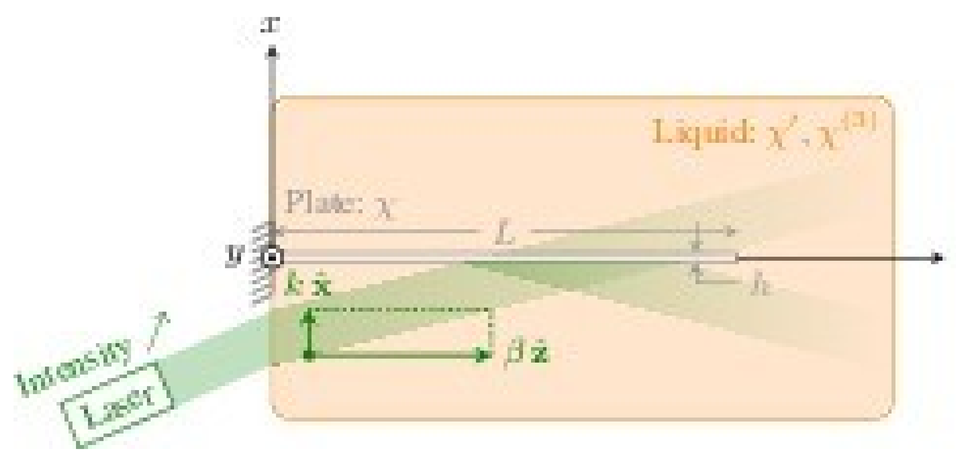
Frictionless flow of superfluid light (I)

All superfluid light experiments so far:

- Planar microcavity device with stationary obstacle in flowing light
- Measure response on the fluid density/momentum pattern
- Obstacle typically is defect embedded in semiconductor material
- Impossible to measure mechanical friction force exerted onto obstacle

Propagating geometry more flexible:

- Obstacle can be solid dielectric slab with different refractive index
- Immersed in liquid nonlinear medium, so can move and deform
- Mechanical force measurable from magnitude of slab deformation



P.-E. Larré and IC, arXiv:1405.2275, in the press on PRA (2015)

Frictionless flow of superfluid light (II)

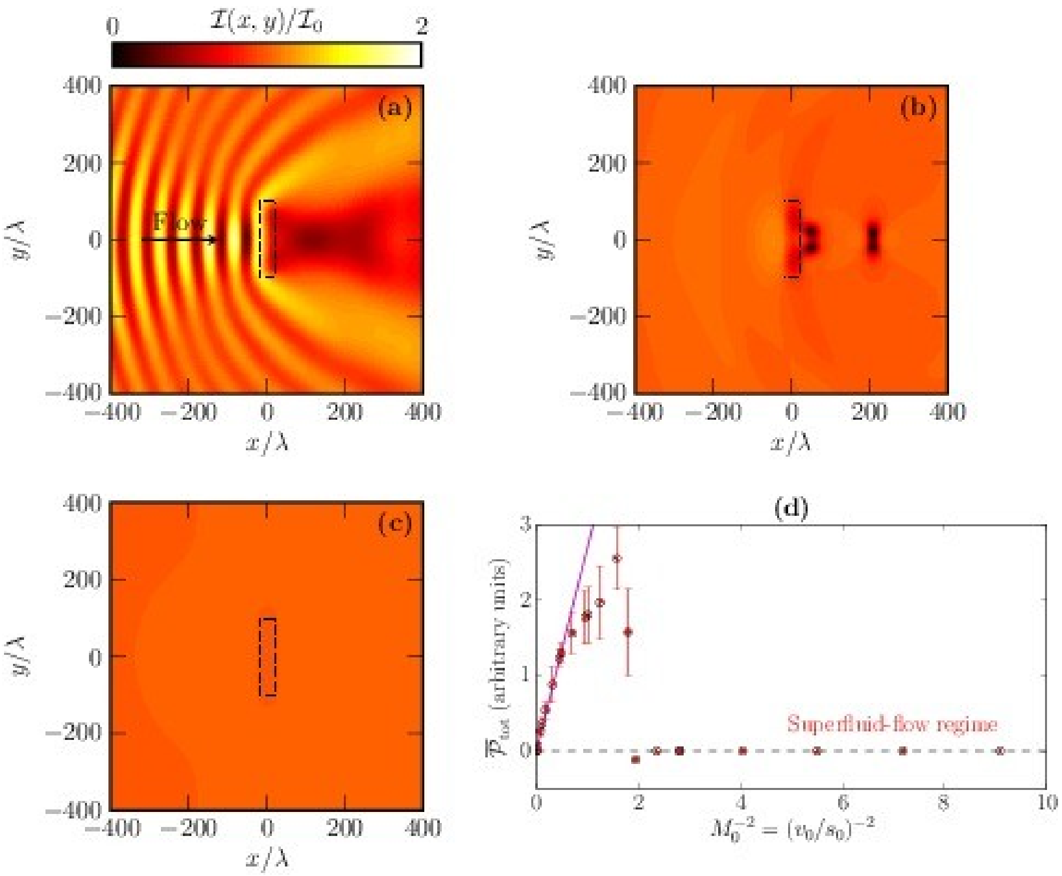
Numerics for propagation GPE of monochromatic laser:

$$i\partial_z E = -\frac{1}{2\beta}(\partial_{xx}+\partial_{yy})E + V(r)E + g\,|E|^2 E$$

with $V(r) = -\beta\,\Delta\varepsilon(r)/(2\varepsilon)$ with rectangular cross section and $g = -\beta\,\chi^{(3)}/(2\varepsilon)$

For growing light power, superfluidity visible:

