Engineering and probing many-body states of light in driven-dissipative arrays

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Acknowledgements

- Thanks to the organizers for scheduling my talk on the evening of the eve of Thanks Giving! ☺
- Thanks to you for sticking around, it seems everyone is on the road to go home today!

Once you are here, it means either that

A) You are really interested in what I will talk about- thus I prepared a slightly more detailed talk of **only 175 slides**…!

B) You are more than 10.000 miles from home, so you are basically stuck …! It is all the same to me ;-)

In any case, I will try to be quick ;-)

Research in our group

*Engineering strongly correlated photonic states in quantum nonlinear set ups <u>for **in and out of**</u> equilibrium many-body simulations.*

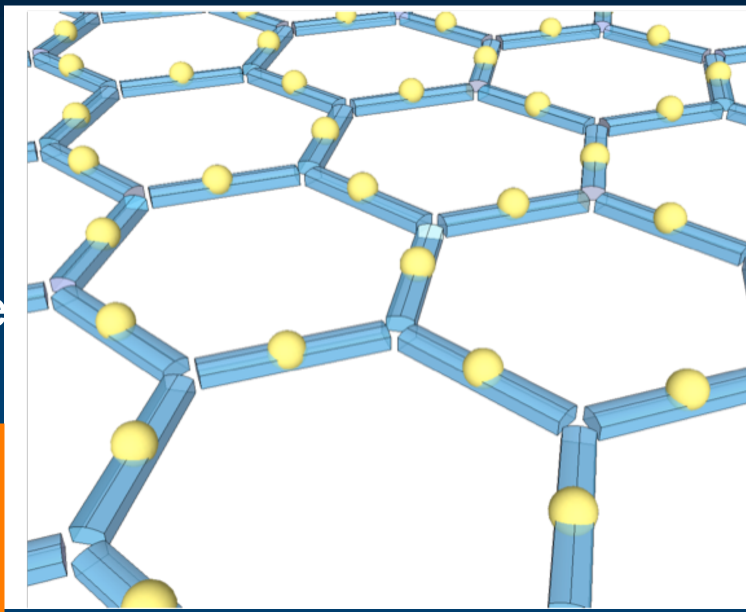
Tools: Open systems approaches, MPS based numerics, Scattering methods

Platforms:

I) Coupled cavity QED arrays (Circuit-QED)

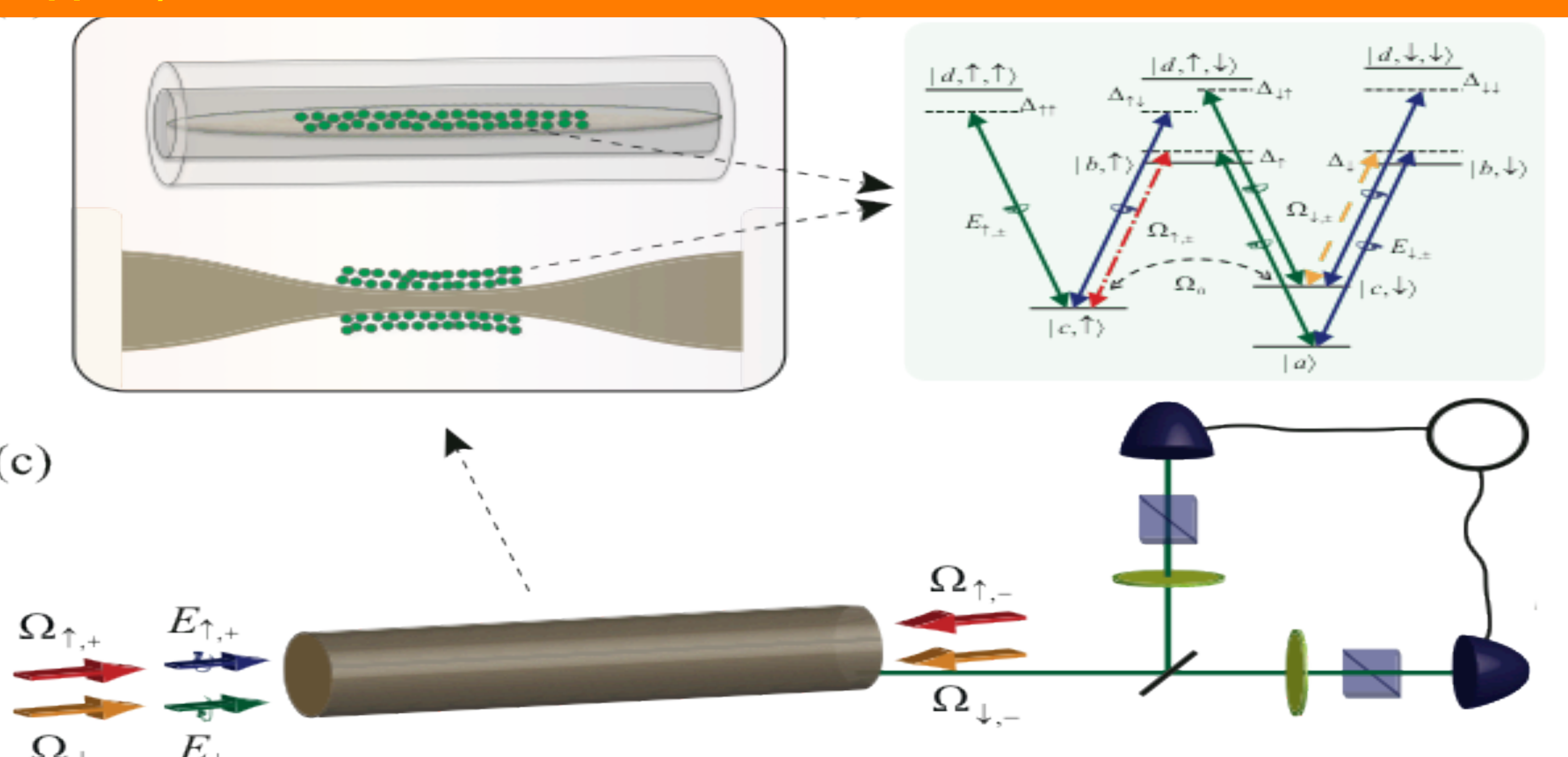
Equilibrium and driven-dissipative regimes

I) Slow light set ups with strong photon nonlinearities for continuous field theorie light: Luttinger liquids and Interacting relativistic theories (Thirring model)

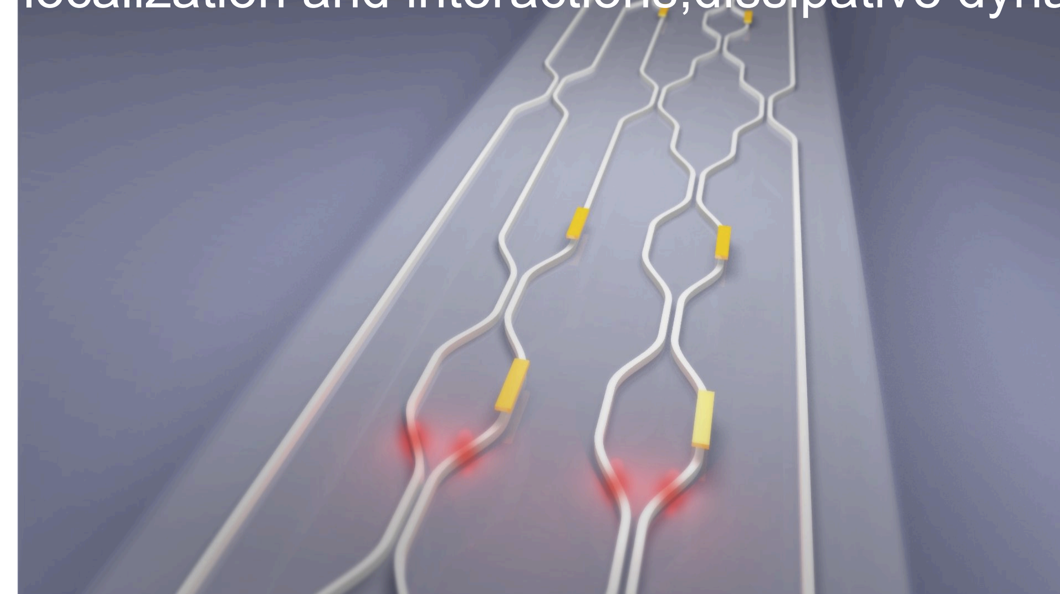


1) C. Noh, D. A, "Many-body physics with strongly-correlated photons" review in Reports in Progress in Physics, 2015(to appear)

2) Book in Springer "Quantum simulations with light", editor D.A, contributions Fazio, Hafezi, Ciuti, Carussoto, Yamamoto, Na, Schmidt, Blatter, Jacksh, Szameit, Solano and othersto appear)



iii) Integrated waveguide arrays for QIP and simulations (Majorana dynamics, Anderson localization and interactions,dissipative dyna



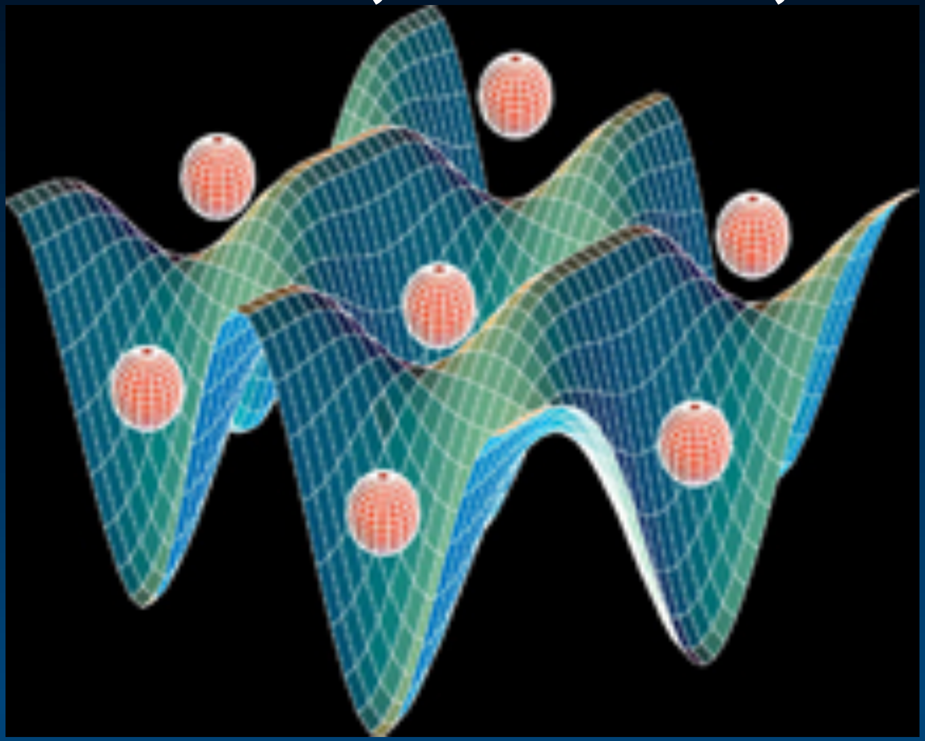
Outlook of this talk

- Quantum simulations of equilibrium many-body physics with cavity arrays: JCH model, photon-blockade Mott transitions,
- Driven dissipative arrays: Out of equilibrium phases and methods
  - Photon fermionization and crystallization in 1D-Master equations with tensor networks
  - Probing via scattering or *how to do spectroscopy of many-body models with photonic quantum simulators*
- Future directions and open questions

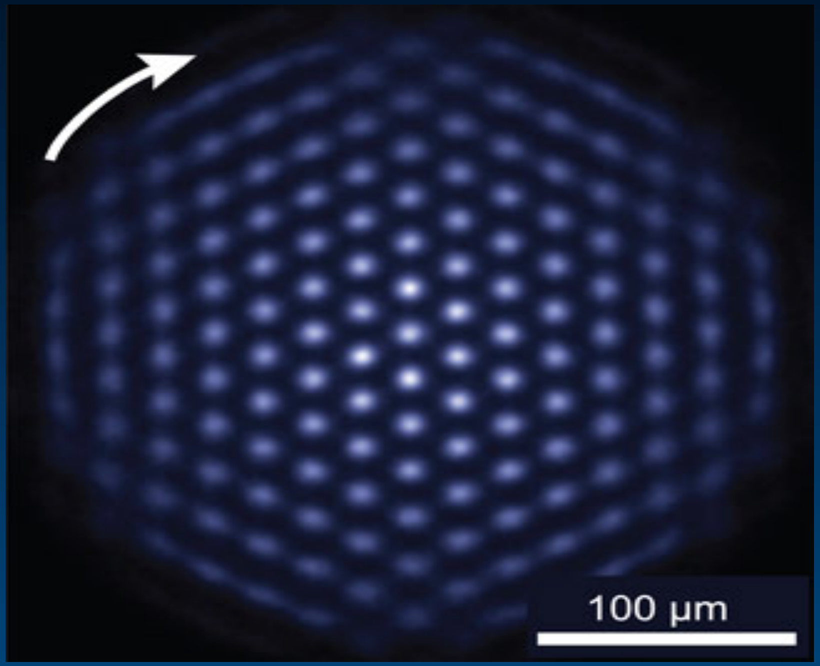
(If time: Exotic physics in photonic chips-Majorana equation and dynamics)

1) C. Noh, D. A, "Many-body physics with strongly-correlated photons " Reports in Progress in Physics, 2015
2) Volume in Springer "Quantum simulations with light", editor D.A, contributions Fazio, Hafezi, Ciuti, Carussoto, Yamamoto, Na, Schmidt, Blatter, Jacksh, and others

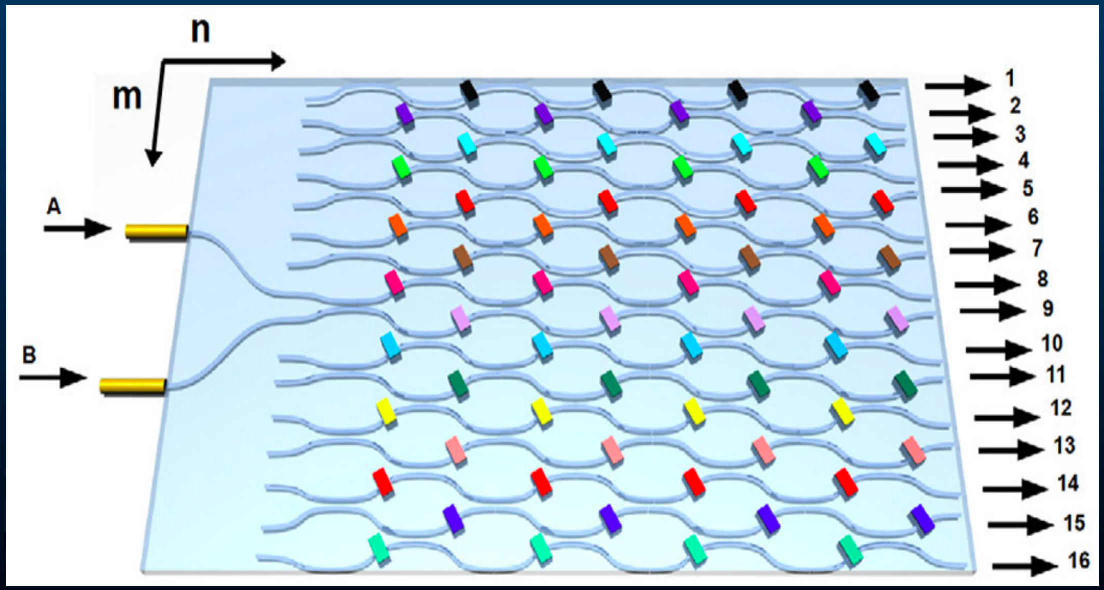
Quantum simulation of many-body physics : Ions, atoms, linear optics, JJ arrays



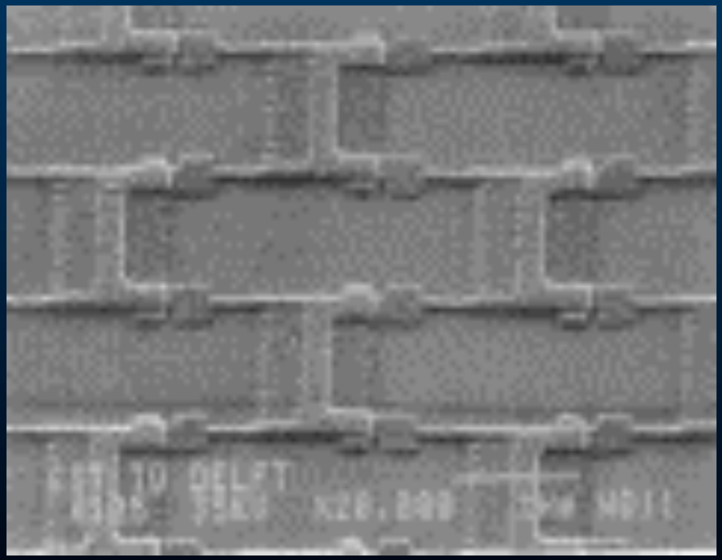
Atoms: Great success in many (site/atoms) Hubbard models



Ions: Great for small scale quantum magnets and effects requiring high local control



Linear optics: Topological effects, Boson sampling, Chemistry



JJ arrays

Quantum simulators

A ``working´´ definition of a quantum simulator could be:

According to Maciej Lewenstei

I. Quantum simulator is an experimental system that mimics a simple model, or a family of simple models of condensed matter, high energy physics, etc.

II. The simulated models have to be of some relevance for applications and/or our understanding of challenges of condensed matter, high energy physics, or more generally quantum many body physics.

III. The simulated models should be computationally very hard for classical computers (meaning= no efficient algorithm exists, or systems are too big). Exceptions from this rule are possible for quantum simulator that exhibit novel, only theoretically predicted and not yet observed phenomena (simulating ≠ simulating and observing).

IV. Quantum simulator should allow for broad control of the parameters of the simulated model, and for control of preparation, manipulation and detection of states of the system. In particular, it should allow for validation!

Quantum simulators

What shall we simulate?

- Statics at zero temperature – ground state and its properties.
- Statics (equilibrium) at non-zero temperature
- Dynamics (Hamiltonian, but out of equilibrium)
- Dissipative dynamics

Simulating the insulator to superfluid quantum phase transition with cold atoms

$U \gg J$ and $N/M = n$ integer

$$|gs\rangle = \prod_j |n\rangle_j$$

Mott insulator

$$\langle a_j^\dagger a_j \rangle = \langle gs | a_j^\dagger a_j | gs \rangle = n$$

$$\left\langle \left(a_j^\dagger a_j - n \right)^2 \right\rangle =$$

$$= \langle gs | \left(a_j^\dagger a_j \right)^2 | gs \rangle - n^2 = 0$$

$U \ll J$

$$|gs\rangle = \frac{1}{\sqrt{N!}} \left(b_0^\dagger \right)^N |0\rangle$$

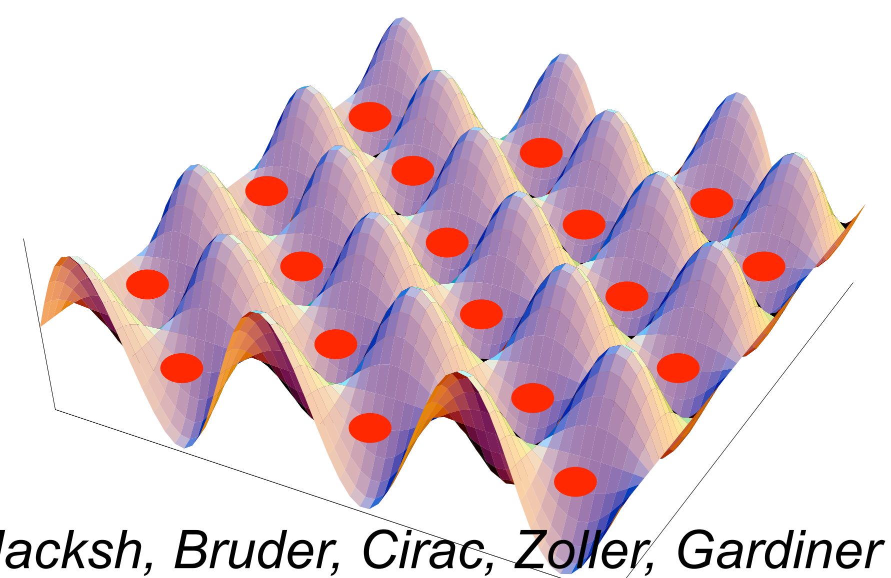
superfluid

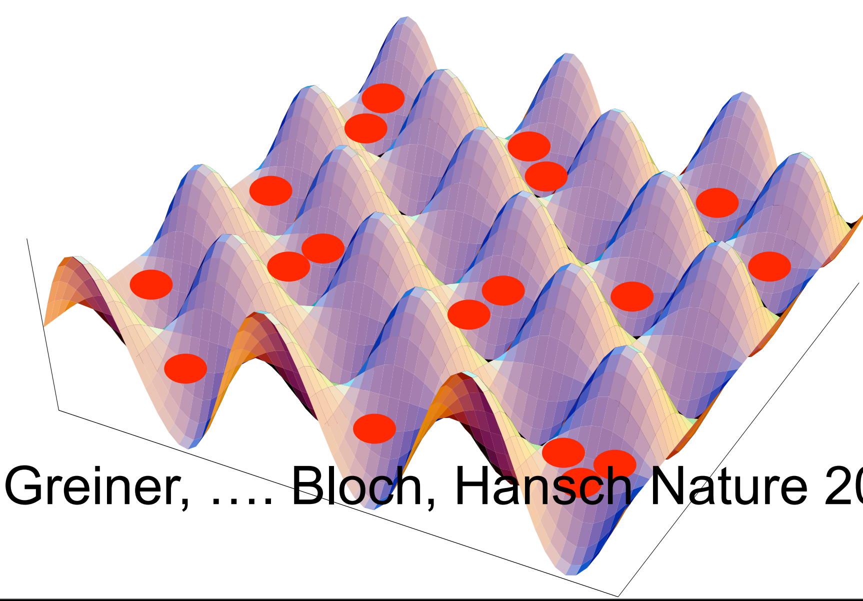
$$\langle a_j^\dagger a_j \rangle = \frac{1}{M} \langle gs | b_0^\dagger b_0 | gs \rangle = n$$

$$\left\langle \left(a_j^\dagger a_j - n \right)^2 \right\rangle =$$

$$= \frac{1}{M^2} \sum_k \langle gs | b_0^\dagger b_k b_k^\dagger b_0 | gs \rangle - n^2 =$$

$$= \frac{N(N-1)}{M^2} \approx n^2$$



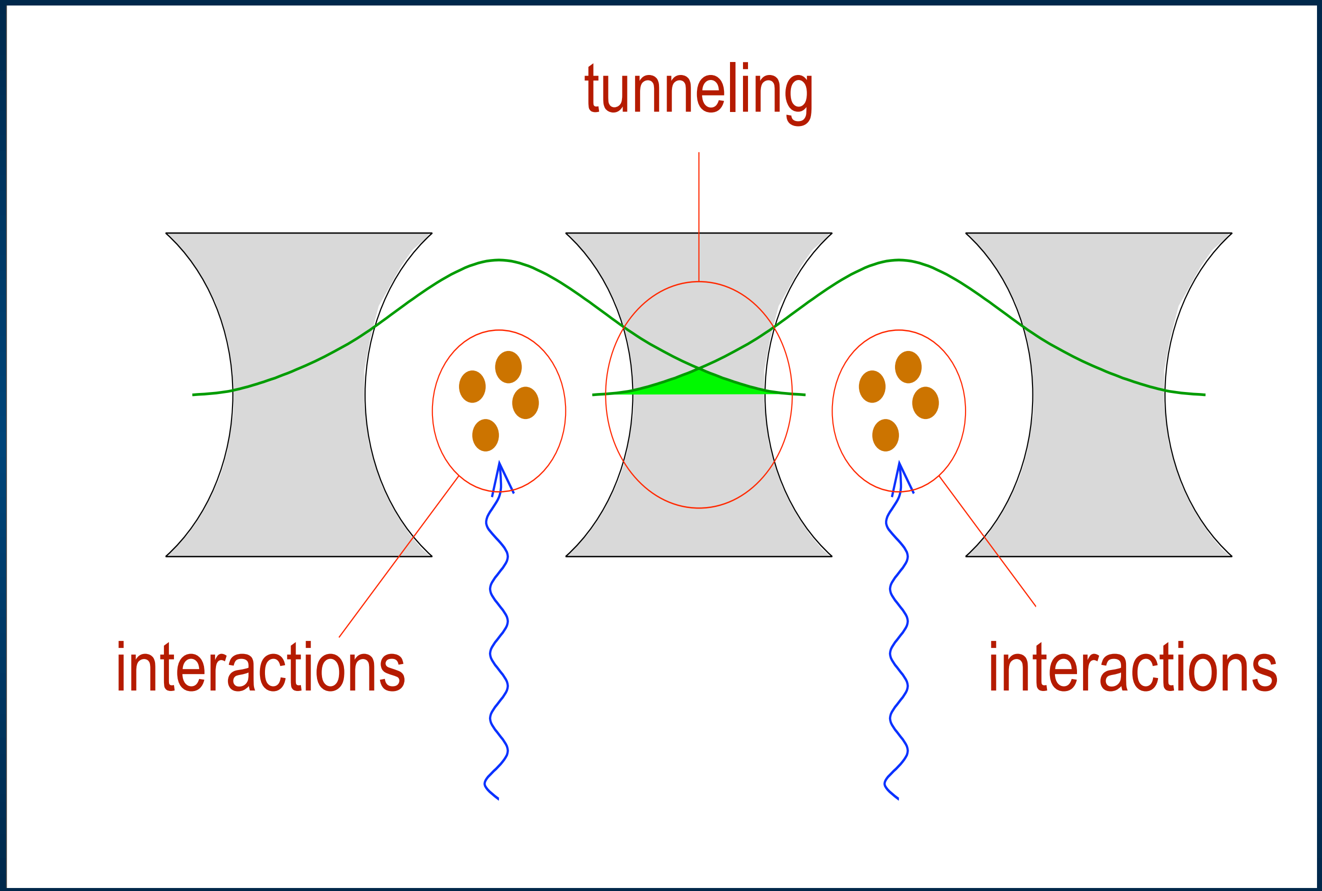


Experiment: Greiner, …. Bloch, Hansch Nature 2004

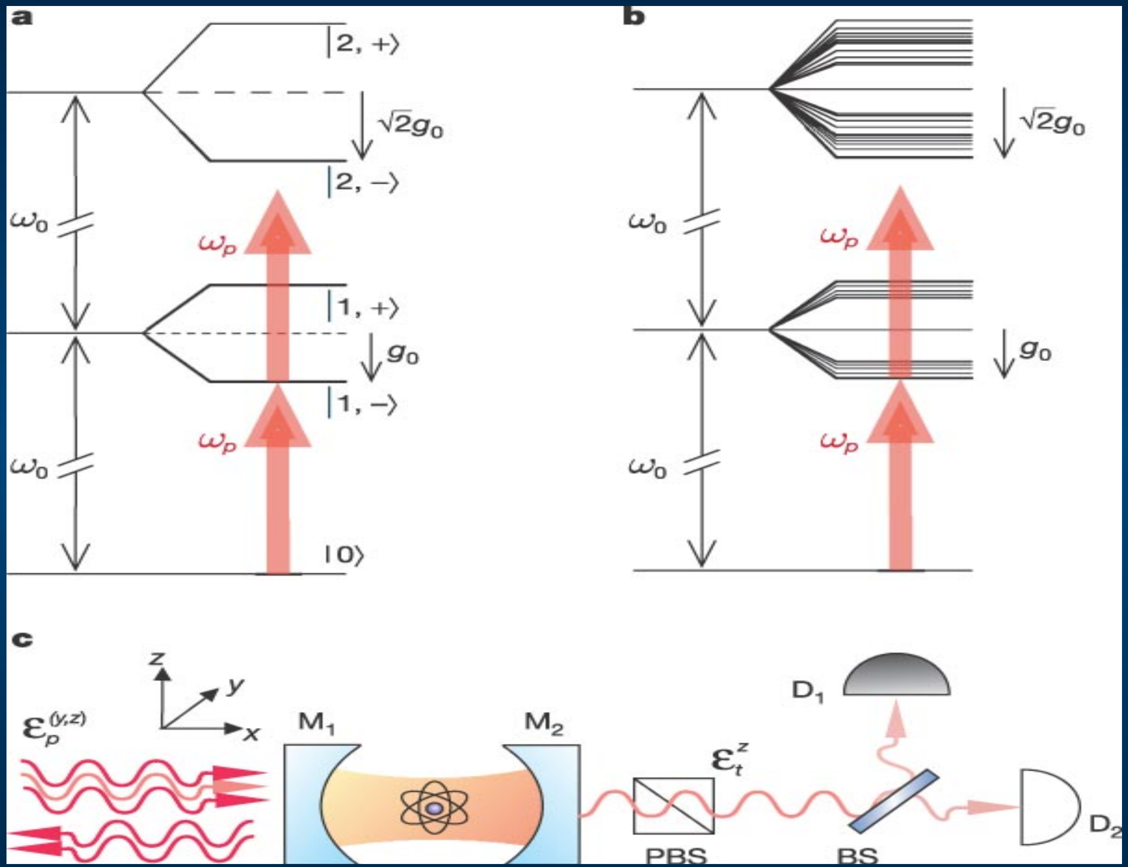
Theory: Jacksh, Bruder, Cirac, Zoller, Gardiner PRL 1998

Photons for quantum simulations of many-body effects?

A "lattice" for photons is needed:
Need to "couple" cavities (tough).



Strong light-matter coupling is needed for
The necessary photon-photon interactions

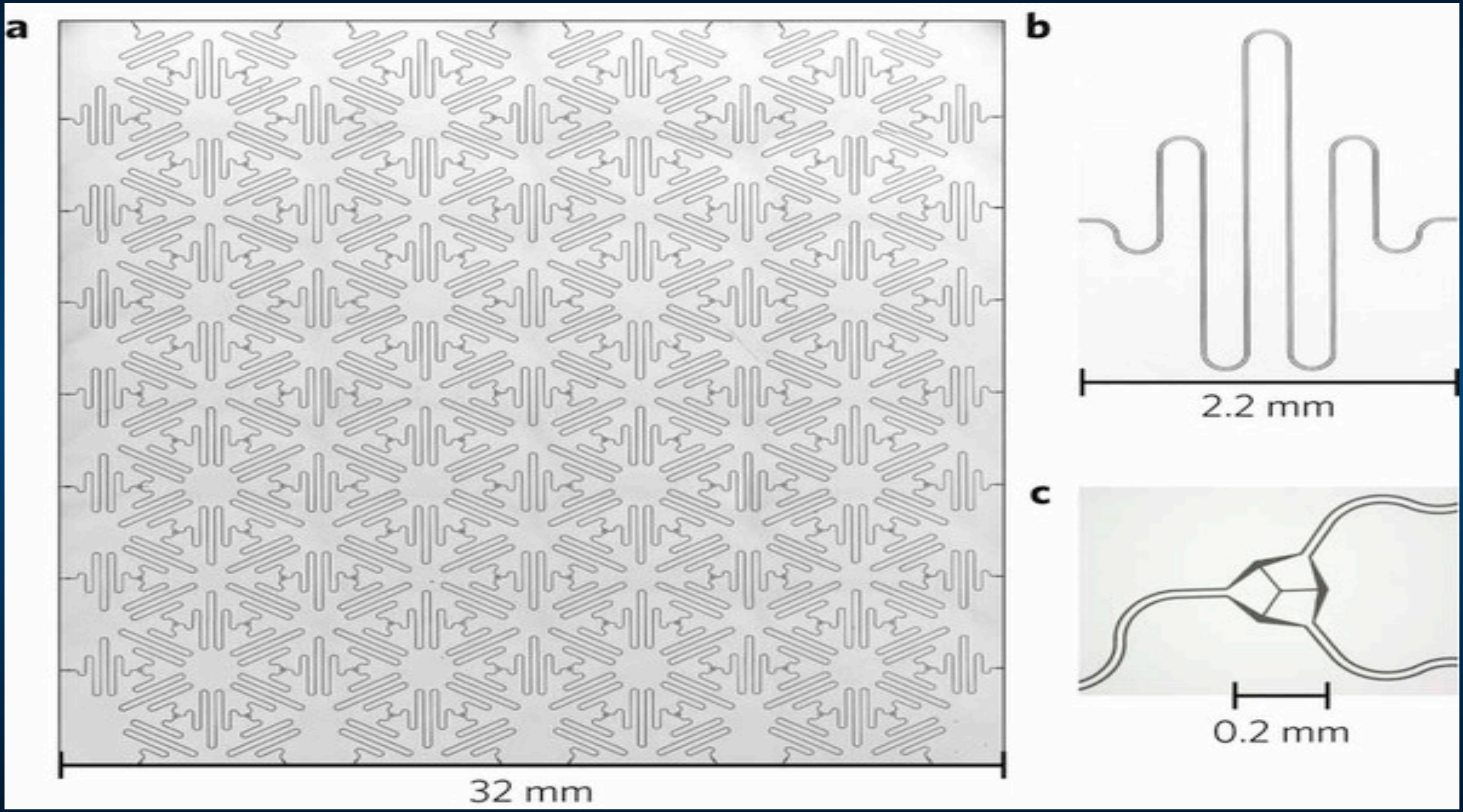


Photon blockade effect was there (Jeff Kimble et al., 2005, Imamoglu et al 1997)but coupling cavities and in the strong-coupling regime was tough until….

DGA , Santos, Bose "Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays" Phys. Rev. A (Rap. Com.) vol. 76, 031805 (2007).

DGA, "Photonic quantum simulators" Innovation Magazine vol. 9, Issue 2, 2010
Mark Mark Buchanan "Engaging photons in light conversation" New Scientist, 13 January 2007, p. 42.

Experimental coupled circuit QED cavities now



More than 200 microwave cavities coupled in a Kagome lattice!

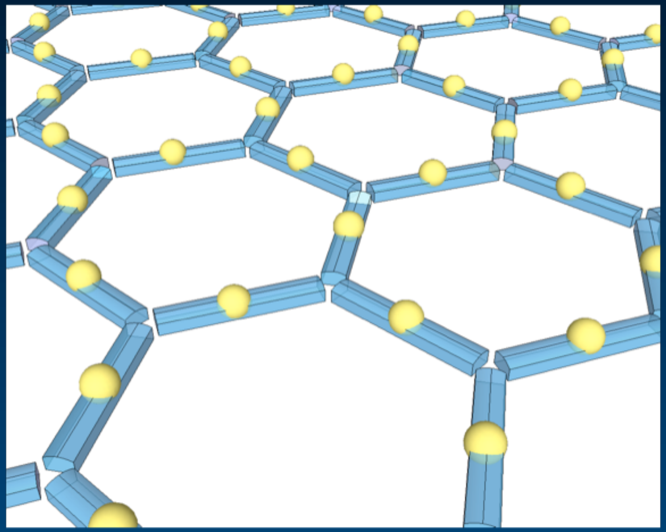
Houck lab, Princeton Univ. No atoms added (yet…)

Retated experimental efforts in small scale (5-10) cavities with atoms/ nonlinearities: ETH, Munich, UCSB (Google), MIT, and many others

Jaynes-Cummings-Hubbard model in cavity arrays

$$H^{JCH} = \omega_d \sum_{k=1}^{N} a_k^{+} a_k + \omega_0 \sum_{k} |e\rangle_k \langle e|_k + g \sum_{k=1}^{N} (a_k^{+} |g\rangle_k \langle e|_k + H.C)$$

$$-\kappa \sum_{k,k'=1}^{N} (a_k^{+} a_{k'+1} + H.C)$$



Jaynes-Cummings Hubbard model:

1)A single two level system coupled to a resonator (Jaynes-Cummings). The two-level system can be a real or artificial one (quantum dots, transmons, etc)

2) Resonators coupled through evanescent/capacitive coupling and form a lattice

DGA , Santos, Bose "Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays" Phys. Rev. A (Rap. Com.) vol. 76, 031805 (2007).

