Der Wissenschaftsfonds.

"PT-symmetry breaking & exceptional points in OM"

Peter Rabl

Collaborations:

- K. Kepesidis, T. Milburn, S. Portolan (-> Industry)
- S. Bennett, M. Lukin (Harvard)
- K. Makris, S. Rotter (TU Wien)
- K. Holms (Queensland)

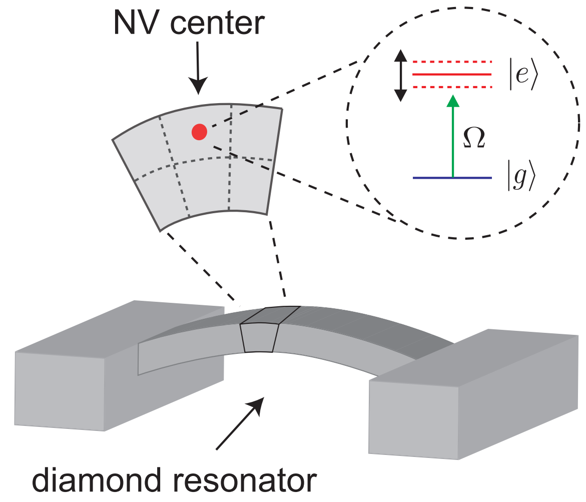
KITP Santa Barbara, 13.10.2015

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

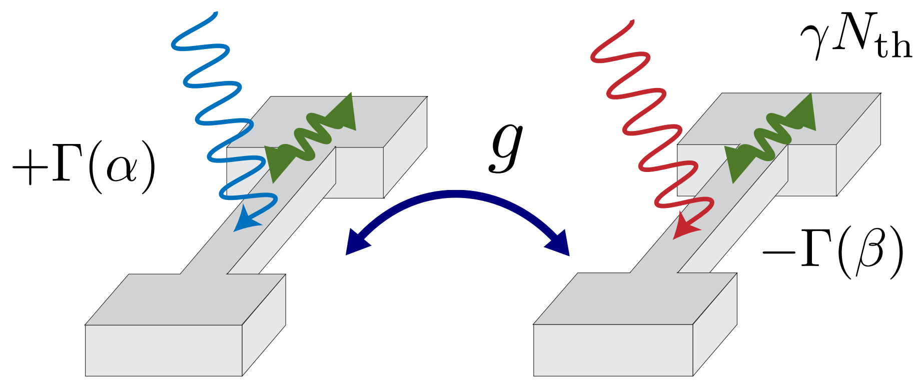
ATOMINSTITUT

Outline

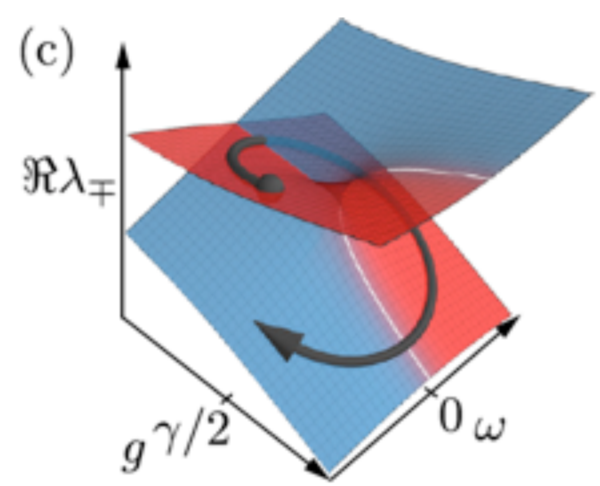
(I) Ground state cooling & phonon lasing in diamond.



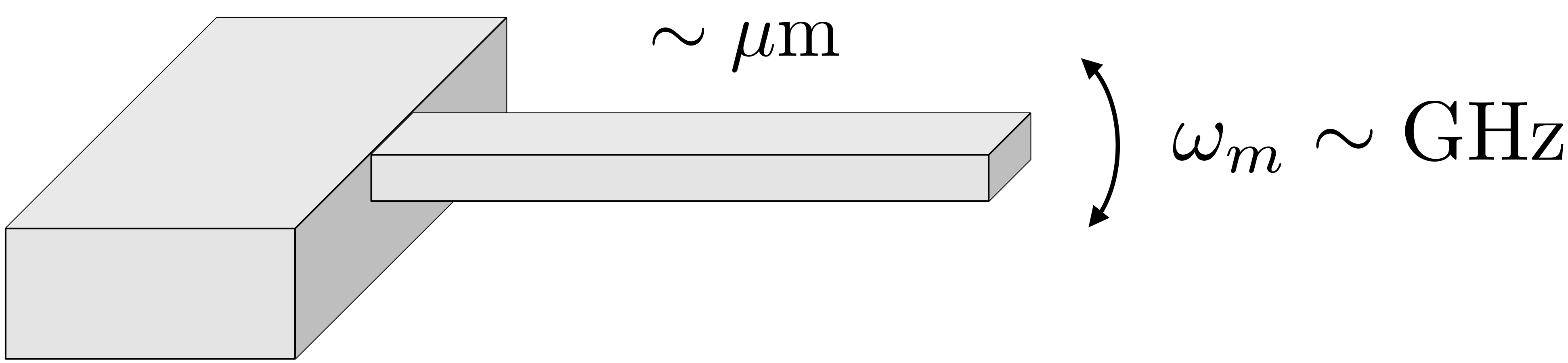
(II) PT-symmetric breaking in coupled phonon systems.



