Interferometric measurement scheme for topological invariants of correlated many-body systems

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Collaborators:

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Tracy Li, Manuel Endres, Immanuel Bloch, Ullrich Schneider

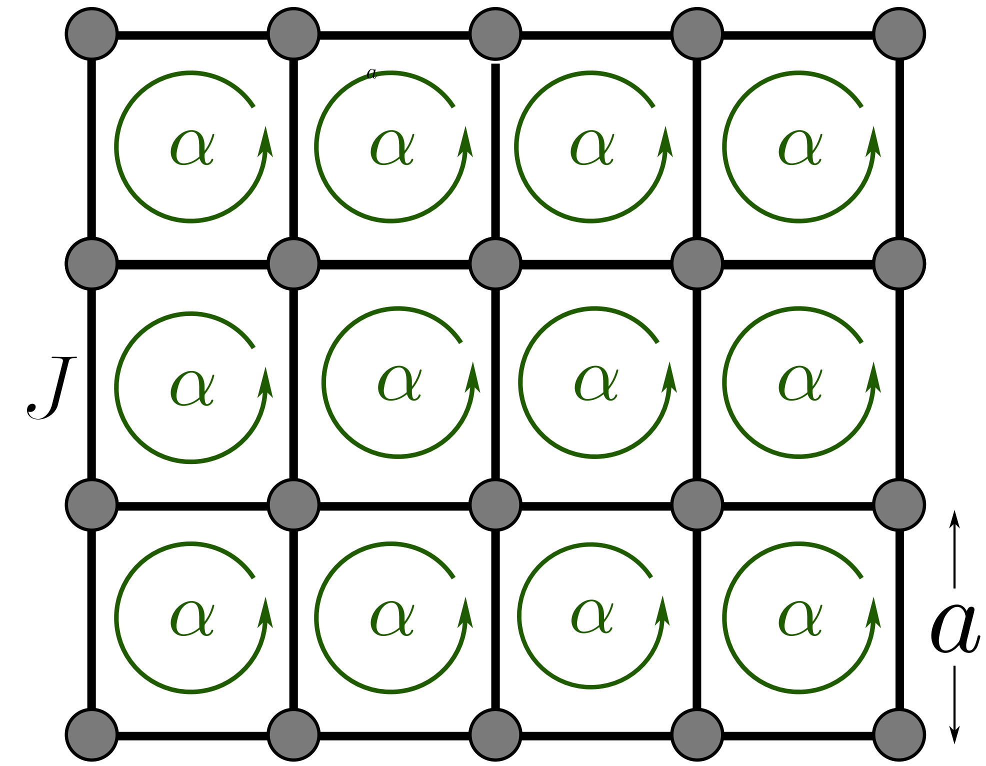
Li et al., arXiv:1509.02185 (2015)

Grusdt et al., arXiv:1512.03407 (2015)

artificial gauge fields

- Hofstadter Hamiltonian

$$\mathcal{H}_0 = -J \sum_{\langle i,j \rangle} e^{i\varphi_{i,j}} \hat{a}_i^\dagger \hat{a}_j$$



$$N_\phi = \alpha \frac{L^2}{a^2}$$

artificial gauge fields

- Hofstadter Hamiltonian

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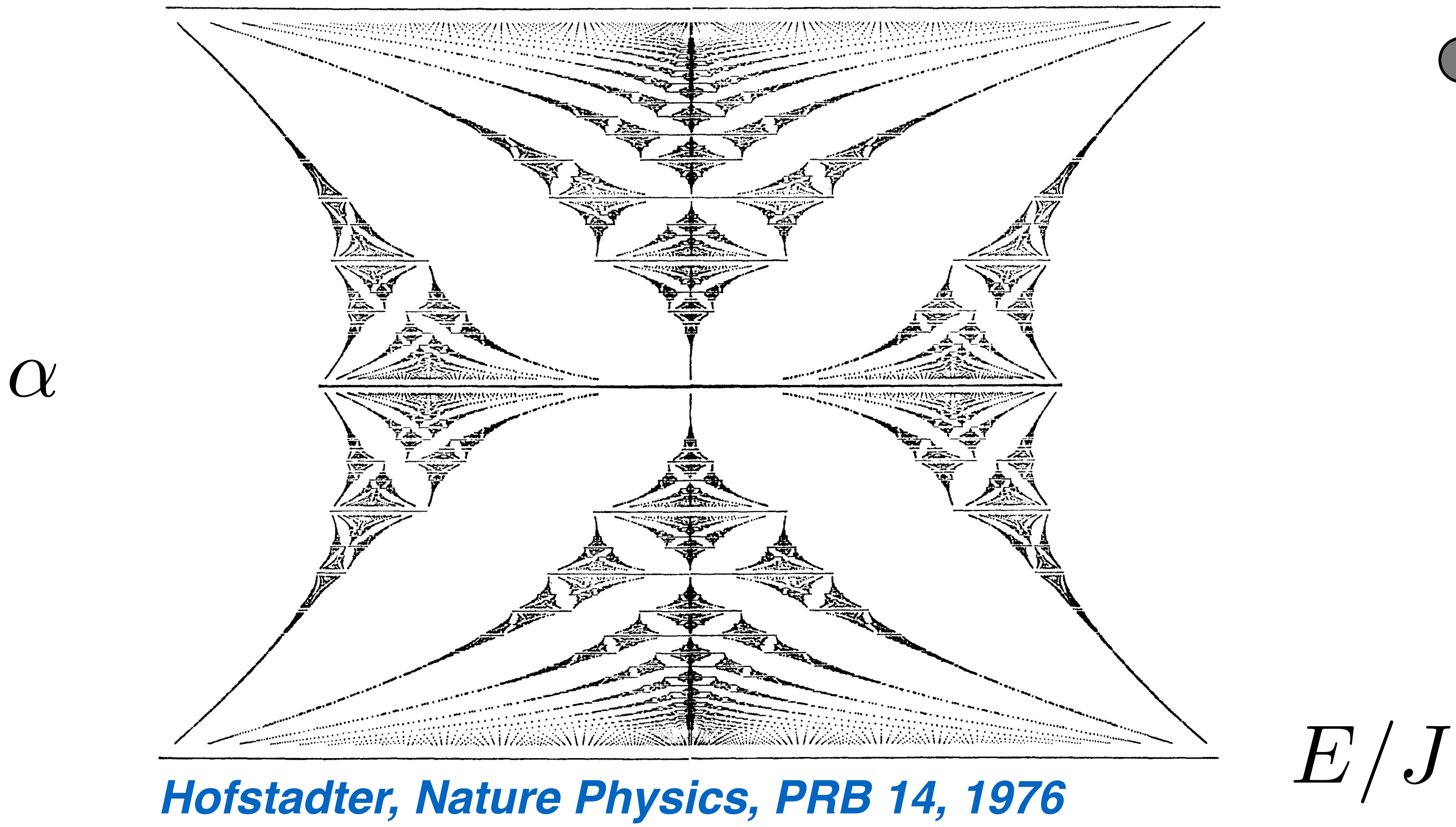
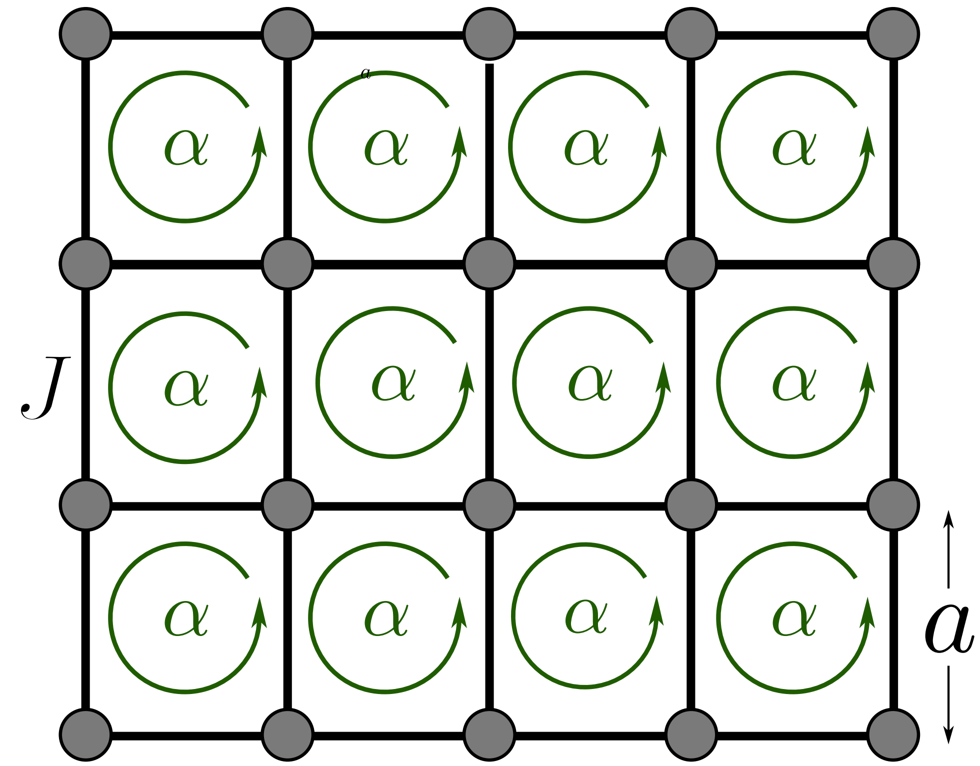
* Non-trivial band structure:



Hofstadter, Nature Physics, PRB 14, 1976



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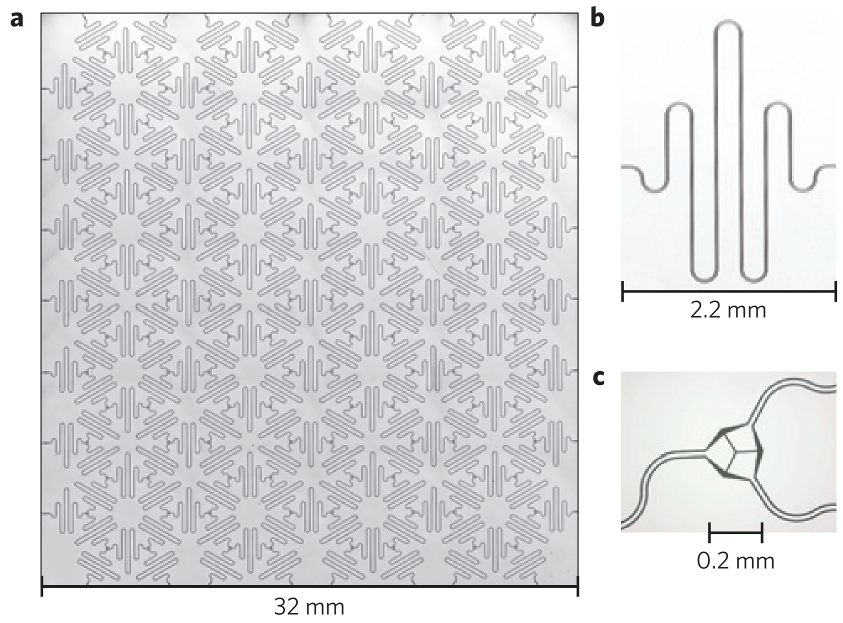
Topological states of light

Houck et.al., Nature Physics, 8, 2012



circuit QED

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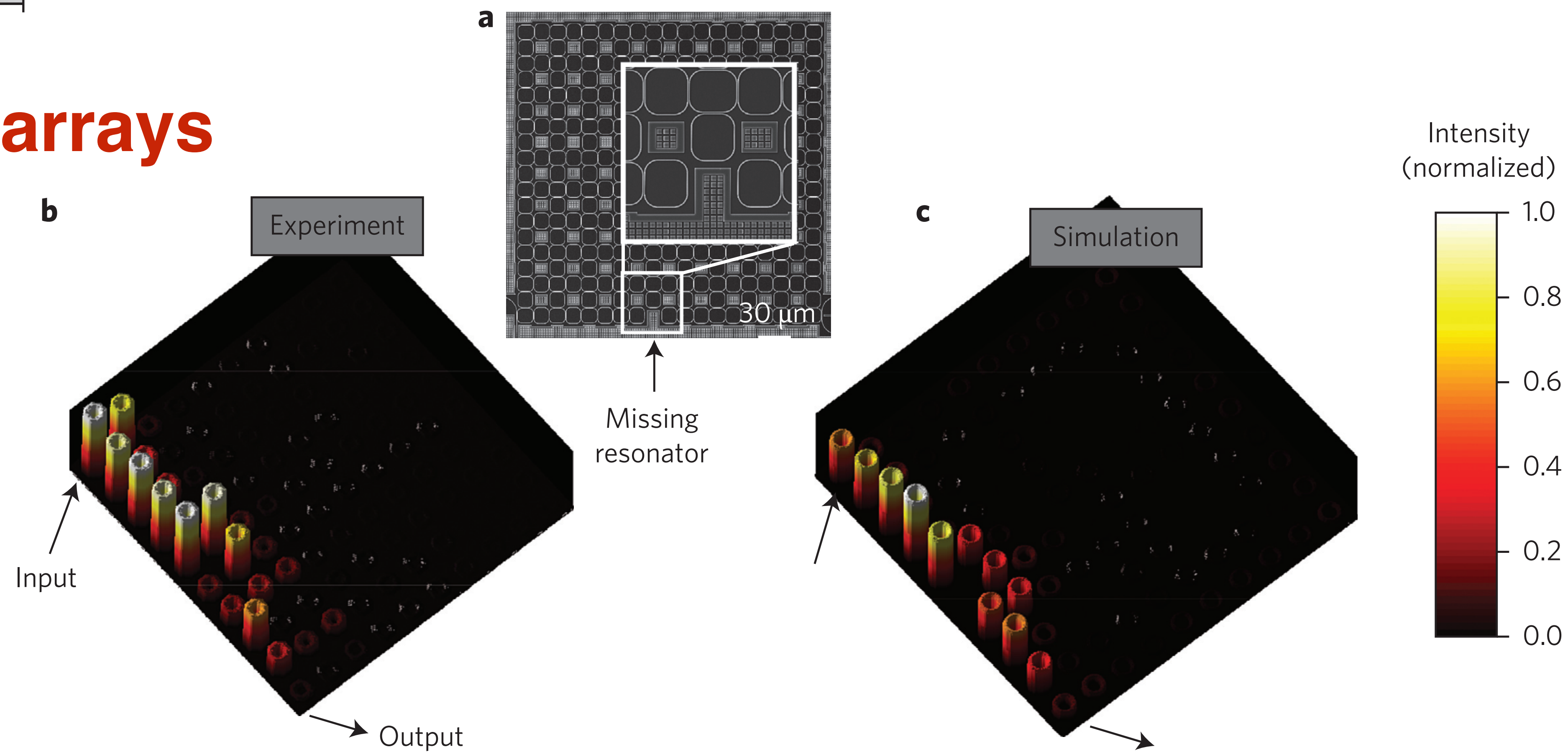
optical cavity arrays



