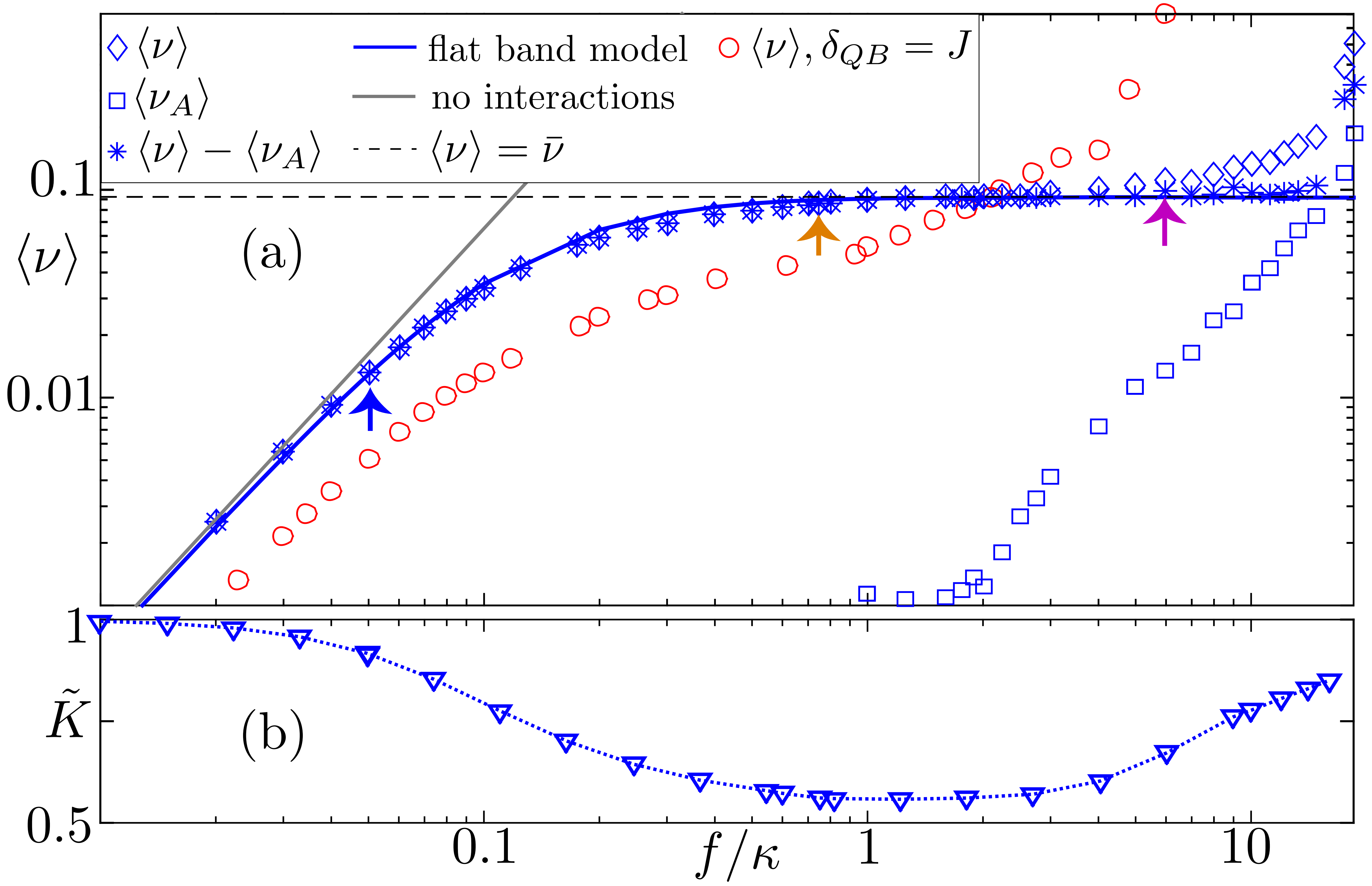
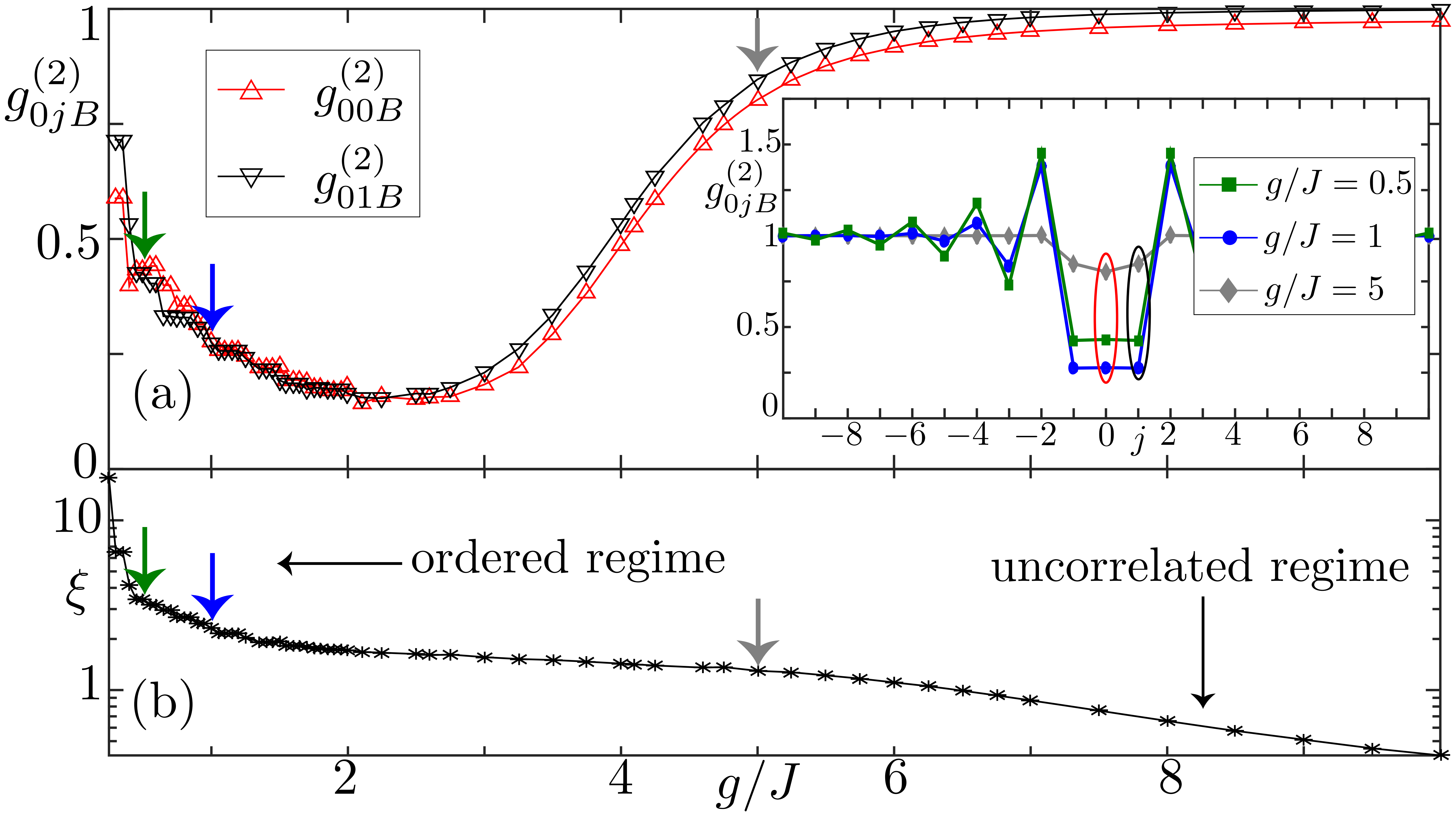
Numerical results: compressibility



Numerical results: dependence on coupling to qubit

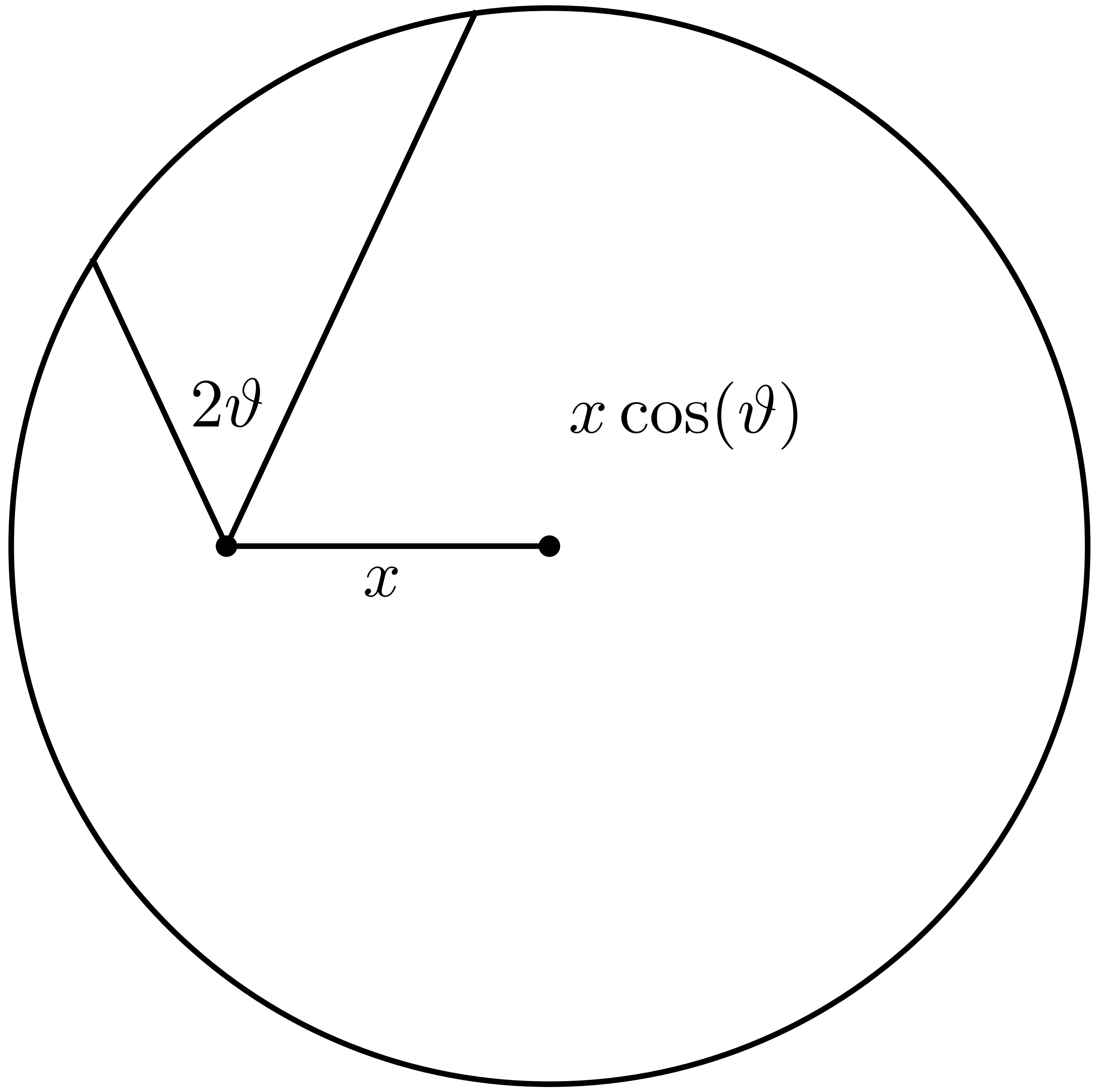


A mechanical “topological insulator”

Roman Süsstrunk
Sebastian Huber

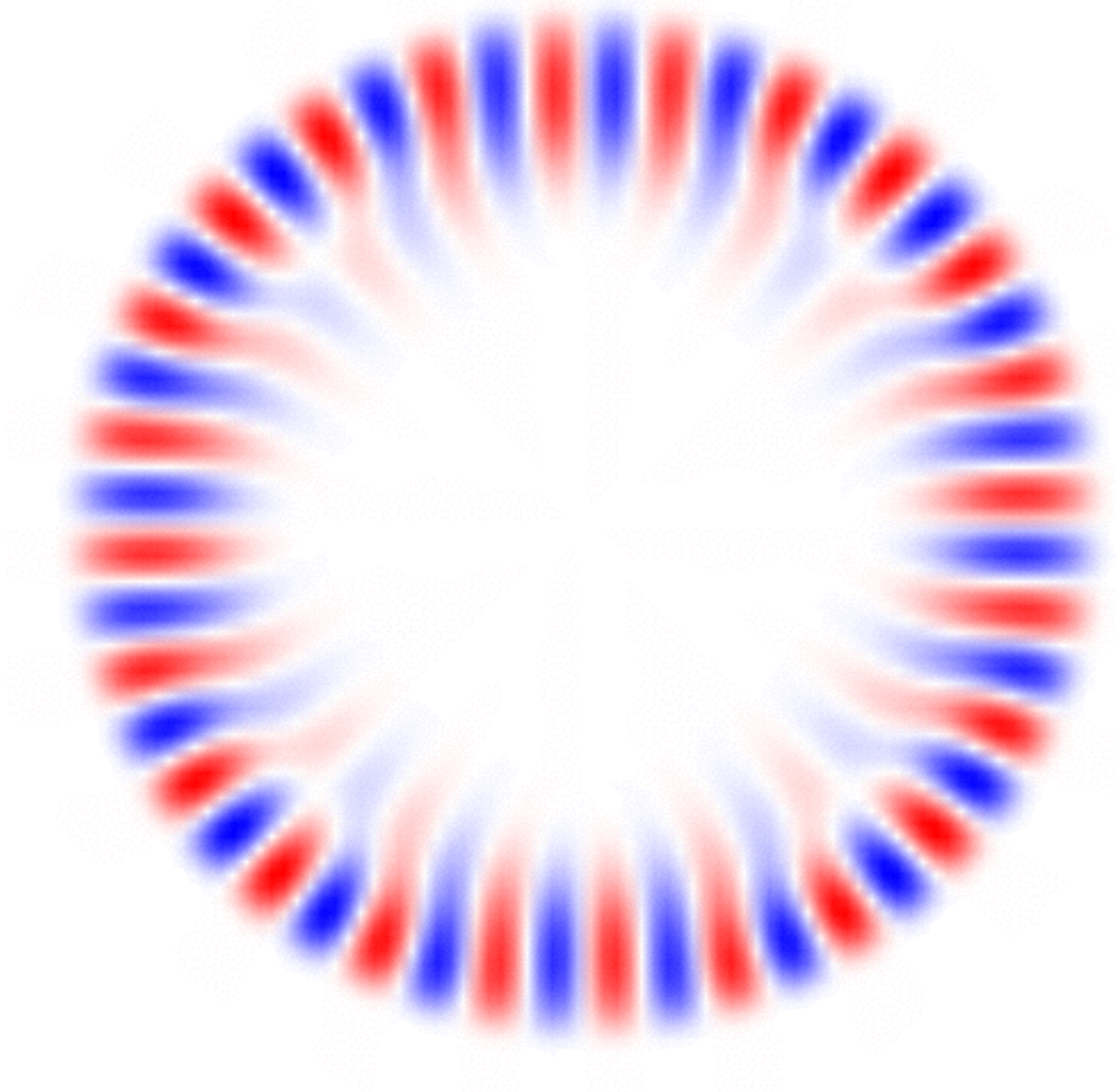
Science **349**, 47 (2015)