(III) Quasi-adiabatic encircling of exceptional points.



Diamond nano-resonators



diamond nano-resonator:

-) high frequencies

-) low intrinsic dissipation, high-Q

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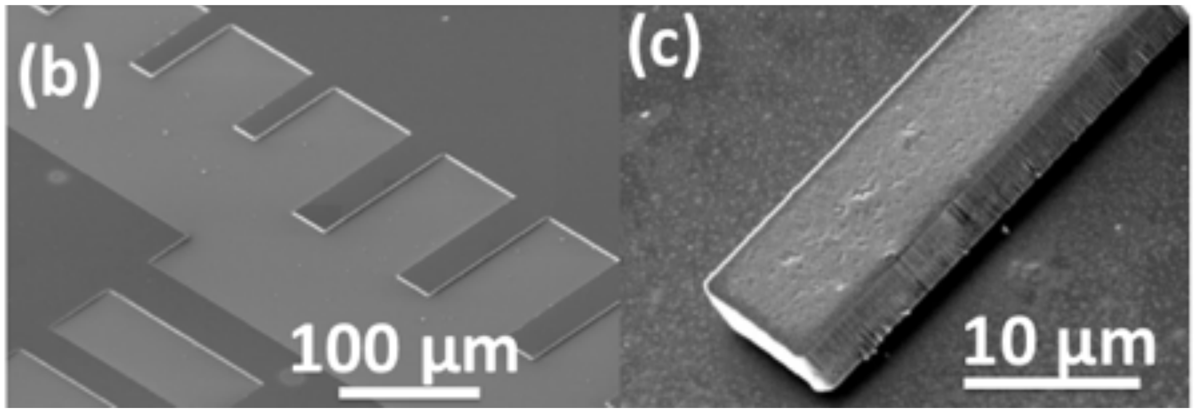
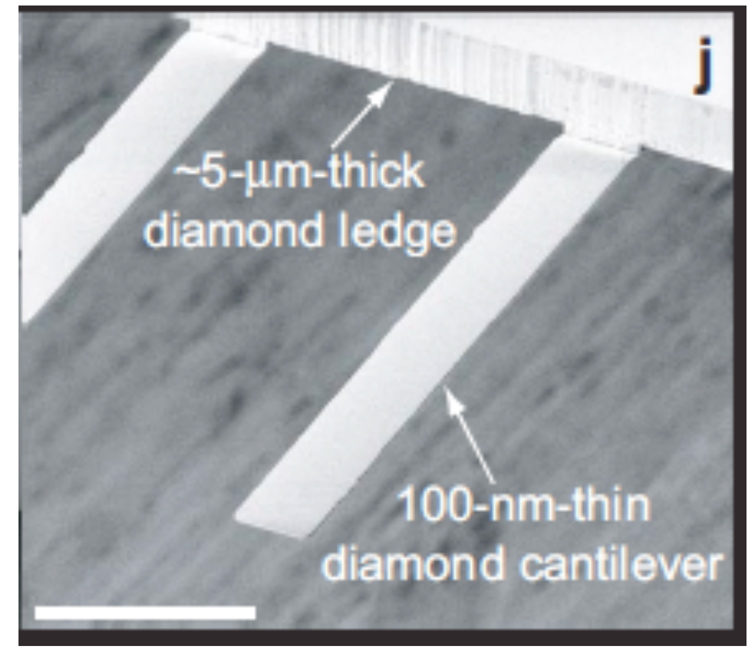
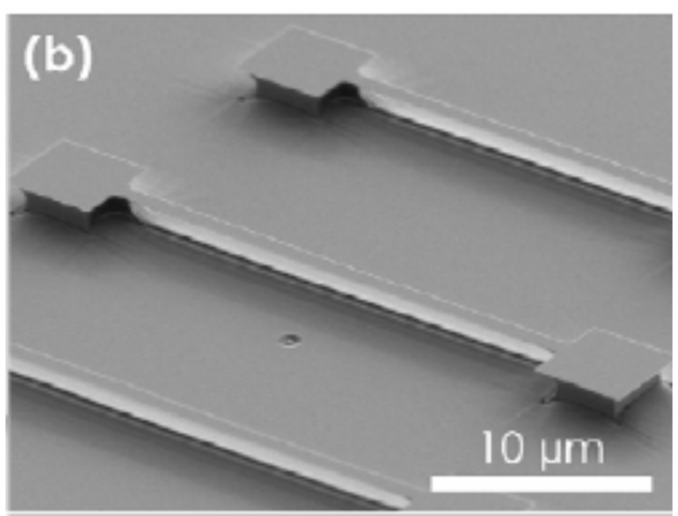