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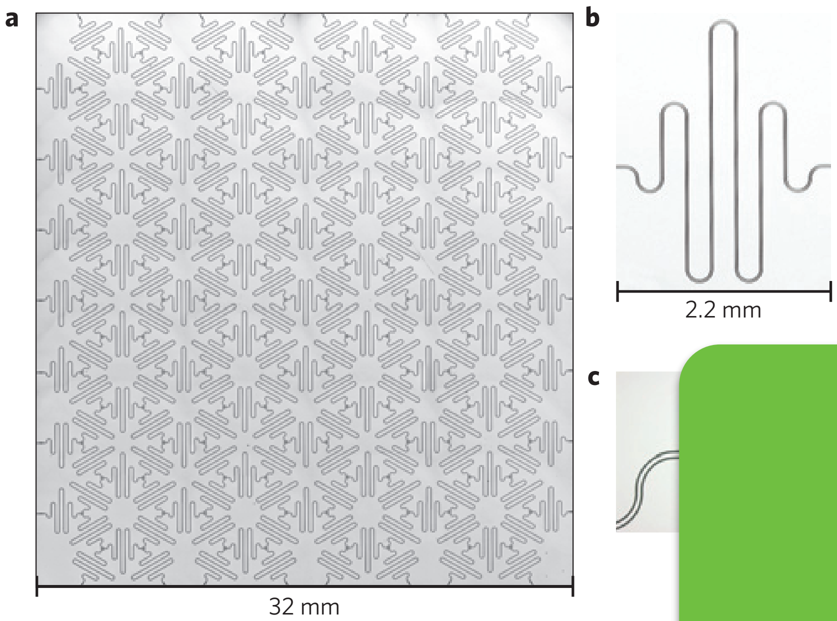

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Hafezi et.al., PRB, 90, 2014

Systems can be supplemented by interactions!

al., Nature Photonics, 7, 2013

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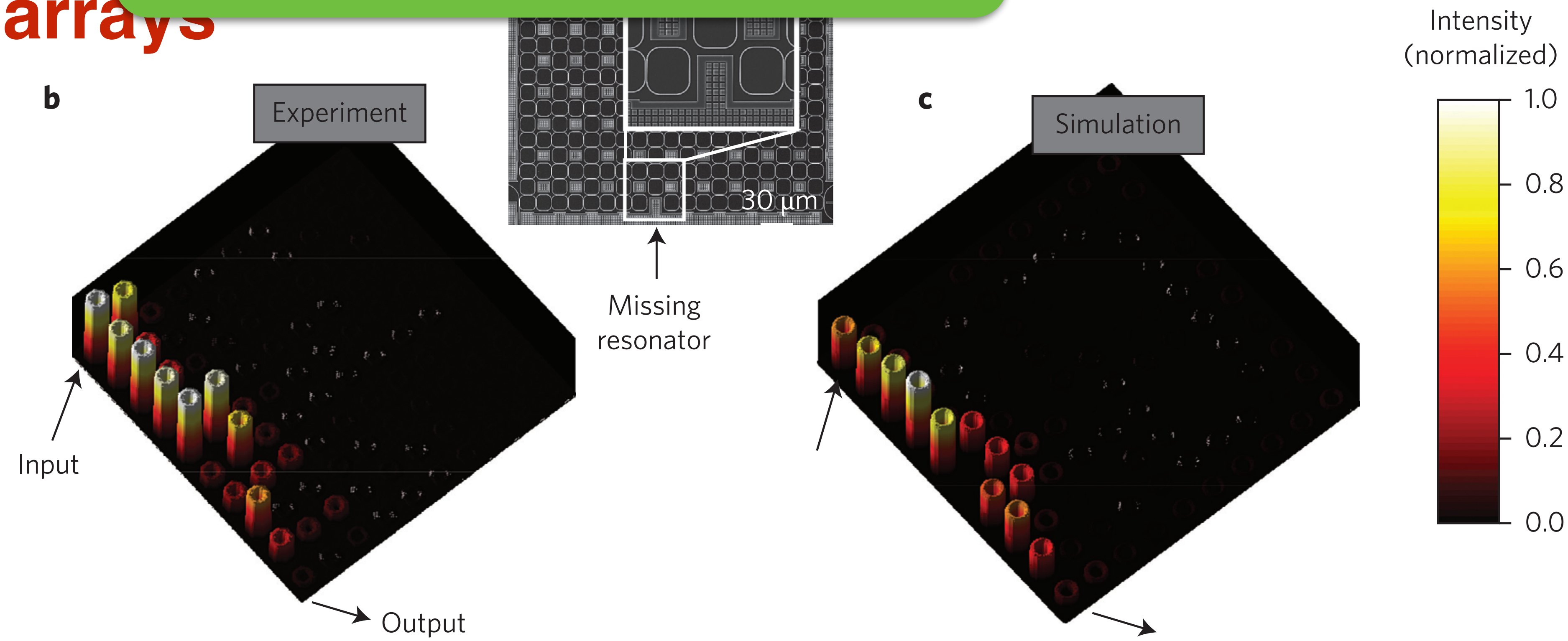


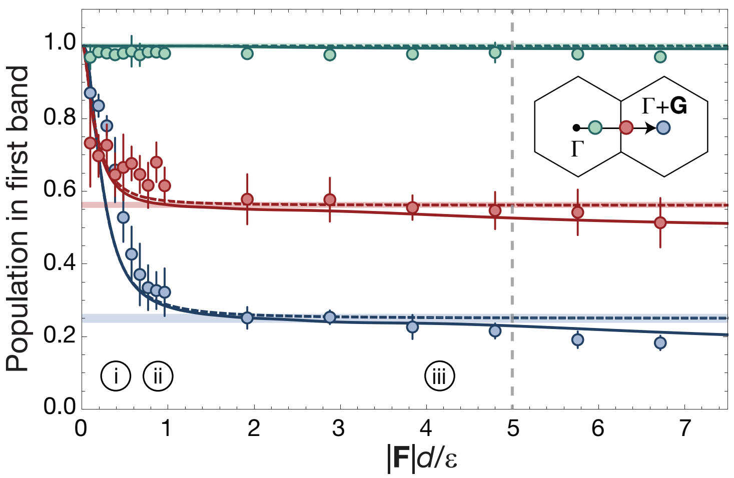
Outline

- **Remarks on topological order**

- **Geometric phases in non-interacting systems**

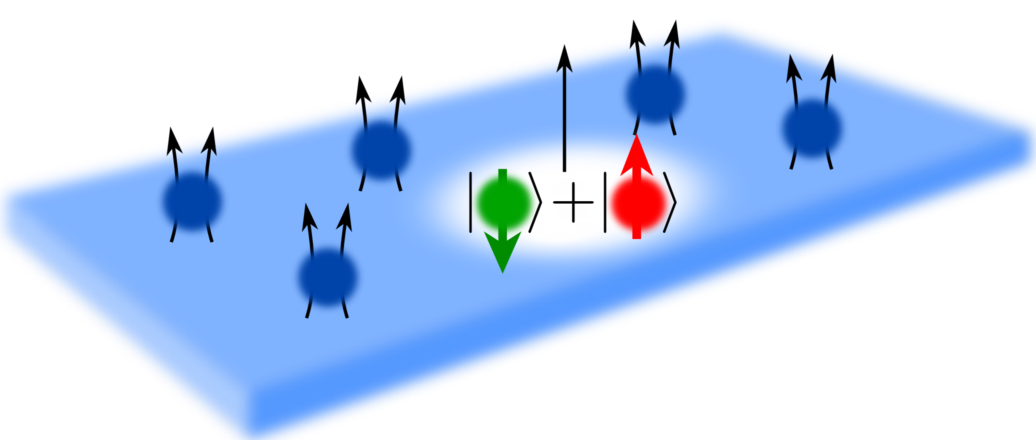
  - Single-band: U(1) Zak phase
    Atala et al., Nat. Phys. 9 (2013)

  - Multi-band: U(2) Wilson-Zak loops
    Li et al., arXiv:1509.02185 (2015)



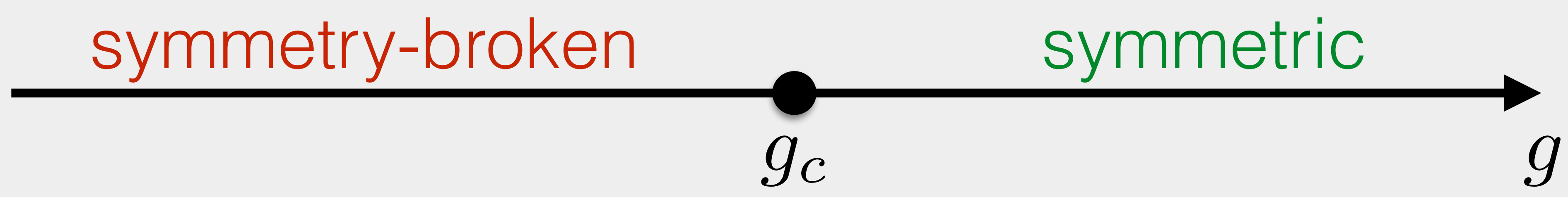
- **Geometric phases in correlated systems**

  - Topological polarons, FQHE
    Grusdt et al., arXiv:1512.03407



Remarks on top. order

Ginzburg-Landau paradigm:



local order parameter,
long-range order

no long-range order

Remarks on top. order

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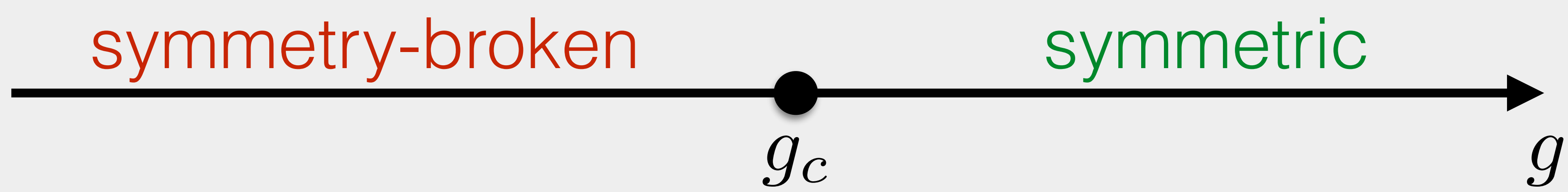

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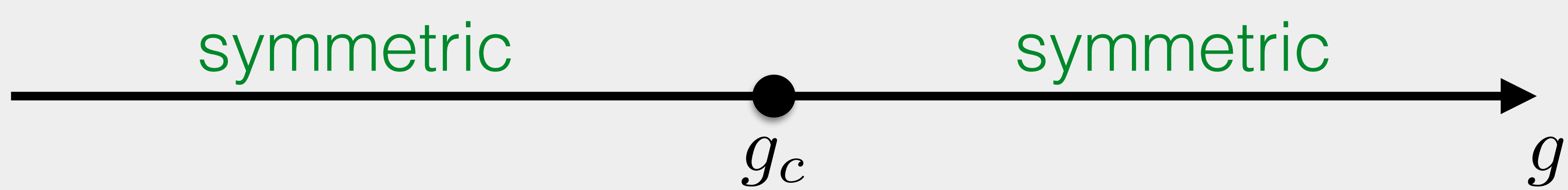
Chen et al., PRB 82 (2010)