Whispering gallery modes



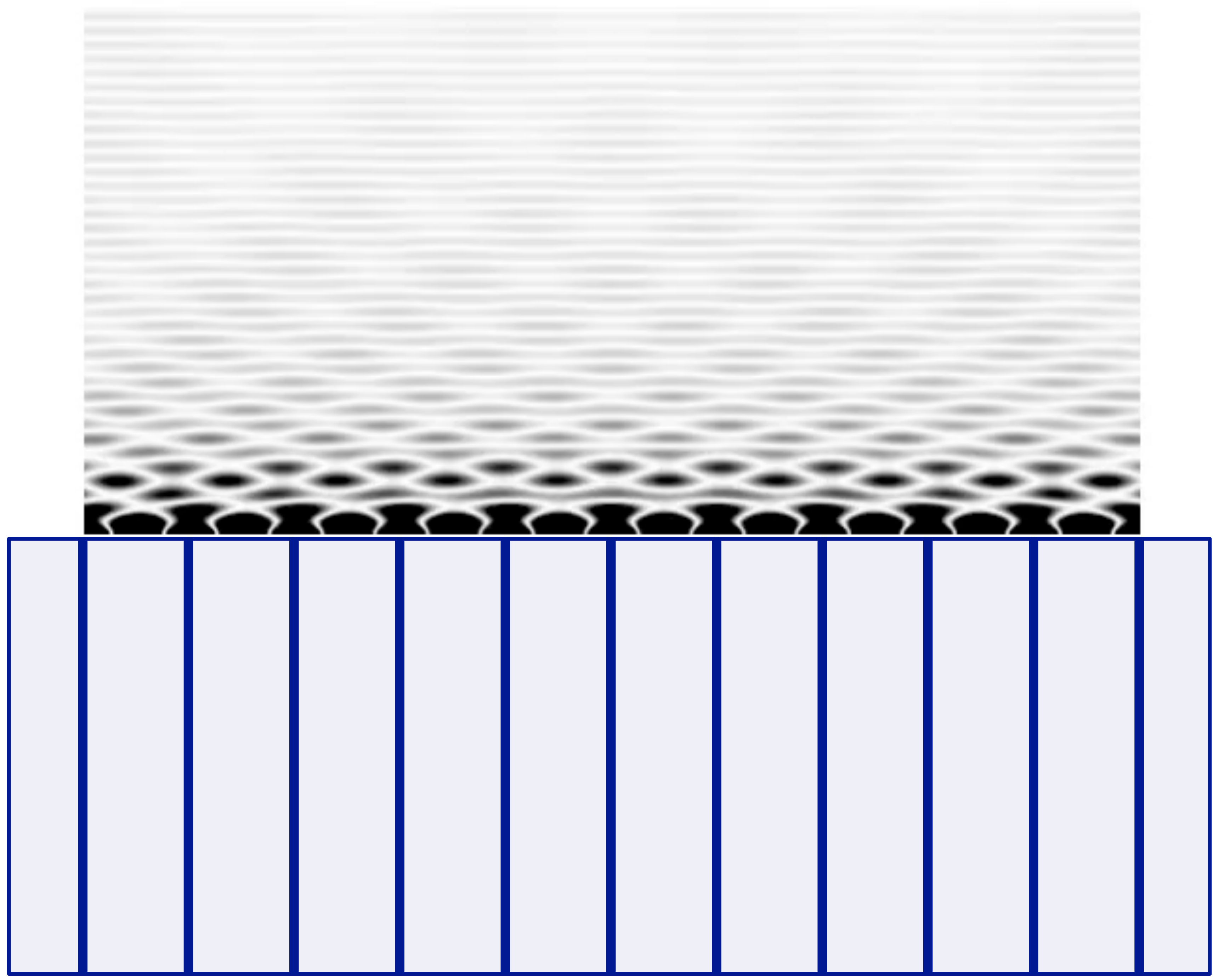
Lord Rayleigh, 1896

$$\psi_n(r,\vartheta) = J_n(kr)\cos(n\vartheta)$$

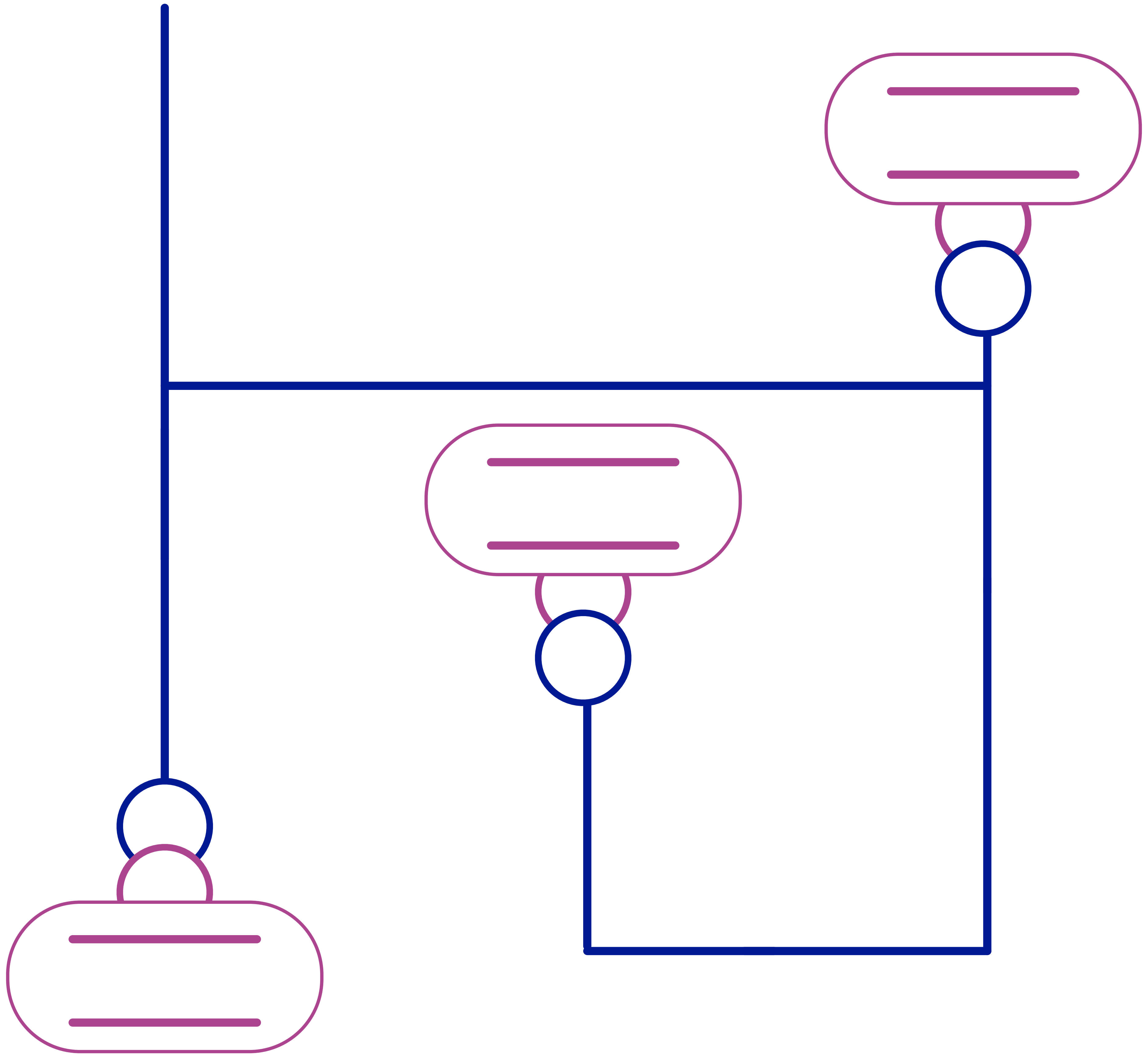


Lord Rayleigh, 1912

Why do we need wave guides (surface states) for phonons?



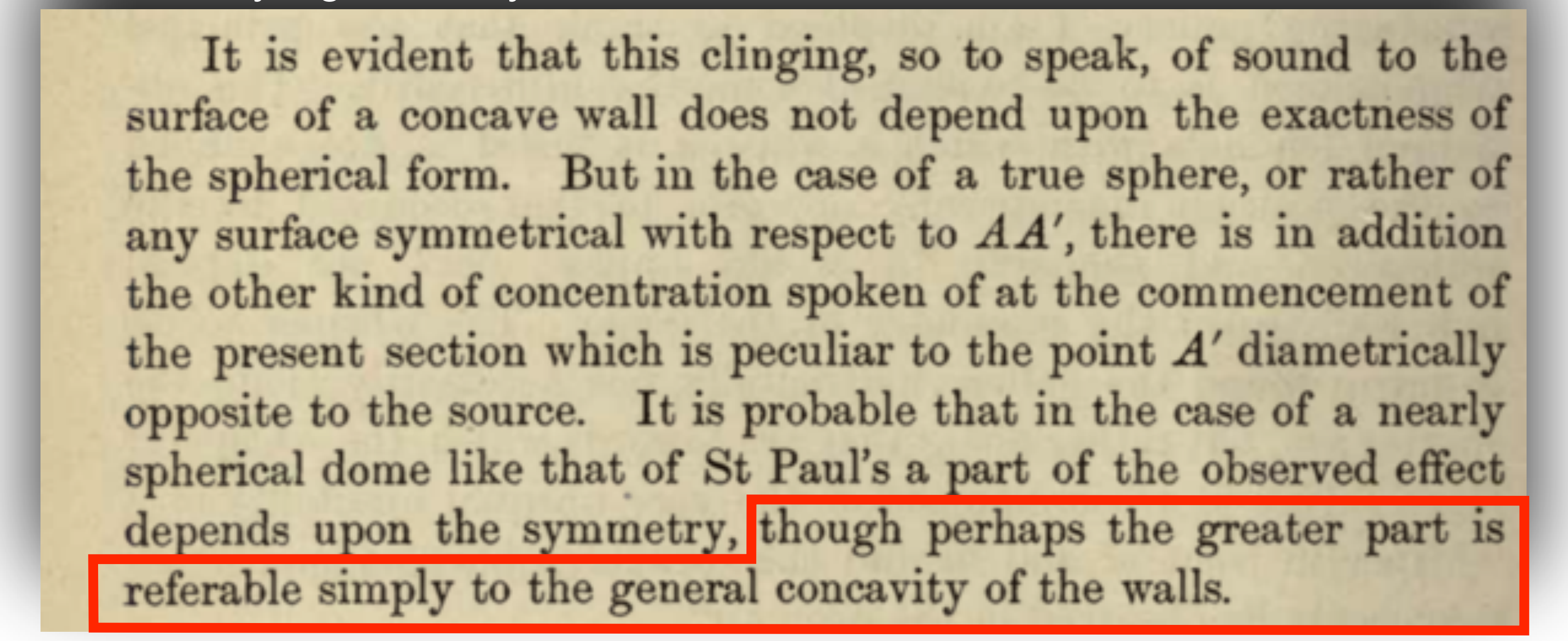
Spadoni et al., PNAS (2010)



Schuetz et al., arXiv (2015)

