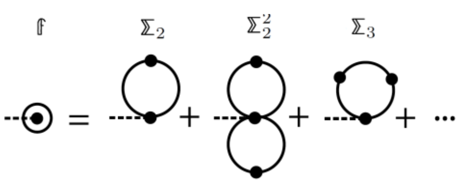
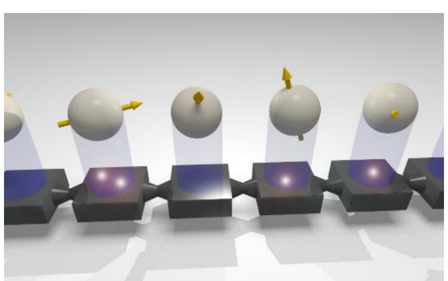
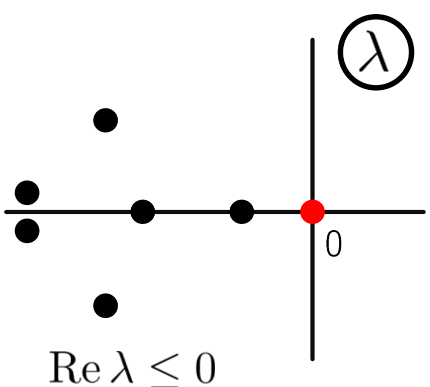
Validating Open Quantum Simulators
by Lindblad Resummation Techniques

Jens Koch



Andy Li (Northwestern U)
Francesco Petruccione (U KwaZulu-Natal)
Andrew Houck (Princeton U, exp.)

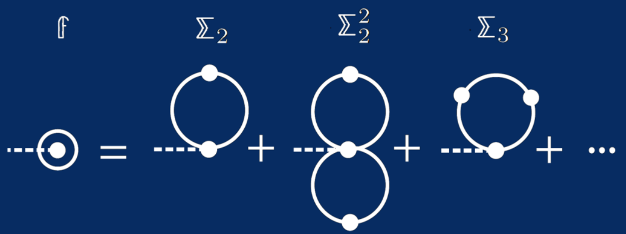




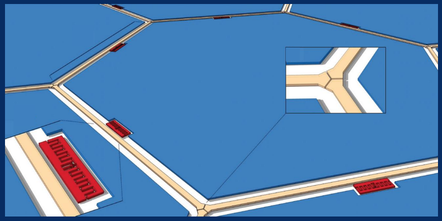
NORTHWESTERN UNIVERSITY

1 Motivation

2 Lindblad perturbation theory

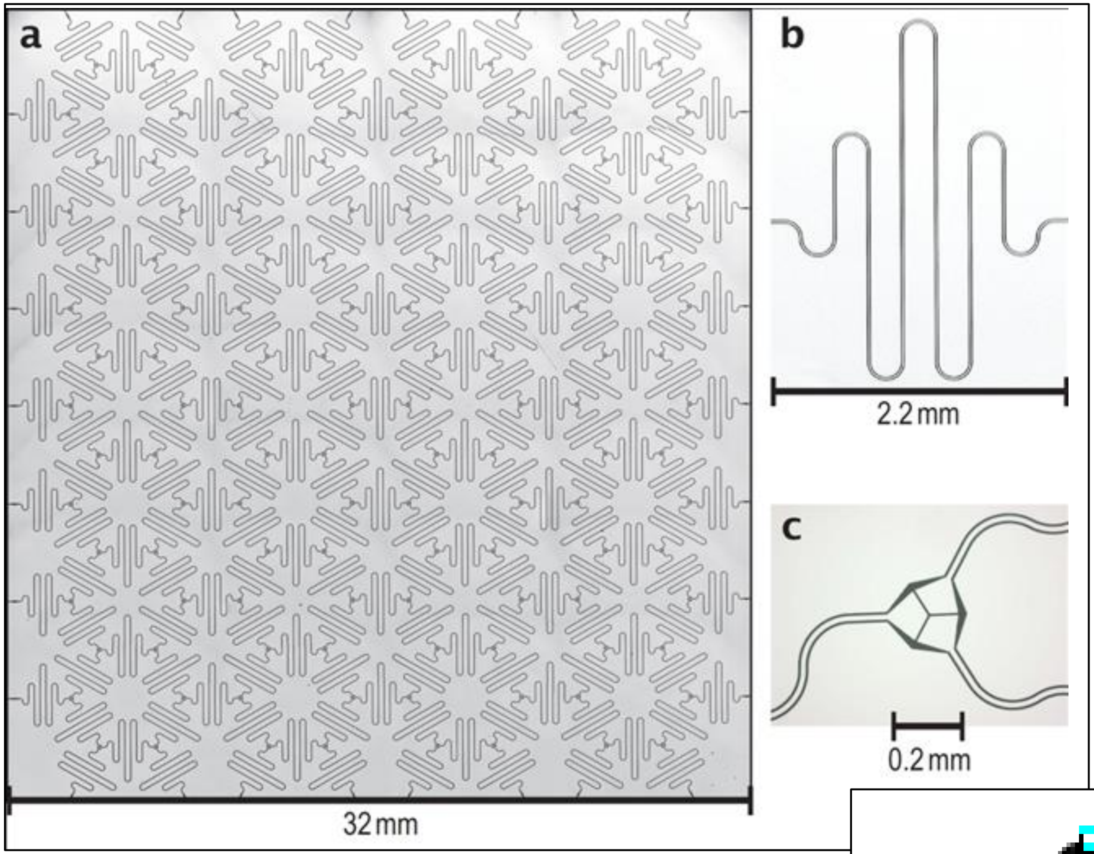


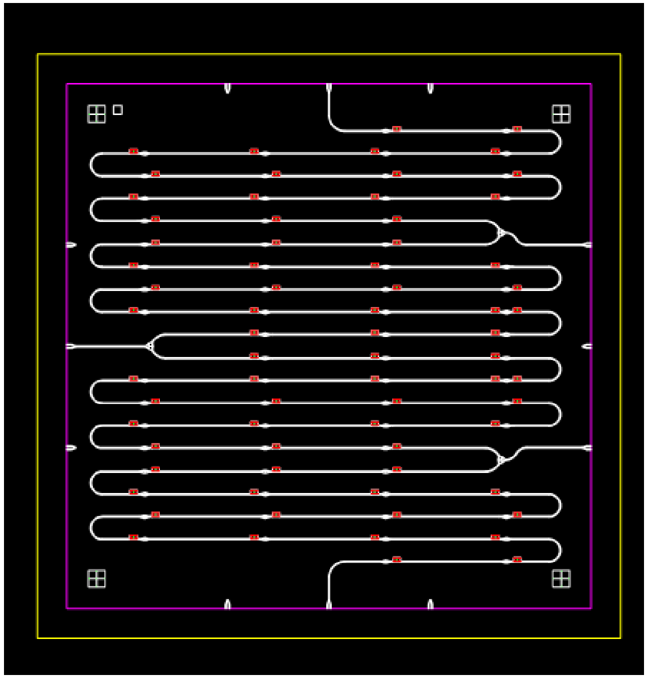
3 Application: Jaynes-Cummings lattice

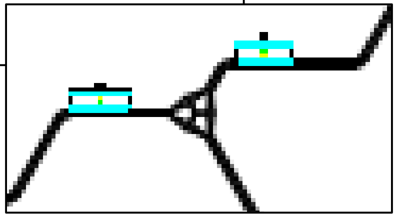


Circuit QED lattices

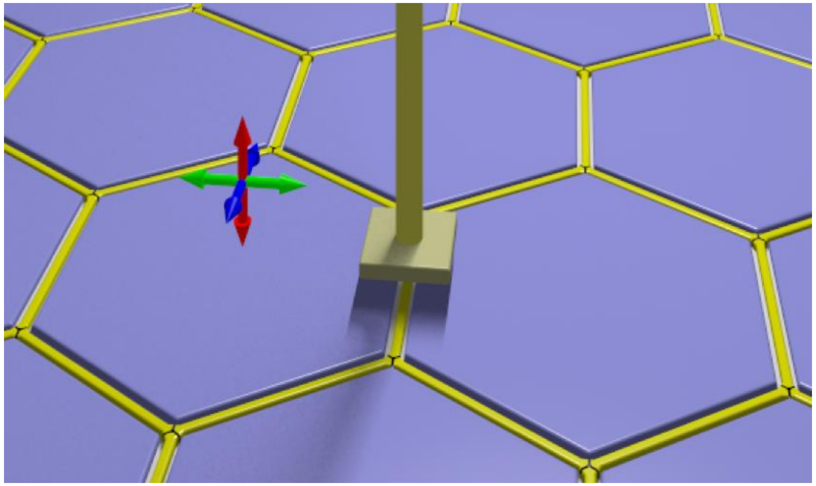
Houck Lab (Princeton): realizing JC lattice models