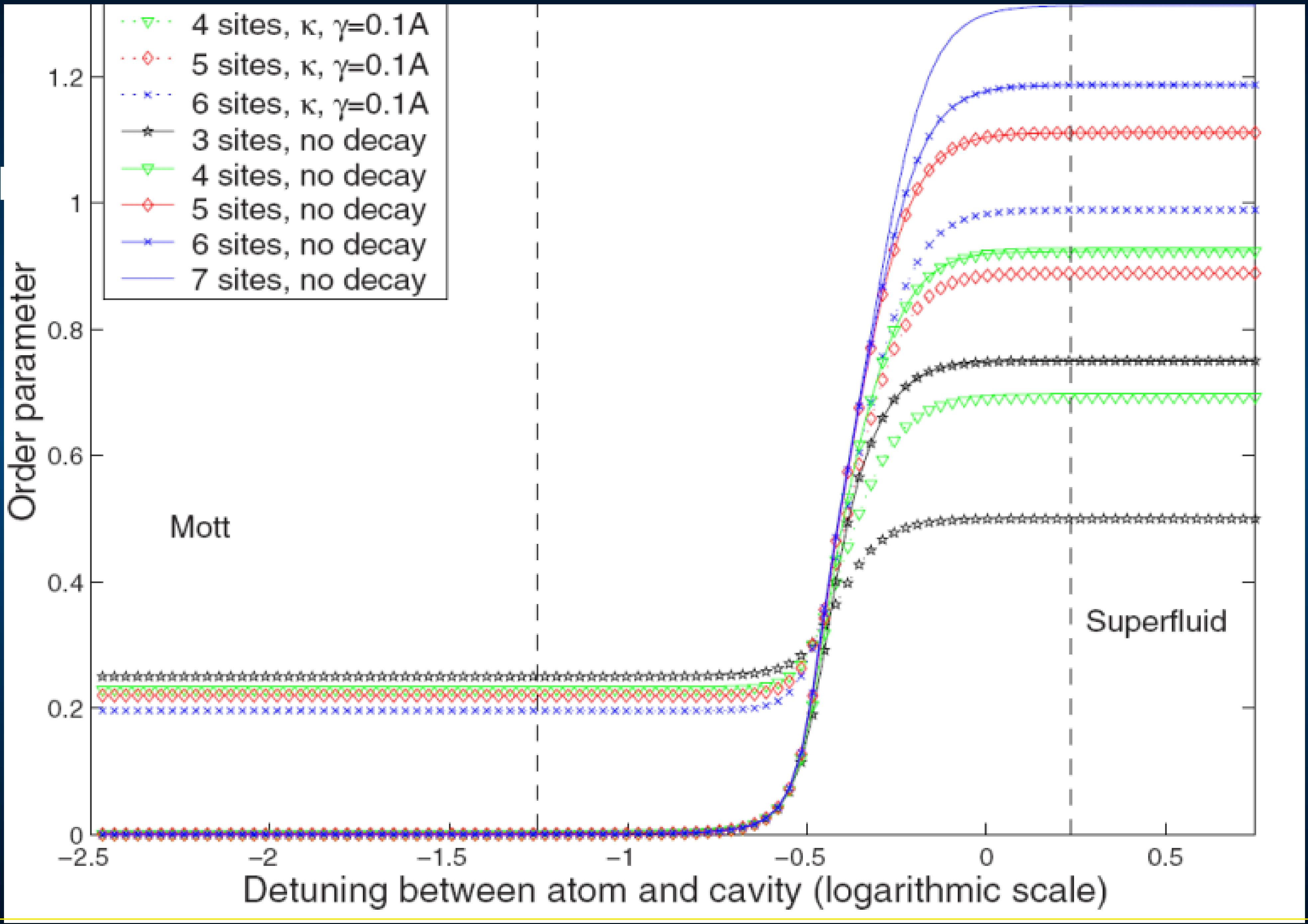
Parallel work by Hartmann et al, using external lasers and many atoms, and later by Greentree et al for 2D with mean-field

Photon blockade induced Mott-transition in coupled QED cavity arrays

Variance of the <u>total</u> excitations per site

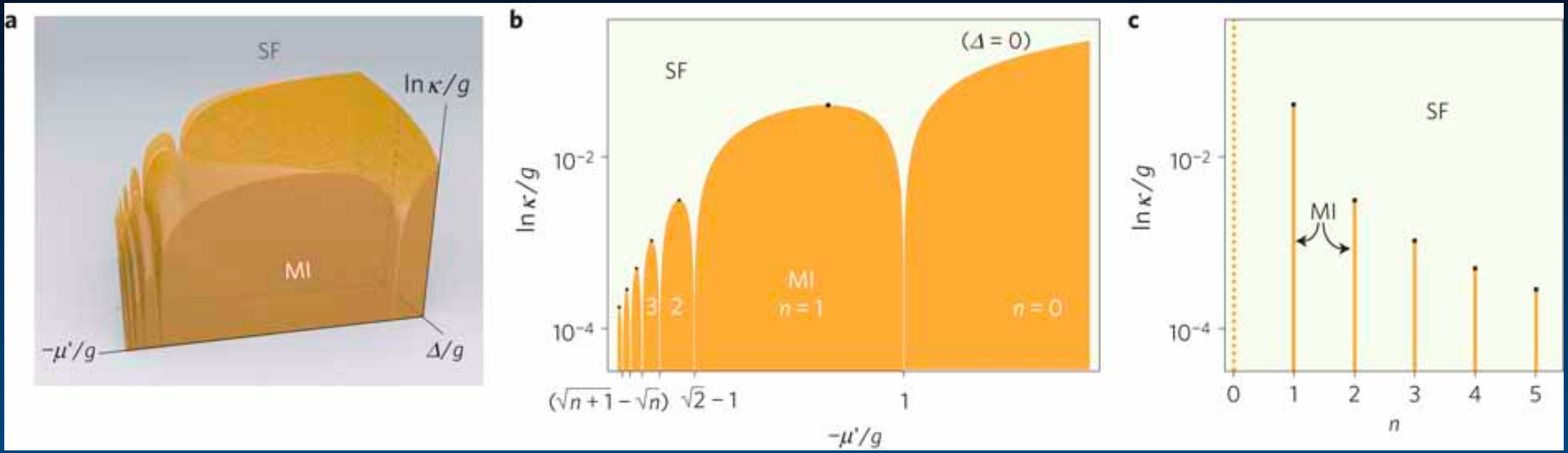
$$Var(N_k)$$

$$N_k = \sum_k a_k^+ a_k + \sigma_\kappa^+ \sigma_\kappa$$



DGA, M. Santos, S. Bose, "Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays" Phys. Rev. A (Rap. Com.) vol. 76, 031805 (2007).

Jaynes-Cummings-Hubbard phases



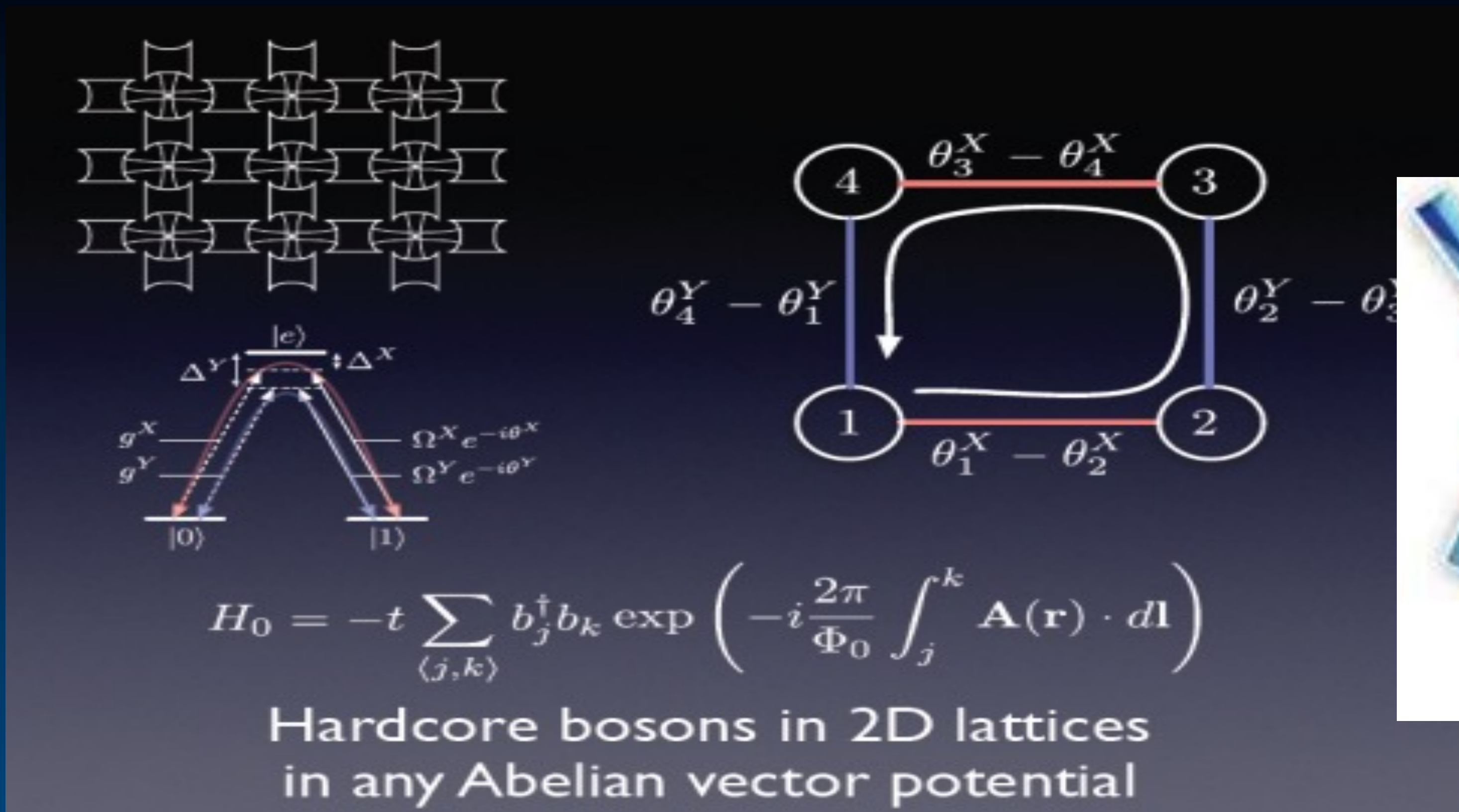
$$H^{JCH} = \omega_d \sum_{k=1}^{N} a_k^+ a_k + \omega_0 \sum_k |e\rangle_k \langle e|_k + g \sum_{k=1}^{N} (a_k^+ |g\rangle_k \langle e|_k + H.C)$$

$$-\kappa \sum_{k,k'=1}^{N} (a_k^+ a_{k'+1} + H.C)$$

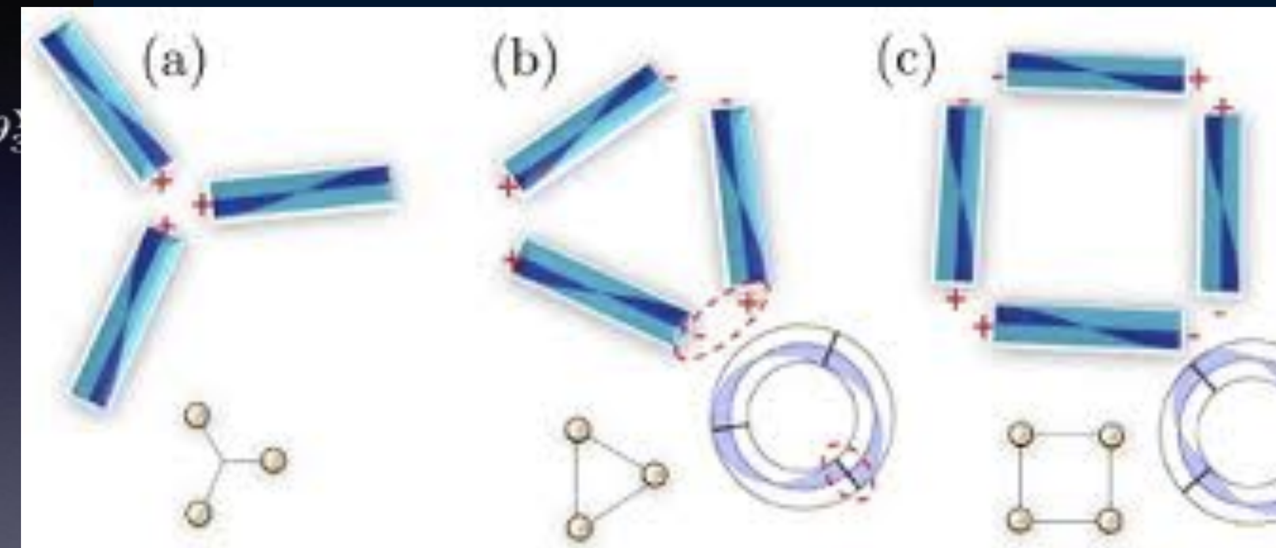
"Photon blockade induced Mott transitions and XY spin models in coupled cavity arrays" (arXiv: ***June 06****) Phys. Rev. A (Rap. Com.) vol. 76, 031805 (2007).*

Reviews: Rossini and Fazio JOSA B, Koch, Schmidt, Tureci Nature Physis 2012,
Angelakis,Noh, Reports in Progress in Physics 2015,
Multi author Volume in Springer 2015

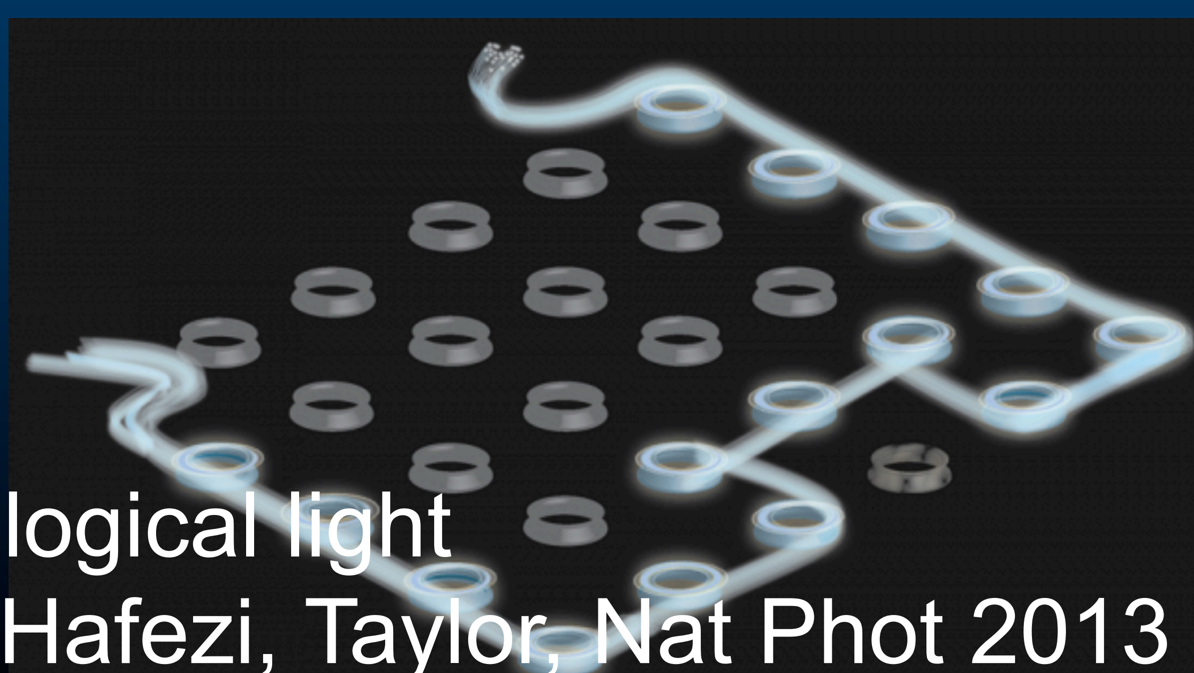
Gauge fields with photons and polaritons



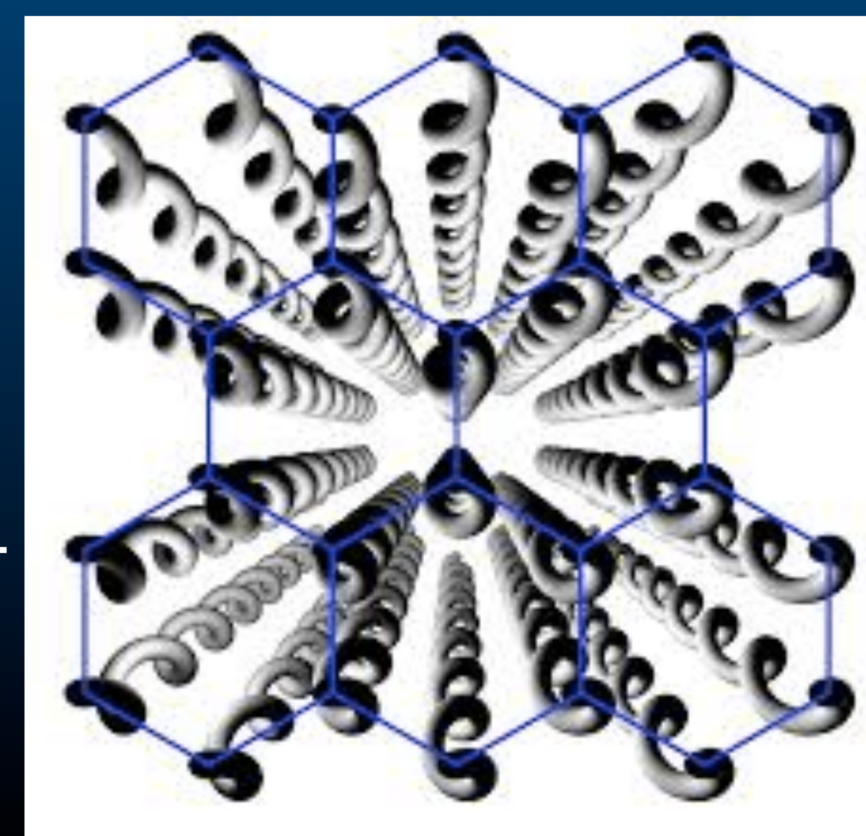
Gauge fields and FQH with cavity arrays Angelakis, Cho PRL 2008.



Gauge fields in Circuit QED
Yale Girvin NJP 20120

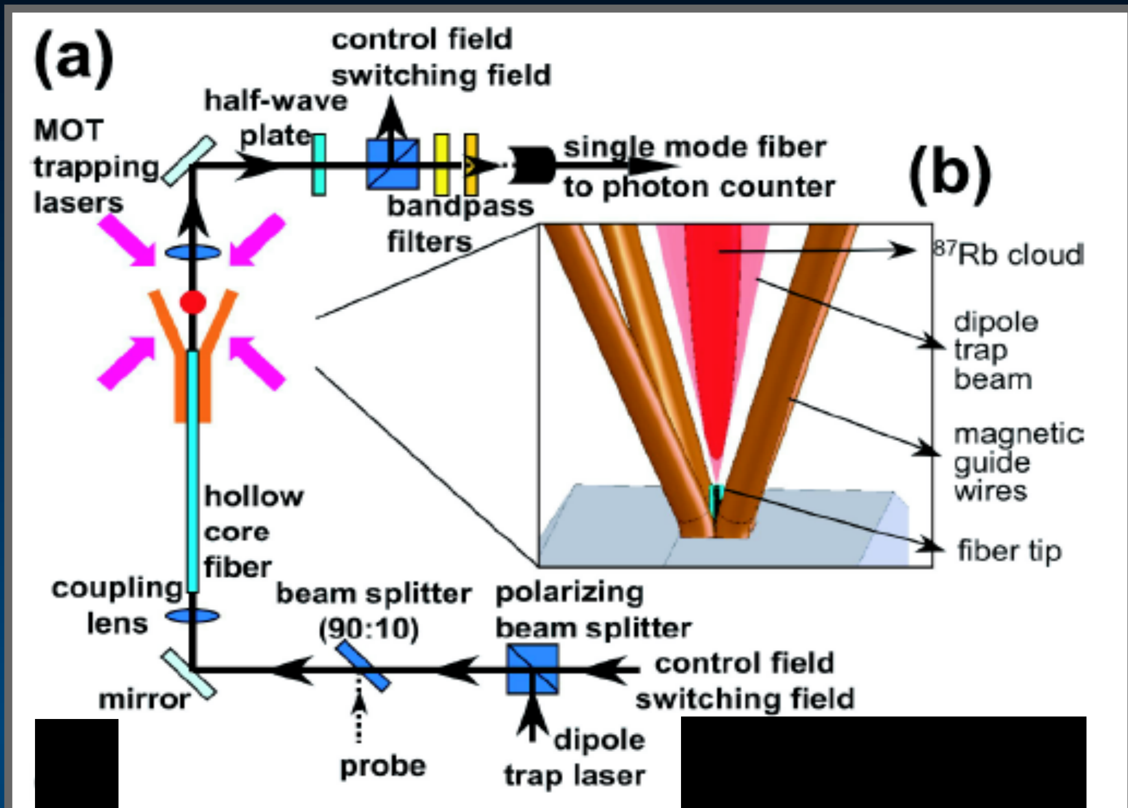


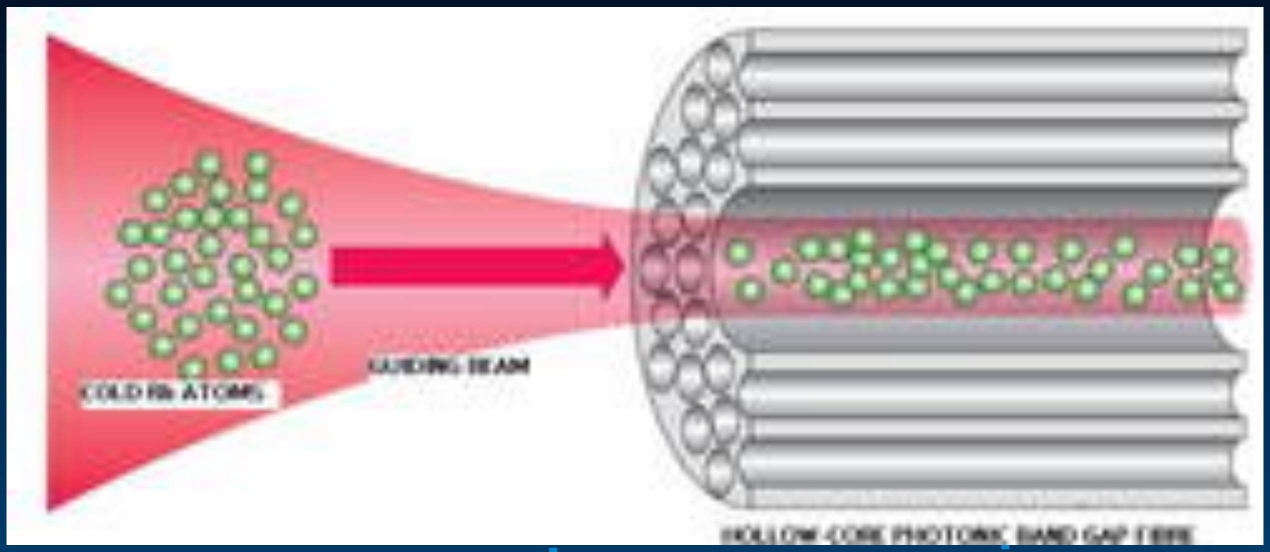
Topological light
JQI, Hafezi, Taylor, Nat Phot 2013



Photonic graphene Szameit, Jena, Nat. 2014

Strong light-matter coupling platforms II: (light interfaced with atoms can exhibit strong nonlinearities without cavities.

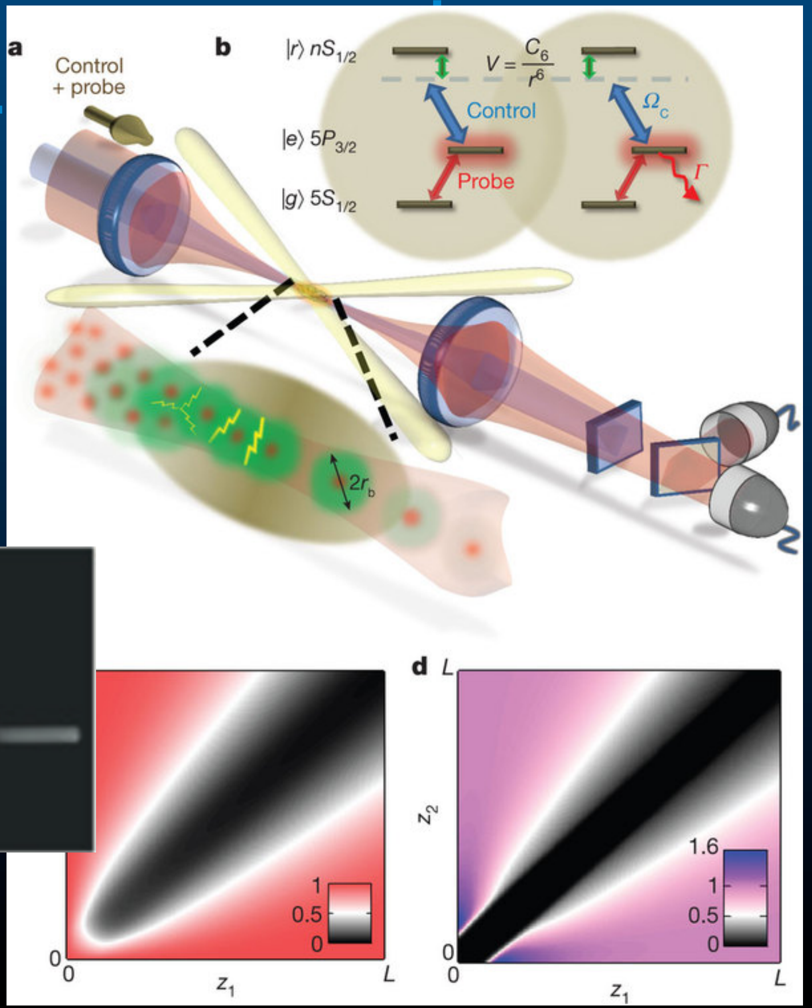


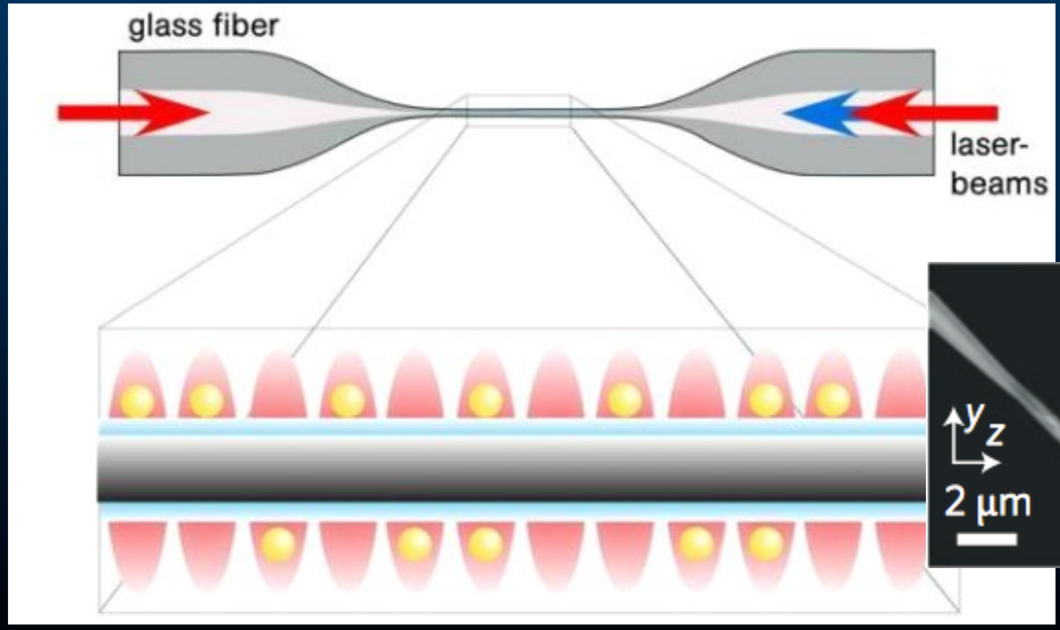


Bajcsy et al. PRL **102**, 203902 (2009)

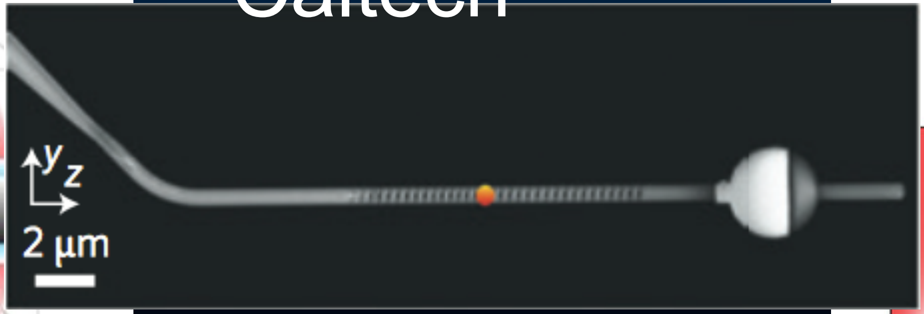
Ghosh et al, PRL **94**, 093902 (2005)

Harvard,
and many others



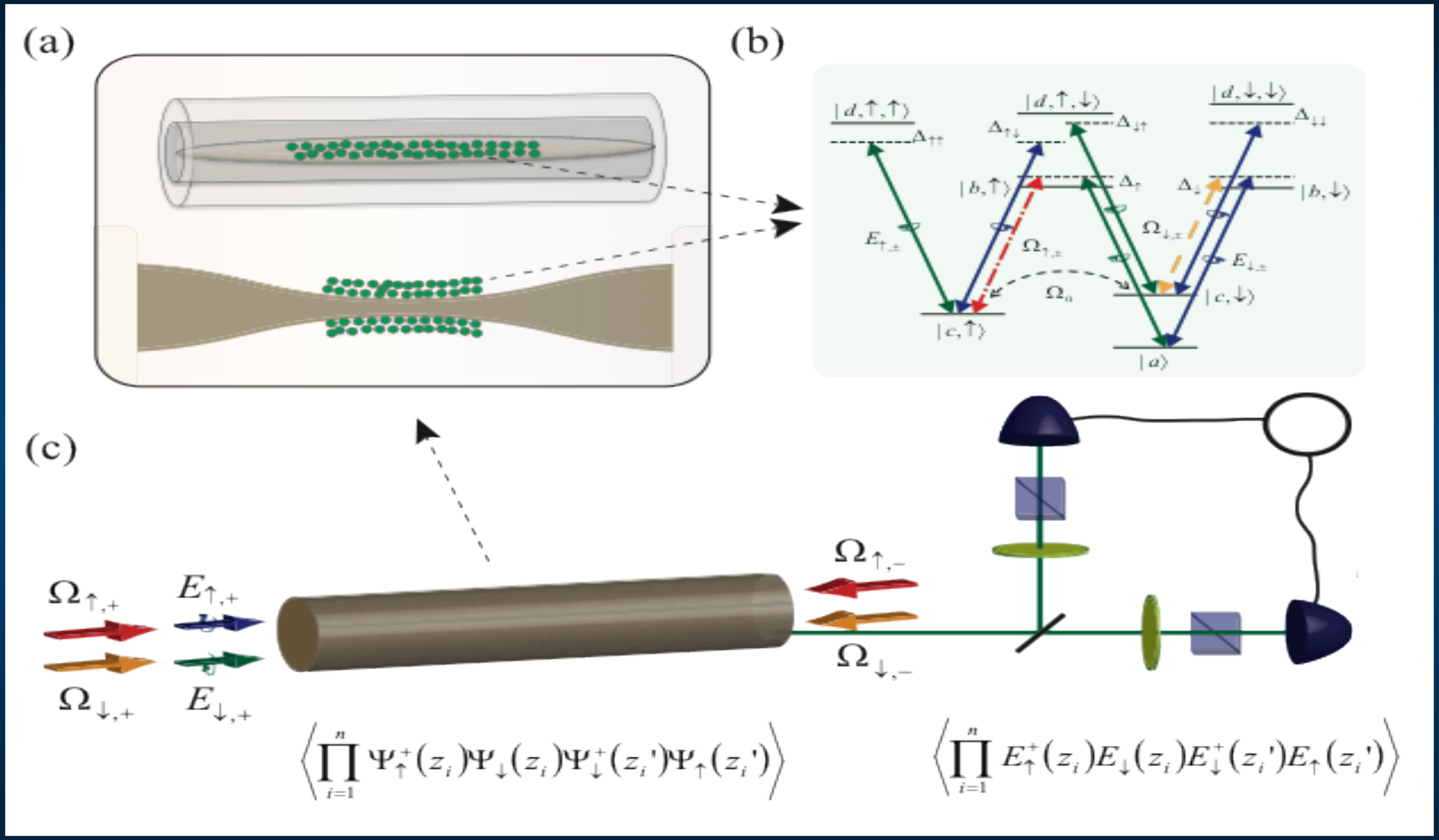


Caltech



A. Rauschenbeutel et al

Many-body physics with light in 1D quantum nonlinear slow light systems:



Physics 4. 30 2011

Atoms-nanophotonic interfaces :

a) Tonks gases and Luttinger liquids of photons (Chang et al. Nat Phys. 2008, DA et al PRL 2011)
b) Thirring model of interacting slow light polaritons (DA et al, PRL 2013)
c) Topological properties of Jackiw-Rebbi model with light (DA et al. Sc. Rep. 2014)

Progress so far in the young field of "Many-body physics with light"

Roughly 600 papers (mainly theory) based on the initial works studying:

- *Effective spin models: XY, XXZ, high-spins, Cluster states,*
- *The phase diagram of JCH*
- *Entanglement and applications in QIP*
- *Gauge fields, Fractional Hall effect...*
- *Quantum transport of correlated photons…*
- *Driven cavities and non-equilibrium many body effects*
- *Fermionized/Tonks-Girardeau photon gases…*
- *Luttinger liquids, sine-Gordon and BH models with stationary light…*
- *Interacting quantum field theories*

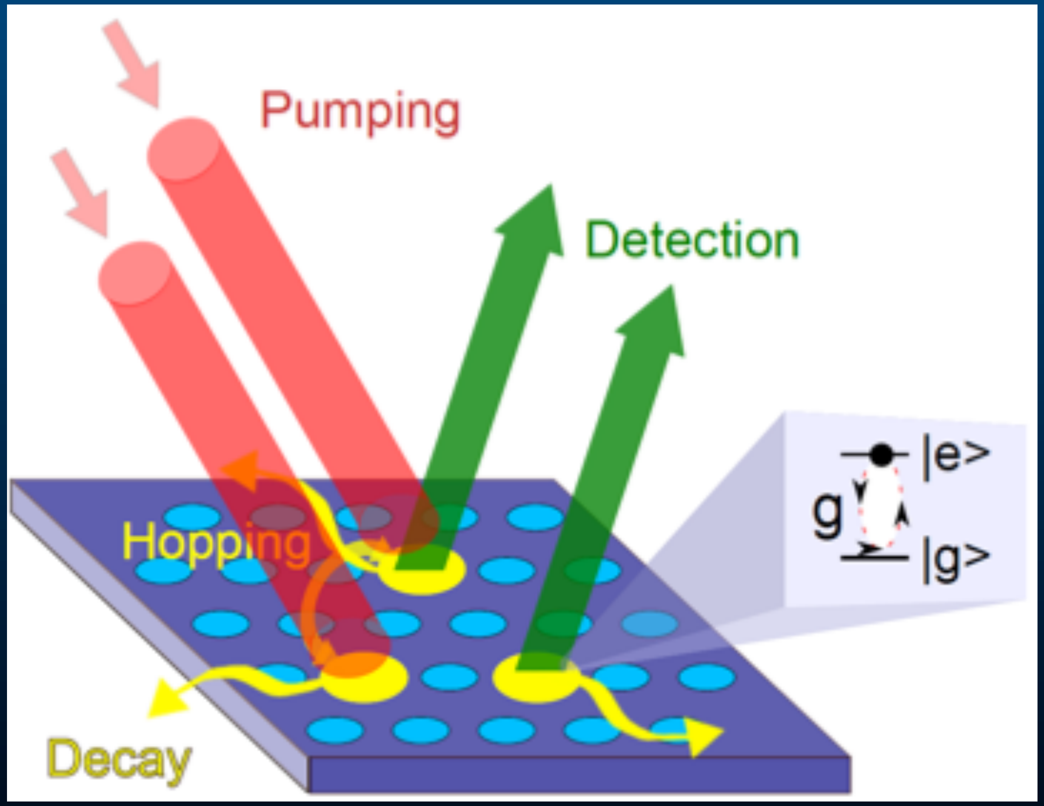
Discrete MB models using coupled cavity arrays (in and out of equilibirum)

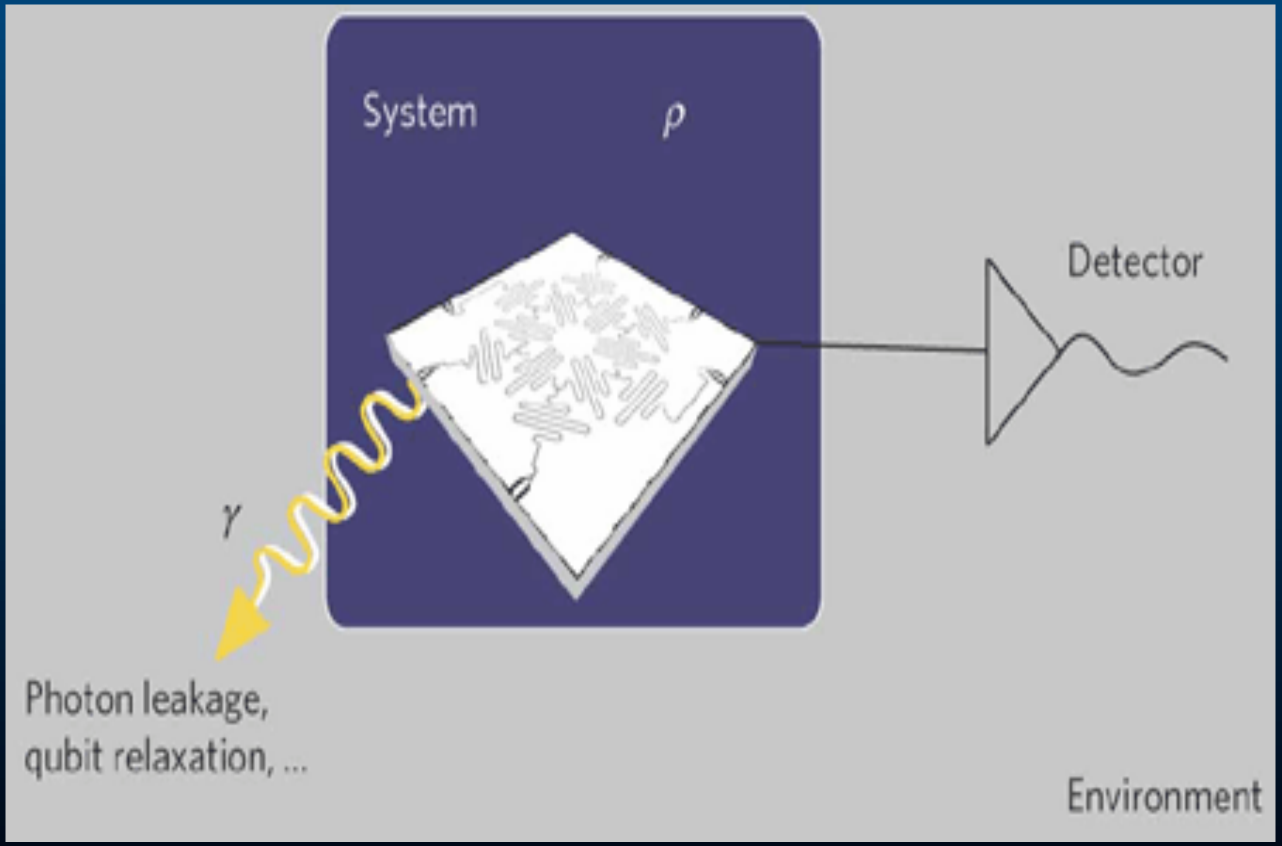
Continuous 1D M models in quant nonlinear set u

Reviews: Hartmann, Plenio LP 2008;Rossini and Fazio JOSA B, Koch, Schmidt, Tureci Nature Physis 2012, Angelakis,Noh, Reports in Progress in Physics 2015, Angelakis editor of a multiauthor review volume in Springer 2015

Rough and incomplete list of groups that have been working in this:
Crete/Singapore, Paris, ICFO, Pisa, Yale, Princeton, JQI, Heriott Watt, Stanford, Beijing, Har
Belfast, Salerno, Zurich, Vienna, Cavendish, Camerino,, Queensland, Grazz,

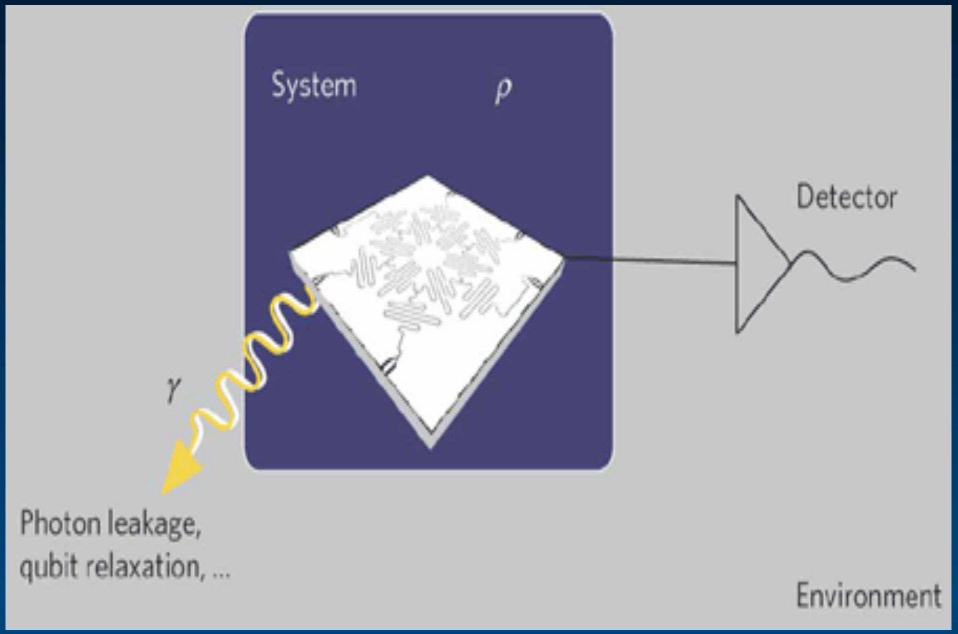
Out of equilbrium phases in driven dissipative QED cavity arrays





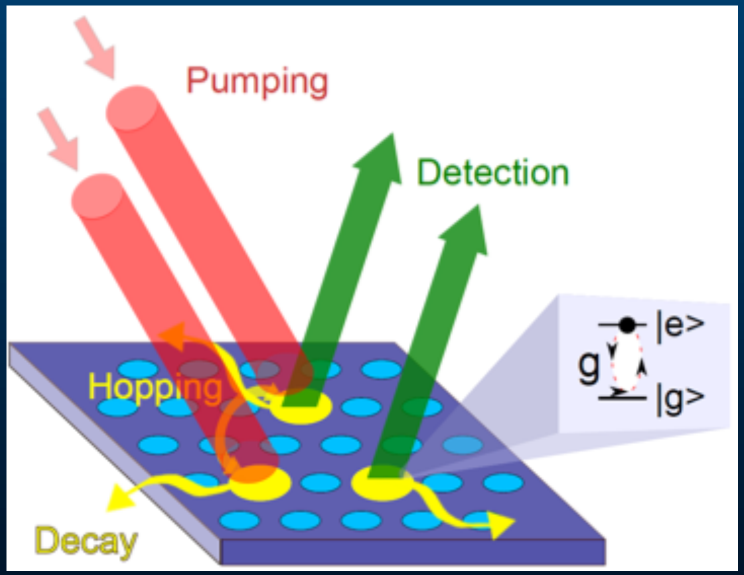
Out of-equilibrium simulations in driven JCH arrays

Taking into account losses, the ground state of JCH is the vacuum! Boring from any aspect…



There is no grand canonical ensemble for photons. No chemical potential for photons!