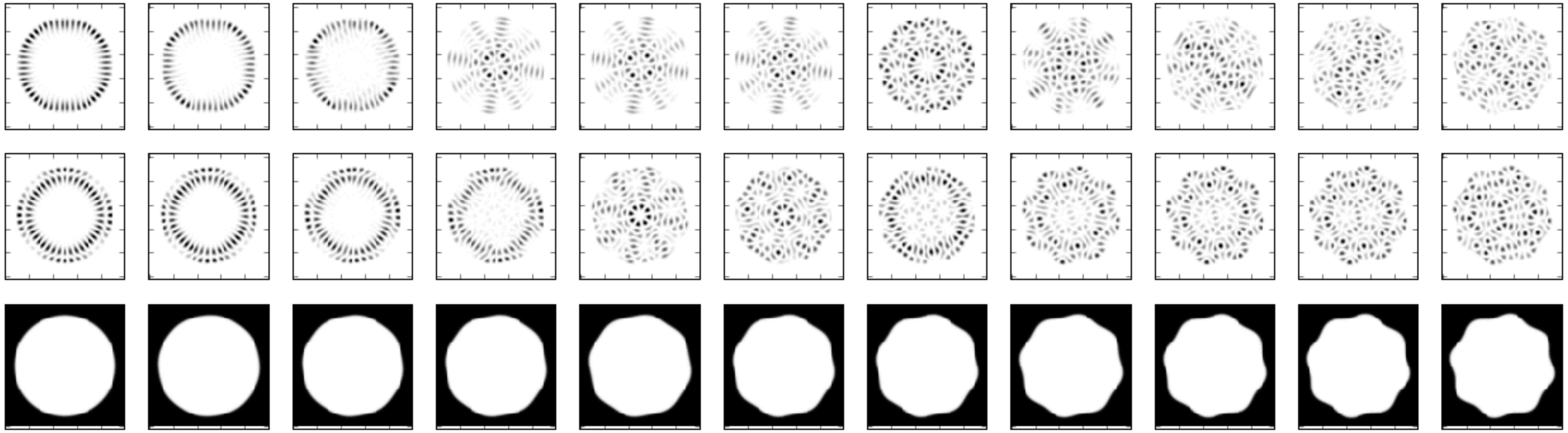
Sensitivity to exact boundary shape

Lord Rayleigh, Theory of Sound, 1896



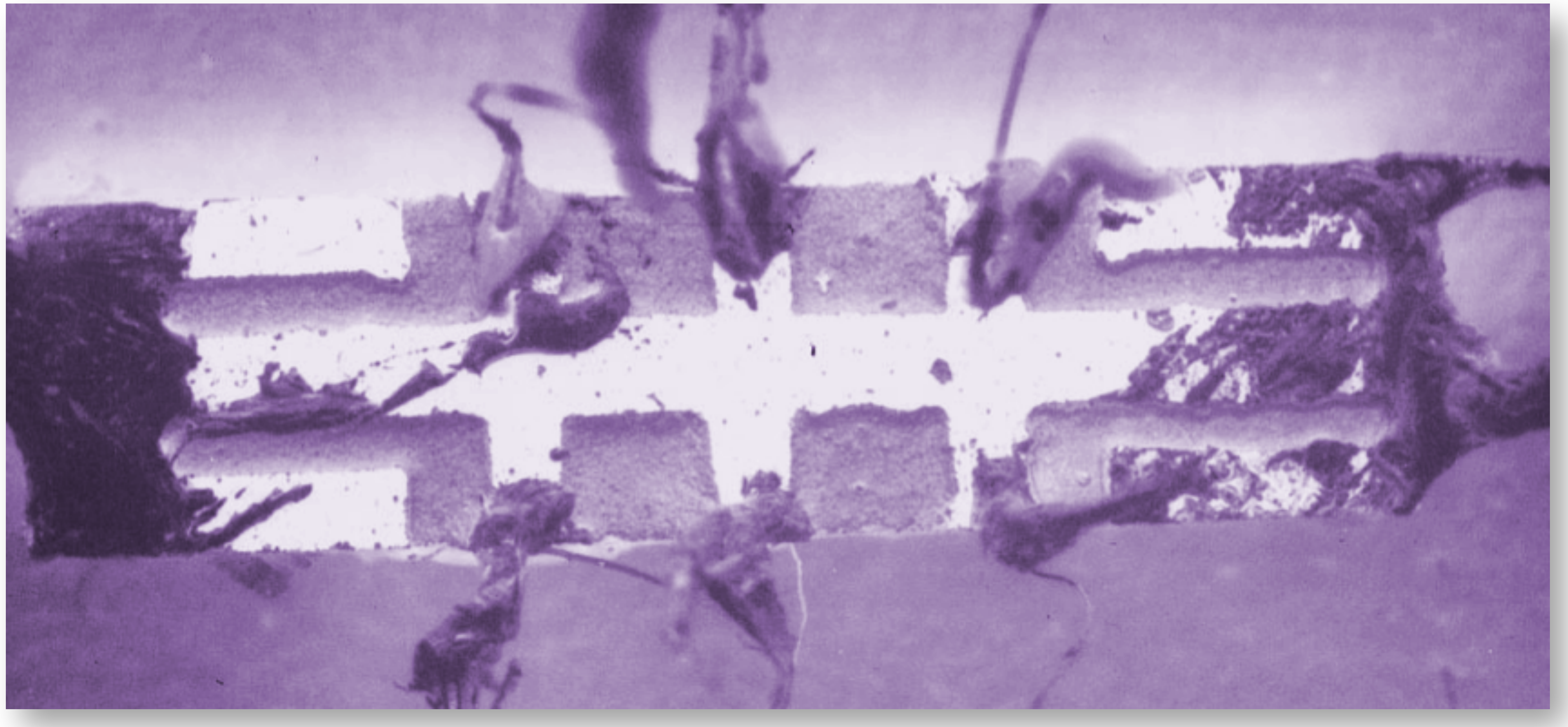
It is evident that this clinging, so to speak, of sound to the surface of a concave wall does not depend upon the exactness of the spherical form. But in the case of a true sphere, or rather of any surface symmetrical with respect to AA', there is in addition the other kind of concentration spoken of at the commencement of the present section which is peculiar to the point A' diametrically opposite to the source. It is probable that in the case of a nearly spherical dome like that of St Paul's a part of the observed effect depends upon the symmetry, though perhaps the greater part is referable simply to the general concavity of the walls.

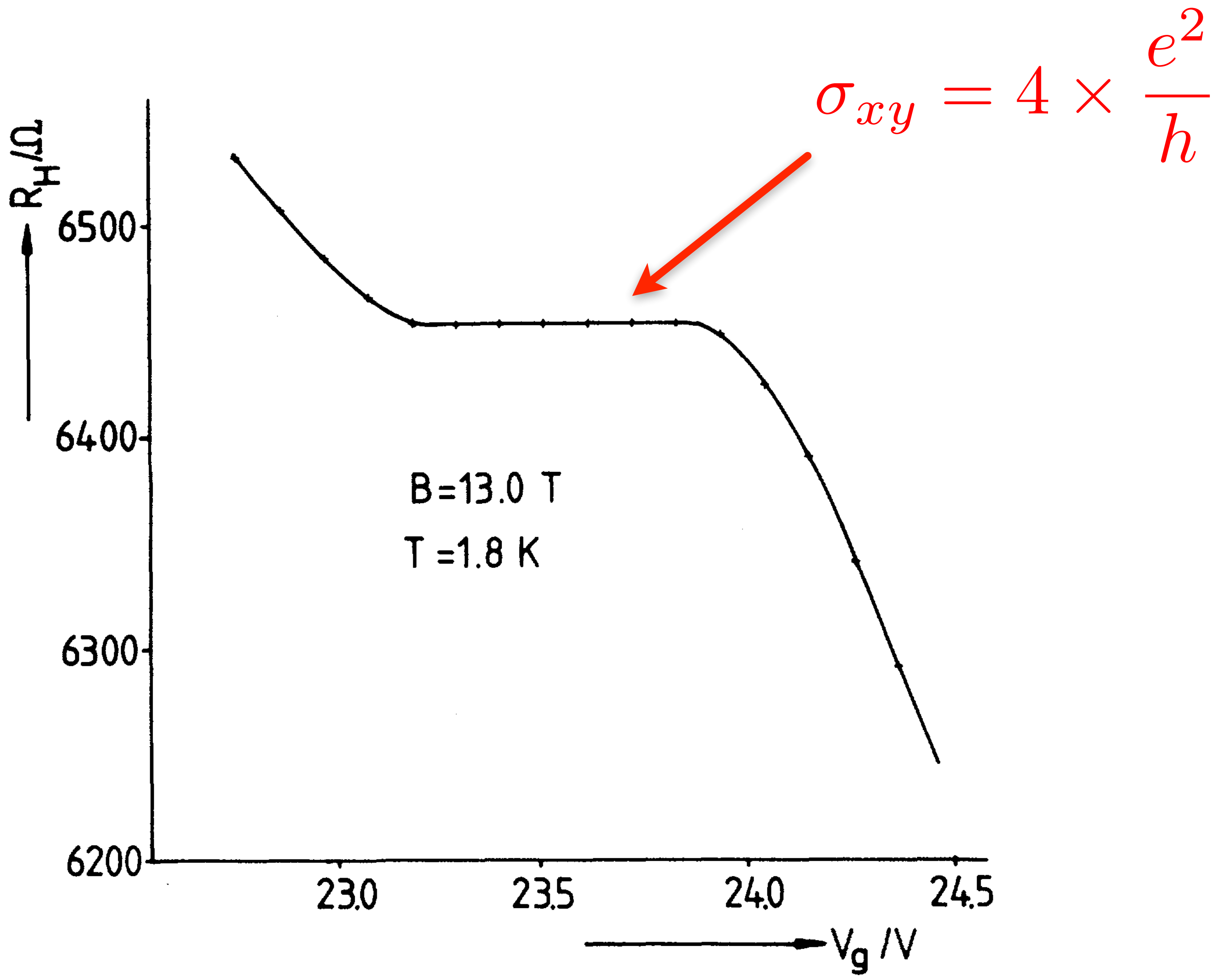
- convexity leads to bulk modes
- no spectral separation



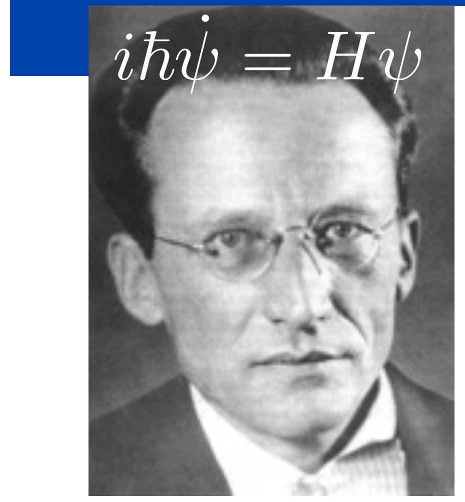
Do we know *stable* surface modes?

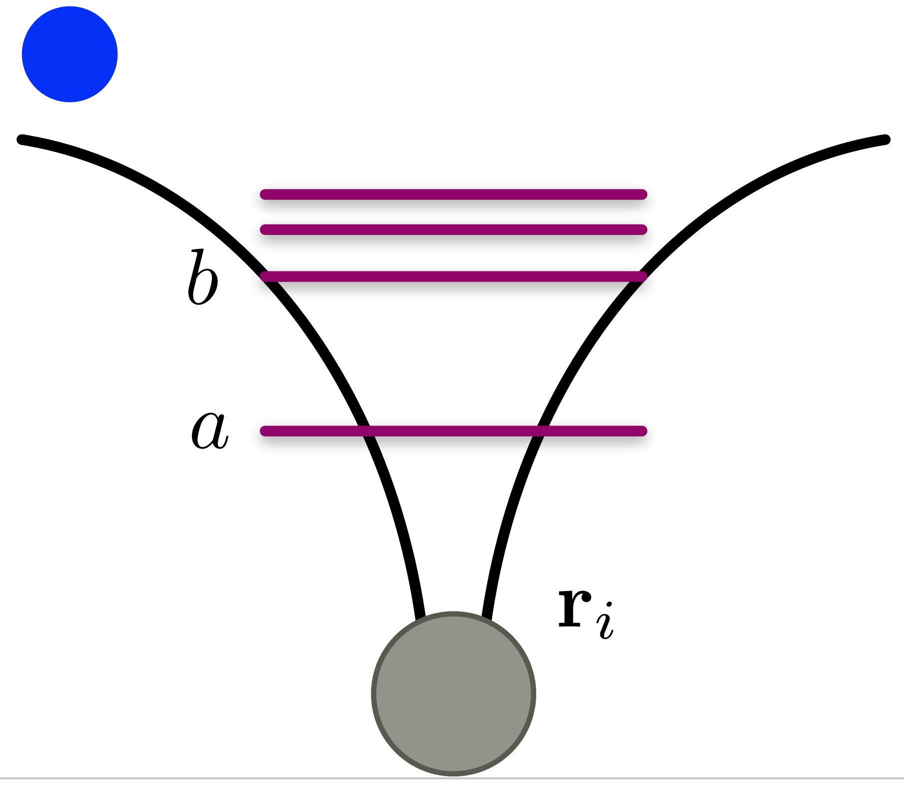
- We need spectral separation of bulk and edge modes: topologically non-trivial fermion systems!