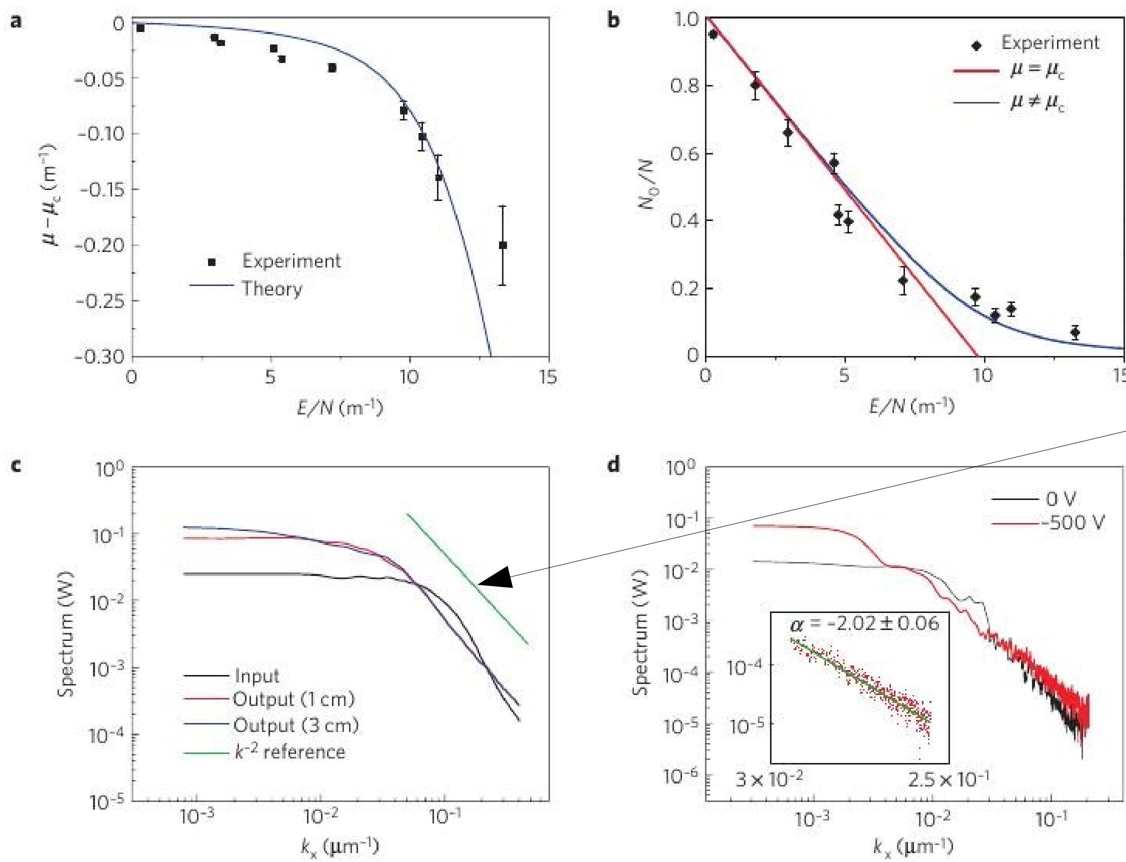
- Intensity modulation disappears
- Suppression of opto-mechanical force



<u>Fused silica slab:</u>
deformation almost in the μm range

P.-E. Larré and IC, PRA (2015)

Condensation of classical waves

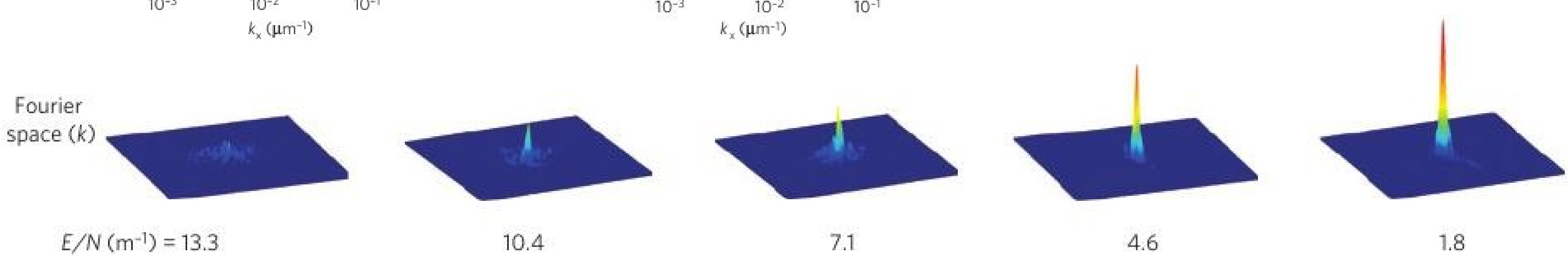


Initial noisy configuration

Evolution during propagation
→ classical GPE

Thermalizes to classical distribution with Rayleigh-Jeans $1/k^2$ high-momentum tail

Strong cut-off dependence
What about quantum effects?
How to recover Planckian?



Sun et al., Nature Physics 8, 470 (2012)

How to include quantum fluctuations beyond MF

Requires going beyond monochromatic beam and explicitly including physical time

Gross-Pitaevskii-like eq. for propagation of quasi-monochromatic field

$$i\frac{\partial E}{\partial z} = -\frac{1}{2\beta_0}\left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2}\right) - \frac{1}{2D_0}\frac{\partial^2 E}{\partial t^2} + V(r)E + g|E|^2 E$$

Propagation coordinate z → time

Physical time → extra spatial variable, dispersion D_0 → temporal mass

Upon quantization → conservative many-body evolution in z: $i\frac{d}{dz}|\psi\rangle = H|\psi\rangle$

with $H = N\iiint dx\ dy\ dt \left[\frac{1}{2\beta_0}\nabla\hat{E}^\dagger\nabla\hat{E} - \frac{D_0}{2}\frac{\partial\hat{E}^\dagger}{\partial t}\frac{\partial\hat{E}}{\partial t} + V\hat{E}^\dagger\hat{E} + \hat{E}^\dagger\hat{E}^\dagger\hat{E}\hat{E}\right]$

Same z commutator $\left[\hat{E}(x.y,t,z), \hat{E}^+(x',y',t',z)\right] = \frac{c\hbar\omega_0 v_0}{\epsilon}\delta(x-x')\,\delta(y-y')\,\delta(t-t')$

P.-E. Larré and IC, arXiv:1412.5405. See also old work by Lai and Haus, PRA 1989

Dynamical Casimir emission at quantum quench (I)

