Photonic gauge field as emerged from dynamic modulation: recent advances

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Outline

- Dynamic modulation approach for an effective magnetic field for photons: a brief review

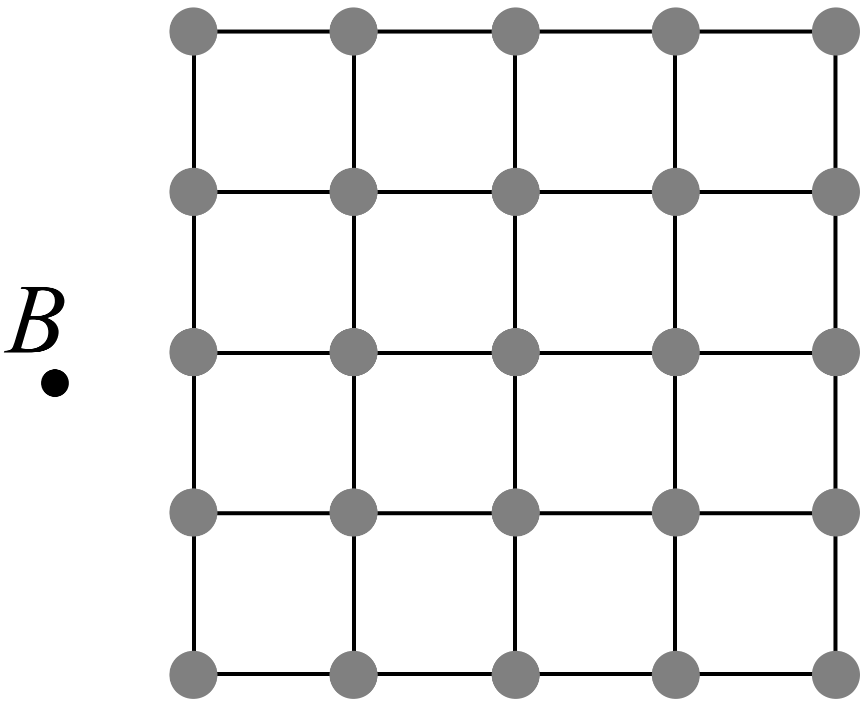
- Novel gauge potential effects
    - Negative refraction
    - Gauge-potential waveguides
    - Dynamic localization in three dimension

- Beyond rotating-wave approximation

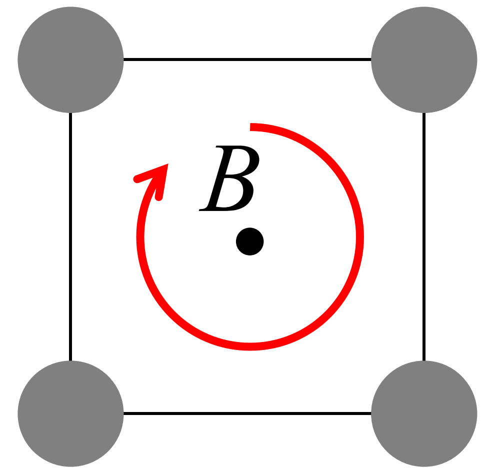
- Resonator-free implementation of photonic gauge potential

Electron on a lattice

Electron hopping on a tight-binding lattice



Single unit cell

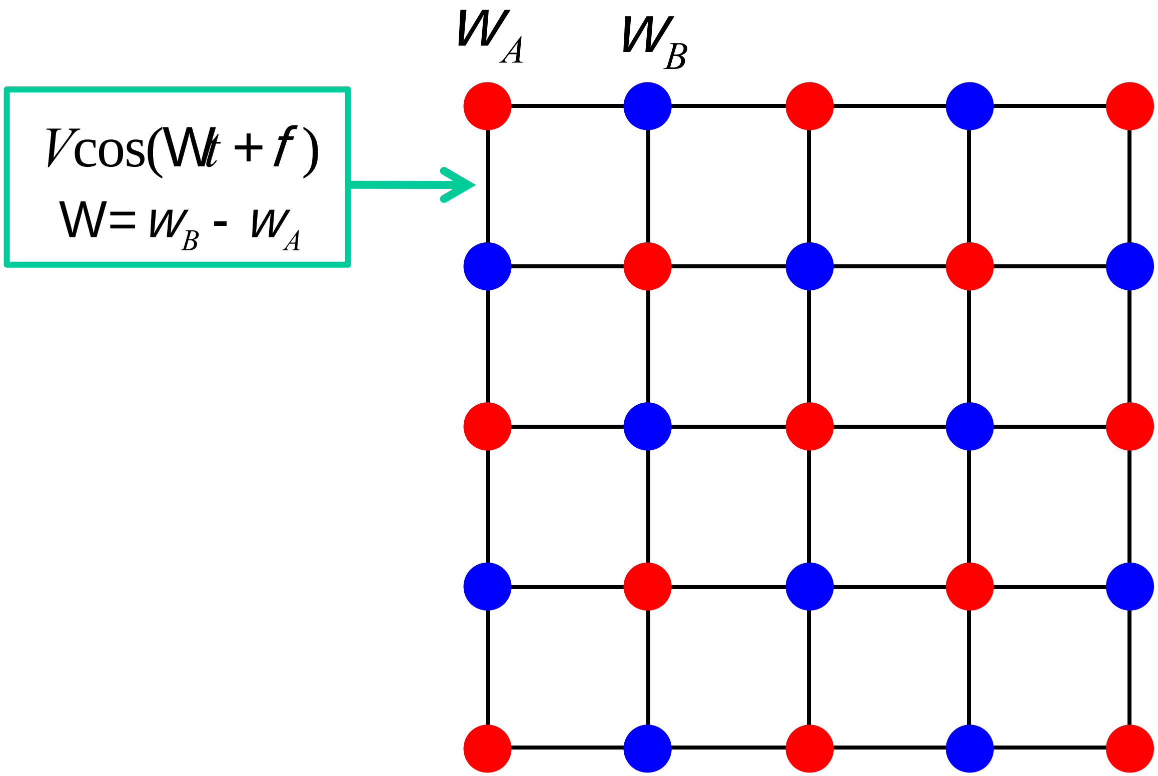


$$\phi = \oint dl \cdot A = \iint ds \cdot B$$

Magnetic field manifests in terms of a non-reciprocal round-trip phase as an electron hops along the edge of a unit cell.

Gauge field for photon: dynamic modulation approach

$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i + V\cos\left(\Omega t + \phi_{ij}\right) \sum_{\langle ij \rangle} \left(a_i^+ b_j + b_j^+ a_i\right)$$



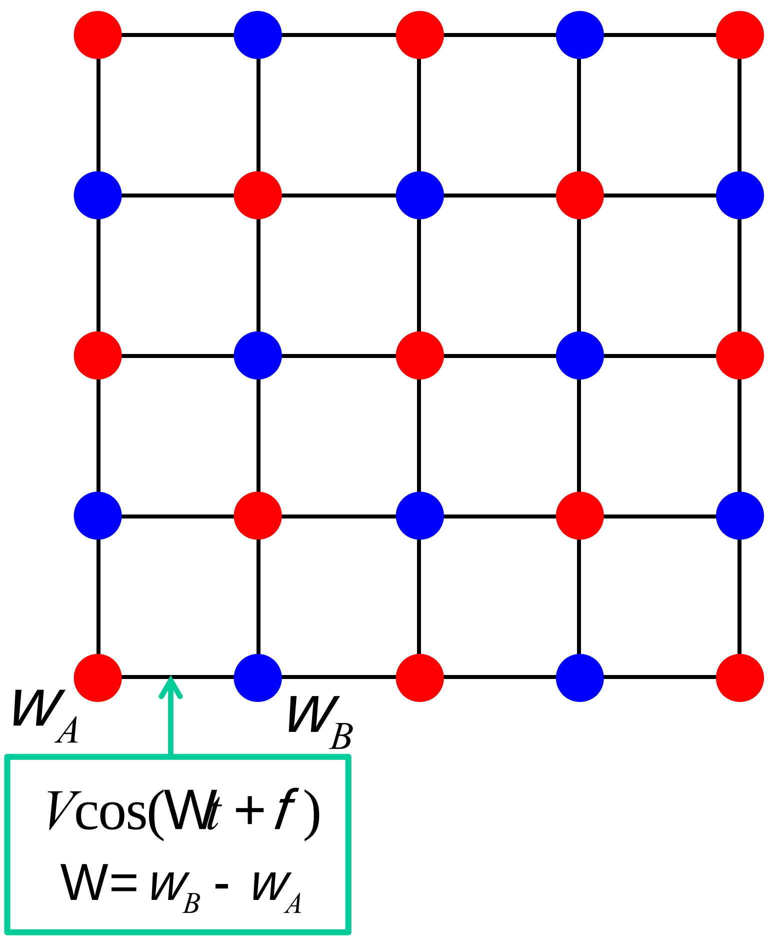
K. Fang, Z. Yu and S. Fan, Nature Photonics 6, 782 (2012).

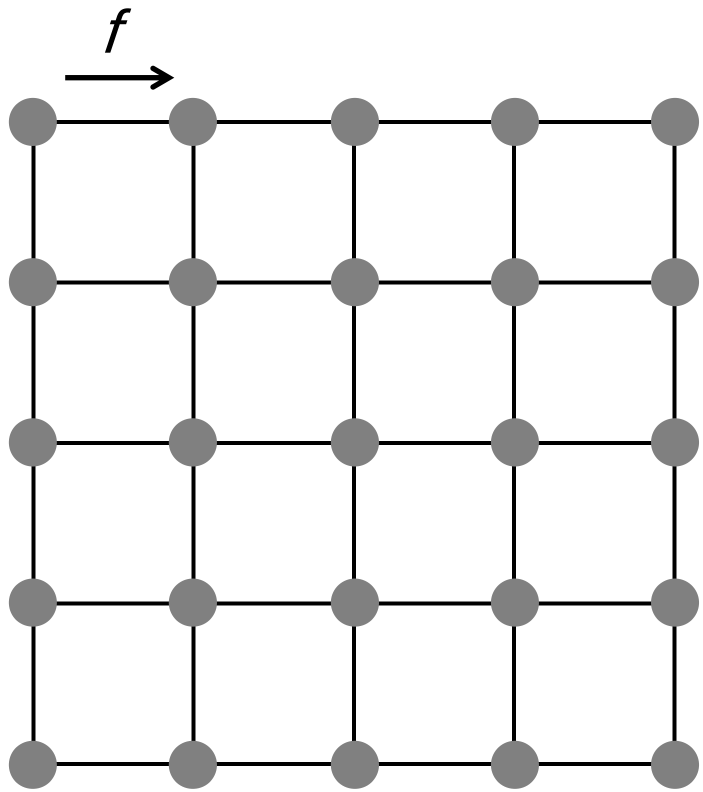
See also M. Hafezi et al, Nature Physics 7, 907 (2011);
R. O. Umucallar and I. Carusotto, Physical Review A 84, 043804 (2011).

With rotating wave approximation

$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i + V\cos\left(\Omega t + \phi_{ij}\right)\sum_{\langle ij\rangle}\left(a_i^+ b_j + b_j^+ a_i\right)$$

$$H = \sum_{\langle ij\rangle}\left(V e^{i\phi_{ij}} c_i^+ c_j + V e^{-i\phi_{ij}} c_j^+ c_i\right)$$

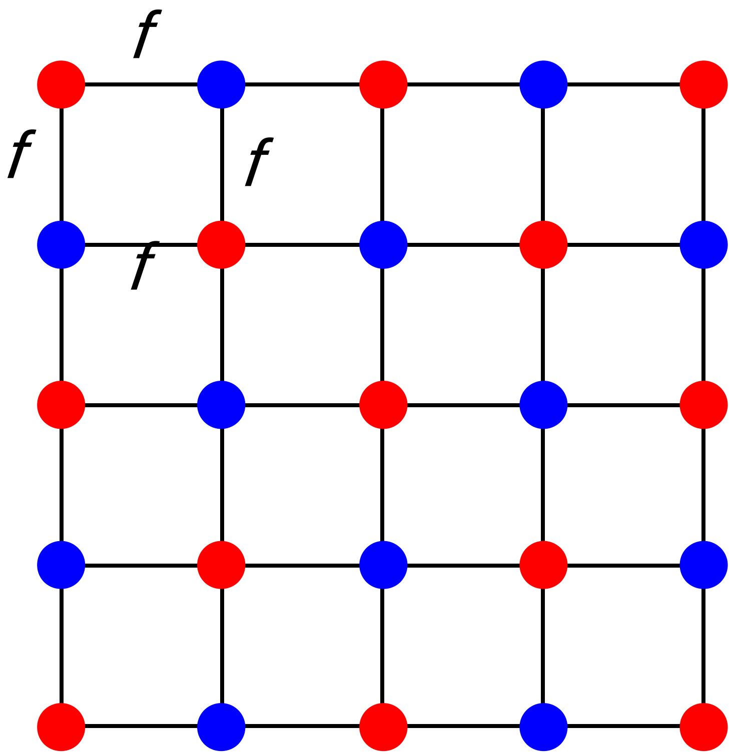




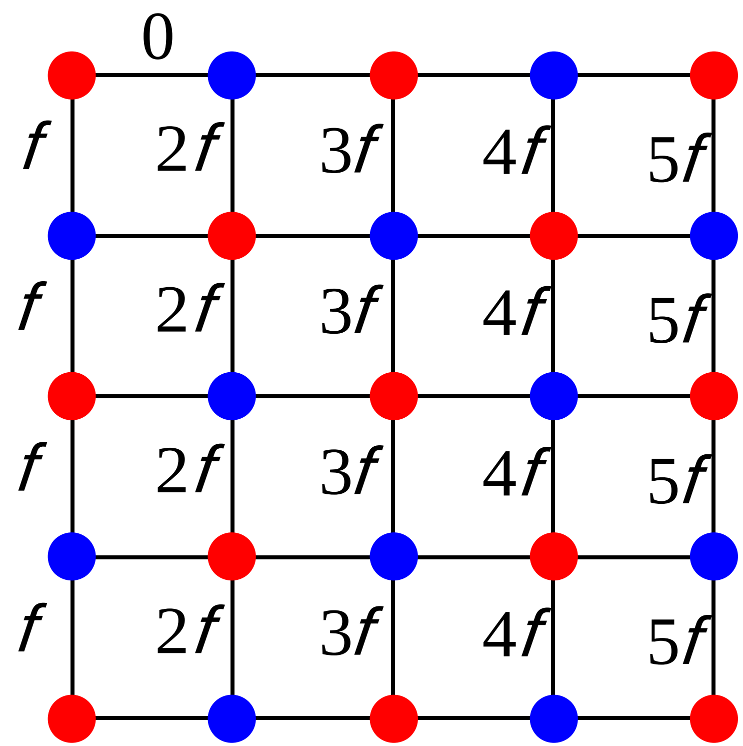
$$\int dl \cdot A_{ij} = \phi_{ij}$$

Uniform effective magnetic field

$B = 0$

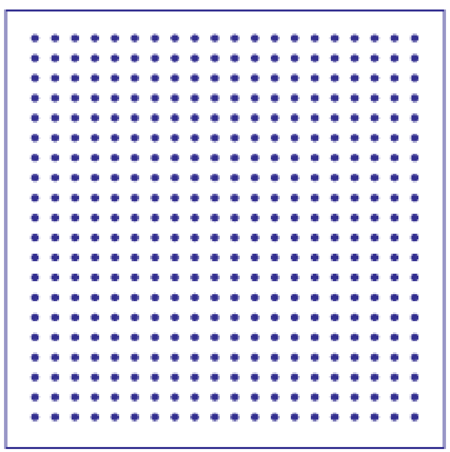


$B \neq 0$



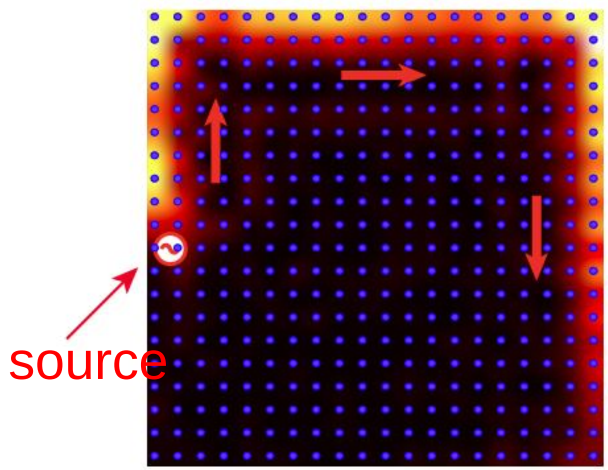
Dynamically induced one-way edge mode

20×20 resonator lattice

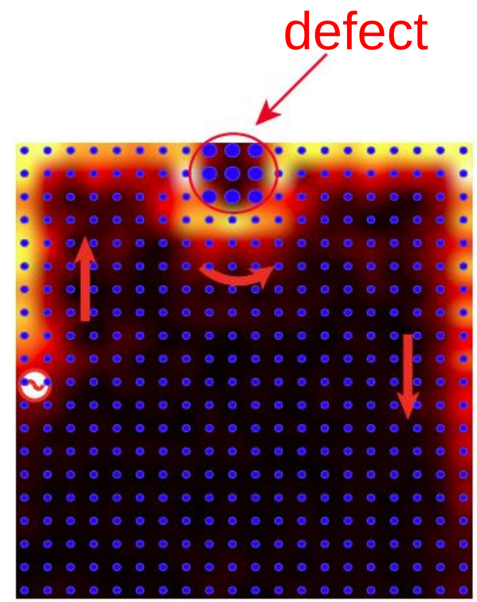


$B \neq 0$

One-way propagation



By-pass defect



K. Fang, Z. Yu and S. Fan, Nature Photonics 6, 782 (2012).

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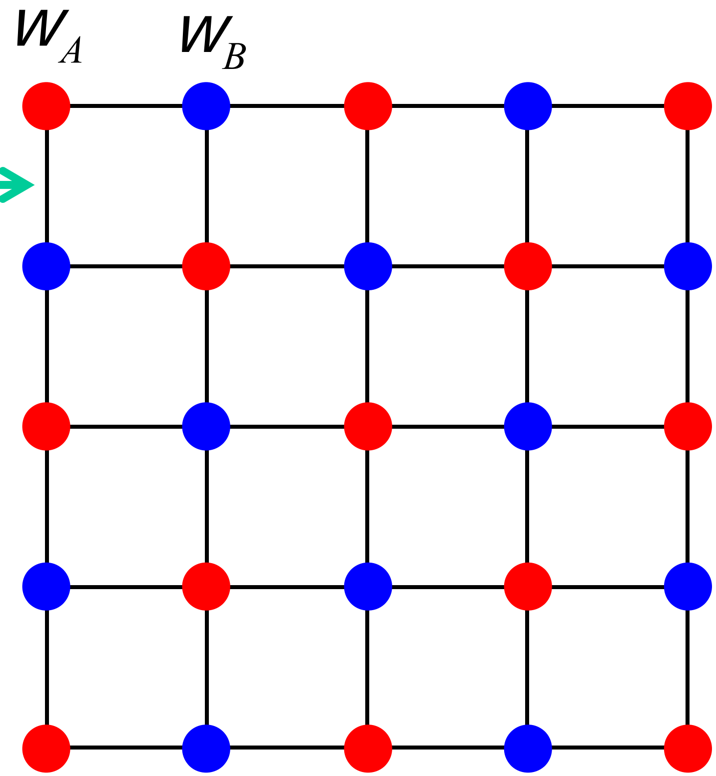
Gauge field for photon: dynamic modulation approach