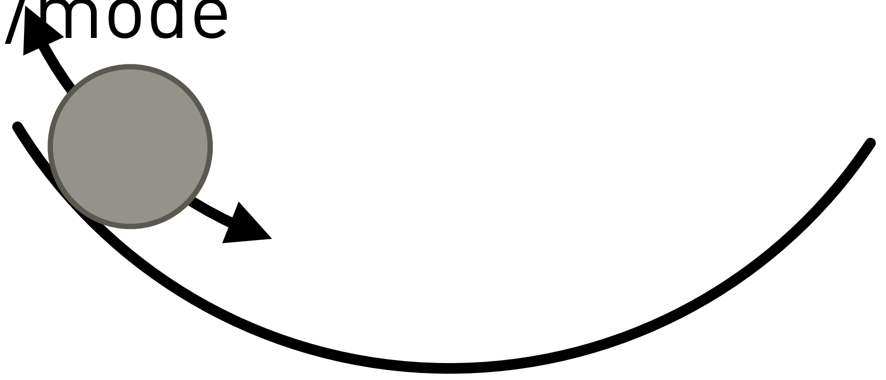


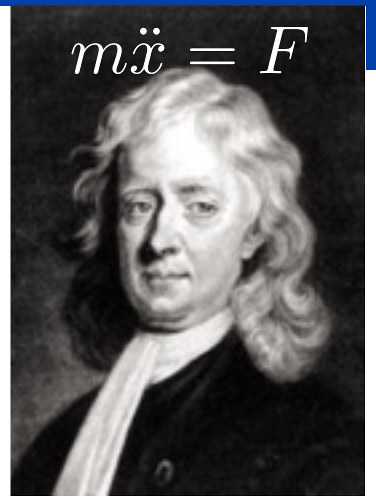
von Klitzing et al, PRL 1980

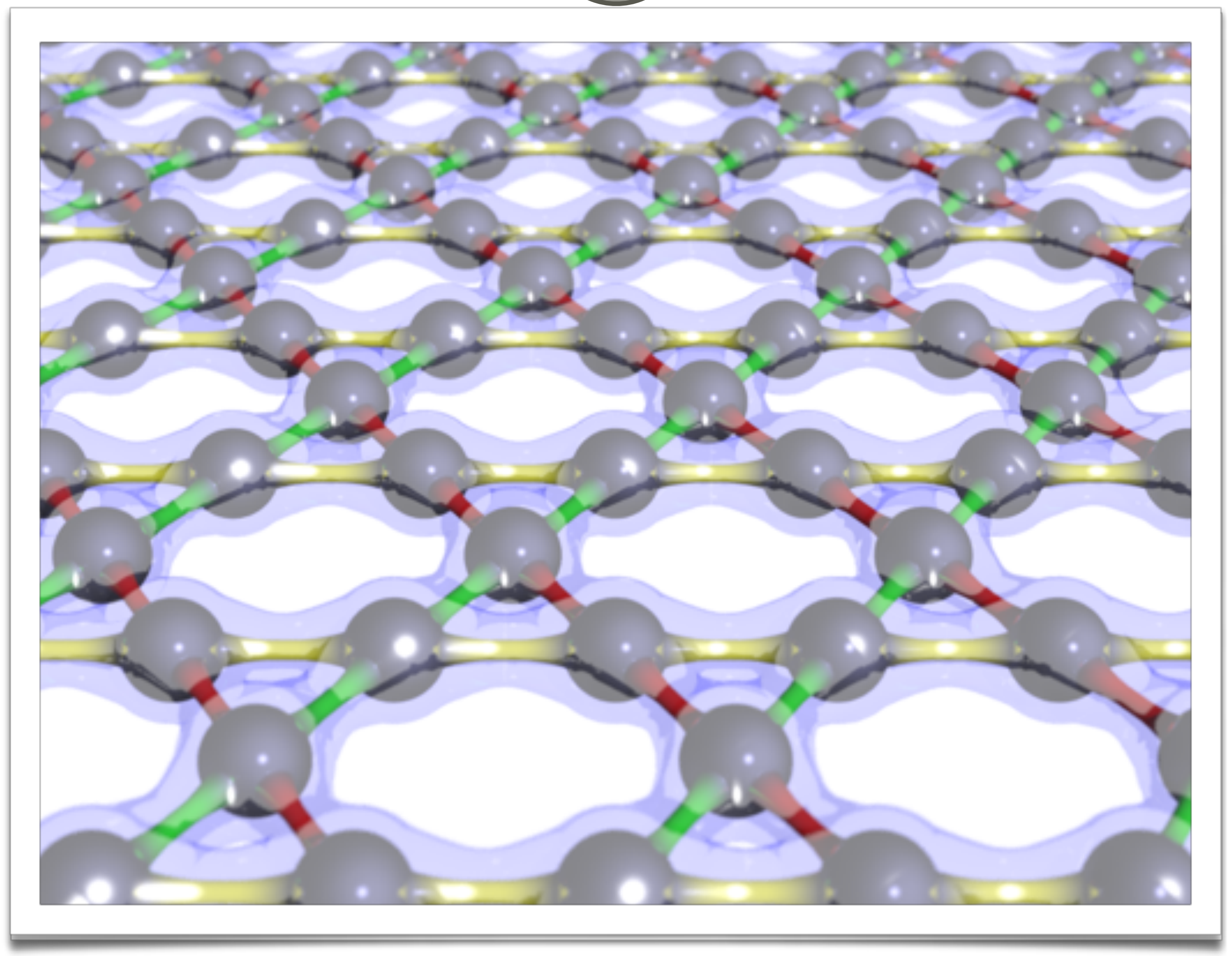




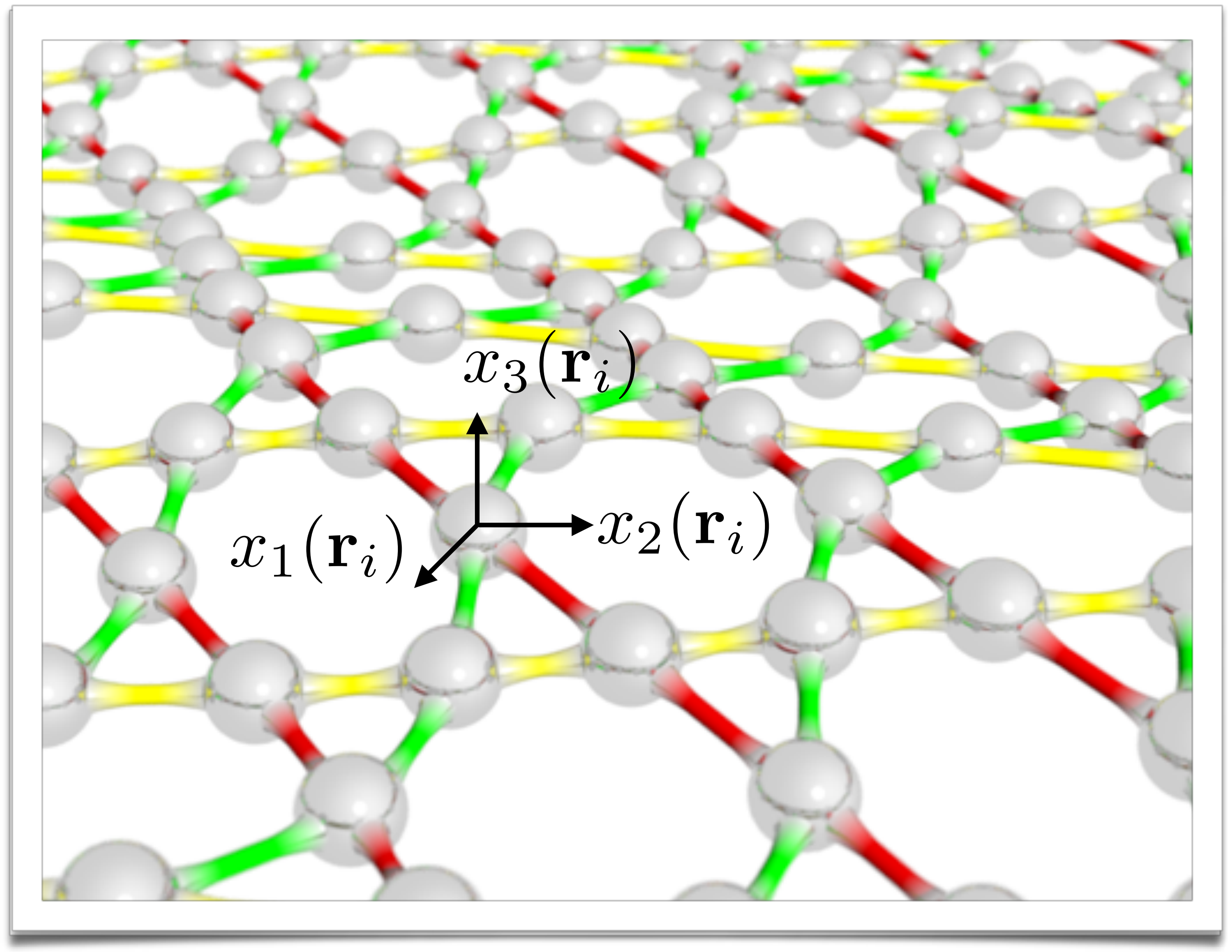
single particle/mode







many modes



$$i\hbar\dot{\psi}_a(\mathbf{r}_i) = \mathcal{H}_{ab}(\mathbf{r}_i, \mathbf{r}_j)\psi_b(\mathbf{r}_j)$$

simple lattice model

$$\ddot{x}_a(\mathbf{r}_i) = -\mathcal{D}_{ab}(\mathbf{r}_i, \mathbf{r}_j)\, x_b(\mathbf{r}_j)$$

Schrödinger vs. Newton

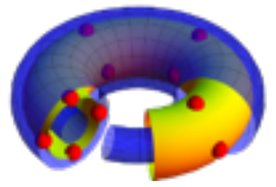
- Schrödinger equation for a lattice model (Hamiltonian)

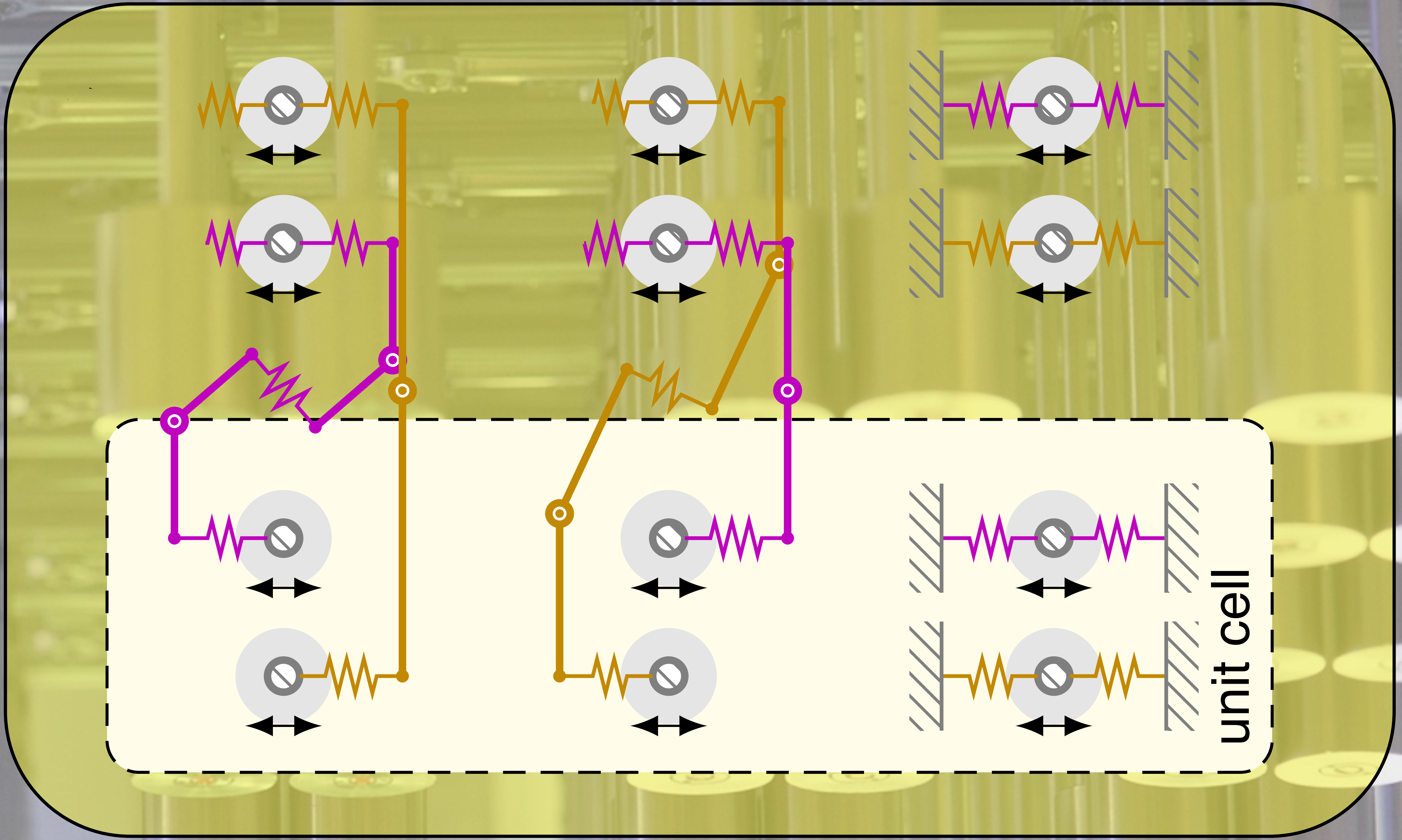
$$i\hbar\dot{\psi}_a(\mathbf{r}_i) = \mathcal{H}_{ab}(\mathbf{r}_i,\mathbf{r}_j)\psi_b(\mathbf{r}_j) \qquad \mathcal{H}_{ab}(\mathbf{r}_i,\mathbf{r}_j) = \mathcal{H}^*_{ba}(\mathbf{r}_j,\mathbf{r}_i)$$

- Newton's equation of motion for coupled lossless mechanical oscillators (Dynamical matrix)

$$\ddot{x}_a(\mathbf{r}_i) = -\mathcal{D}_{ab}(\mathbf{r}_i,\mathbf{r}_j)\,x_b(\mathbf{r}_j) \qquad \mathcal{D}_{ij} \in \mathbb{R}$$

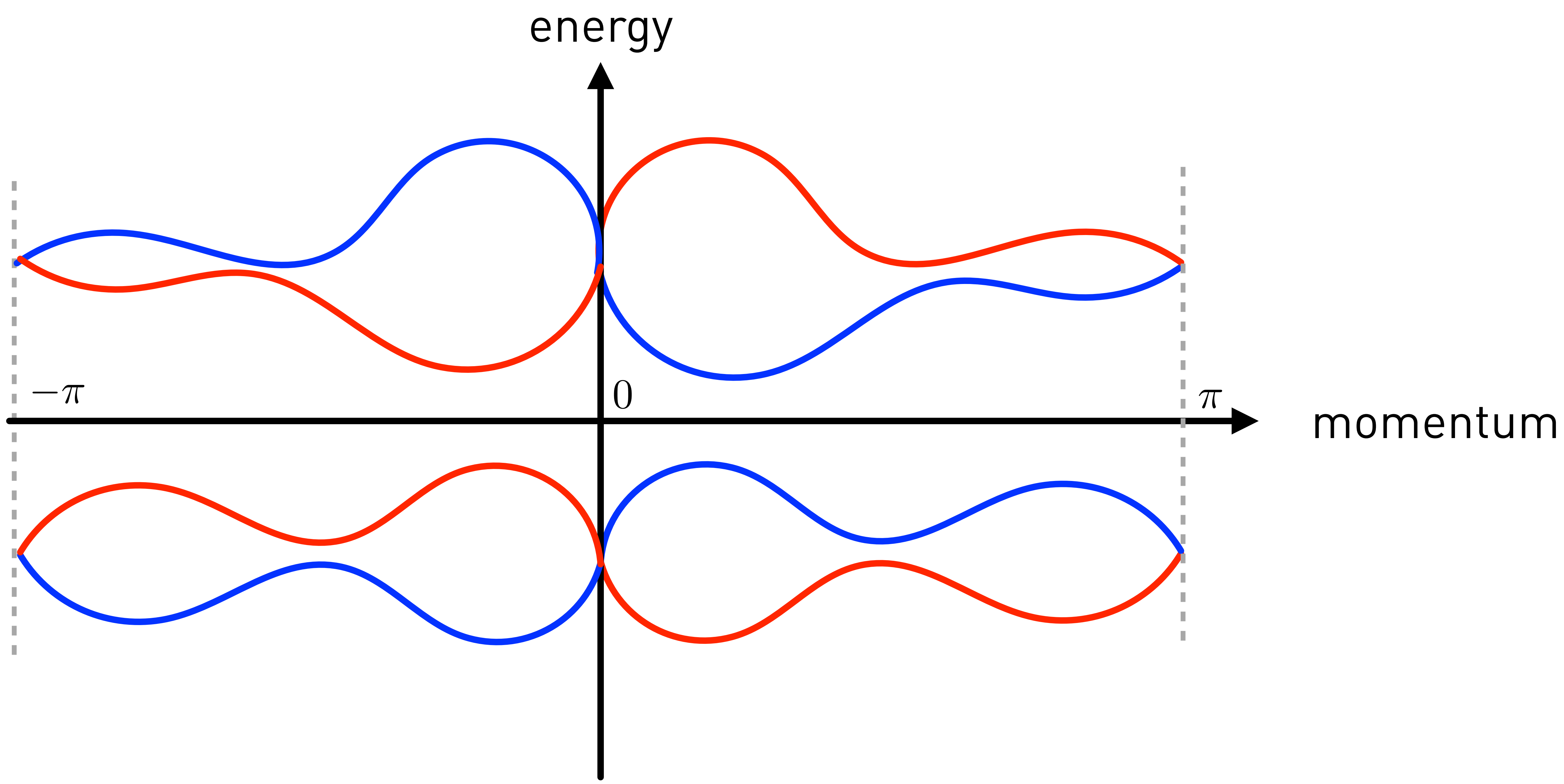
$$\mathcal{D}: \quad \text{positive definite}$$





Implementation with mechanical oscillators

Constraints due to time reversal symmetry, necessary ingredients



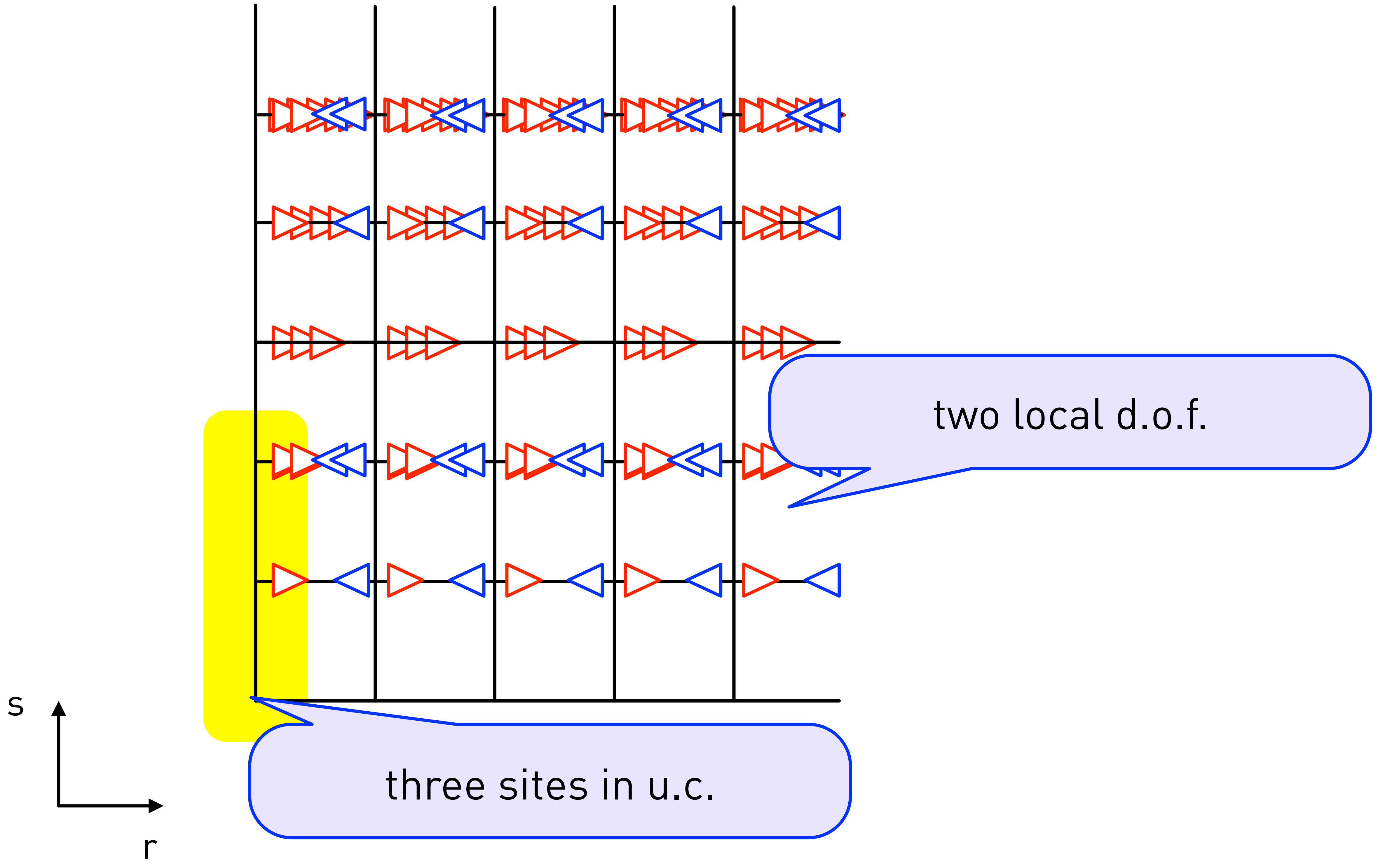
We need four bands: trade local degrees of freedom with a larger unit cell

Local modes should be easy to identify.

