Synthetic dimensions in integrated photonics:

From optical isolation to 4D quantum Hall physics

Hannah Price
INO-CNR BEC Center & University of Trento, Italy

Many-Body Physics with Light
KITP, November 11th 2015

Outline

- Background on synthetic dimensions
- Synthetic dimensions in integrated photonics: how to do it?
- Topological physics in a **1D** ring-resonator chain:
    - Edge states for optical isolation
    - **2D** quantum Hall effect in photon transport
- Topological physics in a in a **3D** resonator array:
    - **4D** quantum Hall effect in photon transport

Synthetic dimensions in integrated photonics: arXiv:1510.03910
Tomoki Ozawa, <u>HMP,</u> Nathan Goldman, Oded Zilberberg & Iacopo Carusotto

INO-CNR BEC Center & University of Trento, Italy

Université Libre de Bruxelles

ETH Zurich

INO-CNR BEC Center & University of Trento, Italy

4D quantum Hall effect with ultracold atoms: Phys. Rev. Lett. 115, 195303 (2015)
<u>HMP</u>, Oded Zilberberg, Tomoki Ozawa, Iacopo Carusotto & Nathan Goldman

Topological Physics for Photons

- Energy bands with topological invariants

  This Talk: *1st and 2nd Chern numbers*

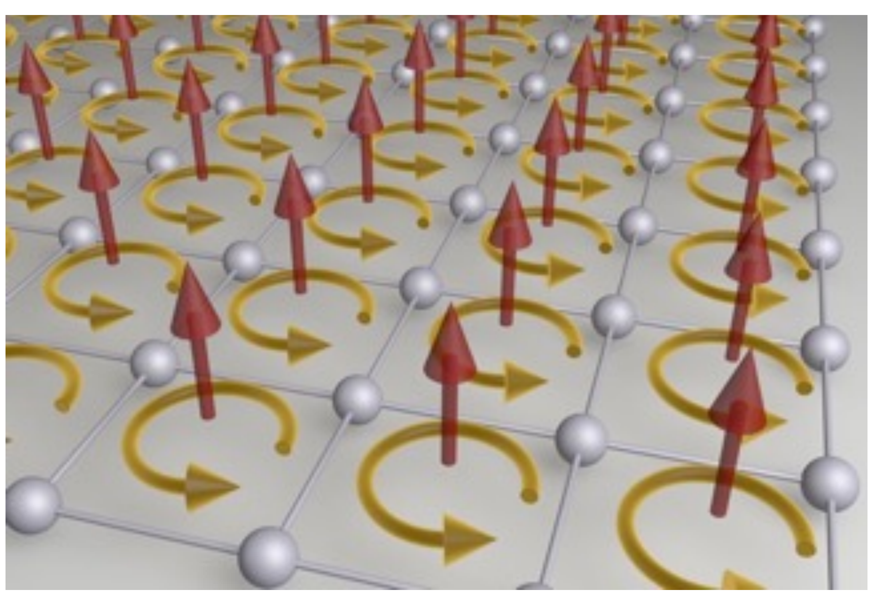
- Topological edge states for unidirectional propagation of light
- New topological physics: 4D quantum Hall effect

Focus on variants of Hofstadter model for charged particle:

Hofstadter, PRB 14, 2239, (1976)

$$H = -J \sum_{x,y} \left(\hat{b}^\dagger_{x+a,y} \hat{b}_{x,y} + e^{i2\pi\Phi x/a} \hat{b}^\dagger_{x,y+a} \hat{b}_{x,y} + \mathrm{h.c.} \right)$$

in Landau gauge



uniform perpendicular magnetic flux per plaquette of $2\pi\Phi$

energy bands with non-zero 1st Chern numbers

Image: http://www.quantum-munich.de/media/realization-of-the-hofstadter-hamiltonian/

Topological Physics for Photons

- Energy bands with topological invariants

  This Talk: *1st and 2nd Chern numbers*

- Topological edge states for unidirectional propagation of light
- New topological physics: 4D quantum Hall effect

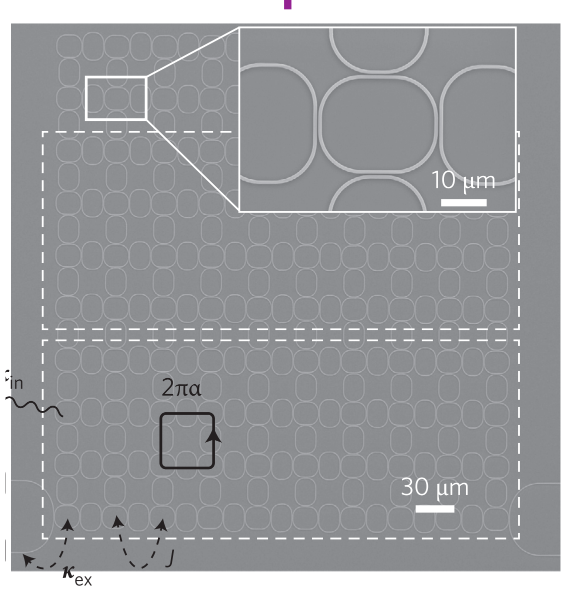
Focus on variants of Hofstadter model for charged particle:

Hofstadter, PRB 14, 2239, (1976)

$$H = -J \sum_{x,y} \left(\hat{b}^\dagger_{x+a,y} \hat{b}_{x,y} + e^{i2\pi\Phi x/a} \hat{b}^\dagger_{x,y+a} \hat{b}_{x,y} + \text{h.c.} \right)$$

in Landau gauge

in silicon photonics

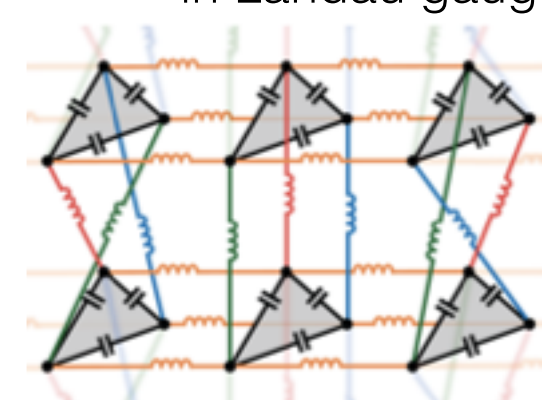


Hafezi et al, Nat. Photon. 7, 1001, (2013)

Also in solid-state superlattices, ultracold atoms…

pendula:
Susstrunk & Huber, Science 349, 47 (2015)

& time-modulation proposal: **Salerno, Ozawa, HMP, Carusotto, arXiv:1510.04697**



circuits:
Expt: **Ningyuan et al., PRX 5, 021031, (2015)**
Theory: **V. V. Albert,et al PRL 114, 173902 (2015).**

Synthetic dimensions in ultracold atoms

Ingredients:

1. Choose degrees of freedom —> site indices in synthetic dimension
2. Couple these degrees of freedom —>“hopping”

1. Spin states
2. Raman laser coupling

Theory:
Boada et al., PRL, 108, 133001 (2012)
Celi et al., PRL, 112, 043001 (2014)
… HMP et al, PRL 115, 195303 (2015)

Experiments:
Mancini et al, Science, 349, 1510 (2015)
Stuhl et al. Science, 349, 1514 (2015)

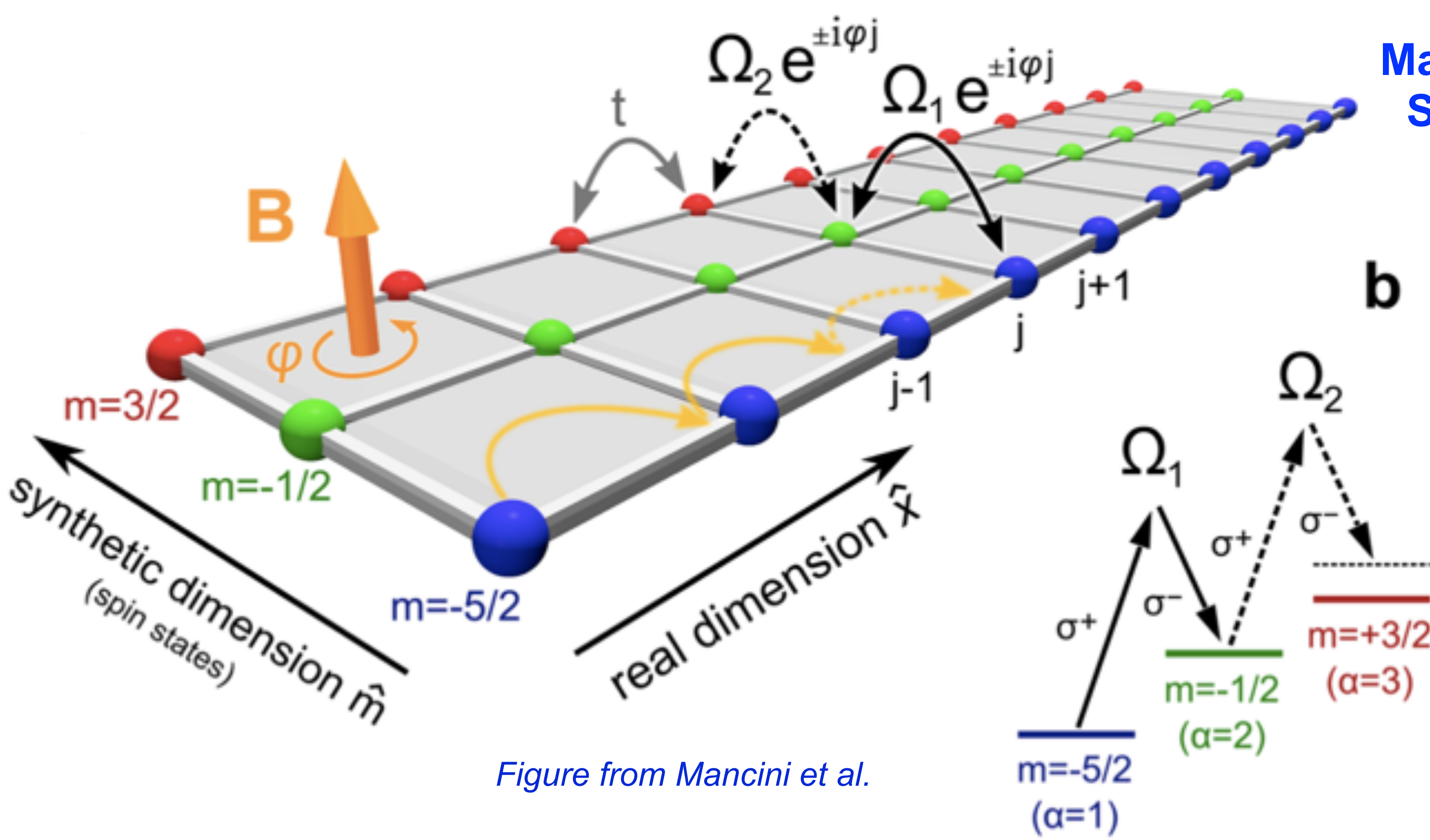


Figure from Mancini et al.