$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i + V\cos\left(\Omega t + \phi_{ij}\right) \sum_{\langle ij \rangle} \left(a_i^+ b_j + b_j^+ a_i\right)$$

$$V\cos(\Omega t + \phi)$$

$$\Omega = \omega_B - \omega_A$$



K. Fang, Z. Yu and S. Fan, Nature Photonics 6, 782 (2012).

The effect of a constant gauge potential

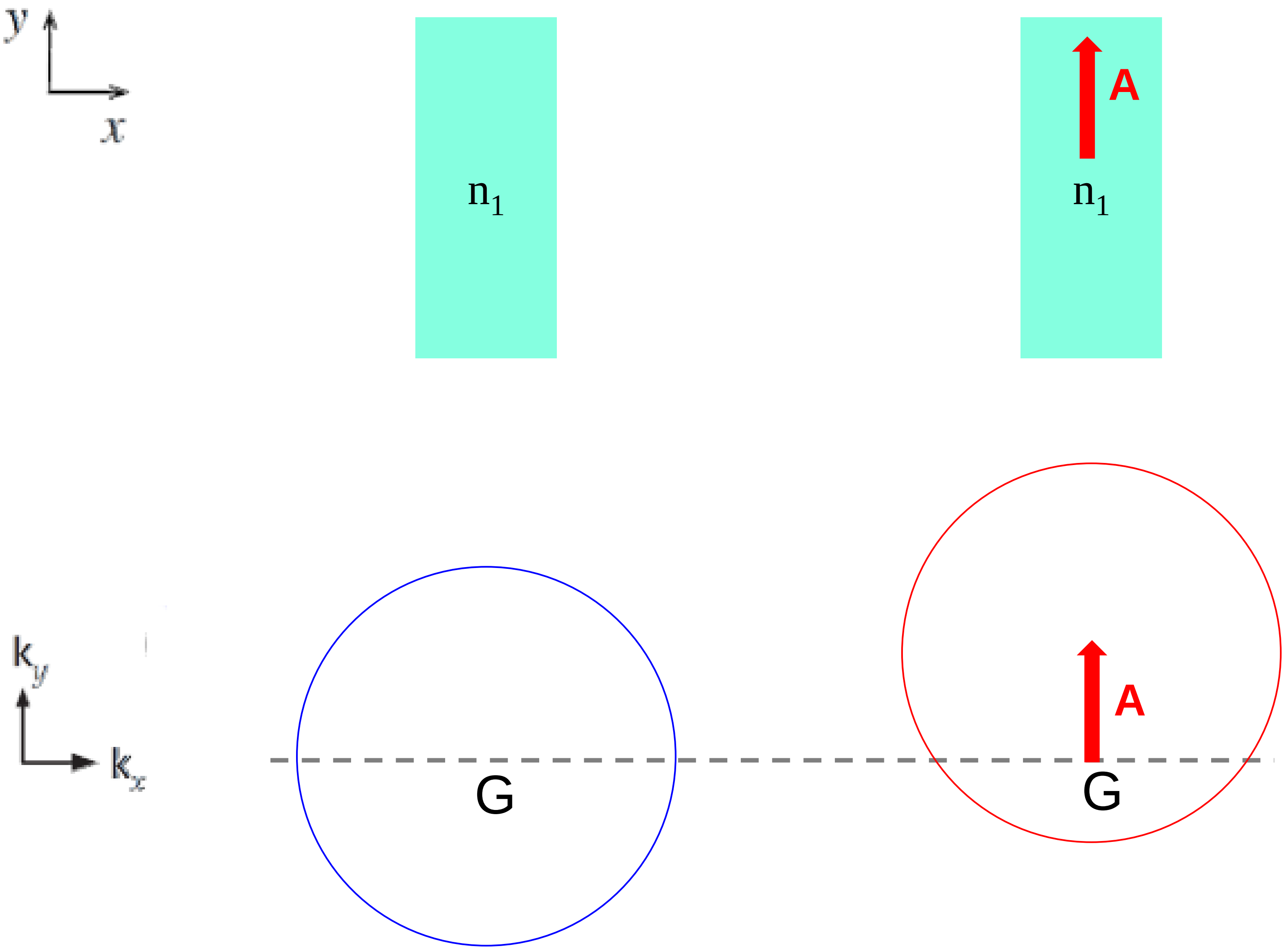
For electrons $(e = \hbar = 1)$

$$H = \frac{1}{2m}(-i\nabla)^2 + V \longrightarrow H = \frac{1}{2m}(-i\nabla - A)^2 + V$$

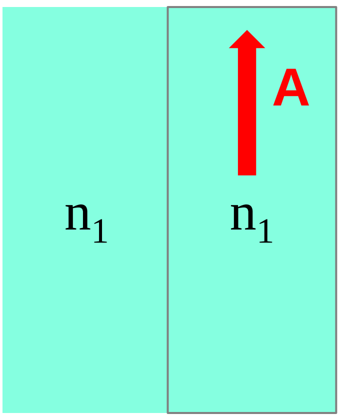
In general, a constant gauge potential shifts the wavevector

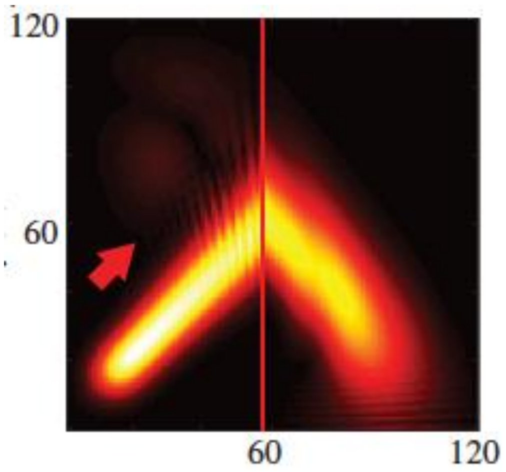
$$-i\nabla \to -i\nabla - A$$
$$w(k) \to w(k - A)$$

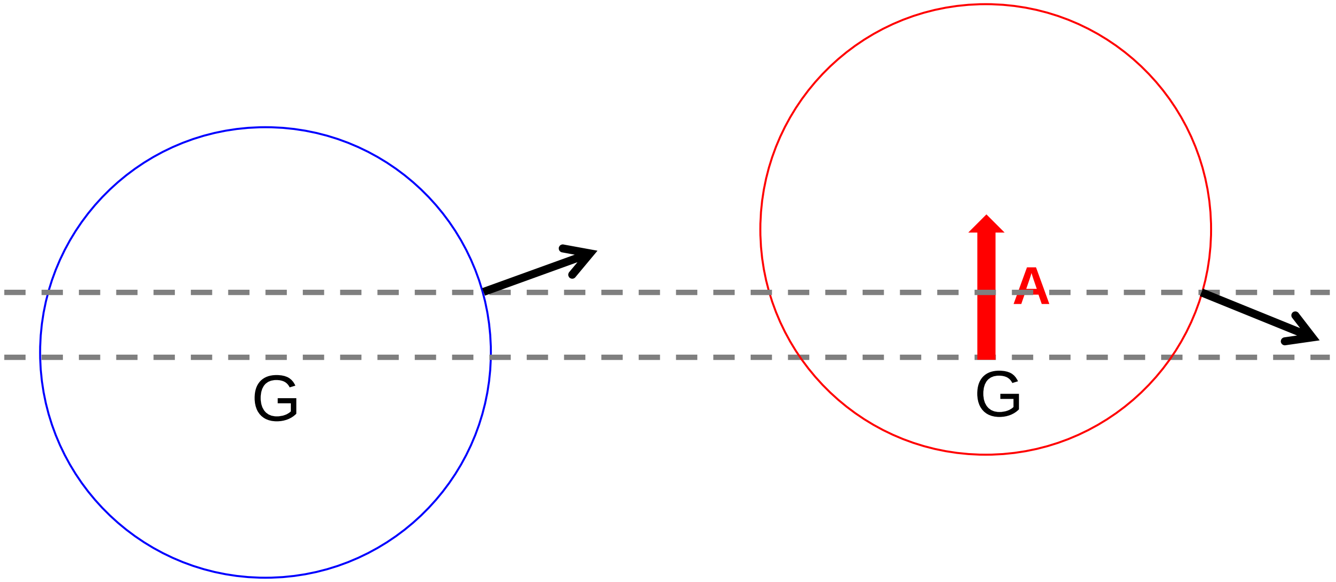
A constant gauge potential shifts the constant frequency contour