Air / nonlinear medium interface

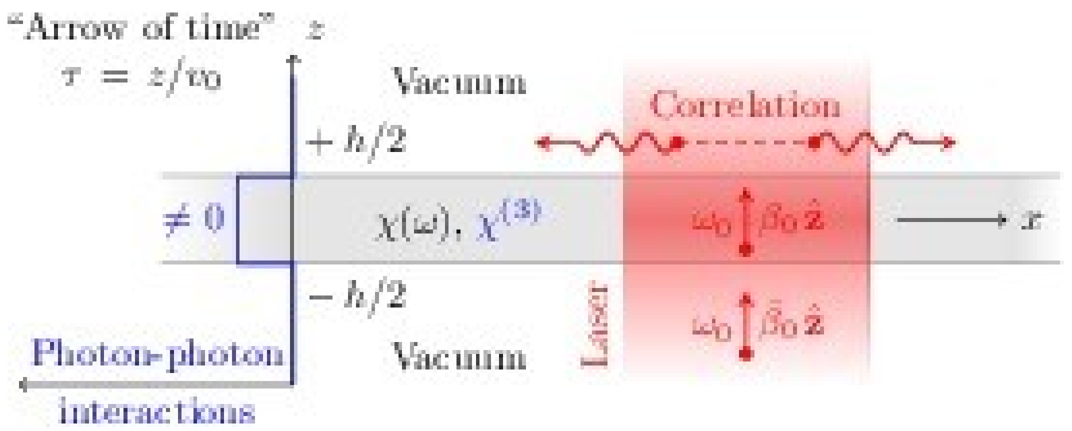
→ sudden jump in interaction constant when moving along z

Monochromatic wave

Normal incidence

Weakly nonlinear medium

→ Weakly interacting Bose gas at rest



Propagation along z

→ conservative quantum dynamics

Mismatch of Bogoliubov ground state in air and in nonlinear medium

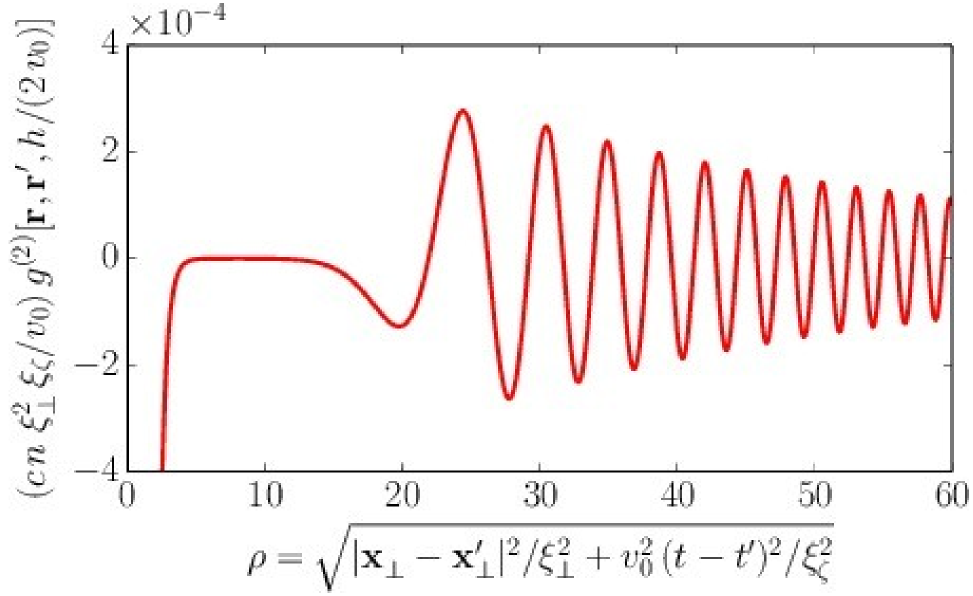
→ emission of phonon pairs at opposite k on top of fluid of light

P.-E. Larré and IC, arXiv:1412.5405

Dynamical Casimir emission at quantum quench (II)

Observables:

- Far-field
  → correlated pairs of photons at opposite angles
- Near-field
  → peculiar pattern of intensity noise correlations



May simulate dynamical Casimir effect & fluctuations in early universe

First peak propagates at the speed of sound c_s
Value of c_s different in spatial (xy) and temporal (t) directions

Experiments on-going to measure c_s^{xy} (Heriot-Watt) and c_s^{t} (Trento)
Quantum dynamics even more interesting in strongly nonlinear media

P.-E. Larré and IC, arXiv:1412.5405

A quite generic *quantum simulator*

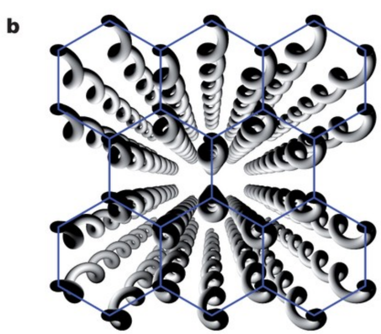
Quantum many-body evolution in *z:*

$$i\frac{d}{dz}|\psi\rangle = H|\psi\rangle \quad \text{with:} \quad H = N\iiint dx\; dy\; dt \left[\frac{1}{2\beta_0}\nabla\hat{E}^\dagger\nabla\hat{E} - \frac{D_0}{2}\frac{\partial\hat{E}^\dagger}{\partial t}\frac{\partial\hat{E}}{\partial t} + V\hat{E}^\dagger\hat{E} + \hat{E}^\dagger\hat{E}^\dagger\hat{E}\hat{E}\right]$$

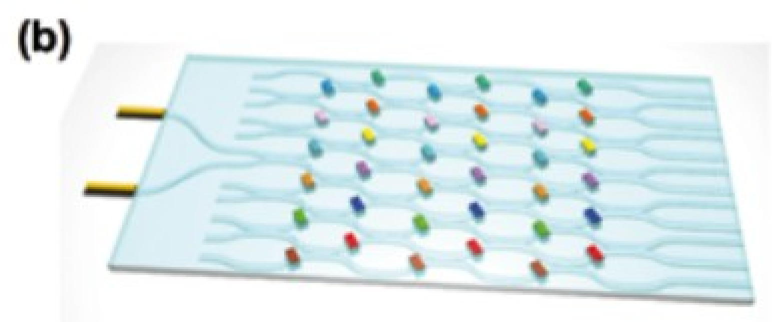
- Physical time t plays role of extra spatial coordinate
- Same z commutator: $\left[\hat{E}(x,y,t,z),\hat{E}^{+}(x',y',t',z)\right] = \frac{c\hbar\omega_0 v_0}{\epsilon}\delta(x-x')\,\delta(y-y')\,\delta(t-t')$

Clever design of V(x,y,z) → simulate wide variety of physical systems:

- Floquet topological insulators
- Arbitrary splitting/recombination of waveguides → quench of tunneling
- In addition to photonic circuit → many-body due to photon-photon interactions
- On top of moving fluid of light → simulate general relativistic QFT



Rechtsman et al., Nature 2012



P.-E. Larré and IC, arXiv:1412.5405

Part II:

Toroidal Landau levels

Berry curvature & quantum mechanics

Chang-Niu's semiclassical equations of motion:

$$\hbar\dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar\dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}}\mathcal{E}_{n,\mathbf{k}} - e\mathbf{E}\times\mathbf{\Omega}_n(\mathbf{k})$$

Can be derived from quantum Hamiltonian

$H = E_n(p) + W[r + \mathcal{A}_n(p)]$ with $W(r) = - e\, E\, r$

Similar to minimal coupling $H = e\,\Phi(r) + [p - e\,A(r)]^2 / 2\,m$ with r ↔ p exchanged

Physical position $r_{ph} = r + A_n(p)$	↔	physical momentum $p - e\,A(r)$
Berry connection $\mathcal{A}_n(p)$	↔	magnetic vector potential $A(r)$
Berry curvature $\Omega_n(p) = \mathrm{curl}_p\, \mathcal{A}_n(p)$	↔	magnetic field $B(r) = \mathrm{curl}_r\, A(r)$
band dispersion $E_n(p)$	↔	scalar potential $e\,\Phi(r)$
trap energy $W(r)$	↔	kinetic energy $p^2/2m$

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , PRL 2014
and references therein (starting from Karplus-Luttinger 1954)

Harper-Hofstadter model + harmonic trap

Magnetic flux per plaquette $\alpha = 1/q$:

- for large q, bands almost flat $E_n(p) \approx E_n$
- lowest bands have C_n=-1 and almost uniform Berry curvature $\Omega_n = a^2/2\pi\alpha$

Within single band approximation:

Momentum space magnetic Hamiltonian $H=E_n(p)+ k[r +\mathcal{A}_n(p)]^2/2$

equivalent to quantum particle in constant B: $H= e\,\Phi(r) + [p - e\,A(r)]^2 / 2\,m$

Mass fixed by harmonic trap strength k

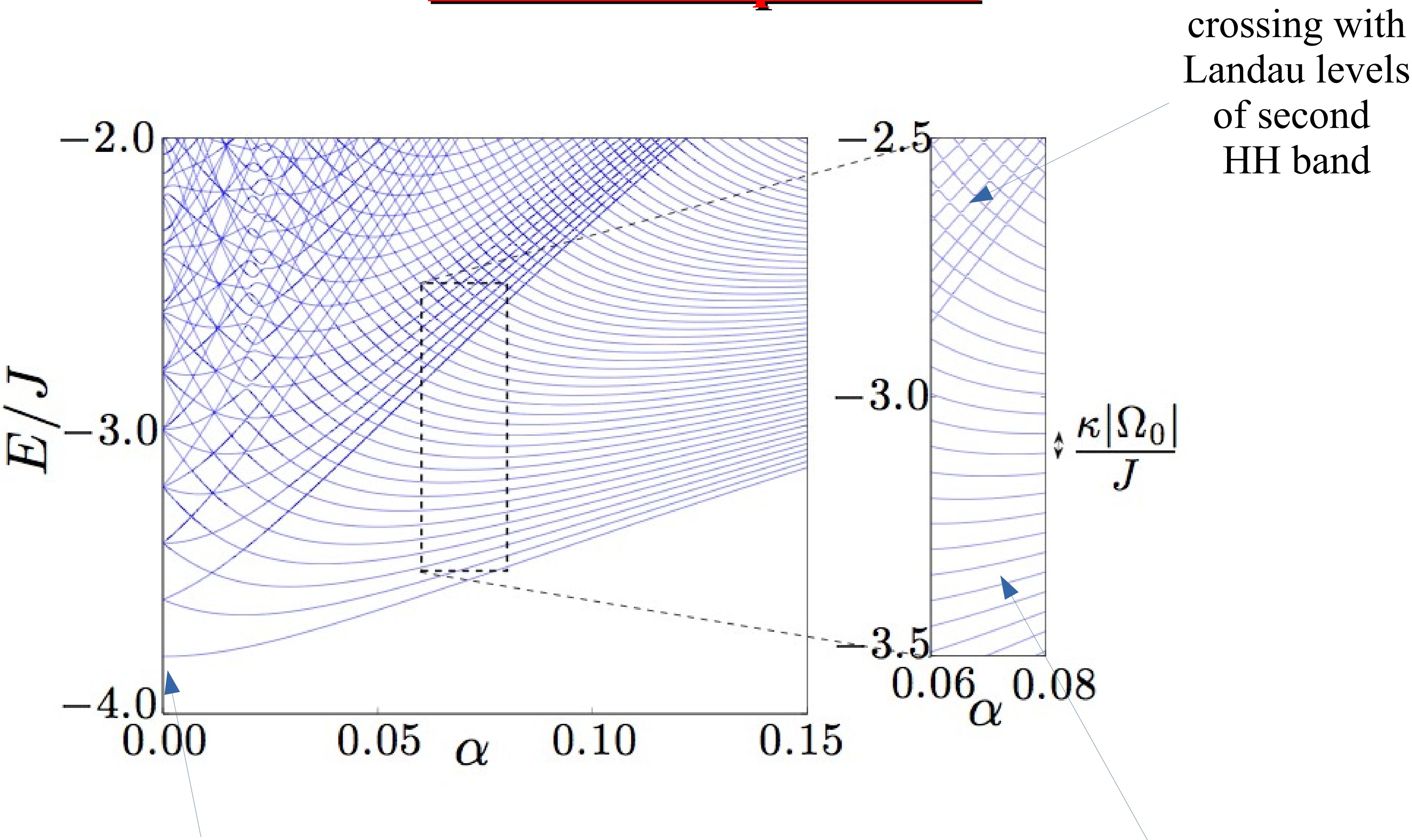
- Landau Levels spaced by "cyclotron" → $k\,|\Omega_n|$
- Global (toroidal) topology of FBZ matters!! Degeneracy of LLs reduced to $|C_n|$

Of course, if:

- Too small α / too strong trap → band too close for single band approx
- Too large α / too weak trap → effect of $E_n(p)$ important