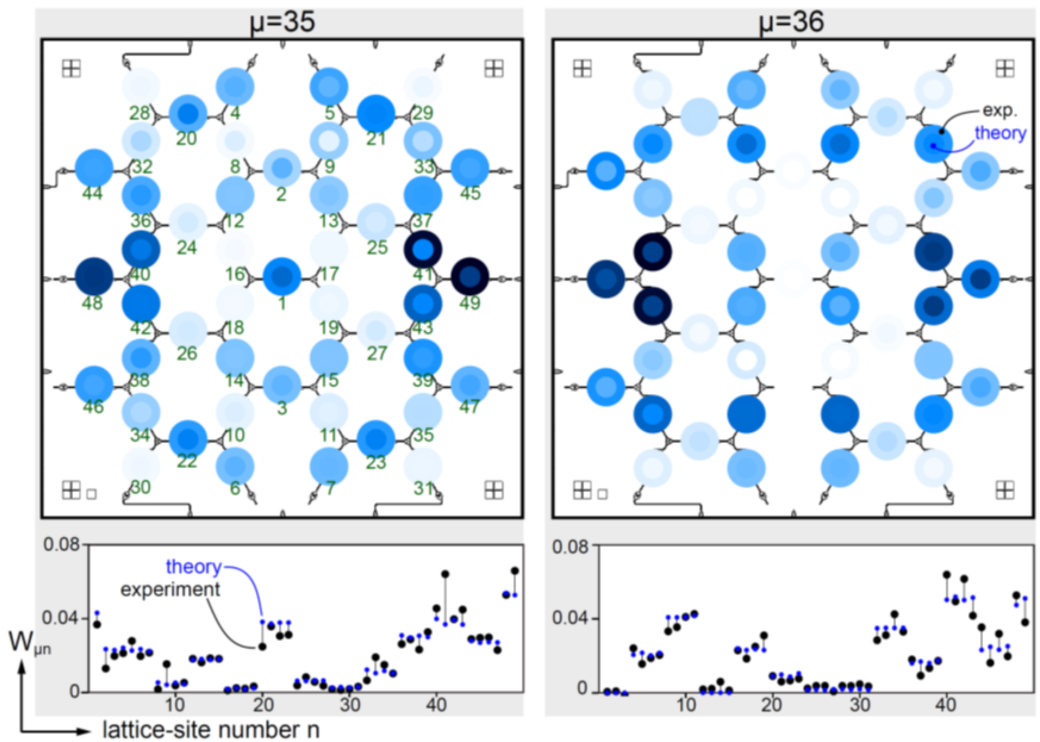




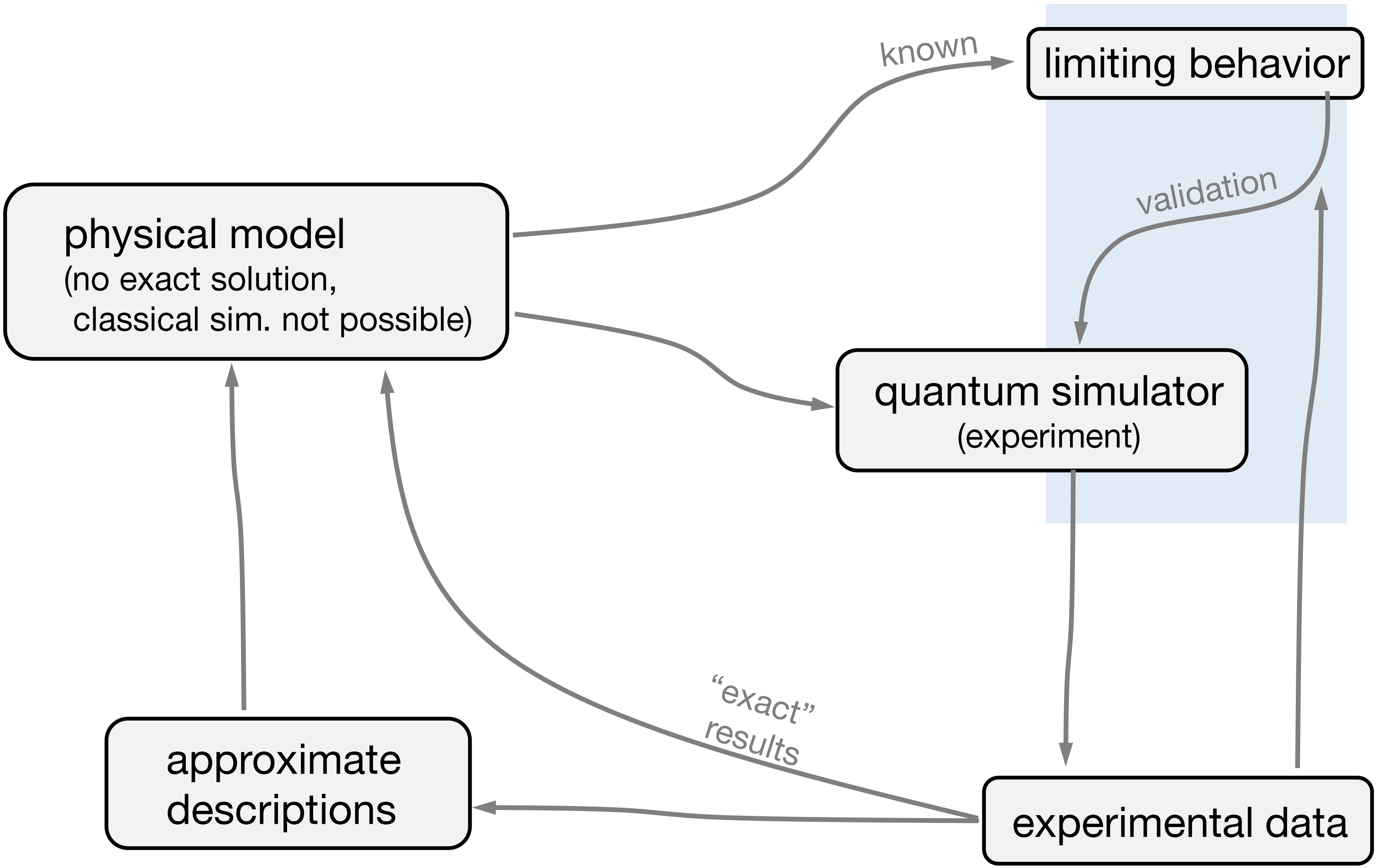


Scanning defect microscopy



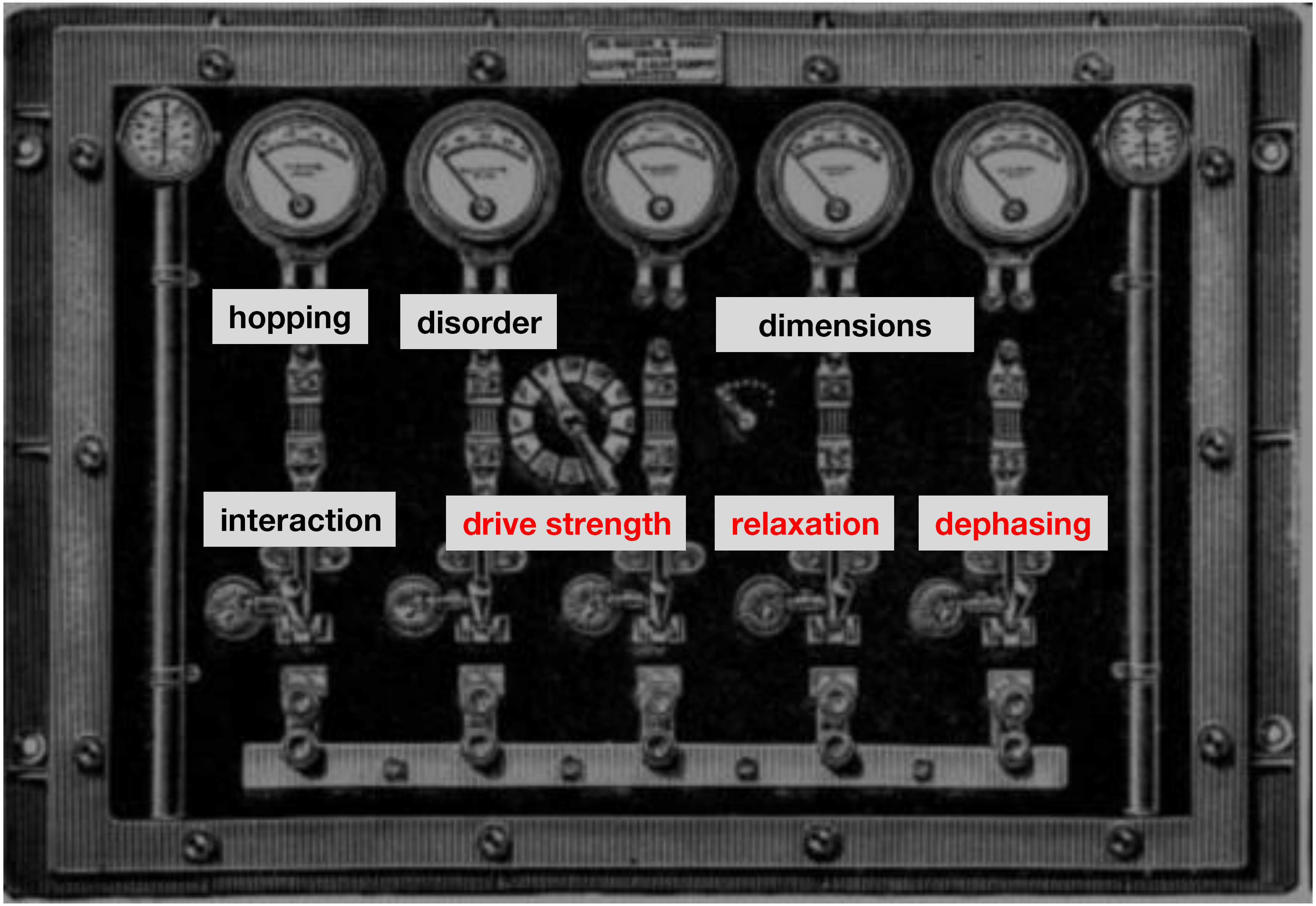


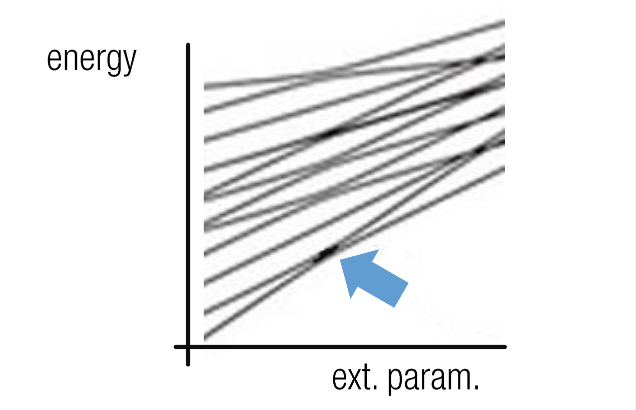
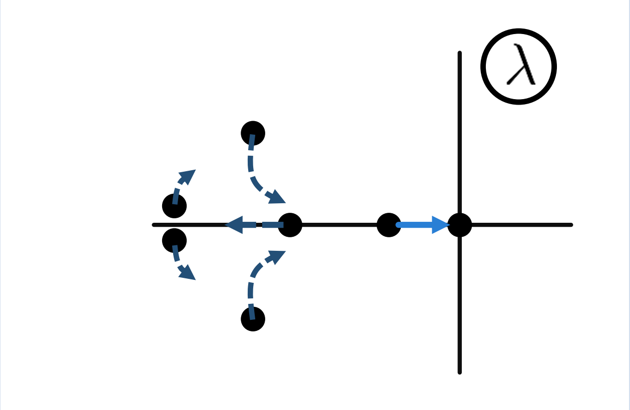
Quantum Simulation



see also: Cirac & Zoller, Nature Phys. 8, 264-266 (2012)

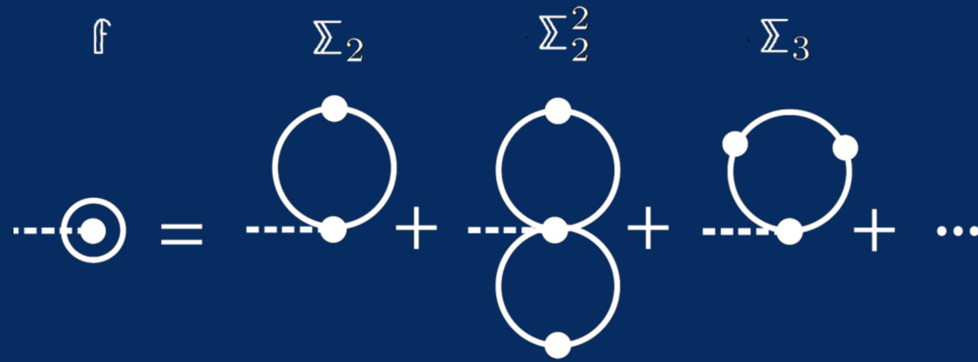