Gaps should be large in order to be stable against unavoidable dissipation.

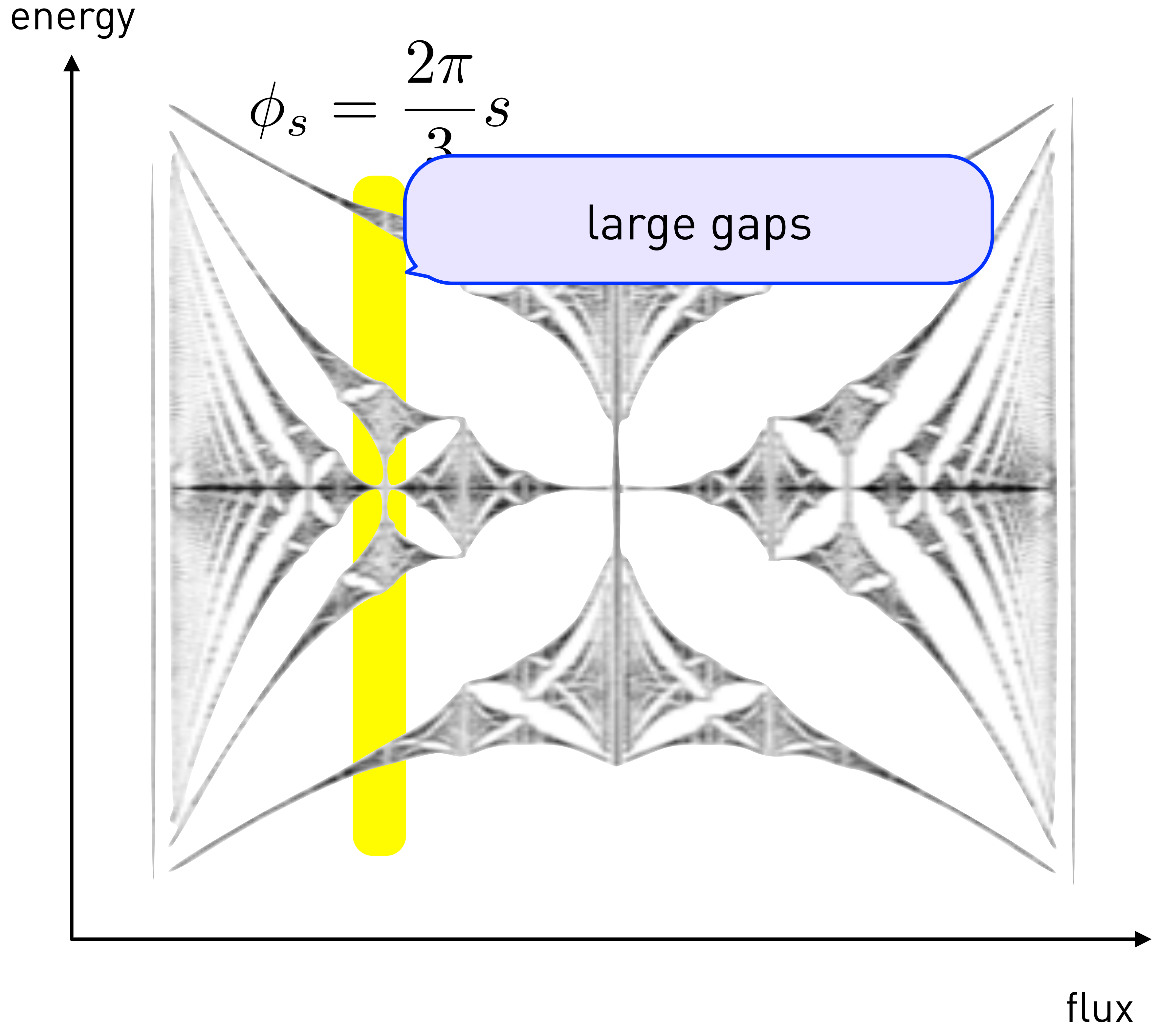
The doubled 1/3-Hofstadter problem

$$\mathcal{H} = f \sum_{r,s,\alpha=\pm} |r,s,\alpha\rangle\langle r,s\pm 1,\alpha| + |r,s,\alpha\rangle\langle r\pm 1,s,\alpha| e^{\pm i\alpha\phi_s}.$$



two local d.o.f.

three sites in u.c.



$\phi_s = \frac{2\pi}{3}s$

large gaps

Hofstadter, PRB (1976)
Hafezi et al., Nature Photonics (2013)

Implementation with mechanical oscillators

- So far, the Hamiltonian is still complex:

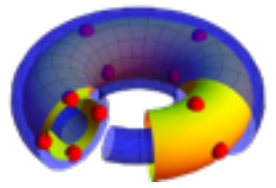
$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_\Phi & 0 \\ 0 & \mathcal{H}_{-\Phi} \end{pmatrix} = \begin{pmatrix} \mathcal{H}_\Phi & 0 \\ 0 & \mathcal{H}_\Phi^* \end{pmatrix}$$

- Go to a new basis: combine local Kramers pairs

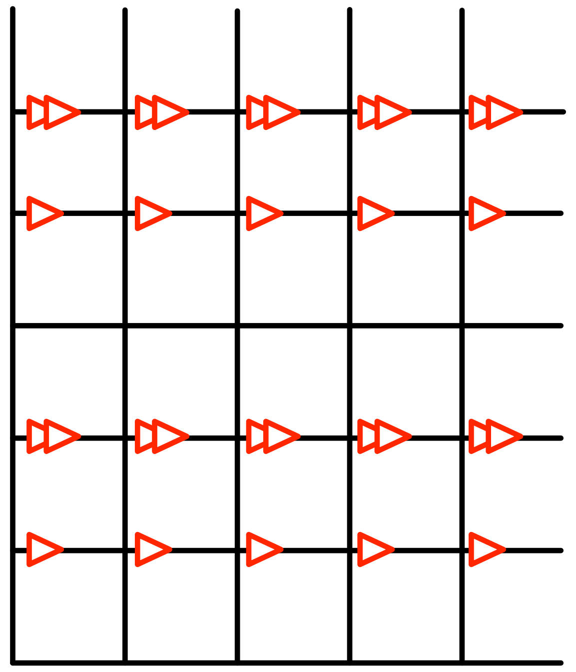
$$\begin{pmatrix} x_{r,s} \\ y_{r,s} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} \psi^+_{r,s} \\ \psi^-_{r,s} \end{pmatrix}$$

The result can be interpreted as a dynamical matrix

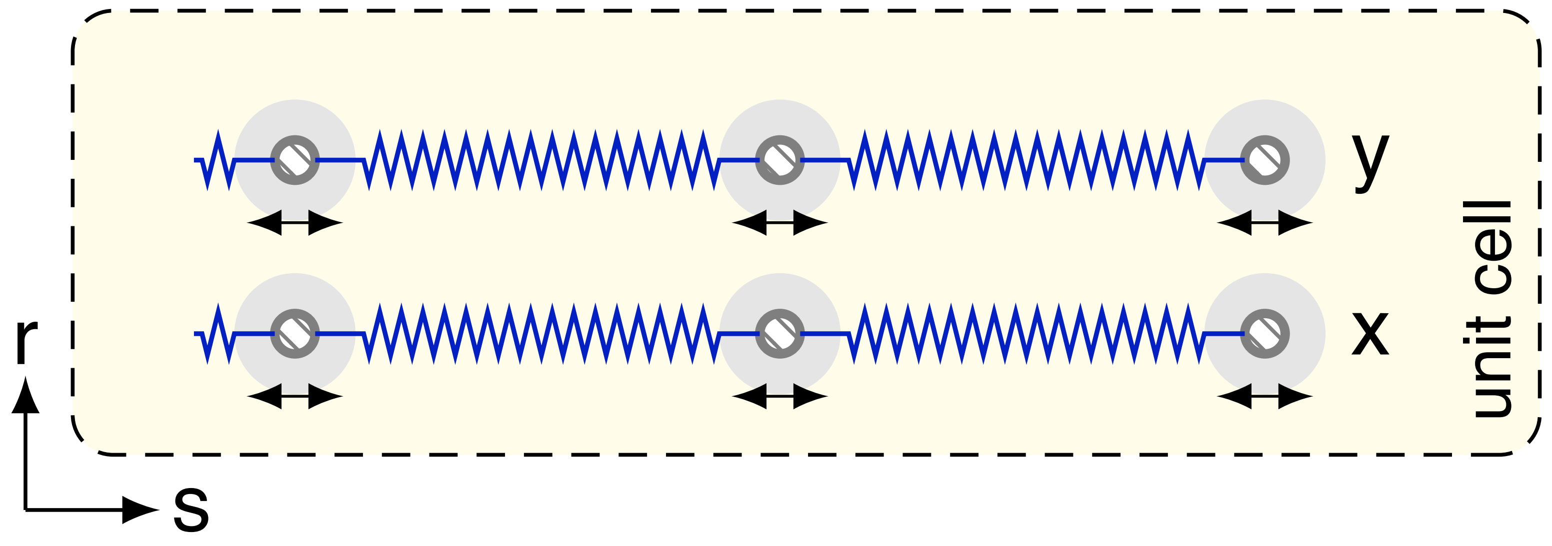
$$U^\dagger \mathcal{H} U = \mathcal{D} = \begin{pmatrix} \mathrm{Re}\mathcal{H}_\Phi & \mathrm{Im}\mathcal{H}_\Phi \\ \mathrm{Im}\mathcal{H}_\Phi & \mathrm{Re}\mathcal{H}_\Phi \end{pmatrix}$$