Topological states of matter:



long-range entangled

$$|\psi\rangle \neq \hat{U}_{\mathrm{LU}} \; |0\rangle \otimes |0\rangle \ldots$$

short-range entangled

$$|\psi\rangle = \hat{U}_{\mathrm{LU}} \; |0\rangle \otimes |0\rangle \ldots$$

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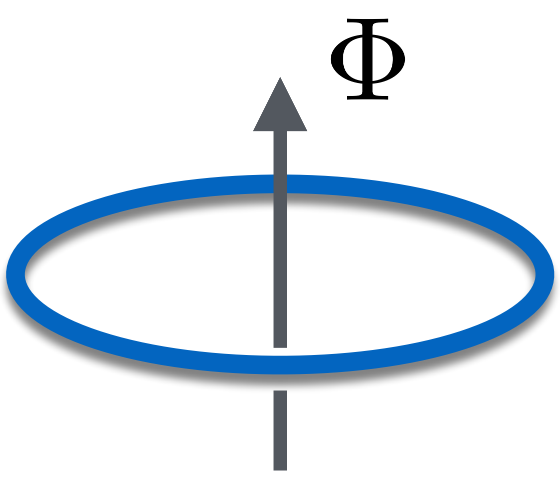
  — consider continuous symmetry

$$\Phi \equiv \Phi + \Phi_0$$



$$\nu = \prod_{j=1}^{N} \langle \psi(\Phi^{(j)}) | \psi(\Phi^{(j+1)}) \rangle$$

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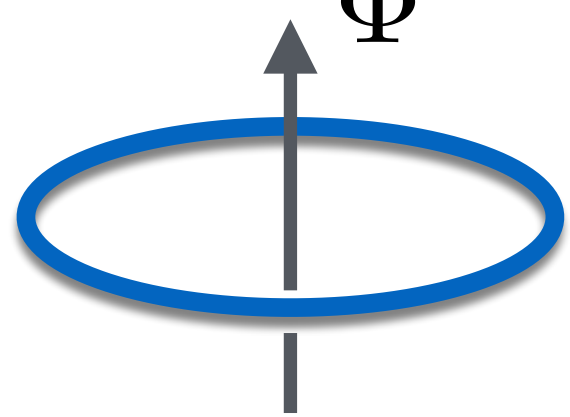

$$\nu = \prod_{j=1}^{N} \langle \psi(\Phi^{(j)}) | \psi(\Phi^{(j+1)}) \rangle = \exp \left[-i \oint d\Phi \langle \psi(\Phi) | i\partial_\Phi | \psi(\Phi) \right]$$

Berry phase

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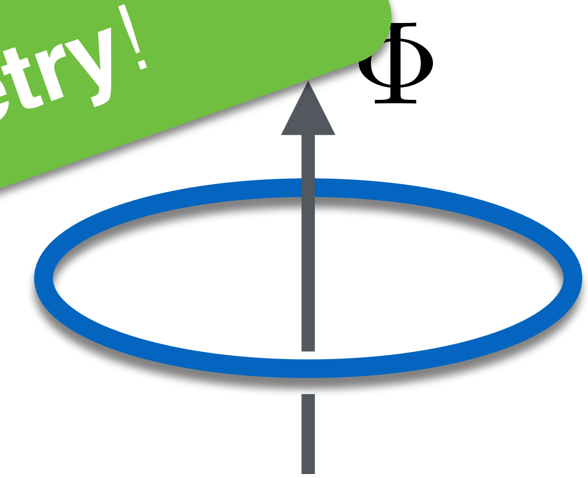


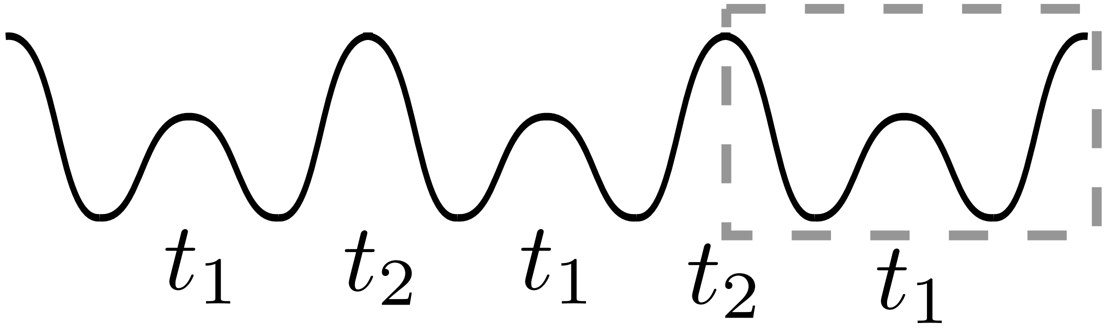
Phase measurement — **interferometry**!

$$\nu = \prod_{j=1} \langle\psi(\Phi^{(j)})|\psi(\Phi^{(j+1)})\rangle = \exp\left[-i \oint d\Phi \langle\psi(\Phi)|i\partial_{\Phi}|\psi(\Phi)\rangle\right]$$

Berry phase

Non-interacting systems

- Su-Schrieffer-Heeger model:



  - U(1) Zak/ Berry phase:

$$\varphi_{\mathrm{Zak}} = \int_{\mathrm{BZ}} dk \; \langle u_k | i\partial_k | u_k \rangle$$

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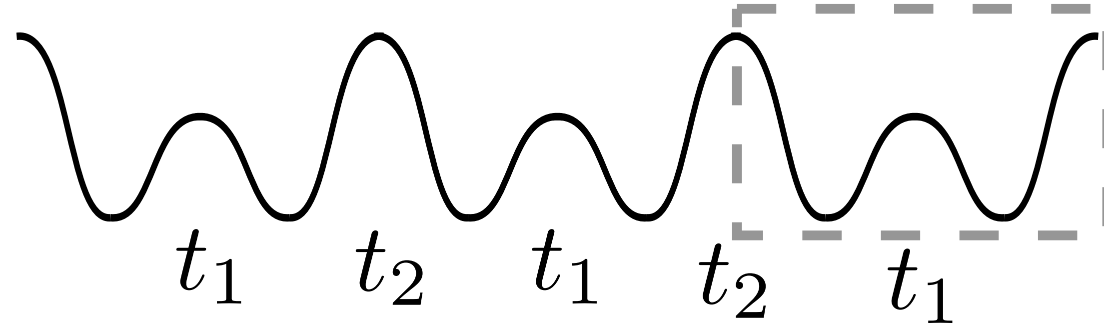

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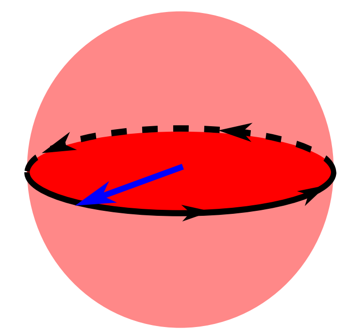

quantized by
inversion symmetry

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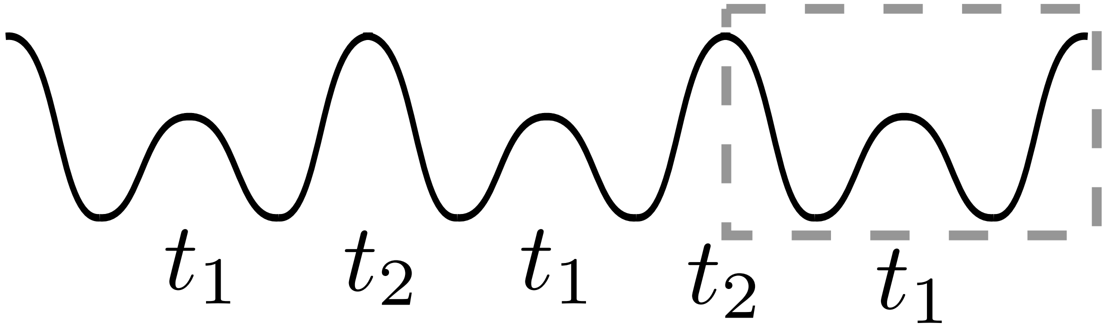

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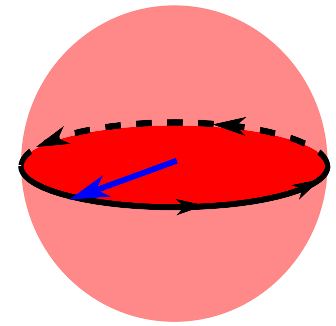
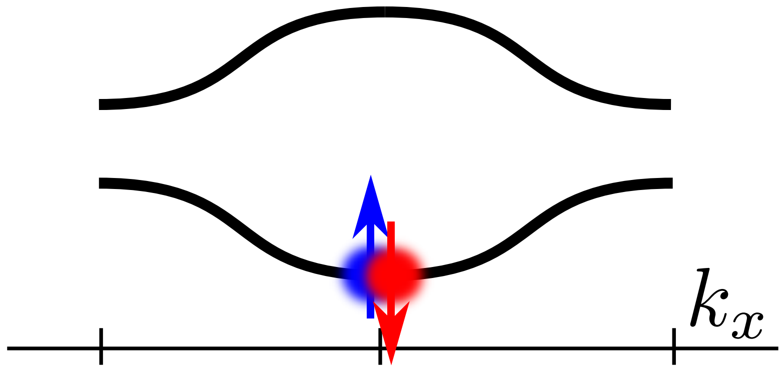


  - Measurement by two-component BEC:

Atala et al., Nat. Phys. 9 (2013)



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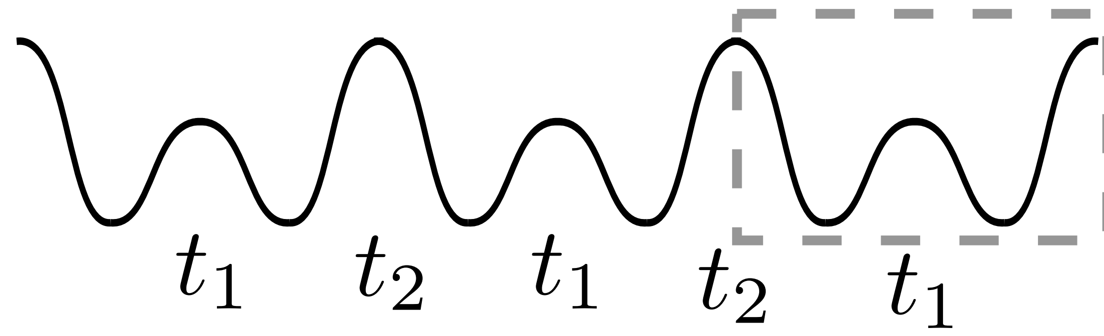

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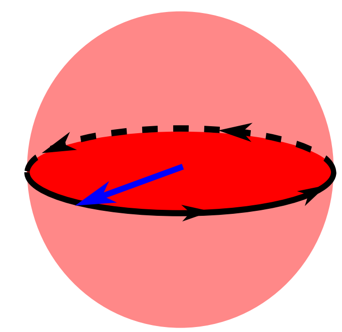

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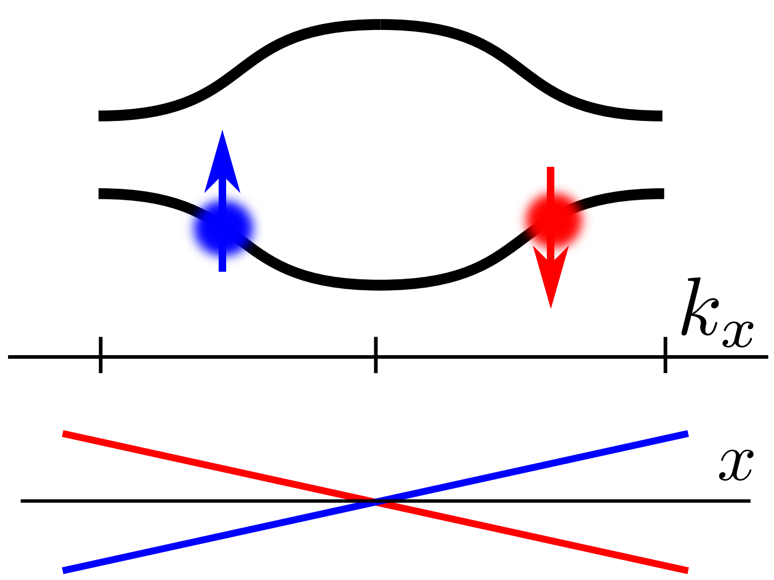

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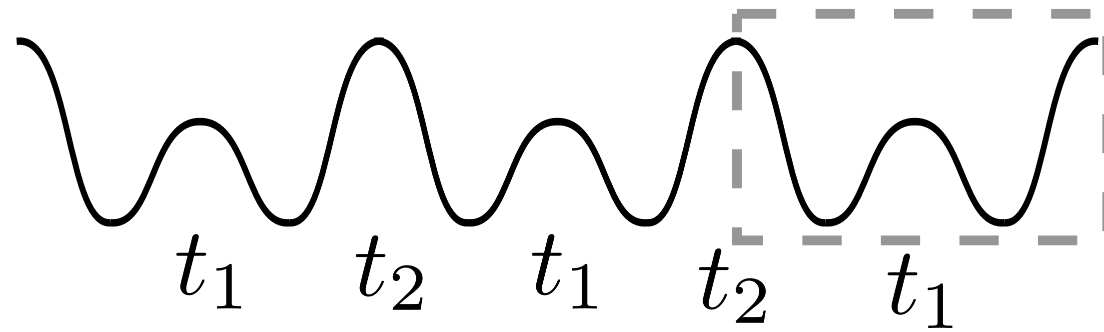

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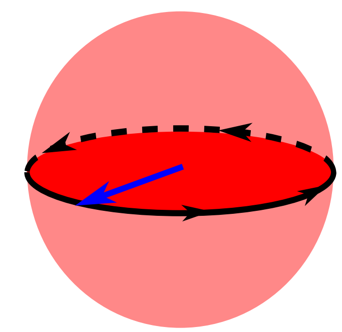