Open-system quantum simulator



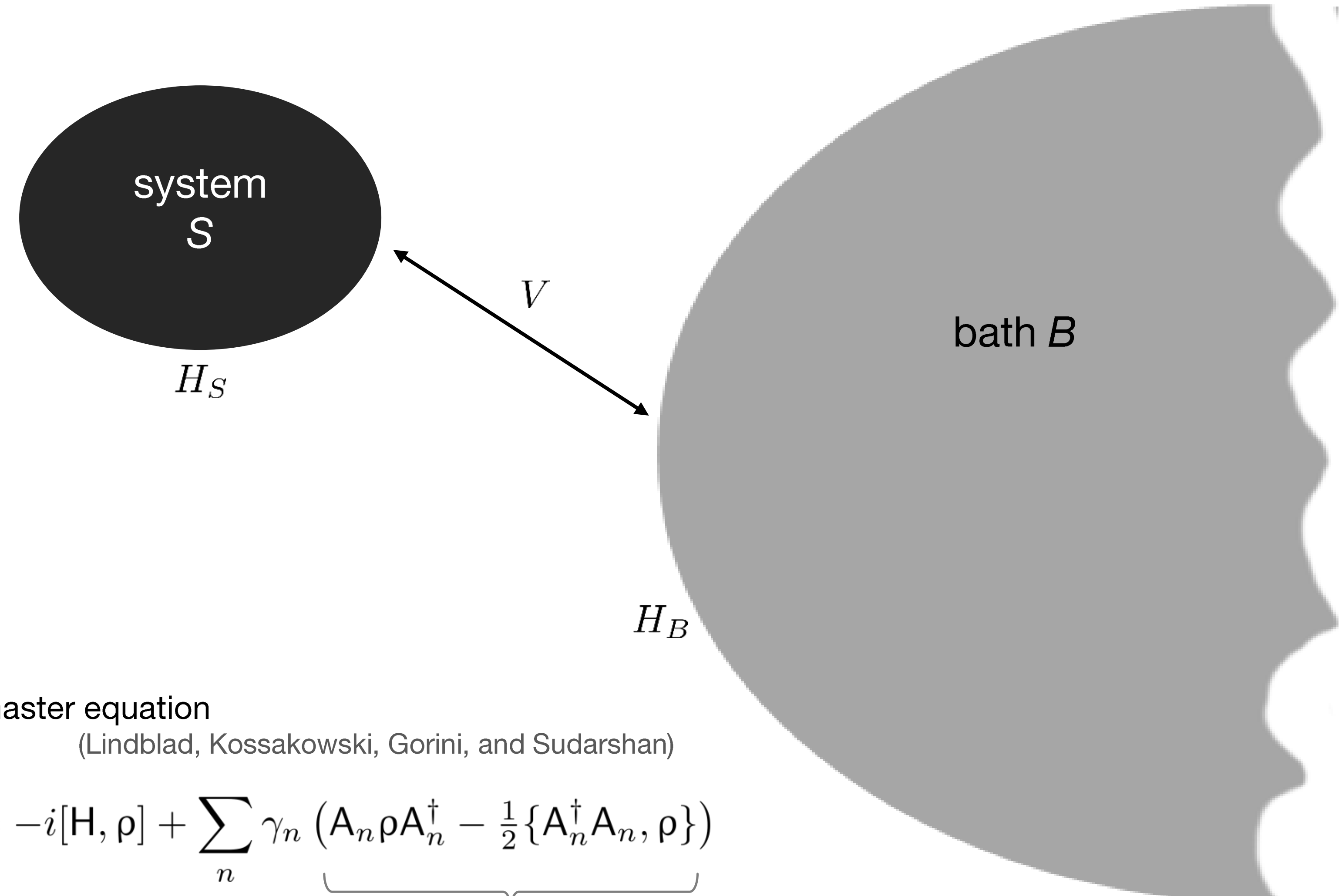
	Quantum Phase Transition	Dissipative Phase Transition
System Operator	Hamiltonian $H = H^\dagger$	Liouville superoperator $\mathbb{L} \neq \mathbb{L}^\dagger$
State	Ground state	Steady state
Transition	Switch to different ground state 	Switch to different steady state 