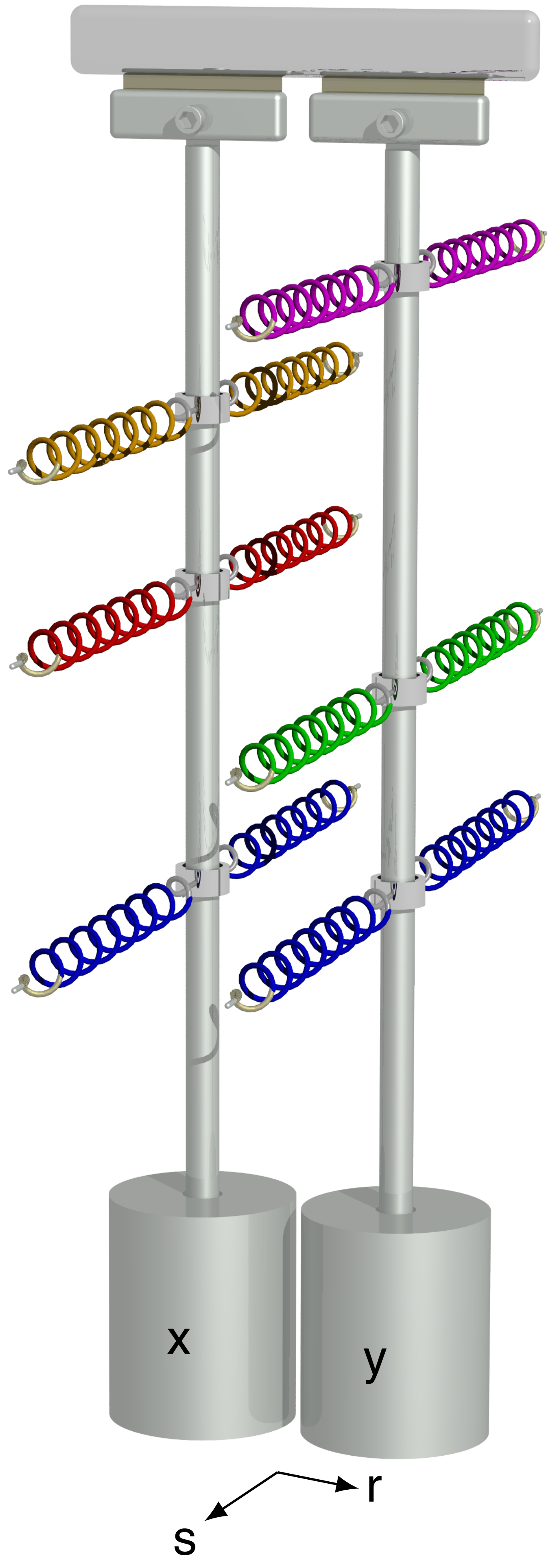


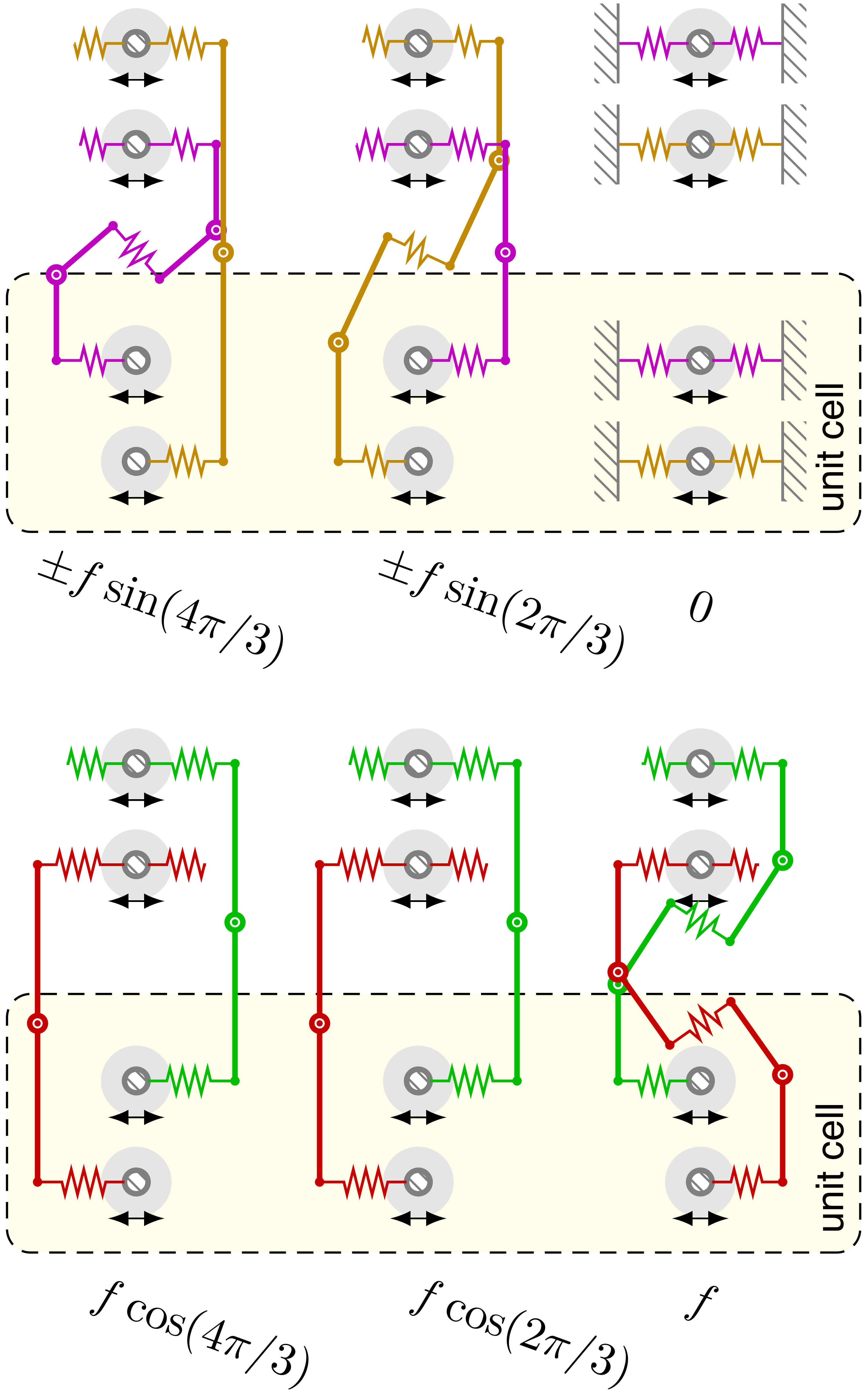
Our implementation

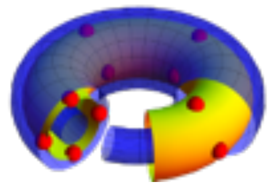


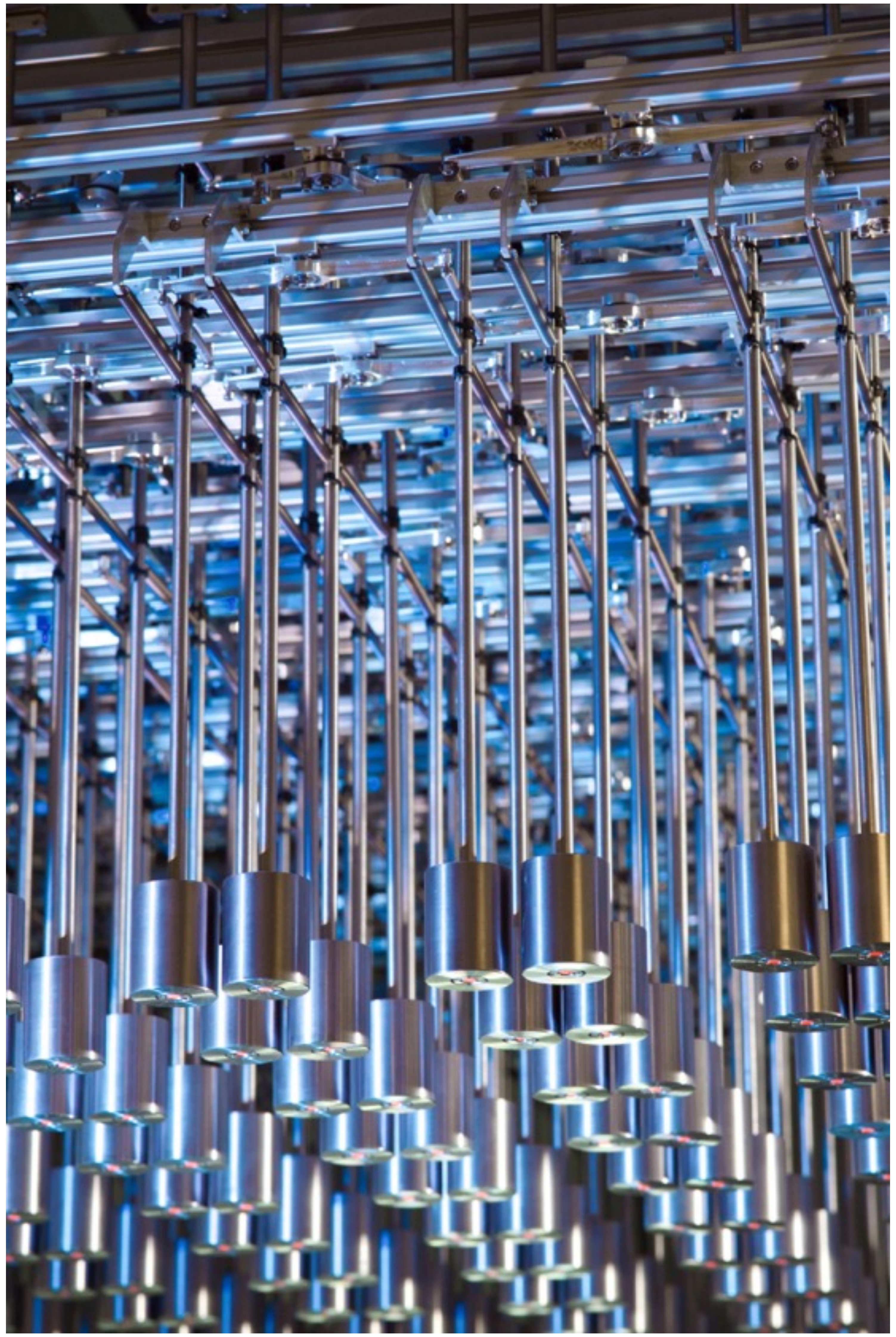
$$\mathcal{D} = \begin{array}{cc} \begin{array}{cc} \mathrm{x} & \mathrm{y} \\ \Downarrow & \Downarrow \end{array} & \\ \begin{pmatrix} \mathrm{Re}\mathcal{H}_\Phi & \mathrm{Im}\mathcal{H}_\Phi \\ \mathrm{Im}\mathcal{H}_\Phi & \mathrm{Re}\mathcal{H}_\Phi \end{pmatrix} & \begin{array}{c} \Leftarrow \mathrm{x} \\ \Leftarrow \mathrm{y} \end{array} \end{array}$$

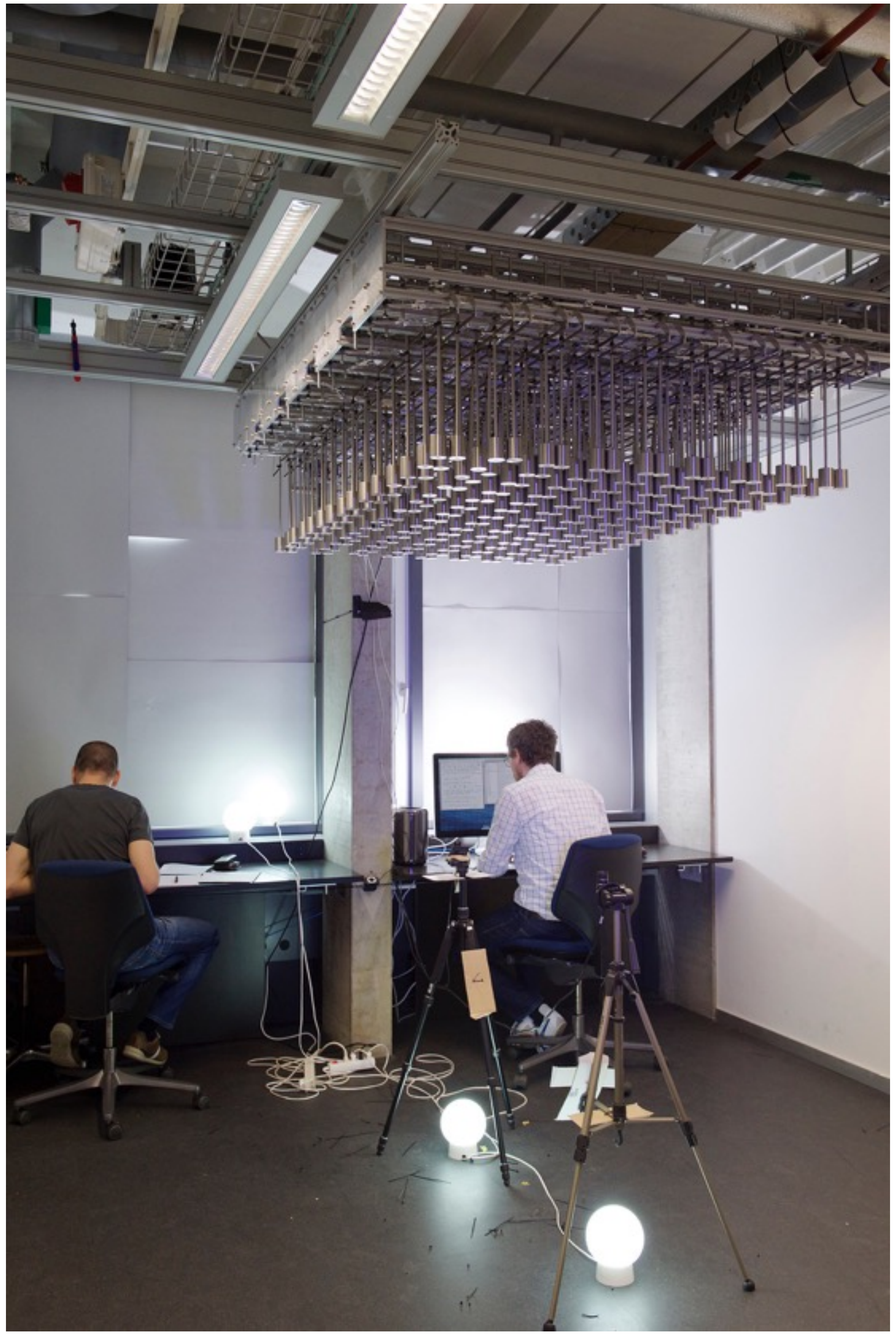












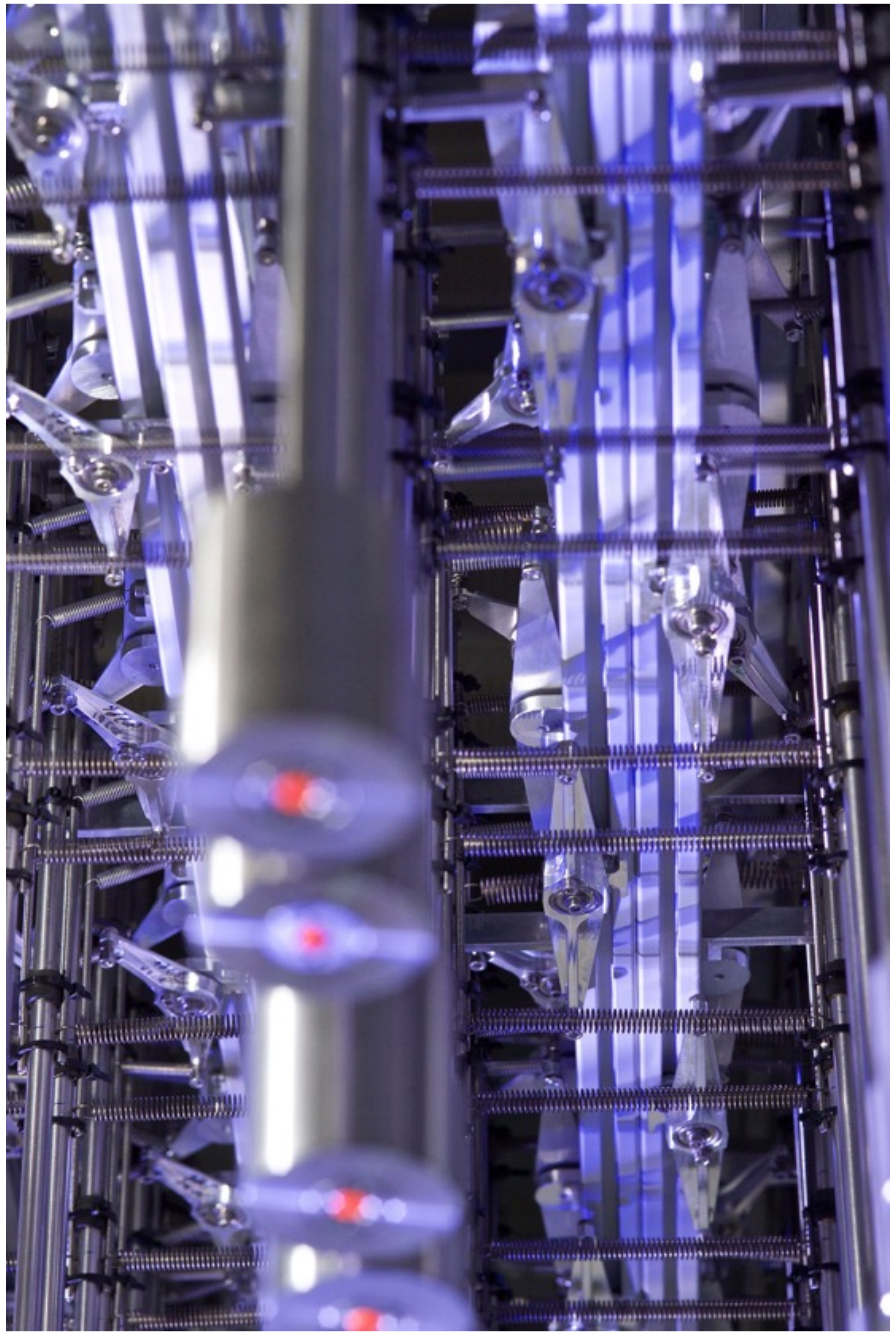
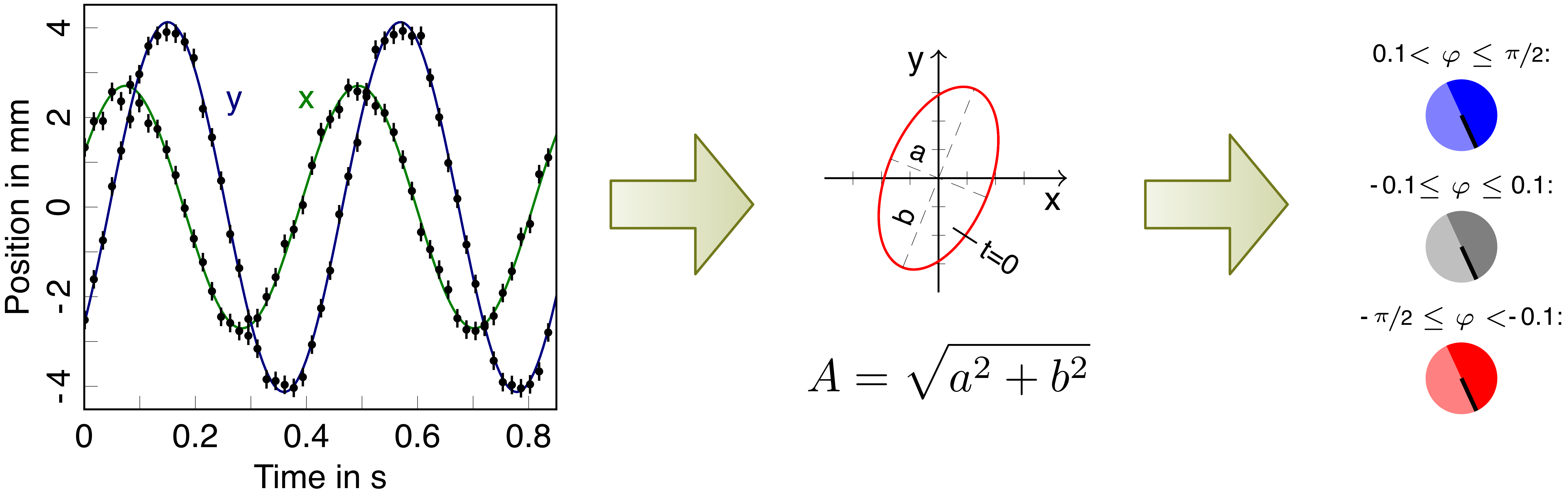