(A.C. Bleszynski-Jayich, Santa Barbara, 2012)



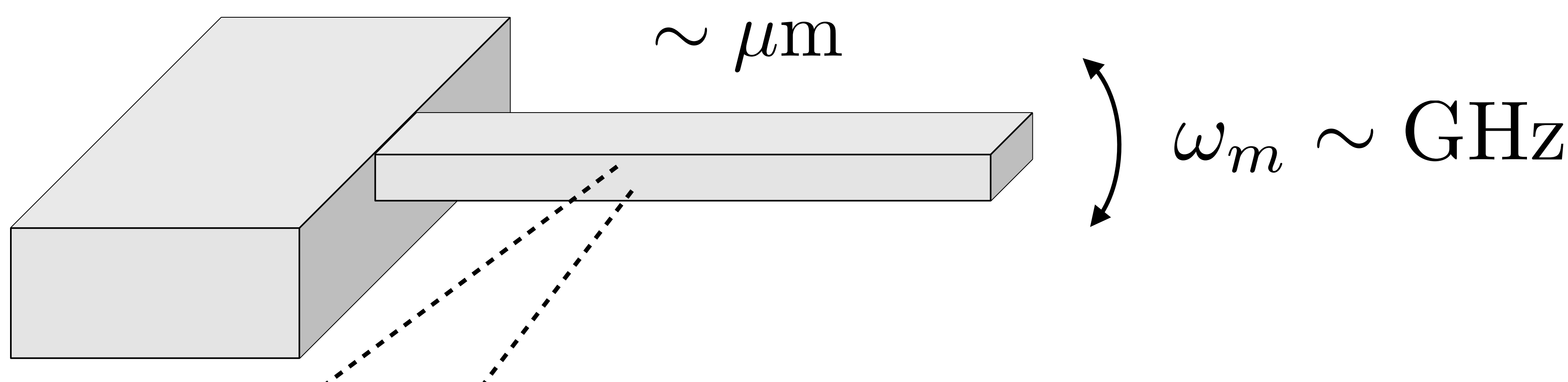
(C. Degen, ETH, 2012)



(M. Loncar, Harvard, 2013)