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , PRL 2014

Numerical spectrum

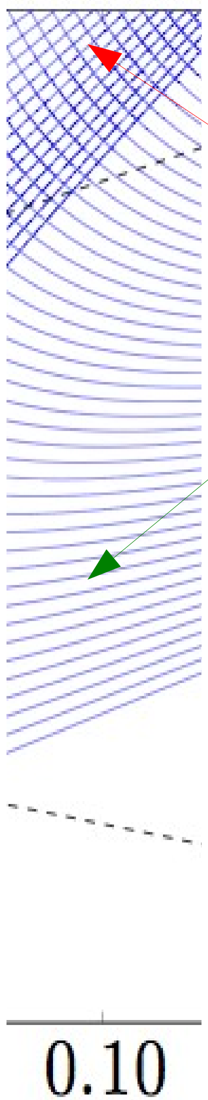


$\alpha \rightarrow 0$ harmonic trap states
(band gap too small)

Landau levels of lowest HH band

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , PRL 2014

Numerical eigenstates



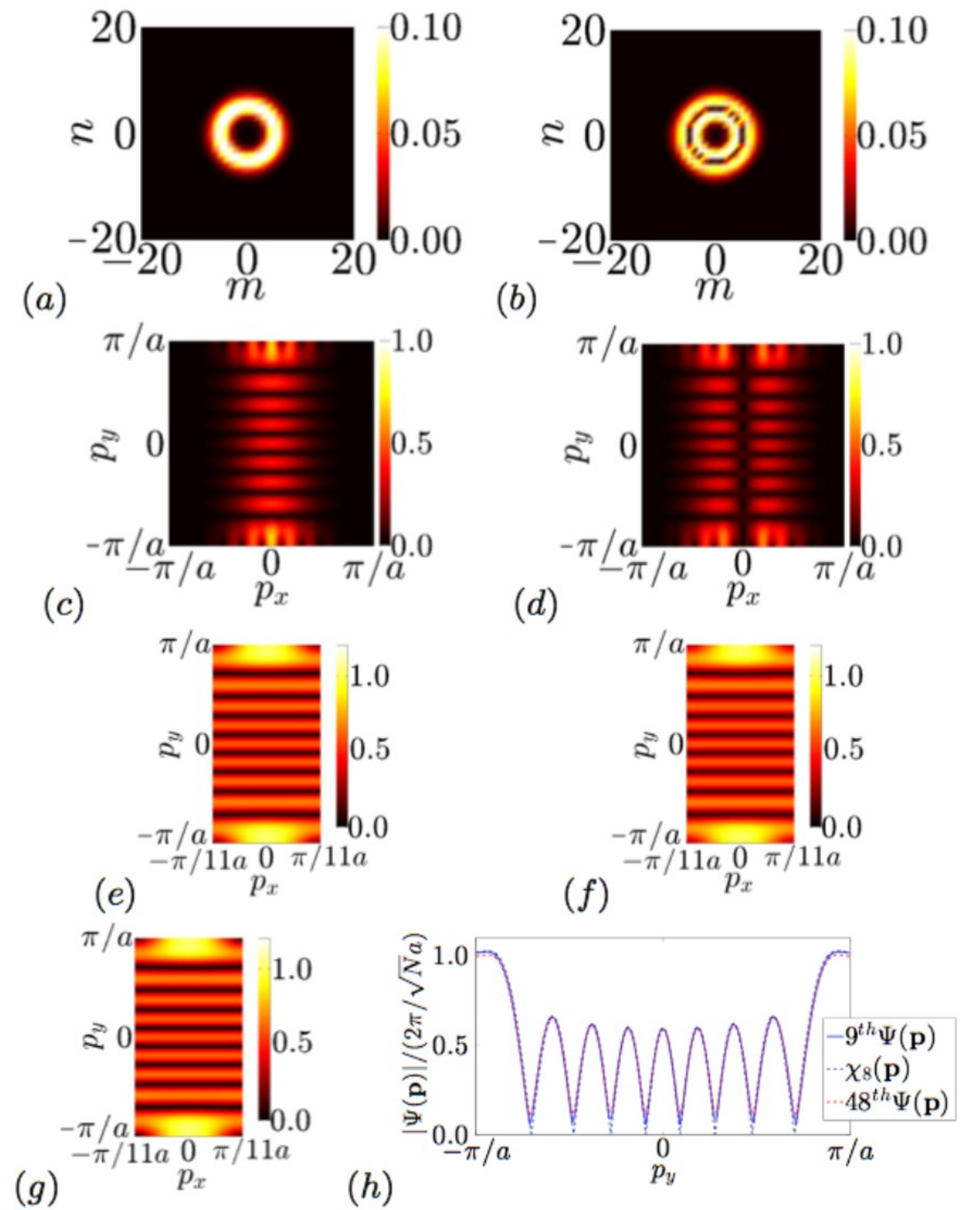
9th and 48th state for $\alpha = 1/11$

eigen-functions recover

β=8 Landau level on torus

for 1st and 2nd HH bands.

Only difference is Bloch function



Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , PRL 2014

Part IV:

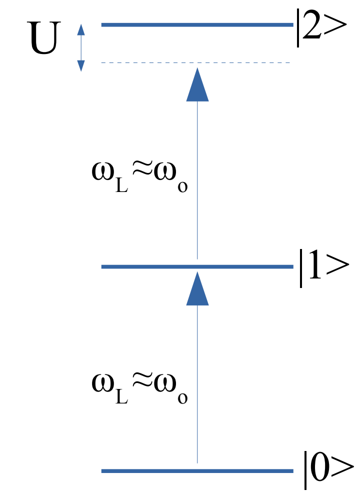
Strongly interacting photons: from Tonks-Girardeau gas to Fractional Quantum Hall states

Photon blockade

Cavity array + strong nonlinearity, e.g. via Rydberg atoms

Bose-Hubbard model:

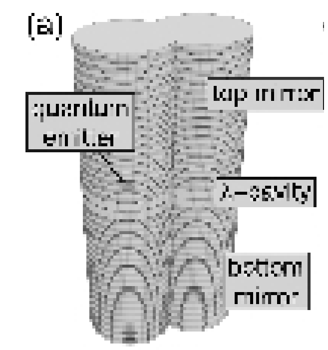
$$H_0 = \sum_i \hbar\omega_o \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar\frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$



- single-mode cavities at ω_o. Tunneling coupling J
- Polariton interactions: on-site repulsion U

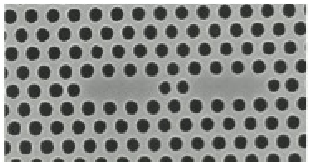
If U >> γ , J, coherent pump resonant with 0→1 transition,
but not with 1→2 transition.