Illustration of couplings

Results

Driving and bare data

- Drive: force position of two local pendula
- Analysis: track all positions



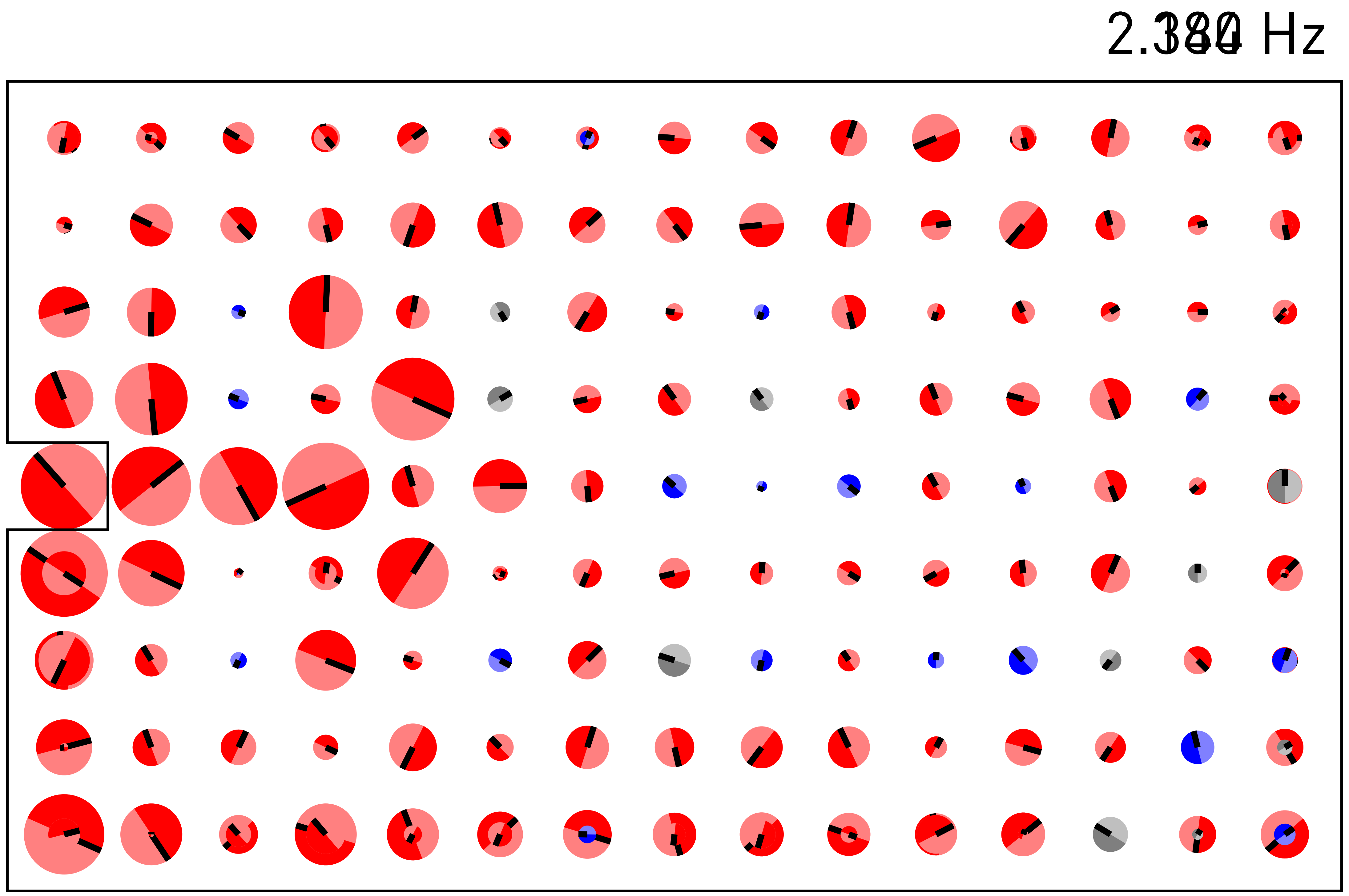
$$A = \sqrt{a^2 + b^2}$$

$0.1 < \varphi \leq \pi/2$:

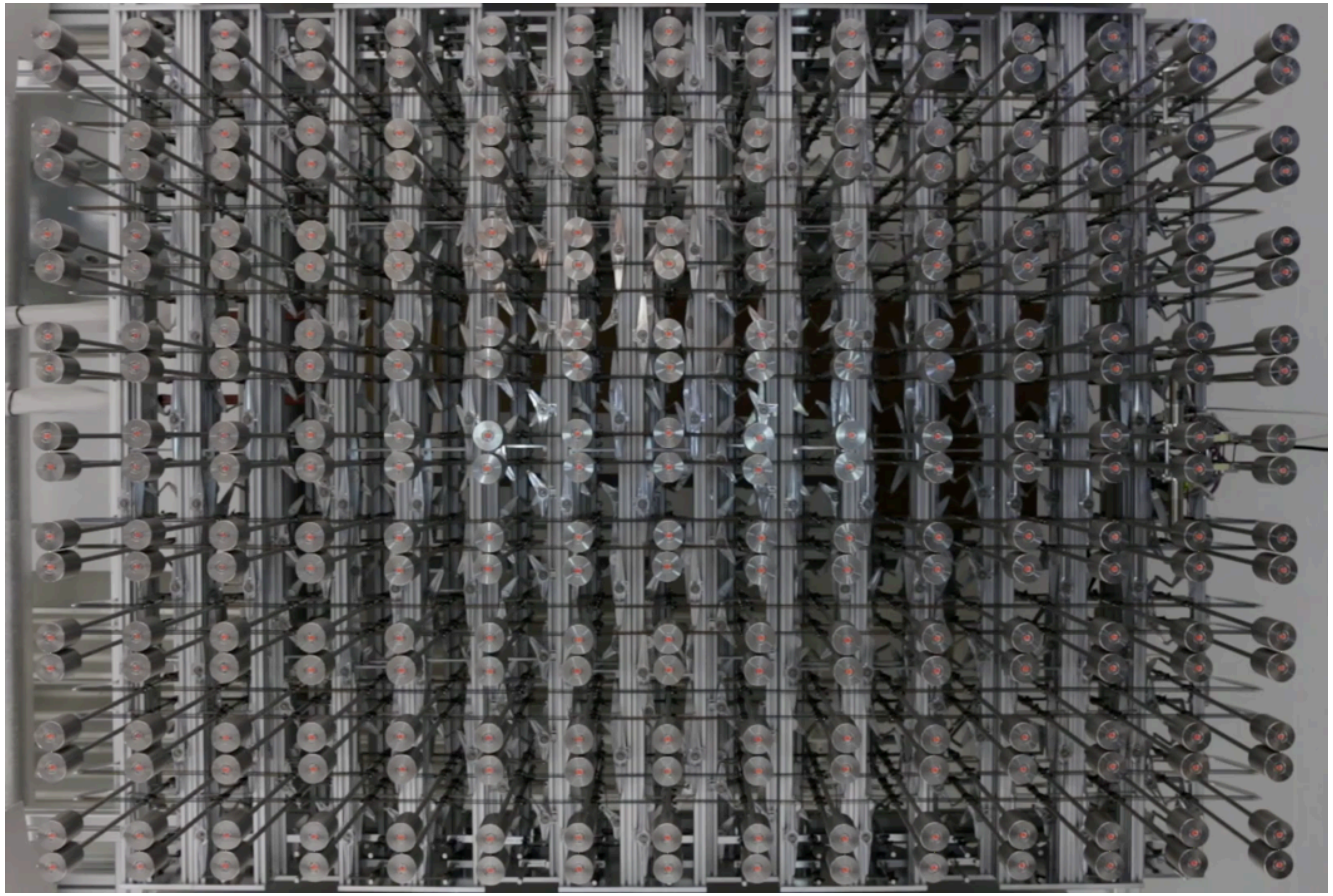
$-0.1 \leq \varphi \leq 0.1$:

$-\pi/2 \leq \varphi < -0.1$:

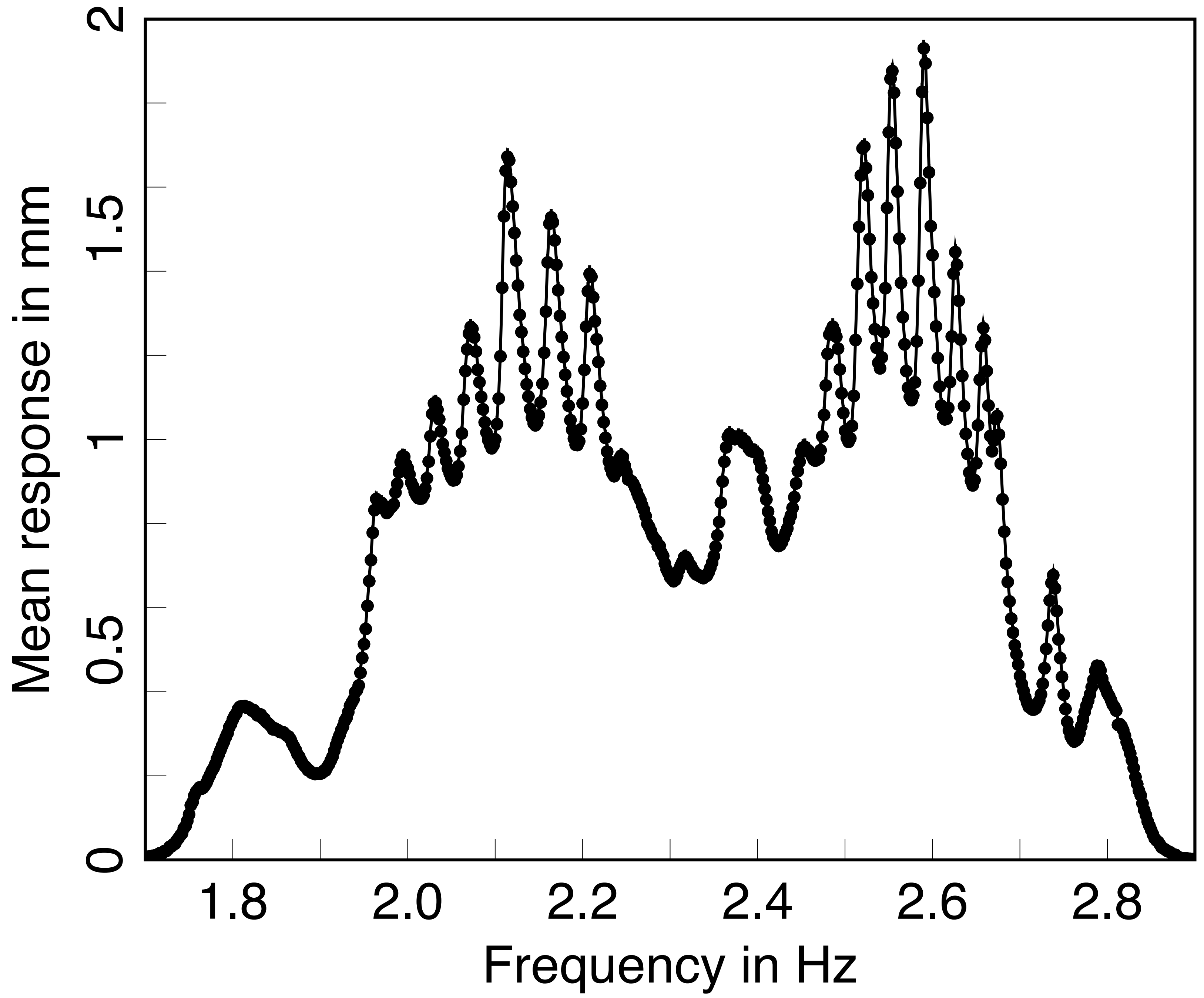
Steady state modes

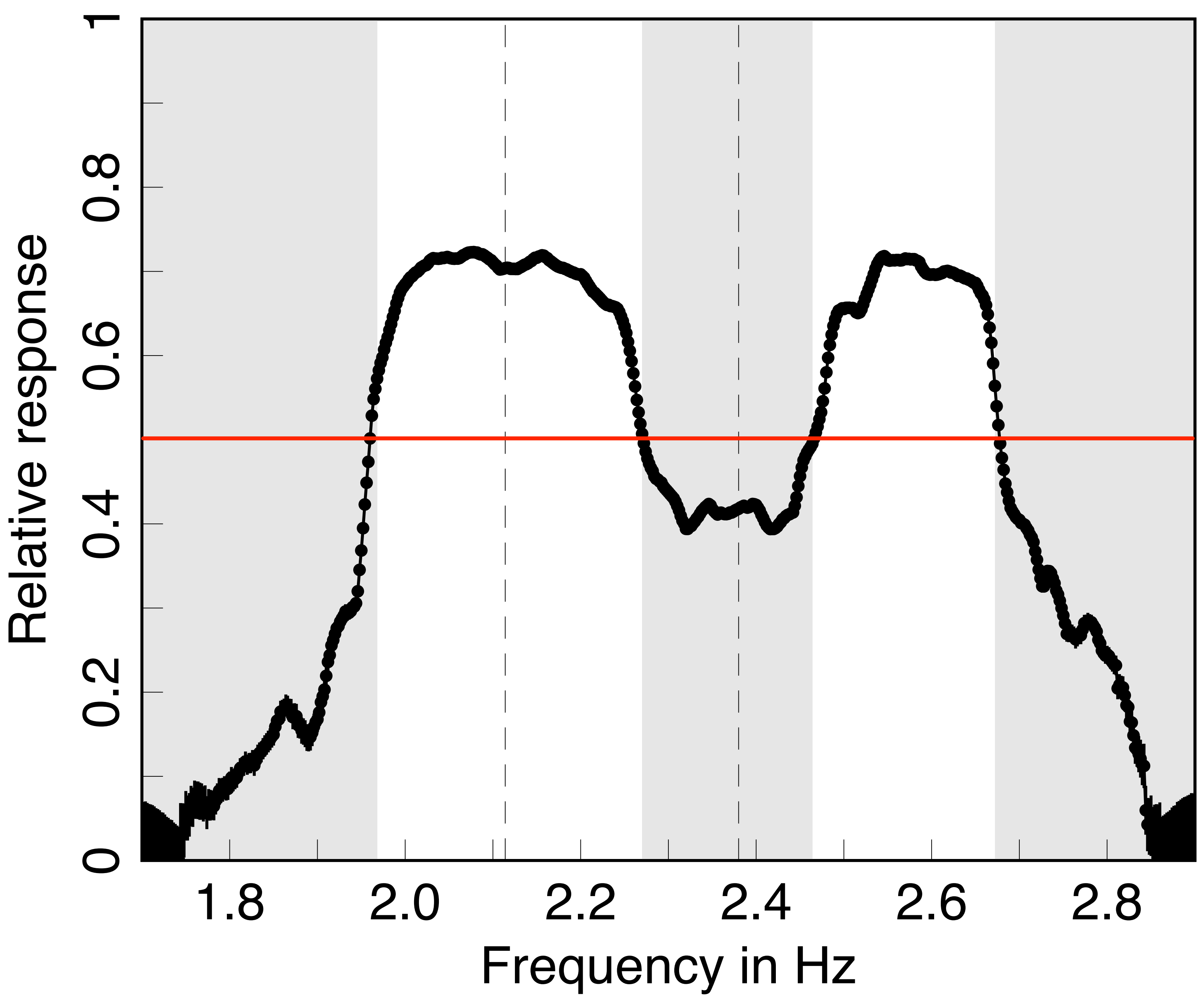


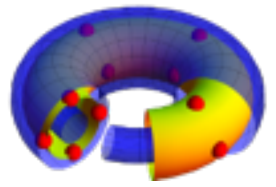
Driving into steady state



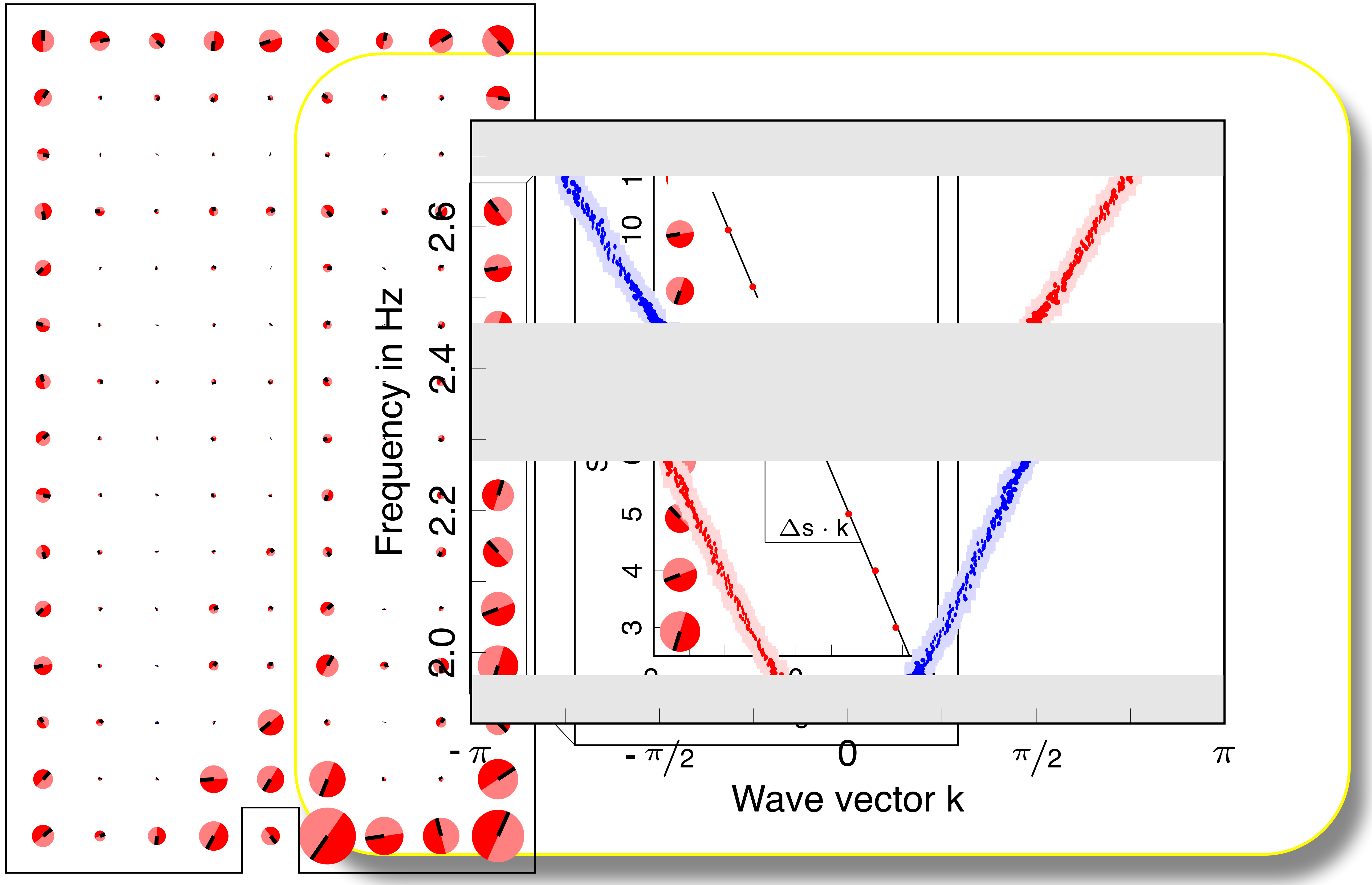
Steady state spectra



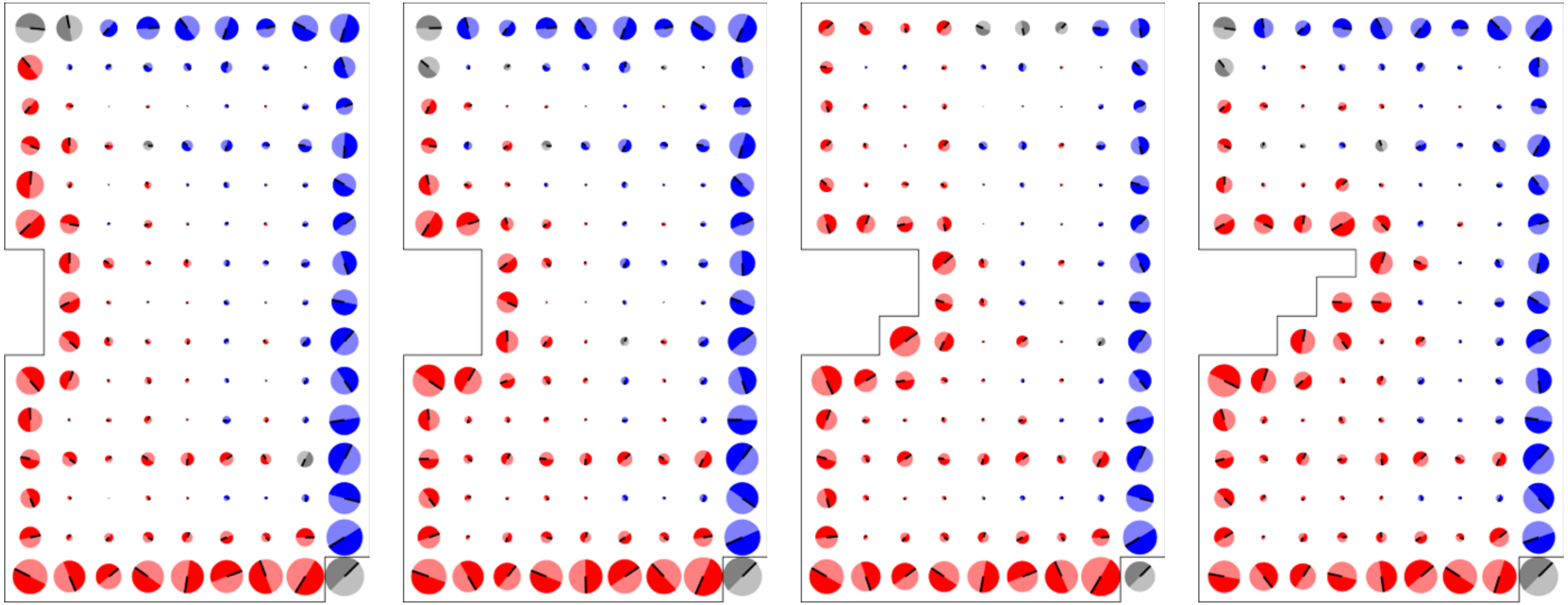




The main result: Helical edge spectrum



Not “just” a whispering gallery mode



What is it, what isn’t it, what is it good for?

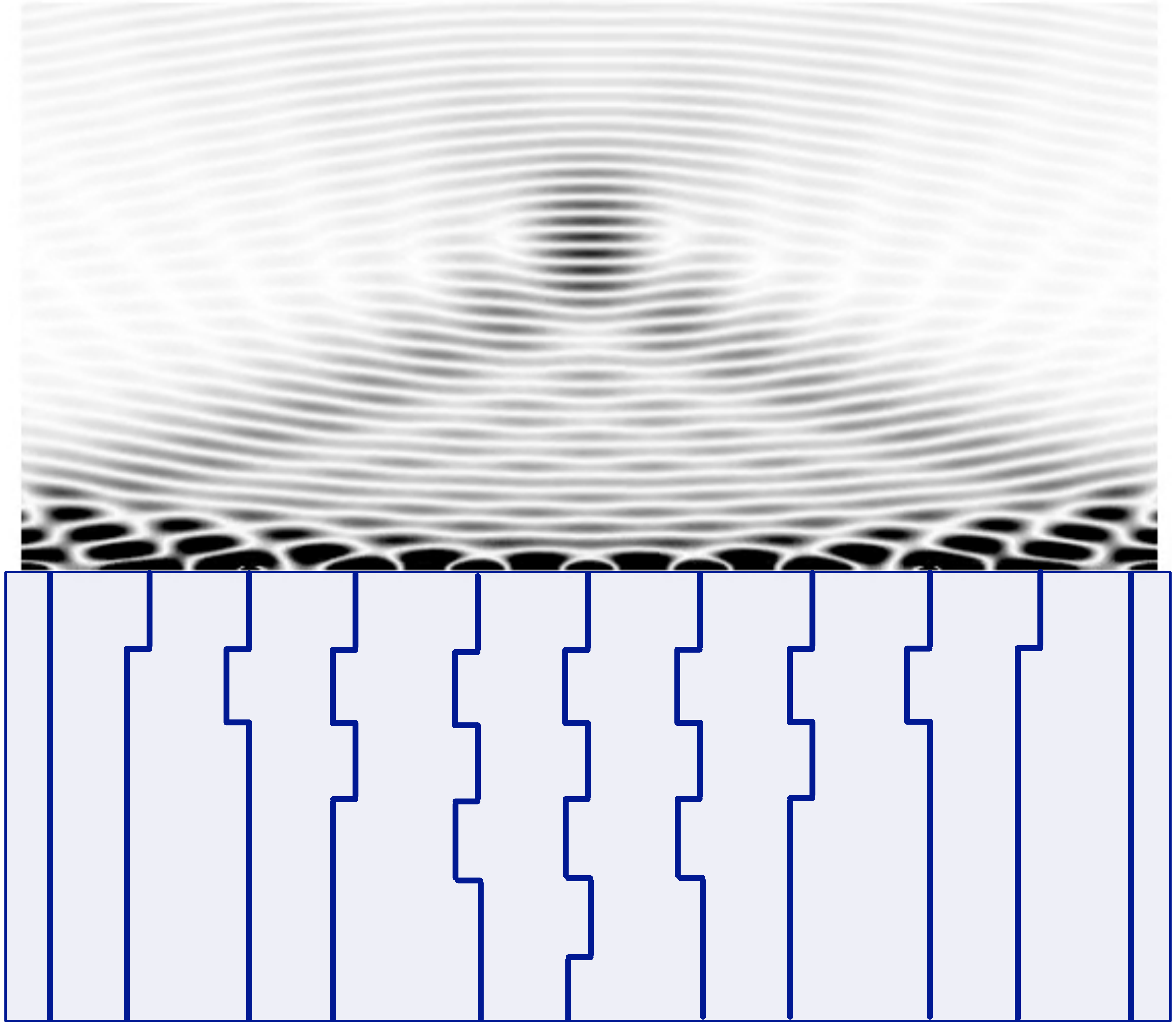
- It is a system with “topologically protected” edge states
- It needs certain symmetries which are not generic

Hafezi et al., Nature Photonics (2013)
Rechtsmann et al. Nature (2013)

- It is not a “topological state of matter”
- No response is quantized

Kane and Lubensky, Nature Physics (2013)
Paulose et al., arXiv (2014)

- Stable phononic wave guides are useful for
  - Acoustic lensing
  - Vibration isolation
  - Acoustic cloaking



Conclusions and outlook

- Stable edge modes for acoustic waves
  - measurement of edge spectrum
  - stable against “good” disorder
  - domain walls guide waves

- In-depth studies of disorder effects:
  - localization length for “bad” disorder
  - stability agains stronger disorder

- Detailed study of classical non-linearities

- Science **349**, 47 (2015)