Can look at quasi-equilibrium settings assuming the time to get there is smaller than dissipation time…
fine but you need to be fast!



Driven arrays is the way to go.
i) Need methods from open systems to treat the system and new theory to quantify the out-of equilibrium phases

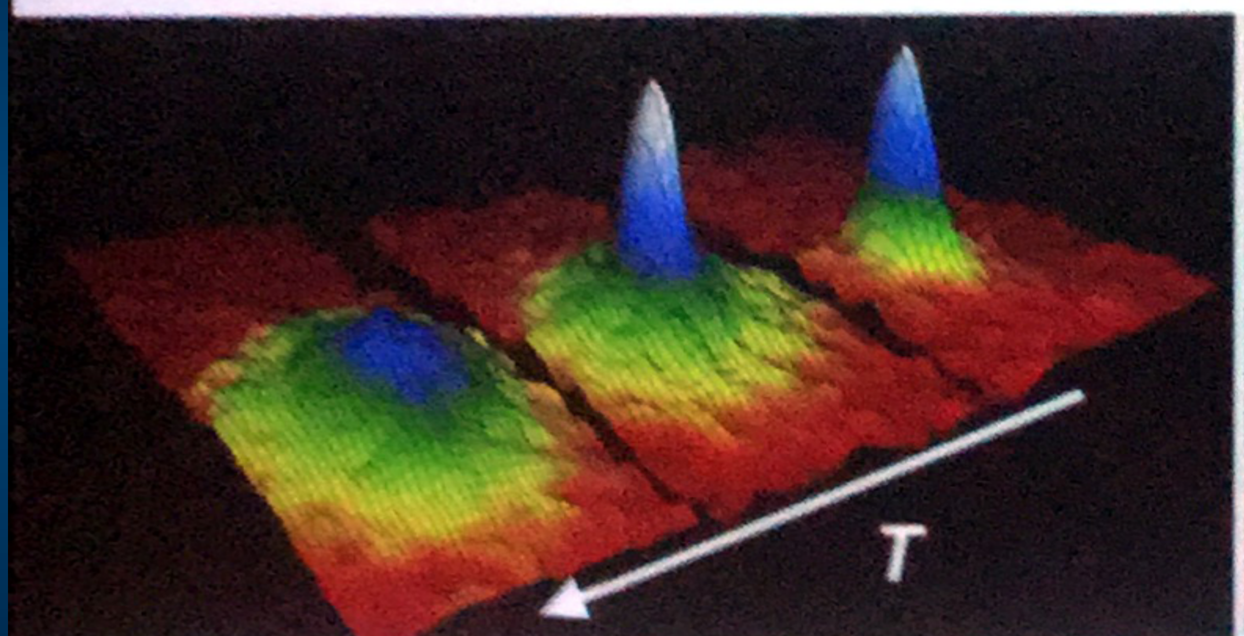
Gruzic Clark, Jacksh, Angelakis, *Many body effects beyond the Bose-Hubbard model in non-equilibrium resonator arrays, EPL 2008, NJP 2012*

Early work by Carussoto, Ciuti et al PRL 2009; Tomadin, Fazio PRA 2009

Driven dissipative Many-body effects with light at the "1st quantization or semi-classical level"

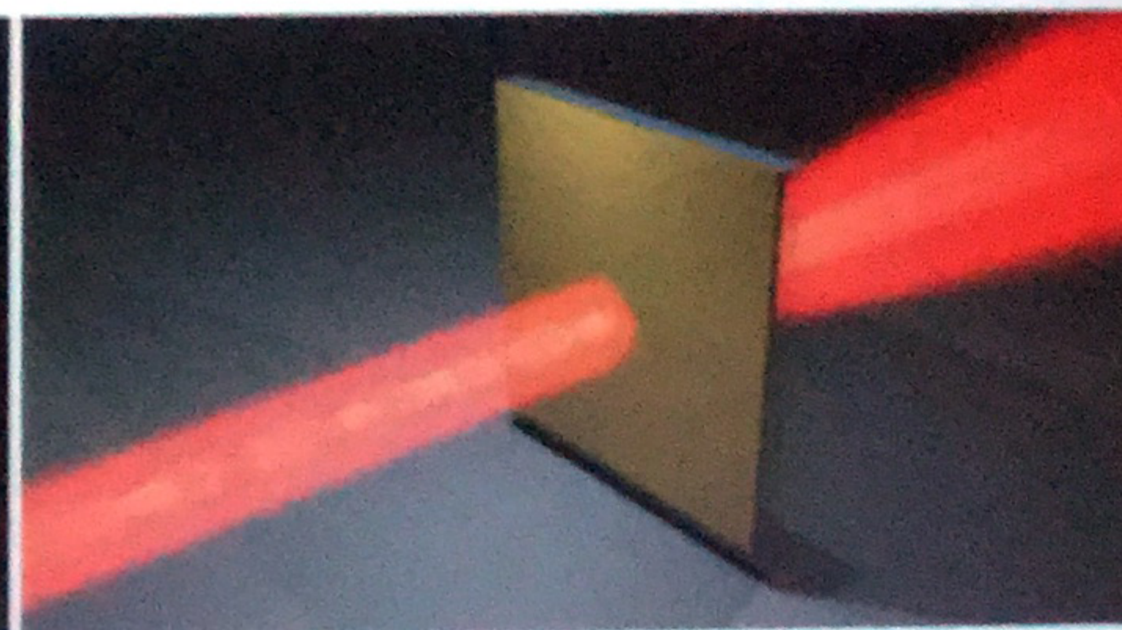
Matter and light

Atomic condensates



http://jilawww.colorado.edu/bec/

Non-linear optics



©Boltasseva group, Purdue

atoms

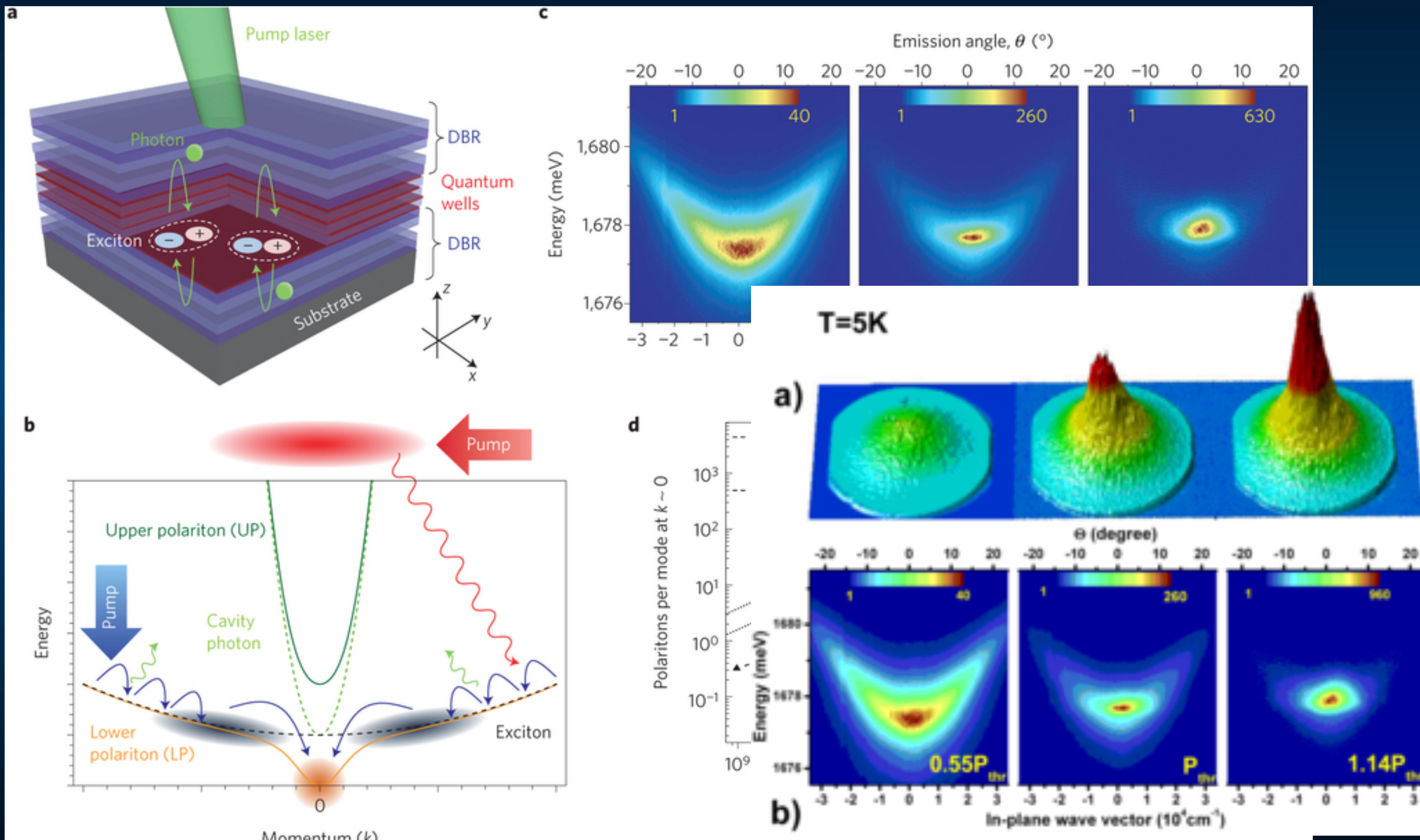
photons

$\chi^{(3)}$

Show common phenomena

$$i\hbar\partial_t\psi(x,t) = \left[\nabla^2 + V(x) + \hbar g|\psi(x,t)|^2 - i\hbar\gamma\right]\psi(x,t)$$

Exciton polariton BECs



CNRS, Stanford, Cambridge,Paris…
Reviews by Ciuti- Carussoto, Deng, Yamamoto

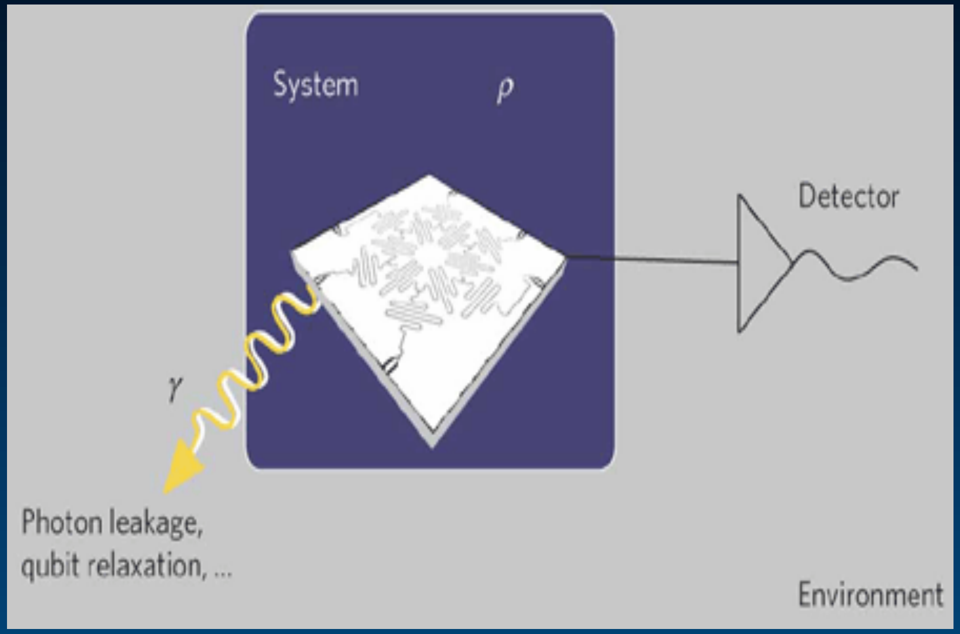
Driven dissipative JCH/BH arrays I: Methods

Numerics are very tough, tougher than the driven BH! Also more interesting-you have more degrees of freedom atom(s)+field in each site.

Analytically: Keldysh (Dicke model ok, possible for full JCH)?

For the strongly correlated regime, so far only numerical methods are used based on tensor-neworks (TEBD) to investigate the steady-state of driven JCH and BH- and generalizations of it (only 1D).

(Recently some methods for 2D working in some regimes have been proposed, see talks by Ciuti, Hartmann, Rossini)



$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H,\rho] + \gamma \sum_j (2\sigma_j^- \rho \sigma_j^+ - \sigma_j^+ \sigma_j^- \rho - \rho \sigma_j^+ \sigma_j^-) + \tilde{\kappa} \sum_j (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j),$$

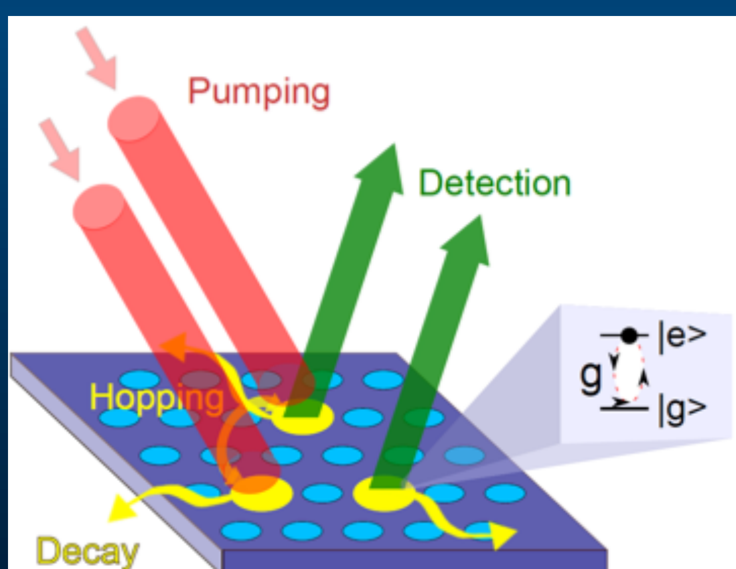
Gruzic Clark, Jacksh, Angelakis, *Many body effects beyond the Bose-Hubbard model in non-equilibrium resonator arrays, New Journal of Physics 2012*

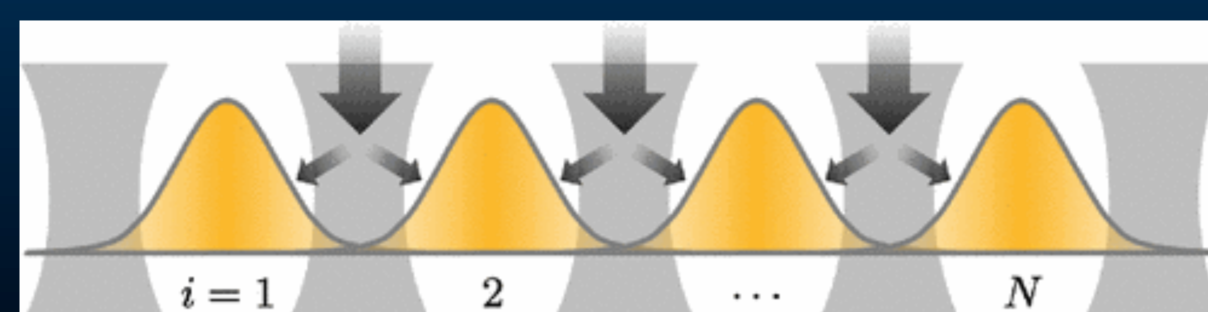
Types of “drives”in driven QED resonator arrays

Freedom to choose from a variety of drives

i) Coherent $\Omega(a^{+} + a)$ ranging from very weak (few photons) to Classical (Carussoto et al 2009, DGA et al 2009)

$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma \sum_j (2\sigma_j^- \rho \sigma_j^+ - \sigma_j^+ \sigma_j^- \rho - \rho \sigma_j^+ \sigma_j^-) + \tilde{\kappa} \sum_j (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j),$$





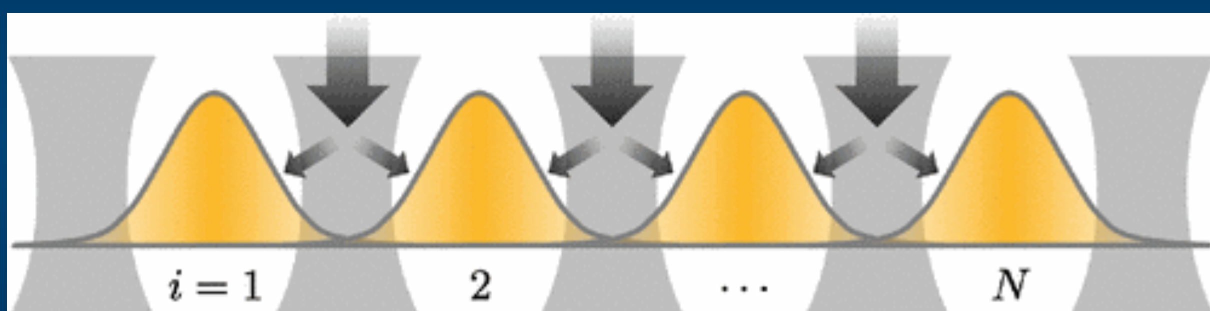
$\Omega(a_i^{+} a_j^{+} + a_i a_j)$

iii) To parametric (Bardyn, Imamoglu 2012)

Driven QED resonator arrays

Freedom to choose from a variety of drives!

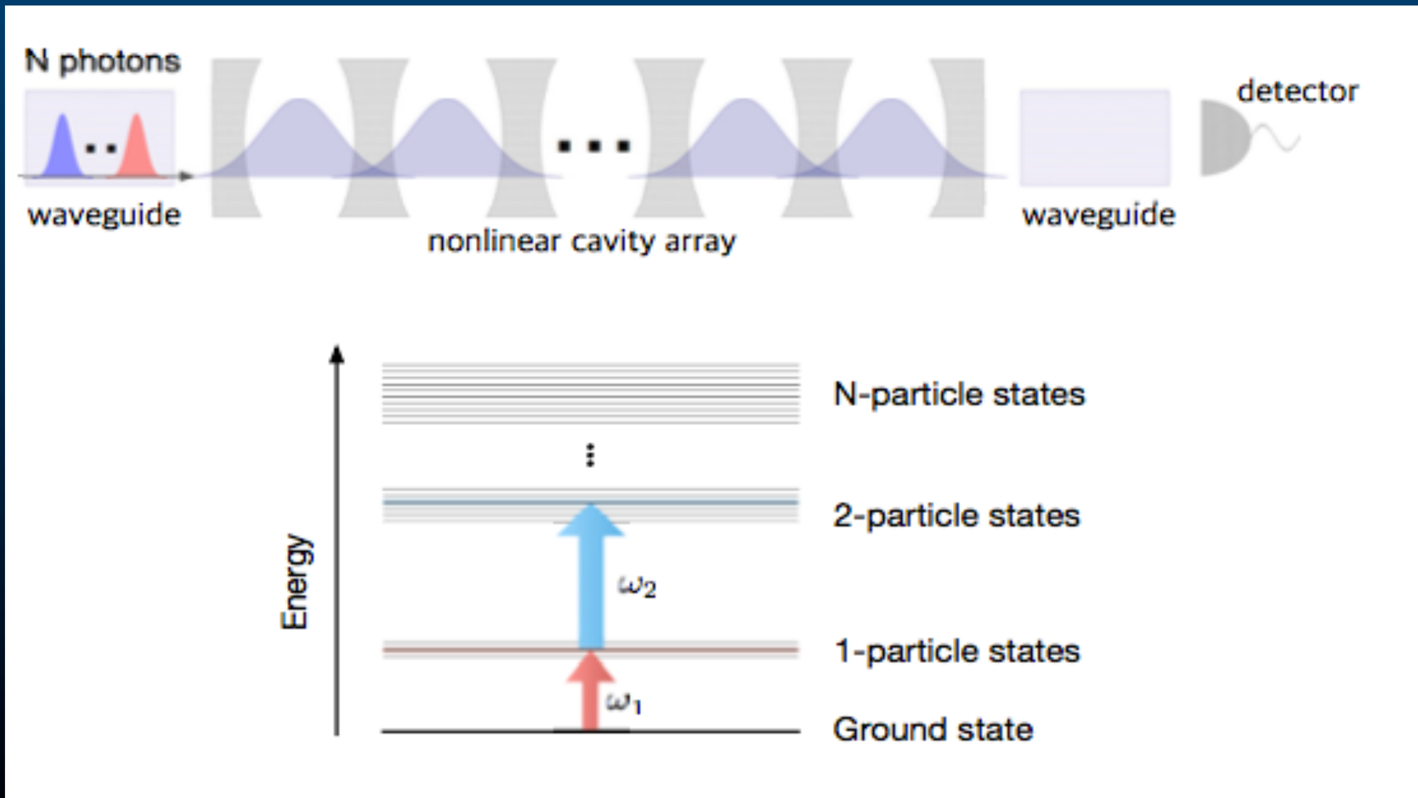
i) Coherent $\Omega(a^{+} + a)$ ranging from very weak (few photons) to Classical



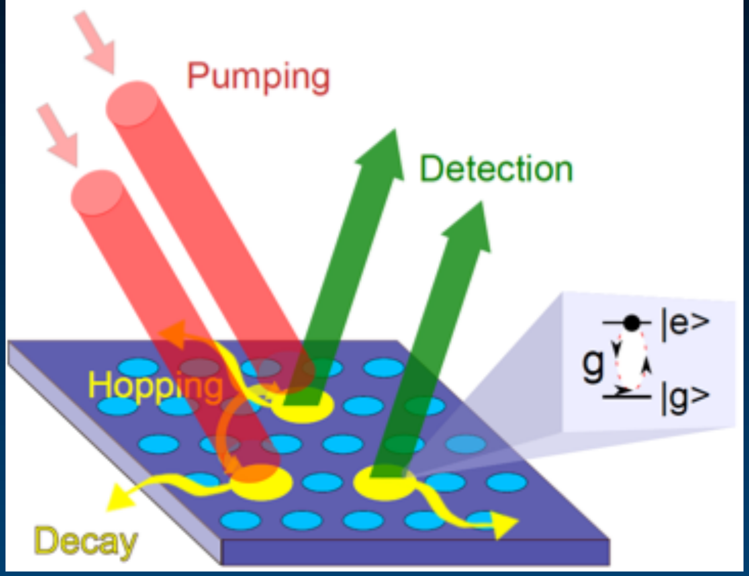
iii) To parametric $\Omega(a_i^{+} a_j^{+} + a_i a_j)$

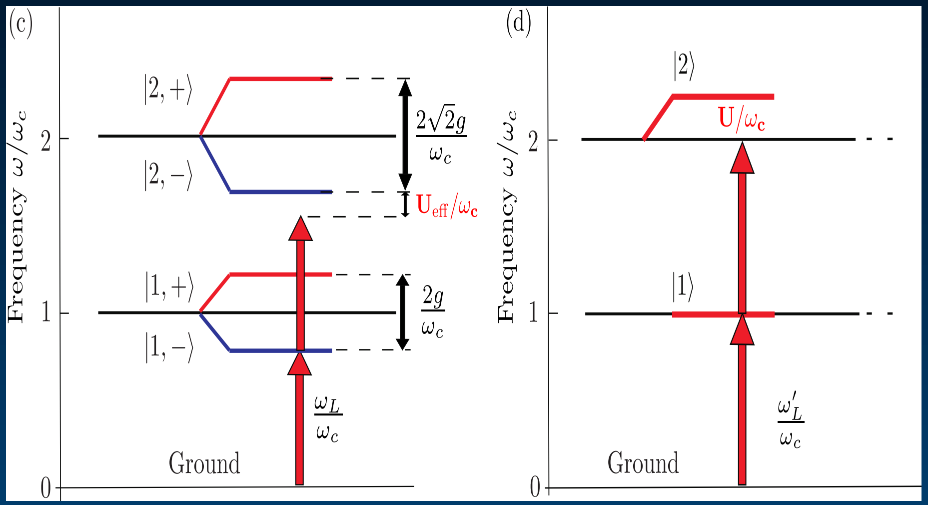
ii) "Fully quantum-driving" with Fock states

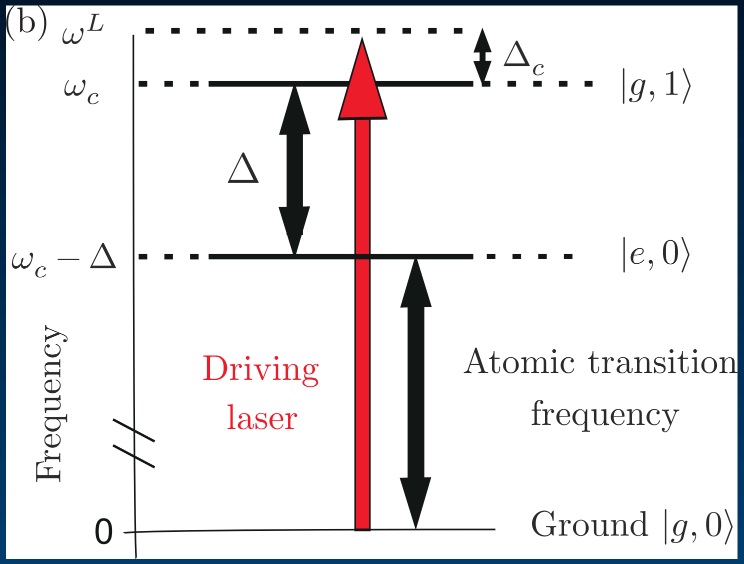
C.Lee , C. Noh, N. Schetakis, DGA ""Few photon scattering in nonlinear cavity arrays : Probing signatures of strongly correlated states" arXiv:1412.8374 (to appear in Phys. Rev. A) "



Coherently driven cavity arrays : Phases and effects within and beyond the BH model







$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma \sum_j (2\sigma_j^- \rho \sigma_j^+ - \sigma_j^+ \sigma_j^- \rho - \rho \sigma_j^+ \sigma_j^-) + \tilde{\kappa} \sum_j (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j), \quad (20)$$

In the photonic limit -D>>g, the dressed state |n-> is mostly photonic but nonlinearity is vanishing…!

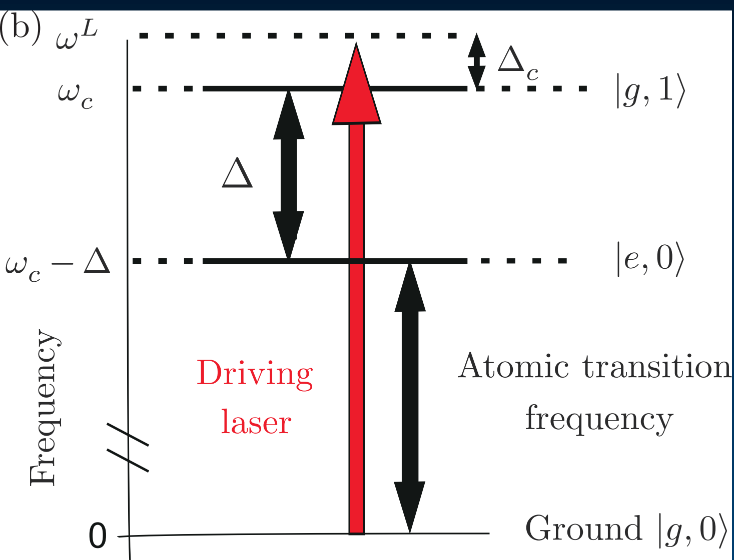
$$H^{JCH} = \omega_d \sum_{k=1}^{N} a_k^+ a_k + \omega_0 \sum_k |e\rangle_k \langle e|_k + g \sum_{k=1}^{N} (a_k^+ |g\rangle_k \langle e|_k + H.C) - \kappa \sum_{k,=1}^{N} (a_k^+ a_{k+1} + H.C) + \Omega(a_k^+ + a_k)$$

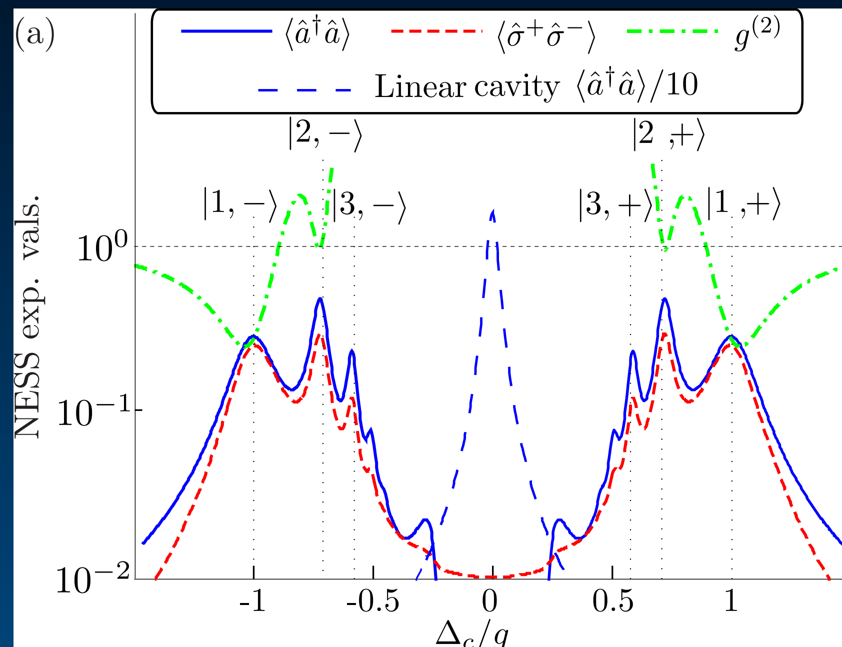
$$\omega_{n,-}(g,\Delta) \approx n\,(\omega_c + g^2/\Delta) + g(g/|\Delta|)^3 n(n-1)$$

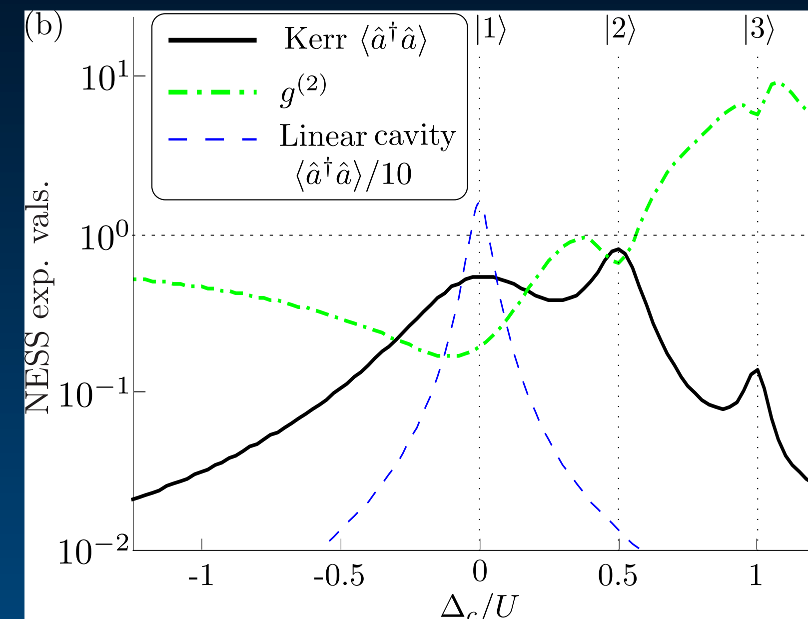
Gruzic Clark, Jacksh, Angelakis, *Many body effects beyond the Bose-Hubbard model in non-equilibrium resonator arrays, NJP* **14** (2012) 103025

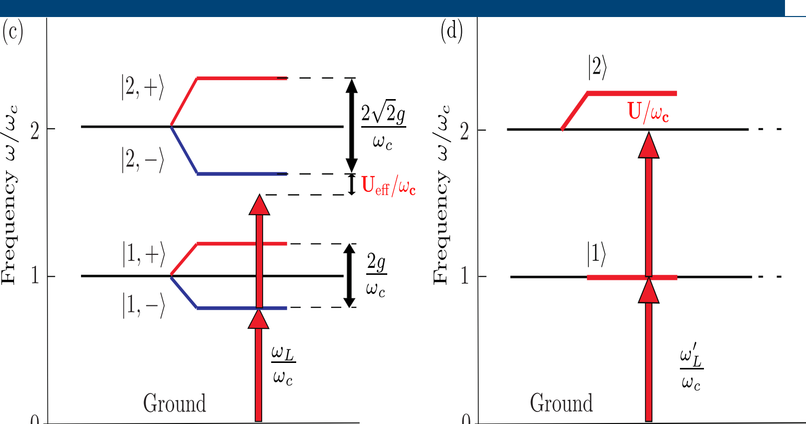
Driven JCH arrays II:

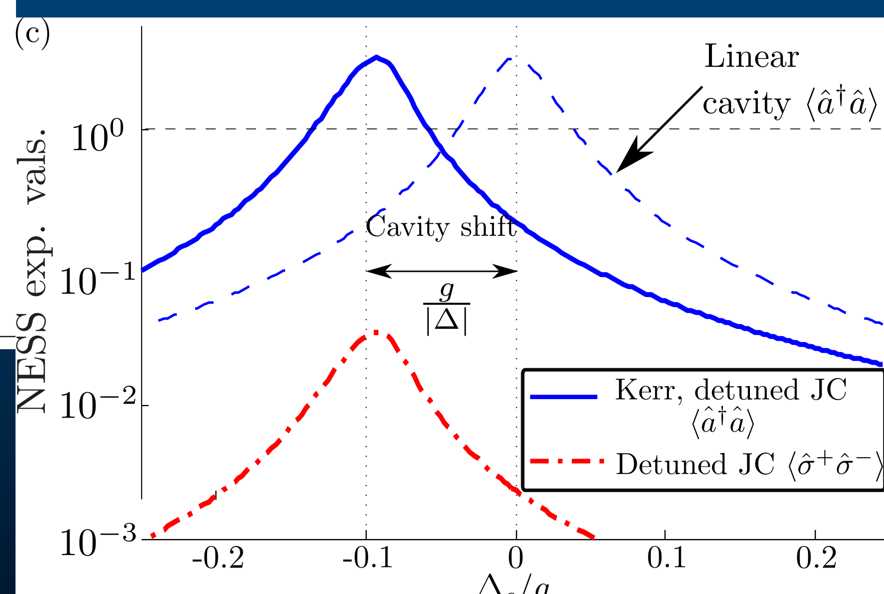
Kerr nonlinearity is pretty different than Jaynes-Cummings!