Gauge field induced negative refraction

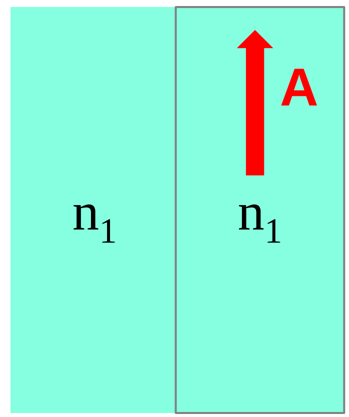


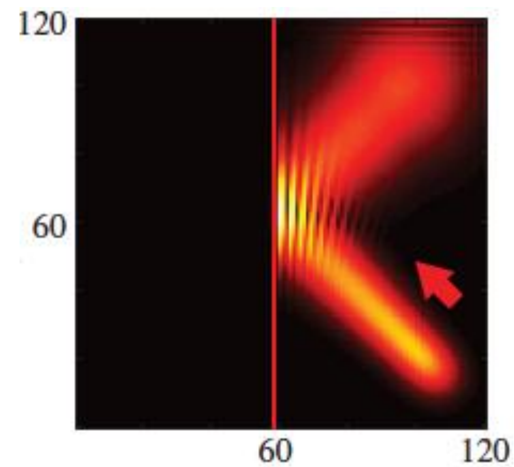


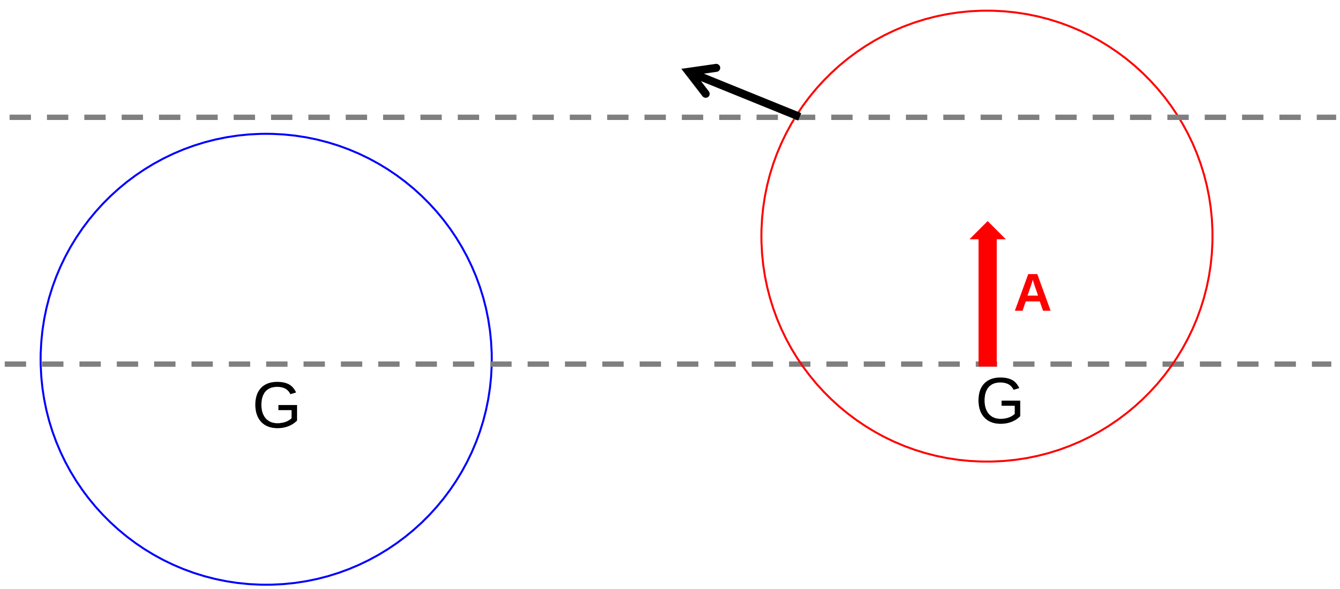


K. Fang, S. Fan, Physical Review Letters 111, 203901 (2013).

Gauge field induced total internal reflection

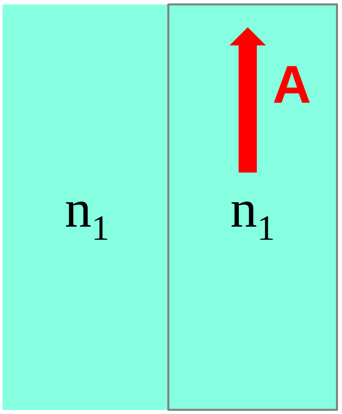


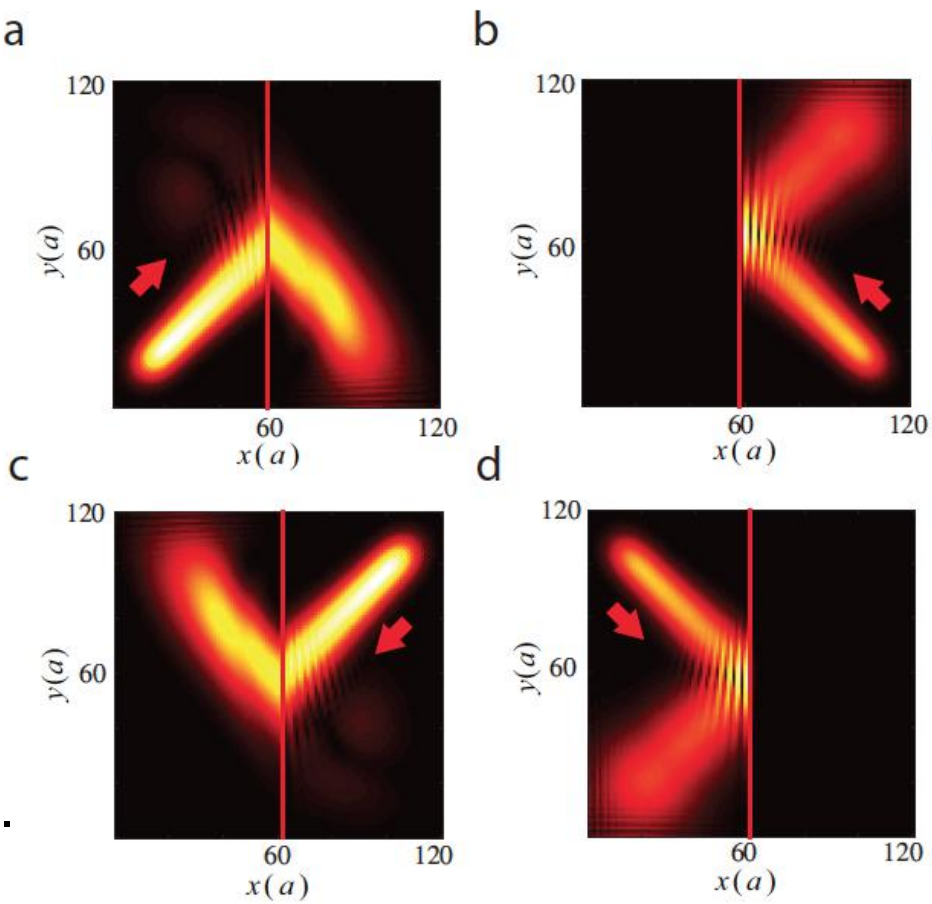




K. Fang, S. Fan, Physical Review Letters 111, 203901 (2013).

A single-interface four-port circulator

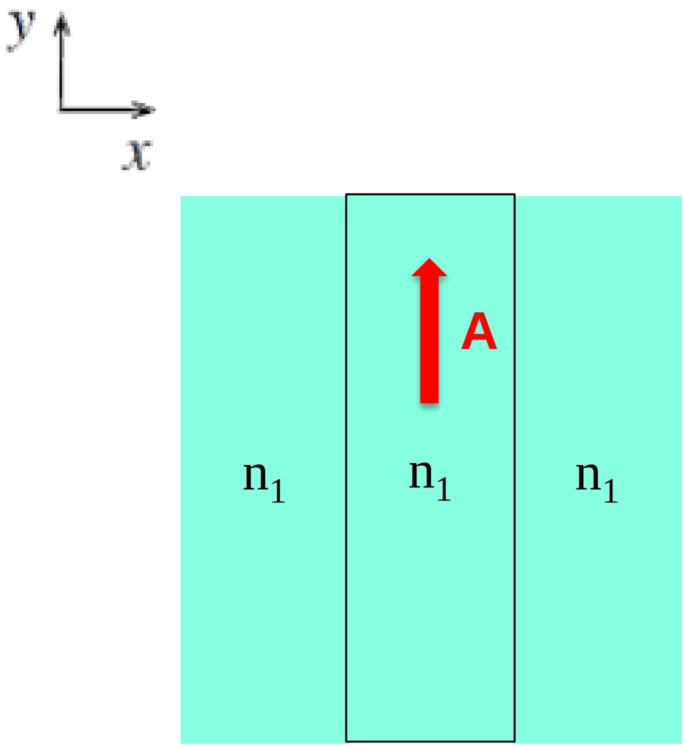


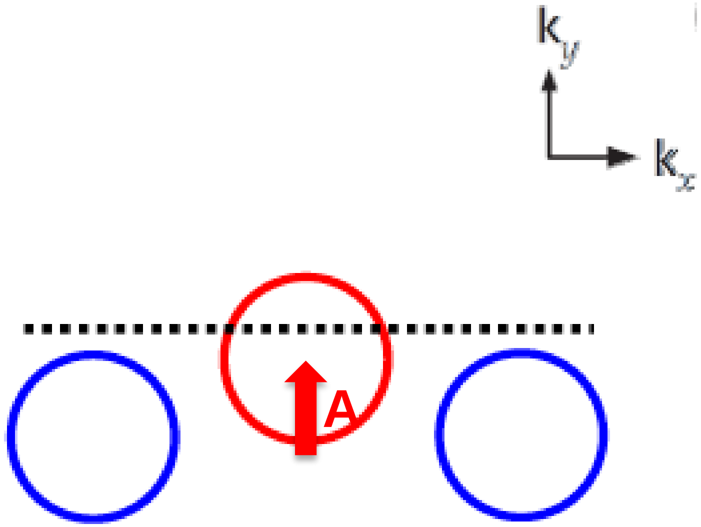


- Both regions have zero effect B-field.
- A B-field sheet at the interface.

K. Fang, S. Fan, Physical Review Letters 111, 203901 (2013).

Gauge-field waveguide for photons

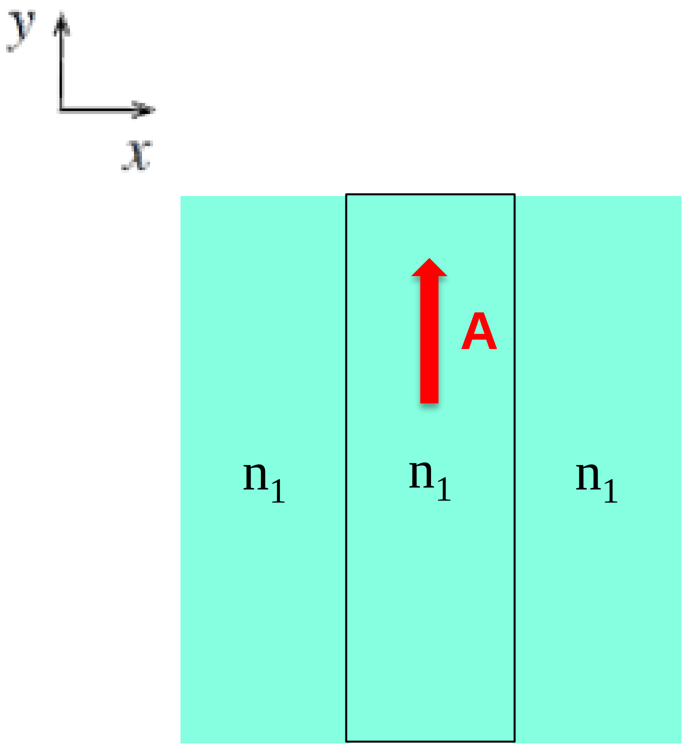




- Total Internal Reflection occurs only for forward going waves.

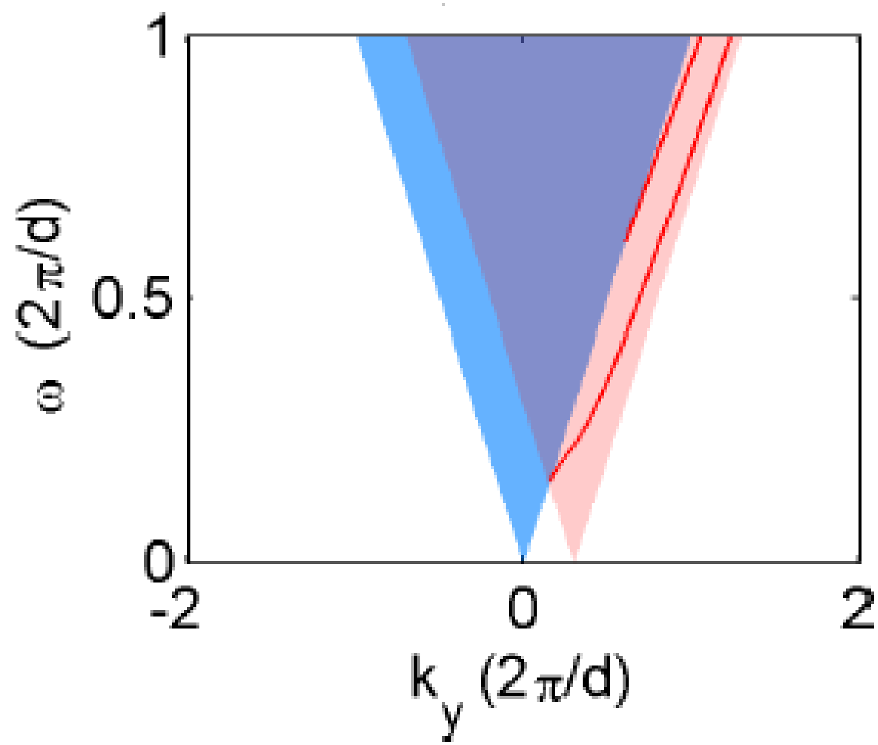
Q. Lin and S. Fan, Physical Review X 4, 031031 (2014).

A novel one-way waveguide



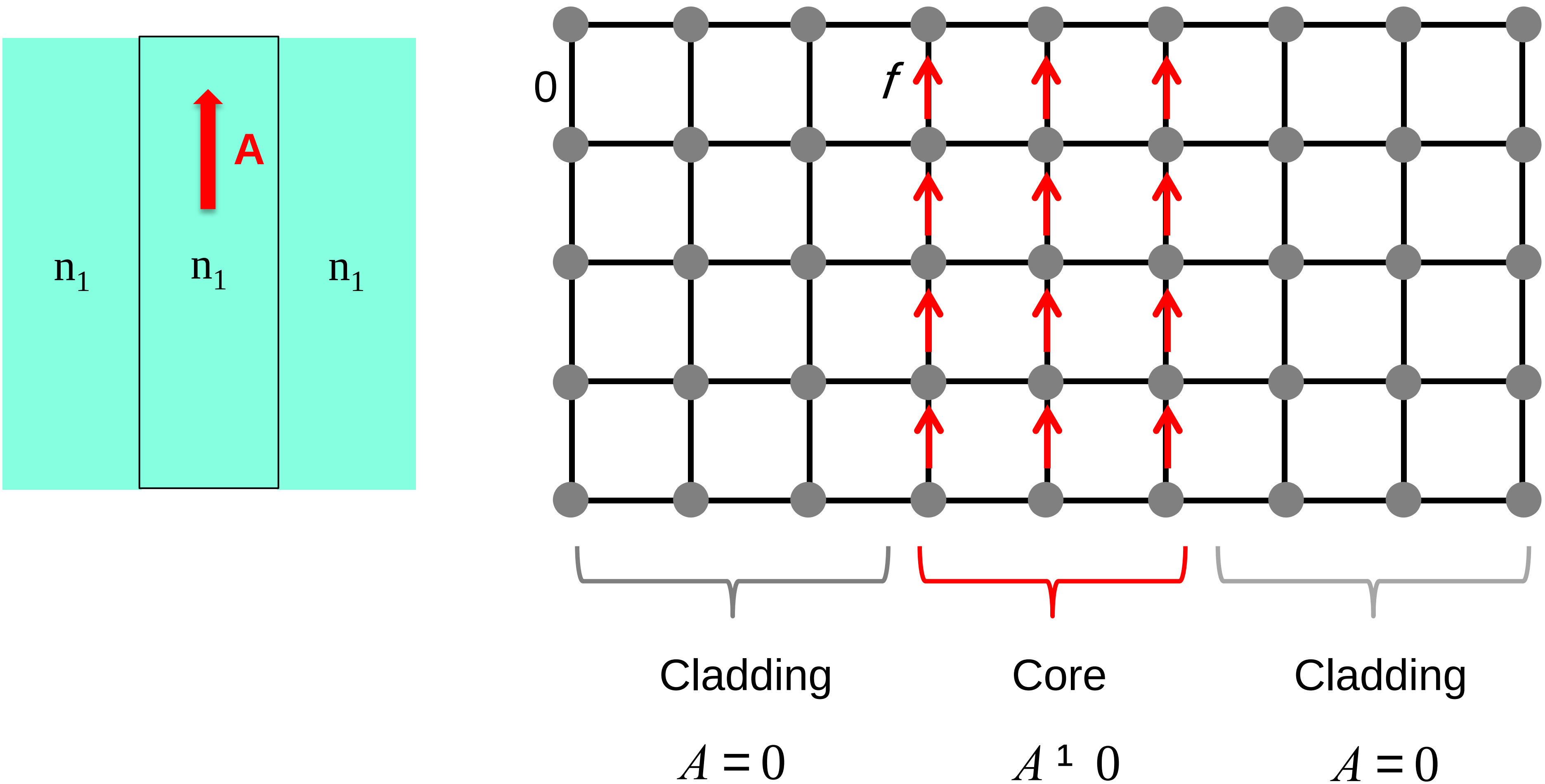
Light cone of the cladding

Light cone of the core



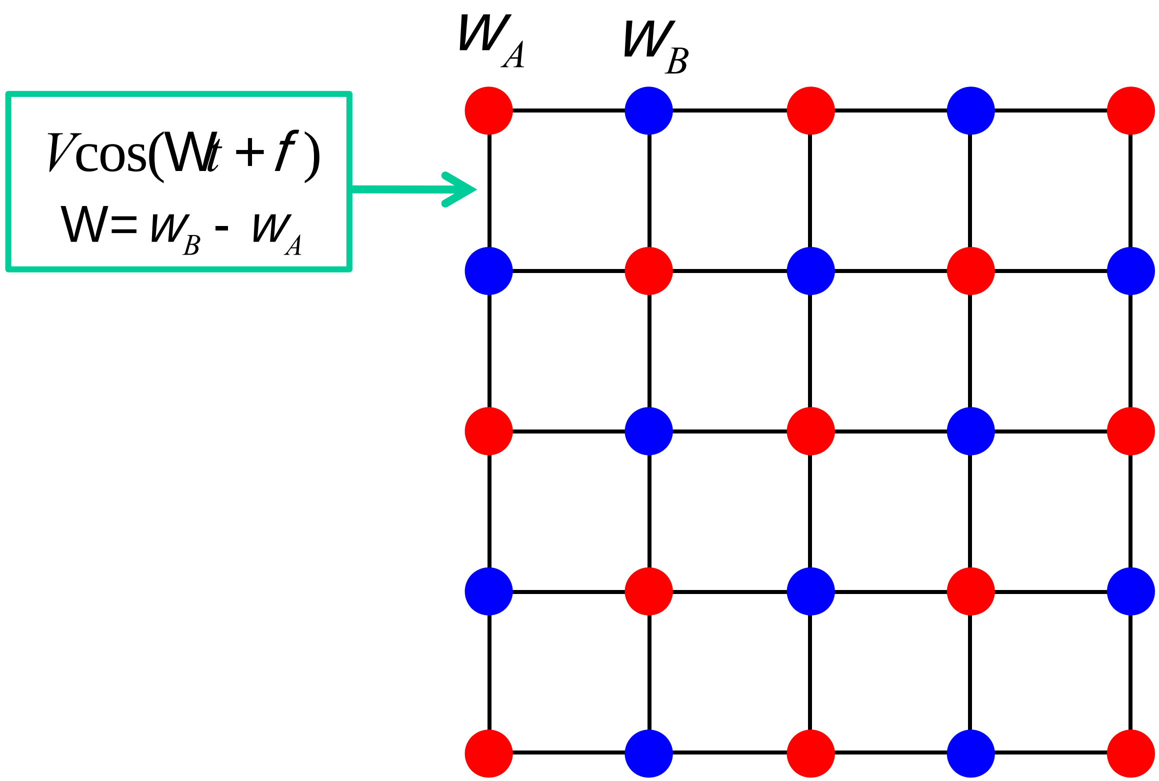
Waveguide mode exists only in the positive k_y region

Gauge-field waveguide in dynamic resonator lattice



Gauge field for photon: dynamic modulation approach

$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i + V\cos\left(\Omega t + \phi_{ij}\right)\sum_{\langle ij \rangle}\left(a_i^+ b_j + b_j^+ a_i\right)$$



K. Fang, Z. Yu and S. Fan, Nature Photonics 6, 782 (2012).

A time-dependent gauge field

$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i + V \cos\left(\Omega t + \phi_{ij}(t)\right) \sum_{\langle ij \rangle} \left(a_i^+ b_j + b_j^+ a_i\right)$$

- Make the modulation phase itself time-dependent

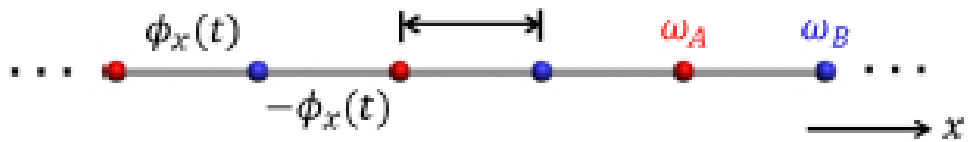
$$\phi\left(t\right) = \alpha \cos(\omega_M t) \sim A\left(t\right)$$

- Since the modulation phase is a gauge potential, this should generate an effective electric field

$$\frac{\partial A}{\partial t} \sim E$$

Effective electric field from a gauge transformation

$$\phi_x(t) = \alpha\cos(\omega_M t)$$



From a spatially periodic Hamiltonian:

$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i + V\cos\left(\Omega t + \phi_{ij}(t)\right) \sum_{\langle ij \rangle} \left(a_i^+ b_j + b_j^+ a_i\right)$$

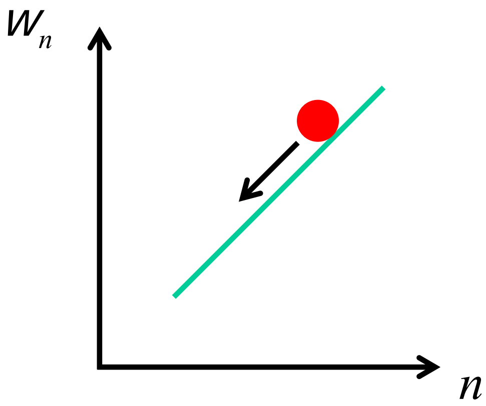
Within rotating wave approximation, and through a local gauge transformation, one can obtain (in one-dimension as an example):

$$H = \sum_{\langle mn \rangle} \frac{V}{2}\left(c_m^+ c_n + c_n^+ c_m\right) - \sum_n \underbrace{n \cdot \alpha\omega_M \sin\left(\omega_M t\right)}_{\text{Position-dependent resonant frequency}} c_n^+ c_n$$