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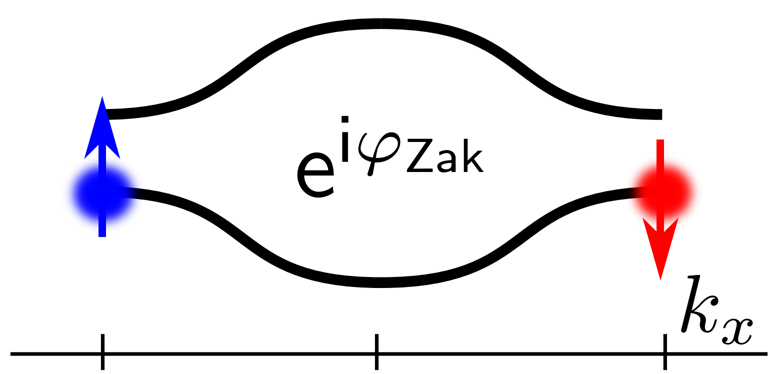
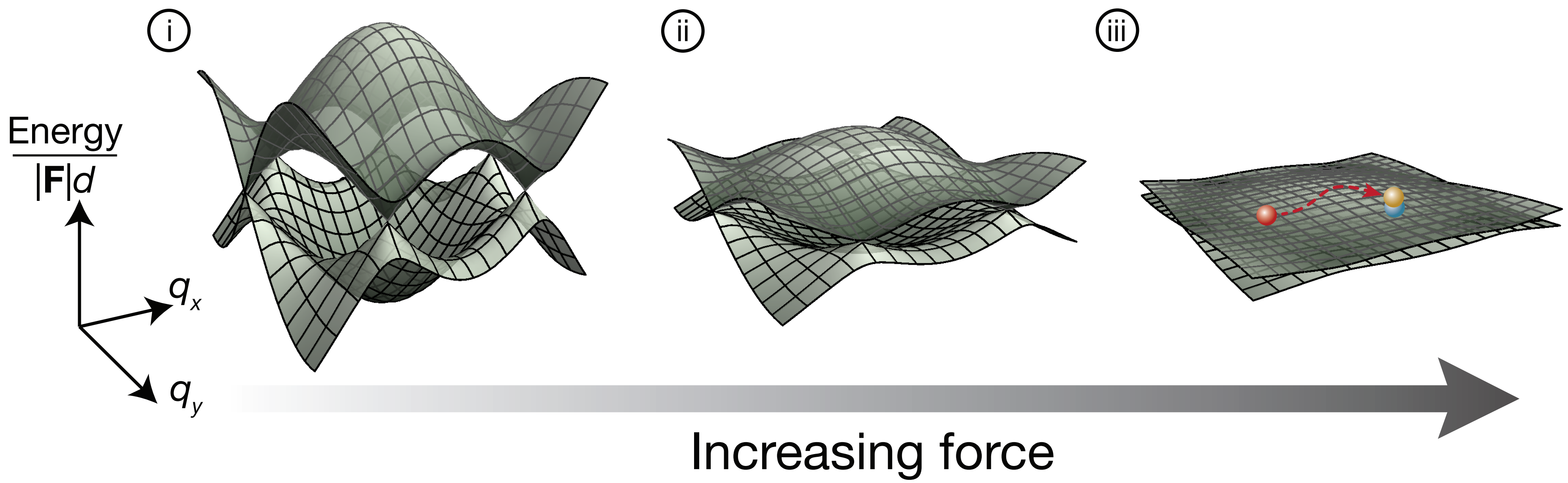


Ramsey interferometry
+
Bloch oscillations

Non-interacting systems

- Multiband generalization of Zak phases: $\vec{k}_t = \vec{k}_0 - \vec{F}t$

$$i\partial_t|\psi(t)\rangle = \left(\hat{h}(\vec{k}_t) - \vec{F}\cdot\hat{\vec{\mathcal{A}}}(\vec{k}_t)\right)|\psi(t)\rangle$$



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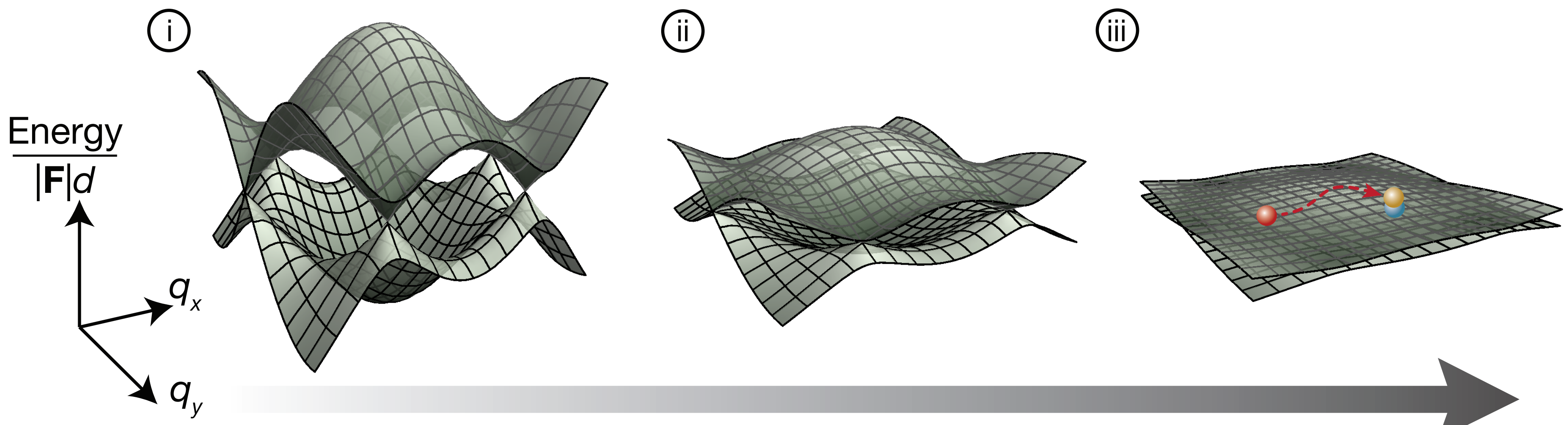


$\vec{F} \to \infty$

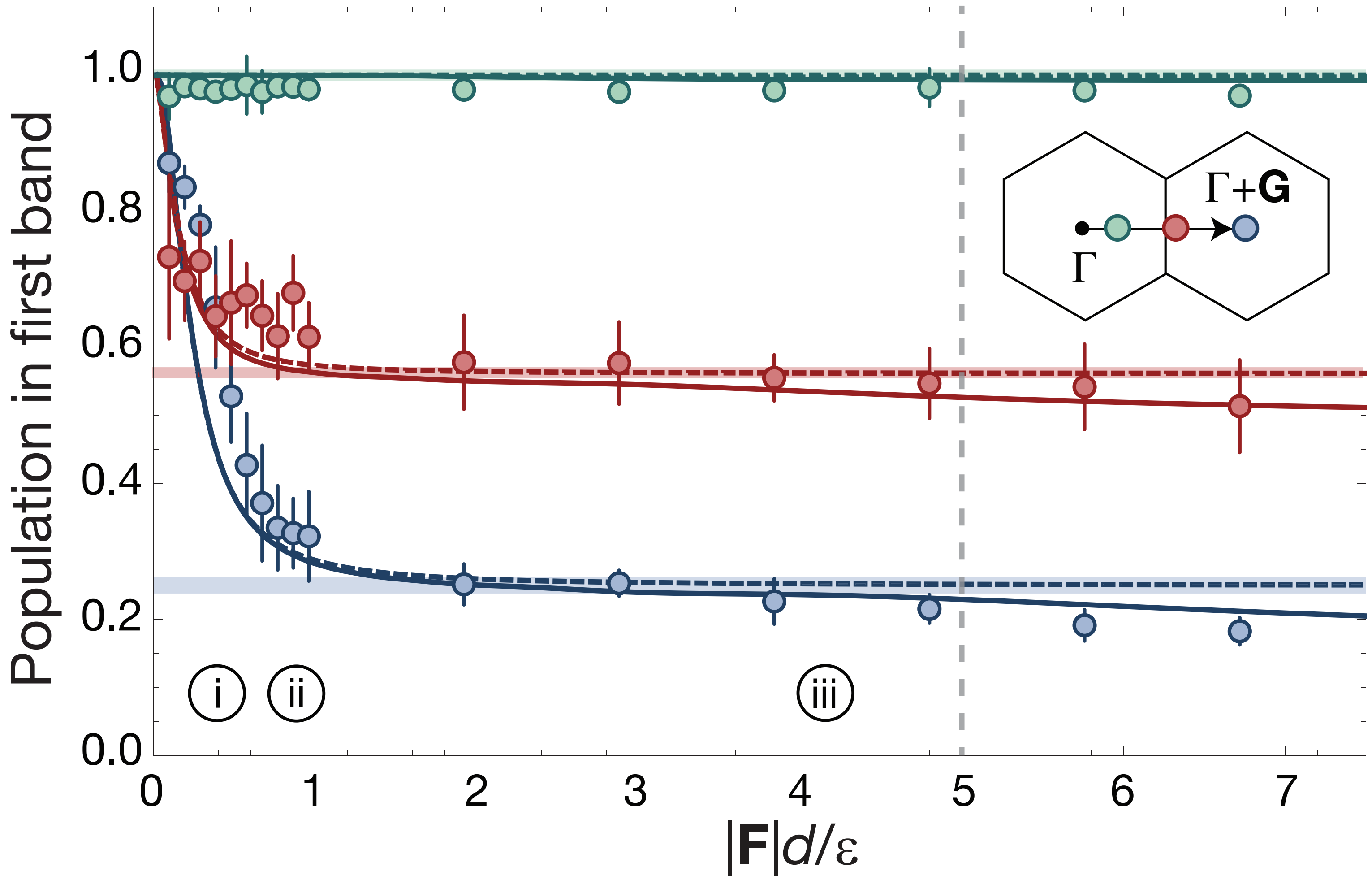
Propagator becomes **Wilson line**:

$$\boxed{\hat{W} = \mathcal{P} \exp\left[-i \int d\vec{k} \cdot \hat{\vec{A}}(\vec{k}) \right]}$$

Non-interacting systems

- Measurement in honeycomb lattice (ultracold atoms):

Li et al., arXiv:1509.02185 (2015)



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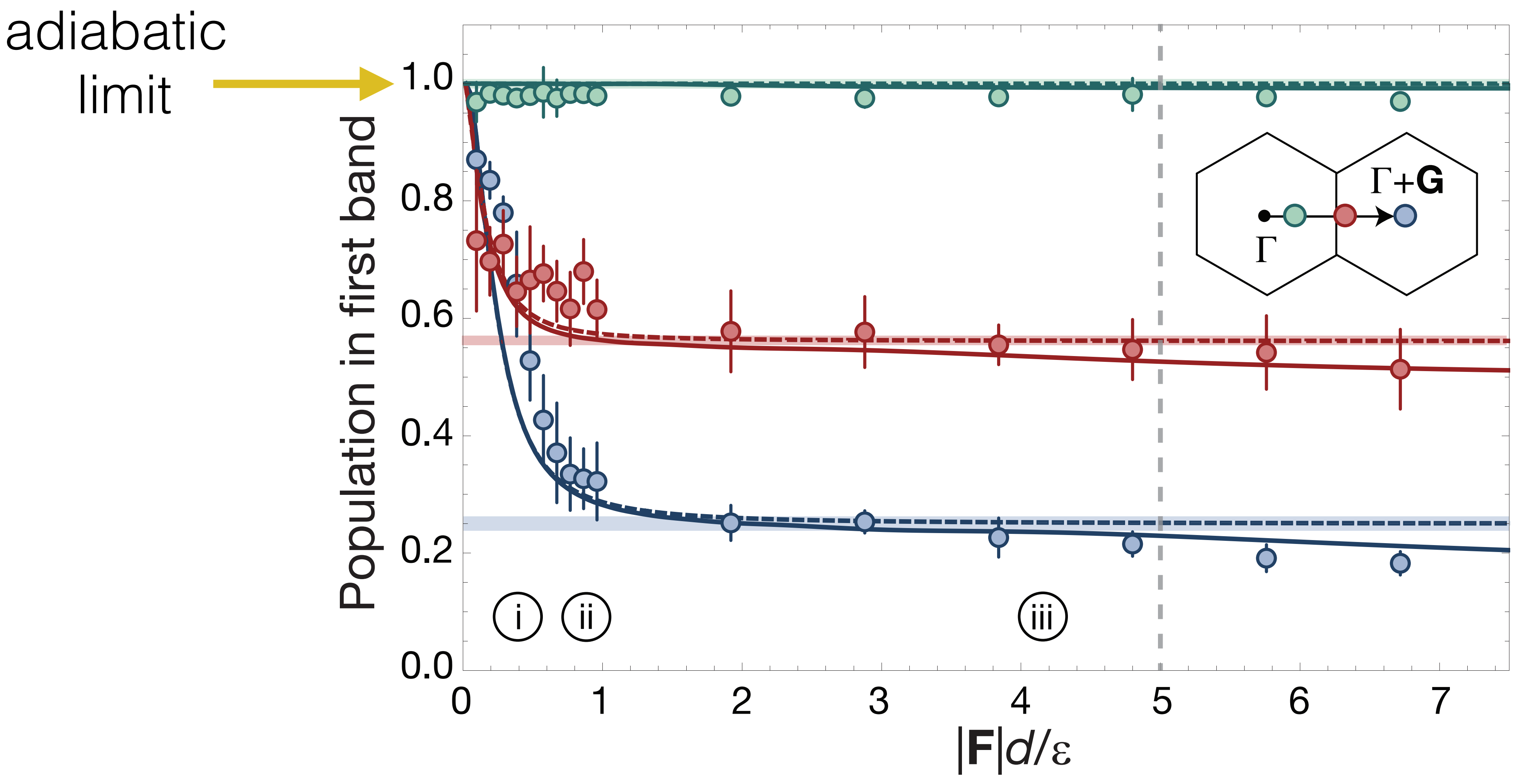