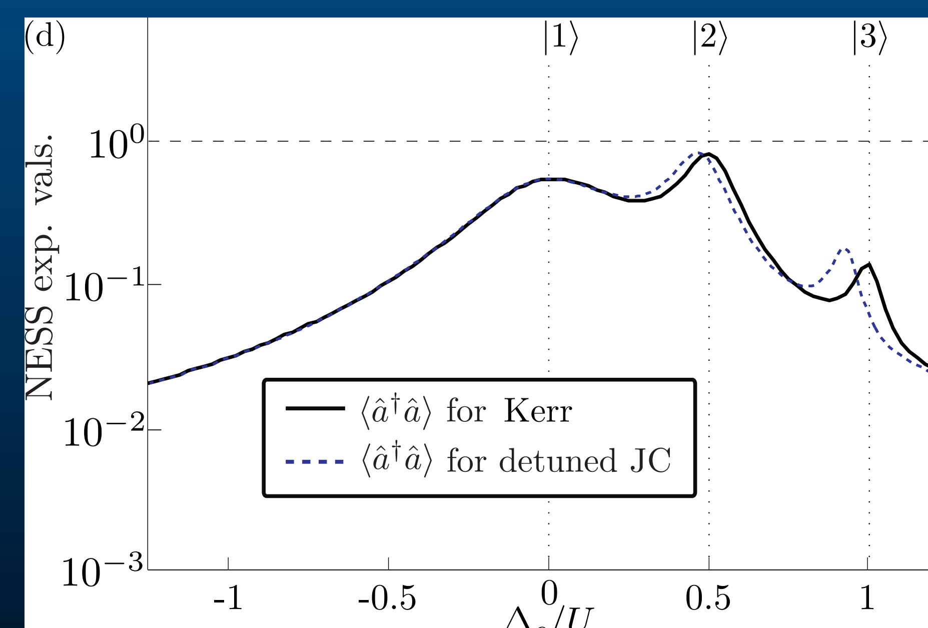












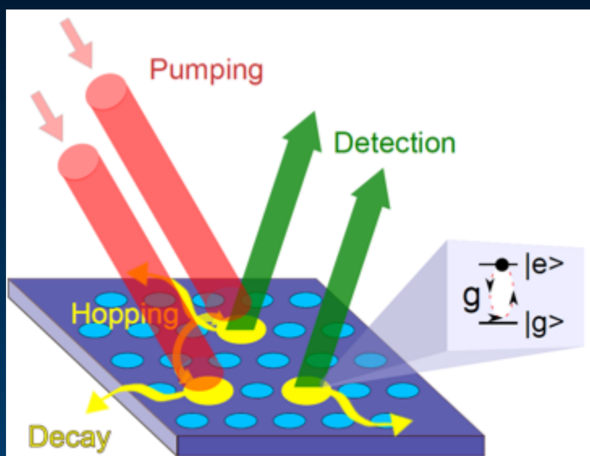
Single site case

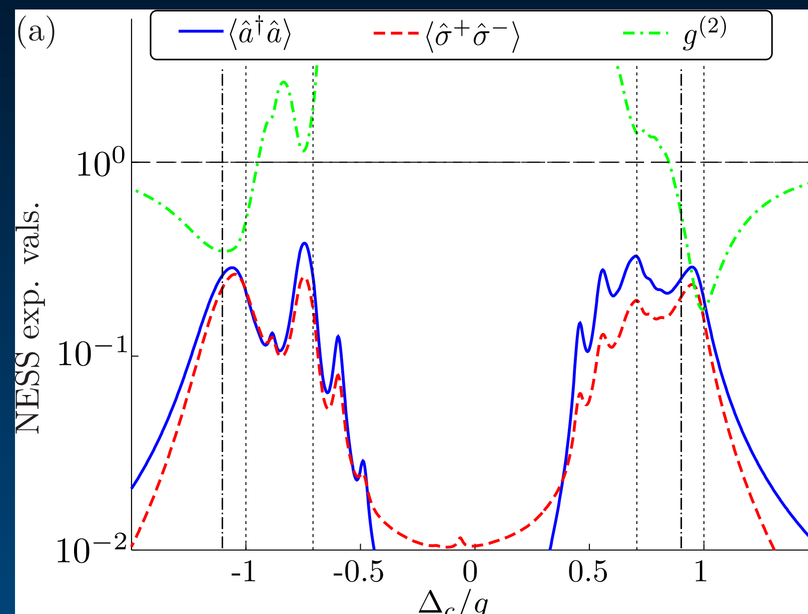
Omega/gamma=2, g/gamma=20 which gives U/gamma=11.7

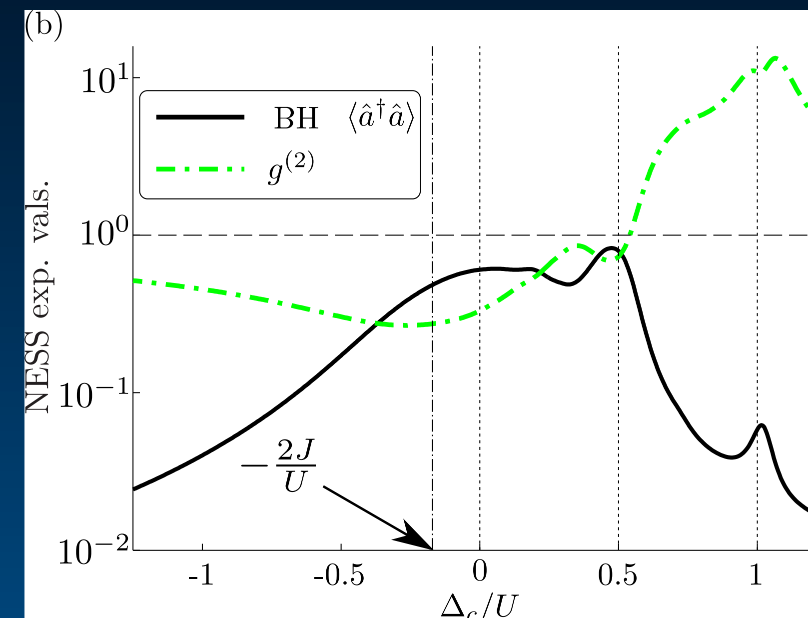
For |D|/g=10, to get the same repulsion and spectra you need g=10.000gamma!!!

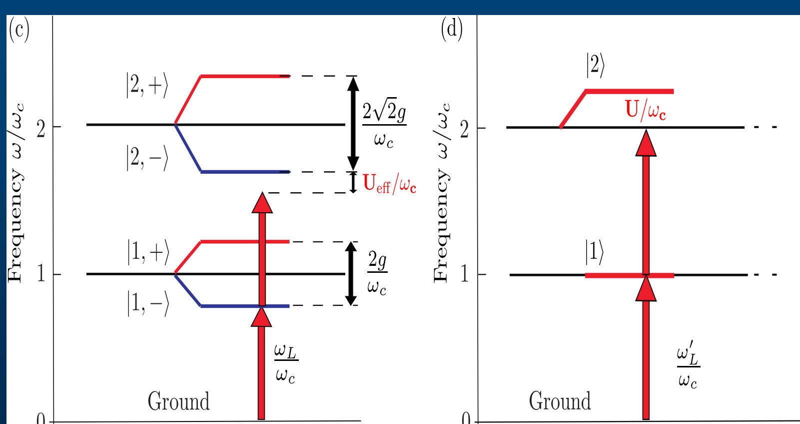
Gruzic, Clark, Jacksh, Angelakis, *Many body effects beyond the Bose-Hubbard model in non-equilibrium resonator arrays, NJP* **14** (2012) 103025

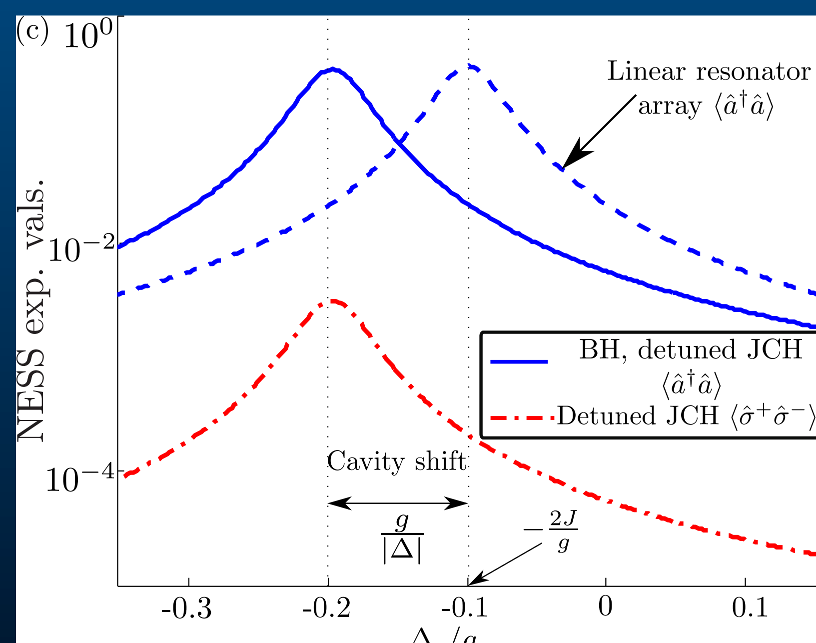
Driven JCH and BH arrays II: Photon/atom occupations at steady state

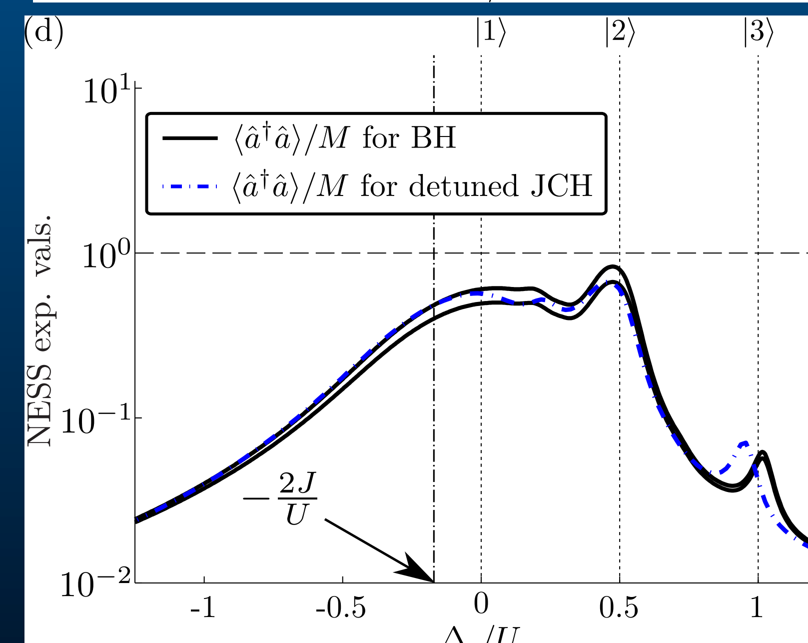












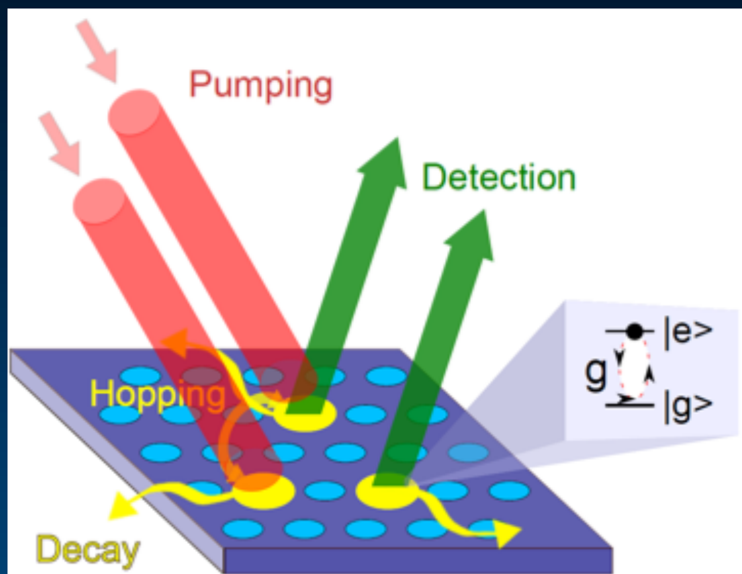
For -D/g=10, to get the same repulsion you need g=10^4gamma!!! (3 sites ring)

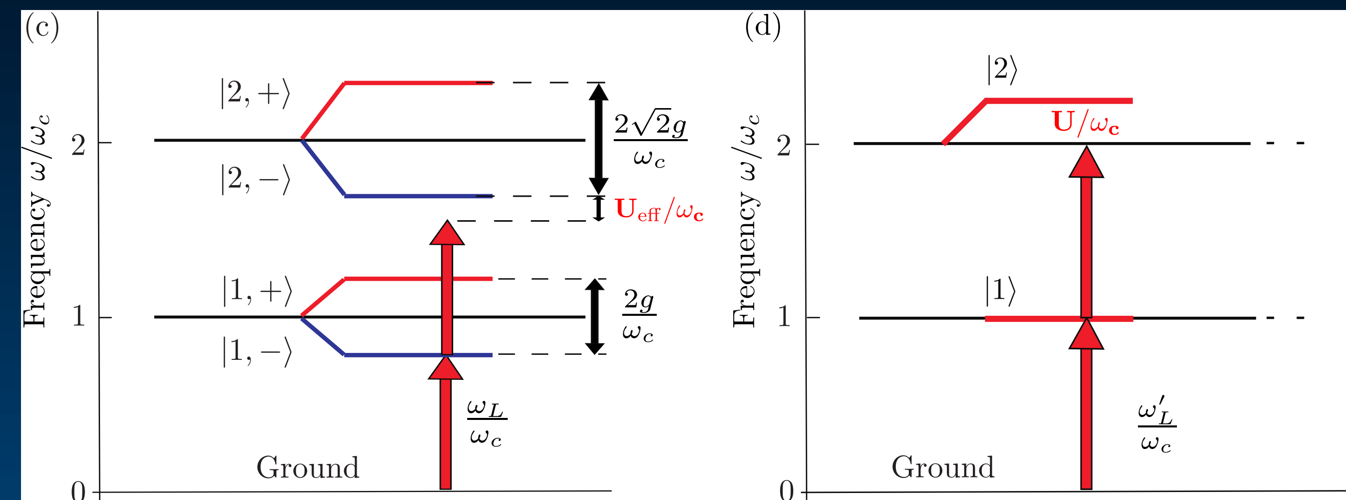
BH spectra even after the "matching" is different than JCH!

Omega/gamma=2, g/gamma=20 which gives U/gamma=11.7

Gruzic Clark, Jacksh, Angelakis, *Many body effects beyond the Bose-Hubbard model in non-equilibrium resonator arrays, NJP* **14** (2012) 103025

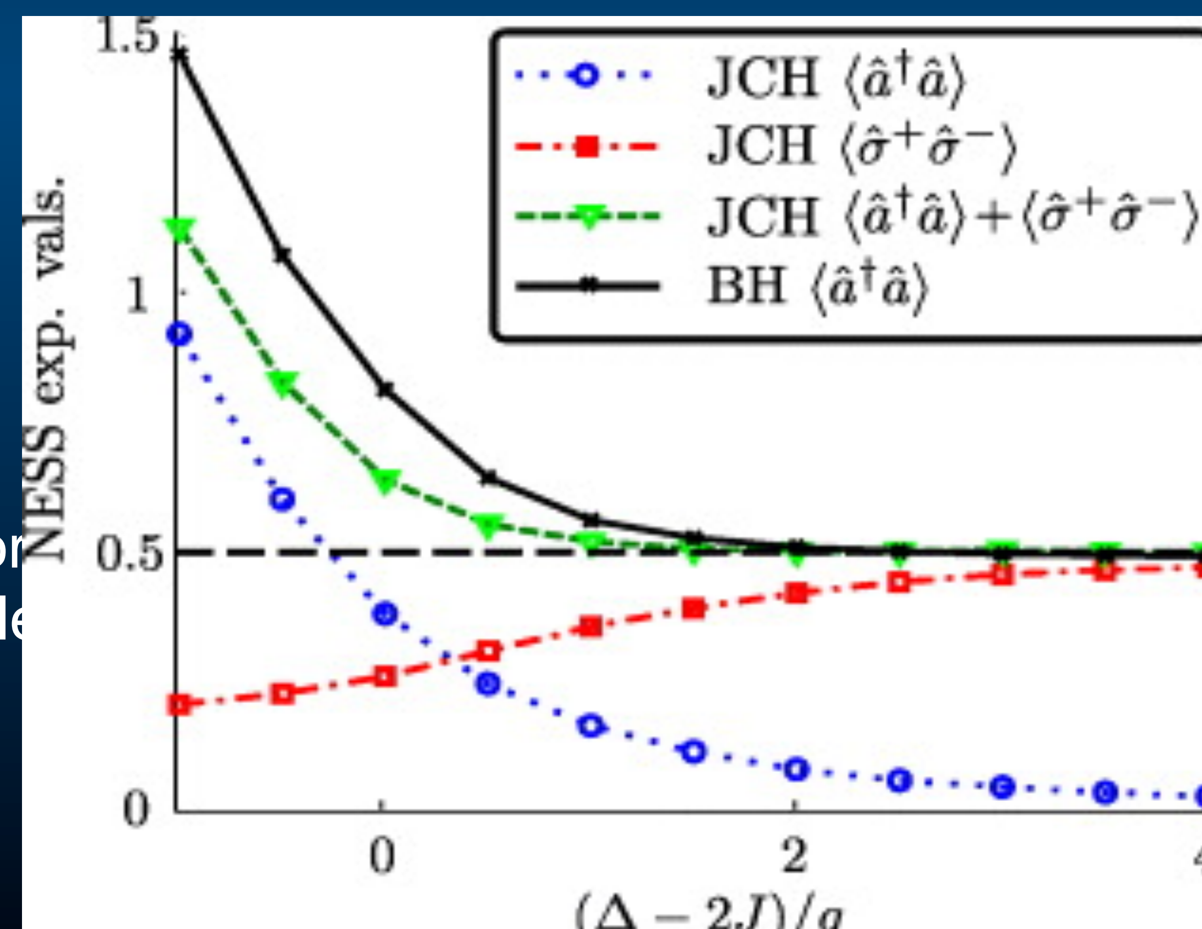
Driven JCH arrays II:
Observables at steady state of BH versus JCH





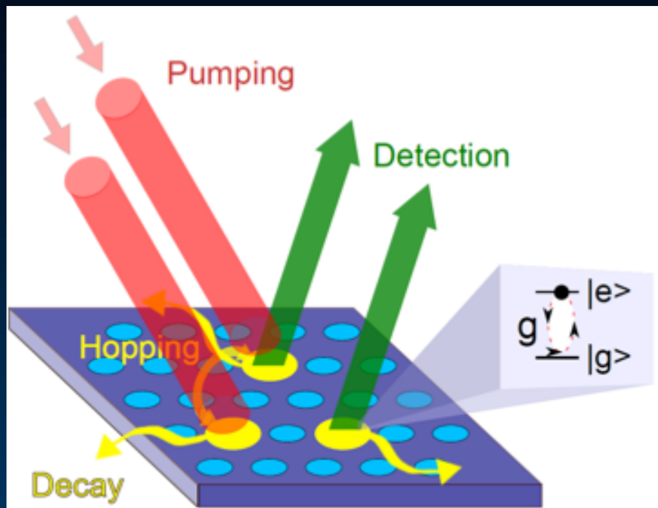
$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma \sum_j (2\sigma_j^- \rho \sigma_j^+ - \sigma_j^+ \sigma_j^- \rho - \rho \sigma_j^+ \sigma_j^-)$$
$$+ \tilde{\kappa} \sum_j (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j), \qquad (20)$$

TEBD calculations: Particle numbers per resonator **<u>for 16-site BH</u>** and JCH arrays driven at their single particle resonances.Ω/γp = 2, g/k = 20, J/k = 1, gamma=0.



Gruzic Clark, Jacksh, Angelakis, *Many body effects beyond the Bose-Hubbard model in non-equilibrium resonator arrays, NJP* **14** (2012) 103025

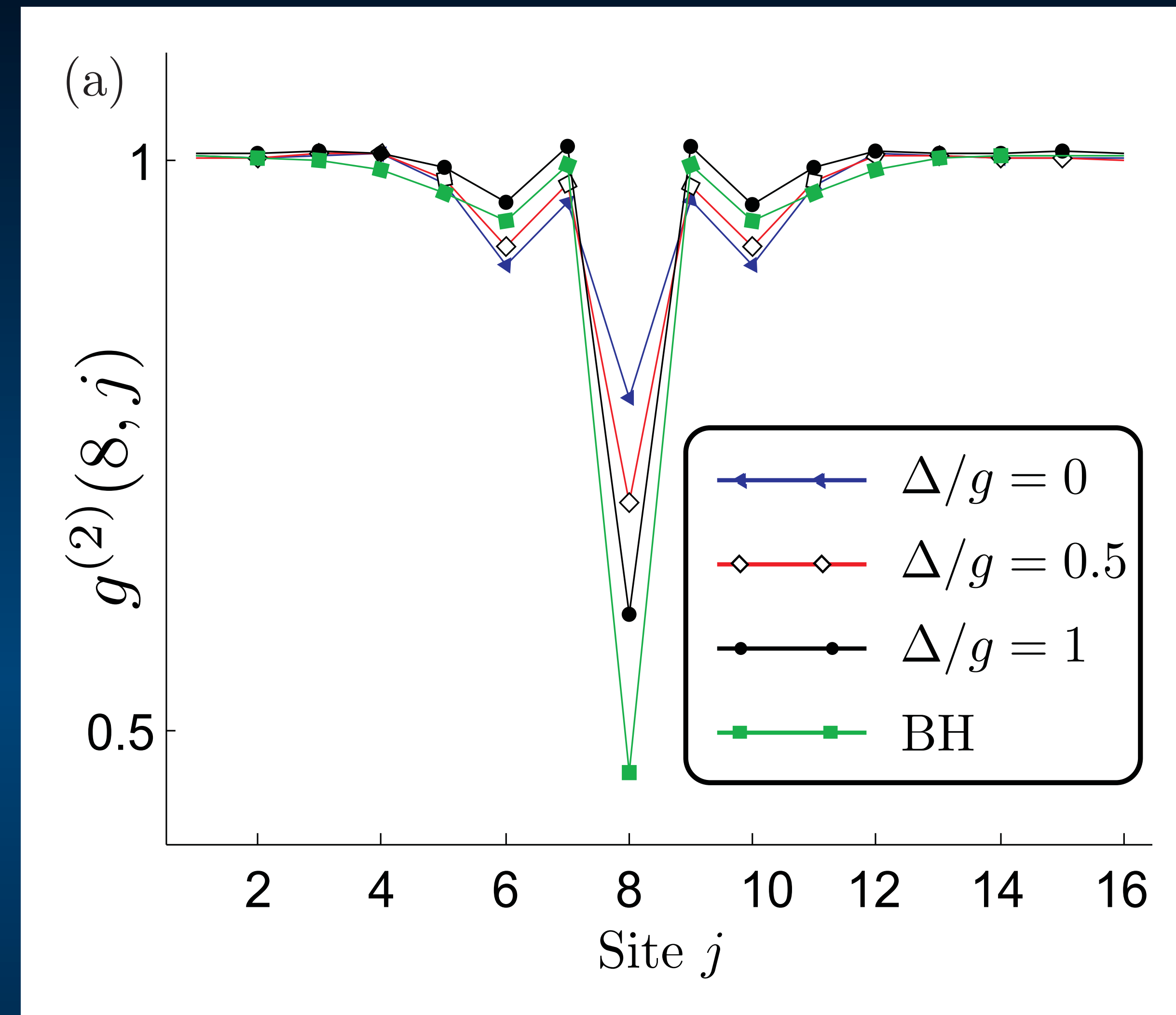
Exotic phases coherently driven JCH arrays: Photon crystallization



$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma \sum_j (2\sigma_j^- \rho \sigma_j^+ - \sigma_j^+ \sigma_j^- \rho - \rho \sigma_j^+ \sigma_j^-) + \tilde{\kappa} \sum_j (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j), \quad (20)$$

$$\Omega_j = \Omega e^{-i\phi_j}$$

$$\phi_j = \frac{\pi}{2} j \; (j = 1, 2, \ldots, N)$$



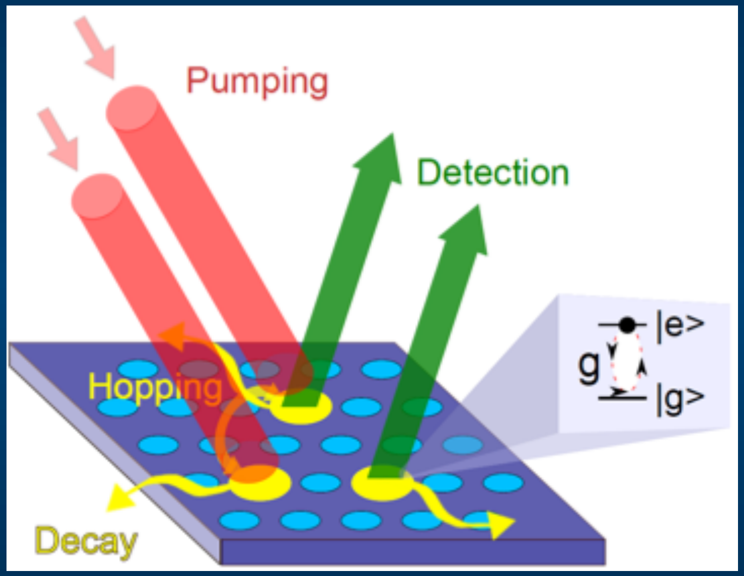
Cross-correlations for 16-site BH and JCH arrays driven at their single particle resonances with appropriate phases in each resonator to create a flow. Ω/k = 2, g/k = 20, J/γp = 1, gamma=0

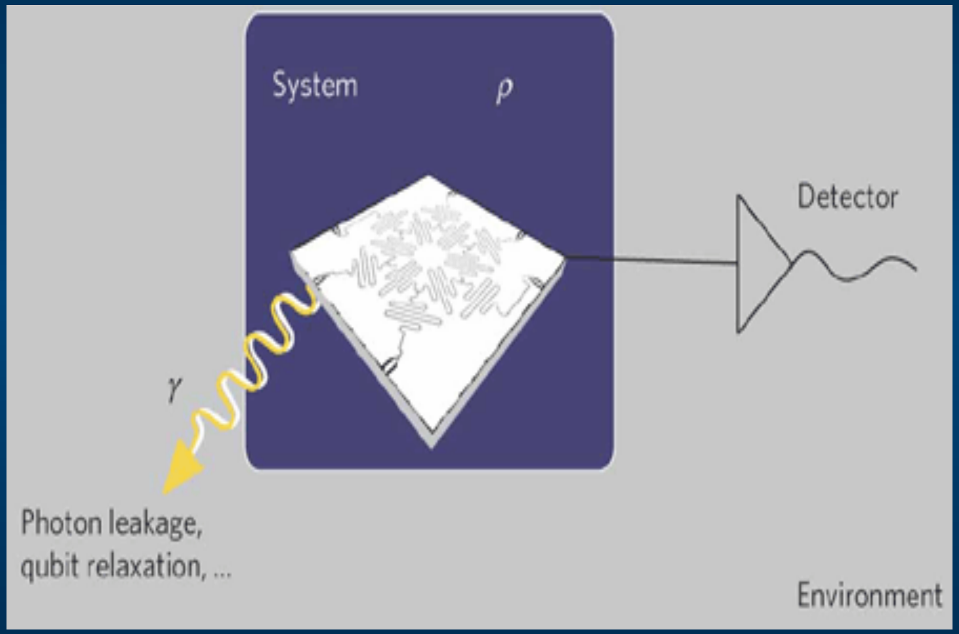
Gruzic Clark, Jacksh, Angelakis, *Many body effects beyond the Bose-Hubbard model in non-equilibrium resonator arrays, NJP 2012, PRA 2013.*

For BH arrays: Hartmann et al. PRL 2010. Photon fermionization Carussoto et al PRL 2008

Quantum (Fock states) driving?

- Coherent and direct/global driving is optimal only for specific situations and implementation platforms

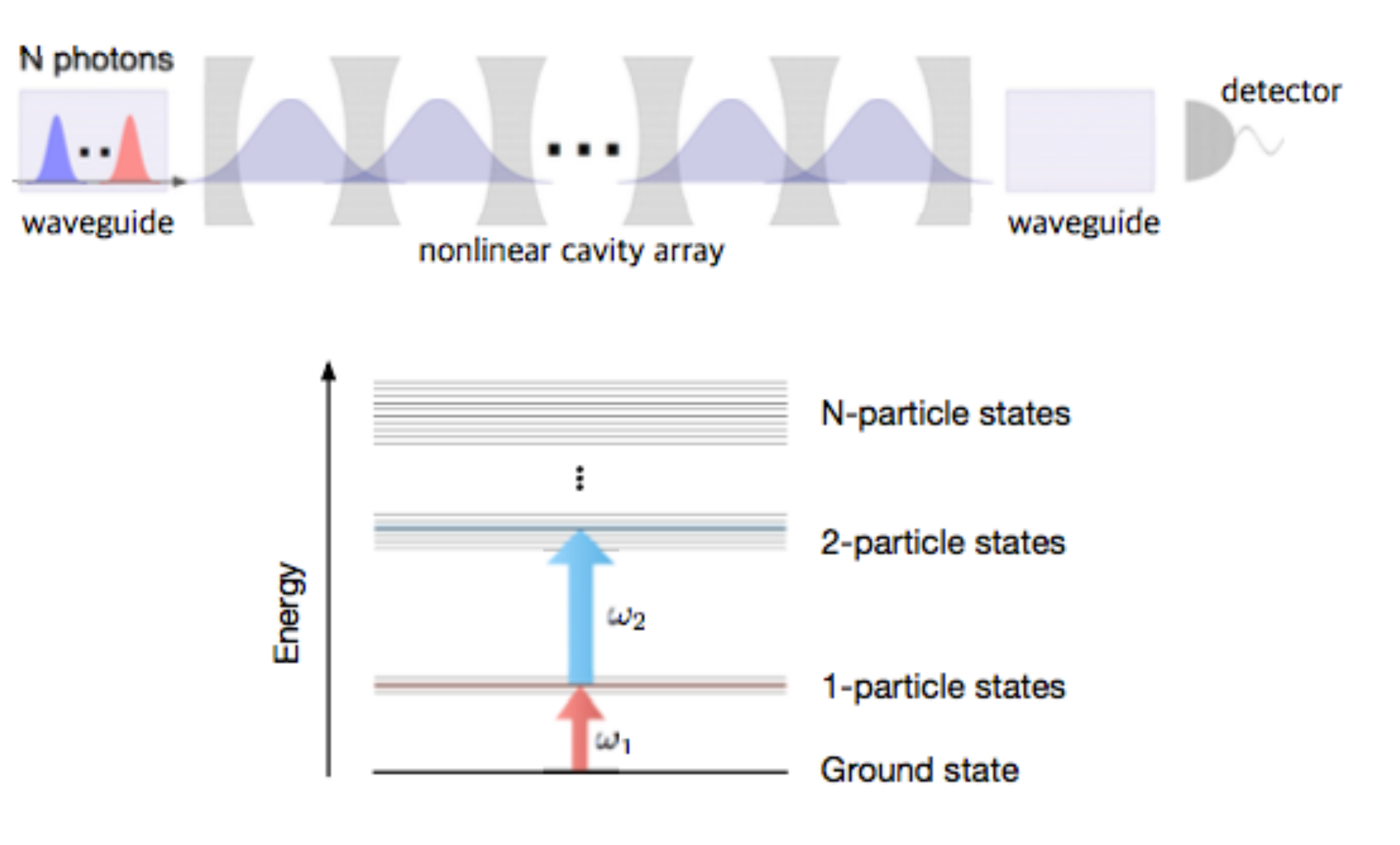




- What is the response when we drive with Fock states and/or when driven from the side-input-output cases?
- Applications in quantum transport?
- What about many-body spectroscopy?

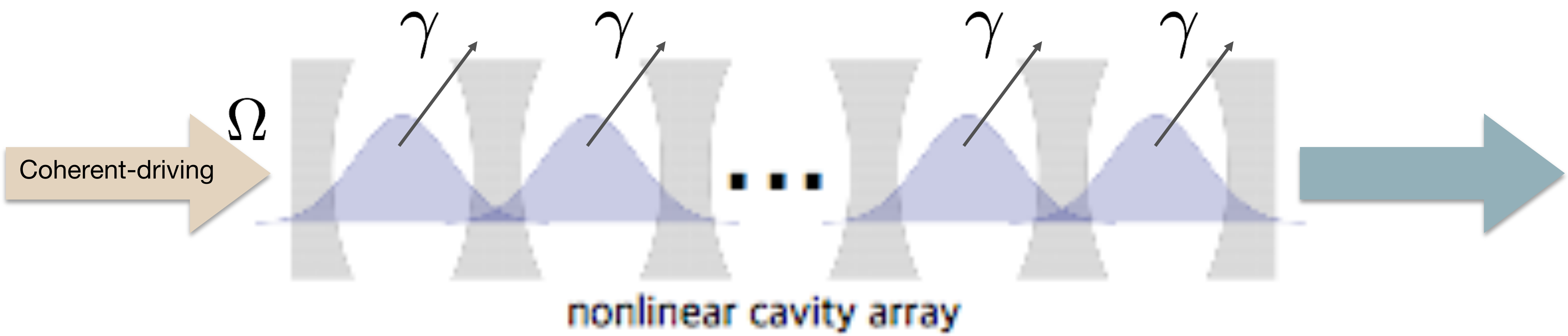
Merging input-output with scattering approaches for many body-spectroscopy

- Use N-photons instead of a coherent field
- Instead of continuous driving, perform a scattering or a quantum transport type of experiment!



Can we probe the many-body structure of the simulated model?
Seems yes and much more faithfully, especially as far as correlations in concerned!