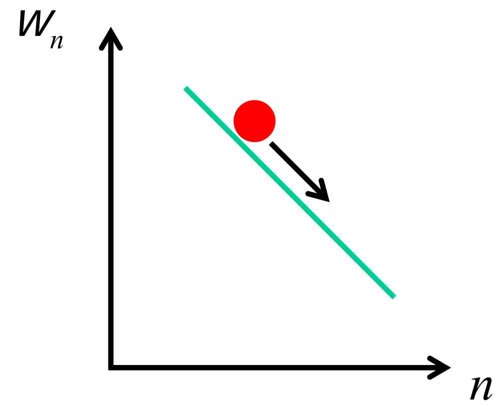
L. Yuan and S. Fan, Physical Review Letters 114, 243901 (2015)

Dynamic localization: a simple picture

t_0



t_1



Every Floquet eigenstate is localized

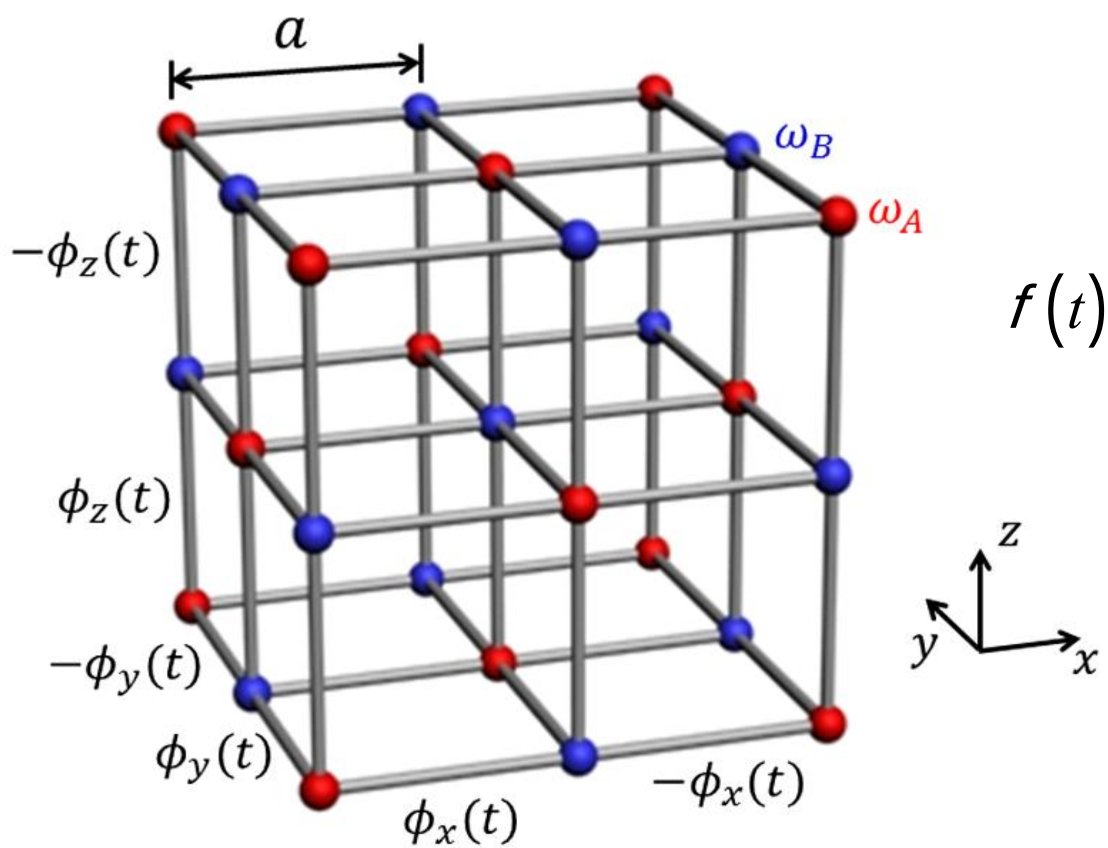
Proposed in semiconductor physics:

D. H. Dunlap and V. M. Kenkre, PRB 34, 3525 (1886); M. Holthaus PRL 69, 351 (1992)

Studied and demonstrated in optics using waveguide array as an analogy:

A. Szameit et al, Nature Physics 5, 271 (2009).

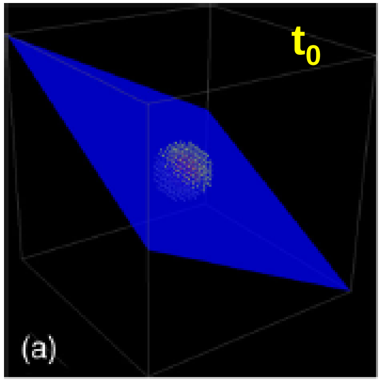
A 3d lattice with a modulated hopping phase

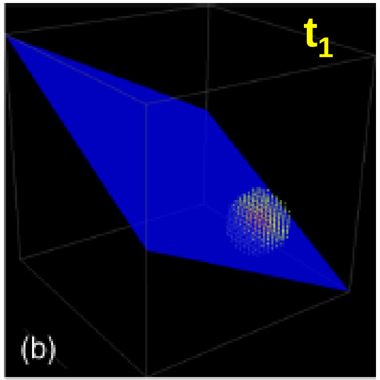


$$f(t) = a\cos(w_M t)$$

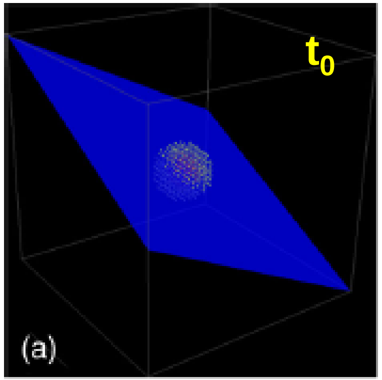
Dynamic localization in three dimension

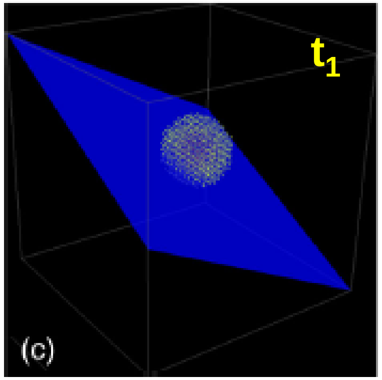
Without phase modulation





With phase modulation





L. Yuan and S. Fan, Physical Review Letters 114, 243901 (2015)

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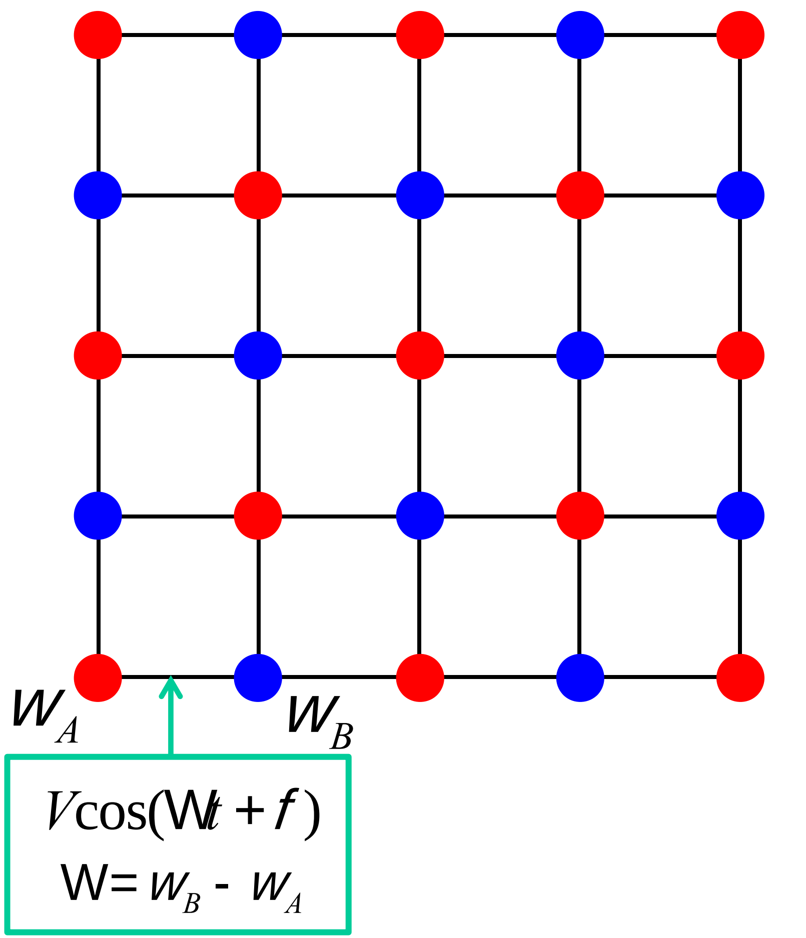
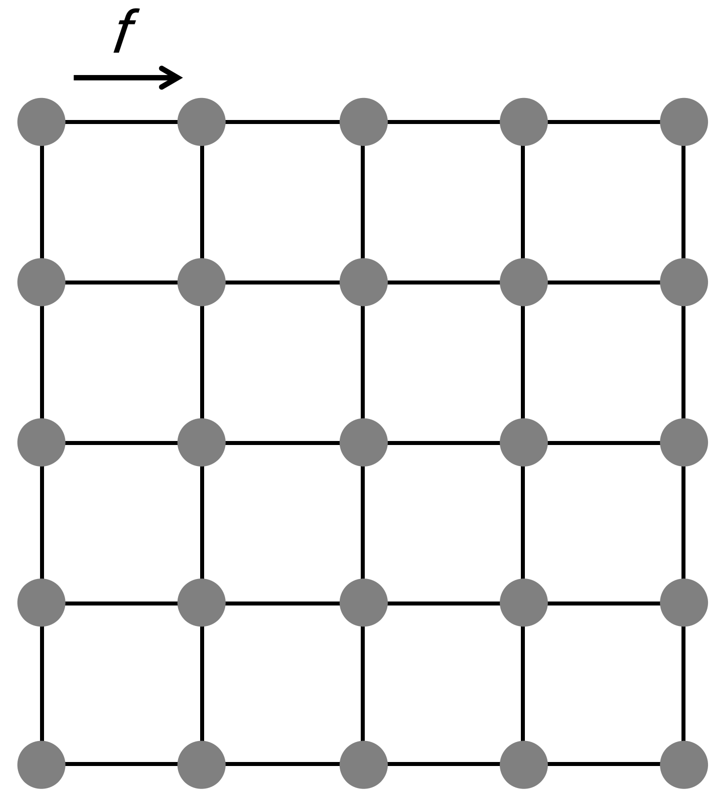
With rotating wave approximation

$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i$$
$$+ V\cos\left(\Omega t + \phi_{ij}\right) \sum_{\langle ij \rangle} \left(a_i^+ b_j + b_j^+ a_i\right)$$

$$H = \sum_{\langle ij \rangle} \left(V e^{i\phi_{ij}} c_i^+ c_j + V e^{-i\phi_{ij}} c_j^+ c_i \right)$$

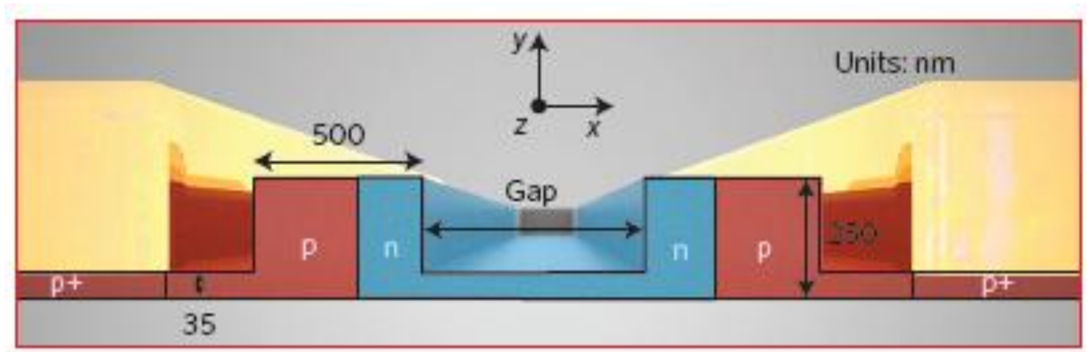




$$\int d\mathbf{l} \cdot \mathbf{A}_{ij} = \phi_{ij}$$

Ultra-strong coupling naturally occur in optical systems

Standard electro-optically modulator on silicon



Refractive index modulation strength $\frac{dn}{n} » 10^{-4}$

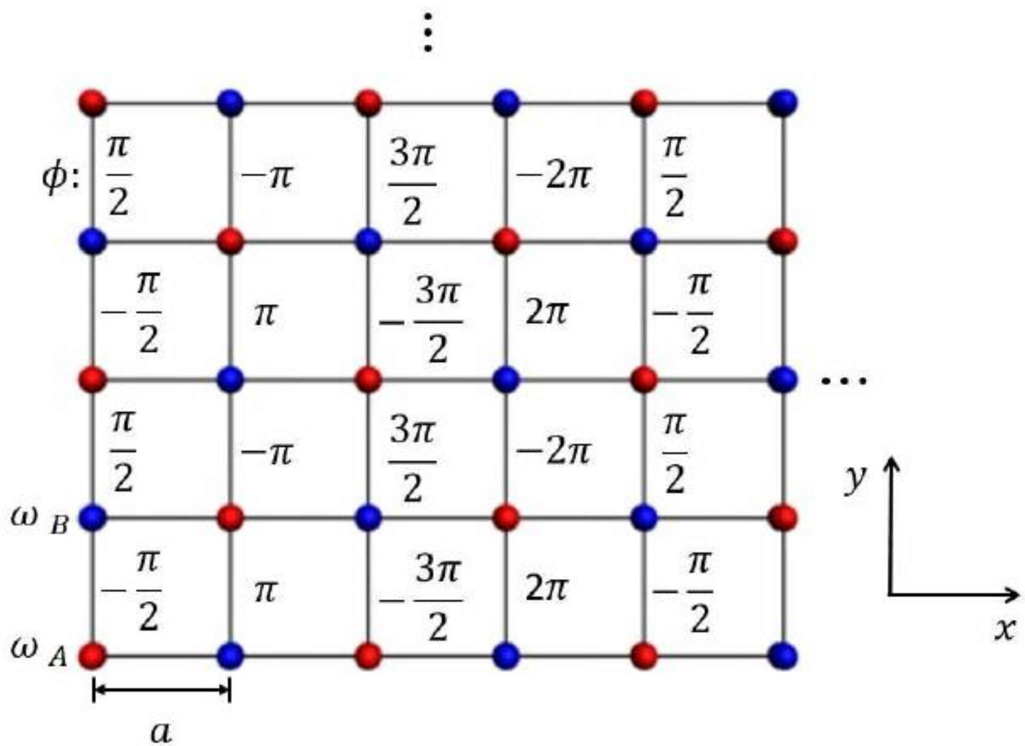
Coupling strength $V \sim \frac{dn}{n} w_0 \sim 10 - 100 GHz$

Modulation frequency $W \sim 10 - 100 GHz$

With standard electro-optic modulation, one is quite likely to be in the ultra-strong coupling regime

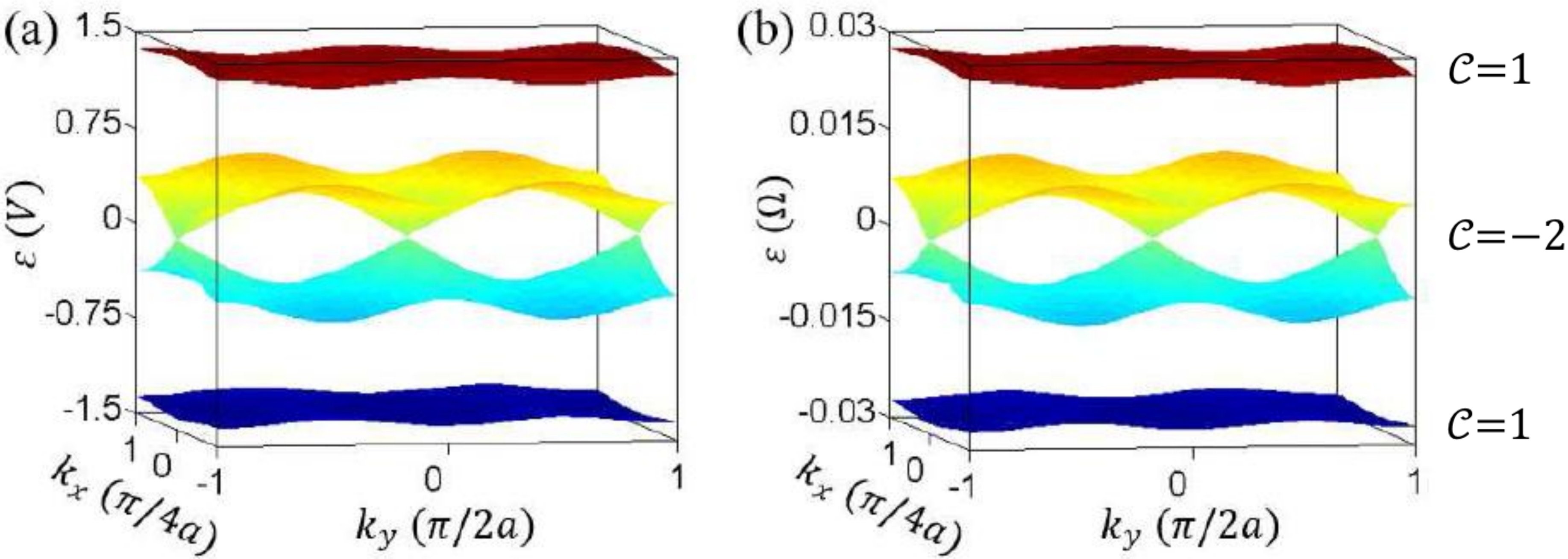
Floquet analysis without rotating wave approximation