after:
E.M. Kessler et al.,
PRA 86, 012116 (2012)

2 Lindblad perturbation theory and Resummation



Open quantum systems



Lindblad master equation

(Lindblad, Kossakowski, Gorini, and Sudarshan)

$$\frac{d}{dt}\rho(t) = -i[\mathrm{H}, \rho] + \sum_n \gamma_n \underbrace{\left(\mathrm{A}_n \rho \mathrm{A}_n^\dagger - \tfrac{1}{2}\{\mathrm{A}_n^\dagger \mathrm{A}_n, \rho\}\right)}_{\mathbb{D}[\mathrm{A}_n]\rho}$$

Observables: $\langle \mathrm{M} \rangle = \mathrm{tr}\left(\mathrm{M}\, \rho_s\right)$

Steady state, stationary Lindblad eq.

$$\frac{d}{dt}\rho(t) = -i[\mathsf{H},\rho] + \sum_n \gamma_n \mathbb{D}[\mathsf{A}]\rho \quad = \mathbb{L}\rho$$

Liouvillian super-operator

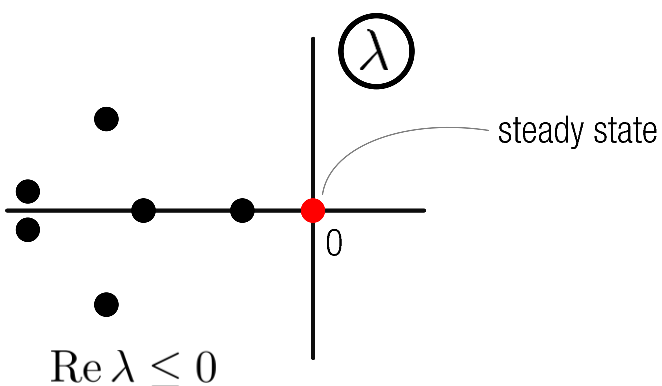
Steady state:

$$0 = \frac{d}{dt}\rho(t) = \mathbb{L}\rho \qquad \to \qquad \mathbb{L}\rho_{\mathrm{s}} = 0$$

stationary sol. to Lindblad master eq.

$$\leadsto \qquad \mathbb{L}|\mathsf{u}_\nu) = \lambda_\nu |\mathsf{u}_\nu)$$

Stationary Lindblad eq.



Lindblad Perturbation Theory (non-deg.)

Stat. Lindblad master eq. $\mathbb{L}|\mathrm{u}_\nu) = \lambda_\nu |\mathrm{u}_\nu)$

decompose Liouvillian: $\boxed{\mathbb{L} = \mathbb{L}_0 + \mathbb{L}_1}$

- controlled, analytical approximation

eigenvalues

$$\lambda_\nu^1 = (\mathrm{w}_\nu^0|\mathbb{L}_1|\mathrm{u}_\nu^0)$$

eigenstates

$$|\mathrm{u}_\nu^1) = \sum_{\mu\neq\nu} \frac{(\mathrm{w}_\mu^0|\mathbb{L}_1|\mathrm{u}_\nu^0)}{\lambda_\mu^0 - \lambda_\nu^0} |\mathrm{u}_\mu^0)$$

- directly study spectrum of Liouville super-operator

- resummation scheme

Benatti et al., J. Phys. A: Math. Theor. 44, 155303 (2011)
Li, Petruccione, JK, Scientific Reports 4, 4887 (2014)

Recursion relations

Eigenvalues:

$$\lambda_\nu^n = (\mathsf{w}_\nu^0|\mathbb{L}_1|\mathsf{u}_\nu^{n-1}) - \sum_{m=1}^{n-1} \lambda_\nu^m (\mathsf{w}_\nu^0|\mathsf{u}_\nu^{n-m})$$

Eigenstates:

$$|u_\nu^n) = -\frac{1}{\mathbb{L}_0 - \lambda_\nu^0}\left[\mathbb{L}_1|\mathsf{u}_\nu^{n-1}) + \sum_{m=1}^{n} \lambda_\nu^m |\mathsf{u}_\nu^{n-m})\right]$$

Steady state: $\lambda_0^j = 0$ (all orders)

$$|\rho_s) = \sum_j |\rho_j) \quad \Rightarrow \quad |\rho_j) = \underbrace{-\mathbb{L}_0^{-1}\mathbb{L}_1}_{\mathbb{U}}|\rho_{j-1}) \quad \Rightarrow \quad |\rho_j) = \mathbb{U}^j|\rho_0)$$

$$|\rho_j) = \underbrace{\mathbb{U}\mathbb{U}\mathbb{U}\cdots\mathbb{U}}_{j \text{ times}}|\rho_0) \qquad \mathbb{1} = \sum_\mu |\mathsf{u}_\mu)(\mathsf{w}_\mu|$$

(assuming completeness)

$$|\mathsf{u}_0^0) \xrightarrow{\mathbb{U}} |\mathsf{u}_{\mu_1}^0) \xrightarrow{\mathbb{U}} |\mathsf{u}_{\mu_2}^0) \cdots\cdots |\mathsf{u}_{\mu_{j-1}}^0) \xrightarrow{\mathbb{U}} |\mathsf{u}_{\mu_j}^0)$$

Resummation scheme

$$|\rho_s) = \sum_{j=0}^{\infty} \mathbb{U}^j |\rho_0) \quad \rightsquigarrow \quad ?$$

Idea: extract diagonal part

- start: $\mathbb{U}^1$ off-diag. $\Rightarrow$ $\Sigma_1 = 0$ and $\mathbb{T}_1 = \mathbb{U}$

diag. part

off-diag. part

- recursion: $\mathbb{T}_{j-1}\mathbb{U} = \Sigma_j + \mathbb{T}_j$

diag. part: $(\Sigma_j)_{\mu\nu} = \delta_{\mu\nu} (\mathbb{T}_{j-1}\mathbb{U})_{\mu\nu}$

$\Sigma_j|\mathsf{u}^0_\mu) = \Sigma_{j;\mu}|\mathsf{u}^0_\mu)$

solution: $\mathbb{T}_j = [\![[\![\cdots [\![[\![\mathbb{U}]\!] \mathbb{U}]\!] \cdots \mathbb{U}]\!] \mathbb{U}]\!]$ (j times)