Photon blockade → Effectively impenetrable photons



Coupled micropillars
de Vasconcellos et al., APL 2011

- Incident laser: coherent external driving $H_d = \sum_i F_i(t)\, \hat{b}_i + h.c.$



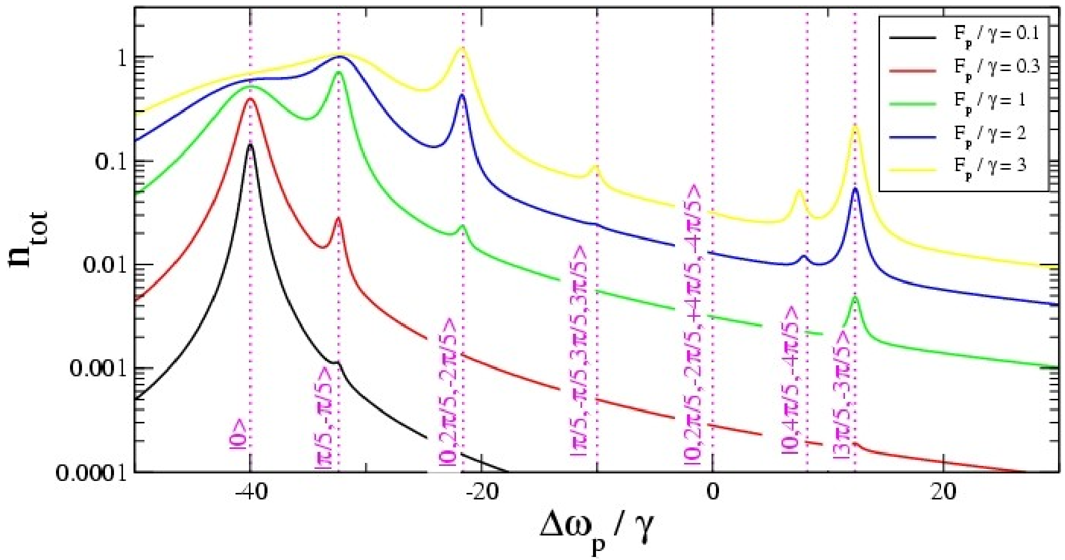
Photonic crystal Cavities
Majumdar et al., arXiv:1201.6244

- Weak losses γ << J, U → Lindblad terms in master eq.
  determine non-equilibrium steady-state
- Strong number fluctuations → dramatic effect on MI, but....

Impenetrable “fermionized” photons in 1D necklaces

Many-body eigenstates of
Tonks-Girardeau gas
of impenetrable photons

Coherent pump
selectively addresses
specific many-body states



Transmission spectrum as a function pump frequency for fixed pump intensity:

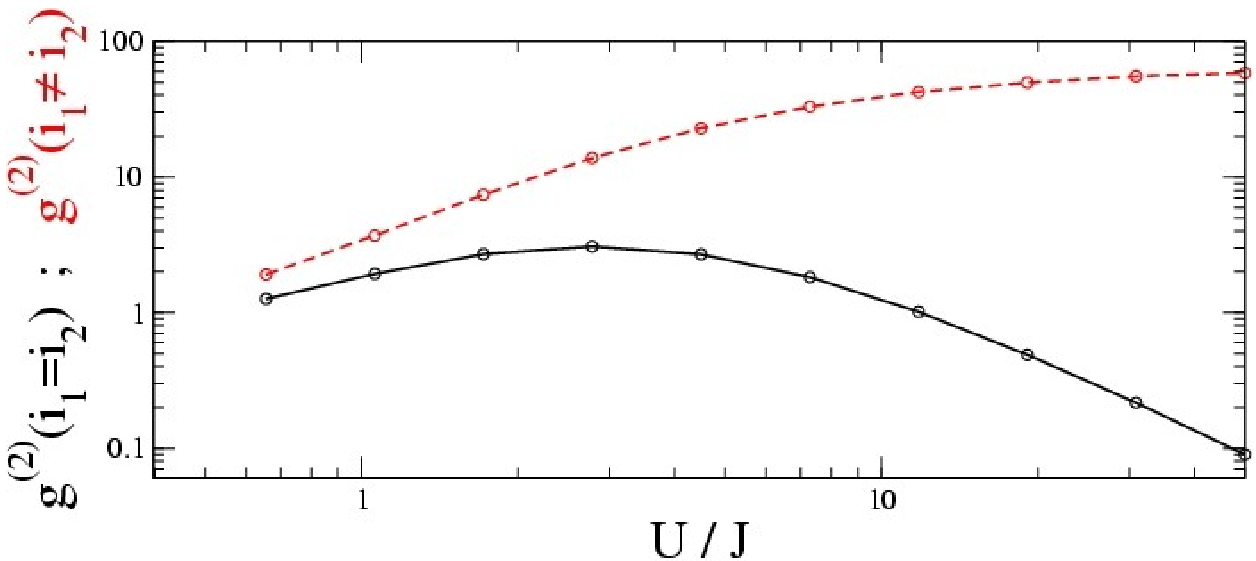
- each peak corresponds to a Tonks-Girardeau many-body state $|q_1,q_2,q_3...>$
- q_i quantized according to PBC/anti-PBC depending on N=odd/even
- $U/J >> 1$: efficient photon blockade, impenetrable photons.

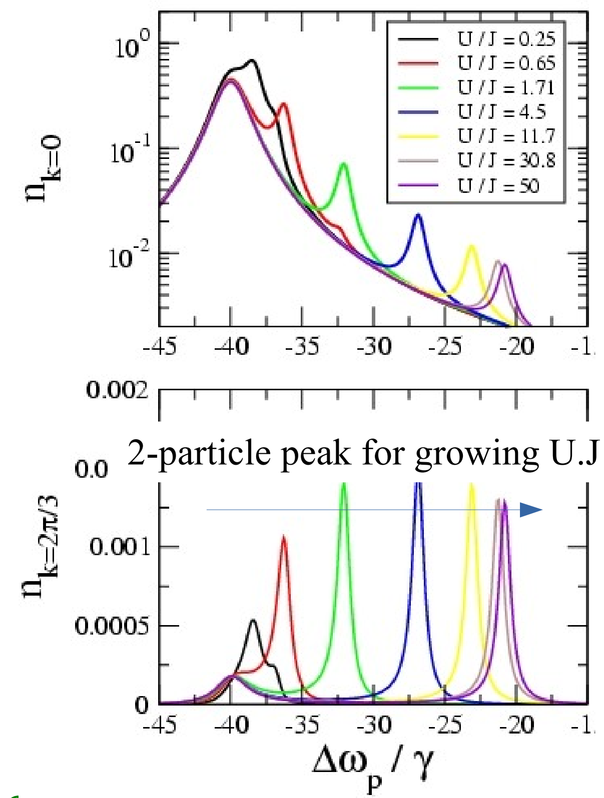
N-particle state excited by N photon transition:

- Plane wave pump with $k_p=0$: selects states of total momentum P=0
- Monochromatic pump at ω_p: resonantly excites states of many-body energy E such that $\omega_p = E / N$

IC, D. Gerace, H. E. Türeci, S. De Liberato, C. Ciuti, A. Imamoglu, PRL **103**, 033601 (2009)
See also related work D. E. Chang et al, Nature Physics (2008)

State tomography from emission statistics





Finite U/J, pump laser tuned on two-photon resonance

- intensity correlation between the emission from cavities i_1, i_2
- at large U/γ, larger probability of having N=0 or 2 photons than N=1
  - low U<<J: bunched emission for all pairs of i_1, i_2
  - large U>>J: antibunched emission from a single site
    positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.

Photon blockade + synthetic gauge field = QHE for light

Bose-Hubbard model: $$H_0 = \sum_i \hbar\omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j e^{i\varphi_{ij}} + \hbar \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$$

gauge field gives phase in hopping terms

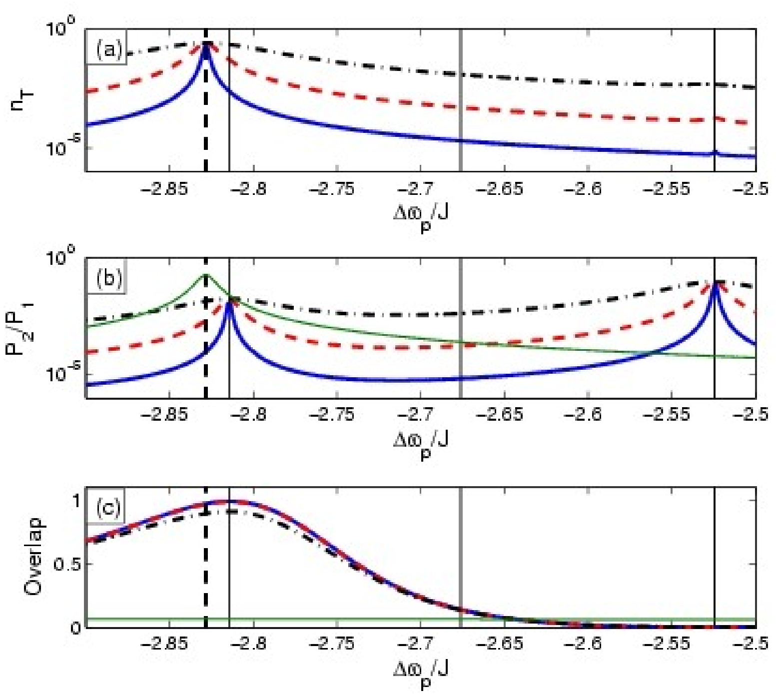
with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$\psi_l(z_1, ..., z_N) = \mathcal{N}_L F_{\rm CM}^{(l)}(Z) e^{-\pi\alpha \sum_i y_i^2} \times \prod_{i<j}^{N} \left(\vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \left(\frac{z_i - z_j}{L} \middle| i \right) \right)^2$$

- no need for adiabatic following, etc....



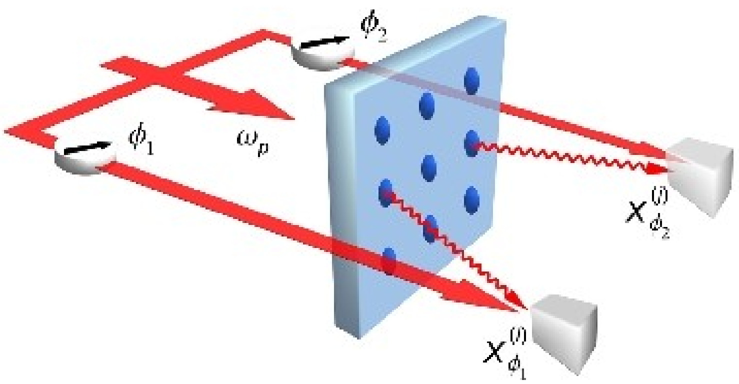
R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)
See also related work by Cho, Angelakis, Bose, PRL 2008; Hafezi et al. NJP 2013; arXiv:1308.0225

Tomography of FQH states

Homodyne detection of secondary emission

→ info on many-body wavefunction

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$



Note: optical signal gauge dependent,

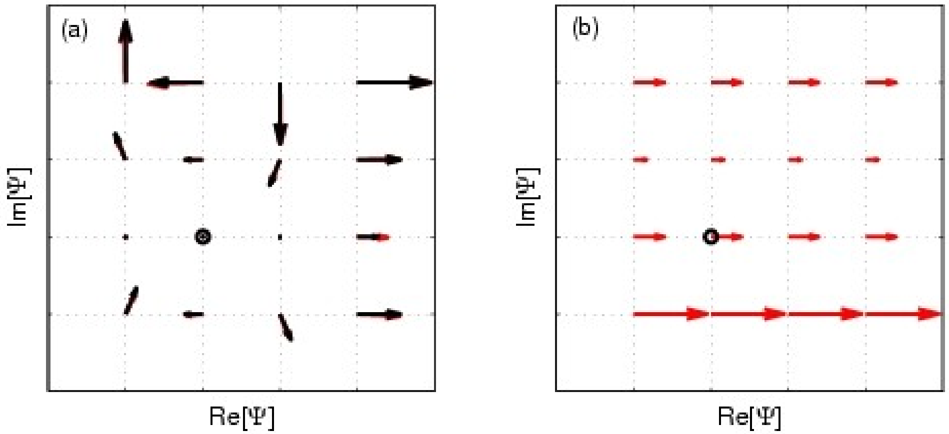
optical phase matters !