$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i + V \cos\left(\Omega t + \phi_{ij}\right) \sum_{\langle ij \rangle} \left(a_i^+ b_j + b_j^+ a_i\right)$$



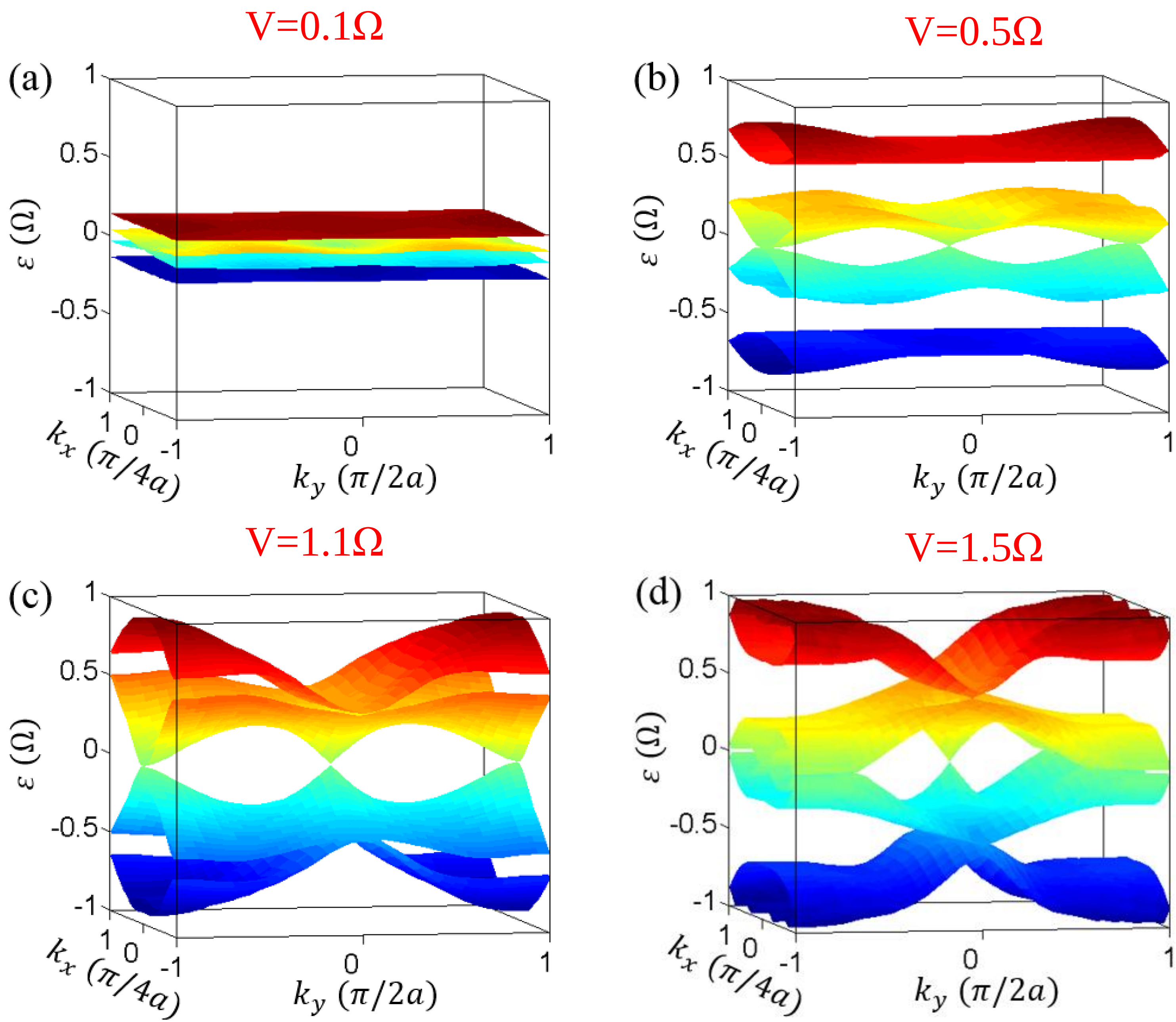
Weak coupling regime

$\tilde{H}_{\mathrm{RWA}}$ Full Hamiltonian with V=0.02Ω



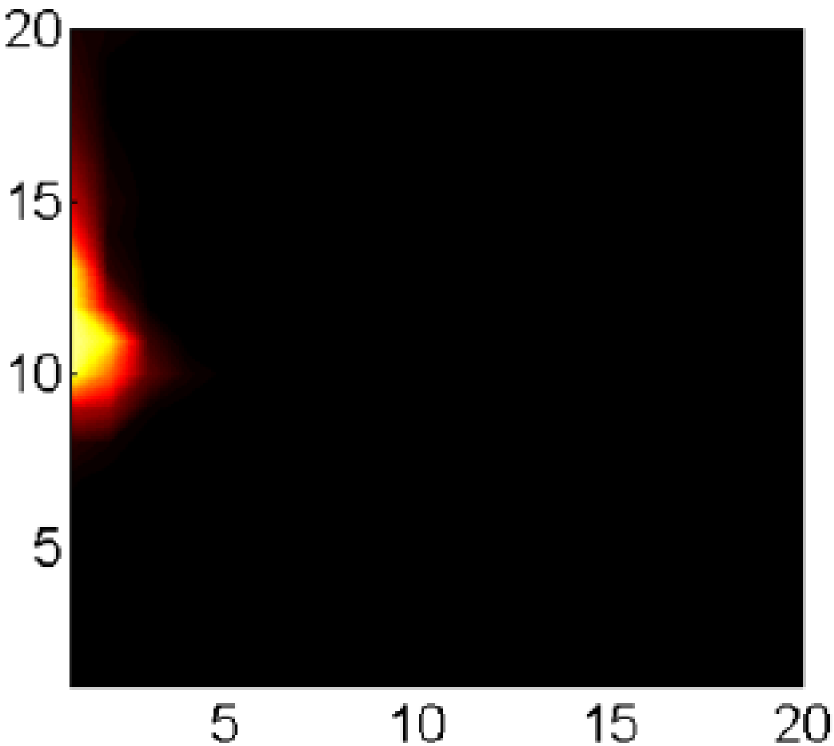
The full Hamiltonian has the same band-structure as the RWA Hamiltonian in the weak-coupling regime

From weak to ultra-strong coupling regime

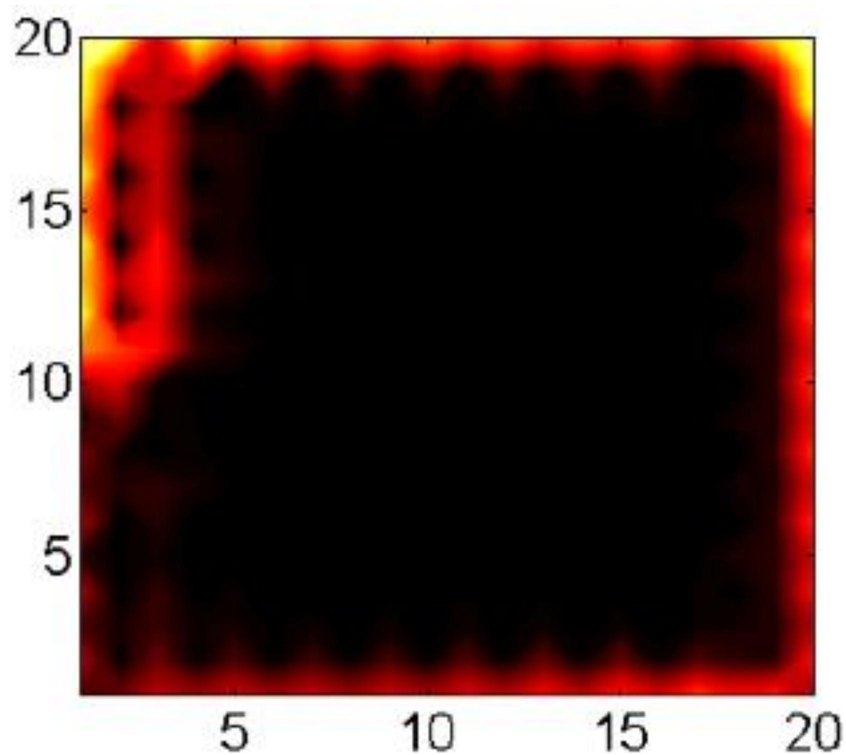


Robustness to absorption loss in the ultra-strong coupling regime

$V = 0.02W$



$V = 0.5W$



Add a damping term for each resonator

L. Yuan and S. Fan, Physical Review A (2015, submitted).

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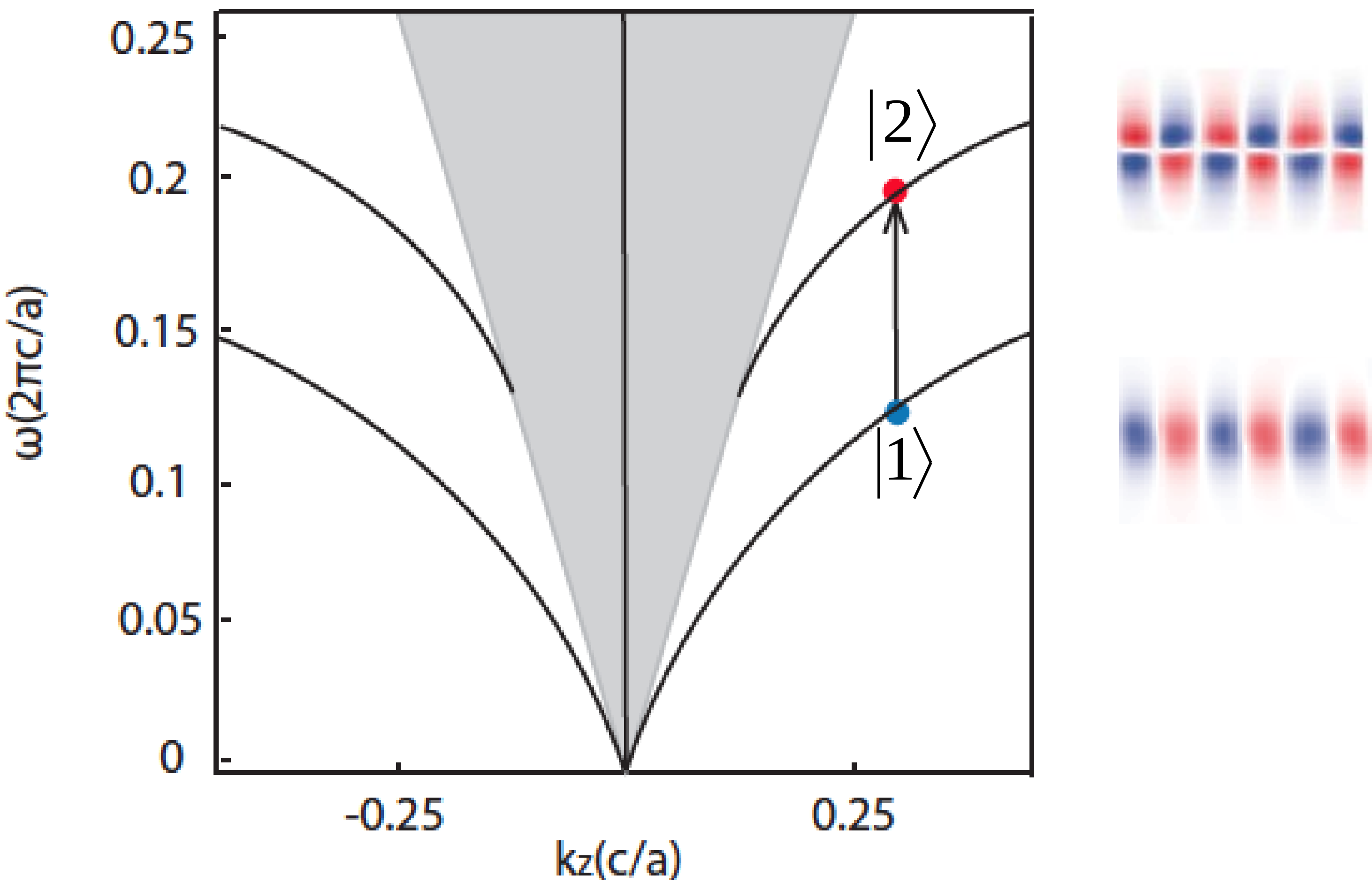
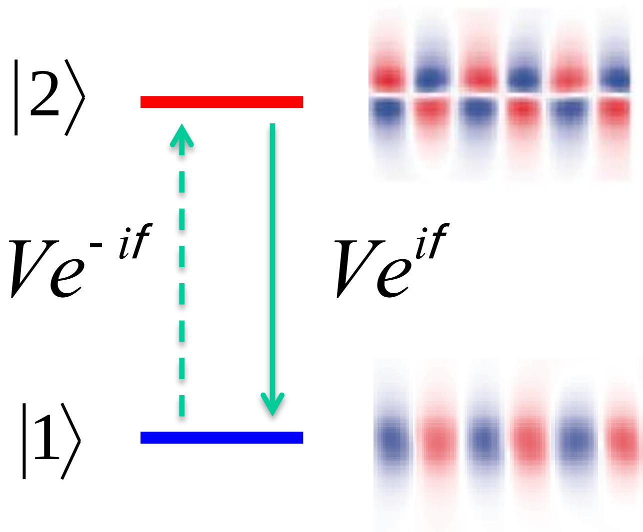
Photonic transition

Uniform modulation along z-direction $\Delta\varepsilon = \delta\cos(\Omega t + \phi)$



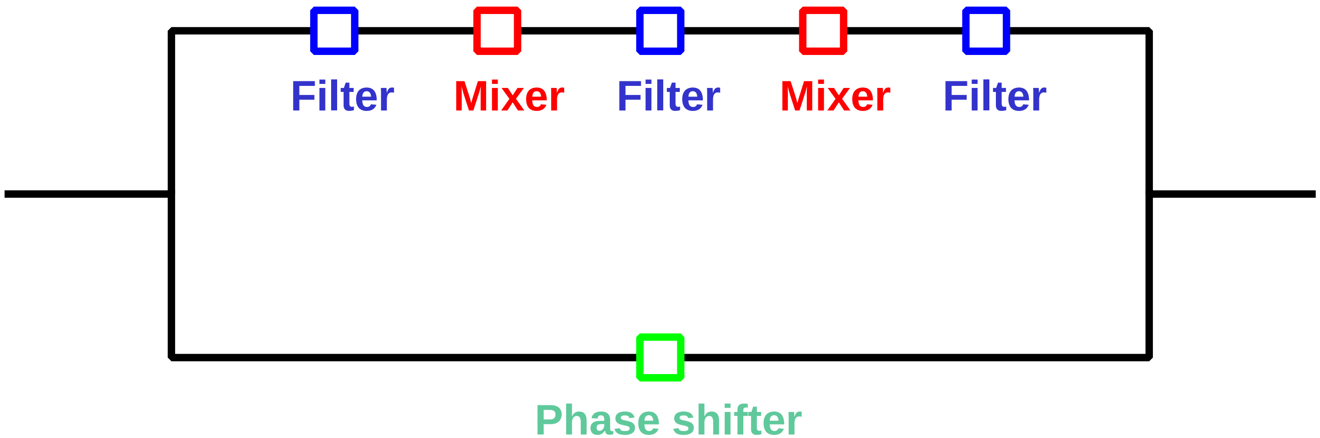
Downward and upper-ward transition acquires a phase difference

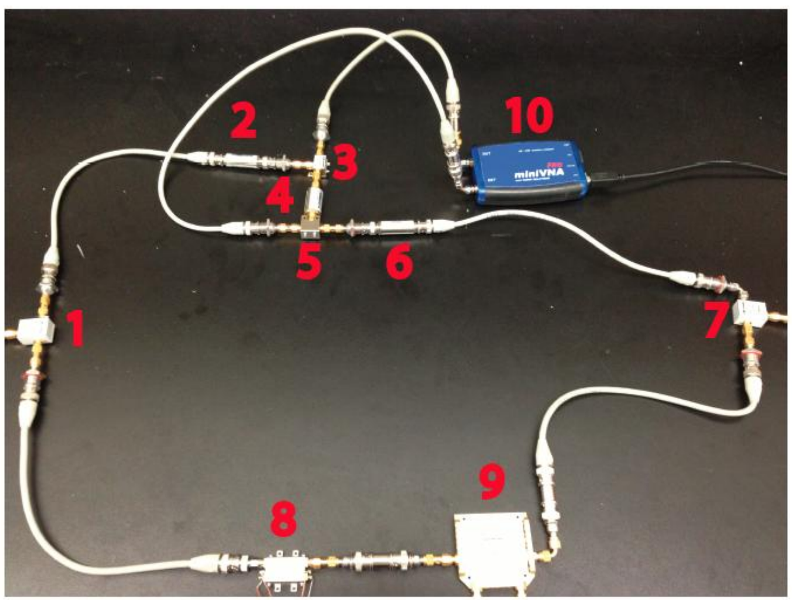


$$\Delta\varepsilon = \delta\cos(\Omega t + \phi)$$

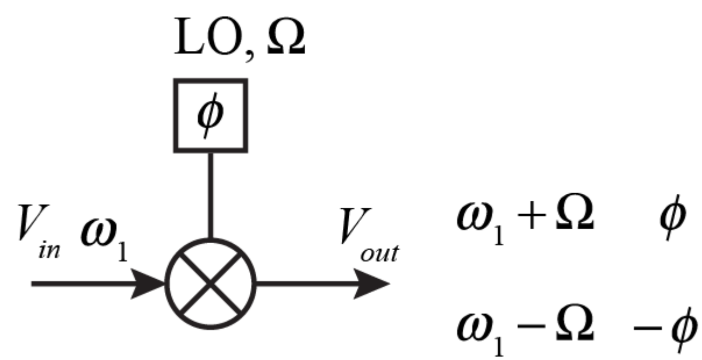
K. Fang, Z. Yu and S. Fan, Physical Review Letters 108, 153901 (2012).