$[\![\mathbb{A}]\!]$: off-diagonal part of $\mathbb{A}$

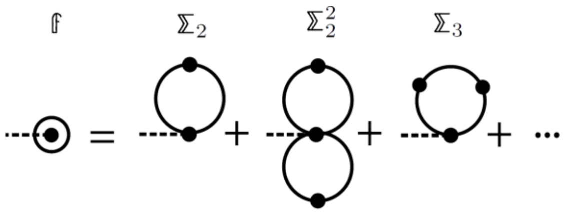
Ambiguity: $\mathbb{U}^3 = \mathbb{U}(\mathbb{U}^2)$ or $\mathbb{U}^3 = (\mathbb{U}^2)\mathbb{U}$

$\mathbb{U}^j = \underbrace{\mathbb{U}^{j-1}}\mathbb{U}$ systematic replacement rule

Resummation scheme (cont'd)

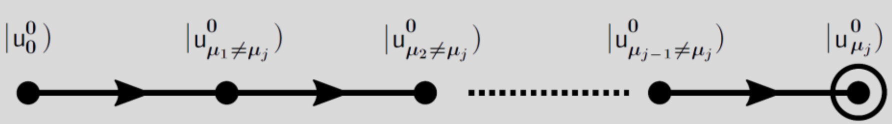
$$\rightsquigarrow \qquad |\rho_\mathrm{s}) = \sum_{j=0}^{\infty} \mathbb{U}^j |\rho_0) = \mathbb{f} \sum_{j=0}^{\infty} \mathbb{T}_j |\rho_0)$$

$$\begin{aligned} \mathbb{f} &= \mathbb{1} + \Sigma_2 + \Sigma_3 + \Sigma_4 + \Sigma_2^2 + \Sigma_5 + \Sigma_2\Sigma_3 + \Sigma_3\Sigma_2 \cdots \\ &= \sum_{n=0}^{\infty} \underbrace{(\Sigma_2 + \Sigma_3 + \cdots)}_{\Sigma \text{ (irred. diagrams only)}}{}^{n} = (\mathbb{1} - \Sigma)^{-1} \end{aligned}$$

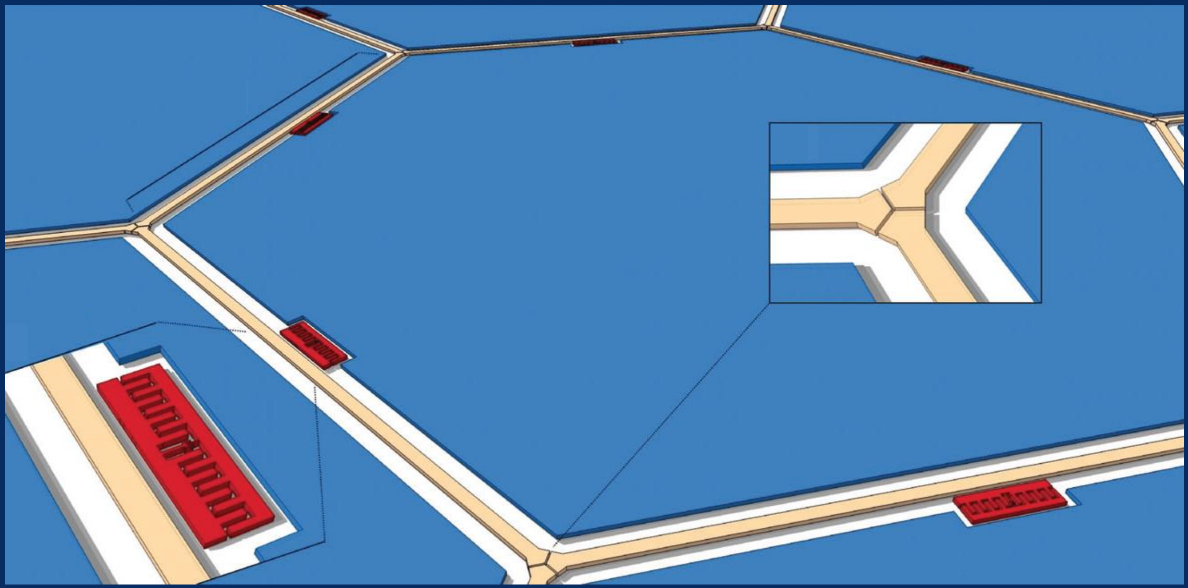


$$|\rho_\mathrm{s}) = \sum_{j=0}^{\infty} \mathbb{U}^j |\rho_0) = \sum_{j=0}^{\infty} \frac{1}{\mathbb{1} - \Sigma} \mathbb{T}_j |\rho_0)$$

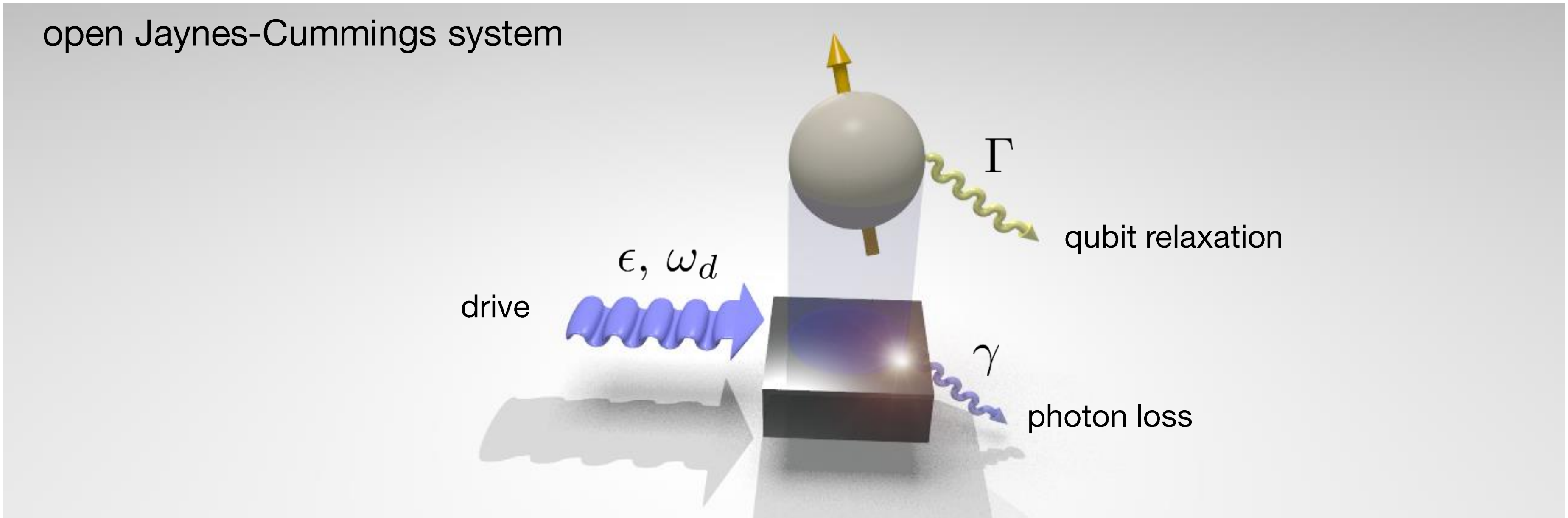
$$|\rho_\mathrm{s}^{(j)}) = \sum_{\mu_j} \sum_{\nu_1,\ldots,\nu_{j-1} \neq \mu_j} |\mathrm{u}_{\mu_j}^0) \left(\frac{1}{\mathbb{1} + \Sigma} \right)_{\mu_j \mu_j} (\mathbb{U})_{\mu_j \nu_{j-1}} (\mathbb{U})_{\nu_{j-1}\nu_{j-2}} \cdots (\mathbb{U})_{\nu_2 \nu_1} (\mathbb{U})_{\nu_1 0}$$