3-leg Hofstadter ladder
for atoms in a 1D
optical lattice

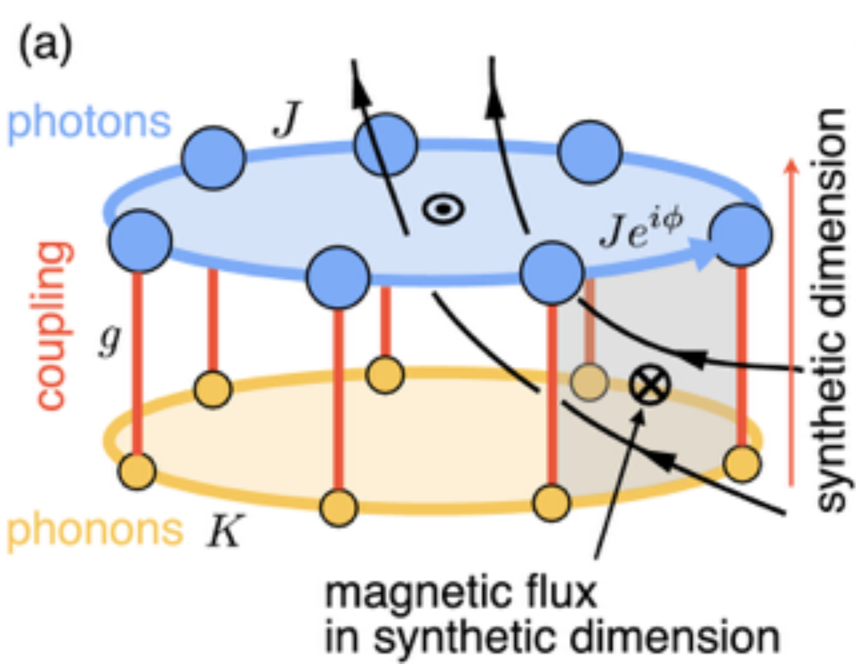
^{173}Yb
$I = 5/2$

Synthetic dimensions in photonics so far…

Ingredients:

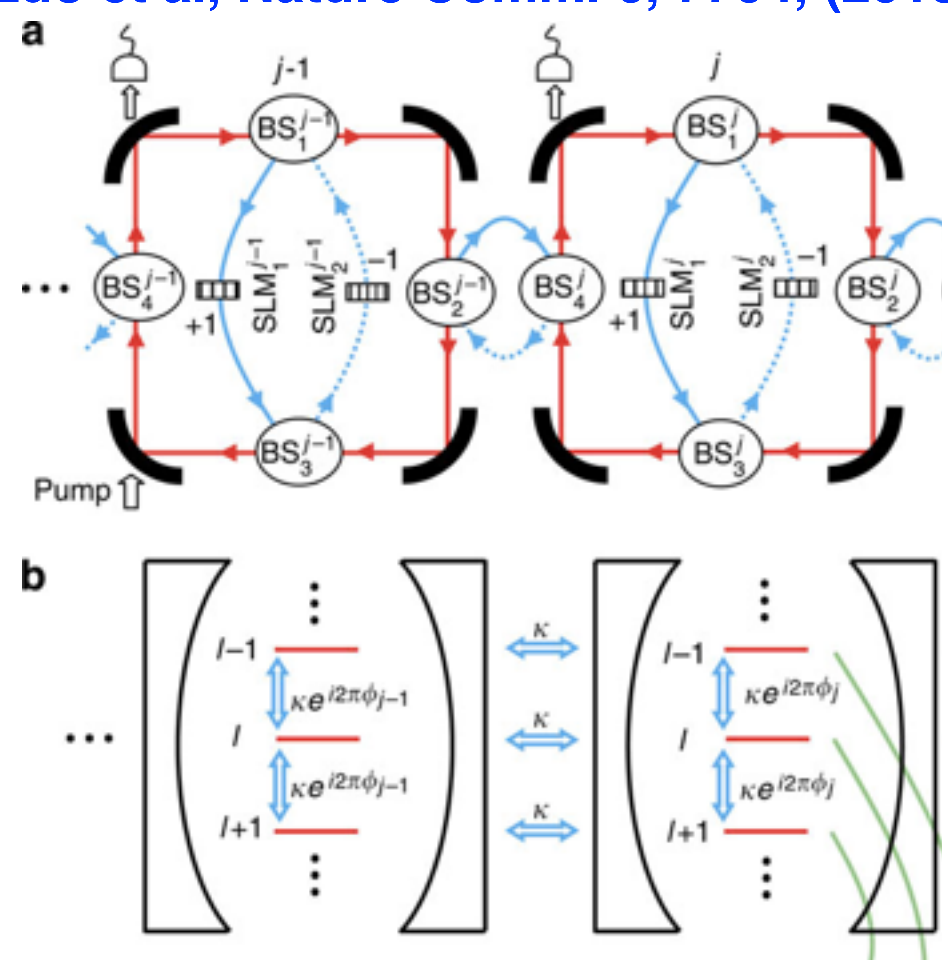
1. Choose degrees of freedom —> site indices in synthetic dimension
2. Couple these degrees of freedom —>"hopping"

Schmidt et al, Optica 2, 7, 635 (2015)



1. Photons & phonons
2. Optomechanical coupling

Luo et al, Nature Comm. 6, 7704, (2015)



1. Orbital angular momentum of cavity modes
2. Spatial light modulators

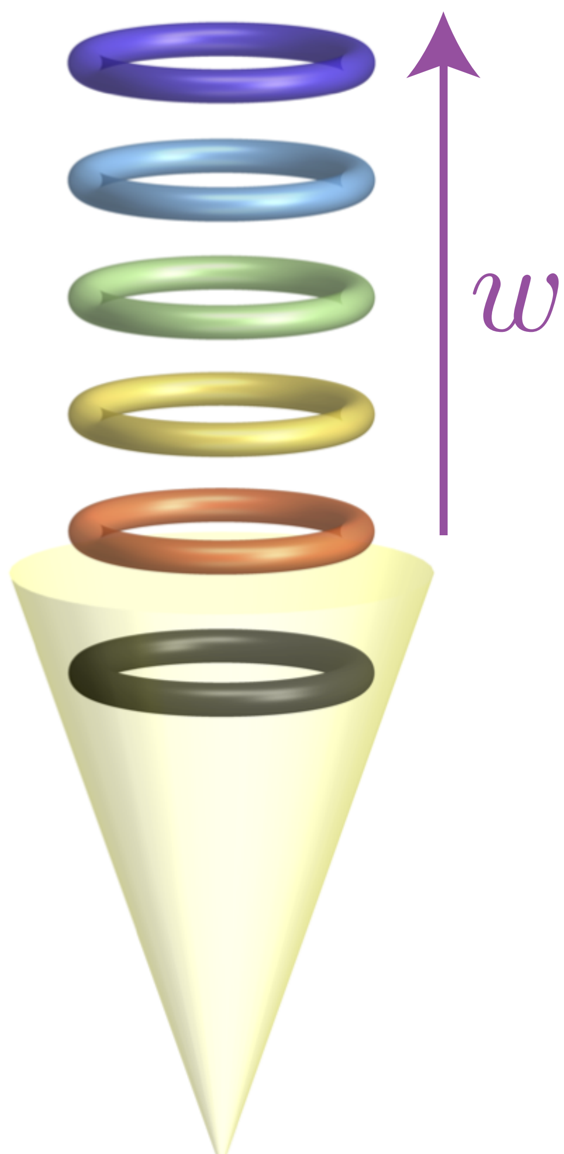
…and also Schleier-Smith et al, KITP DenseLight conference

Synthetic dimensions in integrated photonics

Ingredients:

1. Choose degrees of freedom —> site indices in synthetic dimension
2. Couple these degrees of freedom —>"hopping"

$(\hbar = 1)$



1. Modes of a ring resonator

$$\omega_w = \omega_{w_0} + \Delta\omega\,(|w| - w_0) + \frac{D}{2}(|w| - w_0)^2 + \ldots$$

free spectral range $\Delta\omega = 2\pi c/n_{\text{eff}} R$

2. Strong beam(s) modulates ε_{ij} at $\omega_{\text{mod}} \approx \eta\delta\omega$ via optical nonlinearity

$$H_{\text{mod}} = -\sum_w \Omega_{\mathbf{r}}(t)\,\hat{a}^\dagger_{\mathbf{r},w+\eta}\hat{a}_{\mathbf{r},w} + \text{h.c.},$$

$$\Omega_{\mathbf{r}}(t) = \Omega^o_{\mathbf{r}} e^{-i\omega_{\text{mod}} t}$$

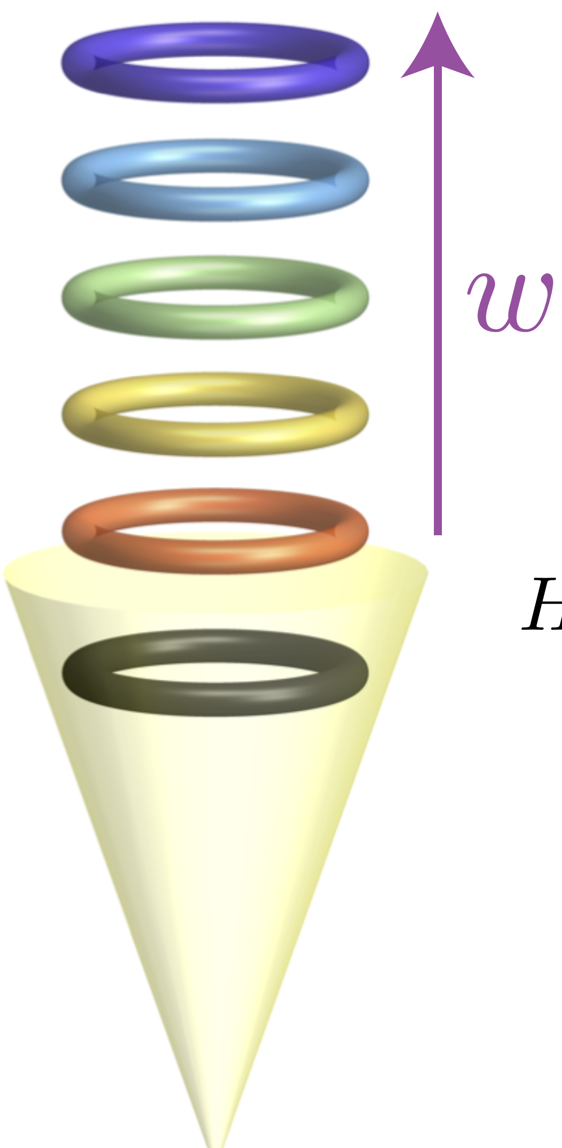
Ozawa et al, arXiv:1510.03910

Synthetic dimensions in integrated photonics

To get static inter-mode coupling from:

$$H_{\text{mod}} = -\sum_{w} \Omega_{\mathbf{r}}(t)\, \hat{a}^{\dagger}_{\mathbf{r},w+\eta} \hat{a}_{\mathbf{r},w} + \text{h.c.},$$

$$\Omega_{\mathbf{r}}(t) = \Omega^{o}_{\mathbf{r}} e^{-i\omega_{\text{mod}} t}$$



Go to rotating frame:

$$\hat{b}_{\mathbf{r},w}(t) \equiv \hat{a}_{\mathbf{r},w}(t) e^{i[\omega_{w_0} + (w-w_0)\omega_{\text{mod}}/\eta]t},$$