Experimental demonstration of photonic AB effect



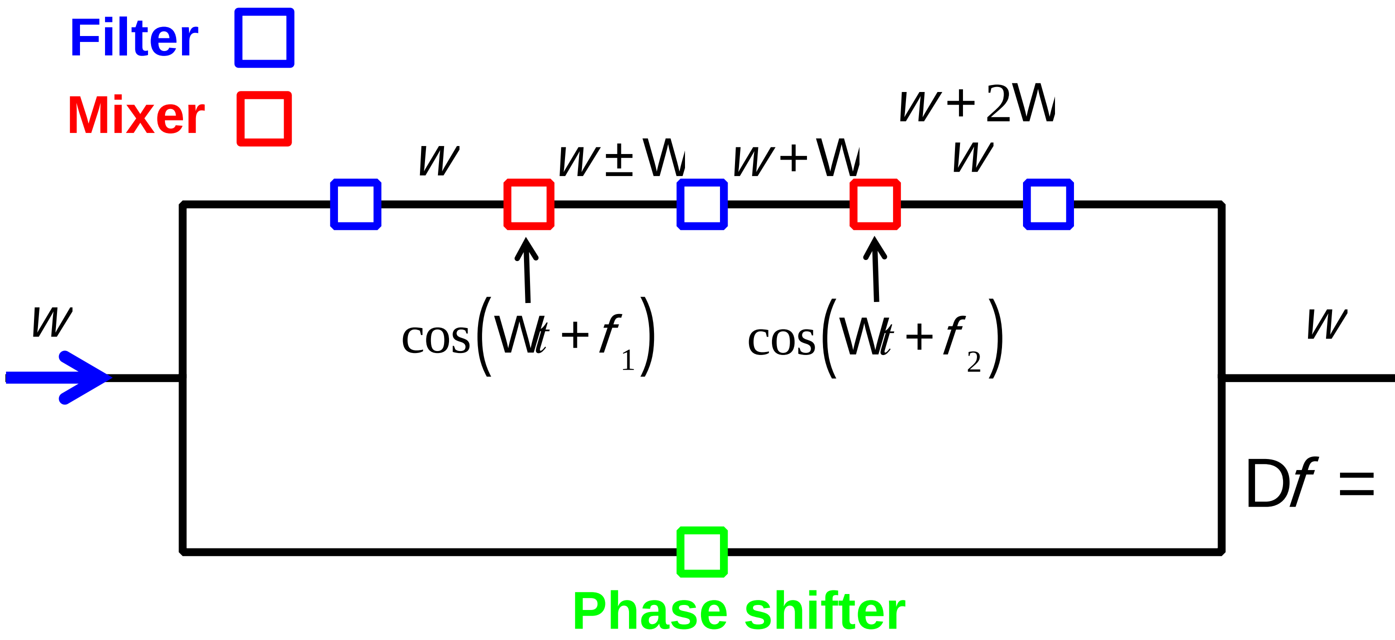


Mixer provides the modulation

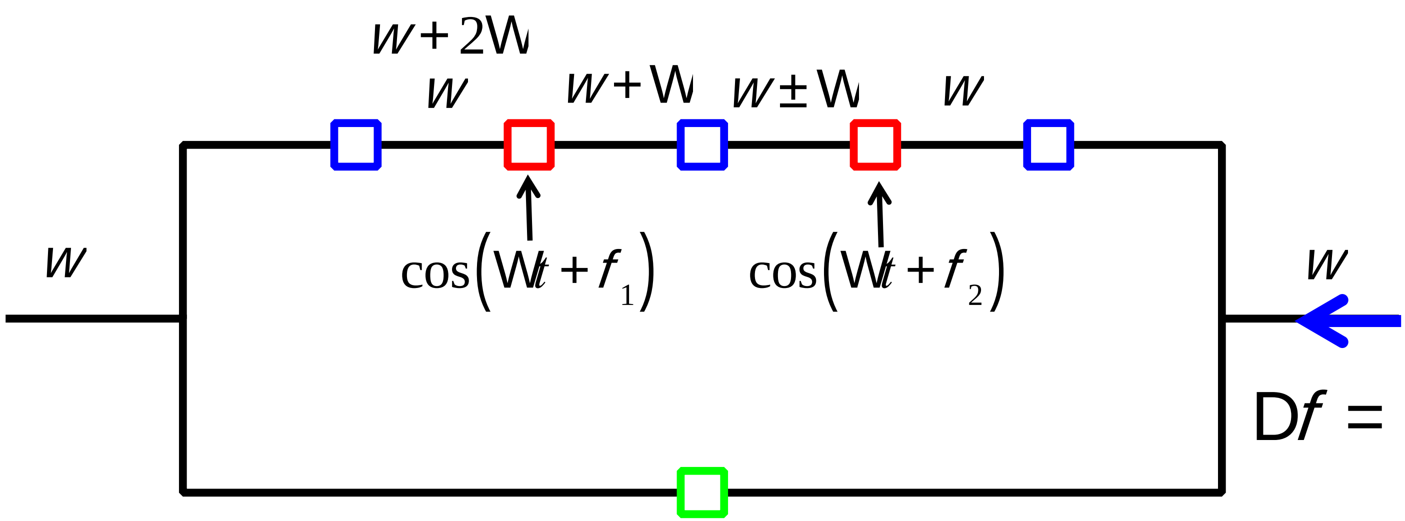


K. Fang, Z. Yu, and S. Fan, Phys. Rev. B Rapid Communications 87, 060301 (2013).

A direction dependent phase for photons

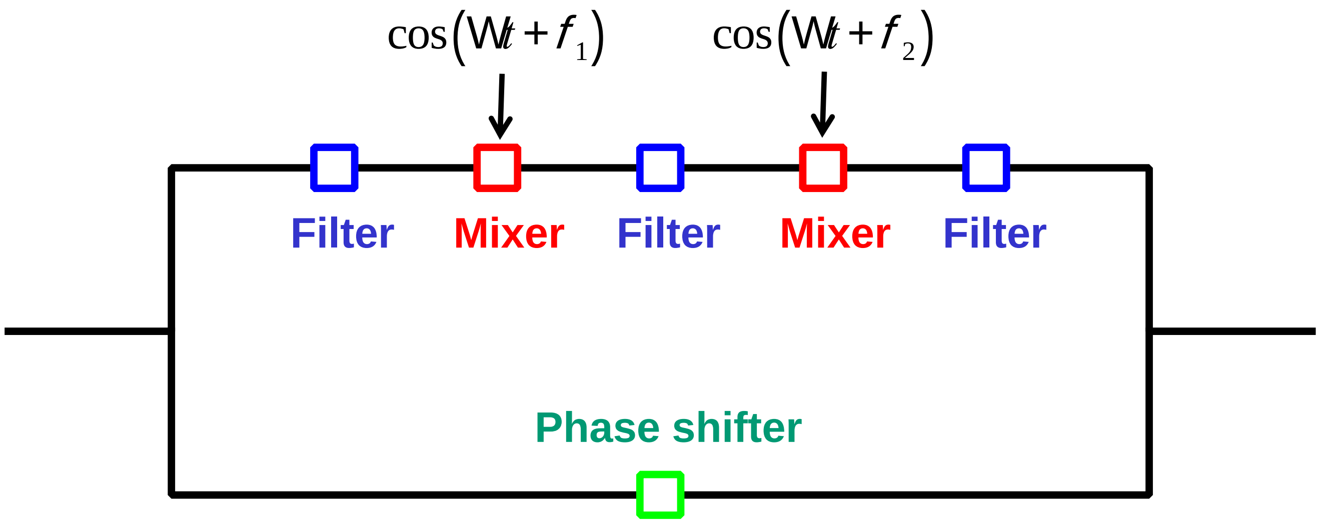


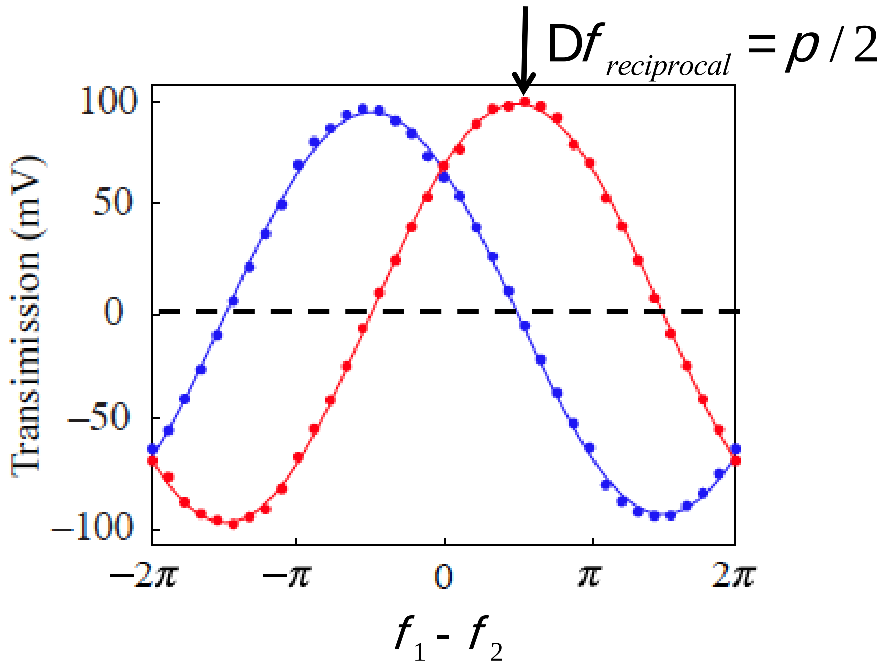
$$\Delta\phi = \Delta\phi_{reciprocal} + \phi_1 - \phi_2$$



$$\Delta\phi = \Delta\phi_{reciprocal} + \phi_2 - \phi_1$$

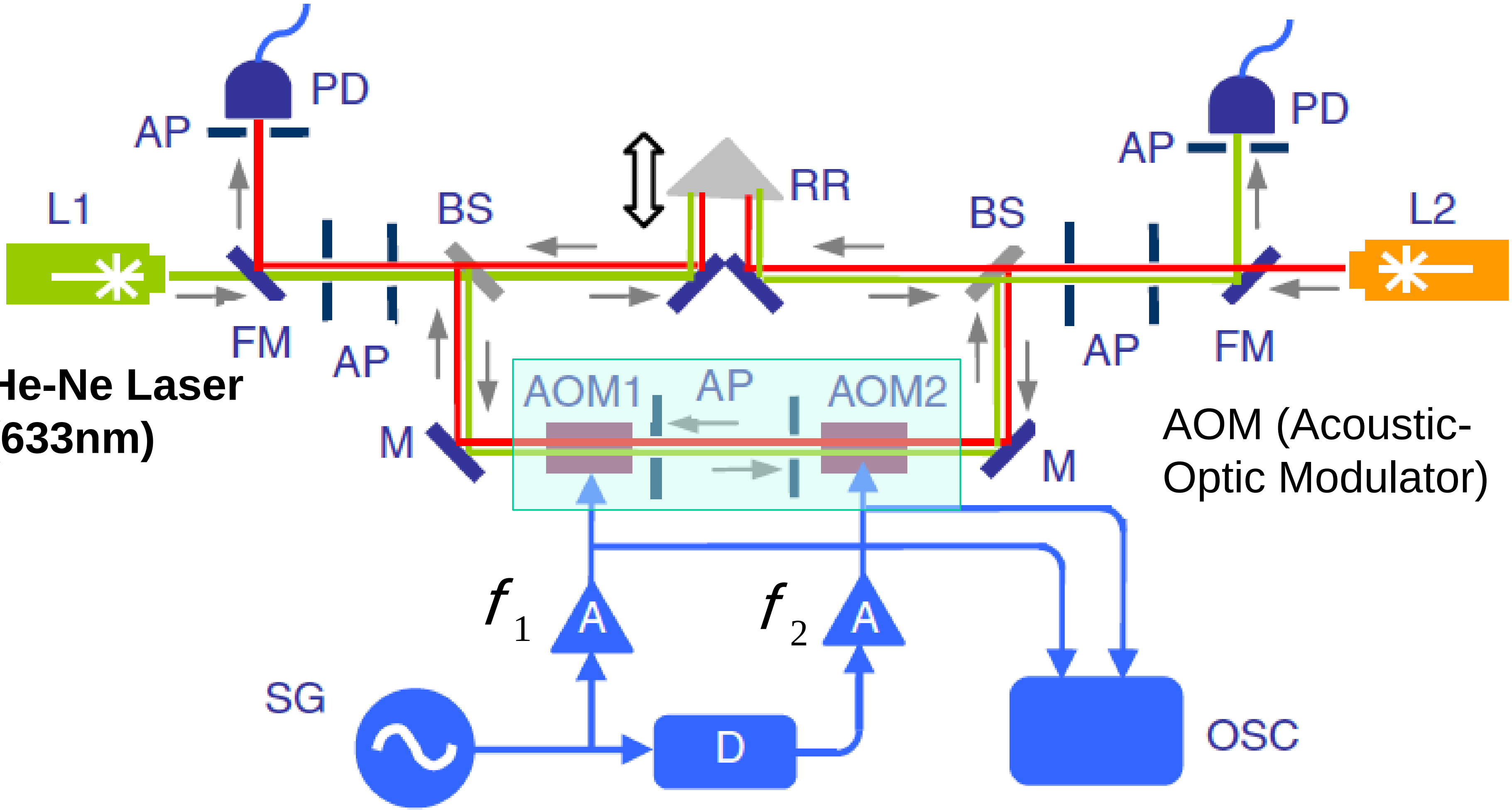
Non-reciprocal oscillation as a function of modulation phase





30dB contrast
From 8-12MHz

AB Interferometer from Photon-Phonon Interaction



He-Ne Laser (633nm)