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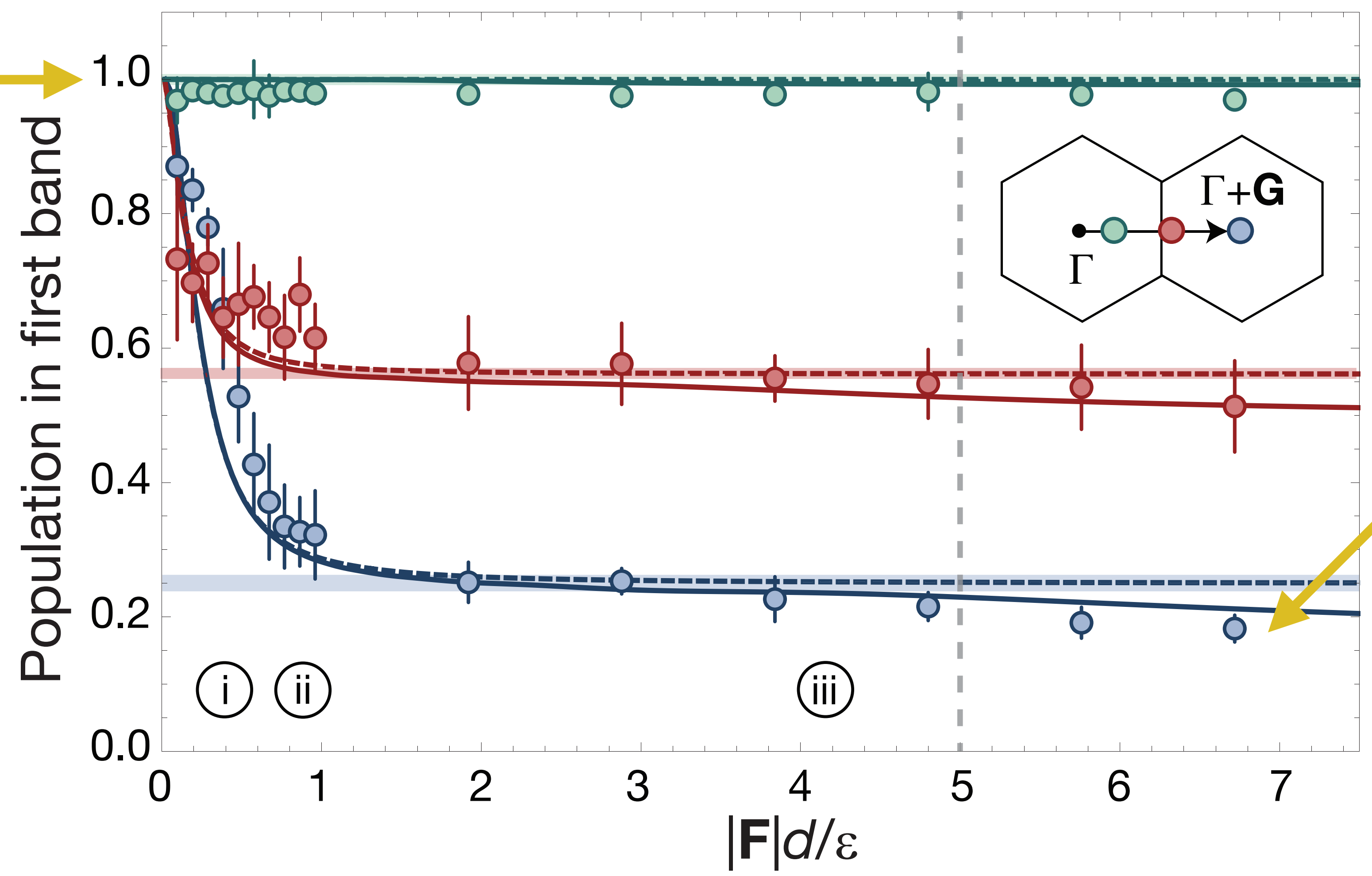
adiabatic limit



transitions into higher bands

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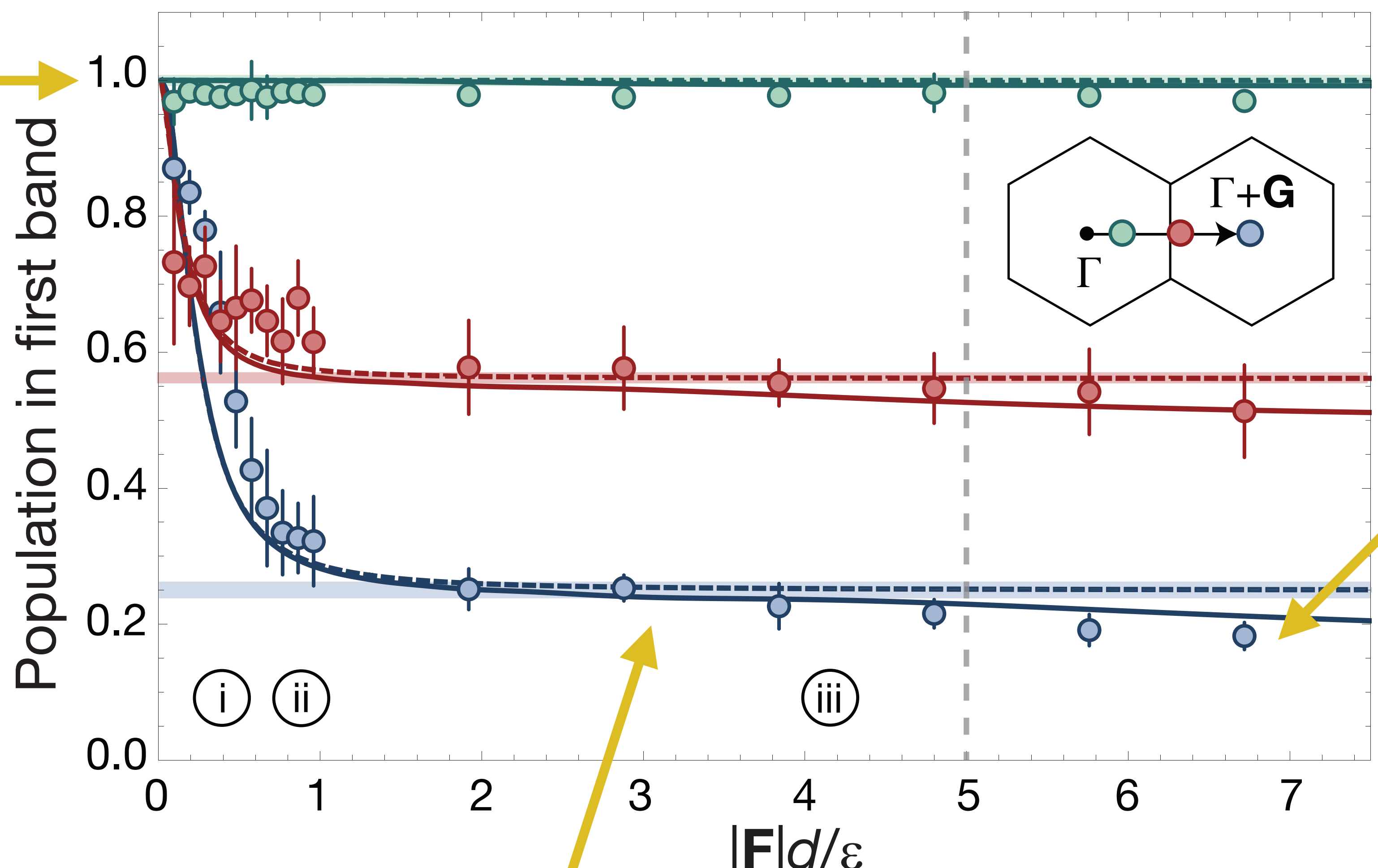

transitions into higher bands

Wilson line regime

different from Landau-Zener dynamics!

Non-interacting systems

- Physical relevance of Wilson loops:

transport by **one reciprocal lattice vector:** $TF = 2\pi/a$

- Single-band: U(1) Zak phase

$$\varphi_{\mathrm{Zak}} = \int_0^T dt \; \vec{F} \cdot \langle \psi(t) | \, \hat{\vec{r}} \, | \psi(t) \rangle$$

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Non-interacting systems

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$$e^{i\varphi_{\mathrm{Zak}}} = e^{i\frac{2\pi}{a}\langle r \rangle} \equiv e^{i\frac{2\pi}{a}P}$$

Polarization

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Polarization

- Multi-band: U(N) Wilson-Zak loops

$$\hat{W} = e^{i\frac{2\pi}{a}\hat{P}}$$

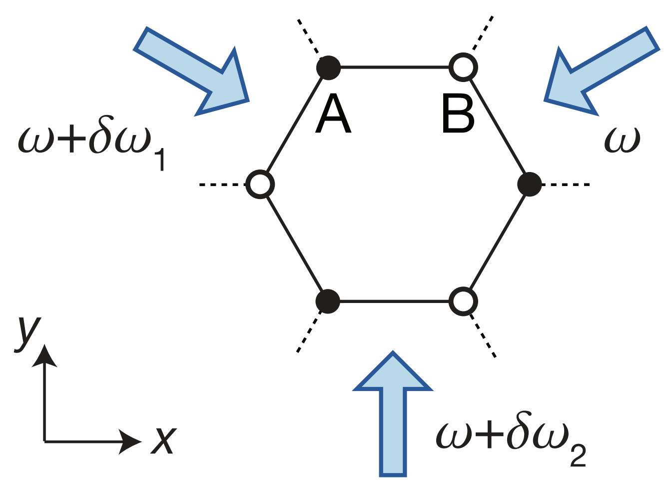
$$P_{\alpha\beta} = \langle w_\alpha | \hat{r} | w_\beta \rangle$$

Wannier functions

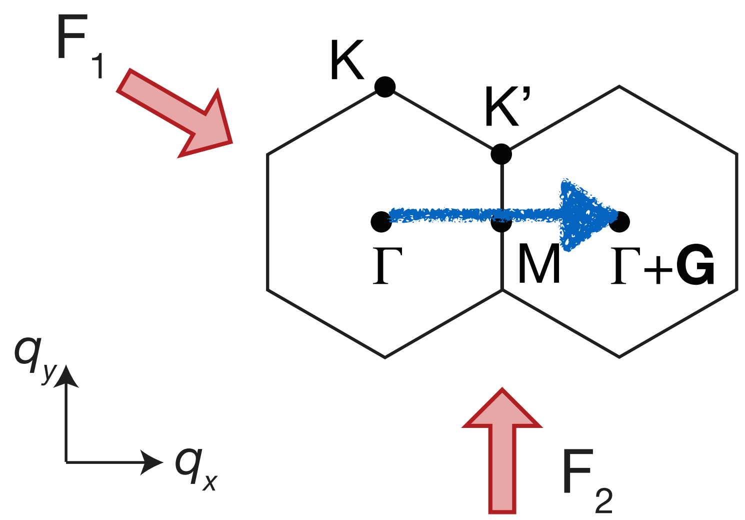
Non-interacting systems

- Measurement of Wilson-Zak loops

Li et al., arXiv:1509.02185 (2015)



Real space



Reciprocal space

Non-interacting systems

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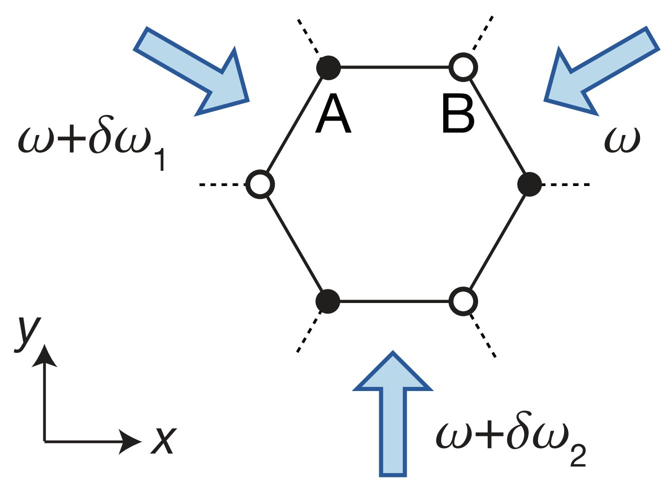
Li et al., arXiv:1509.02185 (2015)



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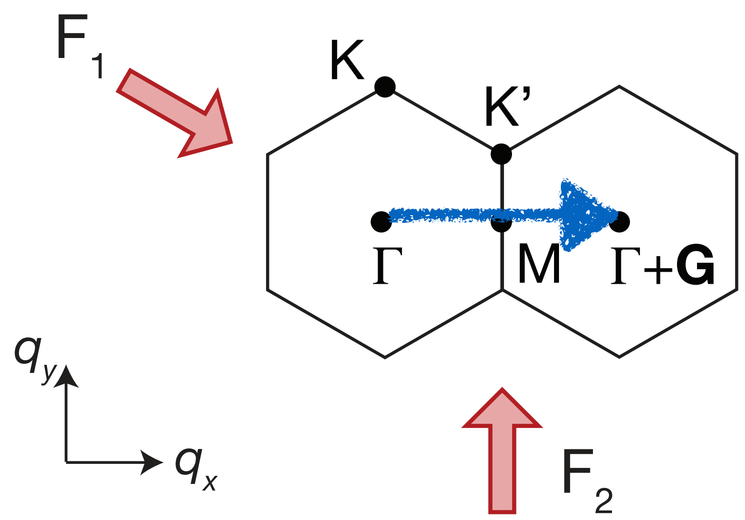

Reciprocal space

- Two-band tight-binding: $$\hat{W} = \mathrm{diag}\left(e^{i\vec{G}\cdot\vec{r}_A}, e^{i\vec{G}\cdot\vec{r}_B}\right)$$