- ☒ ***"Mechanical cavities"***
- ☒ ***Q ~ 10^6, ~ GHz frequencies***

Strain coupling of NV defects in diamond



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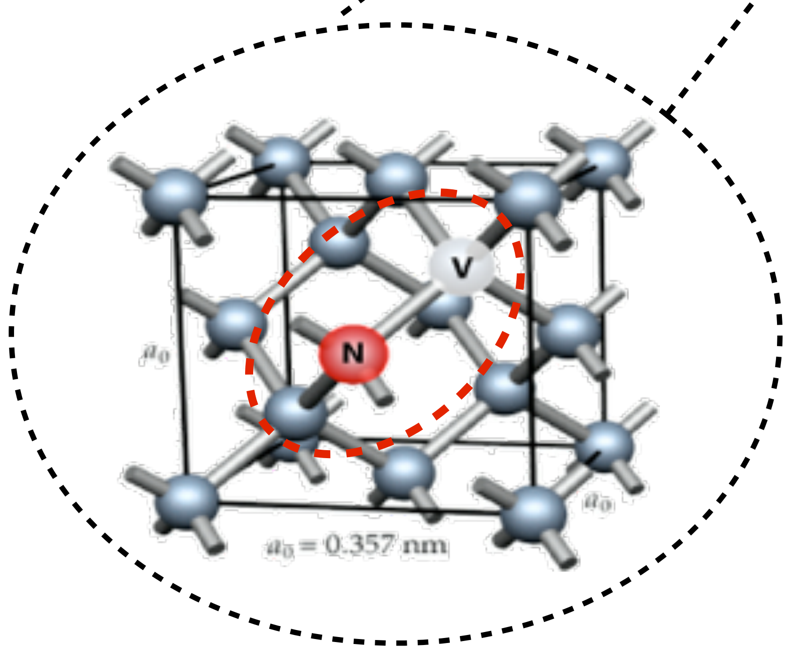
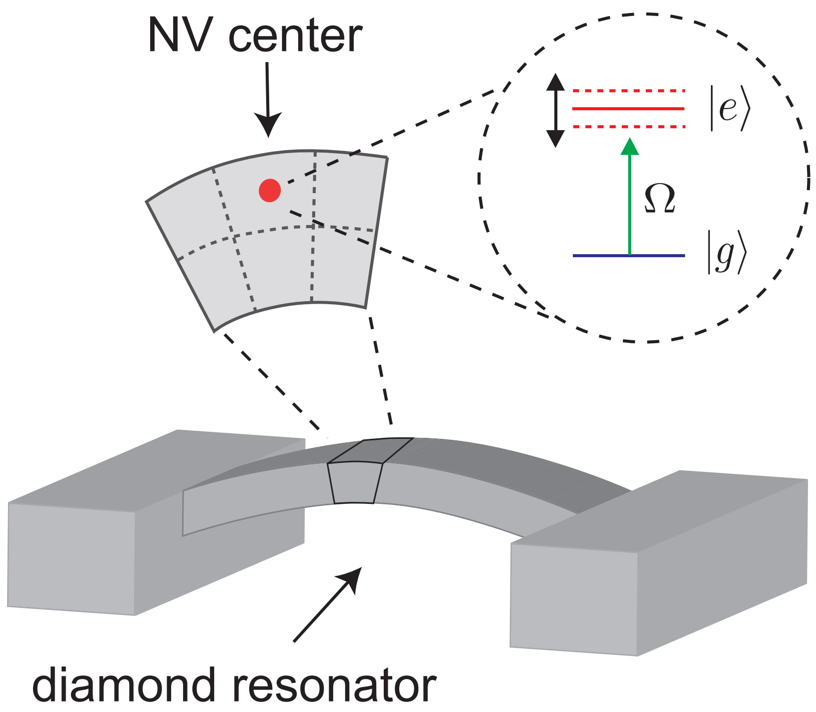
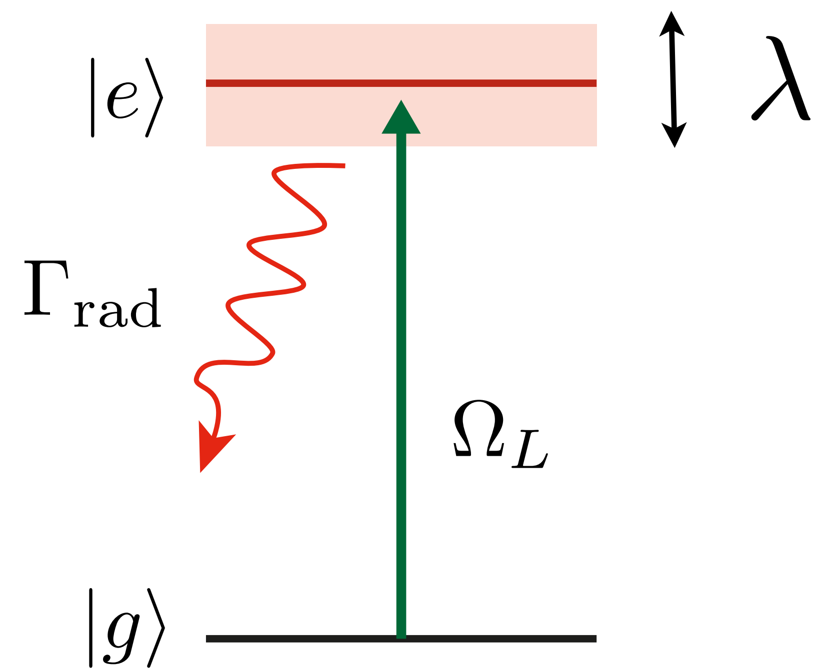


NV center

Nitrogen-vacancy (NV) centers:
-) long spin coherence, ESR control
-) optical control & readout

Strain coupling to defect states





[1] Quantum Dots: I. Wilson-Rae, P. Zoller, A. Imamoglu, PRL (2004)