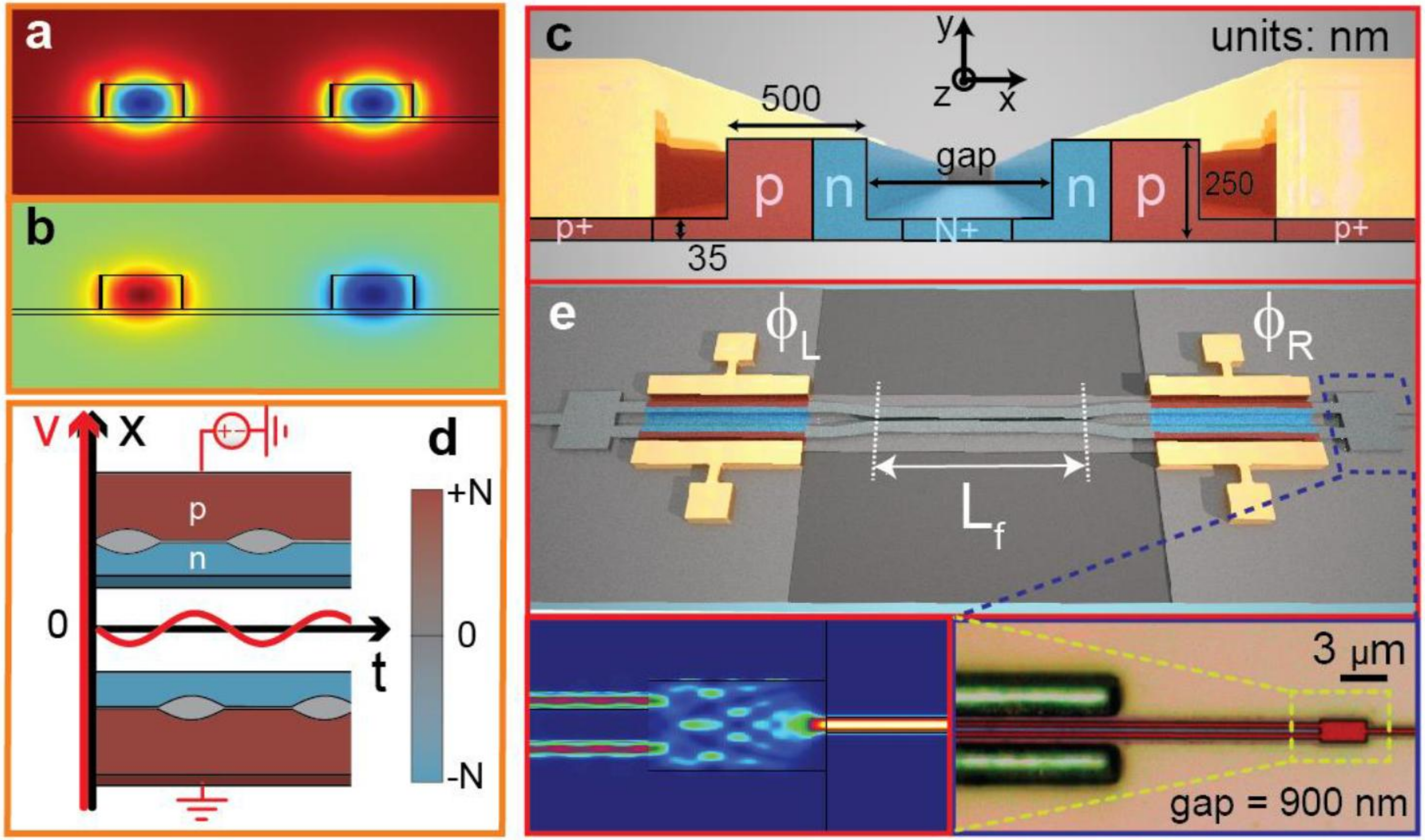
AOM (Acoustic-Optic Modulator)

Local oscillator (50MHz)

13dB forward-backward contrast

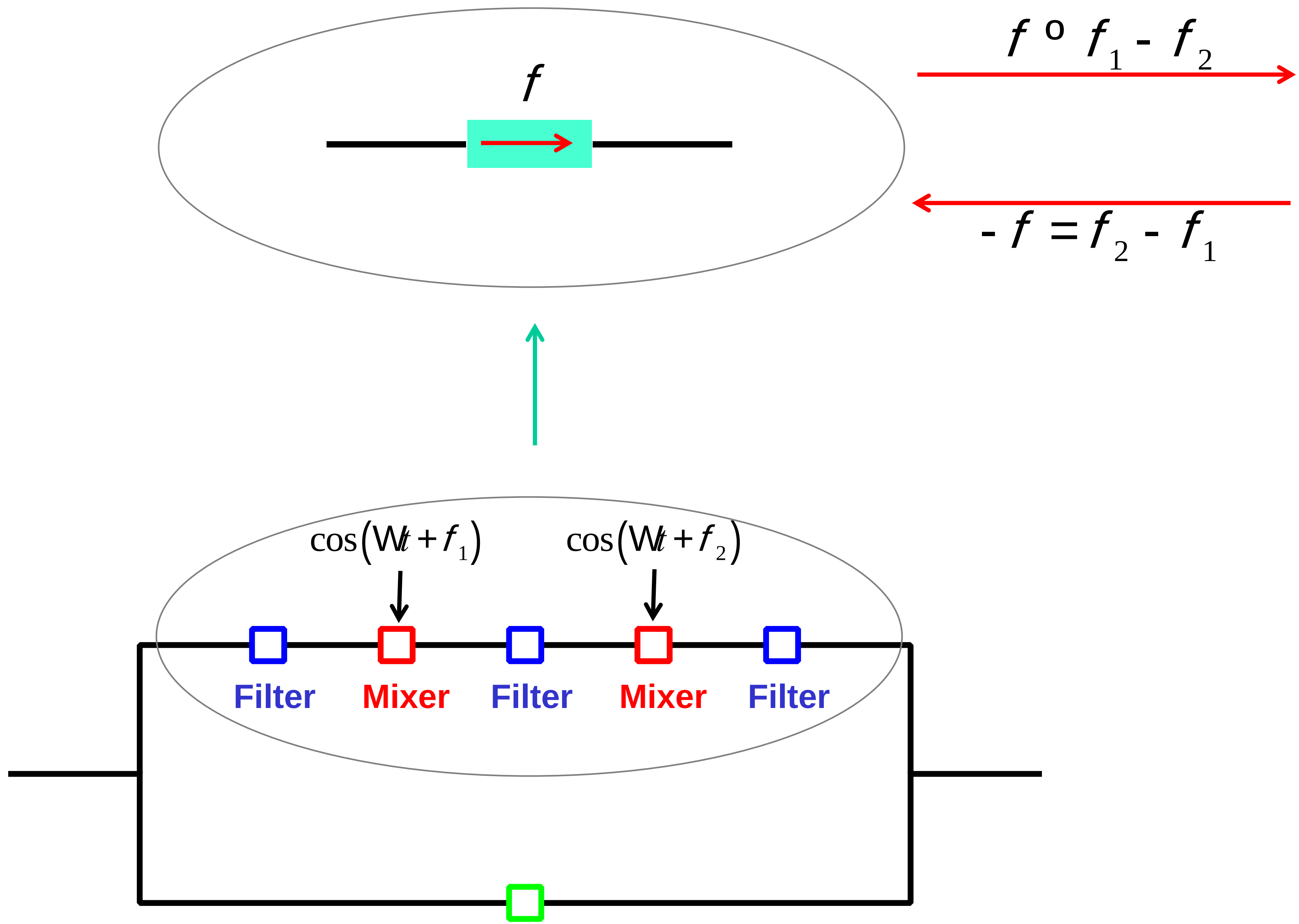
E. Li, B. Eggleton, K. Fang and S. Fan, Nature Communications 5, 3225 (2014).

AB interferometer on a silicon platform



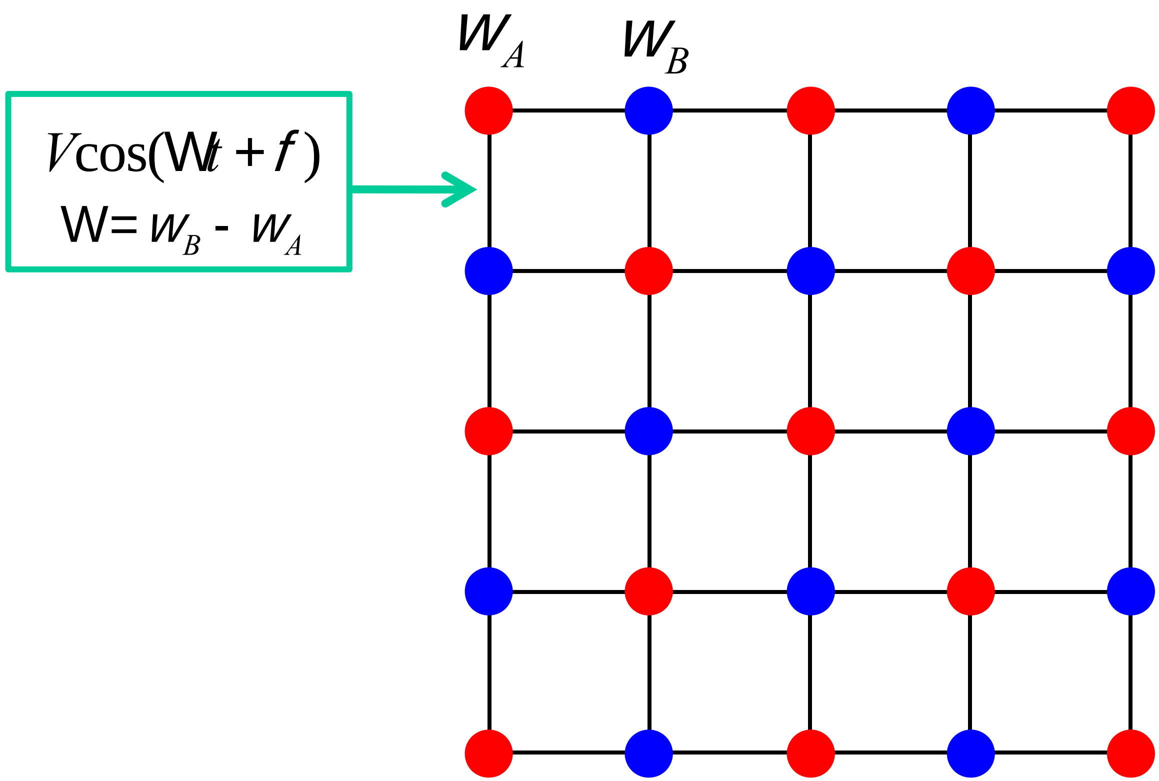
L. Tzuang, K. Fang, P. Nussenzveig, S. Fan, and M. Lipson, Nature Photonics 8, 701 (2014).

Direction-dependent phase shifter



Gauge field for photon: dynamic modulation approach

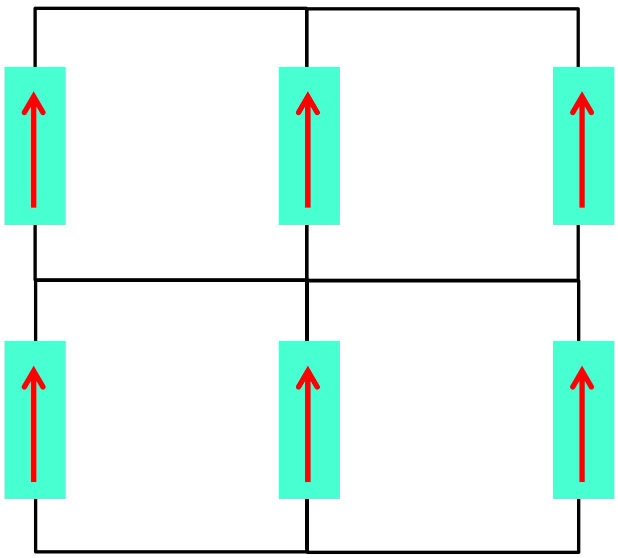
$$H = \omega_A \sum_i a_i^+ a_i + \omega_B \sum_i b_i^+ b_i + V\cos\left(\Omega t + \phi_{ij}\right) \sum_{\langle ij \rangle} \left(a_i^+ b_j + b_j^+ a_i\right)$$



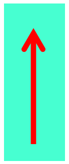
K. Fang, Z. Yu and S. Fan, Nature Photonics 6, 782 (2012).

See also M. Hafezi et al, Nature Physics 7, 907 (2011);
R. O. Umucallar and I. Carusotto, Physical Review A 84, 043804 (2011).

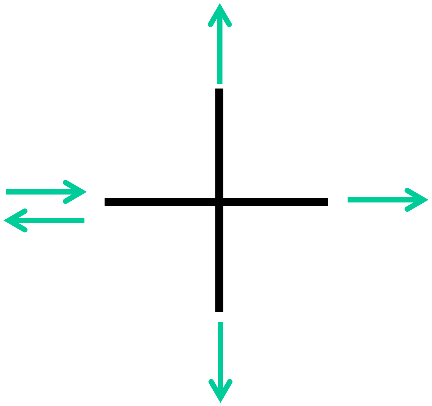
Resonator-free implementation of effective magnetic field for photons



Direction-dependent phase shifter



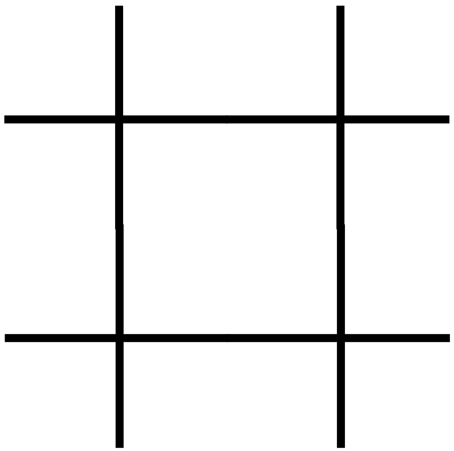
Four-port symmetric waveguide junction

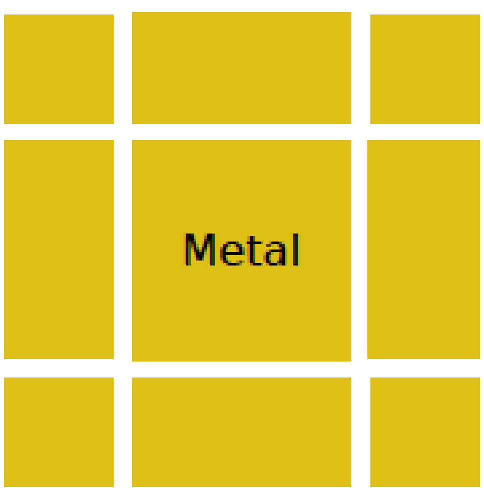


$$S = \begin{pmatrix} -t & t & t & t \\ t & -t & t & t \\ t & t & -t & t \\ t & t & t & -t \end{pmatrix}, \quad t = 1/2$$

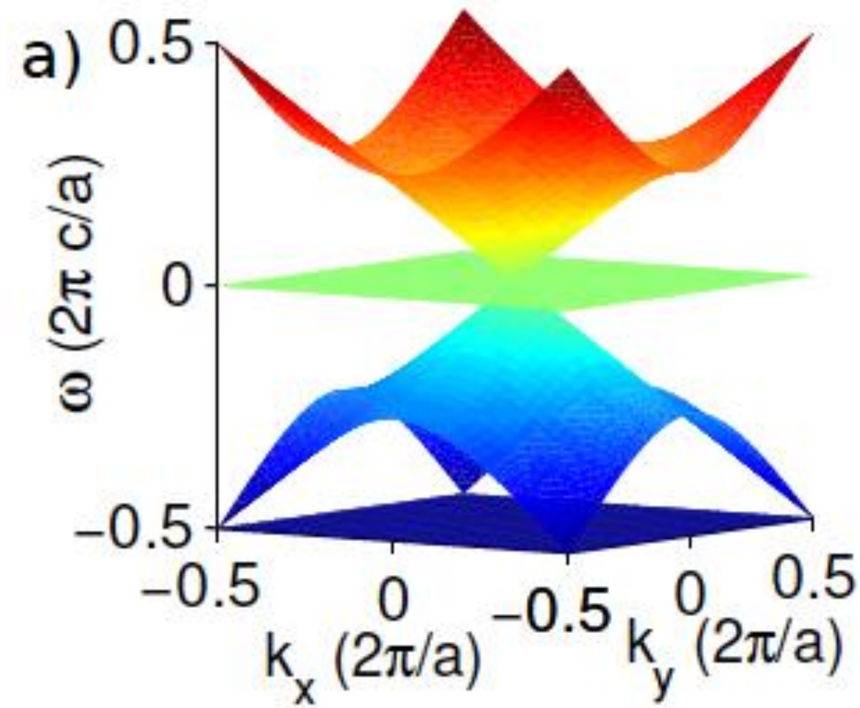
Q. Lin and S. Fan, New Journal of Physics 17, 075008 (2015).

Waveguide network





Dirac dispersion relation



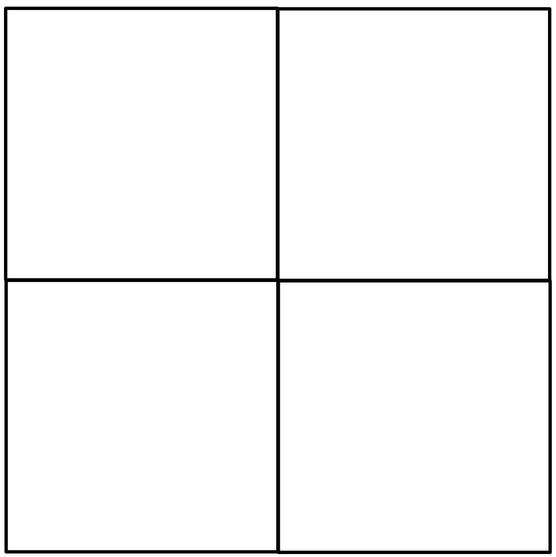
Upper band

Lower band

Feigenbaum and Atwater, PRL 104, 147402 (2010).