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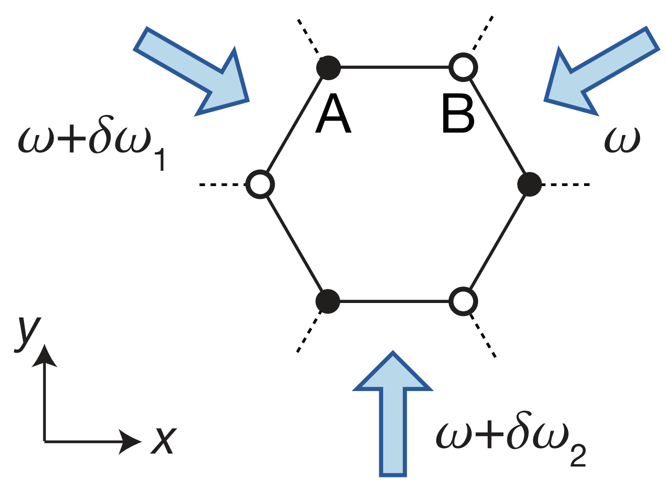

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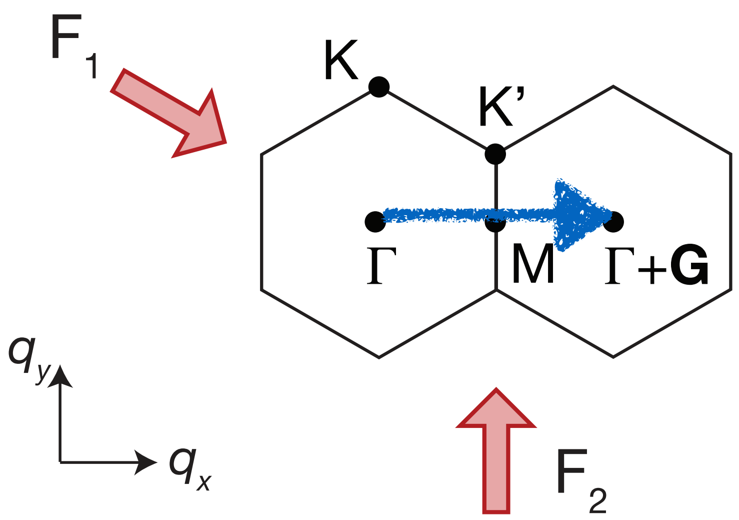

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Theory:

$$\xi = \vec{G}\cdot(\vec{r}_A - \vec{r}_B) = \pi/3$$

Experiment:

$$\xi = 1.03(2)\pi/3$$

Non-interacting systems

- Measurement of Wilson-Zak loops

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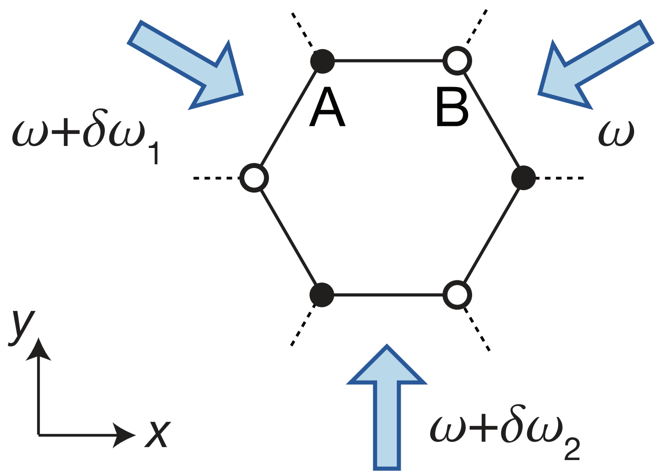

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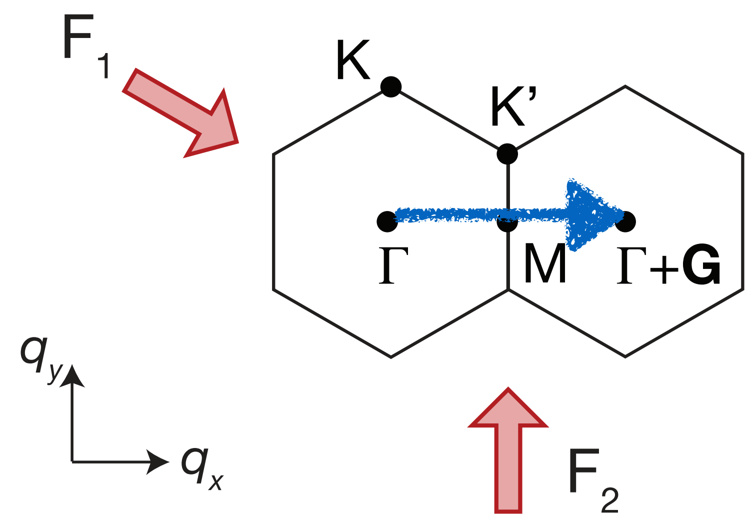

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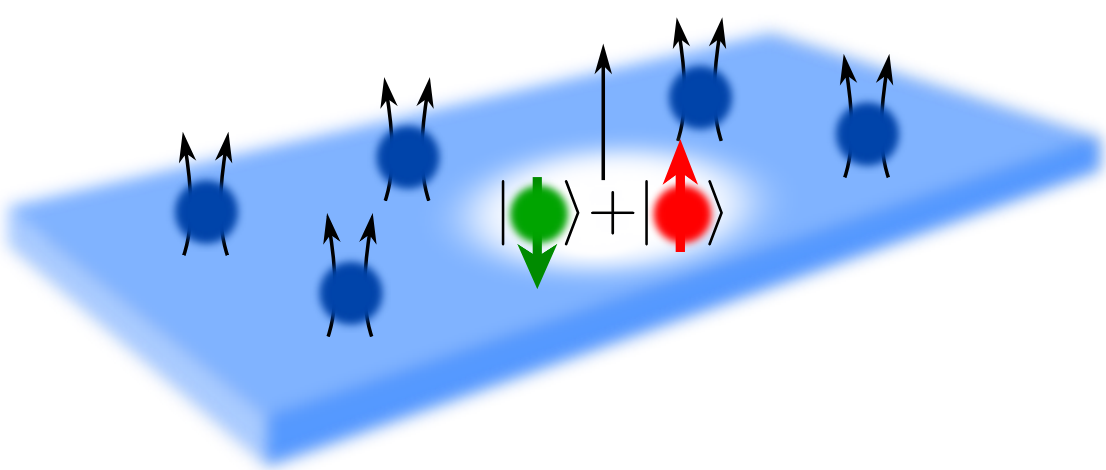
$$\xi = 1.03(2)\pi/3$$

$$\xi = 1.04(4)\pi/3$$

with AB off-set

Geometric phases in correlated systems



- mostly: fractional quantum Hall, fractional Chern insulators
- can be any other gapped, topological system!

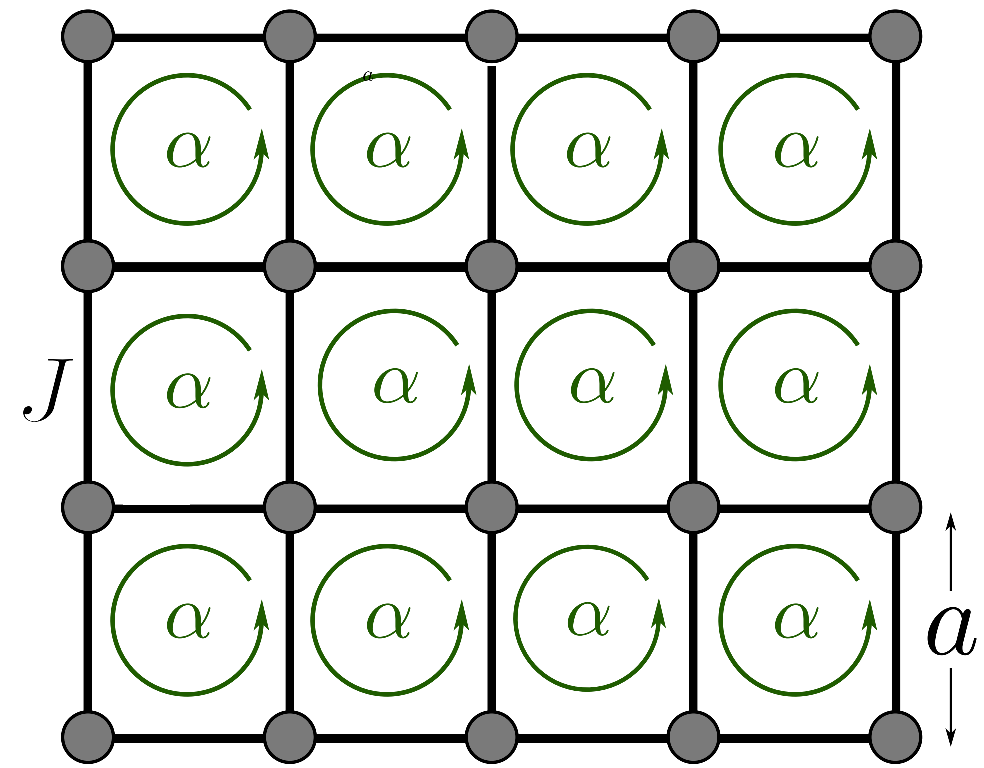
Systems

- Hofstadter Hamiltonian

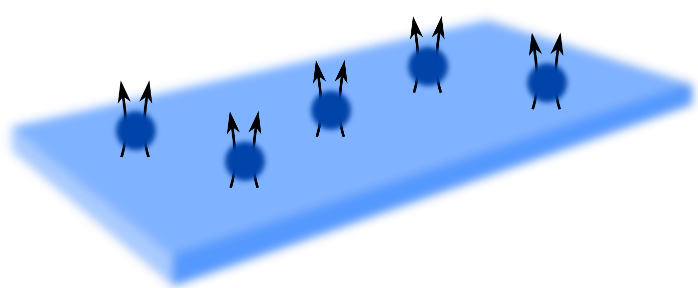
$$\mathcal{H}_0 = -J \sum_{\langle i,j \rangle} e^{i\varphi_{i,j}} \hat{a}_i^\dagger \hat{a}_j$$

- Hubbard term

$$\mathcal{H}_{\text{int}} = \frac{U}{2} \sum_j \hat{a}_j^\dagger \hat{a}_j \left(\hat{a}_j^\dagger \hat{a}_j - 1 \right)$$



✱ Laughlin states



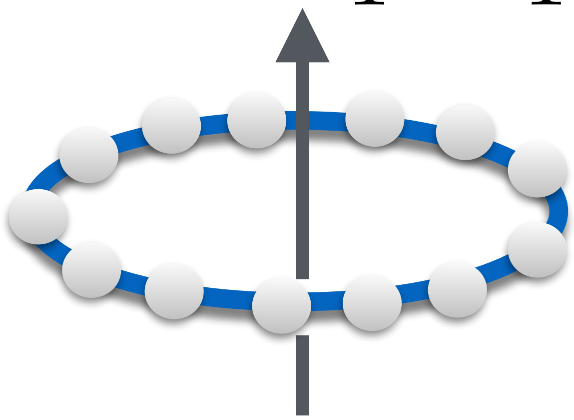
Sorensen et al., PRL 94 (2005)

Hafezi et al., PRA 76 (2015)

Many-body topological invariants

- Twisted periodic boundary conditions:

$$\Phi = \Phi_0 \frac{\theta_x}{2\pi}$$



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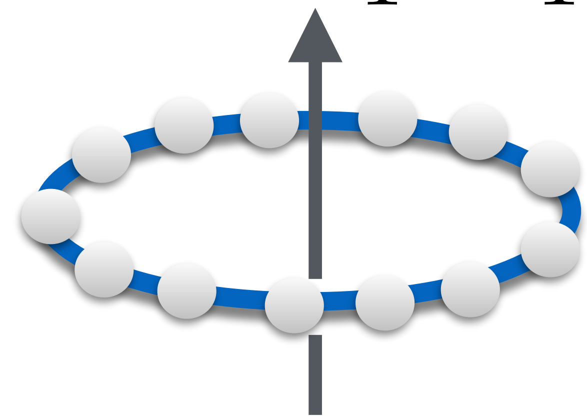

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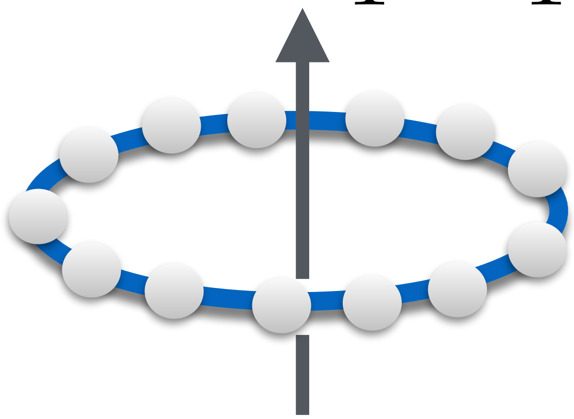

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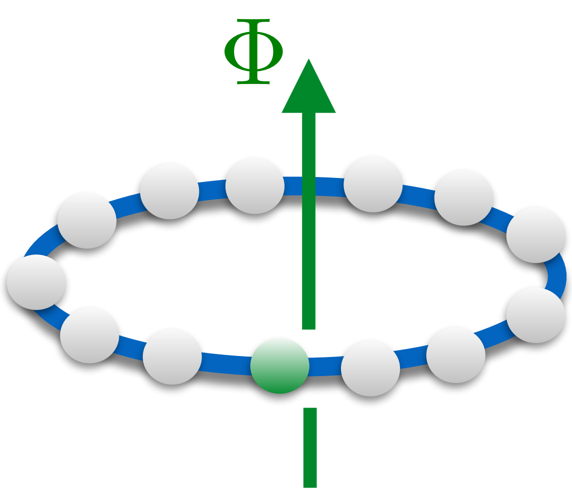

embed a **mobile impurity** in the system

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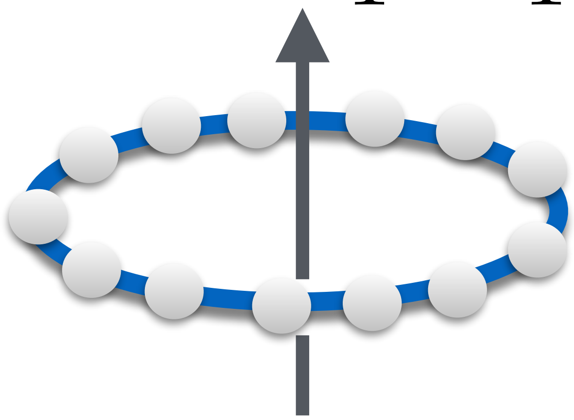