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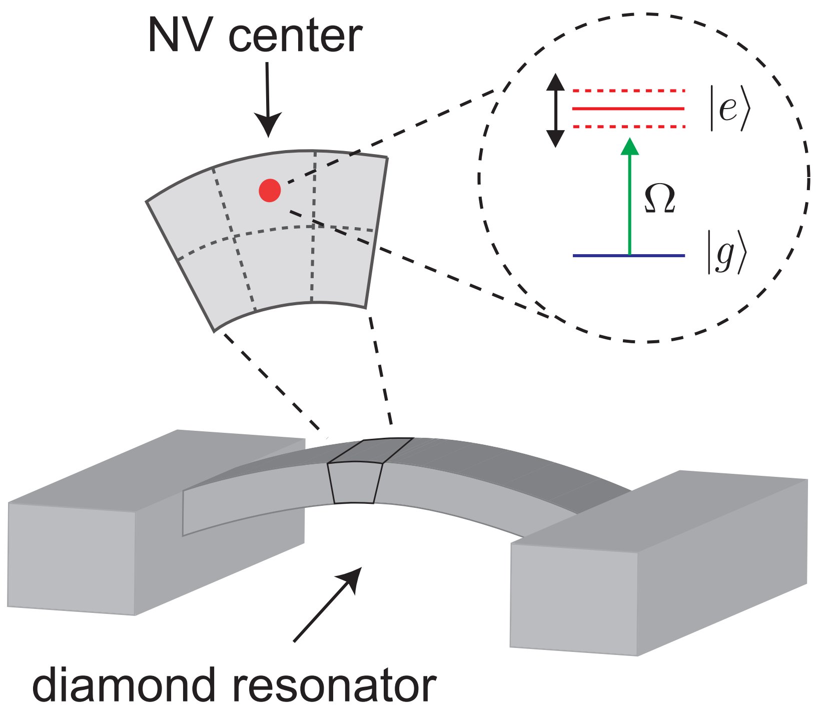

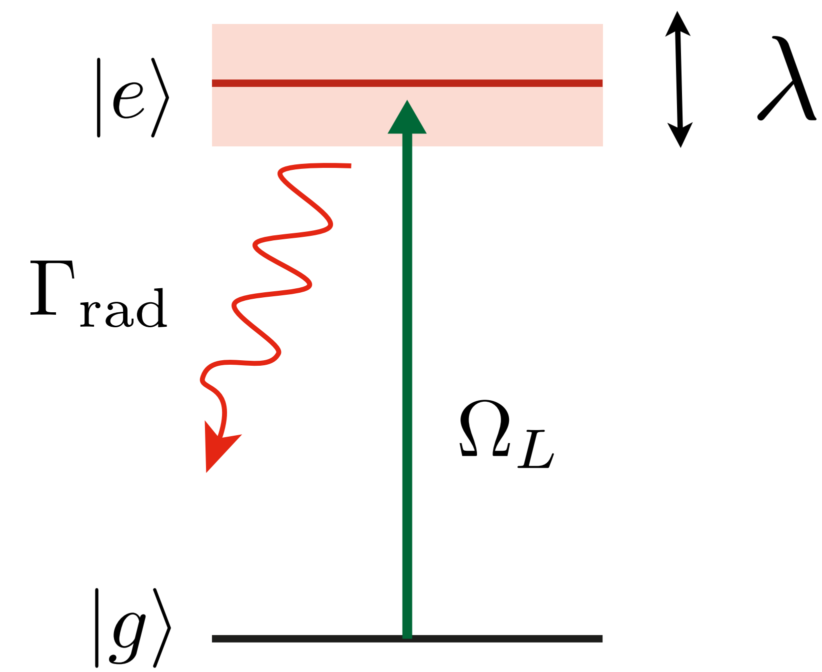

-) "strong" coupling / per phonon $\lambda \sim 10\,\mathrm{MHz} \gg \gamma_m$

-) radiative decay $\Gamma_{\rm rad} \sim \lambda$

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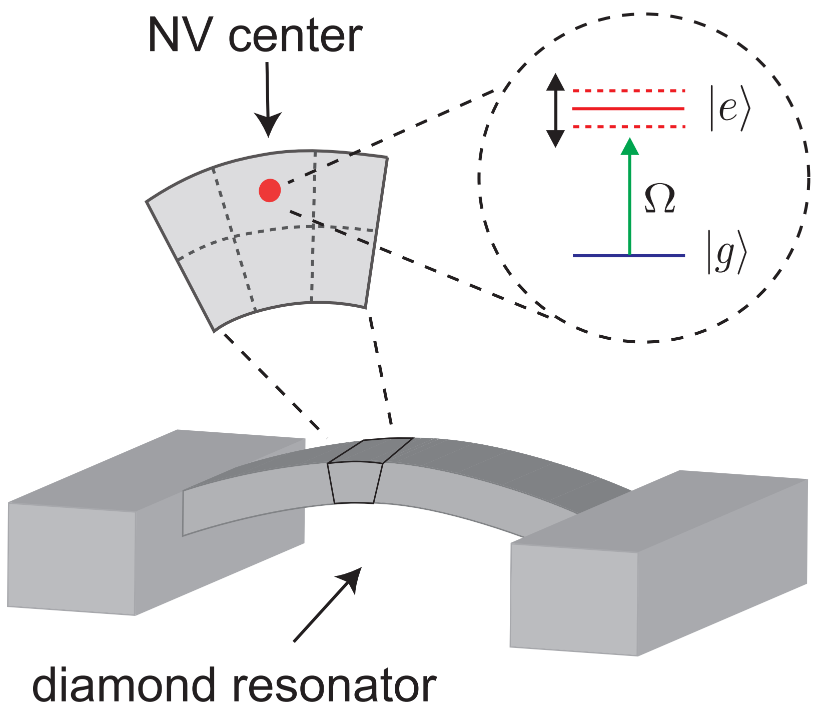