Transport via coherent drive and input-output formalism

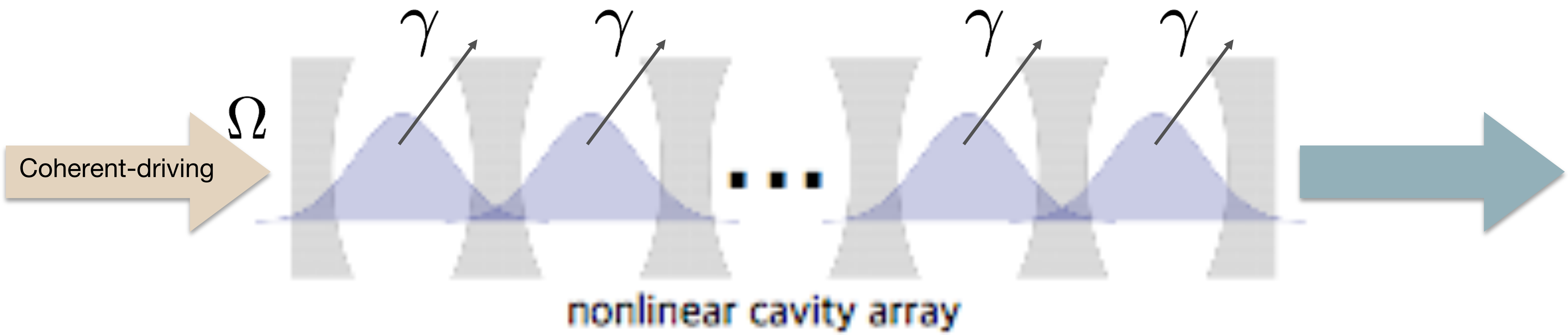


Driven-dissipative approach = Master equation approach

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \frac{\gamma}{2}\sum_j (2\hat{a}_j\rho(t))\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\rho(t) - \rho(t)\hat{a}_j^\dagger\hat{a}_j)$$

$$H = \sum_j \Delta\omega \hat{a}_j^\dagger \hat{a}_j + \sum_j U\hat{a}_j^\dagger\hat{a}_j^\dagger\hat{a}_j\hat{a}_j + \sum_j J(\hat{a}_j^\dagger\hat{a}_{j+1} + \hat{a}_{j+1}^\dagger\hat{a}_j) + \Omega(\hat{a}_1 + \hat{a}_1^\dagger)$$

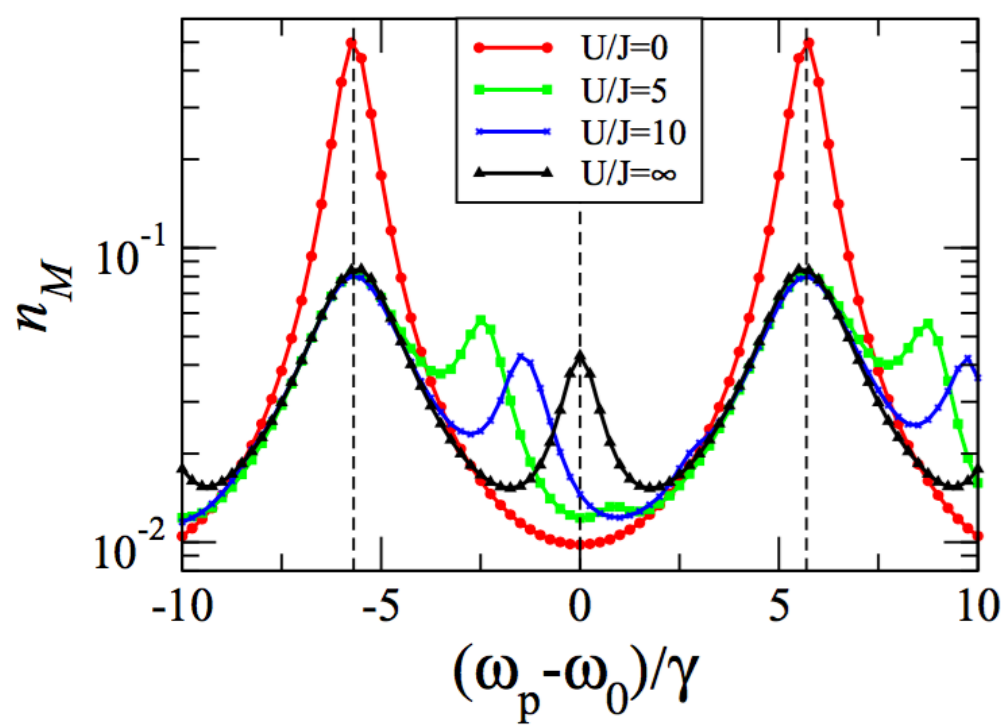
Transport via coherent drive and input-output formalism

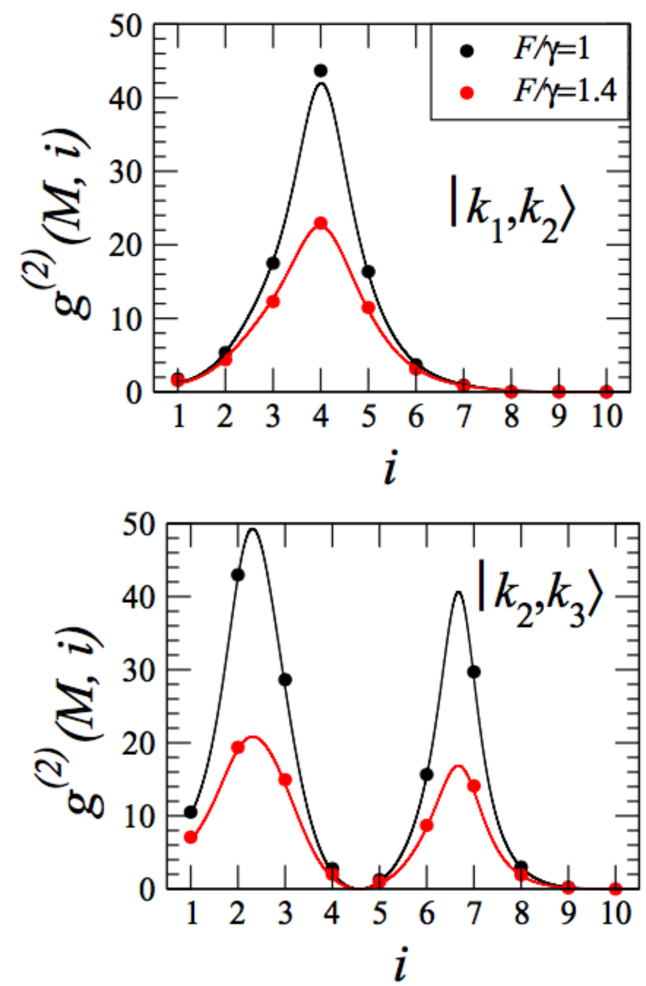


PHYSICAL REVIEW A **91**, 053815 (2015)

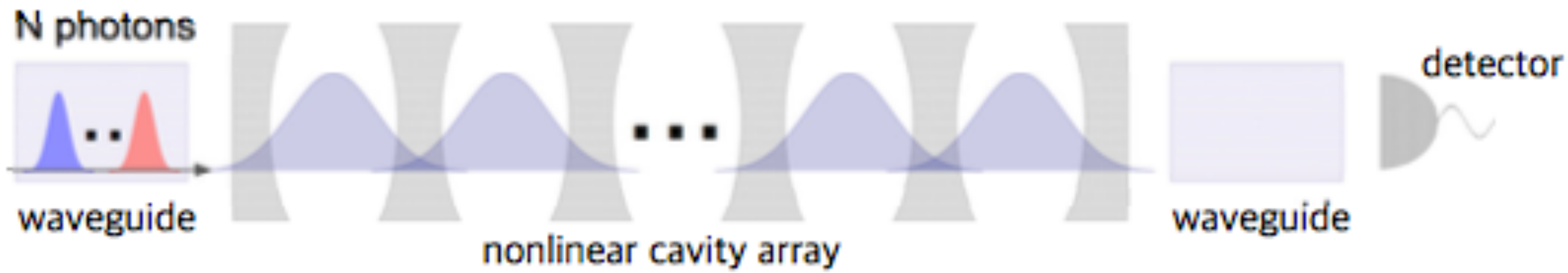
Photon transport in a dissipative chain of nonlinear cavities

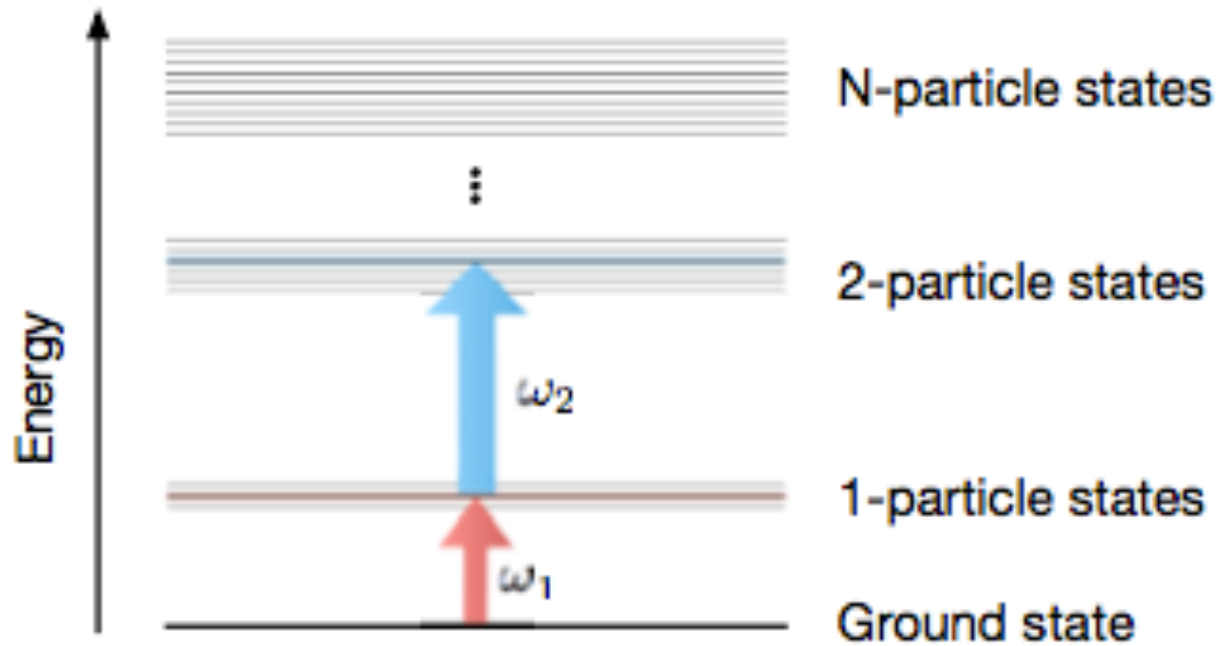
Alberto Biella,[1,*] Leonardo Mazza,[1] Iacopo Carusotto,[2] Davide Rossini,[1] and Rosario Fazio[1]





Transport via quantum drive and scattering formalism



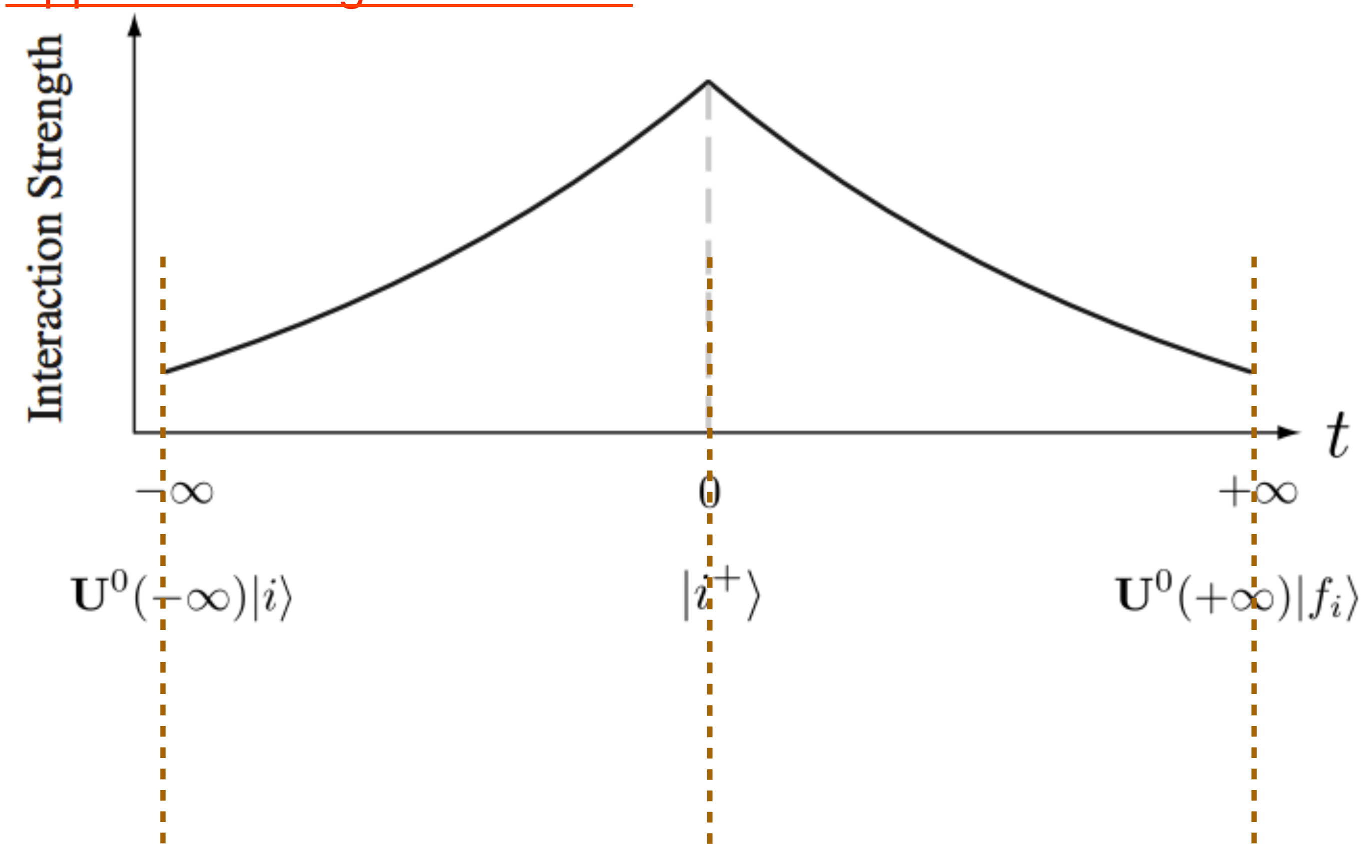


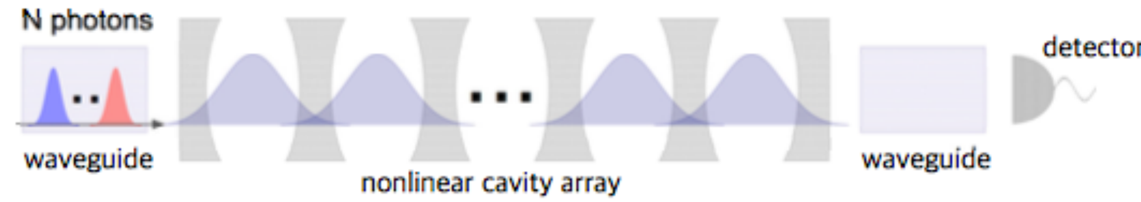
- Use particles (here photons) instead of a coherent field
- Instead of continuous driving, perform a resonant scattering or a quantum transport type of experiment!

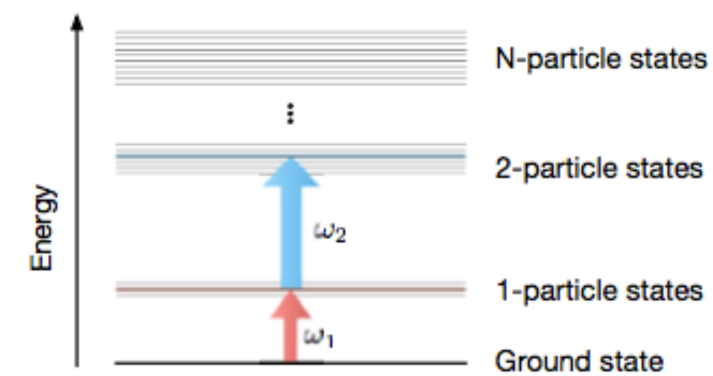
C.Lee , C. Noh, N. Schetakis, DGA “"Few photon transport in nonlinear cavity arrays : Probing signatures of strongly correlated states" arXiv:1412.8374 (to appear in Phys. Rev. A) ”

Scattering via Lippmann Swinger Formalism-Method 1

Lippmann Swinger formalism







$H_0|i\rangle = E|i\rangle$ — Initial state

$H|i^+\rangle = E|i^+\rangle$ — Interacting eigenstate

$H_0|f_i\rangle = E|f_i\rangle$ — Final state

Others: S. Fan , Baranger, M.Pletyukov and others for mainly one or two emitters

More recently Shi-Cirac-Chang, (see talk)

Early theory works by V.Yudson and others

Lippmann Swinger equations

$$|i^+\rangle = |i\rangle + \frac{1}{E - H_0 + i\epsilon} H_{\rm int}|i^+\rangle$$

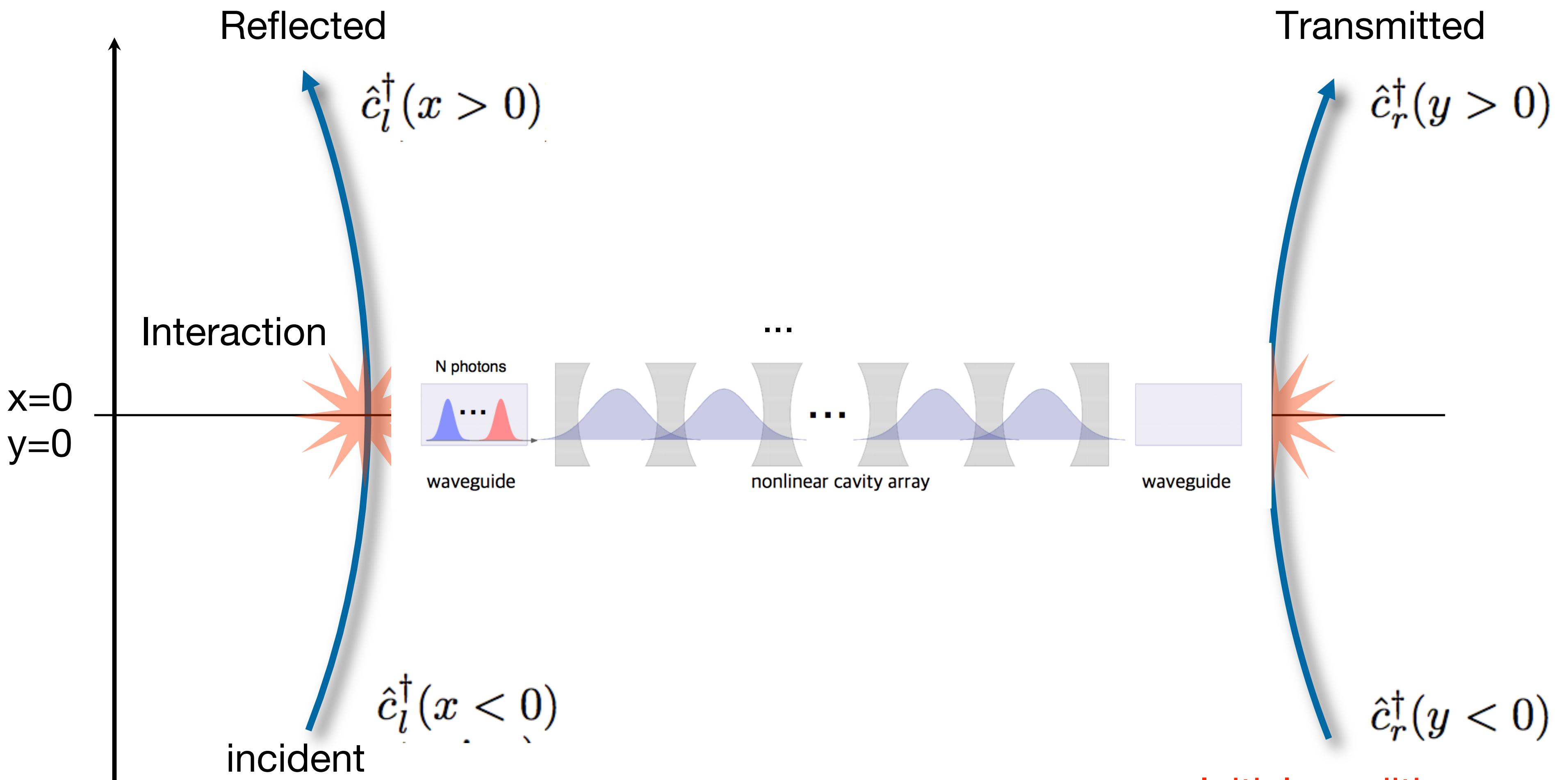
$$|i^+\rangle = |f_i\rangle + \frac{1}{E - H_0 - i\epsilon} H_{\rm int}|i^+\rangle$$

Scattering matrix

$$\mathbf{S} \equiv \sum_i |f_i\rangle\langle i|$$

C.Lee , C. Noh, N. Schetakis, DGA
“"Few photon transport in nonlinear cavity arrays : Probing signatures of strongly correlated states" arXiv:1412.8374 (to appear in Phys. Rev. A) ”

Probing a coupled cavity quantum simulator via scattering



Initial condition

$$\phi_L(x<0)=\frac{1}{\sqrt{2\pi}}e^{ikx} \text{ and } \phi_R(y<0)=0$$

Hamiltonian $H_{\text{tot}} = H_{\text{wg}} + H_{\text{cc}} + H_{\text{wc}}$

$$H_{\text{wg}}=\hbar\int_{-\infty}^{\infty}dx\Big(-iv_g\hat{c}_L^\dagger(x)\frac{\partial}{\partial x}\hat{c}_L(x)\Big)+\hbar\int_{-\infty}^{\infty}dy\Big(-iv_g\hat{c}_R^\dagger(y)\frac{\partial}{\partial y}\hat{c}_R(y)\Big),$$

$$H_{\text{cc}}=\hbar\sum_{j=1}^{N}\big(\omega_j\hat{a}_j^\dagger\hat{a}_j+U_j\hat{a}_j^\dagger\hat{a}_j^\dagger\hat{a}_j\hat{a}_j\big)+\hbar\sum_{j=1}^{N-1}J(\hat{a}_j^\dagger\hat{a}_{j+1}+\hat{a}_j\hat{a}_{j+1}^\dagger),$$

$$H_{\text{wc}}=\hbar\int_{-\infty}^{\infty}dxV_1\delta(x)\big(\hat{c}_L^\dagger(x)\hat{a}_1+\hat{c}_L(x)\hat{a}_1^\dagger\big)+\hbar\int_{-\infty}^{\infty}dyV_2\delta(y)\big(\hat{c}_R^\dagger(y)\hat{a}_N+\hat{c}_R(y)\hat{a}_N^\dagger\big).$$

C.Lee , C. Noh, N. Schetakis, DGA
“"Few photon transport in nonlinear cavity arrays : Probing signatures of strongly correlated states" arXiv:1412.8374 (to appear in Phys. Rev. A) ”

one-photon case (example)

One-photon eigenstate

$$|E^{(1)}\rangle = \int_{-\infty}^{\infty} dx \phi_L(x) \hat{c}_L^\dagger(x)|0\rangle + \int_{-\infty}^{\infty} dy \phi_R(y) \hat{c}_R^\dagger(y)|0\rangle + e_1 \hat{a}_1^\dagger|0\rangle + e_2 \hat{a}_2^\dagger|0\rangle$$

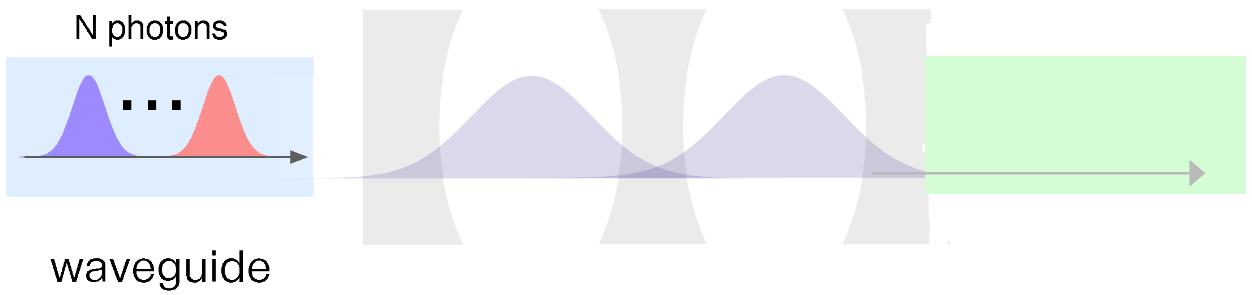
Schrodinger equation $H_{\rm tot}|E^{(1)}\rangle = E^{(1)}|E^{(1)}\rangle$ with $E^{(1)} = k$

Coupled equations

$$\begin{aligned} -i\frac{\partial}{\partial x}\phi_L(x) + V_1\delta(x)e_1 &= E^{(1)}\phi_L(x), \\ -i\frac{\partial}{\partial y}\phi_R(y) + V_2\delta(y)e_2 &= E^{(1)}\phi_R(y), \\ \omega_1 e_1 + Je_2 + V_1\phi_L(0) &= E^{(1)}e_1, \\ \omega_2 e_2 + Je_1 + V_2\phi_R(0) &= E^{(1)}e_2. \end{aligned}$$

Initial condition

$$\phi_L(x<0) = \frac{1}{\sqrt{2\pi}}e^{ikx} \text{ and } \phi_R(y<0) = 0$$



 one-photon case (example)

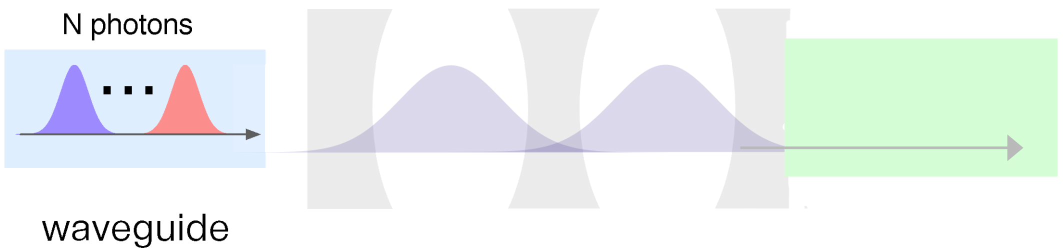
One-photon eigenstate

$$|E^{(1)}\rangle = \int_{-\infty}^{\infty} dx \phi_L(x)\hat{c}_L^\dagger(x)|0\rangle + \int_{-\infty}^{\infty} dy \phi_R(y)\hat{c}_R^\dagger(y)|0\rangle + e_1\hat{a}_1^\dagger|0\rangle + e_2\hat{a}_2^\dagger|0\rangle$$

Solution

$$\phi_L(x) = \frac{1}{\sqrt{2\pi}}(\theta(-x) + r_k\theta(x))e^{ikx}$$

$$\phi_R(y) = \frac{1}{\sqrt{2\pi}}t_k\theta(y)e^{iky}.$$



Scattering matrix

$$\mathbf{S}^{(1)} = \int dk|\phi_{\text{out}}^{(1)}\rangle_k\langle\phi_{\text{in}}^{(1)}|,$$

$$|\phi_{\text{in}}^{(1)}\rangle_k = \int dx\phi_L(x<0)\hat{c}_L^\dagger(x)|0\rangle$$

$$|\phi_{\text{out}}^{(1)}\rangle_k = |\phi_{\text{out}}^{(1)}\rangle_L + |\phi_{\text{out}}^{(1)}\rangle_R$$

$$|\phi_{\text{out}}^{(1)}\rangle_L = \int dx\phi_L(x>0)\hat{c}_L^\dagger(x)|0\rangle \text{ and } |\phi_{\text{out}}^{(1)}\rangle_R = \int dy\phi_R(y>0)\hat{c}_R^\dagger(y)|0\rangle$$

$$e_1 = \frac{\sqrt{\frac{2}{\pi}}V_1(-iV_2^2 - 2E^{(1)} + 2\omega_2)}{4J^2 + (V_1^2 - 2i(E^{(1)} - \omega_1))(V_2^2 - 2i(E^{(1)} - \omega_2))},$$

$$e_2 = -\frac{2J\sqrt{\frac{2}{\pi}}V_1}{4J^2 + (V_1^2 - 2i(E^{(1)} - \omega_1))(V_2^2 - 2i(E^{(1)} - \omega_2))}$$

$$r_k = \frac{4J^2 - (V_1^2 + 2i(E^{(1)} - \omega_1))(V_2^2 - 2i(E^{(1)} - \omega_2))}{4J^2 + (V_1^2 - 2i(E^{(1)} - \omega_1))(V_2^2 - 2i(E^{(1)} - \omega_2))},$$

$$t_k = \frac{4iJV_1V_2}{4J^2 + (V_1^2 - 2i(E^{(1)} - \omega_1))(V_2^2 - 2i(E^{(1)} - \omega_2))}.$$

LS approach: two-photon case

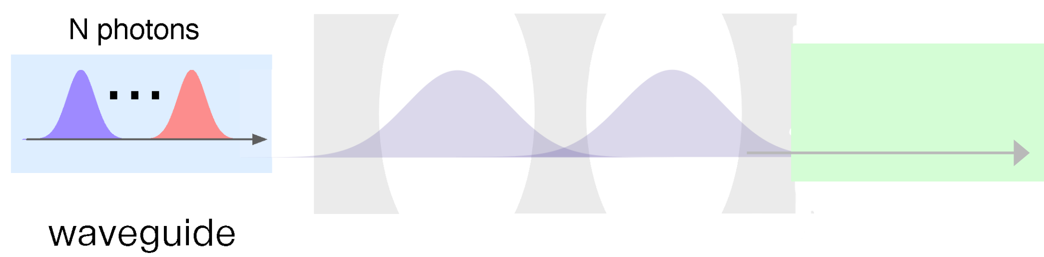
two-photon eigenstate $|E^{(2)}\rangle = |E_1^{(2)}\rangle + |E_2^{(2)}\rangle + |E_3^{(2)}\rangle$

$$|E_1^{(2)}\rangle = \int_{-\infty}^{\infty} dx_1 dx_2 \phi_{LL}(x_1, x_2) \frac{1}{\sqrt{2}} \hat{c}_L^\dagger(x_1)\hat{c}_L^\dagger(x_2)|0\rangle + \int_{-\infty}^{\infty} dy_1 dy_2 \phi_{RR}(y_1, y_2) \frac{1}{\sqrt{2}} \hat{c}_R^\dagger(y_1)\hat{c}_R^\dagger(y_2)|0\rangle + \int_{-\infty}^{\infty} dx_1 dy_1 \phi_{LR}(x_1, y_1)\hat{c}_L^\dagger(x_1)\hat{c}_R^\dagger(y_1)|0\rangle,$$

$$|E_2^{(2)}\rangle = e_{11}\frac{1}{\sqrt{2}}\hat{a}_1^\dagger\hat{a}_1^\dagger|0\rangle + e_{12}\hat{a}_1^\dagger\hat{a}_2^\dagger|0\rangle + e_{22}\frac{1}{\sqrt{2}}\hat{a}_2^\dagger\hat{a}_2^\dagger|0\rangle,$$

$$|E_3^{(2)}\rangle = \int_{-\infty}^{\infty} dx_1\Big(\phi_{L1}(x_1)\hat{c}_L^\dagger(x_1)\hat{a}_1^\dagger + \phi_{L2}(x_1)\hat{c}_L^\dagger(x_1)\hat{a}_2^\dagger\Big)|0\rangle + \int_{-\infty}^{\infty} dy_1\Big(\phi_{R1}(y_1)\hat{c}_R^\dagger(y_1)\hat{a}_1^\dagger + \phi_{R2}(y_1)\hat{c}_R^\dagger(y_1)\hat{a}_2^\dagger\Big)|0\rangle.$$

Schrodinger equation $H|E^{(2)}\rangle = E^{(2)}|E^{(2)}\rangle$



$$H_{\text{tot}} = H_{\text{wg}} + H_{\text{cc}} + H_{\text{wc}}$$

$$H_{\text{wg}} = \hbar\int_{-\infty}^{\infty} dx\Big(-iv_g\hat{c}_L^\dagger(x)\frac{\partial}{\partial x}\hat{c}_L(x)\Big) + \hbar\int_{-\infty}^{\infty} dy\Big(-iv_g\hat{c}_R^\dagger(y)\frac{\partial}{\partial y}\hat{c}_R(y)\Big),$$

$$H_{\text{cc}} = \hbar\sum_{j=1}^{N}\Big(\omega_j\hat{a}_j^\dagger\hat{a}_j + U_j\hat{a}_j^\dagger\hat{a}_j^\dagger\hat{a}_j\hat{a}_j\Big) + \hbar\sum_{j=1}^{N-1} J(\hat{a}_j^\dagger\hat{a}_{j+1} + \hat{a}_j\hat{a}_{j+1}^\dagger),$$

$$H_{\text{wc}} = \hbar\int_{-\infty}^{\infty} dxV_1\delta(x)\Big(\hat{c}_L^\dagger(x)\hat{a}_1 + \hat{c}_L(x)\hat{a}_1^\dagger\Big) + \hbar\int_{-\infty}^{\infty} dyV_2\delta(y)\Big(\hat{c}_R^\dagger(y)\hat{a}_N + \hat{c}_R(y)\hat{a}_N^\dagger\Big).$$

Scattering matrix: two-photon case

Coupled equations

$$
\begin{aligned}
-i\frac{\partial}{\partial x_2}\phi_{LL}(x_1,x_2) - i\frac{\partial}{\partial x_1}\phi_{LL}(x_1,x_2) + \frac{V_1}{\sqrt{2}}(\delta(x_1)\phi_{L1}(x_2) + \delta(x_2)\phi_{L1}(x_1)) &= E^{(2)}\phi_{LL}(x_1,x_2), \\
-i\frac{\partial}{\partial x_1}\phi_{LR}(x_1,y_1) - i\frac{\partial}{\partial y_1}\phi_{LR}(x_1,y_1) + V_1\delta(x_1)\phi_{R1}(y_1) + V_2\delta(y_1)\phi_{L2}(x_1) &= E^{(2)}\phi_{LR}(x_1,y_1), \\
-i\frac{\partial}{\partial y_2}\phi_{RR}(y_1,y_2) - i\frac{\partial}{\partial y_1}\phi_{RR}(y_1,y_2) + \frac{V_2}{\sqrt{2}}(\delta(y_1)\phi_{R2}(y_2) + \delta(y_2)\phi_{R2}(y_1)) &= E^{(2)}\phi_{RR}(y_1,y_2), \\
-i\frac{\partial}{\partial x_1}\phi_{L1}(x_1) + \phi_{L1}(x_1)\omega_1 + \phi_{L2}(x_1)J + V_1\delta(x_1)e_{11}\sqrt{2} + V_1\frac{1}{\sqrt{2}}(\phi_{LL}(x_1,0) + \phi_{LL}(0,x_1)) &= E^{(2)}\phi_{L1}(x_1), \\
-i\frac{\partial}{\partial x_1}\phi_{L2}(x_1) + \phi_{L2}(x_1)\omega_2 + \phi_{L1}(x_1)J + V_1\delta(x_1)e_{12} + V_2\phi_{LR}(x_1,0) &= E^{(2)}\phi_{L2}(x_1), \\
-i\frac{\partial}{\partial y_1}\phi_{R1}(y_1) + \phi_{R1}(y_1)\omega_1 + \phi_{R2}(y_1)J + V_2\delta(y_1)e_{12} + V_1\phi_{LR}(0,y_1) &= E^{(2)}\phi_{R1}(y_1), \\
-i\frac{\partial}{\partial y_1}\phi_{R2}(y_1) + \phi_{R2}(y_1)\omega_2 + \phi_{R1}(y_1)J + V_2\delta(y_1)e_{22}\sqrt{2} + V_2\frac{1}{\sqrt{2}}(\phi_{RR}(y_1,0) + \phi_{RR}(0,y_1)) &= E^{(2)}\phi_{R2}(y_1), \\
\omega_1 e_{11}\sqrt{2} + Je_{12} + U_1 e_{11}\sqrt{2} + V_1\phi_{L1}(0) &= E^{(2)}e_{11}\frac{1}{\sqrt{2}}, \\
\omega_1 e_{12} + \omega_2 e_{12} + Je_{22}\sqrt{2} + Je_{11}\sqrt{2} + V_1\phi_{L2}(0) + V_2\phi_{R1}(0) &= E^{(2)}e_{12}, \\
\omega_2 e_{22}\sqrt{2} + Je_{12} + U_2 e_{22}\sqrt{2} + V_2\phi_{R2}(0) &= E^{(2)}e_{22}\frac{1}{\sqrt{2}}.
\end{aligned}
$$

Initial condition

$$
\begin{aligned}
\phi_{LL}(x_1<0, x_2<0) &= \frac{1}{\sqrt{2}}\frac{1}{2\pi}(e^{ik_1x_1+ik_2x_2} + e^{ik_2x_1+ik_1x_2}), \\
\phi_{LR}(x_1<0, y_1<0) &= 0, \\
\phi_{RR}(y_1<0, y_2<0) &= 0.
\end{aligned}
$$

Boundary conditions

$$
\begin{aligned}
\phi_{LL}(0_+,x_2) &= \phi_{LL}(0_-,x_2) - i\frac{V_1}{\sqrt{2}}\phi_{L1}(x_2), \\
\phi_{LL}(x_1,0_+) &= \phi_{LL}(x_1,0_-) - i\frac{V_1}{\sqrt{2}}\phi_{L1}(x_1), \\
\phi_{LR}(0_+,y_1) &= \phi_{LR}(0_-,y_1) - iV_1\phi_{R1}(y_1), \\
\phi_{LR}(x_1,0_+) &= \phi_{LR}(x_1,0_-) - iV_2\phi_{L2}(x_1), \\
\phi_{RR}(0_+,y_2) &= \phi_{RR}(0_-,y_2) - i\frac{V_2}{\sqrt{2}}\phi_{R2}(y_2), \\
\phi_{RR}(y_1,0_+) &= \phi_{RR}(y_1,0_-) - i\frac{V_2}{\sqrt{2}}\phi_{R2}(y_1).
\end{aligned}
$$

$$
\begin{aligned}
\phi_{L1}(0_+) &= \phi_{L1}(0_-) - iV_1e_{11}\sqrt{2}, \\
\phi_{L2}(0_+) &= \phi_{L2}(0_-) - iV_1e_{12}, \\
\phi_{R1}(0_+) &= \phi_{R1}(0_-) - iV_2e_{12}, \\
\phi_{R2}(0_+) &= \phi_{R2}(0_-) - iV_2e_{22}\sqrt{2}.
\end{aligned}
$$

$$
\begin{aligned}
\phi_{LL}(0,x) &= \phi_{LL}(x,0) = \frac{1}{2}(\phi_{LL}(0_+,x) + \phi_{LL}(0_-,x)), \\
\phi_{LR}(0,y) &= \frac{1}{2}(\phi_{LR}(0_+,y) + \phi_{LR}(0_-,y)), \\
\phi_{LR}(x,0) &= \frac{1}{2}(\phi_{LR}(x,0_+) + \phi_{LR}(x,0_-)), \\
\phi_{RR}(0,y) &= \phi_{RR}(y,0) = \frac{1}{2}(\phi_{RR}(0_+,y) + \phi_{RR}(0_-,y)).
\end{aligned}
$$

Scattering matrix: two-photon case

Solution

$$\phi_{LL}(x_1,x_2) = \frac{1}{\sqrt{2}}\frac{1}{2\pi}\Big(\sum_P (\theta(-x_1)\theta(-x_2)+\theta(x_1)\theta(x_2)r_{k_{P_1}}r_{k_{P_2}})e^{ik_{P_2}x_1+ik_{P_1}x_2}$$
$$+\sum_Q e^{i(k_1+k_2)x_{Q_1}}\Big(B_{LL_-}e^{i\lambda_-(x_{Q_1}-x_{Q_2})}+B_{LL_+}e^{i\lambda_+(x_{Q_1}-x_{Q_2})}\Big)\theta(x_{Q_2}-x_{Q_1})\theta(x_{Q_1})\Big),$$

$$\phi_{LR}(x_1,y_1) = \frac{1}{2\pi}\Big(\sum_P \theta(y_1)\theta(y_1)t_{k_{P_1}}r_{k_{P_2}}e^{ik_{P_2}x_1+ik_{P_1}y_1}$$
$$+e^{i(k_1+k_2)x_1}\Big(B_{LR1_-}e^{i\lambda_-(x_1-y_1)}+B_{LR1_+}e^{i\lambda_+(x_1-y_1)}\Big)\theta(y_1-x_1)\theta(x_1)$$
$$+e^{i(k_1+k_2)y_1}\Big(B_{LR2_-}e^{i\lambda_-(y_1-x_1)}+B_{LR2_+}e^{i\lambda_+(y_1-x_1)}\Big)\theta(x_1-y_1)\theta(y_1)\Big),$$

$$\phi_{RR}(y_1,y_2) = \frac{1}{\sqrt{2}}\frac{1}{2\pi}\Big(\sum_P \theta(y_1)\theta(y_2)t_{k_{P_1}}t_{k_{P_2}}e^{ik_{P_2}y_1+ik_{P_1}y_2}$$
$$+\sum_Q e^{i(k_1+k_2)y_{Q_1}}\Big(B_{RR_-}e^{i\lambda_-(y_{Q_1}-y_{Q_2})}+B_{RR_+}e^{i\lambda_+(y_{Q_1}-y_{Q_2})}\Big)\theta(y_{Q_2}-y_{Q_1})\theta(y_{Q_1})\Big),$$