Can be complex: easily tuneable synthetic gauge field

"hopping" in synthetic dimension

$$H_{\Omega} = \sum_{w,\mathbf{r}} \Big[-\Omega^{o}_{\mathbf{r}} \hat{b}^{\dagger}_{\mathbf{r},w+\eta} \hat{b}_{\mathbf{r},w} + \text{h.c.}$$
$$+ \left((\Delta\omega - \omega_{\text{mod}}/\eta)(w - w_0) + \frac{D}{2}(w - w_0)^2 \right) \hat{b}^{\dagger}_{\mathbf{r},w} \hat{b}_{\mathbf{r},w} \Big].$$

"uniform force" along synthetic dimension

"harmonic potential" along synthetic dimension

Extending to a lattice of resonators

$$H = H_J + H_\Omega$$

spatial tunnelling

$$H_J = -\sum_{w,\mathbf{r},j} J_j\, \hat{b}^\dagger_{\mathbf{r}+\mathbf{a}_j,w} \hat{b}_{\mathbf{r},w} + \text{h.c.}$$

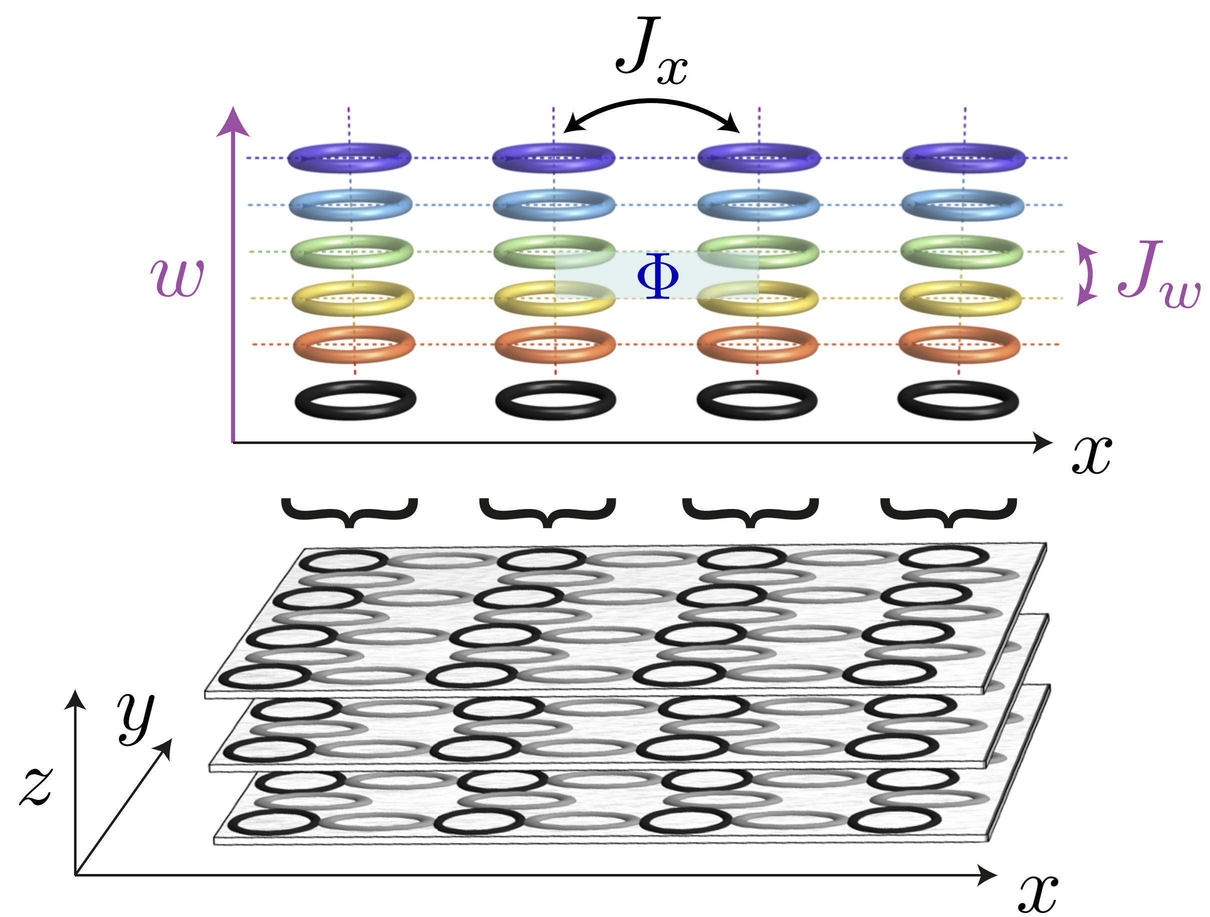
coupling in
synthetic dimension

$$H_\Omega \approx -\sum_{w,\mathbf{r}} \Omega^o_{\mathbf{r}} \hat{b}^\dagger_{\mathbf{r},w+\eta} \hat{b}_{\mathbf{r},w} + \text{h.c.}$$

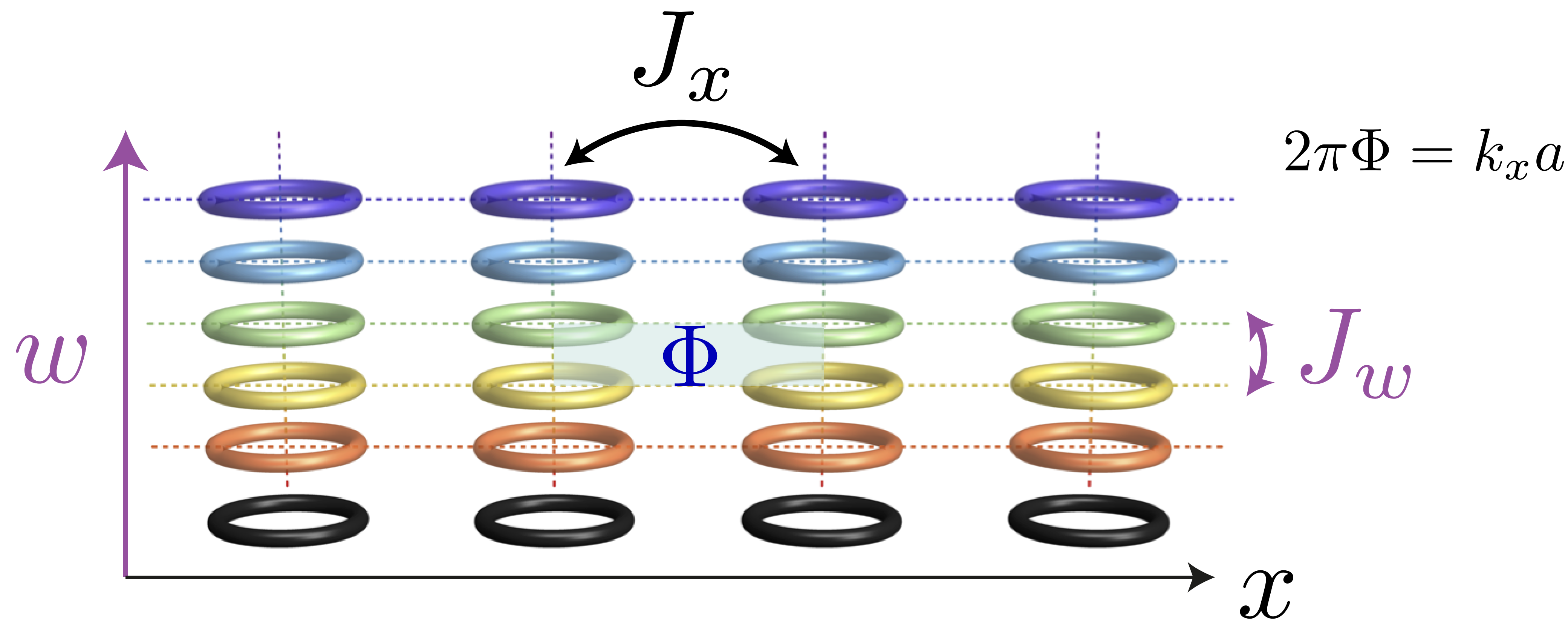
Now to specific systems:

1. 1D chain —> effective 2D lattice

2. 3D array —> effective 4D lattice



1D resonator chain with the synthetic dimension



$$H = H_J + H_\Omega$$

Choose explicitly: $\Omega_x^o = |\Omega_x^o| e^{ik_x x}$

$$H = -\sum_{x,w} \left(J_x \hat{b}^\dagger_{x+a,w} \hat{b}_{x,w} + |\Omega_x^0| e^{ik_x x} \hat{b}^\dagger_{x,w+\eta} \hat{b}_{x,w} + \text{h.c.} \right)$$

Hofstadter Model

Ultracold atoms: **Celi et al., PRL 112, 043001 (2014)** (Theory)
Mancini et al, Science, 349, 1510 (2015) (Expt)
Stuhl et al. Science, 349, 1514 (2015) (Expt)
Optical cavities: **Luo et al, Nature Comm. 6, 7704, (2015)** (Theory)