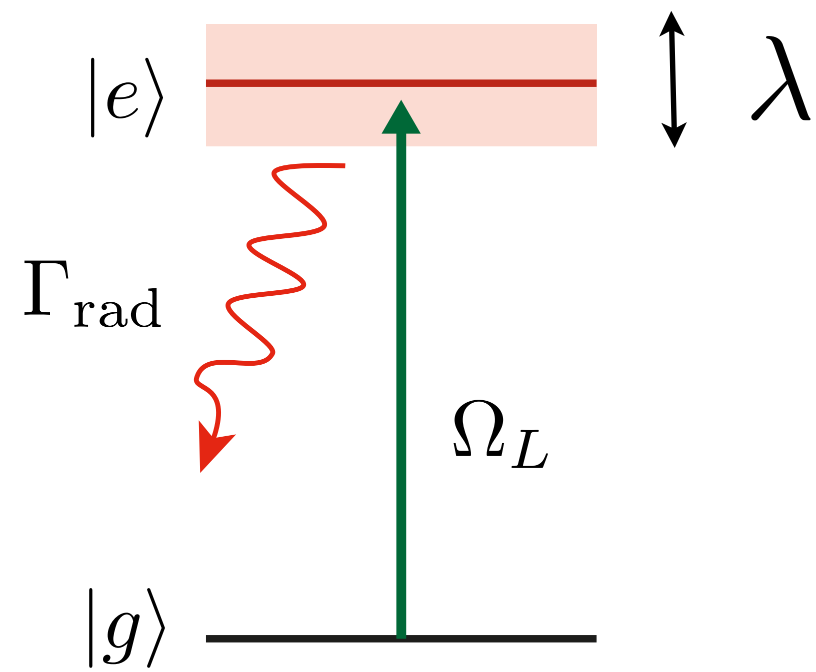

-) "strong" coupling / per phonon $\lambda \sim 10\,\mathrm{MHz} \gg \gamma_m$

-) radiative decay $\Gamma_{\mathrm{rad}} \sim \lambda$

- Significant backaction on the mechanical state !

  $\Rightarrow$ *ground state cooling [1], phonon lasing, ...*

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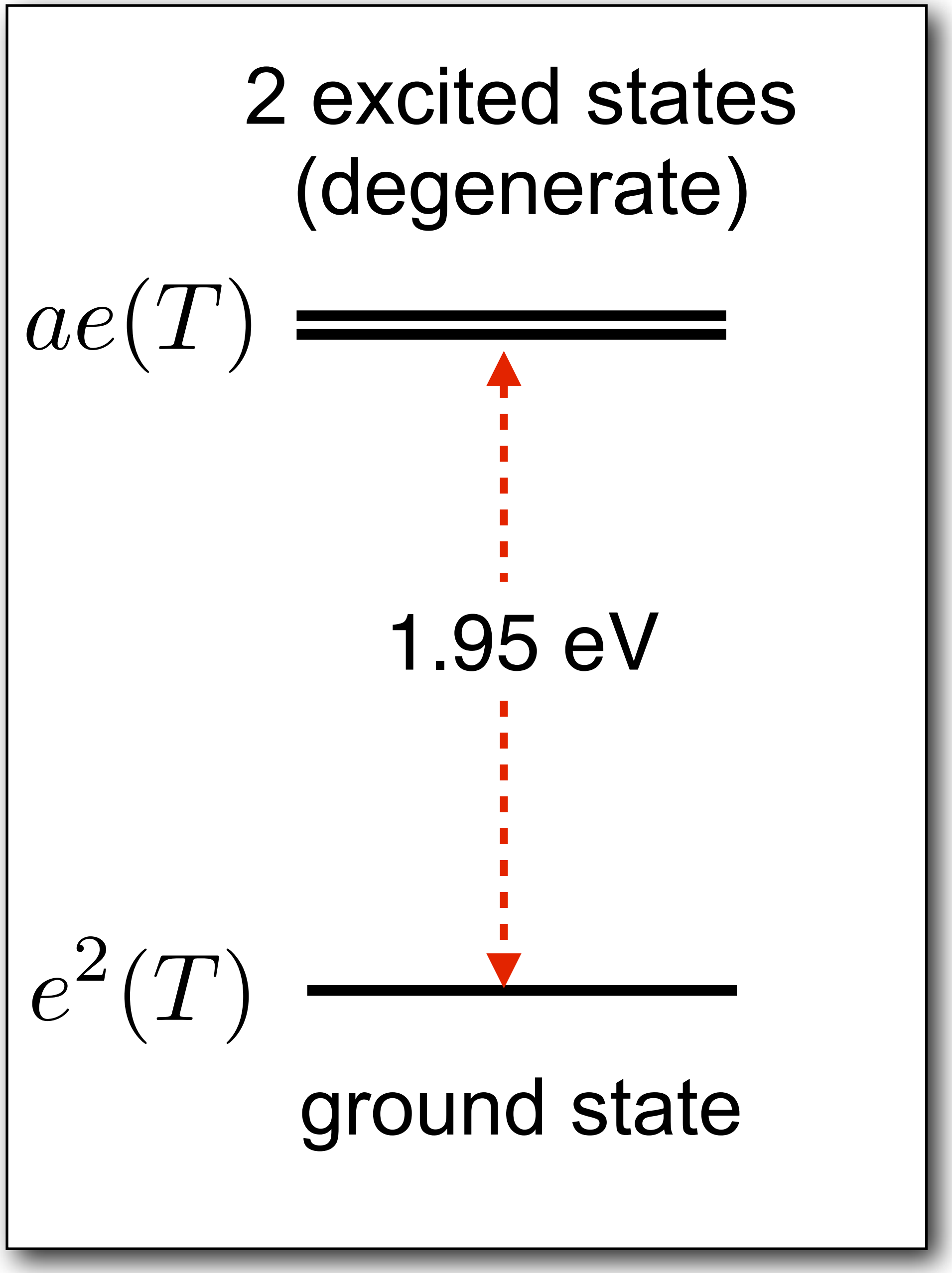
NV-phonon interactions