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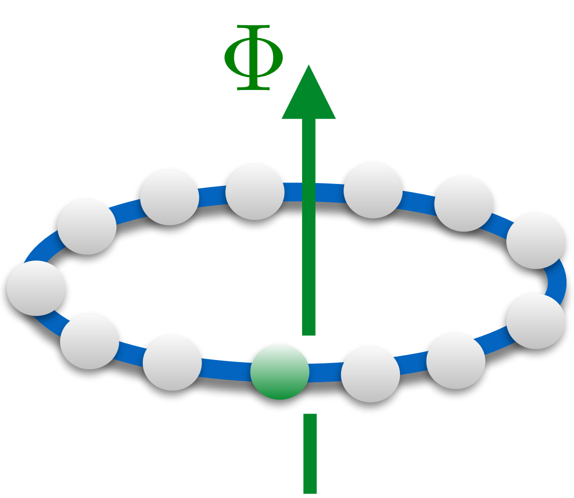

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Topological qp excitation + mobile impurity = topological polaron (TP)

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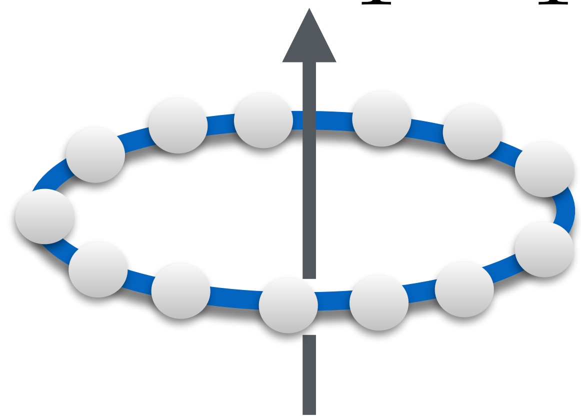

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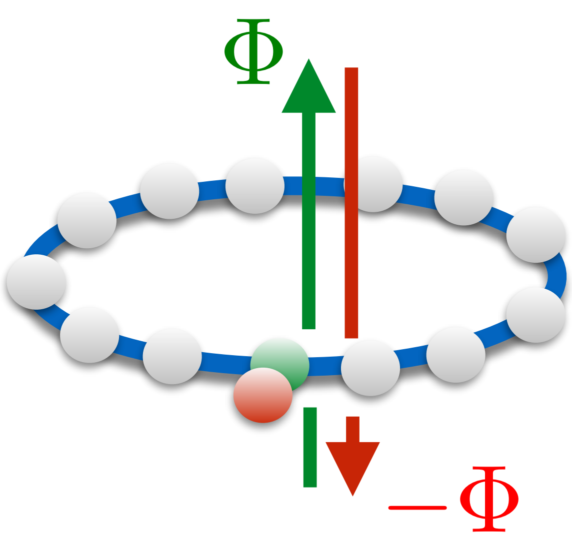

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Strong-Coupling Theory

- Microscopic model: $$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\text{int}} + \hat{\mathcal{H}}_{\text{I}}^0 - \sigma^z \vec{F} \cdot \hat{\vec{r}}$$

host system

free impurity

PHYSICS

Strong-Coupling Theory

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- Strong-coupling wavefunction:

$$|\psi_{\rm TP}(\boldsymbol{q})\rangle = |\psi_{\rm qp}(\boldsymbol{q})\rangle \otimes |\phi_{\rm I}\rangle$$

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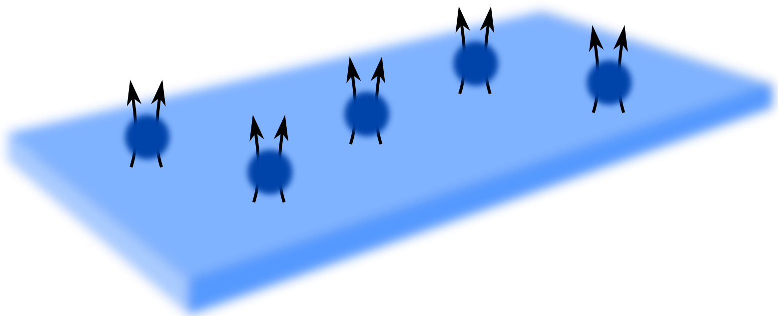
represents topological order
of bulk groundstate

Fractional Quantum Hall Effects

- Laughlin states $\nu = \dfrac{1}{m}$



  - many-body Chern number (winding of Zak phase)

$$\mathcal{C} = \frac{1}{m} = \frac{e^*}{e}$$ (*m*-fold degenerate gs)

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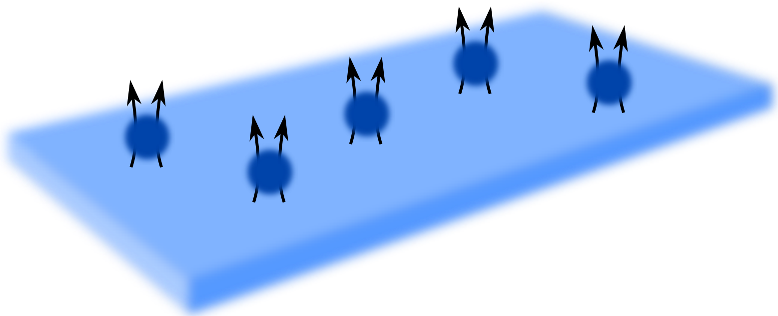

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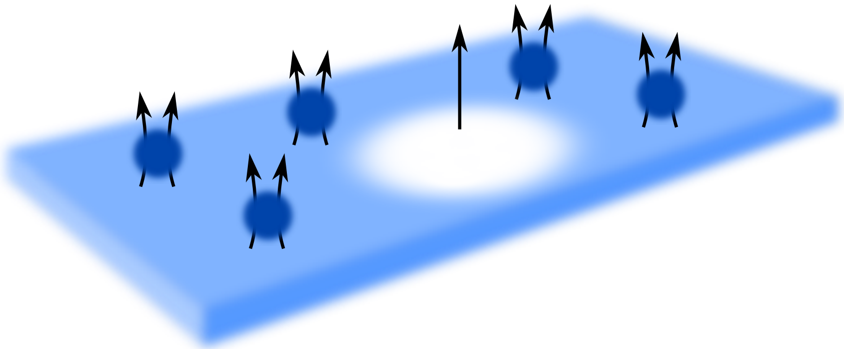

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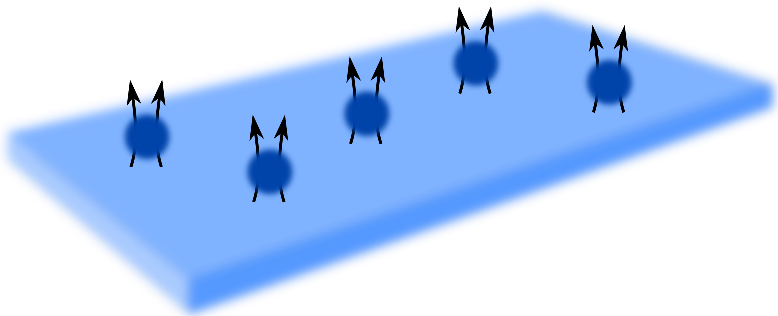

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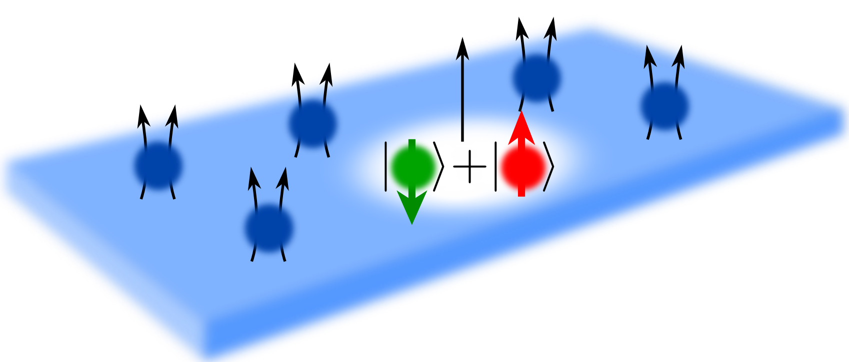
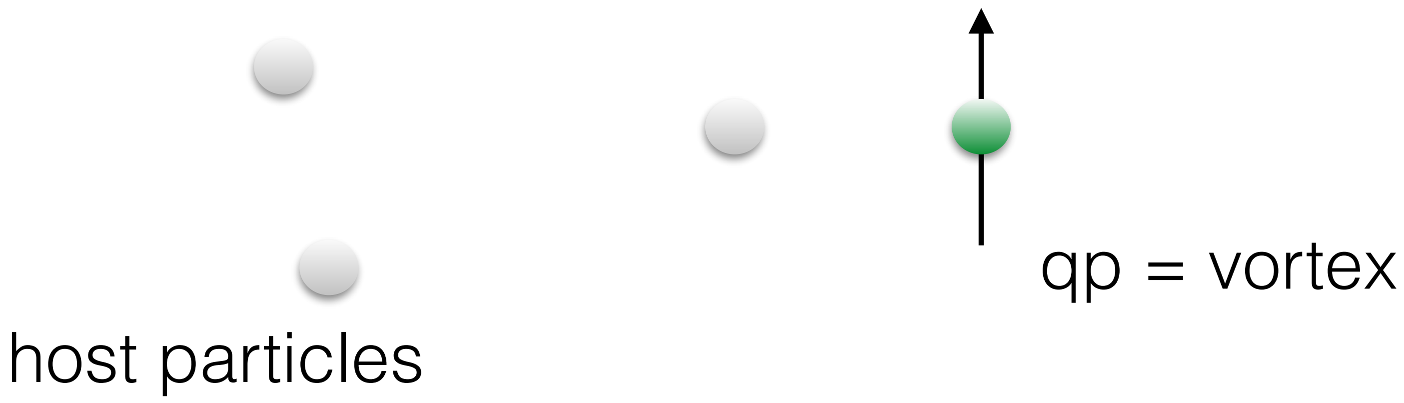


$$\mathcal{C}_{\mathrm{TP}} = \frac{e}{e^*} = m$$

(non-degenerate)

Fractional Quantum Hall Effects

- Charge e/m vortex sees particles as a source of flux



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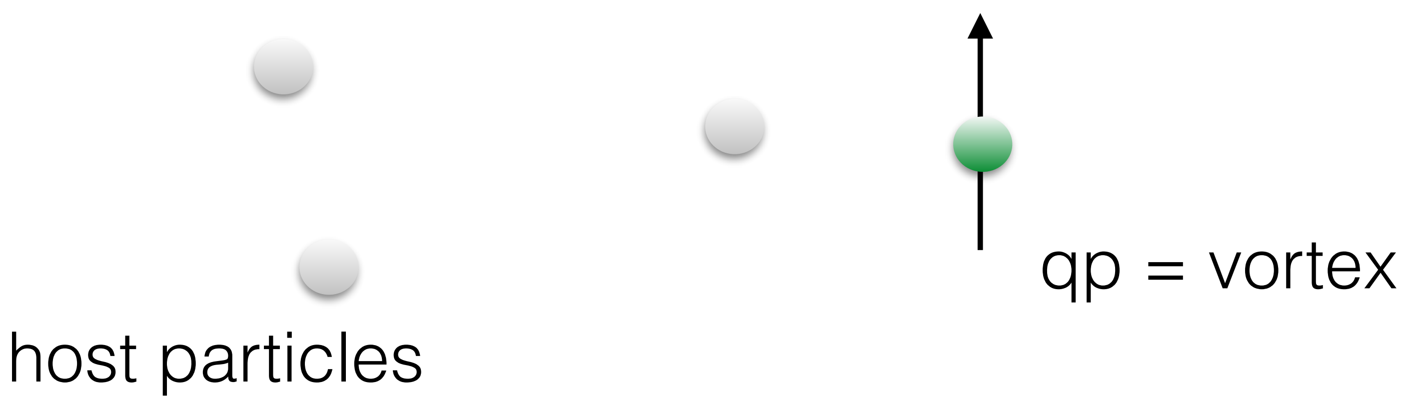