A few more details about the Hofstadter model

$$H = -\sum_{x,w}\left(J_x \hat{b}^{\dagger}_{x+a,w}\hat{b}_{x,w} + |\Omega^0_x| e^{ik_x x}\hat{b}^{\dagger}_{x,w+\eta}\hat{b}_{x,w} + \text{h.c.}\right)$$

$$2\pi\Phi = k_x a$$

$$\Phi = p/q$$

Topological invariant

$$\mathcal{F}^{xw} = i\left(\langle\partial_{k_x}u|\partial_{k_w}u\rangle - \langle\partial_{k_x}u|\partial_{k_w}u\rangle\right)$$

geometrical Berry curvature

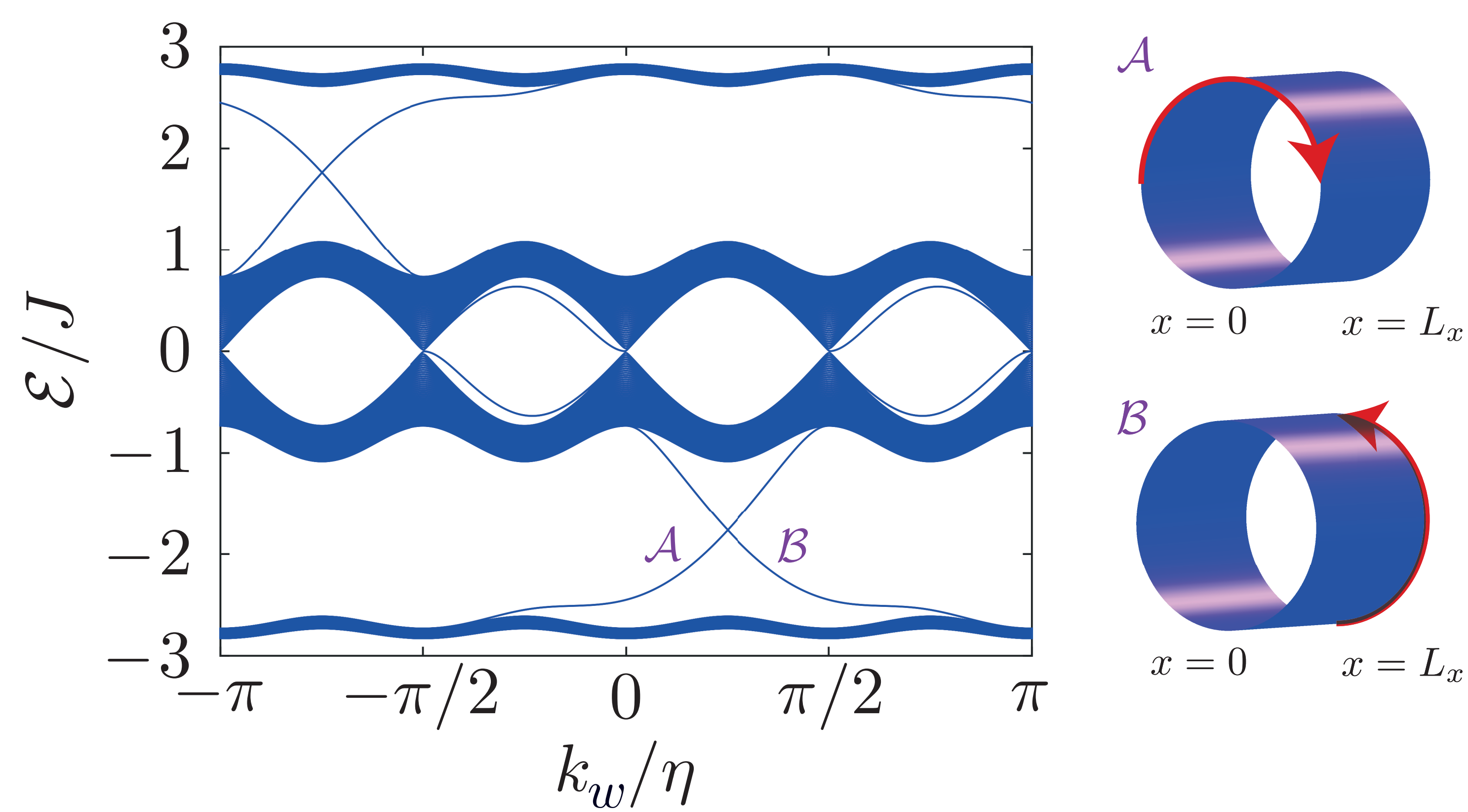
$$\nu_1^{xw} = \int_{BZ}\mathcal{F}^{xw}\mathrm{d}^2k \;\in \mathbb{Z}$$

topological 1st Chern number

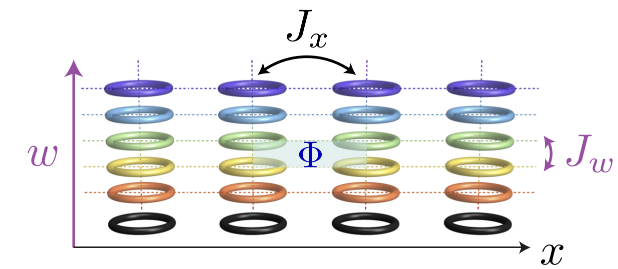
Topological edge States

$$\Phi = 1/4$$

Energy spectrum with PBCs around w and finite length along x



Driven-dissipative experiments



Include **losses** $\gamma_{x,w}$

and **coherent driving**

Ozawa & Carusotto, PRL, 112, 133902, (2014)

$$H_{\text{drive}} = \sum_{x,w} \left(f_{x,w}(t)\hat{b}_{x,w} + f^*_{x,w}(t)\hat{b}^\dagger_{x,w} \right)$$

assume $f_{x,w}(t) \propto e^{-i\omega_{\text{drive}} t}$

i.e. in non-rotating frame

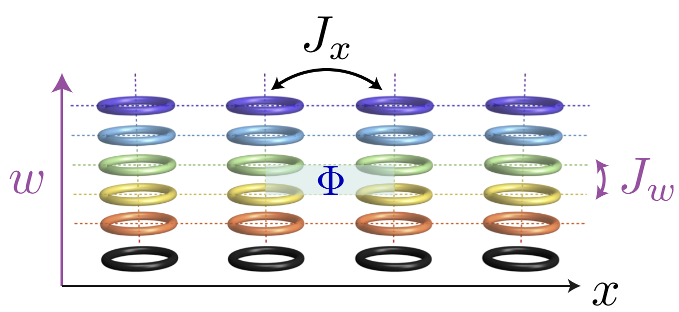
$\omega_w + \omega_{\text{drive}}$

Look for long-time **steady-state**

- Consider single-site driving, couples to all momenta in the Brillouin zone
- Vary position and frequency of drive to excite different supermodes

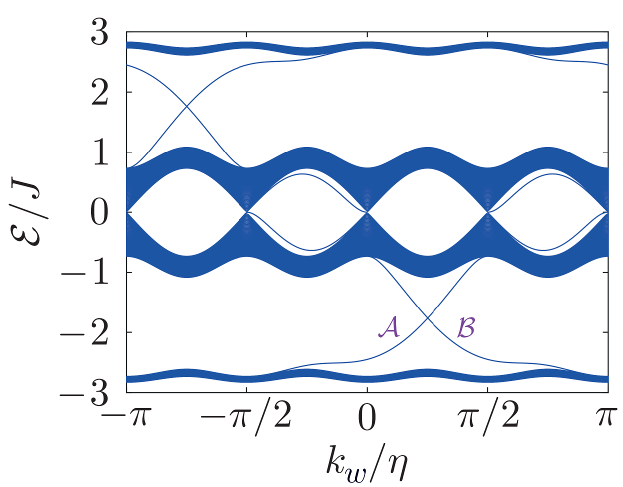
Carusotto & Ciuit, RMP, 85, 299, (2013)

Topological edge states



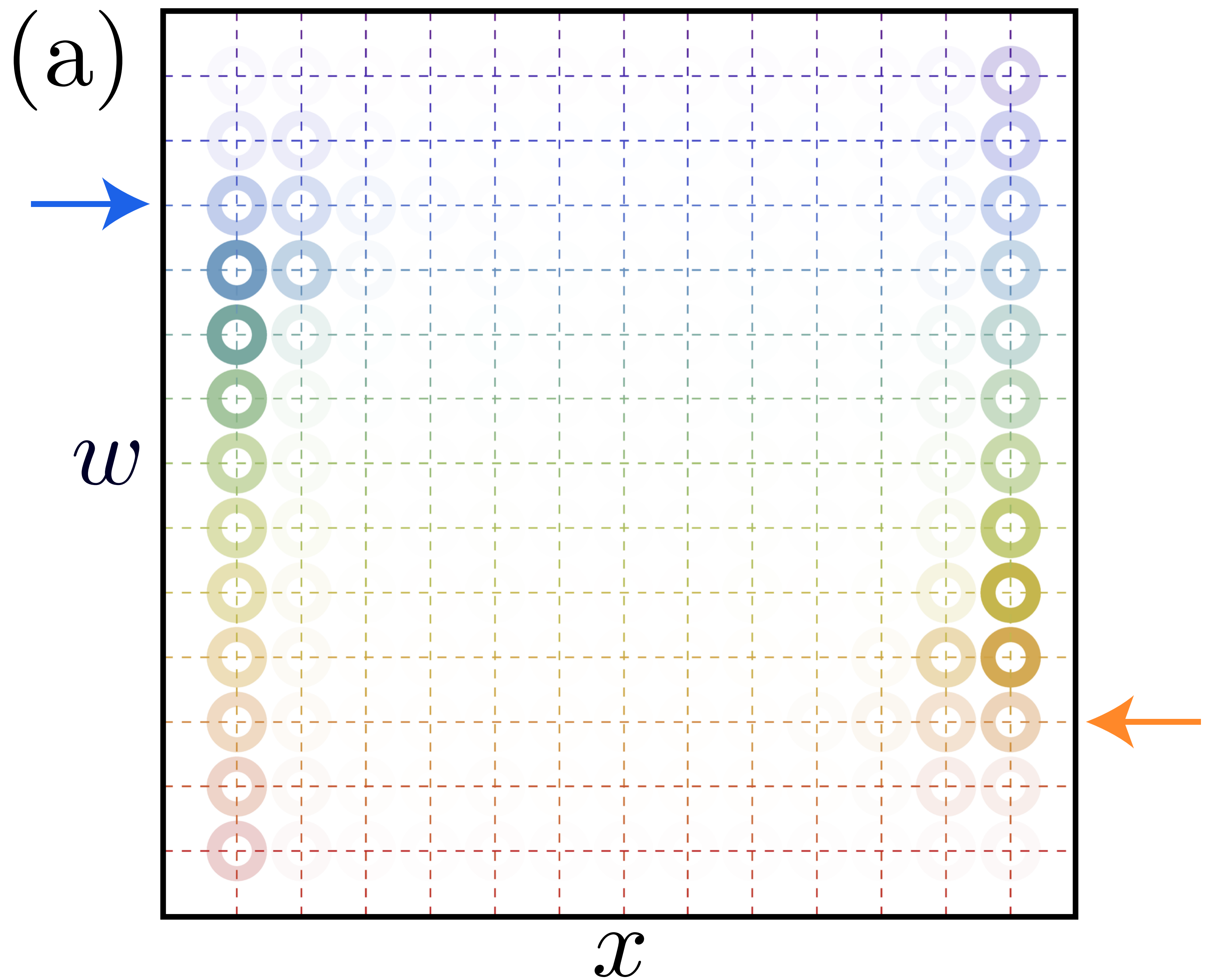
Excite edge states:

$$\omega_{\rm drive} = -2J$$



with uniform loss rate:

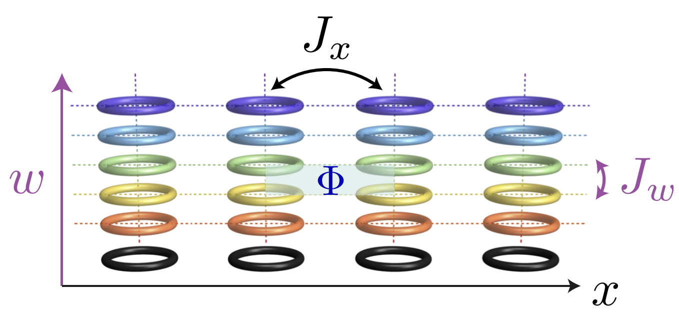
$$\gamma = 0.1J$$



Also see Luo et al, Nature Comm. 6, 7704, (2015)

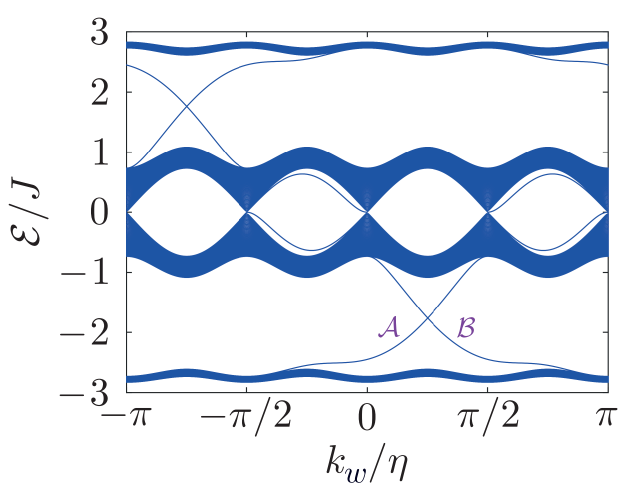
Topological edge states

How to make an effective boundary in the synthetic dimension?



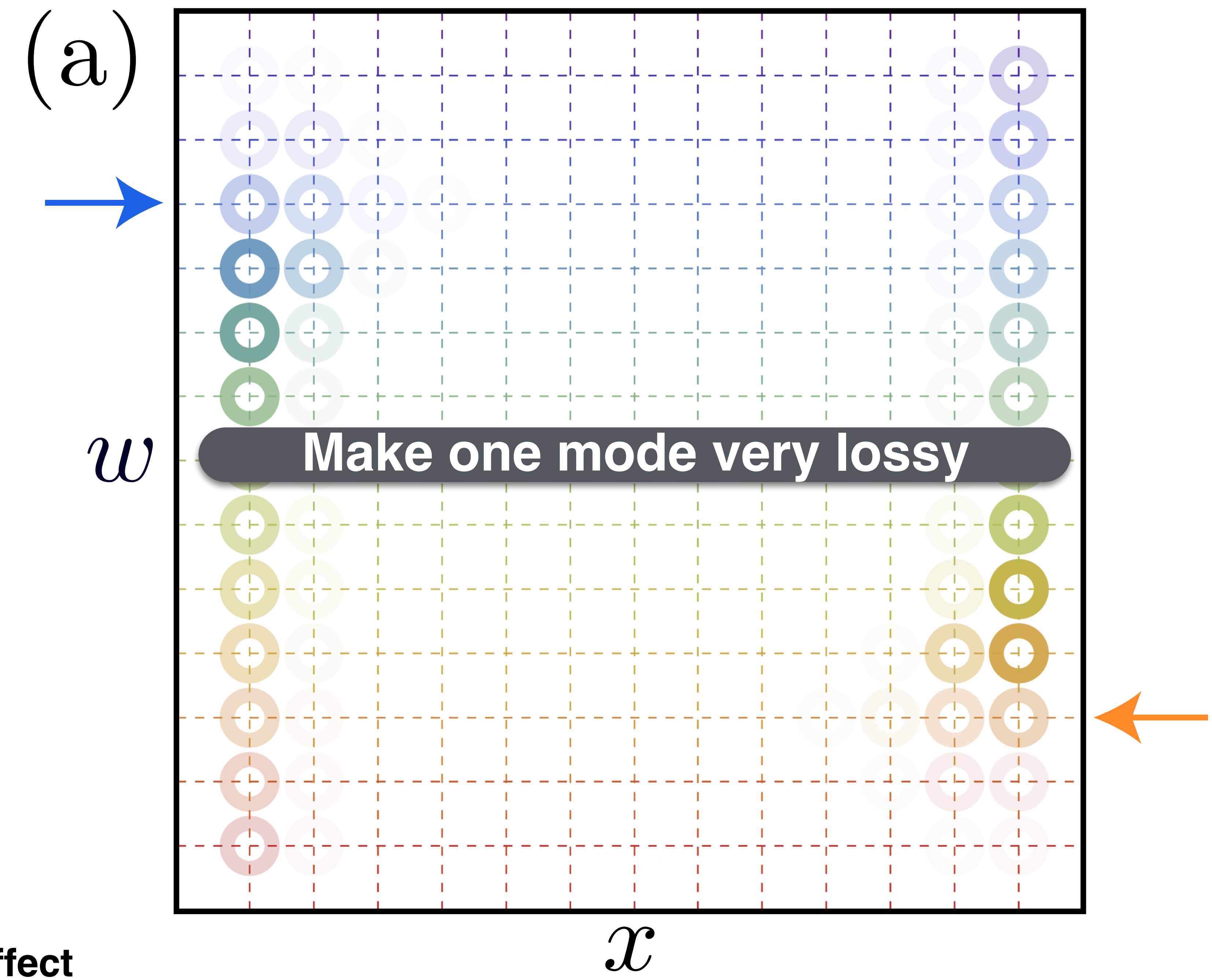
Excite edge states:

$$\omega_{\text{drive}} = -2J$$



with uniform loss rate:

$$\gamma = 0.1J$$

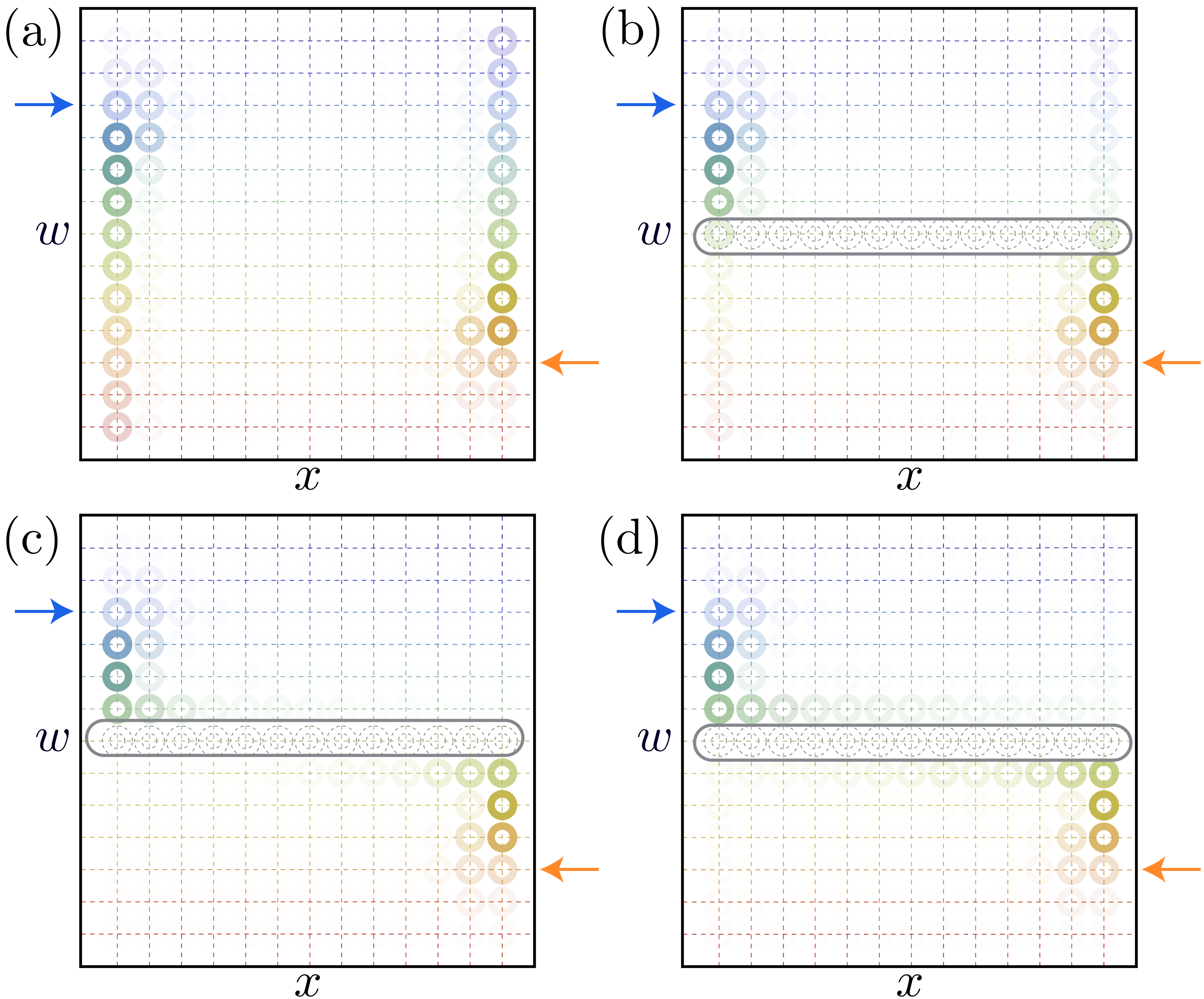


the Zeno effect

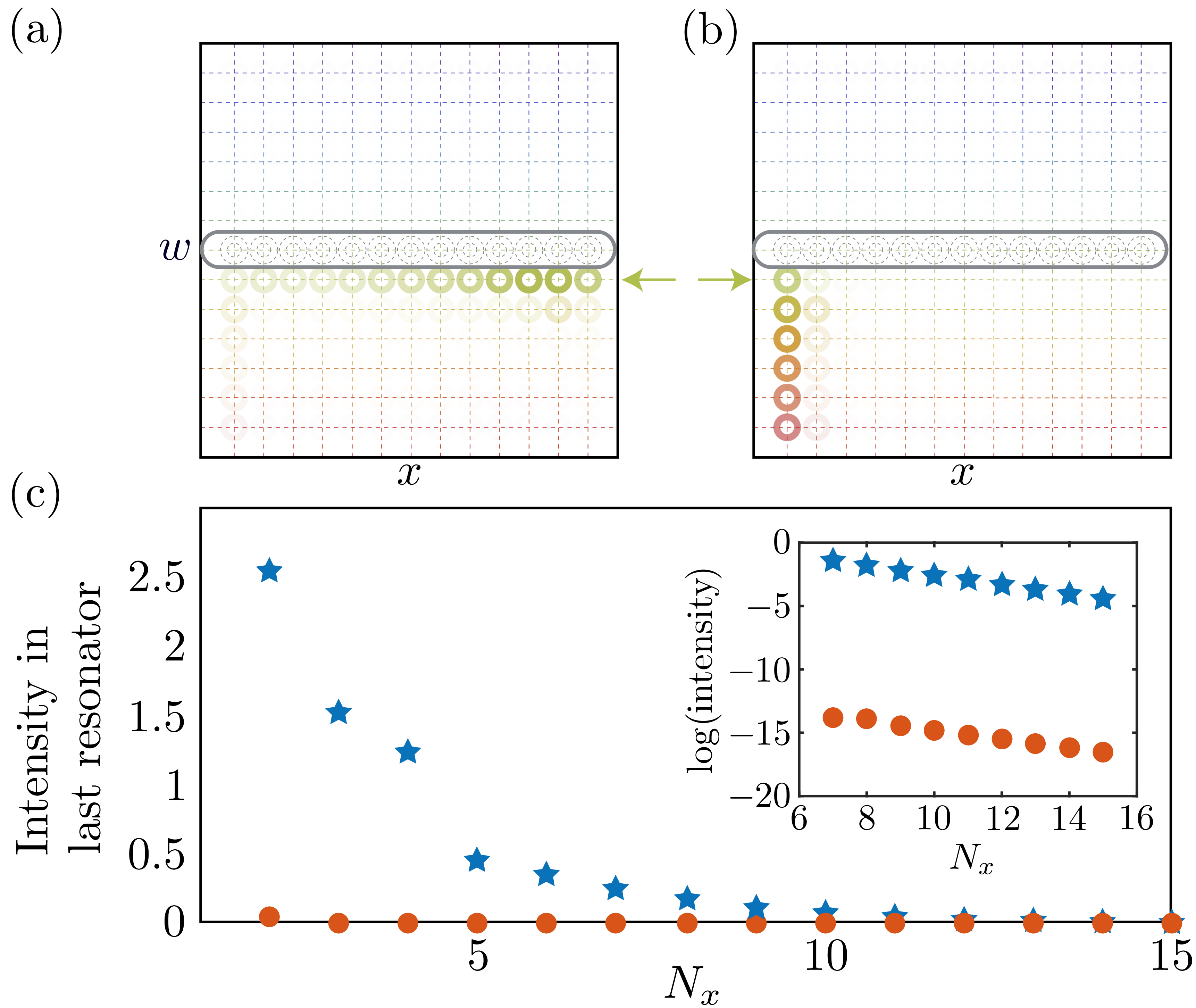
e.g. Barontini et al., PRL, 110, 035302 (2014)

Topological edge states

Losses in the central row equal to:
(a) γ
(b) 10γ
(c) 100γ
(d) 1000γ



Topological edge states for optical isolation



- Extension of works on spatiotemporal modulation to include topological protection

Yu & Fan, Nat. Photon., 3, 91, (2009)
Lira et al., PRL 109, 033901 (2012)
Tzuang et al., Nat. Photon., 8, 701, (2014)

2D quantum Hall effect in photon transport in 1D chain

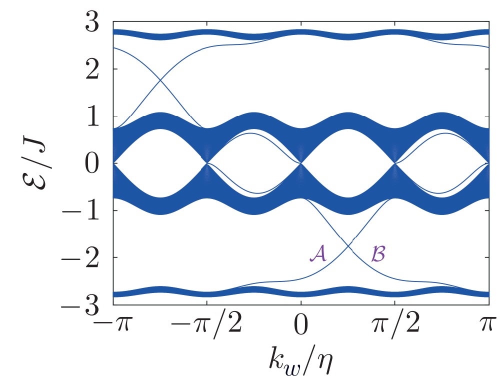
2D quantum Hall for a filled band of fermions:

$$j^x = -\frac{eE_w}{2\pi}\nu_1^{xw}$$

Optical bosonic analogue when drive on resonance with a band and $\Delta\mathcal{E}_{\mathrm{band}} < \gamma < \Delta\mathcal{E}_{\mathrm{gap}}$

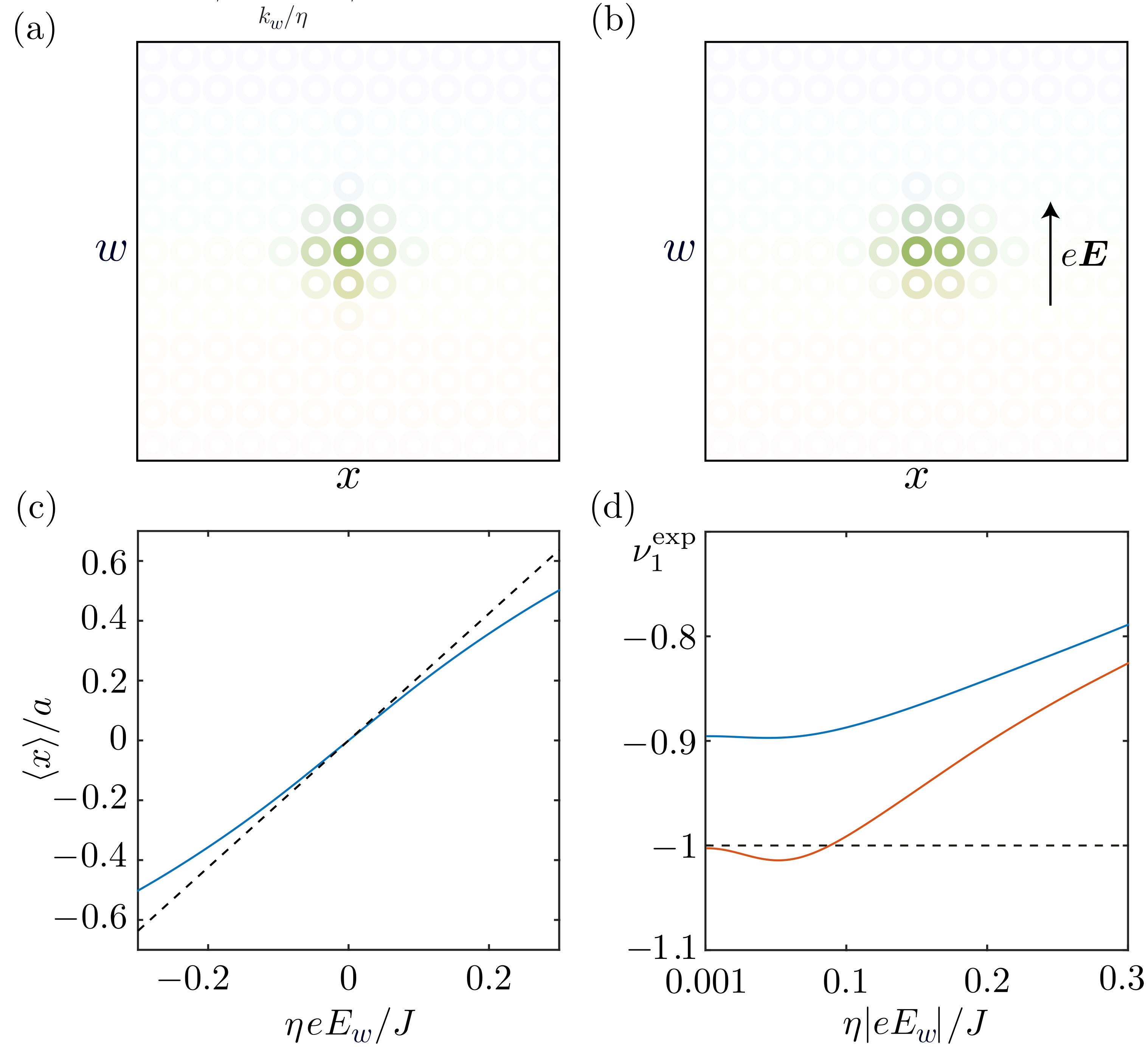
$$\langle x \rangle = -q(\eta a)\frac{eE_w}{2\pi\gamma}\nu_1^{xw} + \mathcal{O}(\gamma^0),$$