NV level-structure (without spin) [1]:



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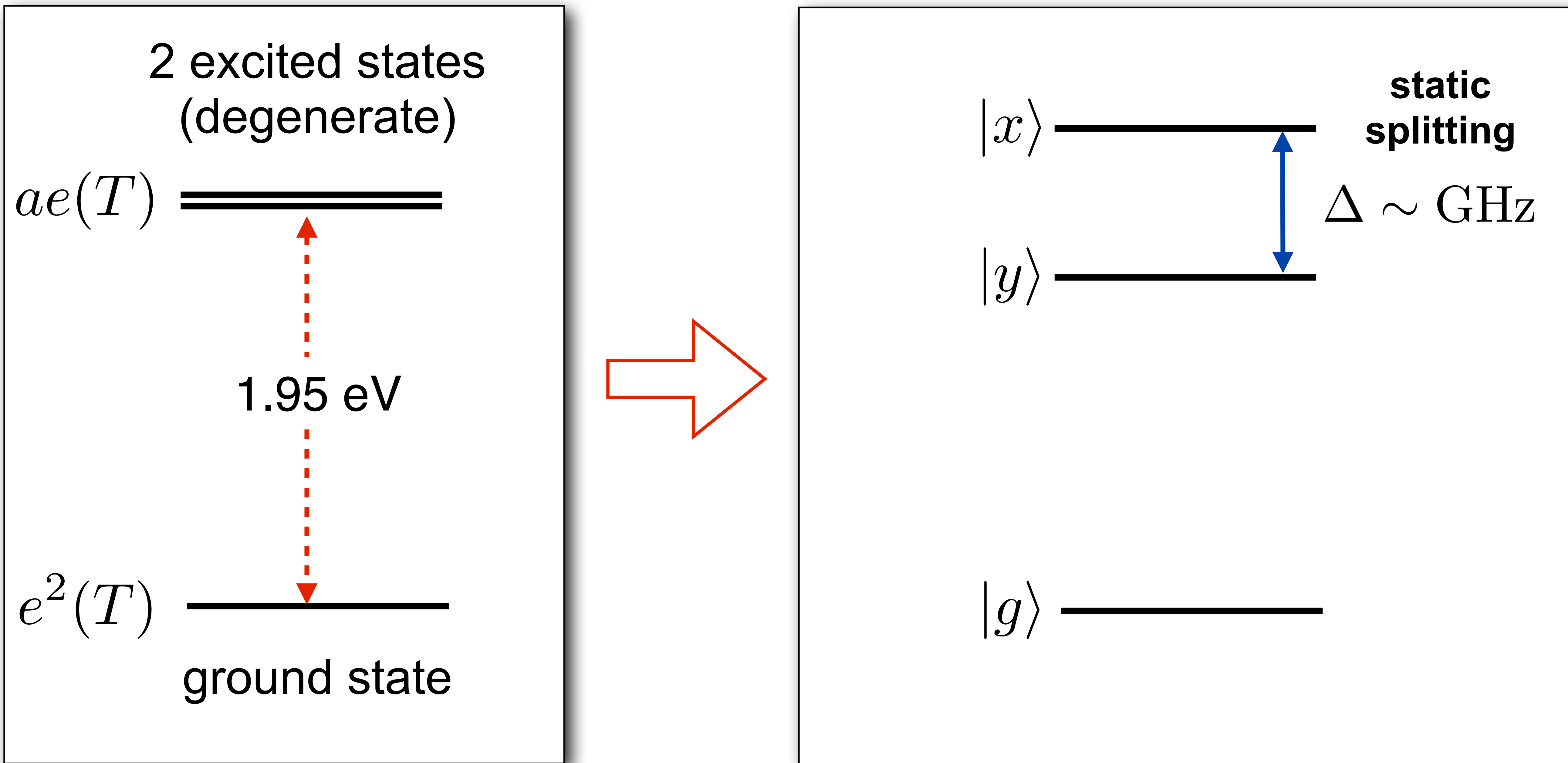