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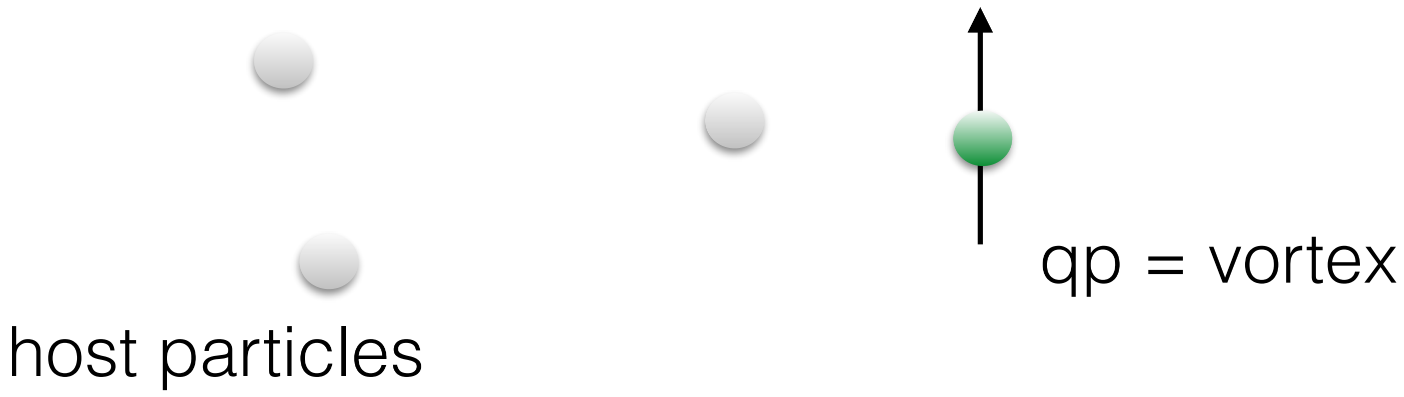

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reduced effective magnetic field: $b_z^* = b_z/m$

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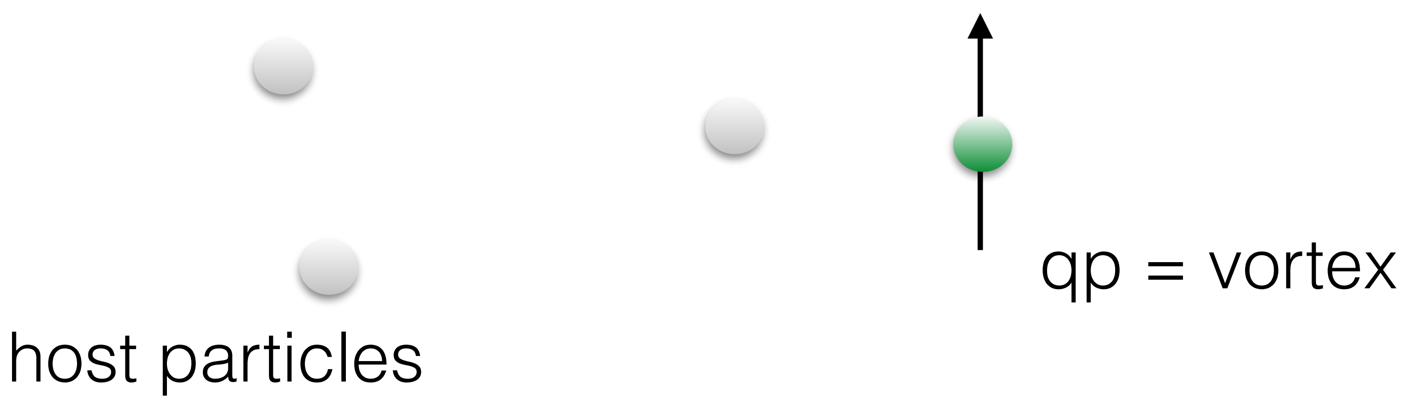

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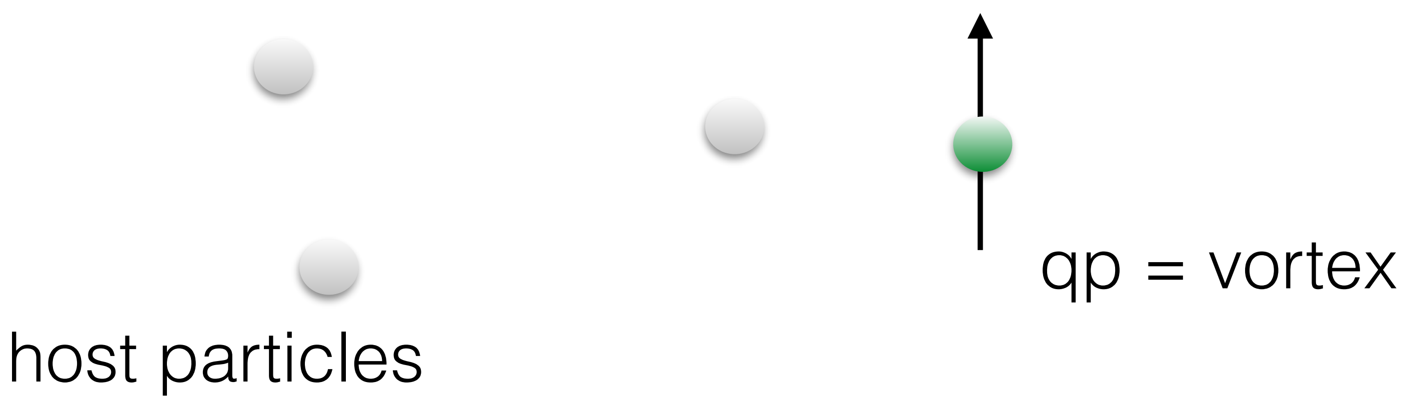

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Fractional Quantum Hall Effects

- Effective field theory description of bulk: *Wen, Adv.Phys. 44 (1995)*

$$\mathcal{L} = \frac{1}{4\pi} \underline{a}_\mu^T \underline{\underline{K}} \partial_\nu \underline{a}_\lambda \epsilon^{\mu\nu\lambda} - \frac{e}{2\pi} A_\mu \underline{t}^T \partial_\nu \underline{a}_\lambda \epsilon^{\mu\nu\lambda} + \\ + \sum_{I=1}^{n} a_{I\mu} \ell_I j_{I\mu} + \text{kin. energy}.$$

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K-matrix

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quasiparticle current

- Euler-Lagrange equations w.r.t a_μ:

$$J_\mu = \mathcal{C} \frac{e^2}{2\pi} \epsilon^{\mu\nu\lambda} \partial_\nu A_\lambda \qquad \mathcal{C} = \sum_{I,J=1}^{n} t_I (K^{-1})_{IJ} = \sum_J \frac{e_J^*}{e}$$

Fractional Quantum Hall Effects

* Binding of qp to impurity

$$j_{I\mu} = \ell_I \; j_\mu$$

qp current — impurity current

$$\mathcal{L}_{\text{imp}} = qB_\mu \; j_\mu$$

integrate out a_μ

J.Moore, Oxford Univ.Press (2014)

$$\mathcal{L}_{\text{eff}} = -\frac{e^2}{4\pi} \underline{t}^T \underline{\underline{K}}^{-1} \underline{t} \, \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + \pi \underline{\ell}^T \underline{\underline{K}}^{-1} \underline{\ell} \, j_\mu \frac{\epsilon^{\mu\nu\lambda}\partial_\nu}{\partial^2} j_\lambda + \left(qB_\mu + eA_\mu \, \underline{t}^T \underline{\underline{K}}^{-1} \underline{\ell} \right) j_\mu$$

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fractional statistics

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enhanced TP Chern number: $$\mathcal{C}_{\mathrm{TP}}^{(J)} = \frac{e}{e_J^*}$$

Fractional Chern insulators

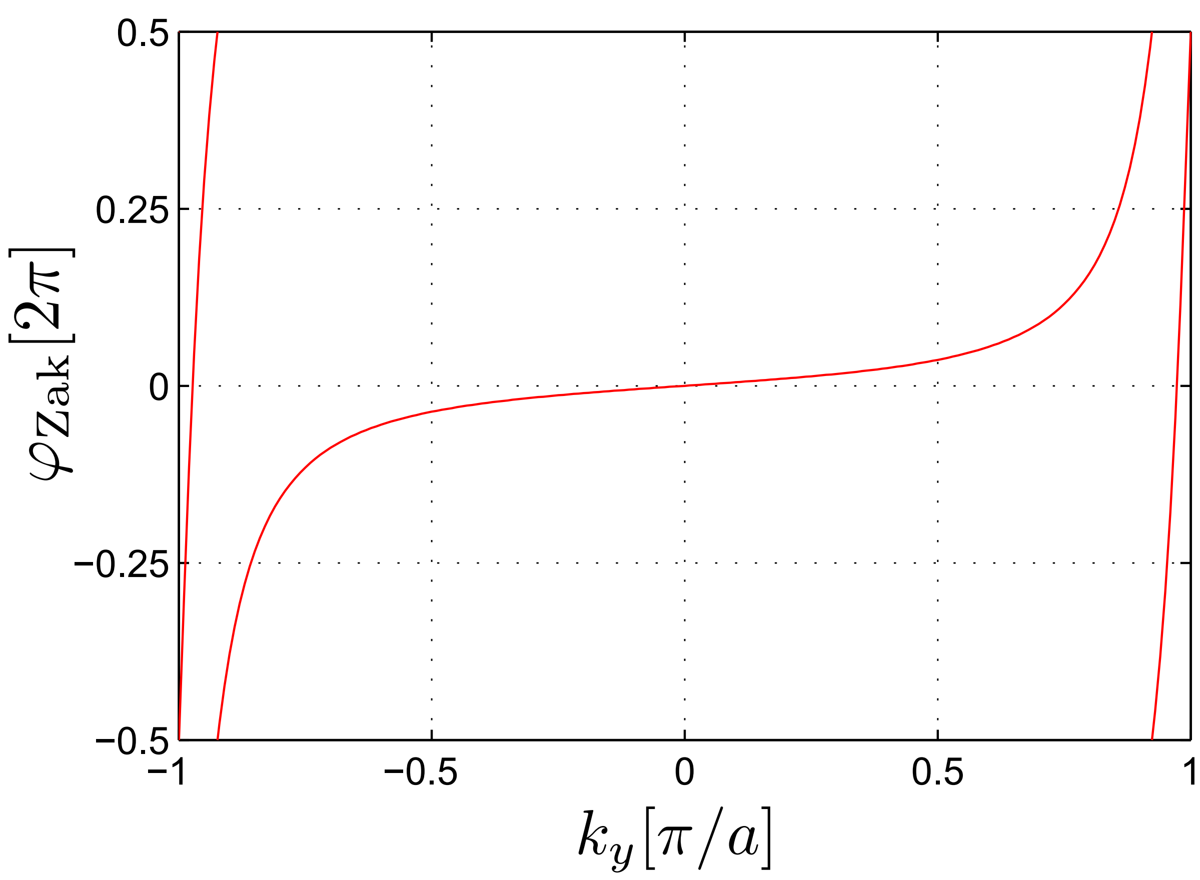
- Hofstadter Fermi-Hubbard model:

$$\hat{\mathcal{H}}_0 = -t\sum_{m,n}\left[e^{-i2\pi\alpha n}\hat{c}^\dagger_{m+1,n}\hat{c}_{m,n} + \hat{c}^\dagger_{m,n+1}\hat{c}_{m,n} + \text{h.c.}\right] +$$
$$+ U \sum_{\langle (m,n),(m',n')\rangle} \hat{c}^\dagger_{m,n}\hat{c}_{m,n}\hat{c}^\dagger_{m',n'}\hat{c}_{m',n'}.$$

- Numerical simulation:

$$\alpha = 1/4, \qquad \nu = 1/3$$

$$\mathcal{C}_{\text{TP}} = 3$$



Summary and Outlook

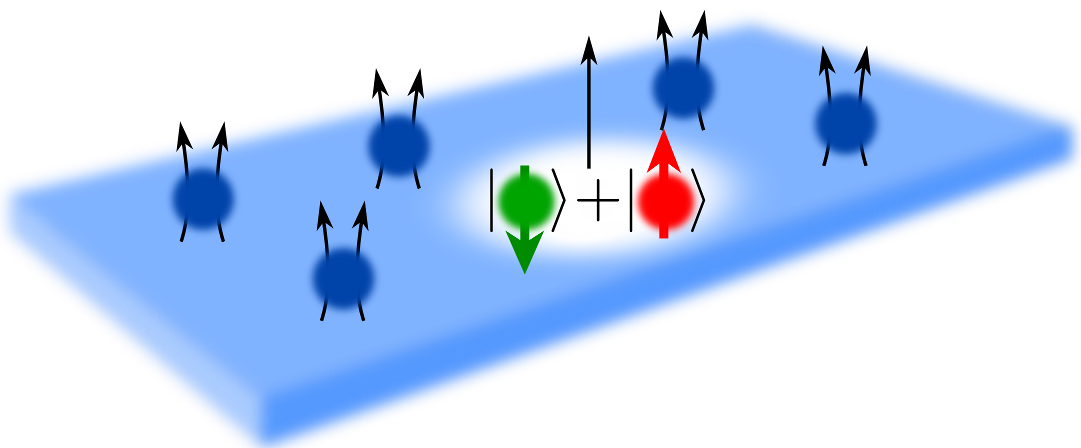
Summary

- Measure geometrical phases:

  Bloch-oscillations + Ramsey interferometry

- Many-body:

  Measure topology of elementary qp excitations

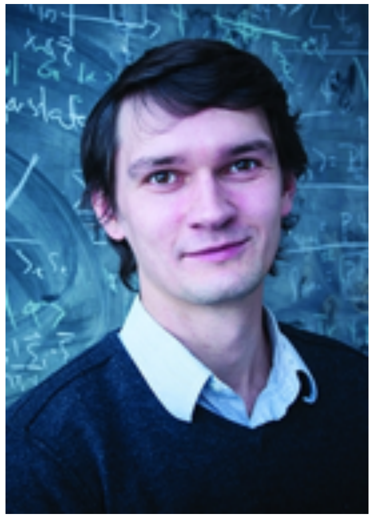


Outlook

- Quantum Spin liquids?
- Investigation of other correlated phases?

…

Eugene Demler
(Harvard)



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(Geneve)

Michael Fleischhauer
(Kaiserslautern)

Norman Yao
(Berkeley)

… and thanks for your attention!