for $\Phi = p/q$



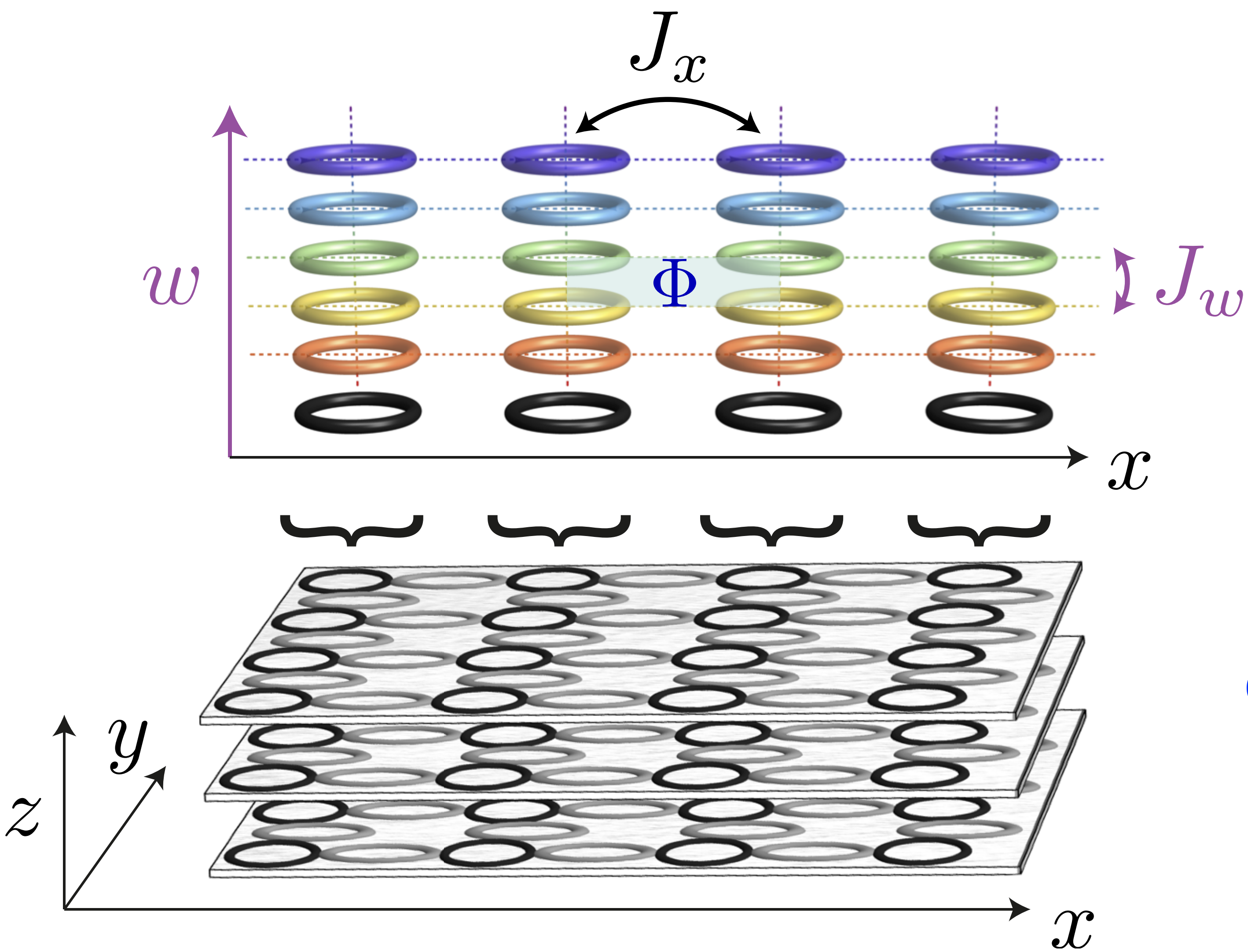
$$\omega_{\mathrm{drive}} = -2.7J$$
$$\gamma = 0.3J$$

Synthetic E field from $(\Delta\omega - \omega_{\mathrm{mod}}/\eta)$



Ozawa & Carusotto, PRL, 112, 133902, (2014)

3D resonator array with the synthetic dimension



What about 4D quantum Hall physics?

S.-C.Zhang & J. Hu, Science 294, 823, (2001),
Qi, Hughes & Zhang, PRB 78, 195424, (2008)…

What do we need for interesting 4D topological physics?

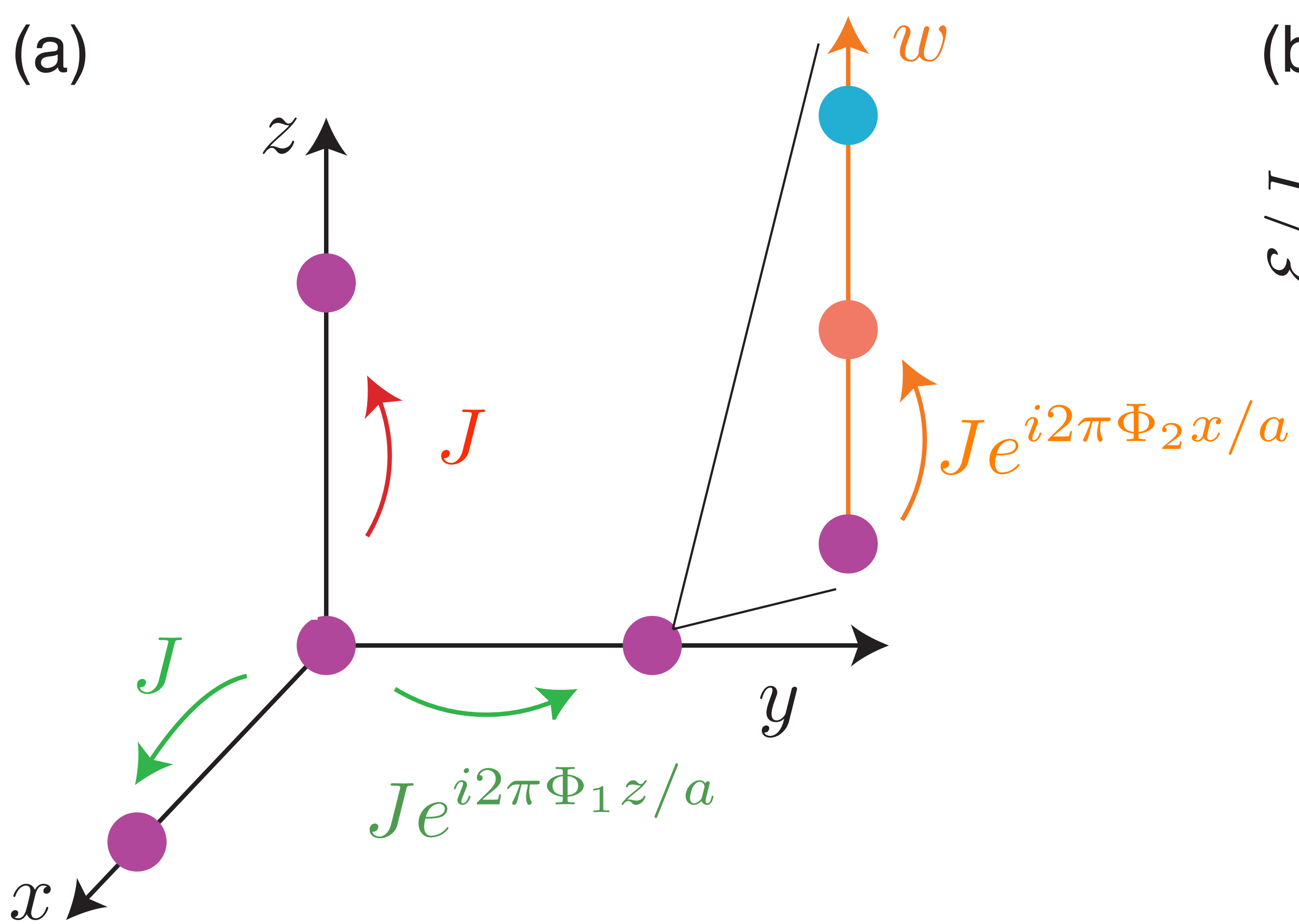
$$\nu_2 \equiv \frac{1}{(2\pi)^2} \int_{\mathrm{BZ}} \left(\mathcal{F}^{xy}\mathcal{F}^{zw} + \mathcal{F}^{wx}\mathcal{F}^{zy} + \mathcal{F}^{zx}\mathcal{F}^{yw} \right) d^4k \;\; \in \mathbb{Z},$$

topological 2nd Chern number for TRS-breaking model

Minimal 4D topological lattice model

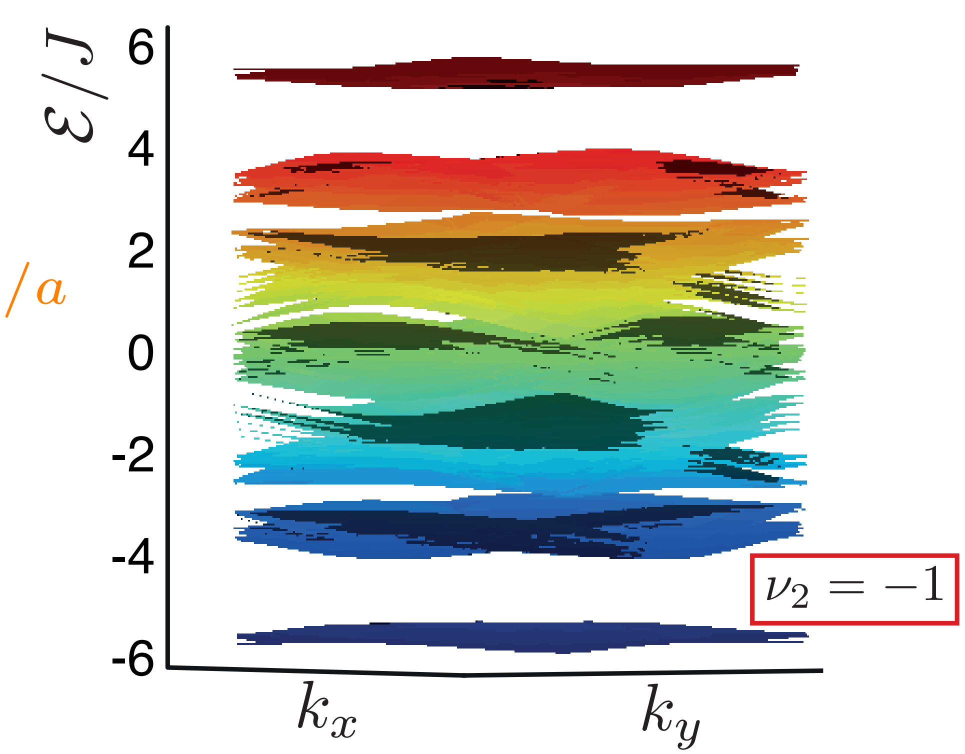
Two copies of Hofstadter models in “disconnected” planes

(a)



(b)

e.g. $\Phi_1 = 1/4, \Phi_2 = 1/4$



$$H = -J \sum_{\mathbf{r},w} \hat{b}^\dagger_{\mathbf{r}+a\mathbf{e_x},w}\hat{b}_{\mathbf{r},w} + \hat{b}^\dagger_{\mathbf{r}+a\mathbf{e_z},w}\hat{b}_{\mathbf{r},w}$$

$$+ e^{i2\pi\Phi_1 z/a}\hat{b}^\dagger_{\mathbf{r}+a\mathbf{e_y},w}\hat{b}_{\mathbf{r},w} + e^{i2\pi\Phi_2 x/a}\hat{b}^\dagger_{\mathbf{r}+\eta\mathbf{e_w},w}\hat{b}_{\mathbf{r},w} + \text{h.c.}$$

HMP et al., PRL, 115, 195303 (2015)