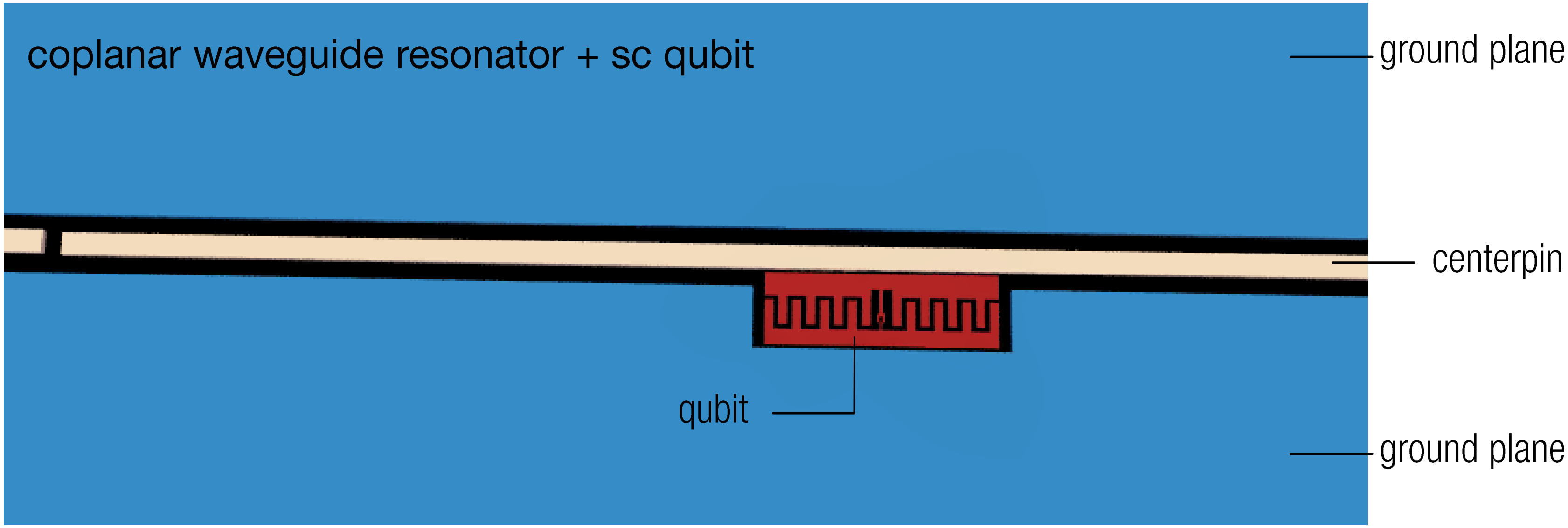


3 Application: Jaynes-Cummings lattice

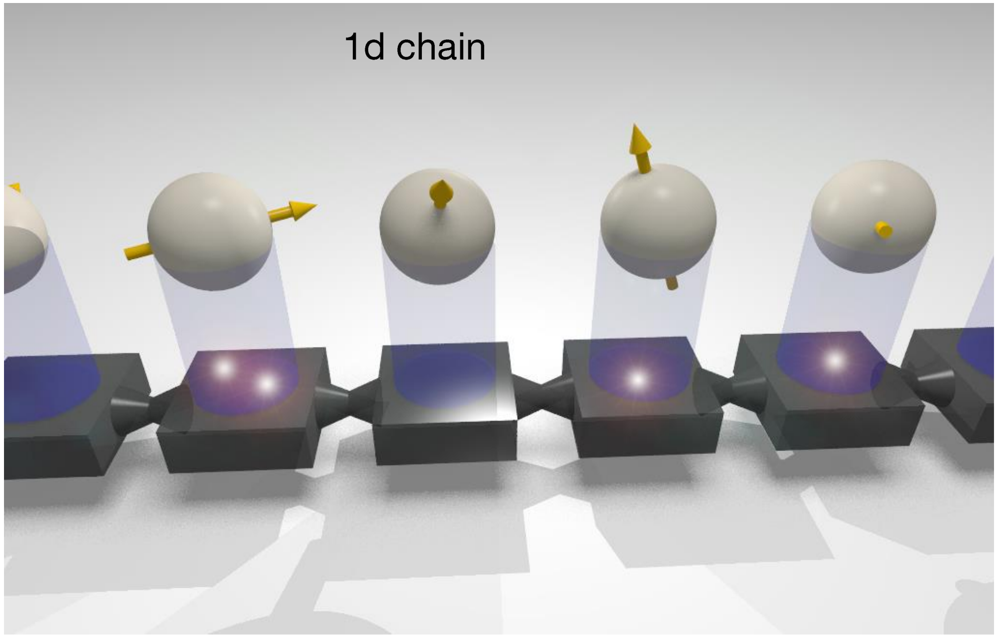


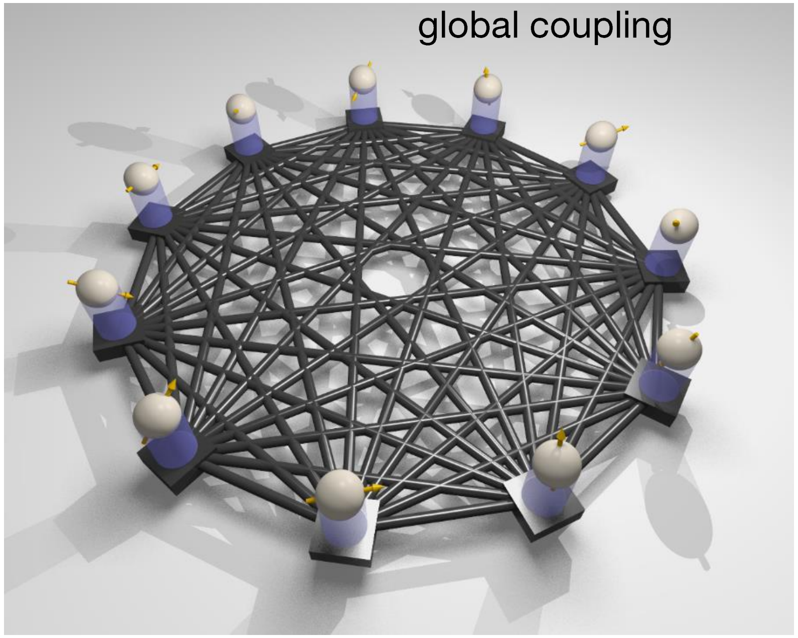
JC building block

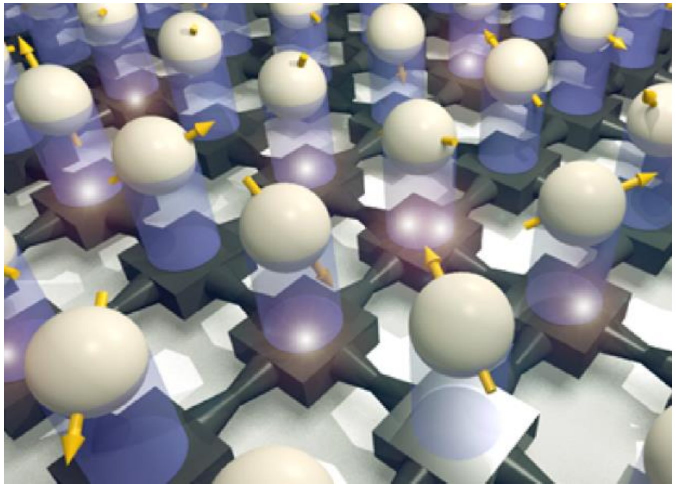




JC lattices







2d square lattice

REVIEWS

A. Tomadin and R. Fazio
J. Opt. Soc. Am. B 27, A130 (2010)

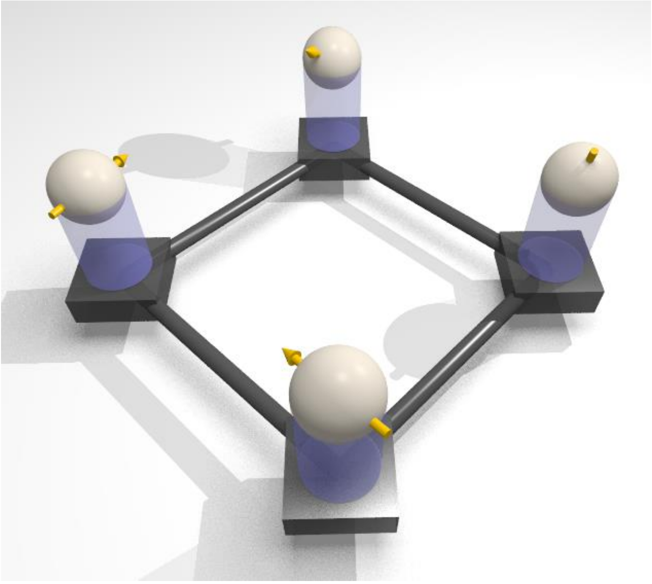
Houck, Tureci, JK
Nature Phys. 15, 115002 (2012)

Schmidt, JK
Ann. Physik 525, 395 (2013)

Exact numerics?