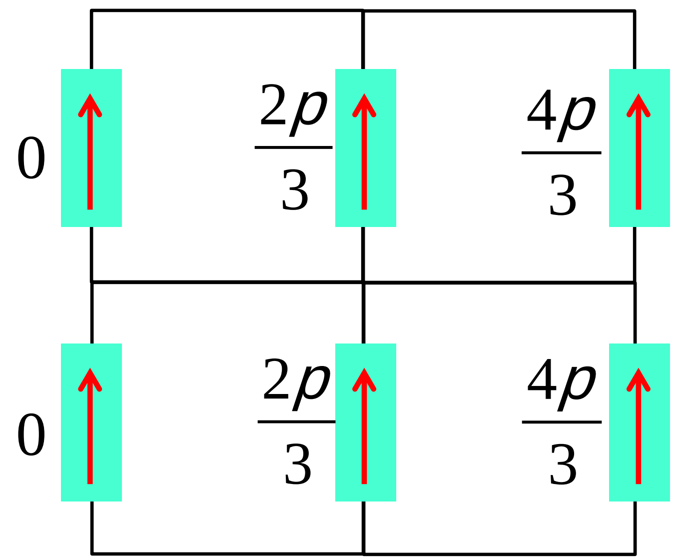
Waveguide-network with directional dependent phase shifter on the waveguide

Reciprocal waveguide network



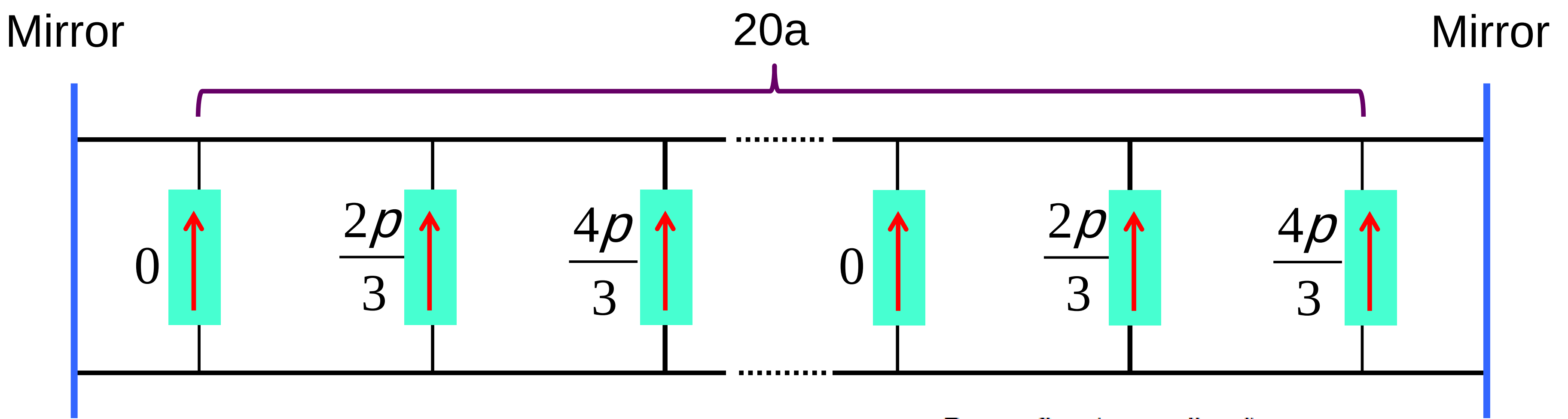
Zero effective magnetic field

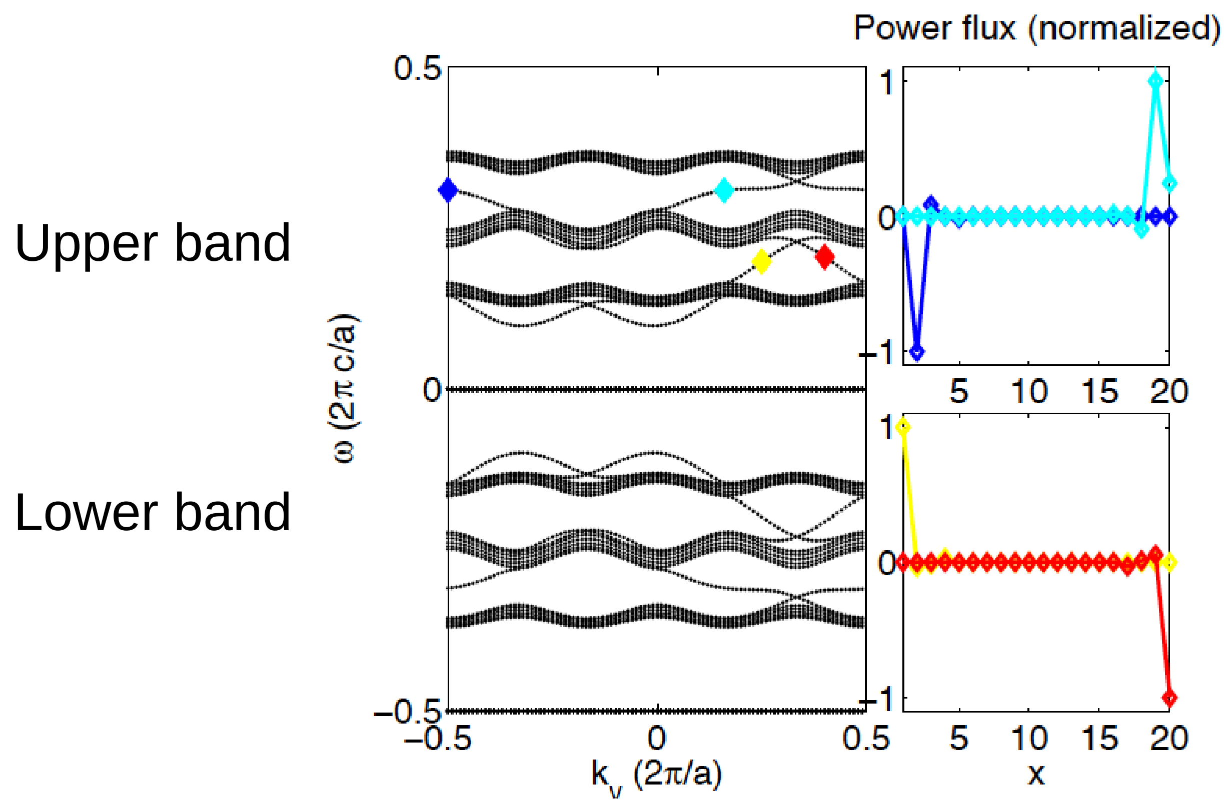
Non-reciprocal waveguide network



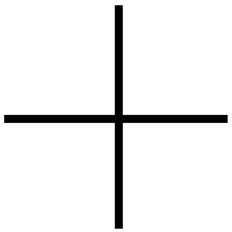
An effective magnetic field for photons

One-way edge state





Four-port symmetric waveguide network



$$S = \begin{pmatrix} -t + \mathrm{i}r & t & t & t \\ t & -t + \mathrm{i}r & t & t \\ t & t & -t + \mathrm{i}r & t \\ t & t & t & -t + \mathrm{i}r \end{pmatrix}$$

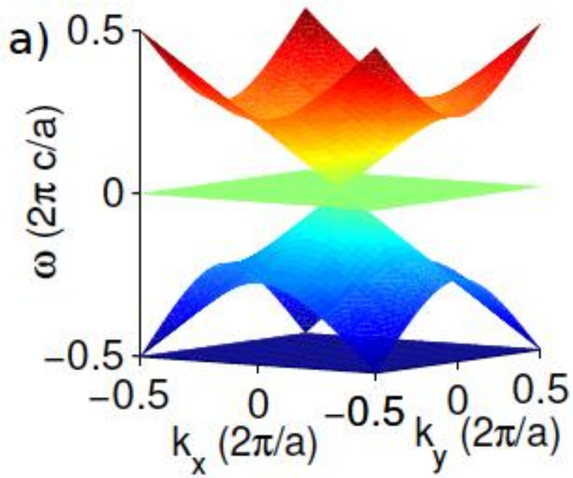
Energy conservation

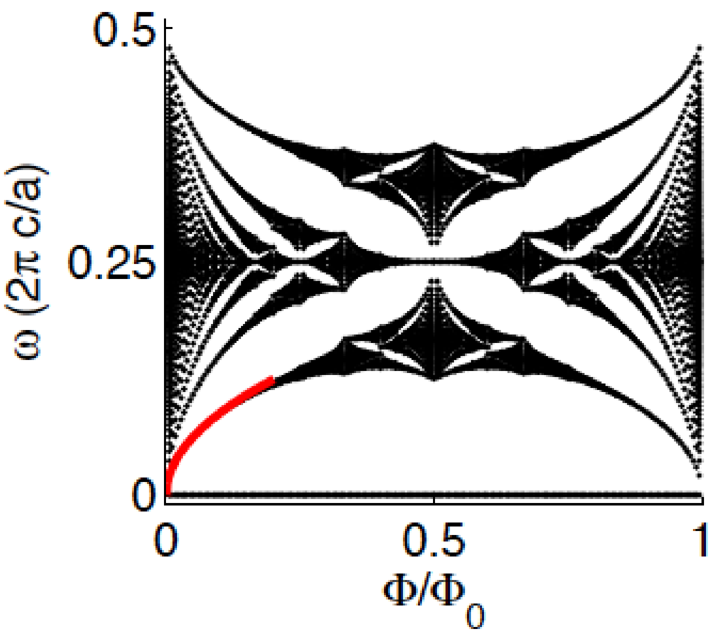
$$t^2 + 4r^2 = 1$$

The property of the network depends on *t*

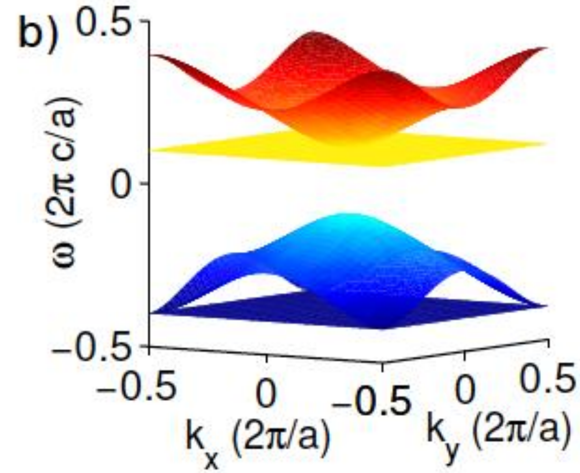
Effective magnetic field for massless and massive particles

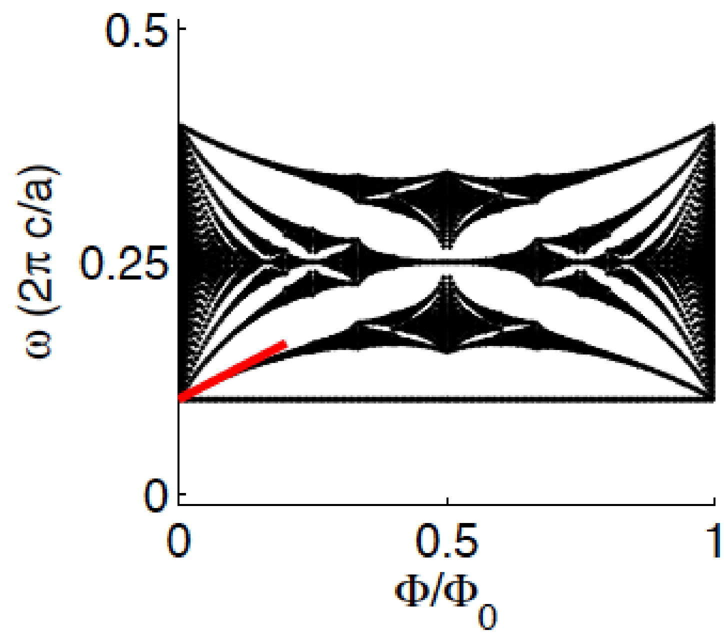
t=0.5



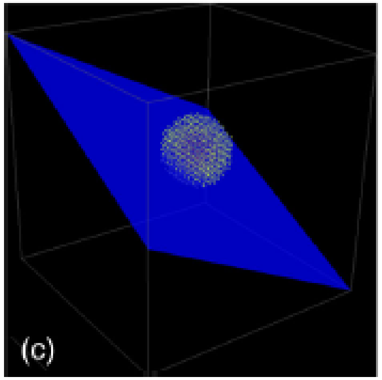


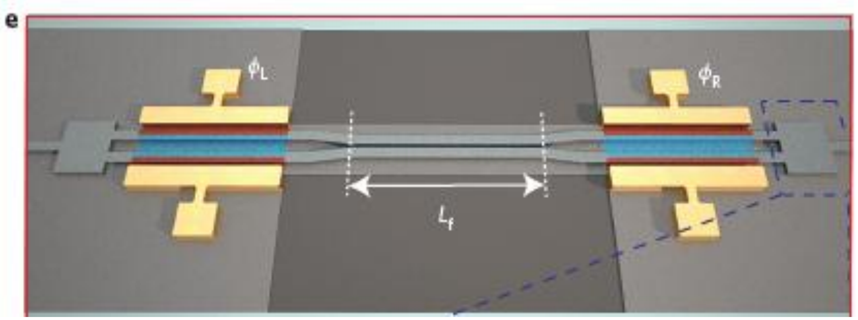
t=0.4





Summary





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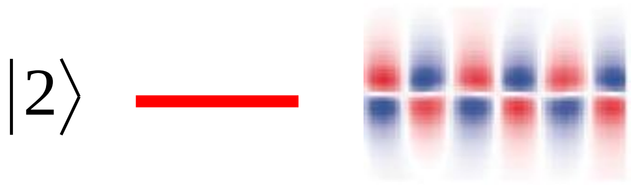
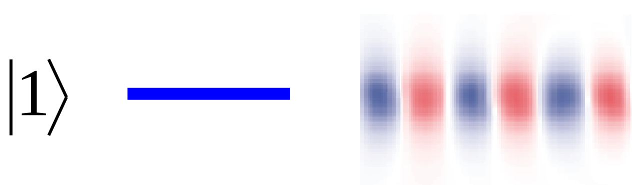
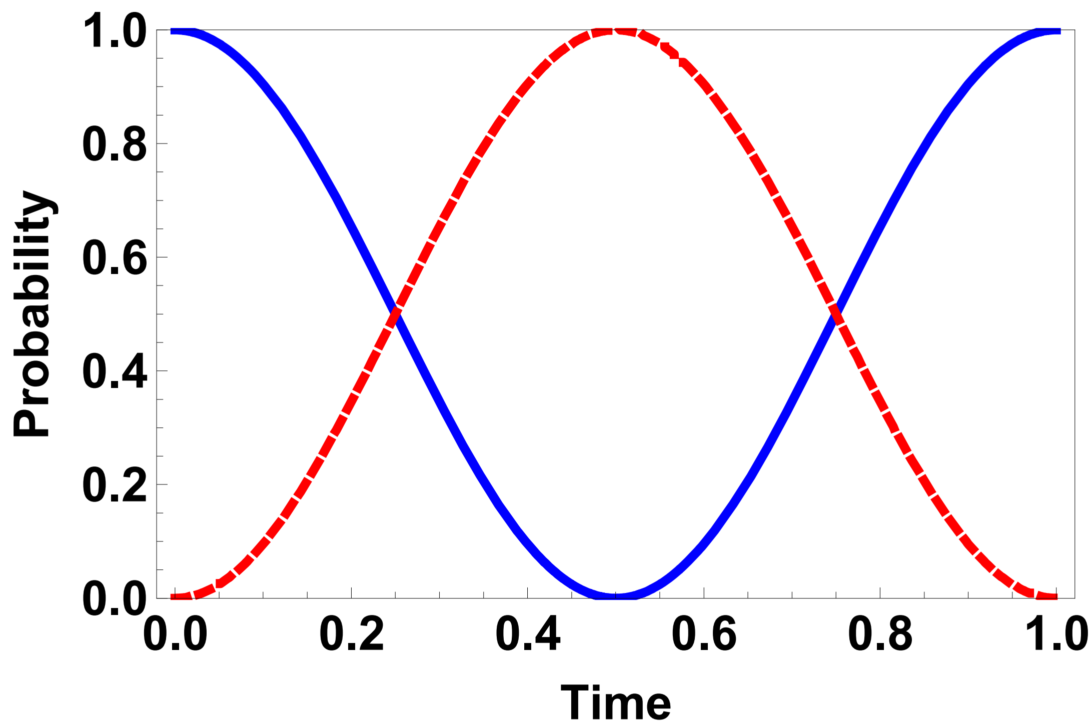
Prof. Michal Lipson
Dr. Enbang Li, Prof. Ben Eggleton

Dynamic modulation breaks time-reversal symmetry



$$\Delta\epsilon = \delta\cos(\Omega t + \phi)$$





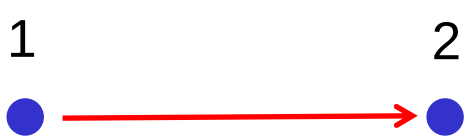
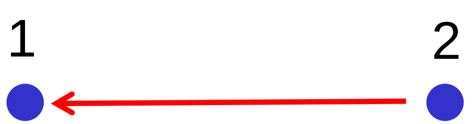
Note that

$$\Delta\epsilon(t) = \delta\cos(\Omega t + \phi) \neq \Delta\epsilon(-t)$$

Gauge potential is equivalent to a direction-dependent phase

Consider electron interacting with a magnetic field B

$$B = \nabla \times A$$

	Propagation phase $A = 0$	Propagation phase $A \neq 0$
	ϕ_s	$\phi_s + \underline{q \int_1^2 ds \cdot A}$
	ϕ_s	$\phi_s + \underline{q \int_2^1 ds \cdot A}$

Contrast with Conventional Waveguide