4D quantum Hall effect in photon transport

4D quantum Hall for a filled (non-degenerate) band of fermions:

perturbing magnetic field
$\delta B_{\rho\sigma} \equiv \partial_\rho A_\sigma - \partial_\sigma A_\rho$

$$j^\mu = -eE_\nu \int_{\mathrm{BZ}} \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathcal{F}^{\mu\nu}(\mathbf{k}) + e^2 E_\nu \delta B_{\rho\sigma} \frac{\nu_2}{(2\pi)^2} \epsilon^{\mu\nu\rho\sigma},$$

essentially 2D quantum Hall physics

the genuine 4D quantum Hall physics

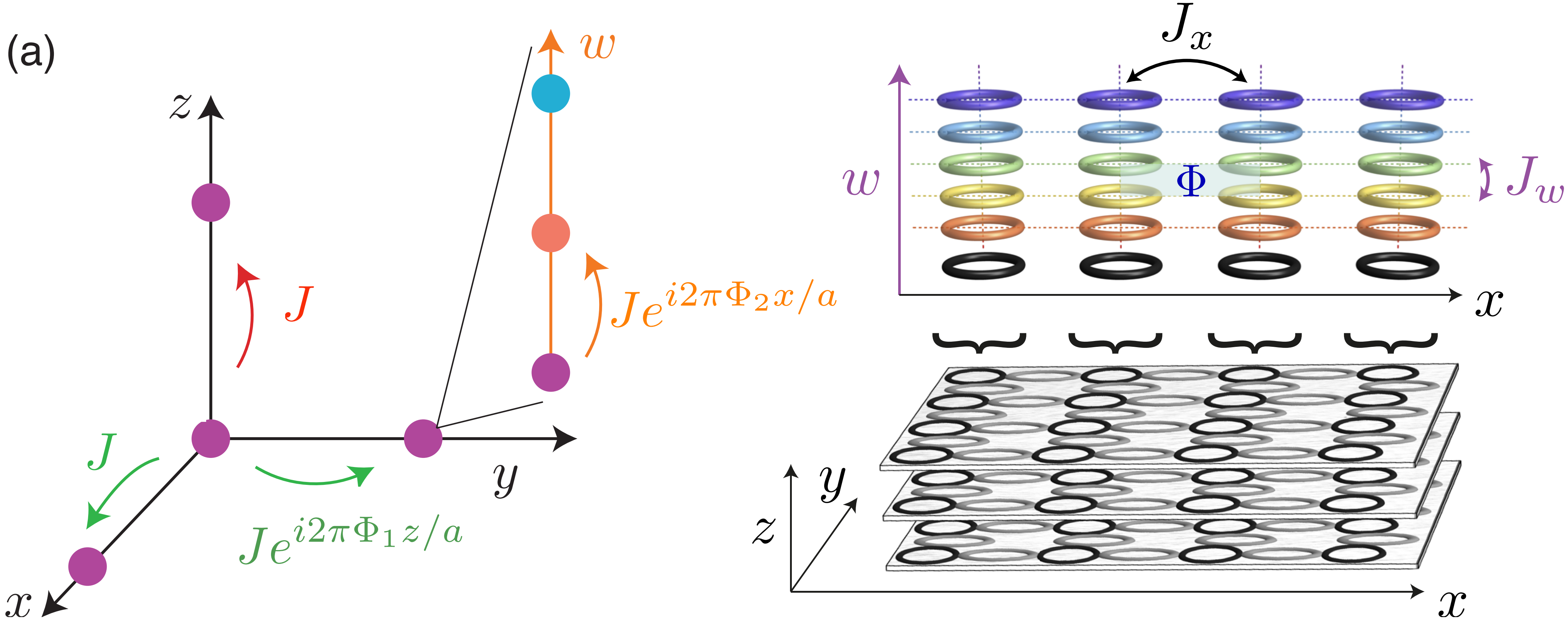
Generalising 2D optical analogue effect to 4D, again when on resonance and when $\Delta\mathcal{E}_{\mathrm{band}} < \gamma < \Delta\mathcal{E}_{\mathrm{gap}}$

$$\langle r^\mu \rangle = \frac{-eE_\nu \int_{\mathrm{BZ}} \frac{d^4 k}{(2\pi)^4} \Omega^{\mu\nu}(\mathbf{k}) + e^2 E_\nu \delta B_{\rho\sigma} \frac{\nu_2}{(2\pi)^2} \epsilon^{\mu\nu\rho\sigma}}{\gamma \int_{\mathrm{BZ}} d^4 k D(\mathbf{k})}.$$

Ozawa et al arXiv:1510.03910

for case discussed here, modified density of states is simply $D(\mathbf{k}) = 1/(2\pi)^4$

4DQH in 3D array with a synthetic dimension



1. Synthetic flux in *xw* plane: as for the 1D chain of resonators through $\Omega^o_{\mathbf{r}} = |\Omega^o_{\mathbf{r}}|e^{ik_x x}$

2. Synthetic flux in *yz* plane: non-resonant links with a z-dependent displacement

like in Hafezi et al., Nat. Photon. 7, 1001, (2013)

3. Synthetic perturbing electric field E_x : e.g. let cavity size vary uniformly

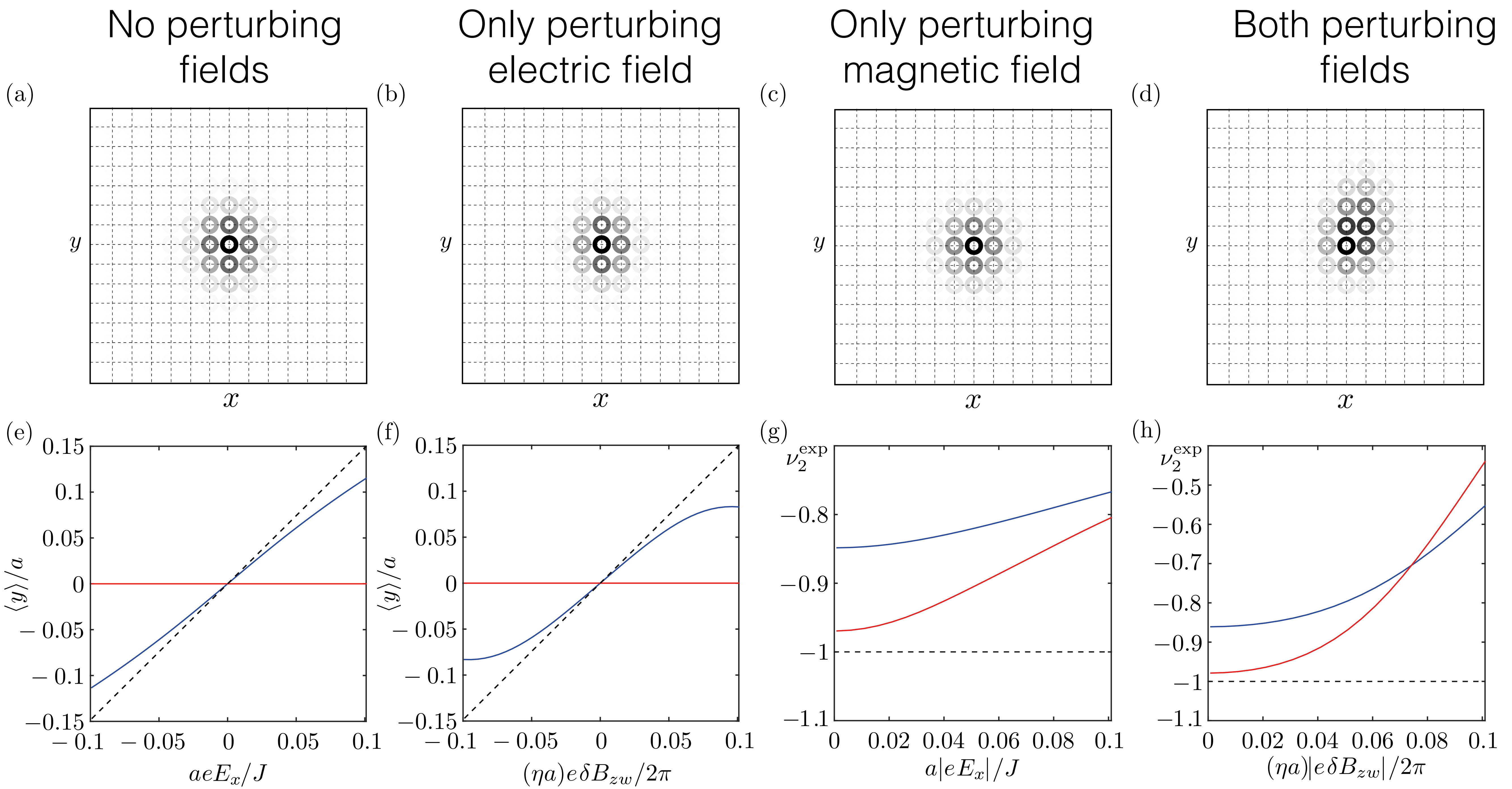
4. Synthetic perturbing magnetic field δB_{zw} : additional slow z-dependence

$$\Omega^o_{\mathbf{r}} = |\Omega^o_{\mathbf{r}}|e^{ik_x x}e^{ik_x z}$$

4DQH in 3D array with a synthetic dimension

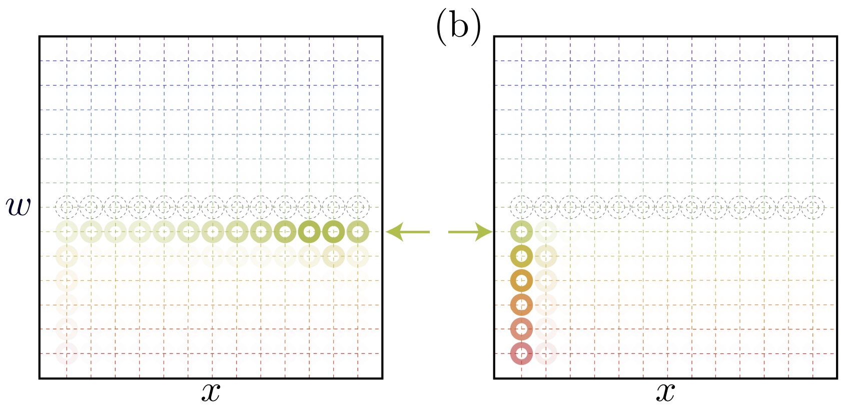
$$\langle y \rangle = -(\eta a^3) e^2 \frac{q^{yz} q^{xw}}{\gamma} \frac{\nu_2}{(2\pi)^2} E_x \delta B_{zw} + \mathcal{O}(\gamma^0),$$

$$\omega_{\mathrm{drive}} = -6.3J$$
$$\nu_2 = -1$$



In summary

- Different modes of a silicon ring-resonator can be exploited as an extra synthetic dimension for photons
- Topological physics in a **1D** ring-resonator chain:
    - Optical isolation with topological edge states
    - **2D** quantum Hall effect



- Topological physics in a in a **3D** resonator array:
    - **4D** quantum Hall effect