Scattering matrix

Sum_P -> (P1,P2)=(1,2) or (2,1)
Sum_Q -> (Q1,Q2)=(1,2) or (2,1)

$$\mathbf{S}^{(2)} = \int dk_1 dk_2 \frac{1}{2!}|\phi^{(2)}_{\rm out}\rangle_{k_1,k_2}\langle\phi^{(2)}_{\rm in}|$$

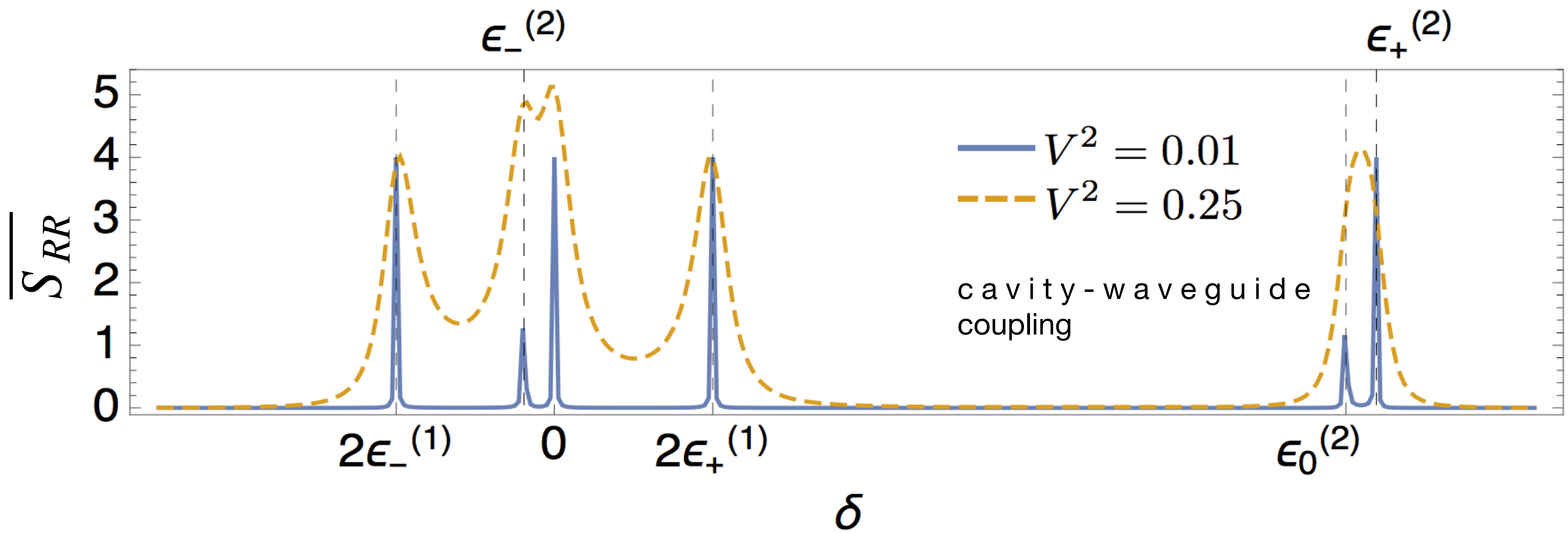
Scattering matrix: two-photon case

Scattering matrix $\mathbf{S}^{(2)} = \int dk_1 dk_2 \frac{1}{2!} |\phi^{(2)}_{\text{out}}\rangle_{k_1,k_2} \langle \phi^{(2)}_{\text{in}}|$

$${}_{RR}\langle p_1, p_2|\mathbf{S}^{(2)}|k_1, k_2\rangle = S_{RR}\delta(k_1 + k_2 - p_1 - p_2) + (t_{k_1} t_{k_2} \delta(k_1 - p_1)\delta(k_2 - p_2) + (k_1 \leftrightarrow k_2))$$

$$\overline{S_{RR}} = \int d\Delta k d\Delta p |S_{RR}|^2$$

$$\Delta k = k_1 - k_2$$
$$\Delta p = p_1 - p_2$$
$$2\omega_0 + \delta = k_1 + k_2 = p_1 + p_2$$



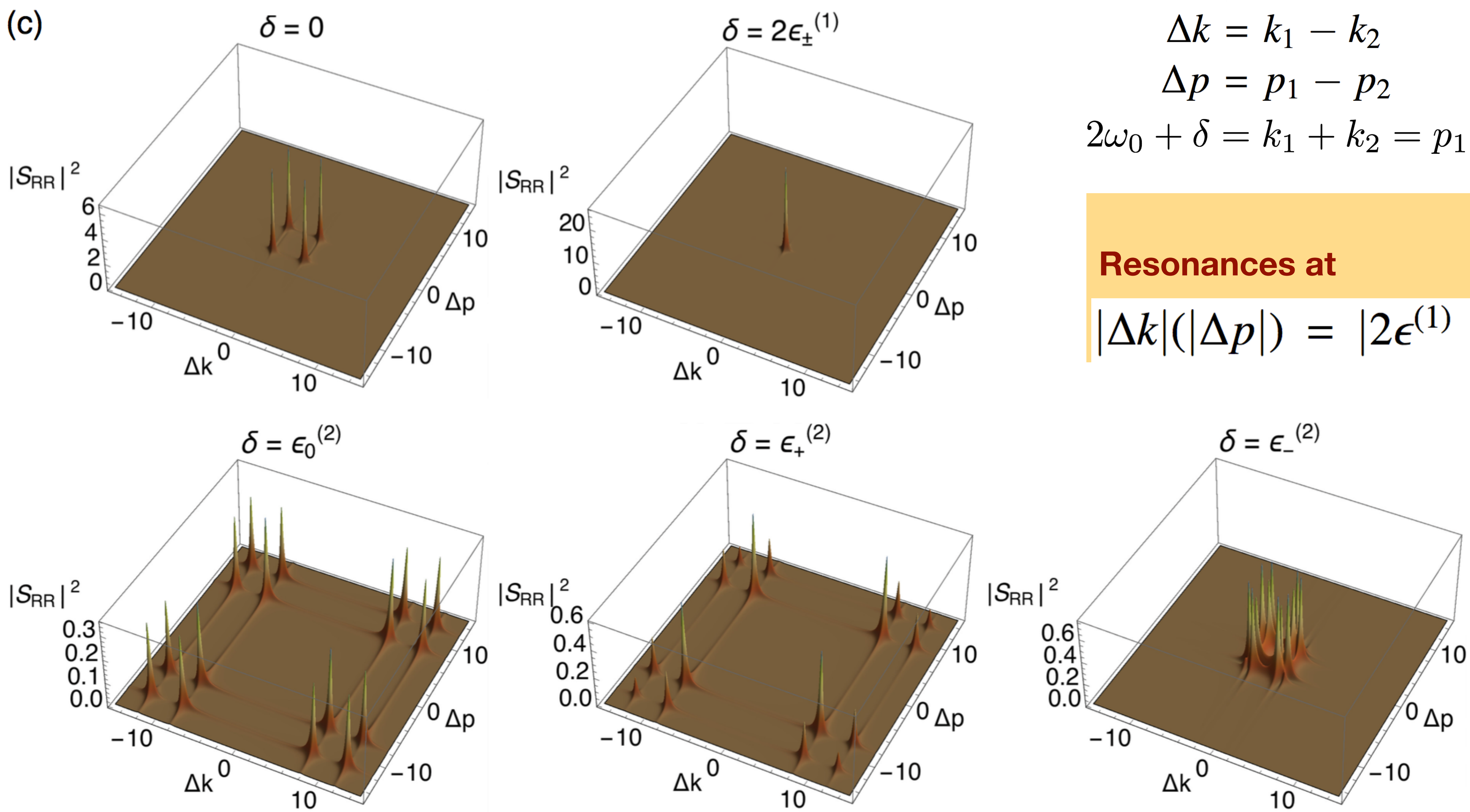
For $\frac{U}{J} = 0,1,5$ and assuming sending in *narrow* gaussian pulses of width σ/J=0.005

two-photon case

Scattering matrix

$$\mathbf{S}^{(2)} = \int dk_1 dk_2 \frac{1}{2!} |\phi^{(2)}_{\text{out}}\rangle_{k_1,k_2} \langle \phi^{(2)}_{\text{in}}|$$

$$_{RR}\langle p_1, p_2|\mathbf{S}^{(2)}|k_1, k_2\rangle = S_{RR}\delta(k_1 + k_2 - p_1 - p_2) + (t_{k_1} t_{k_2} \delta(k_1 - p_1)\delta(k_2 - p_2) + (k_1 \leftrightarrow k_2))$$



$$\Delta k = k_1 - k_2$$
$$\Delta p = p_1 - p_2$$
$$2\omega_0 + \delta = k_1 + k_2 = p_1 + p_2$$

Resonances at

$$|\Delta k|(|\Delta p|) = |2\epsilon^{(1)} - \delta|$$

Scattering matrix: # two-photon case

Scattering matrix
$$\mathbf{S}^{(2)} = \int dk_1 dk_2 \frac{1}{2!} |\phi^{(2)}_{\text{out}}\rangle_{k_1,k_2} \langle \phi^{(2)}_{\text{in}}|$$

$${}_{RR}\langle p_1, p_2|\mathbf{S}^{(2)}|k_1, k_2\rangle = S_{RR}\delta(k_1 + k_2 - p_1 - p_2) + (t_{k_1} t_{k_2}\delta(k_1 - p_1)\delta(k_2 - p_2) + (k_1 \leftrightarrow k_2))$$

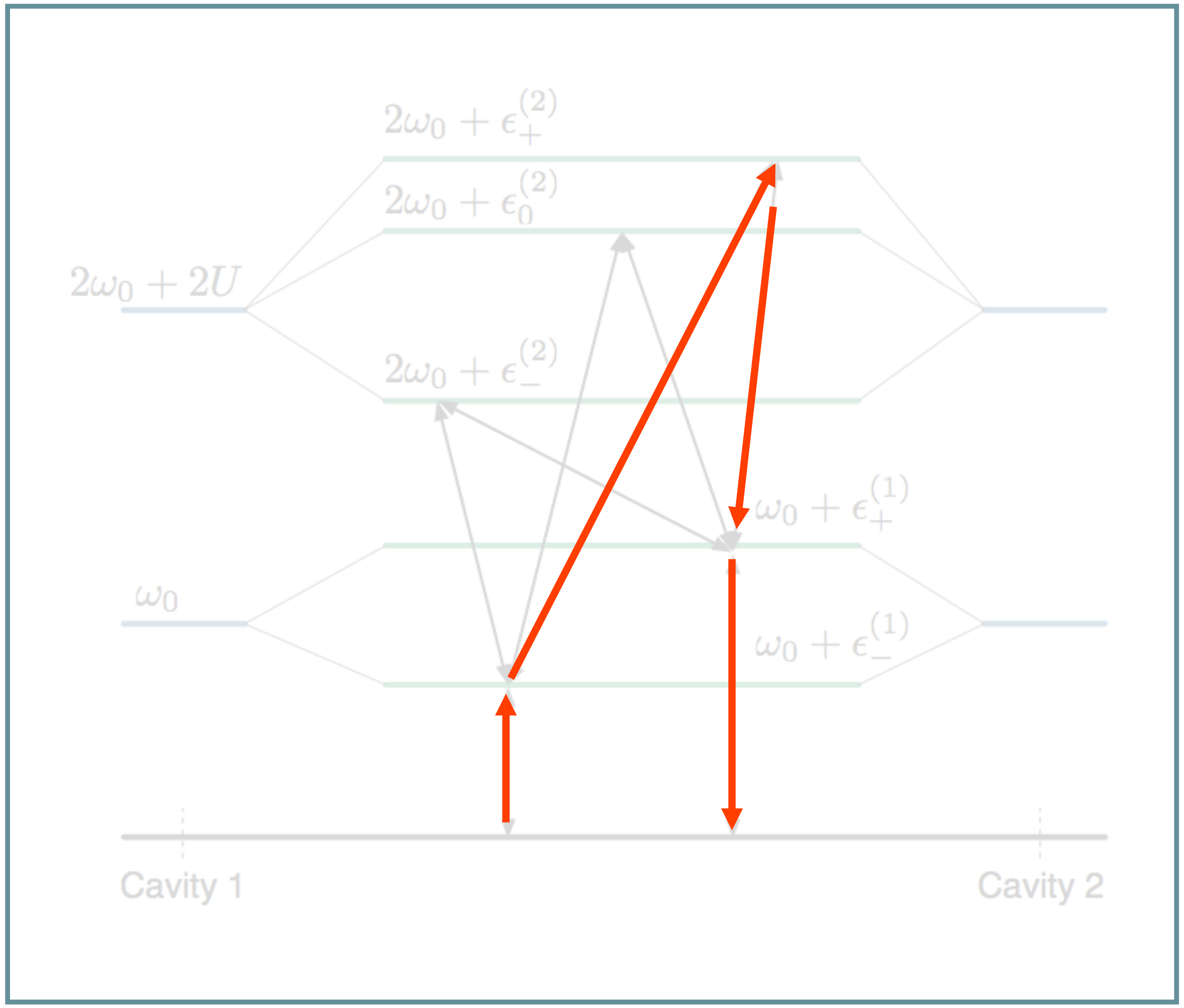
Resonances at

$$|\Delta k|(|\Delta p|) = |2\epsilon^{(1)} - \delta|$$

For example,

$$k_1 = \omega_0 + \epsilon_-^{(1)}$$
$$k_2 = \omega_0 + \epsilon_+^{(2)} - \epsilon_-^{(1)}$$
$$p_1 = \omega_0 + \epsilon_+^{(2)} - \epsilon_+^{(1)}$$
$$p_2 = \omega_0 + \epsilon_+^{(1)}$$

$$|2_0\rangle \sim |20\rangle - |02\rangle,$$
$$|2_\pm\rangle \sim |20\rangle + |02\rangle - \frac{U \mp \sqrt{4J^2 + U^2}}{\sqrt{2}J}|11\rangle$$
$$\epsilon_\pm^{(1)} = \pm J$$
$$\epsilon_\pm^{(2)} \equiv U \pm \sqrt{4J^2 + U^2}$$
$$\epsilon_0^{(2)} \equiv 2U$$



The nonlinear effects are significant only if one of the input or output photons is resonant with one of the single-photon eigenstates.

Scattering matrix: two-photon case, resonant versus off-resonant cases

Scattering matrix

$$\mathbf{S}^{(2)} = \int dk_1 dk_2 \frac{1}{2!} |\phi^{(2)}_{\text{out}}\rangle_{k_1,k_2} \langle \phi^{(2)}_{\text{in}}|$$

$$_{RR}\langle p_1, p_2|\mathbf{S}^{(2)}|k_1, k_2\rangle = S_{RR}\delta(k_1 + k_2 - p_1 - p_2) + (t_{k_1} t_{k_2} \delta(k_1 - p_1)\delta(k_2 - p_2) + (k_1 \leftrightarrow k_2))$$

Resonances at

$$|\Delta k|(|\Delta p|) = |2\epsilon^{(1)} - \delta|$$

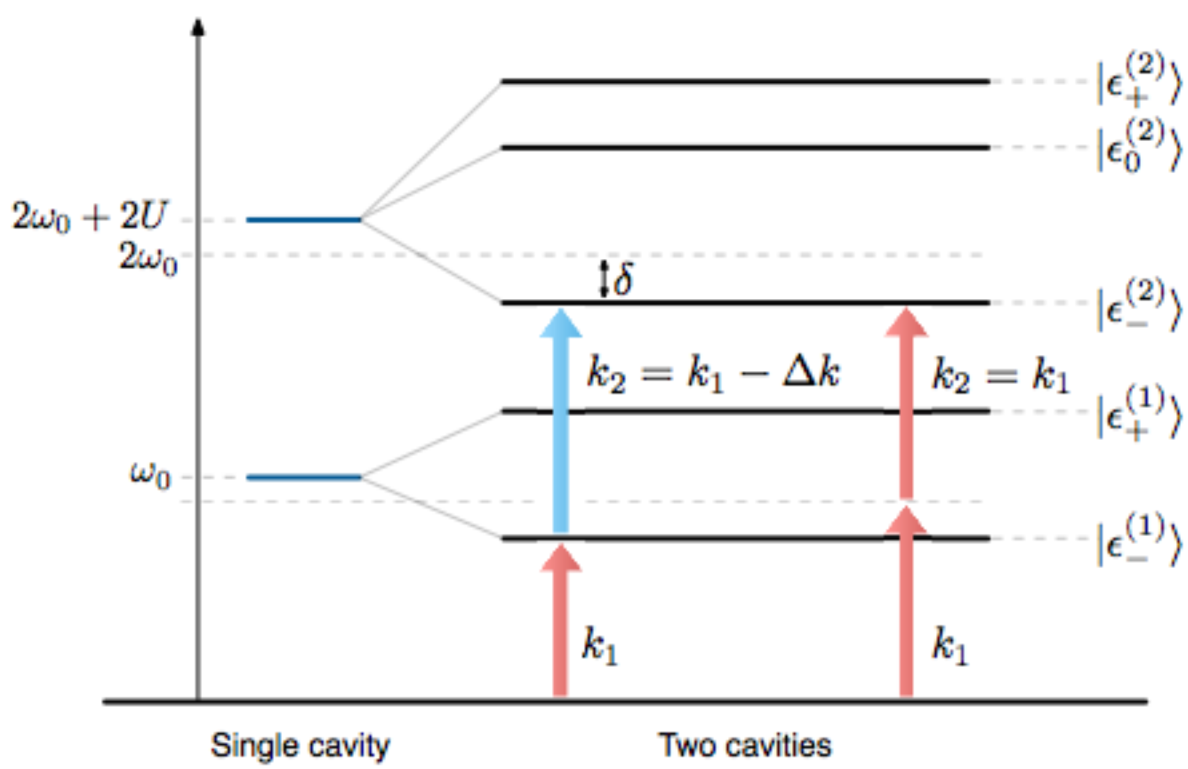
$$|2_0\rangle \sim |20\rangle - |02\rangle,$$

$$|2_\pm\rangle \sim |20\rangle + |02\rangle - \frac{U \mp \sqrt{4J^2 + U^2}}{\sqrt{2}J}|11\rangle$$

$$\epsilon^{(1)}_\pm = \pm J$$

$$\epsilon^{(2)}_\pm \equiv U \pm \sqrt{4J^2 + U^2}$$

$$\epsilon^{(2)}_0 \equiv 2U$$



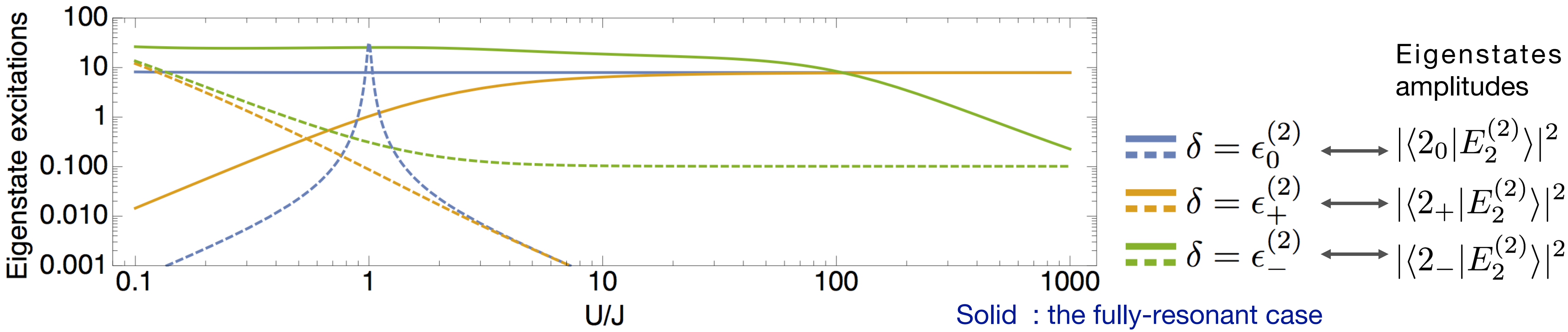
fully-resonant — off-resonant

$$\Delta k = \delta - 2\epsilon^{(1)}_- \rightarrow k_1 = \delta - \epsilon_- \qquad \Delta k = 0 \rightarrow k_1 = k_2$$

$$k_2 = \epsilon_-$$

Scattering matrix: two-photon case and difference between resonant and off resonant driving

$$|E_2^{(2)}\rangle = e_{11}\frac{1}{\sqrt{2}}\hat{a}_1^\dagger\hat{a}_1^\dagger|0\rangle + e_{12}\hat{a}_1^\dagger\hat{a}_2^\dagger|0\rangle + e_{22}\frac{1}{\sqrt{2}}\hat{a}_2^\dagger\hat{a}_2^\dagger|0\rangle$$



Solid : the fully-resonant case
Dotted: the off-resonant case

$$|2_0\rangle \sim |20\rangle - |02\rangle,$$

$$|2_\pm\rangle \sim |20\rangle + |02\rangle - \frac{U \mp \sqrt{4J^2+U^2}}{\sqrt{2}J}|11\rangle$$

$$\epsilon_\pm^{(1)} = \pm J$$

$$\epsilon_\pm^{(2)} \equiv U \pm \sqrt{4J^2+U^2}$$

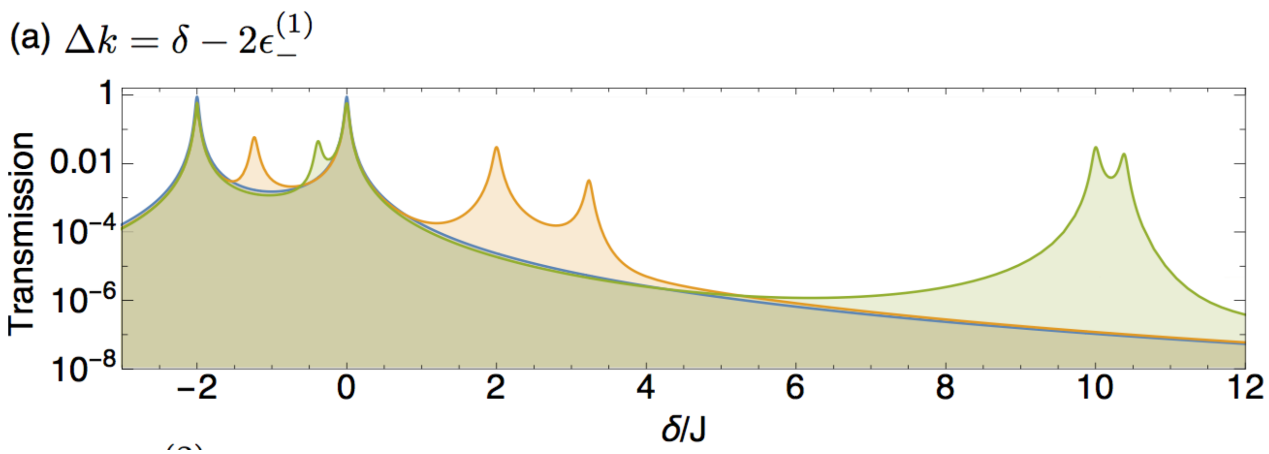
$$\epsilon_0^{(2)} \equiv 2U$$

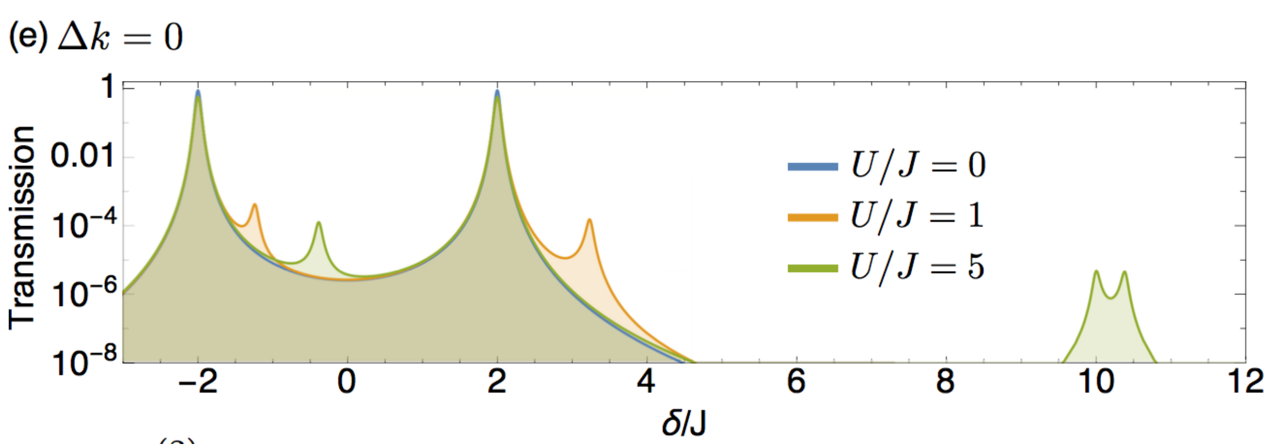
The desired eigenstates are much more efficiently excited when the fully-resonant condition is met as opposed to the off-resonant case!

Results - scattering probabilities

fully-resonant

off-resonant

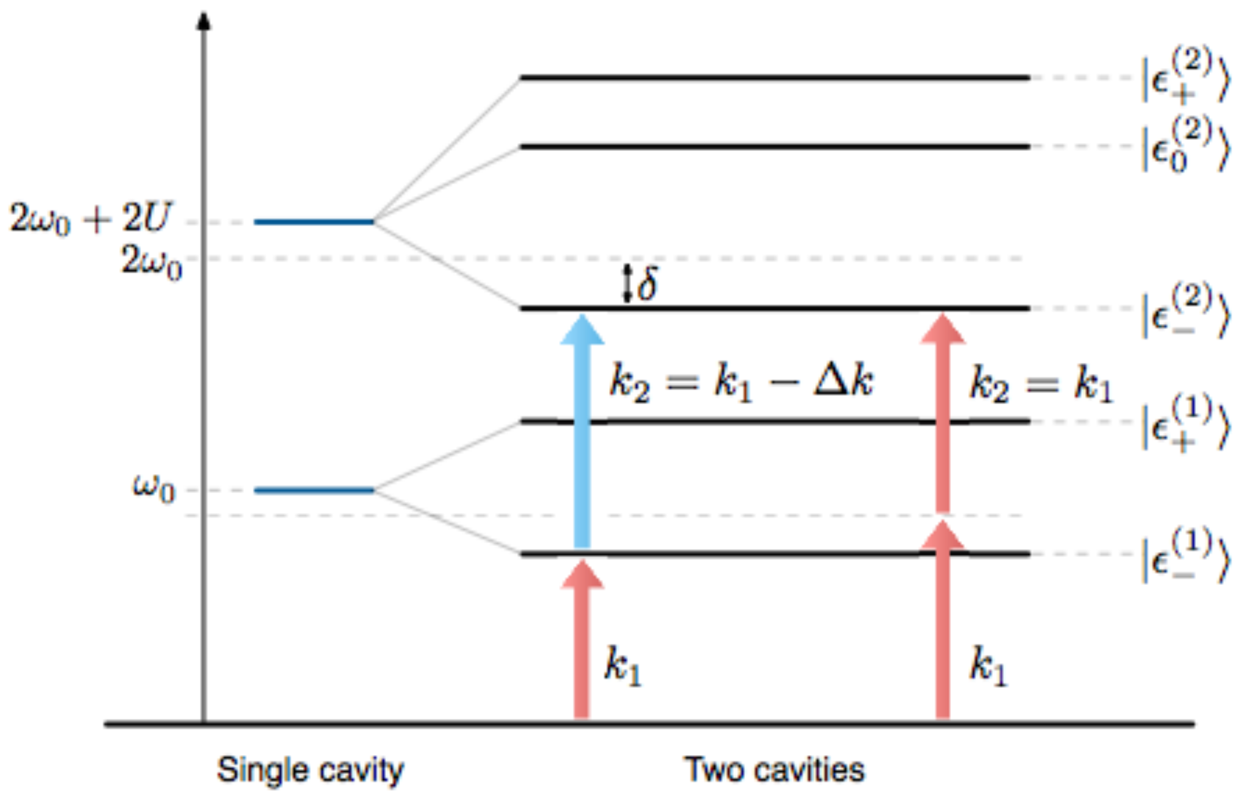




For $\frac{U}{J} = 0,1,5$ and assuming sending in

narrow gaussian pulses of width σ/J=0.005, $\frac{V^2}{J} = 0.04$

In the fully-resonant case, the transmission is generally significantly larger which makes the approach experimentally more efficient.



- intensity-intensity correlations

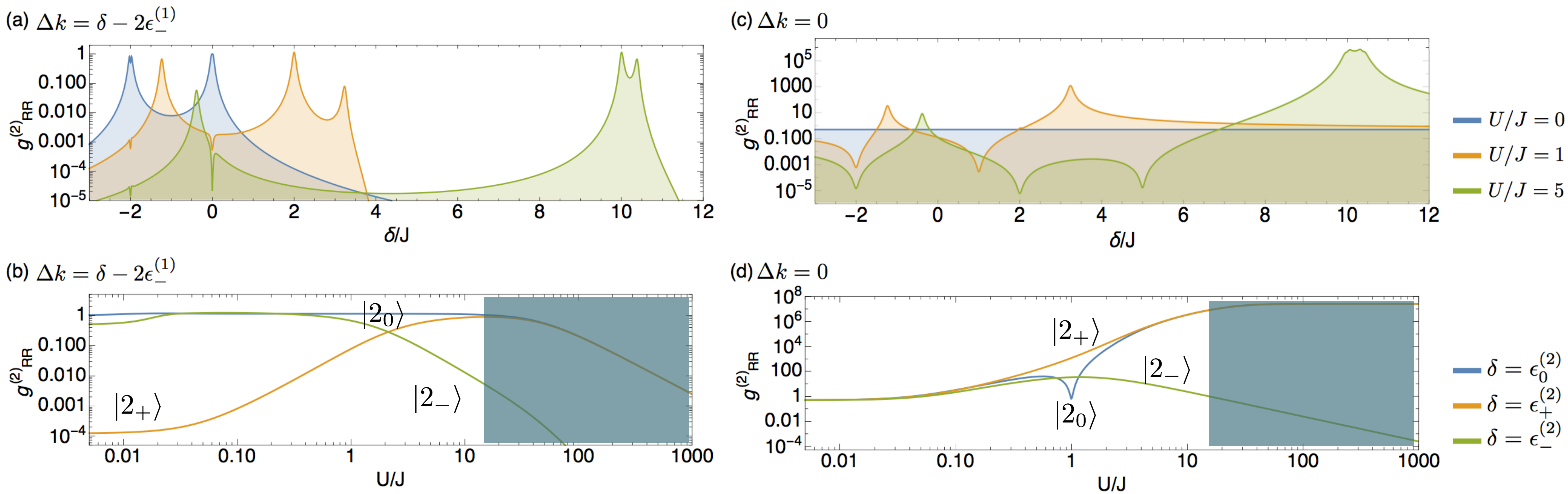
$$g^{(2)}_{RR}(z_1, z_2) = \frac{\langle \mathrm{out}^{(2)}_{\{\xi\}} | \hat{c}^{\dagger}_R(z_1) \hat{c}^{\dagger}_R(z_2) \hat{c}_R(z_2) \hat{c}_R(z_2) | \mathrm{out}^{(2)}_{\{\xi\}} \rangle}{\langle \mathrm{out}^{(2)}_{\{\xi\}} | \hat{c}^{\dagger}_R(z_1) \hat{c}_R(z_1) | \mathrm{out}^{(2)}_{\{\xi\}} \rangle \langle \mathrm{out}^{(2)}_{\{\xi\}} | \hat{c}^{\dagger}_R(z_2) \hat{c}_R(z_2) | \mathrm{out}^{(2)}_{\{\xi\}} \rangle}$$

$$= \frac{2\left| \int_{\{\xi(k)\}} \phi_{RR}(z_1, z_2) \right|^2}{\frac{1}{M_2} \int dx \Big(\left| \int_{\{\xi(k)\}} \phi_{LR}(x, z_1) \right|^2 + 2 \left| \int_{\{\xi(k)\}} \phi_{RR}(x, z_1) \right|^2 \Big) \int dx \Big(\left| \int_{\{\xi(k)\}} \phi_{LR}(x, z_2) \right|^2 + 2 \left| \int_{\{\xi(k)\}} \phi_{RR}(x, z_2) \right|^2 \Big)}$$

- intensity-intensity correlations

fully-resonant

off-resonant



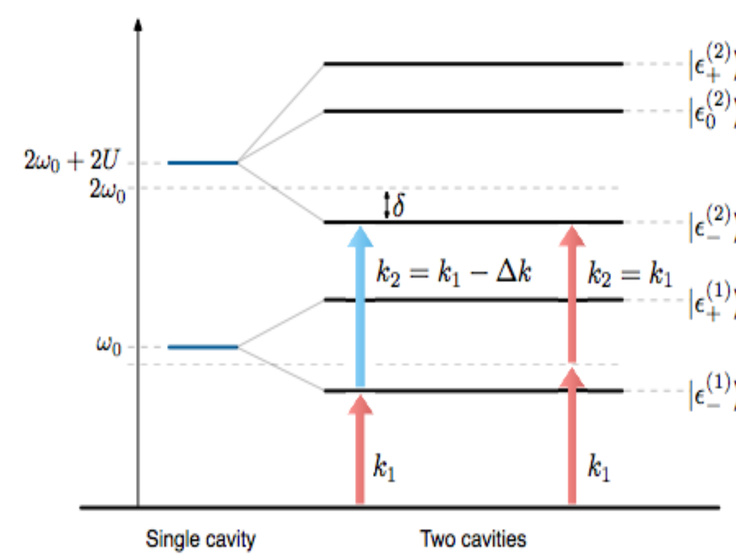
$$|2_0\rangle \sim |20\rangle - |02\rangle,$$

$$|2_\pm\rangle \sim |20\rangle + |02\rangle - \frac{U \mp \sqrt{4J^2 + U^2}}{\sqrt{2}J}|11\rangle$$

$$\epsilon_\pm^{(1)} = \pm J.$$

$$\epsilon_\pm^{(2)} \equiv U \pm \sqrt{4J^2 + U^2}$$

$$\epsilon_0^{(2)} \equiv 2U$$



The fully-resonant scattering scenario maps out the underlying many-body states better compared to the identical-photon scattering scenario.