Non-trivial structure of Laughlin state

compared to non-interacting photons

Laughlin

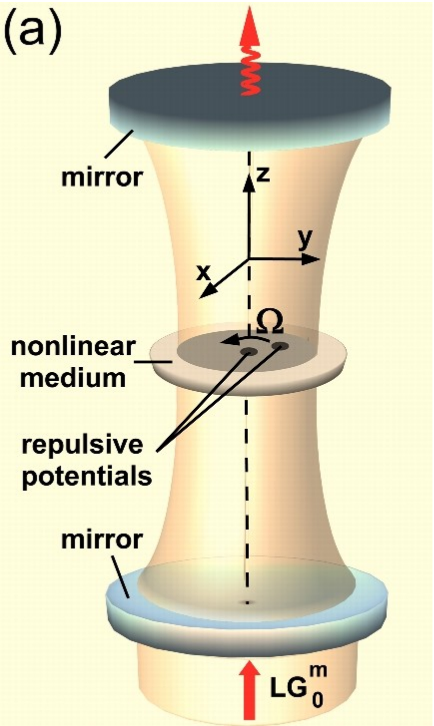
Landau (non-interacting)



R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)

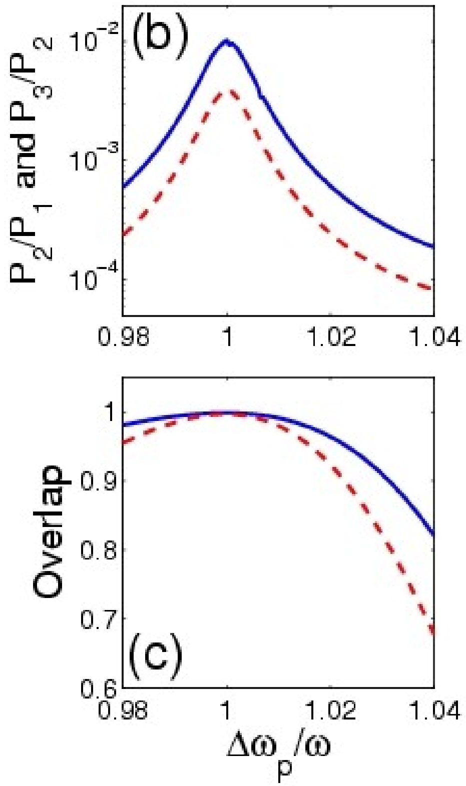
A simpler design: rotating photon fluids

Rotating system at angular speed Ω. No need for cavity array



same form → Coriolis $F_c = -2m\Omega \times v$

→ Lorentz $F_L = e\, v \times B$



Rotating photon gas injected by LG pump

with finite orbital angular momentum

Strong repuls. interact., e.g. layer of Rydberg atoms

Resonant peak in transmission due to Laughlin state:

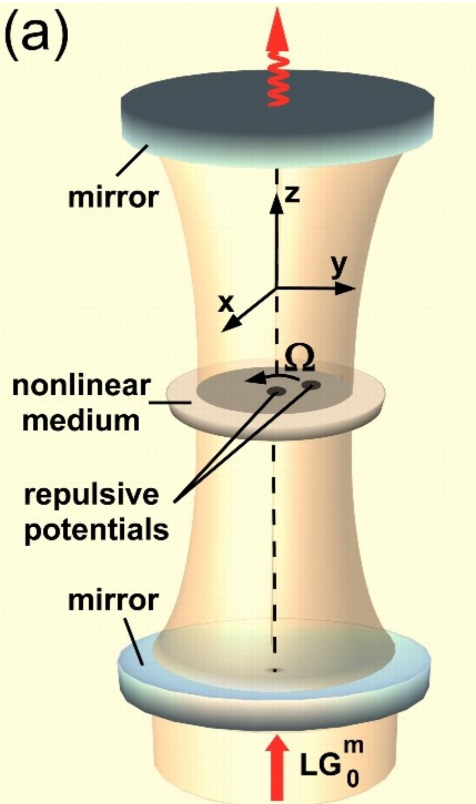
$$\psi(z_1,\ldots,z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i<j} (z_i - z_j)^2$$

Overlap measured from quadrature noise of transmitted light

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$

R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

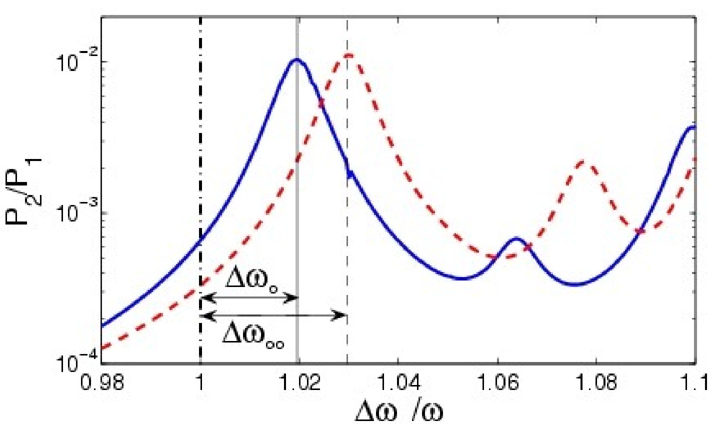
Anyonic braiding phase

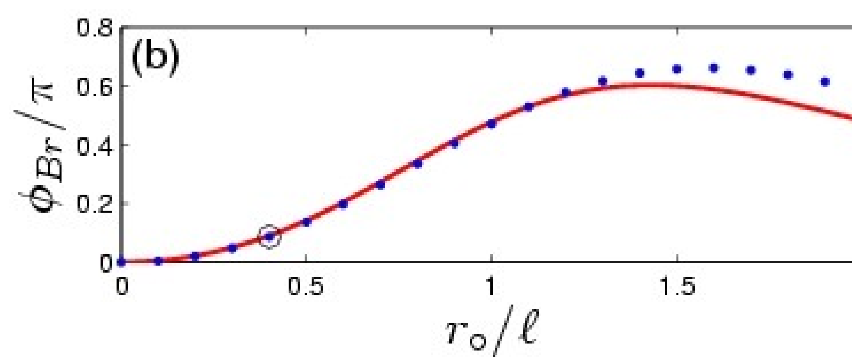


- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
  → create quasi-hole excitation in quantum Hall liquid
  → position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase ϕ_{Br}
  when positions swapped during braiding
- Berry phase extracted from shift of transmission resonance
  while repulsive potential moved with period T_{rot} along circle

$$\phi_{Br} \equiv (\Delta\omega_{oo} - \Delta\omega_{o})\, T_{rot} \quad [2\pi]$$

- so far, method restricted to low particle number





R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070