Lindblad master eq. $\frac{d}{dt}\rho = \mathbb{L}\rho = 0$

$$\mathrm{H}: \; N \times N \quad \Rightarrow \quad \mathbb{L}: \; N^2 \times N^2$$



"worse" than exact diagonalization
for closed system

e.g., 4 resonators
(up to 3 photons each)
4 qubits

❯ $\mathbb{L}$: 16 millions × 16 millions

Promising numerical schemes:
Cluster-MFT, DMRG, TEBD, Variational Methods, …

JC lattice model: pert. treatment

$$\mathsf{H} = \sum_{\mathbf{r}} \mathsf{h}^{\mathrm{JC}}_{\mathbf{r}} + t \sum_{\langle \mathbf{r},\mathbf{r}' \rangle} (\mathsf{a}^\dagger_{\mathbf{r}} \mathsf{a}_{\mathbf{r}'} + \mathrm{h.c.})$$

photon hopping

$$\mathsf{h}^{\mathrm{JC}}_{\mathbf{r}} = \delta\omega\, \mathsf{a}^\dagger_{\mathbf{r}} \mathsf{a}_{\mathbf{r}} + \epsilon(\mathsf{a}^\dagger_{\mathbf{r}} + \mathsf{a}_{\mathbf{r}}) + \delta\Omega\, \sigma^+_{\mathbf{r}} \sigma^-_{\mathbf{r}} + g(\mathsf{a}_{\mathbf{r}} \sigma^+_{\mathbf{r}} + \mathsf{a}^\dagger_{\mathbf{r}} \sigma^-_{\mathbf{r}})$$

resonators — drive — qubits — JC coupling

$$\dot{\rho} = -i[\mathsf{H}, \rho] + \gamma \sum_{\mathbf{r}} \mathbb{D}[\mathsf{a}_{\mathbf{r}}]\rho + \Gamma \sum_{\mathbf{r}} \mathbb{D}[\sigma^-_{\mathbf{r}}]\rho$$

photon loss — qubit relax.

Perturbation: JC interaction g or photon hopping t

done here

problem: no exact solution for driven, damped JC site

Structure of PT corrections

$\mathbb{L}_0$: Damped resonator spectrum: third-quantization formalism

T. Prosen, NJP 10, 043026 (2008)

$$|\mathrm{r}^{\mathbf{k}}_{mn}) = (\mathbb{b}^{\dagger\dagger}_{\mathbf{k}})^m (\beta^{\dagger\dagger}_{\mathbf{k}})^n ||0\rangle\langle 0|)/\sqrt{m!n!},$$

$$\lambda^{\mathbf{k}}_{mn} = -i\,\delta\omega_{\mathbf{k}}(m-n) - \tfrac{\gamma}{2}(m+n)$$

$\bigotimes_{\mathbf{r}}$ driven, damped spin $|\mathrm{u}^{\mathbf{r}}_{\mu})$

0th order:

$$|\rho_0) = \bigotimes_{\mathbf{k}} |\mathrm{r}^{\mathbf{k}}_{00}) \otimes \bigotimes_{\mathbf{r}} |\mathrm{u}^{\mathbf{r}}_0)$$

1st order:

$$|\rho_1) = \sum_{\mathbf{k},\mathbf{r}} |\rho^1_{\mathbf{kr}}) \otimes |\rho^0_{\cancel{\mathbf{k}}\cancel{\mathbf{r}}})$$

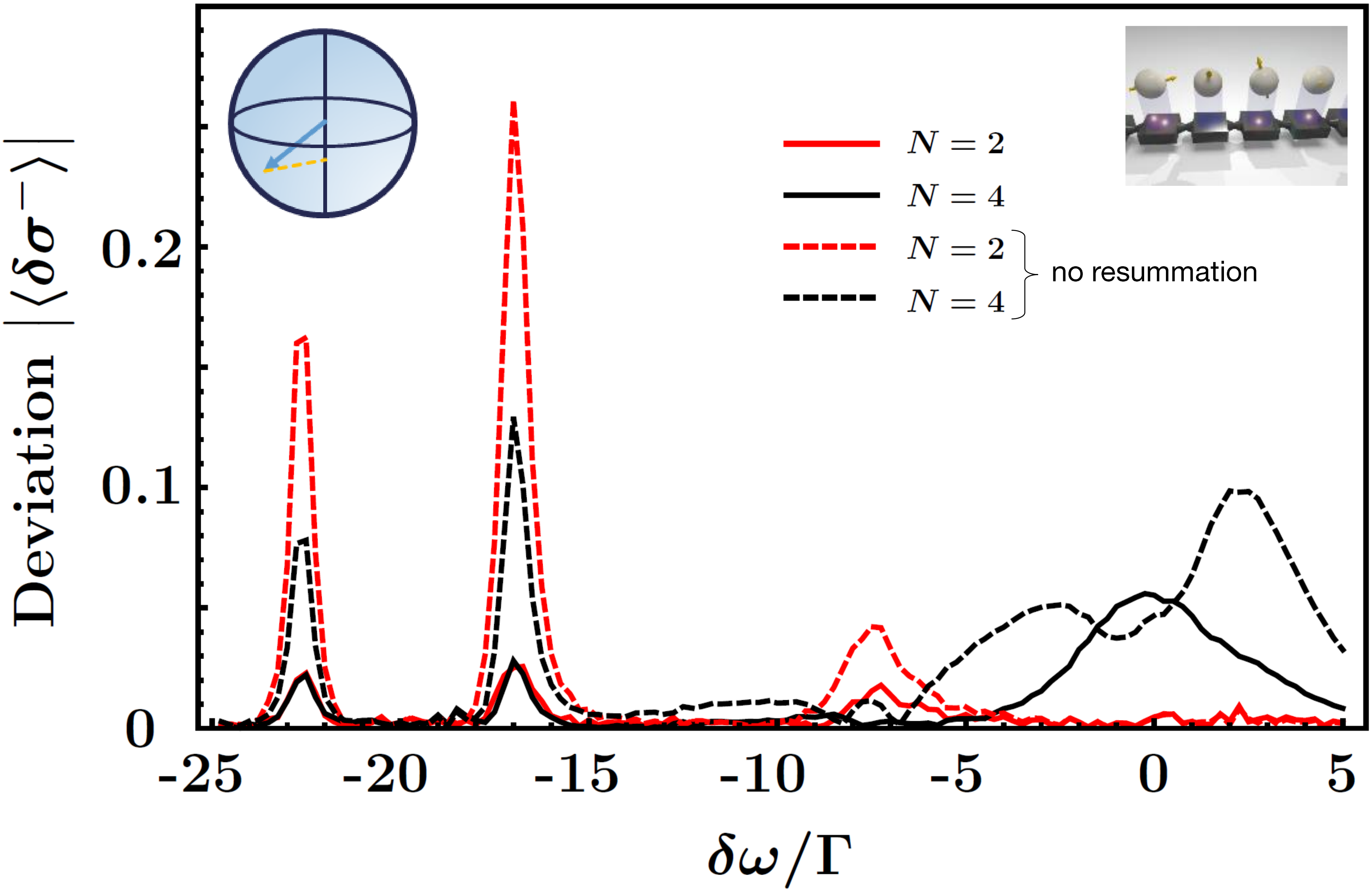
cluster

2nd order:

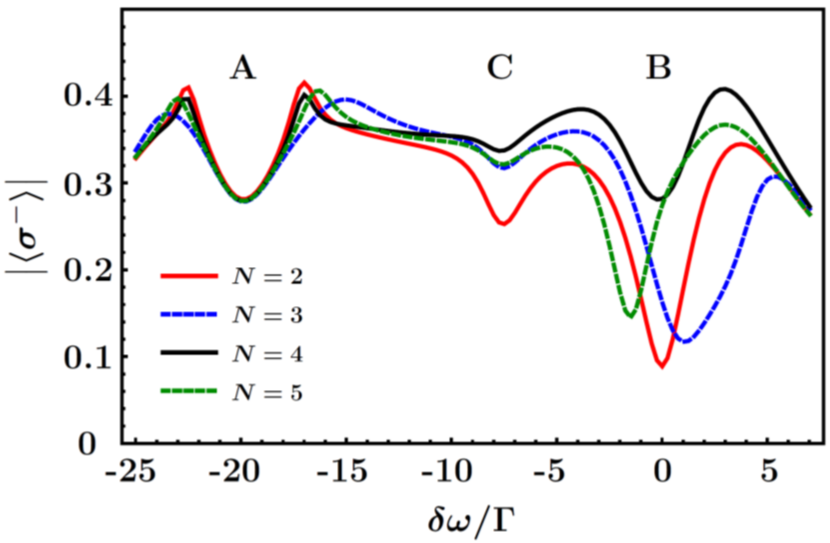
$$|\rho_2) = \sum_{\mathbf{k},\mathbf{k}',\mathbf{r},\mathbf{r}'} |\rho^2_{\mathbf{kk}'\mathbf{rr}'}) \otimes |\rho^0_{\cancel{\mathbf{k}}\cancel{\mathbf{k}'}\cancel{\mathbf{r}}\cancel{\mathbf{r}'}})$$

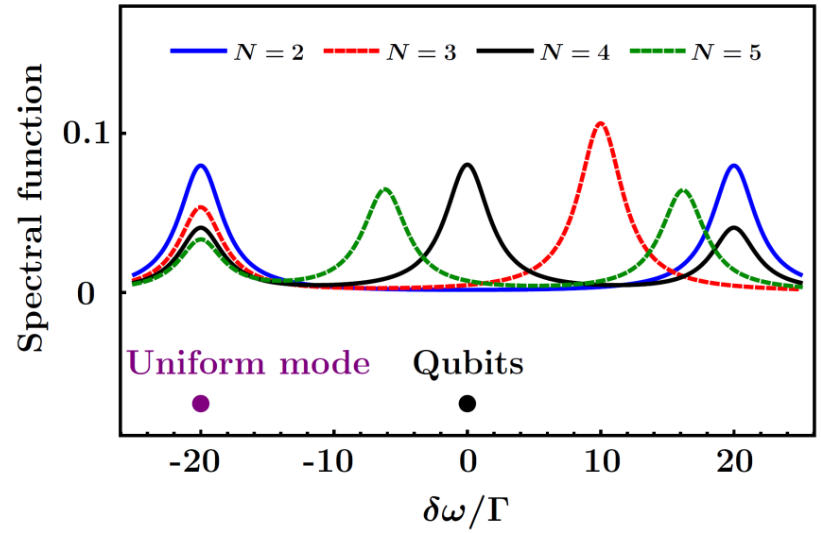
$\vdots$

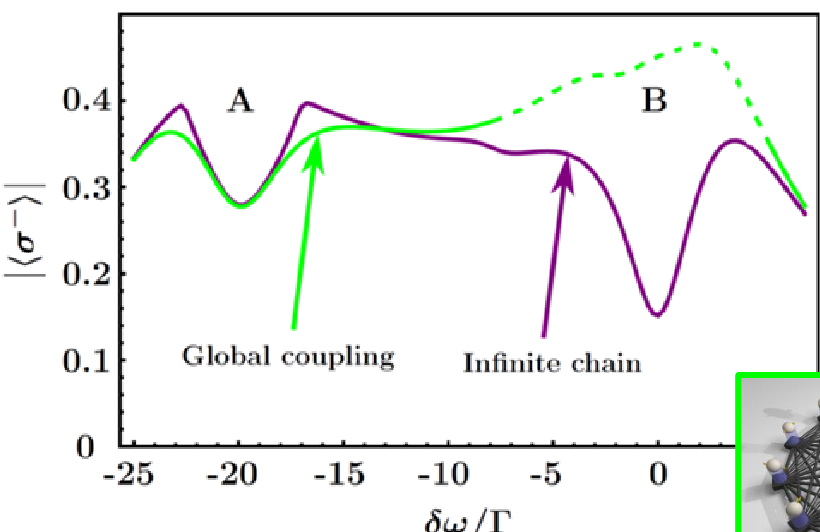
Comparison of results

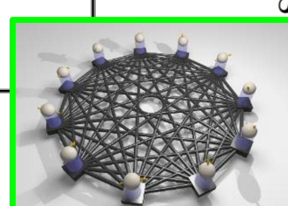


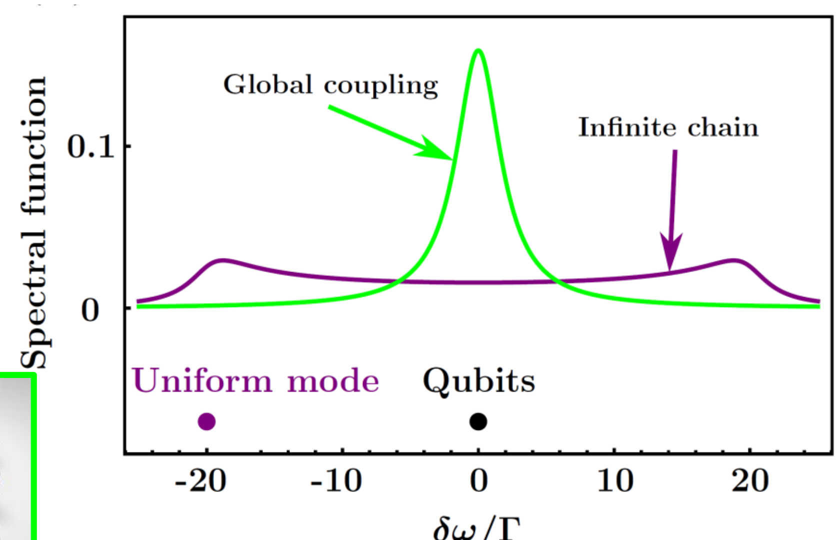
JC lattice results











Summary

Motivation: validation of circuit QED quantum simulators (Houck Lab)

Andrew Houck
(Princeton)

1

- simple stationary Lindblad perturbation theory
- use resummation to go beyond finite order

Andy Li
(Northwestern)

2

- application to JC lattices
- comparison w/ exact solutions, results

Francesco Petruccione
(U of KwaZulu-Natal)

Goal: validation of circuit QED quantum simulators

FIG. 5. Comparison between the perturbative results and the exact results obtained by quantum trajectories methods. **(a)** The steady-state expectation value $\left|\langle\sigma^-\rangle\right|$ of the spin lowering operator is plotted as a function of the detuning $\delta\omega$ for $\delta\Omega = \delta\omega$, $g/\Gamma = 3$, $\epsilon/\Gamma = 20$, $\kappa_0/\Gamma = 10$, $\gamma/\Gamma = 4$. The exact results (points) for $N = 2$ and $N = 4$ are well approximated by the perturbative results with SE corrections (solid lines). **(b)** The deviation $\left|\langle\delta\sigma^-\rangle\right|$ of the steady-state expectation value $\left|\langle\sigma^-\rangle\right|$ from the exact results is plotted as a function of the detuning $\delta\omega$ using the same set of parameters. In general, the perturbative results without SE corrections (dashed lines) show a much larger deviation from the exact results than the one with SE corrections (solid lines).