Driving with a coherent field

Coherent state

$$|\alpha\rangle = e^{\hat{c}_\alpha^\dagger - \hat{c}_\alpha}|0\rangle$$

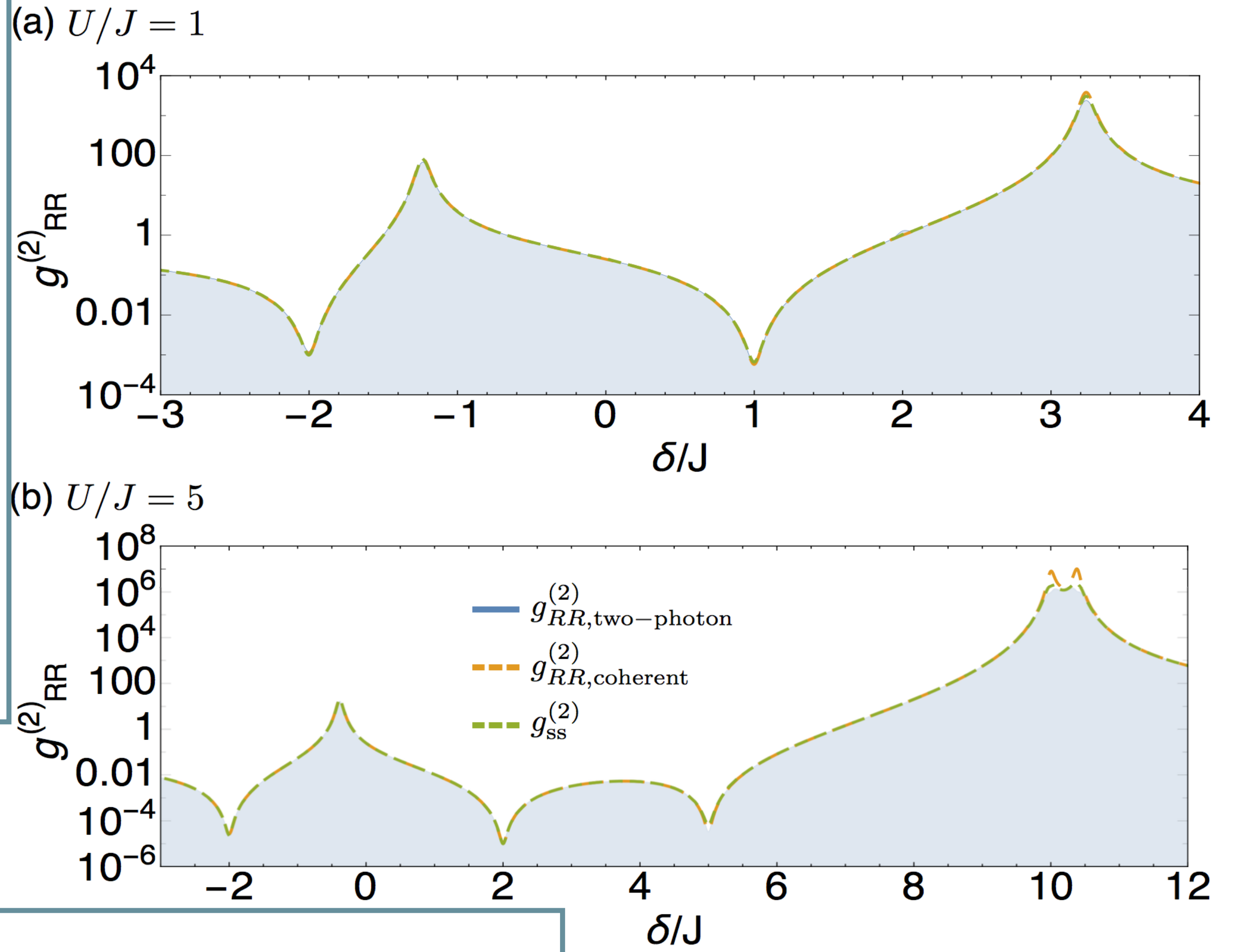
Output state

$$|\text{out}_\alpha\rangle = \sum_n \mathbf{S}^{(n)}|\alpha\rangle$$
$$\approx e^{-\bar{n}/2}(|0\rangle + \mathbf{S}^{(1)}\hat{c}_\alpha^\dagger|0\rangle + \frac{1}{2}\mathbf{S}^{(2)}(\hat{c}_\alpha^\dagger)^2|0\rangle)$$

Correspondence

$$g^{(2)}_{RR,\text{two-photon}} \approx \frac{1}{2} g^{(2)}_{RR,\text{coherent}}$$

off-resonant



Driven-dissipative approach

$$\frac{d\rho}{dt} = -i[\rho, H] + \frac{\gamma_{\text{total}}}{2}\sum_j(2\hat{a}_j\rho\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\rho - \rho\hat{a}_j^\dagger\hat{a}_j).$$

$$H = \sum_j \Delta\omega\hat{a}_j^\dagger\hat{a}_j + \sum_j U\hat{a}_j^\dagger\hat{a}_j^\dagger\hat{a}_j\hat{a}_j + \sum_j J(\hat{a}_j^\dagger\hat{a}_{j+1} + \hat{a}_{j+1}^\dagger\hat{a}_j) + \Omega(\hat{a}_1 + \hat{a}_1^\dagger)$$

$$g^{(2)}_{ss} = \frac{\text{Tr}[\hat{a}_{\text{out}}^\dagger\hat{a}_{\text{out}}^\dagger\hat{a}_{\text{out}}\hat{a}_{\text{out}}\rho_{\text{ss}}]}{\text{Tr}[\hat{a}_{\text{out}}^\dagger\hat{a}_{\text{out}}\rho_{\text{ss}}]^2} = \frac{\text{Tr}[\hat{a}_2^\dagger\hat{a}_2^\dagger\hat{a}_2\hat{a}_2\rho_{\text{ss}}]}{\text{Tr}[\hat{a}_2^\dagger\hat{a}_2\rho_{\text{ss}}]^2}$$

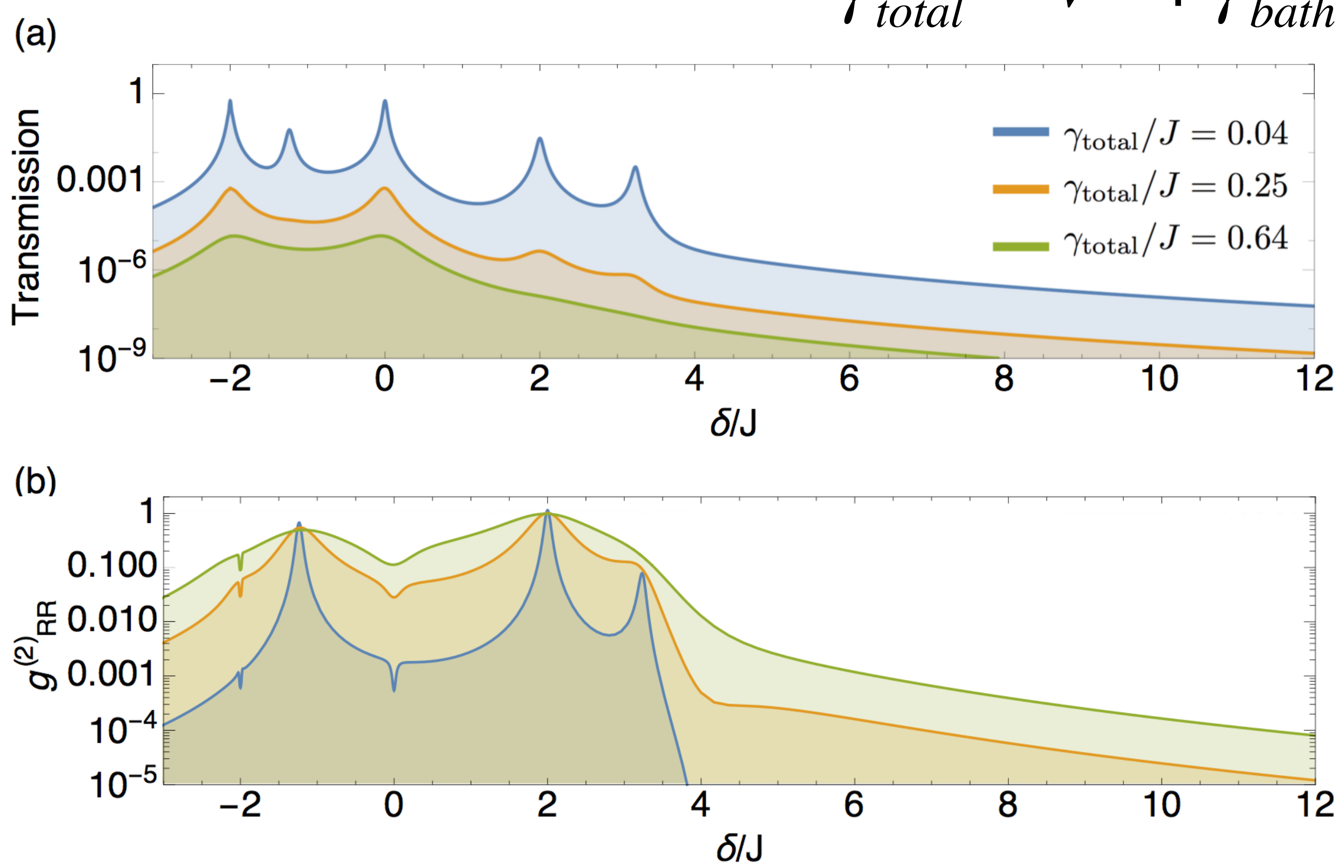
Treatment of photon loss in a cavity

$\omega_j - i\gamma_{\mathrm{bath}}/2$

$$\hat{H}_{\mathrm{cc}} = \hbar \sum_{j=1}^{N} \left(\omega_j \hat{a}_j^{\dagger} \hat{a}_j + U_j \hat{a}_j^{\dagger} \hat{a}_j^{\dagger} \hat{a}_j \hat{a}_j \right) + \hbar \sum_{j=1}^{N-1} J(\hat{a}_j^{\dagger} \hat{a}_{j+1} + \hat{a}_j \hat{a}_{j+1}^{\dagger})$$

fully-resonant and U/J=1

$$\gamma_{total} = V^2 + \gamma_{bath}$$

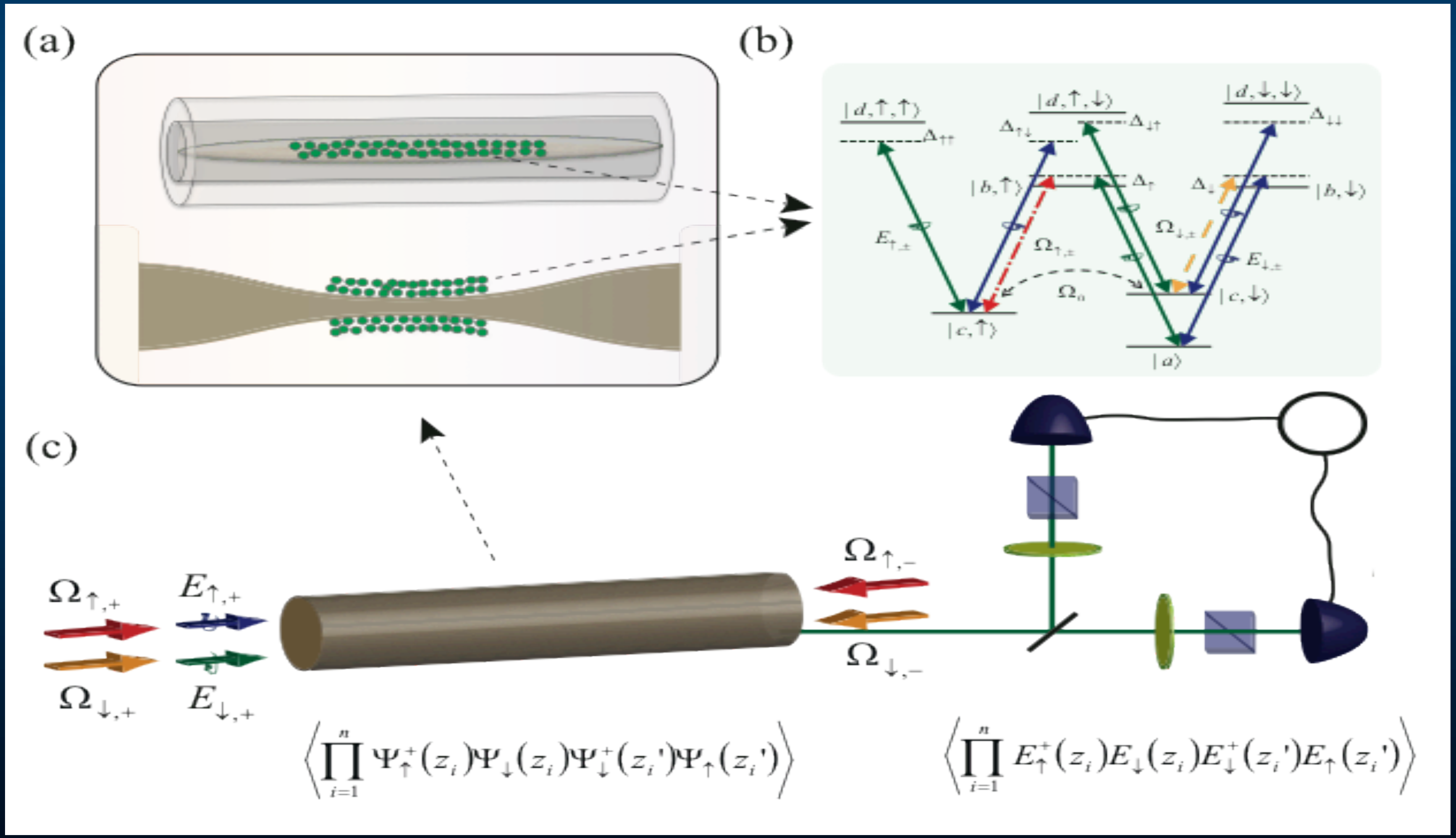


“Interesting” phases in driven but linear photonic set ups: Topological and exotic models

1) Jackiw-Rebbi model in slow light

2) The Majoranon in integrated waveguide arrays
(also see poster by Changsuk Noh)

Probing the topological properties of Jackiw-Rebbi model with slow light (no nonlinearities needed!)



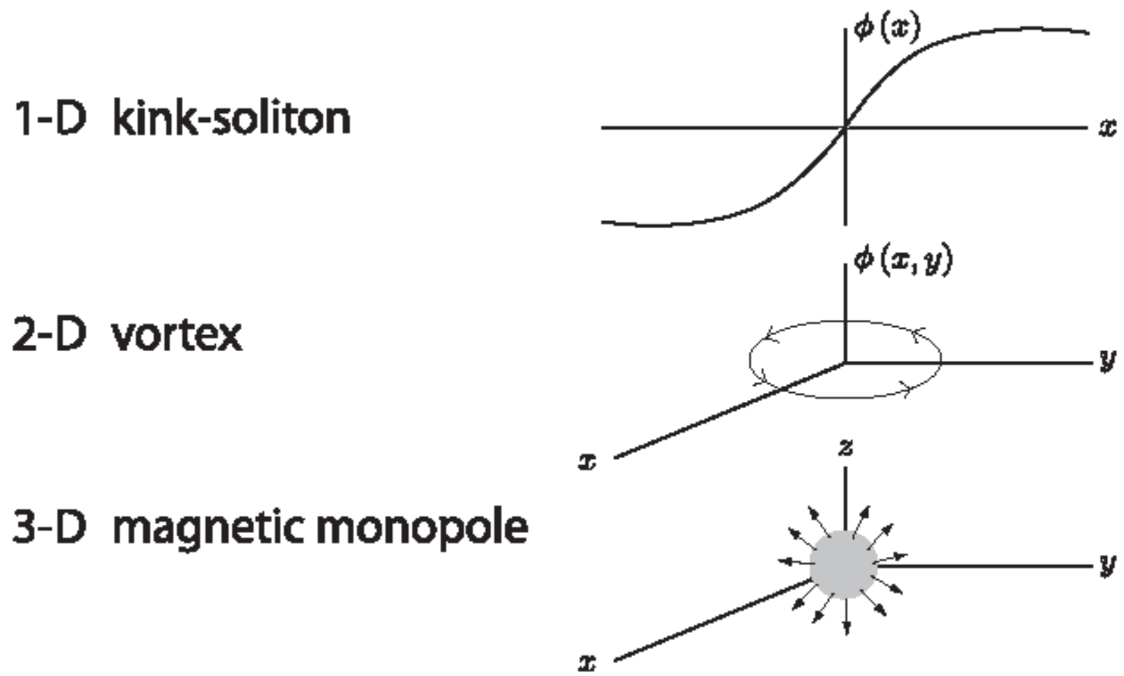
D. G. A, P. Das, C, Noh, Nature Scientific Reports 2014

Jackiw-Rebbi model

One dimensional Dirac particle coupled to a scalar field

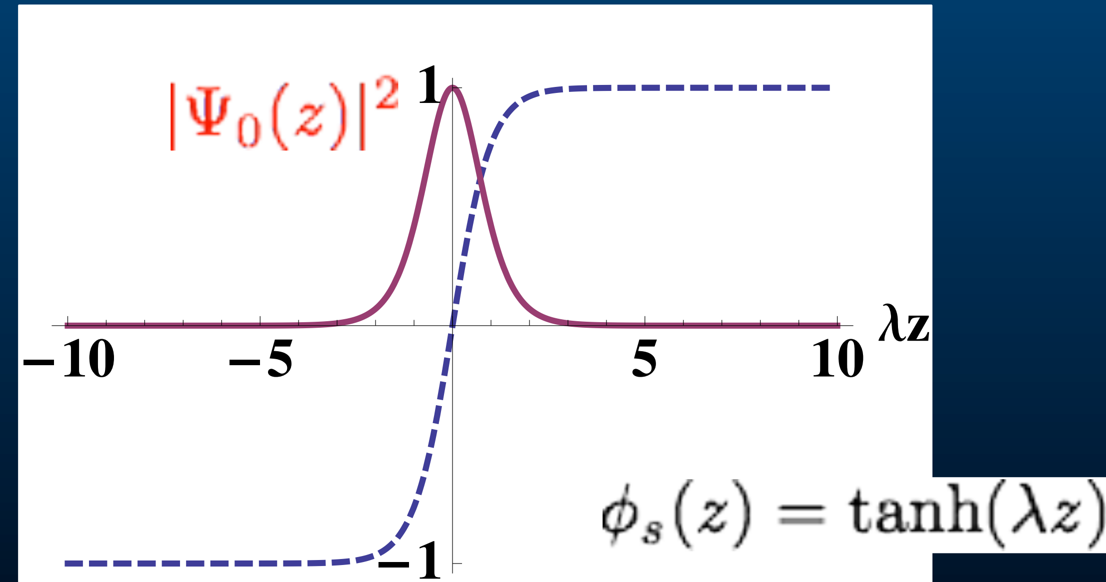
$$i\partial_t \Psi = (\alpha p_z + \beta g \phi(z))\, \Psi$$

Continuum solutions $E > 0$ and $E < 0$, and the isolated $E = 0$ solution are given below:



$$\Psi_0(z) = \exp\left(g\int_0^z dx\,\phi_s(x)\right)\chi = \exp\left[-\frac{g}{\lambda}\ln(\cosh\lambda z)\right]\chi$$

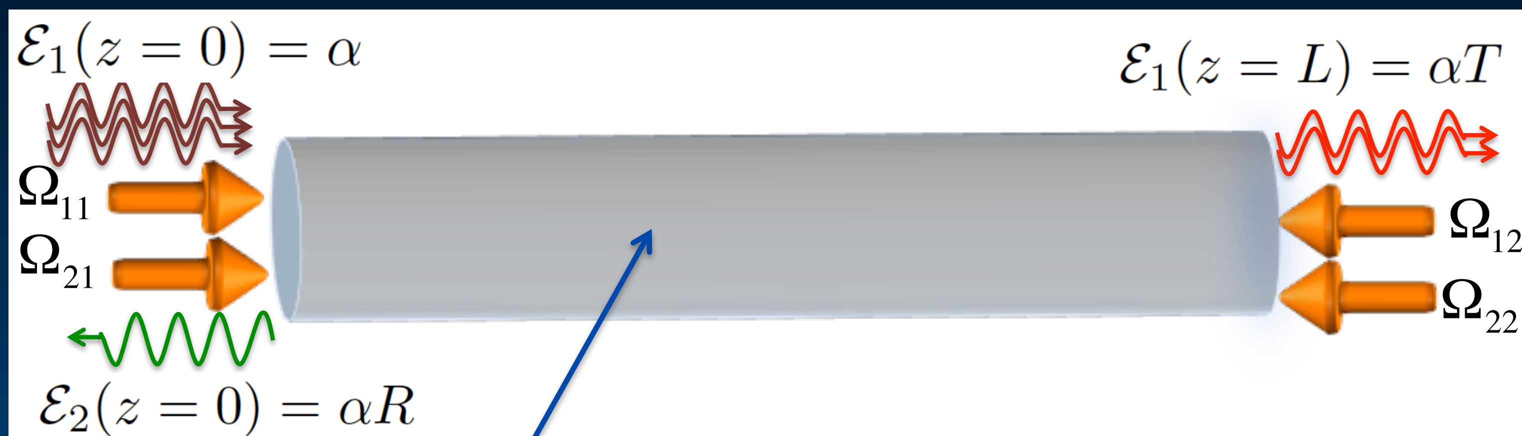
1D-Zero-energy solution

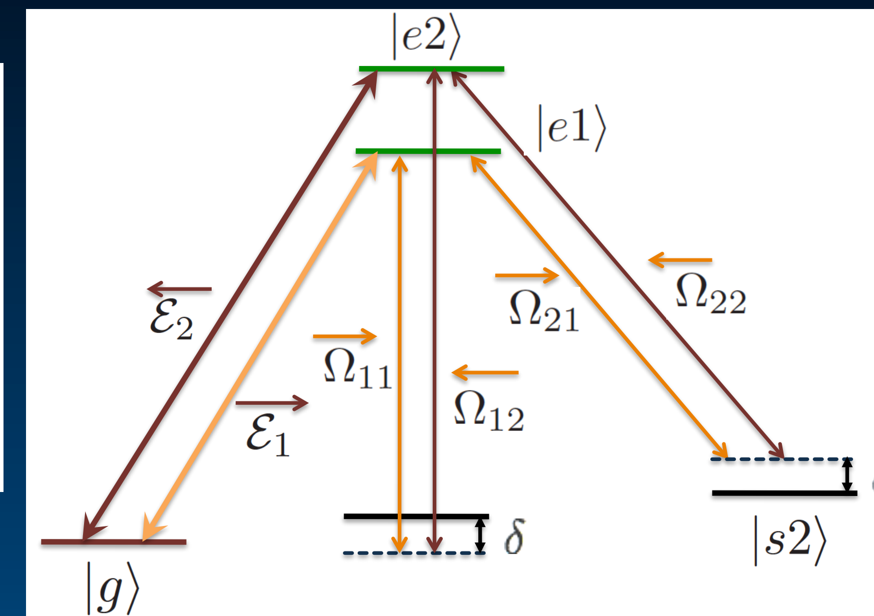


The zero energy solutions depend only on the topology of the soliton background (mass profile).

R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976).

JR realization in a slow-light setup.





Waveguide coupled to an ensemble of atoms

$$\Omega_{1,2}=\Omega_{2,1}=\Omega/\sqrt{2}\exp(iS)$$

$$\Omega_{1,1}=\Omega_{2,2}=\Omega/\sqrt{2}$$

Supports two independent slow-light modes (Electromagnetically induced transparency)

$$\tilde{\mathcal{E}}=(\tilde{\mathcal{E}}_1,\tilde{\mathcal{E}}_2)^{\mathrm{T}}$$

Two-photon detuning

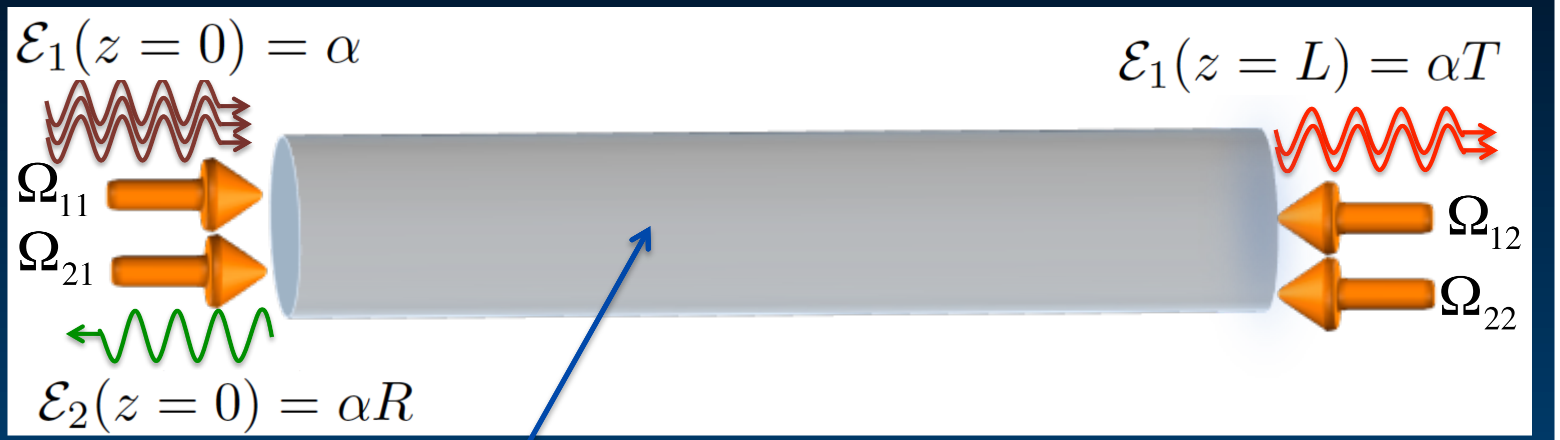
$$\left[\left(1+\frac{1}{v_0}\frac{1}{\sin^2 S}\right)\sigma_z-i\frac{1}{v_0}\frac{\cos S}{\sin^2 S}\sigma_y\right]\frac{\partial}{\partial_t}\tilde{\mathcal{E}}+\frac{\partial}{\partial_z}\tilde{\mathcal{E}}=-\frac{\delta}{v_0\sin S}\sigma_x\tilde{\mathcal{E}}$$

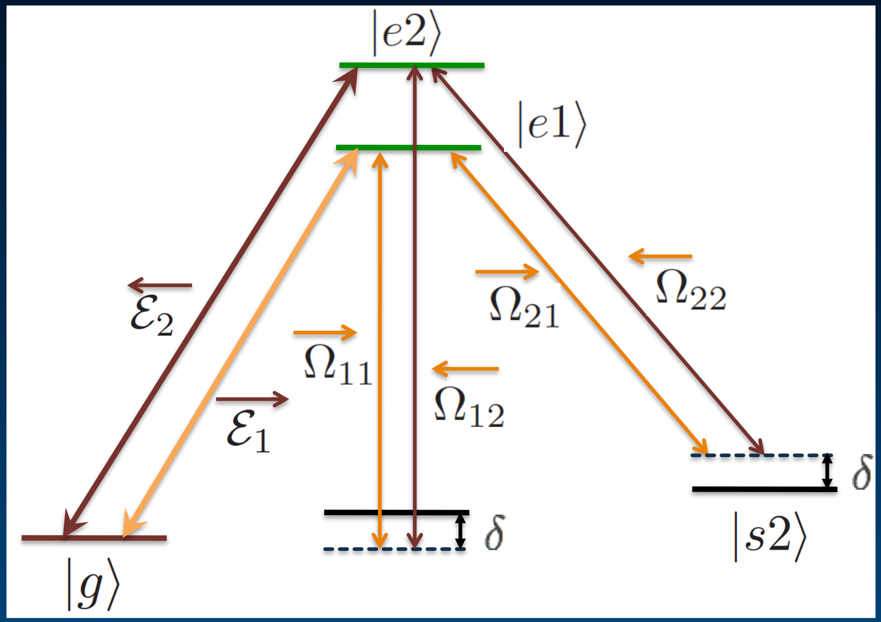
$\Omega^2/g^2 n \ll 1$

Slowly-varying components

Unanyan et al., PRL 105, 173603 (2010). Ruseckas et al., PRA 83, 063811 (2011)
Angelakis. Das, Noh, Scientific Report 2014.

JR realization in a slow-light setup.





Waveguide coupled to an ensemble of atoms

$$\Omega_{1,2}=\Omega_{2,1}=\Omega/\sqrt{2}\exp(i\pi/2)$$

$$\Omega_{1,1}=\Omega_{2,2}=\Omega/\sqrt{2}$$

Position dependent two-photon detuning

Check the existence of the zero mode and its topological stability

$$L=0.3mm$$

$$\left(i\partial_t+iv_0\sigma_z\partial_z-\delta\sigma_y\right)\tilde{\mathcal{E}}=0$$

$$\Omega^2/g^2n\ll 1$$

$$v_0=1.7m/s$$

Within EIT window

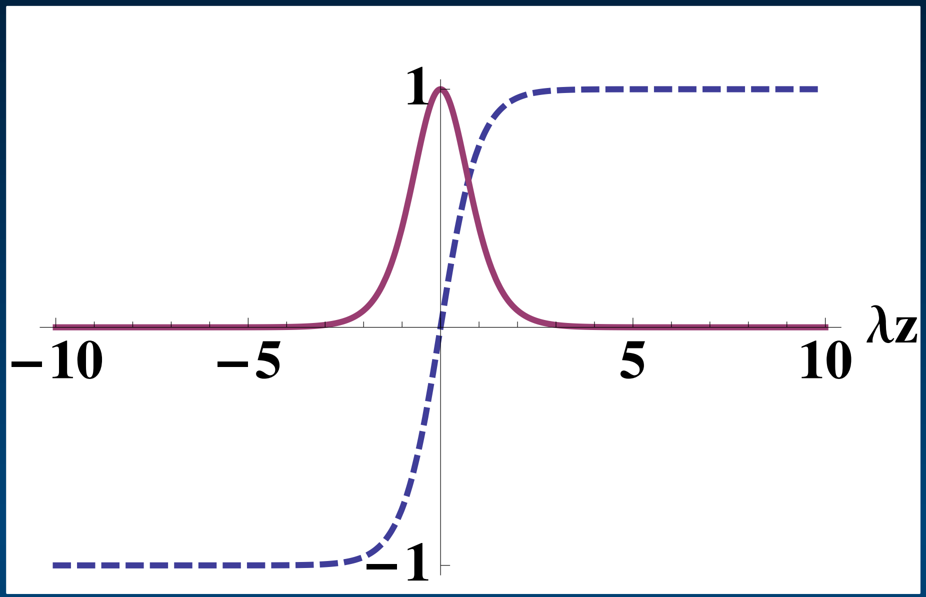
$$\delta_{max}=0.25v_0/L$$

$$\tilde{\mathcal{E}}=(\tilde{\mathcal{E}}_1,\tilde{\mathcal{E}}_2)^{\mathrm{T}}$$

Unanyan et al., PRL 105, 173603 (2010). Ruseckas et al., PRA 83, 063811 (2011).

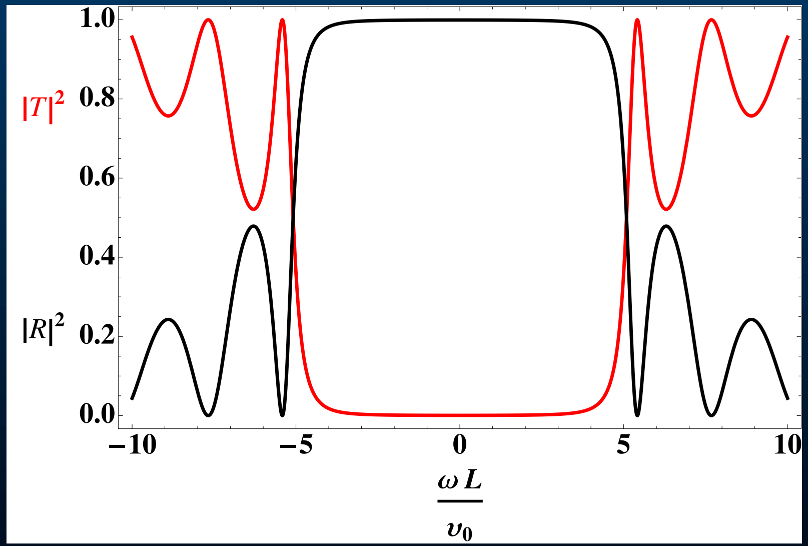
Zero mode in transmission

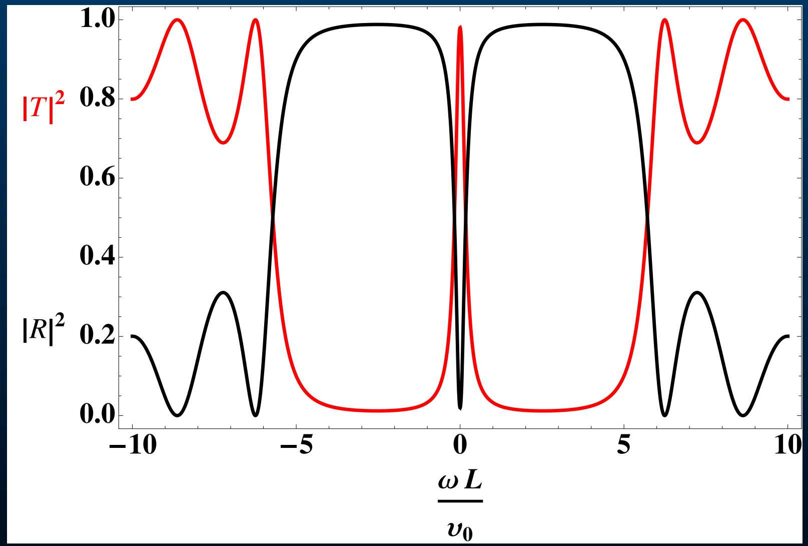
Probe with an incident field $\tilde{\mathcal{E}}_1 = \alpha \exp(-i\Delta\omega t)$ and look at the transmission and reflection spectrum.

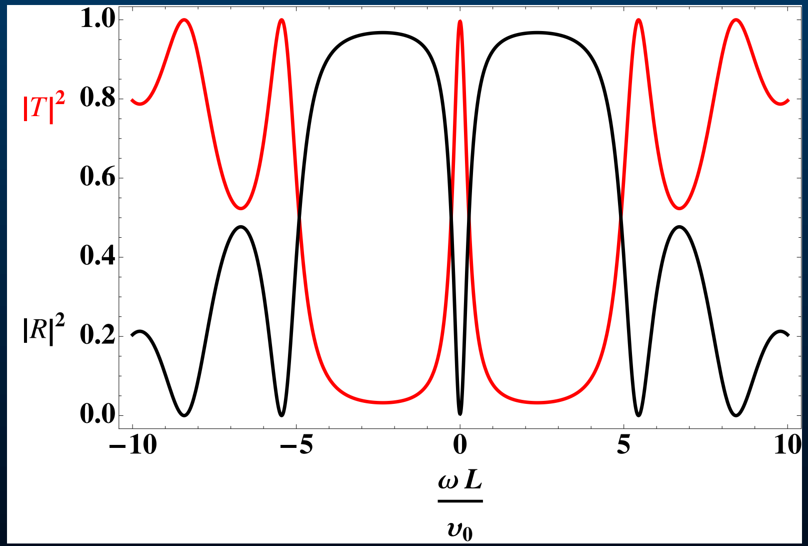


Without soliton

With soliton







$\delta = \tanh(\lambda z)$

$\delta(z) = \sin(\lambda z)$

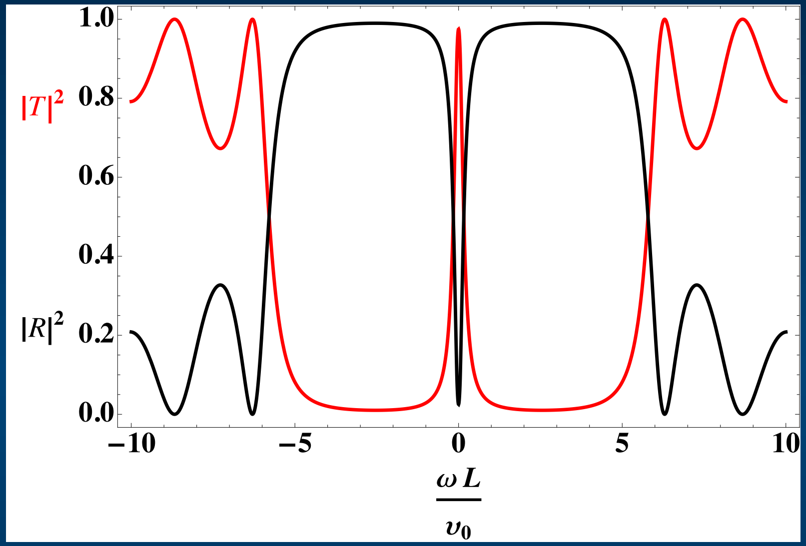
Angelakis, Das, Noh, Nature Scientific Report 2014.

Robustness with respect to fluctuations in the effective potential:

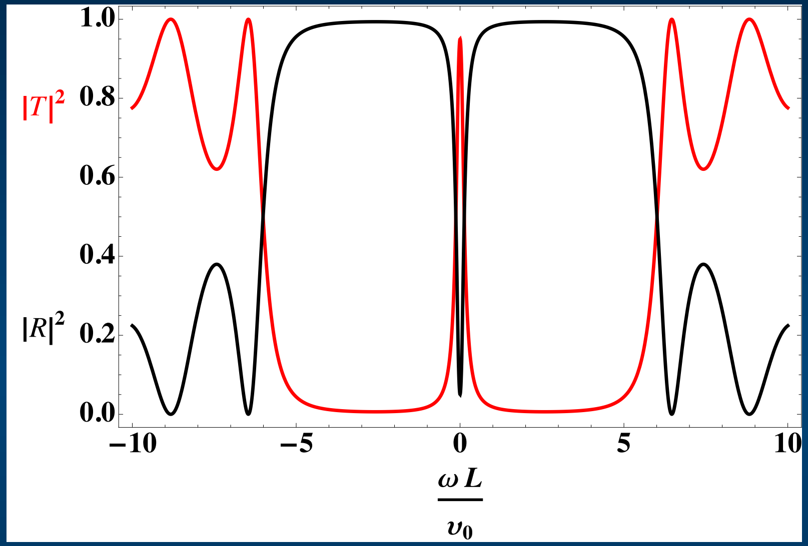
$$\delta(z) = \tanh(\lambda z)(1 + \epsilon(z))$$

$$\epsilon(z) = [-\epsilon_0(z), \epsilon_0(z)]$$

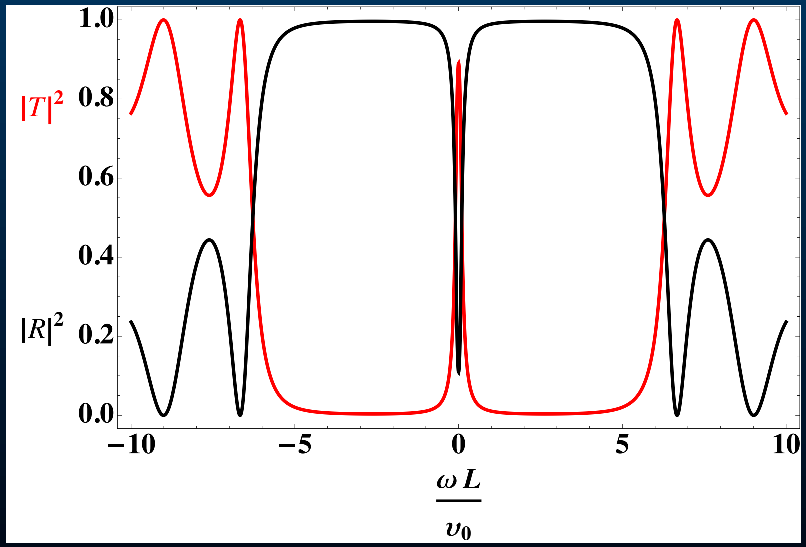
$\epsilon_0 = 0.05$



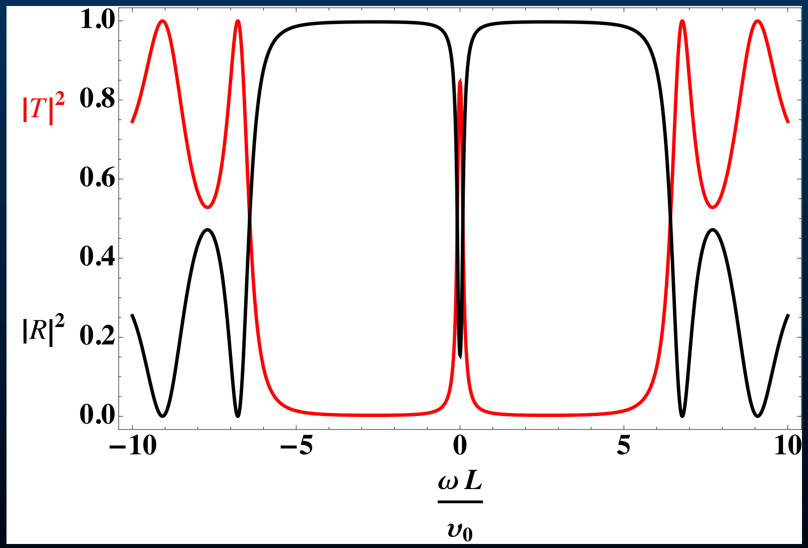
$\epsilon_0 = 0.2$



$\epsilon_0 = 0.4$

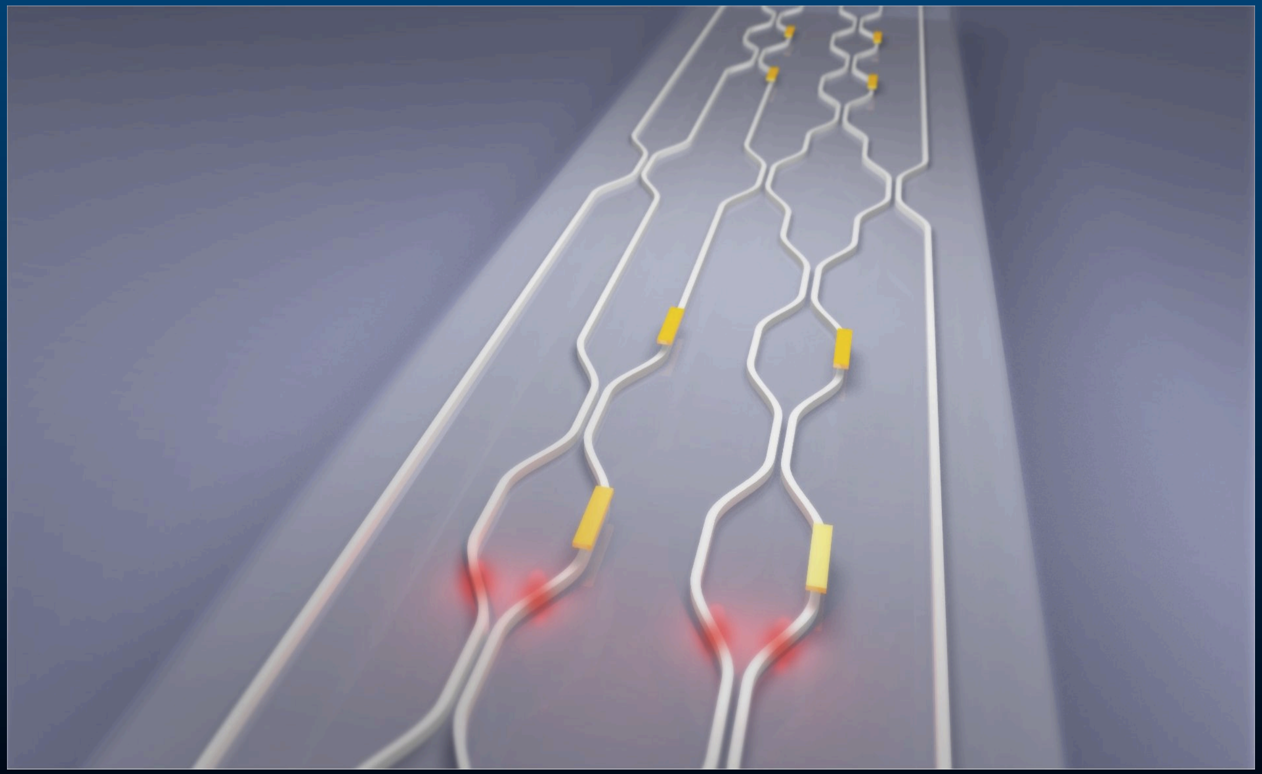


$\epsilon_0 = 0.5$



Angelakis, Das, Noh, Nature Scientific Report 2014.

Majorana dynamics in photonic lattices
(an experiment)



Majorana equation

General Lorentz covariant equation

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi = m\psi_c$$

Obeyed by the hypothetical "Majoranons"

The difference with the Dirac equation is that Majorana contains the operation of charge conjugation leading to dynamics violating charge conservation. Thus the Majoranon is unphysical!

We will show here how to effectively simulate Majoranon dynamics with photons!

Majorana fermions are neutral and their own antiparticles. Neutrino?

$$\psi = \psi_c$$

A range of theories beyond the standard model exist, which connect the violation of charge conservation with the existence of higher dimensions of spacetime or describe alternative models where the photon acquires a non-zero photon mass.

E. Majorana, Nuovo Cim. 9, 335 (1932); ibid 14, 171(1937)

F. Wilczek, Nat. Phys. 5, 614 (2009)

Majorana equation – Quantum simulation

Observation: Majoranon can be decomposed as superposition of Majorana fermions with opposite masses

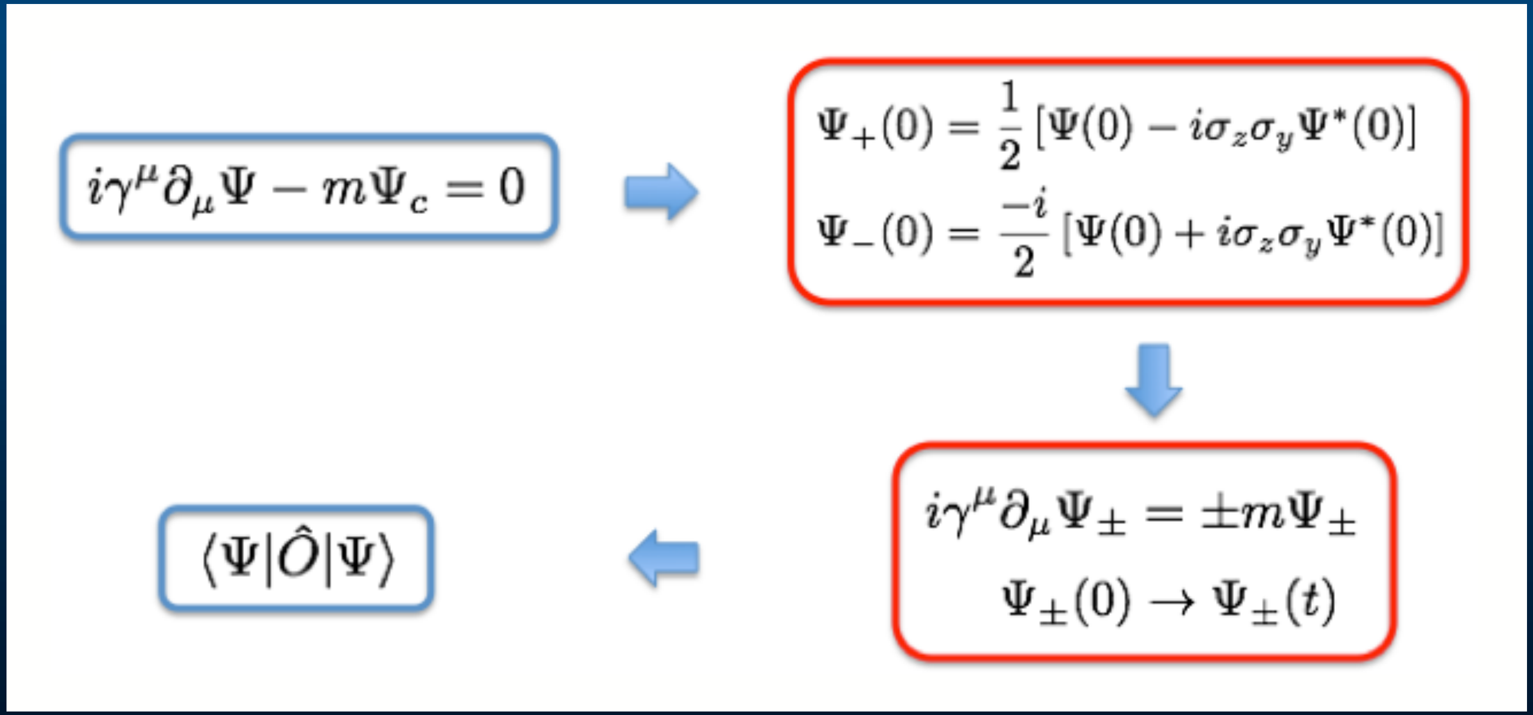
$$\Psi = \Psi_{+m} + i\Psi_{-m}$$

↓ Majorana field

$$i\partial_t \psi_\pm - \sigma_x p_x \psi_\pm \mp m\sigma_z \psi_\pm = 0$$

↘ Dirac fields

Steps for Quantum simulation

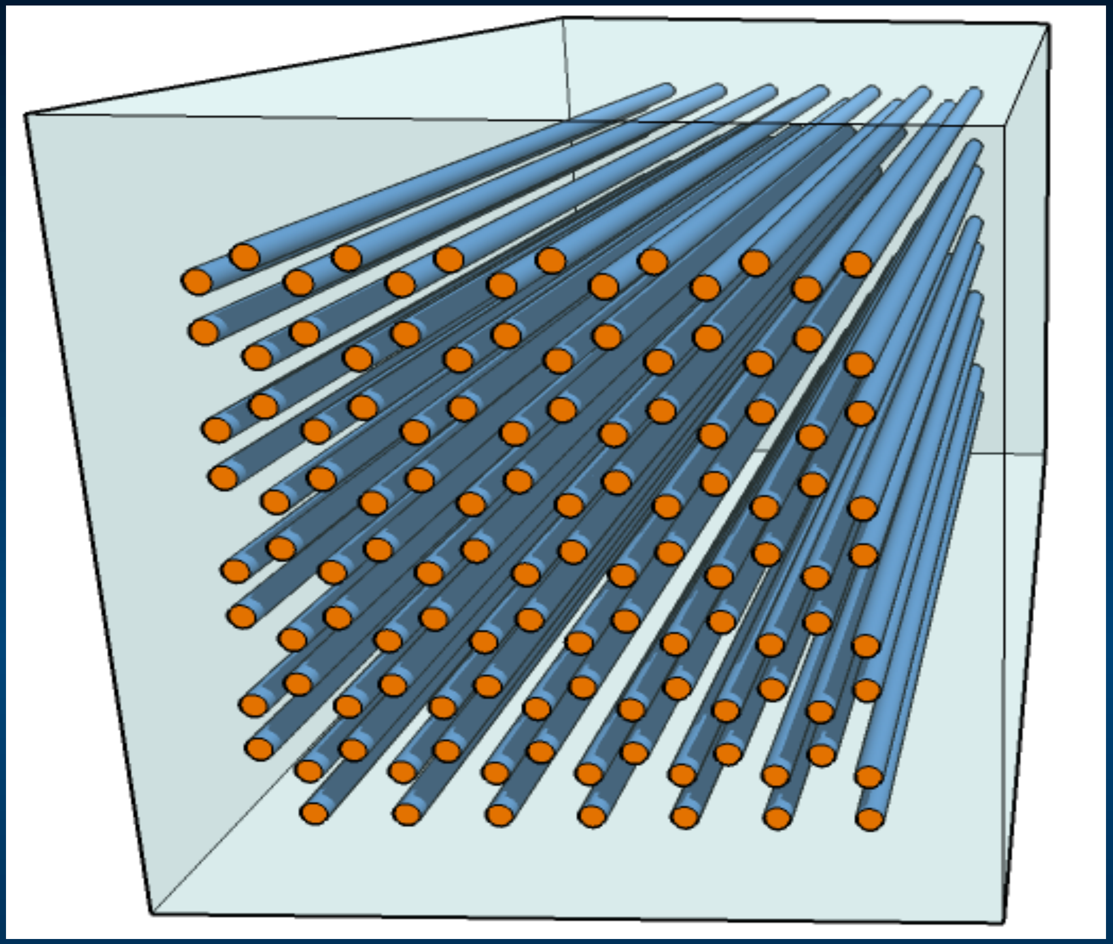



Can be done with single ion, No spin-spin interaction

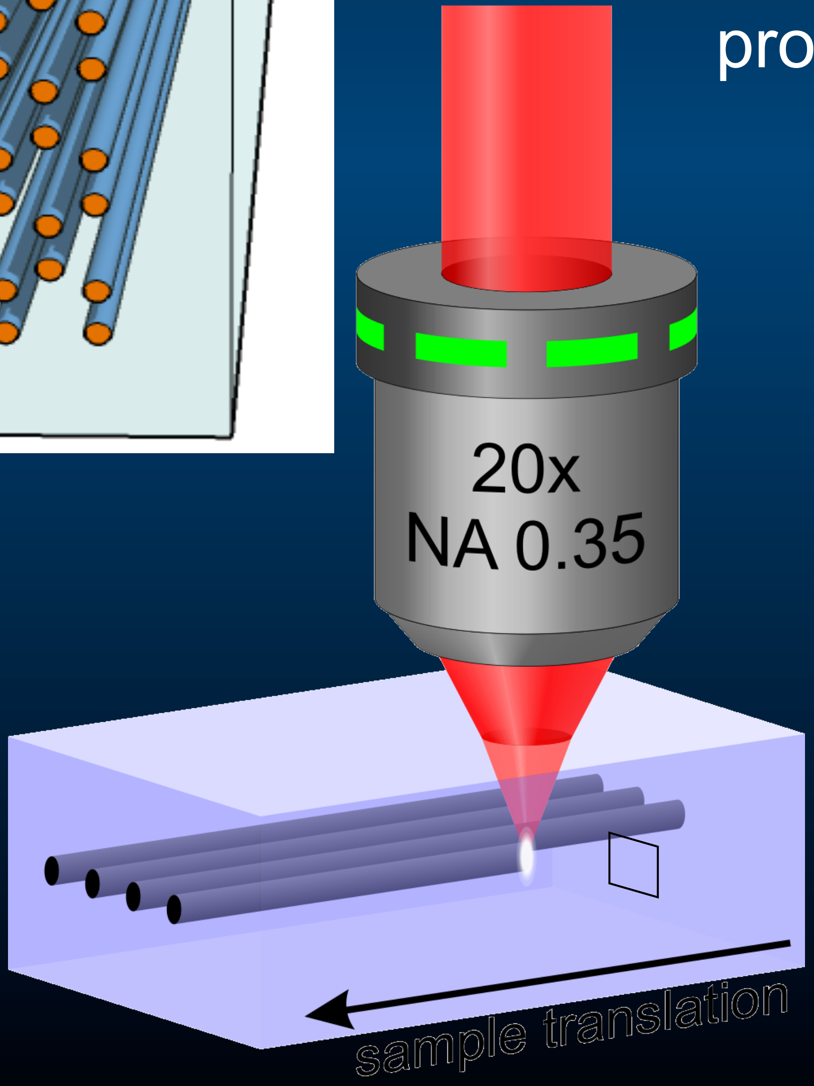

Full tomography including the phonons-Possible with photons?

C.Noh, B. M. Rodriguez-Lara, and D. G. Angelakis, PRA 2012

Photonic waveguide arrays



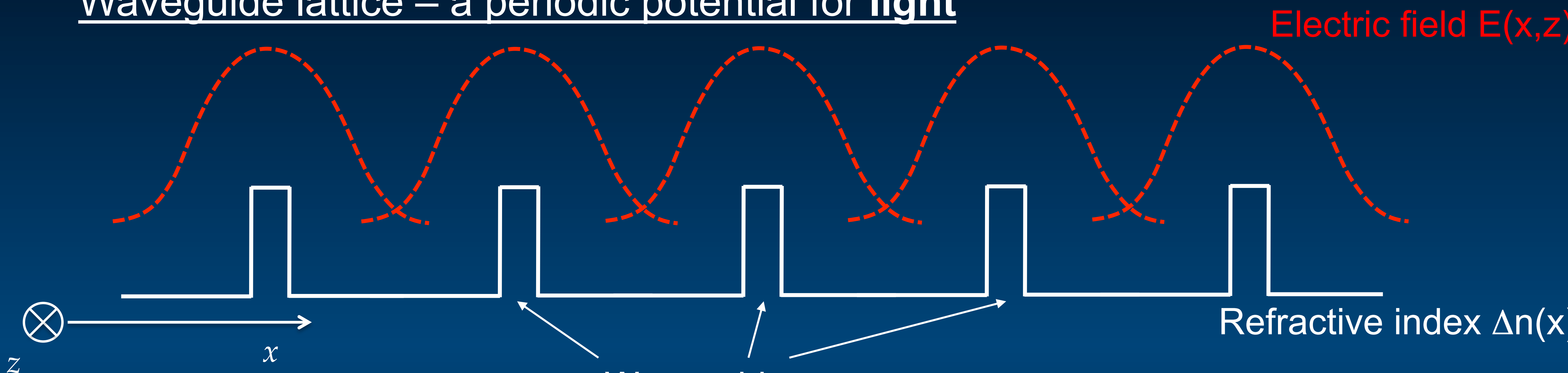
- **Direct waveguide inscription** by ultrashort laser pulses
- Permanent refractive index increase
- Waveguide separation → coupling
- Power/Translation velocity → on-site propagation constant σ



Szameit & Nolte, *J. Phys. B* **43,** 163001 (2010)

Optics as quantum-mechanical analogue

<u>Waveguide lattice – a periodic potential for **light**</u>



→Schrödinger equation:

$$i\hbar\partial_t\Psi + \frac{\hbar^2}{2m}\partial_x^2\Psi - V(x)\Psi = 0$$

→Paraxial **Helmholtz** equation:

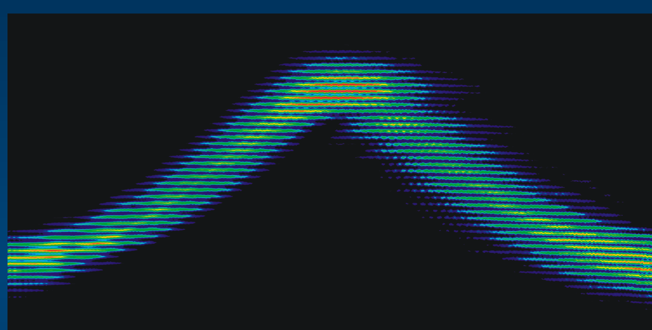
$$i\frac{\lambda}{2\pi}\partial_z E + \frac{\lambda^2}{8\pi^2 n_0}\partial_x^2 E + \Delta n(x)E = 0$$

$$V(x) \leftrightarrow -\Delta n(x)$$
$$\Psi(x,t) \leftrightarrow E(x,z)$$
$$t \leftrightarrow z$$
$$m \leftrightarrow n_0$$
$$h \leftrightarrow \lambda$$

Optical analogues to the Schrödinger equation

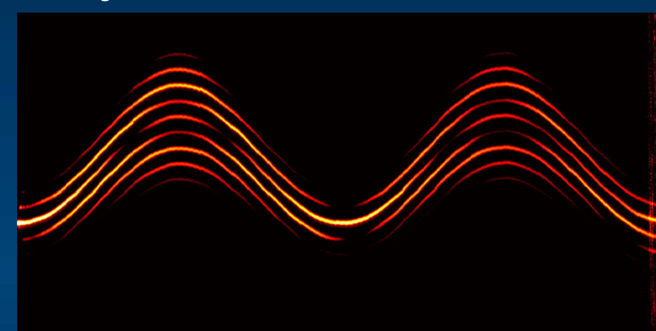
$$i\hbar \frac{\partial \Psi}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi = 0$$

Bloch oscillations



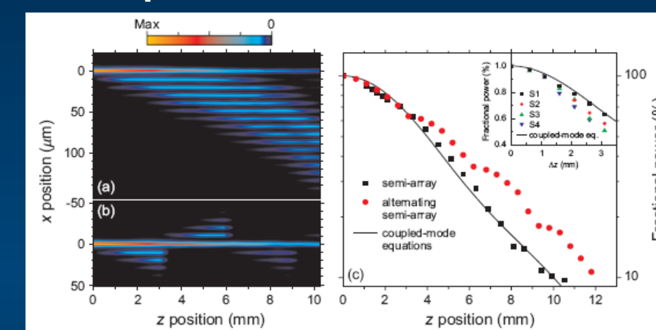
Phys. Rev. Lett. **83**, 4752–4755 (1999).
Phys. Rev. Lett. **83**, 4756–4759 (1999).

Dynamic Localization



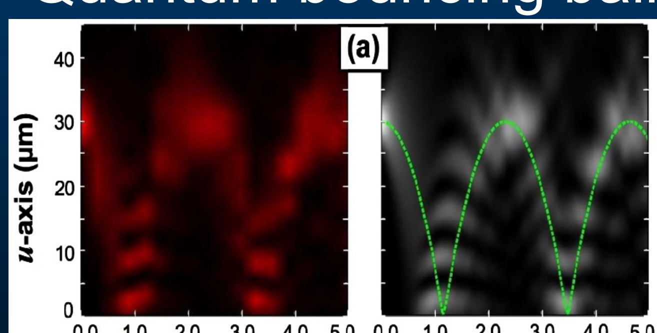
Phys. Rev. Letters **96**, 243901 (2006).
Nature Physics **5**, 271-275 (2009).

Optical Zeno effect



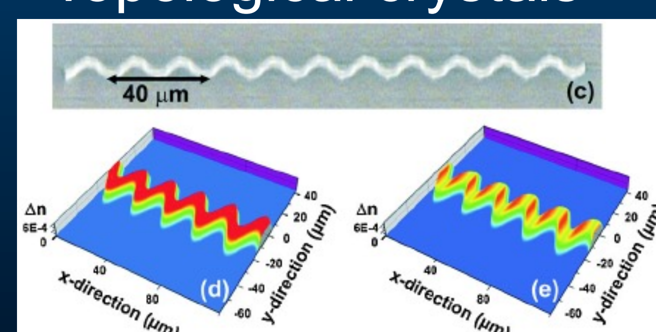
Optics Express **16**, 3762-3767 (2008).
Phys. Rev. Lett. **101**, 143602 (2008).

Quantum bouncing ball



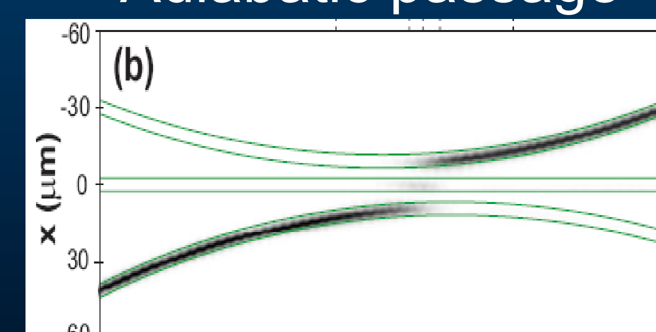
Phys. Rev. Lett. **102**, 180402 (2009).

Topological crystals



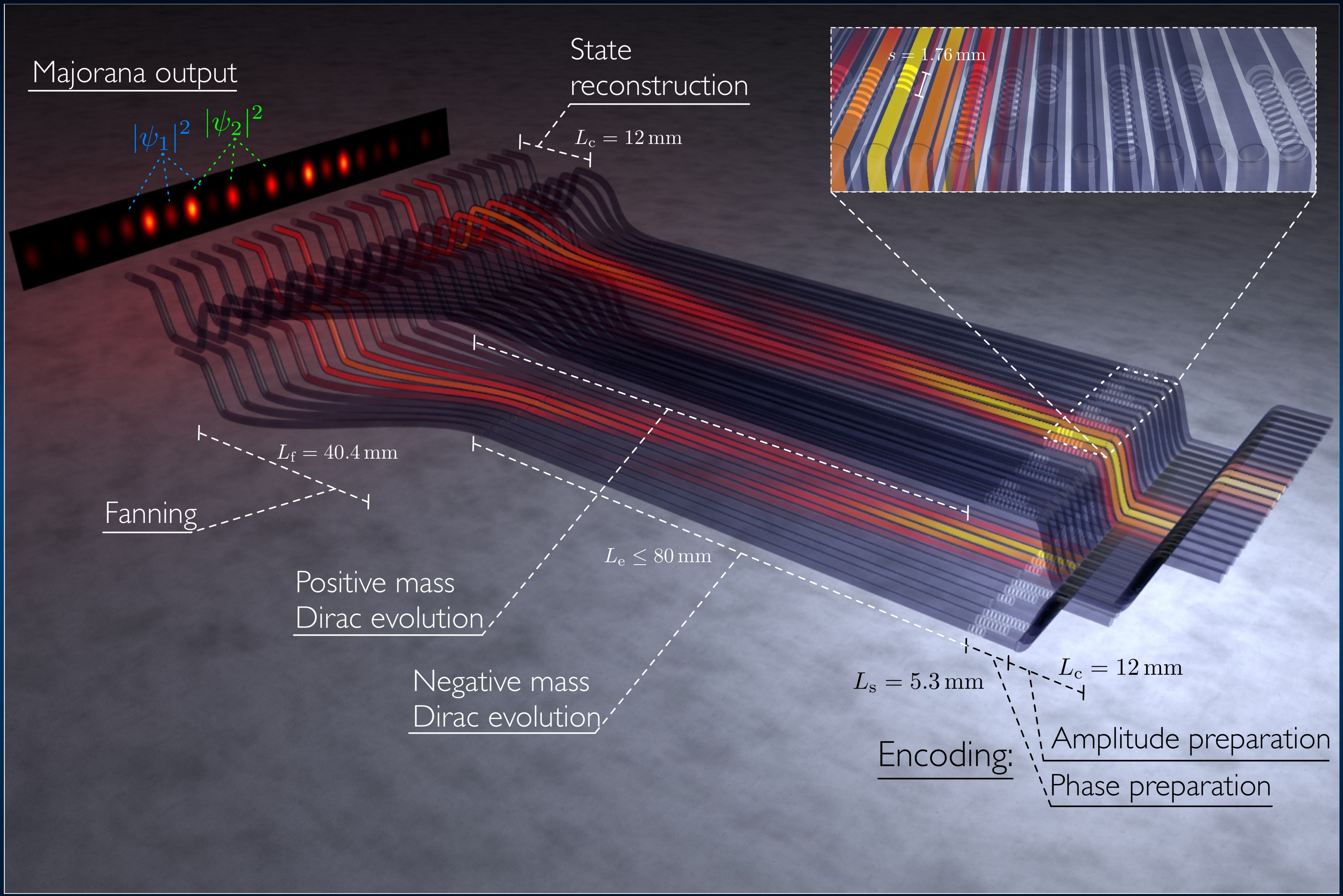
Phys. Rev. Lett. **104**, 150403 (2010).

Adiabatic passage



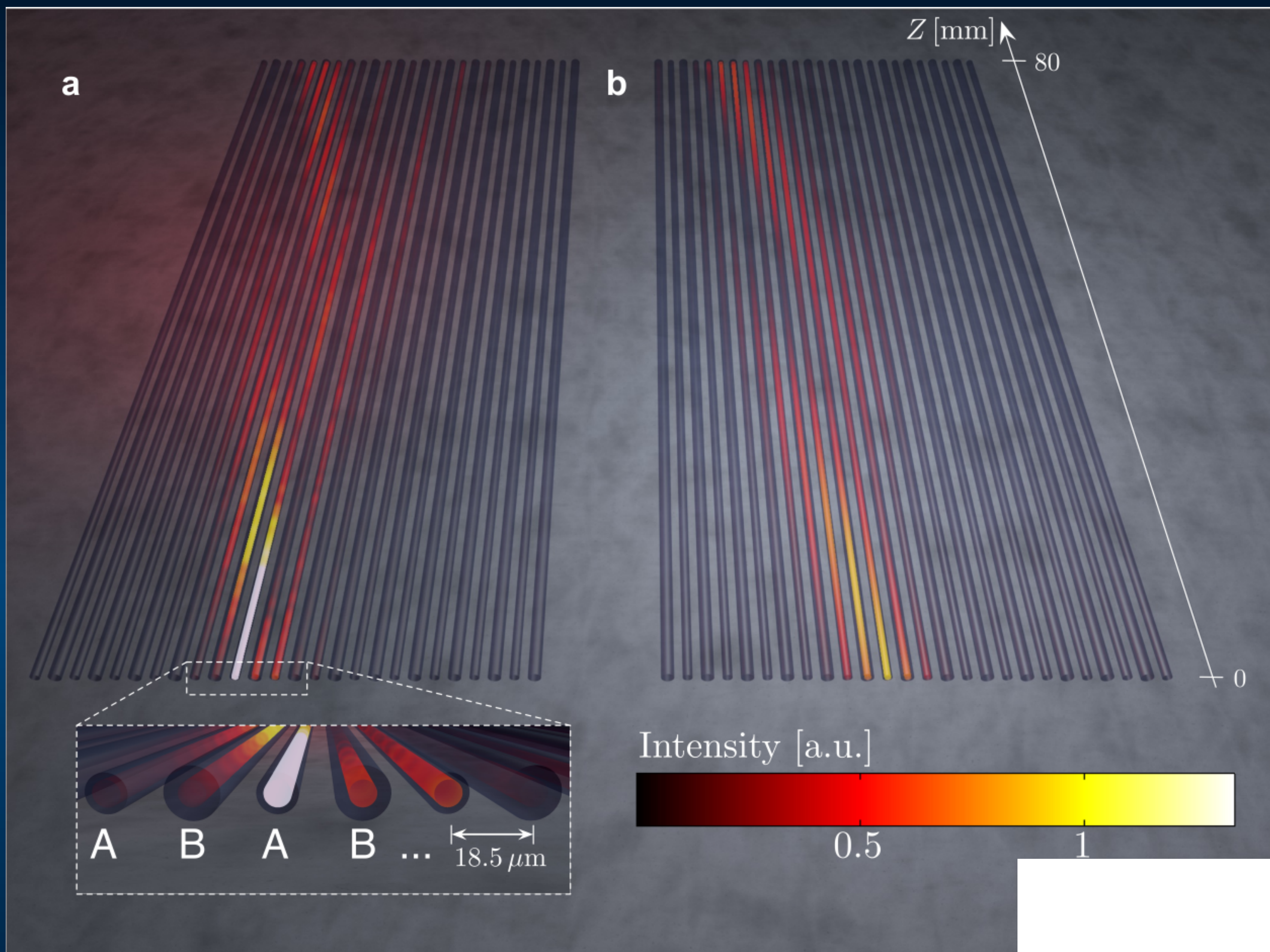
Phys. Rev. B **76**, 201101(R) (2007).
Phys. Rev. Lett. **101**, 193901 (2008).

Photonic "Majoranon"



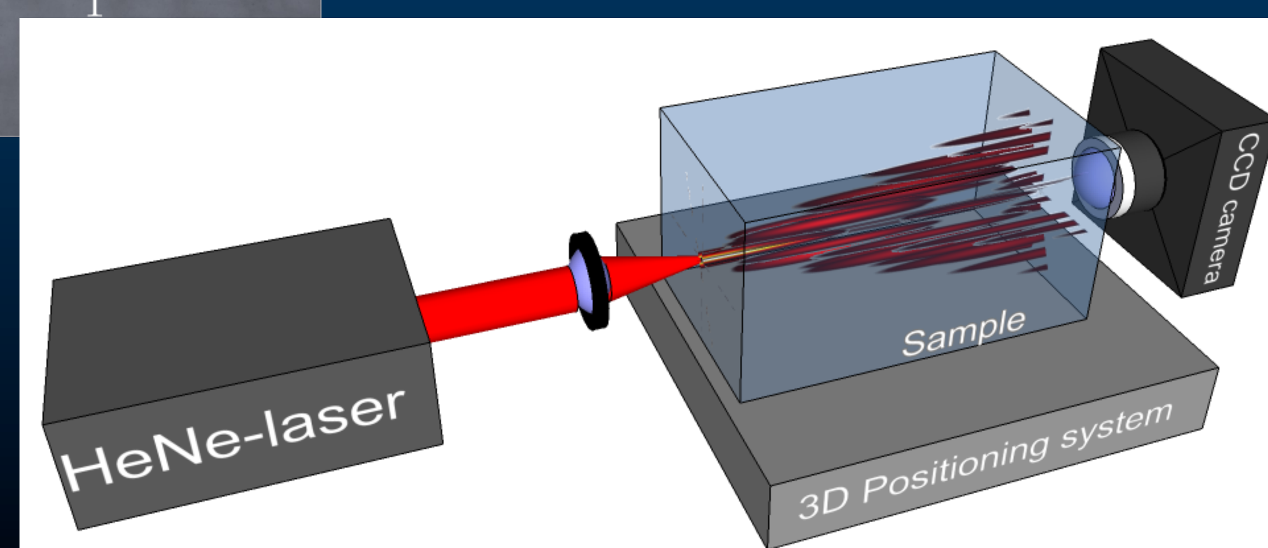
The fabricated waveguide sample, where two Dirac equations with opposite masses are simulated in two parallel planar lattices. Combining them creates the Majoranon at the end!

Photonic "Majoranon"



View from above and observation of photonic Zitterbewegung.

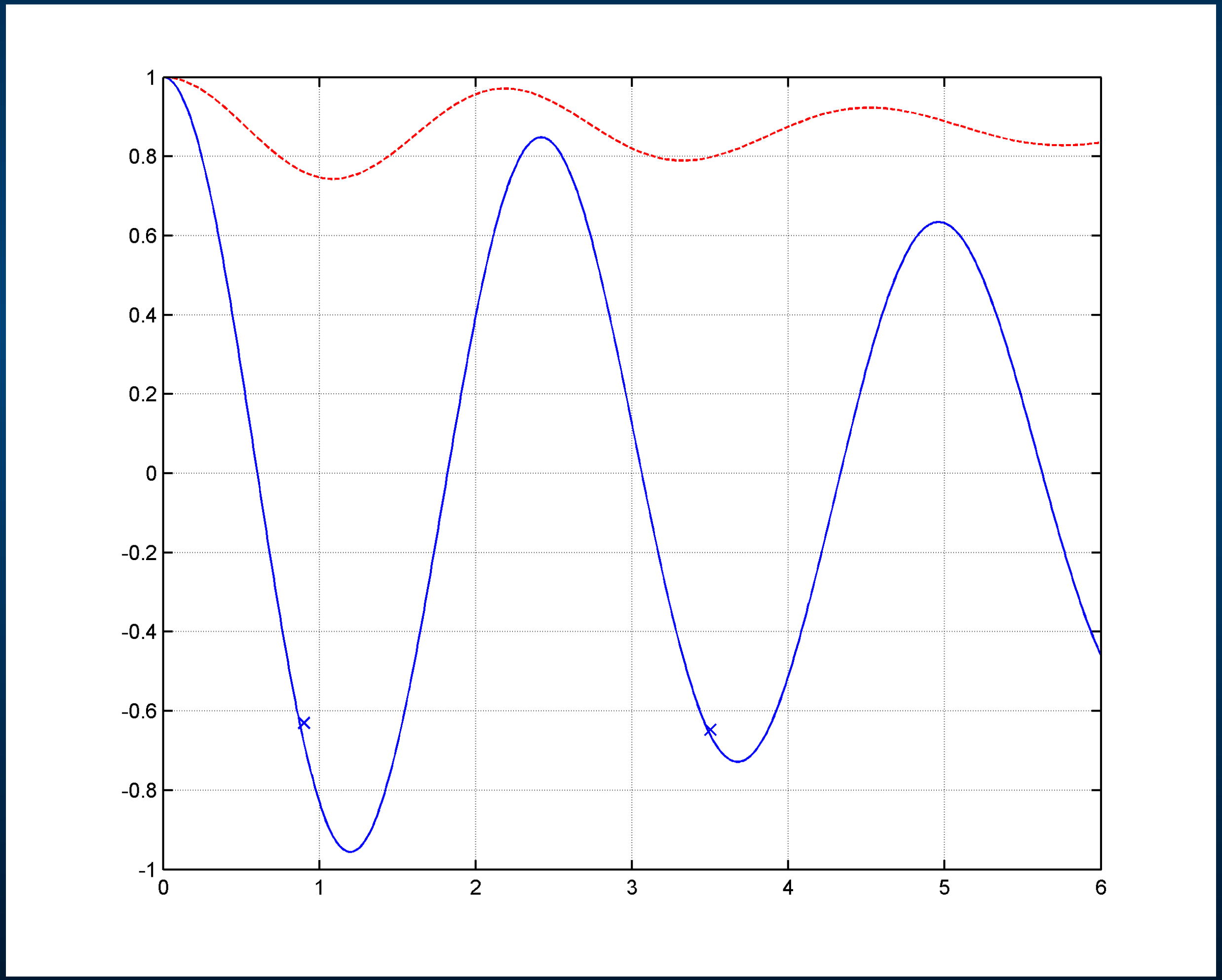
Left is theory and right is the experiment



3D positioning and measuring

Majoranon v/s its Dirac cousin

$$\langle \sigma_z \rangle = \sum_n |\psi_{1,n}|^2 - |\psi_{2,n}|^2$$



Measures the population difference between the positive and negative energy branches.

For a Dirac particle at rest or equivalently of very large mass is a conserved quantity(finite non-zero momentum components cause small oscillation)

For the same initial conditions, the Majoranon oscillates wildly!

The Dirac particle oscillated due to finite non-zero momentum components in the initial wave packet, while the oscillation for the Majoranon is mainly due to the unphysical mass term!

Summary

A range of “simulable” many body effects with strongly correlated photons in quantum nonlinear media in closed and driven-dissipative schenarios

1. Mott transitions-JCH
2. Spin models
3. Q.Hall effects
4. Luttinger liquids
5. QFTs
6. Quantum scattering

Future

- Focus on specific promising platforms for driven many body physics: circuit QED arrays.
- Scattering for probing many-body states in CCQED and nanophotonics interfaces
- Integrated waveguide arrays for relativistic and topological emulations

Future

- Which are the scalable platforms?
- How can we prepare and efficiently, characterize and probe driven dissipative many body states?
- What are the relevant observables?
- What are the unique features in driven dissipative light matter systems
- What techniques we need to adapt/invent to study the systems (Keldysh, MPS, 2D numerics, scattering, QFT tools?)

QUANTUM SIMULATIONS AND MANY-BODY PHYSICS WITH LIGHT

5-12 June 2016, Chania, Crete, Greece

Organiser:
Dimitris G. Angelakis

Scientific Advisory committee:
Dimitris Angelakis, Darrick Chang, Peter Rabl, Rosario Fazio, Atac Imamoglu

***KEY TOPICS** to be addressed in the meeting include:*

- Experimental platforms for strong photon-photon interactions at quantum level Circuit QED cavity arrays, atom nano-photonics interfaces, exciton-polaritons systems, Rydberg polaritons, optomechanical systems
- Numerical and analytical methods for treating driven-dissipative many-body photonic systems
- Generation and detection of many-body states of photons and polaritons
- Out of equilibrium exotic phases, optimal observables and characterization of many-body photonic systems
- Quantum transport, scattering and many-body quantum dynamics of multi-photon states in strongly coupled light-matter systems
- Topologically protected phases and gauge fields in driven-dissipative photonics
- Applications in quantum technologies and optical circuitry
- Quantum simulation and computation with strongly coupled light-matter systems

Invited Speakers:
Iacopo Carussoto
Ignacio Cirac
Sebastian Diehl
Irene d'Amico
Vladimir Gritsev
Mohammad Hafezi
Dieter Jaksh
Jens Koch
Jonathan Keeling
G. Morrigi
Karyn le Hur
Martin Plenio
Helmut Ritsch
Alex Szameit
Sebastian Schmidt
Hakan Tureci
Martin Weitz
Andreas Wallraff
Andrew White
Andre Xuereb
....

Thank you for your attention!

PhD and Postdoc positions available

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CQT Centre for Quantum Technologies