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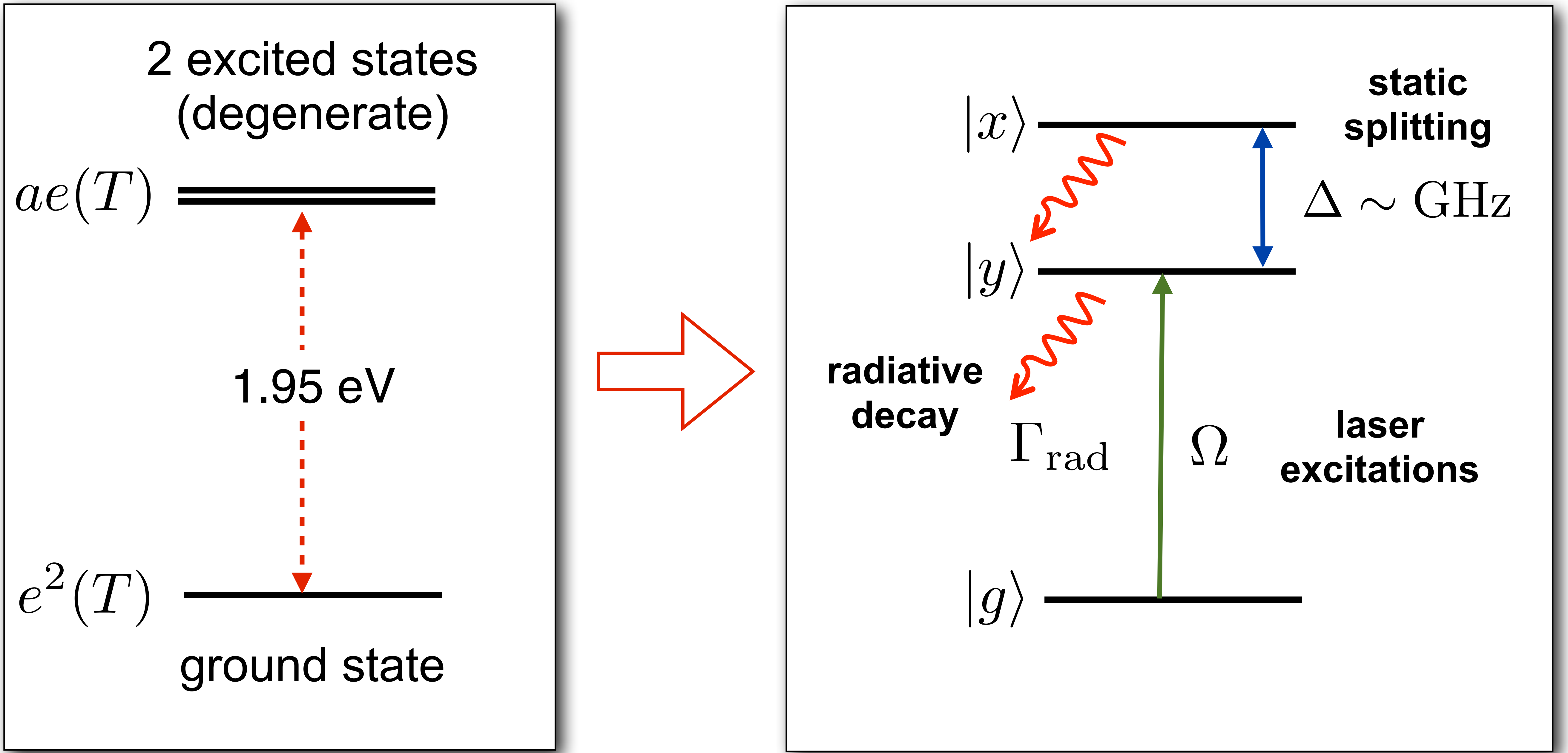

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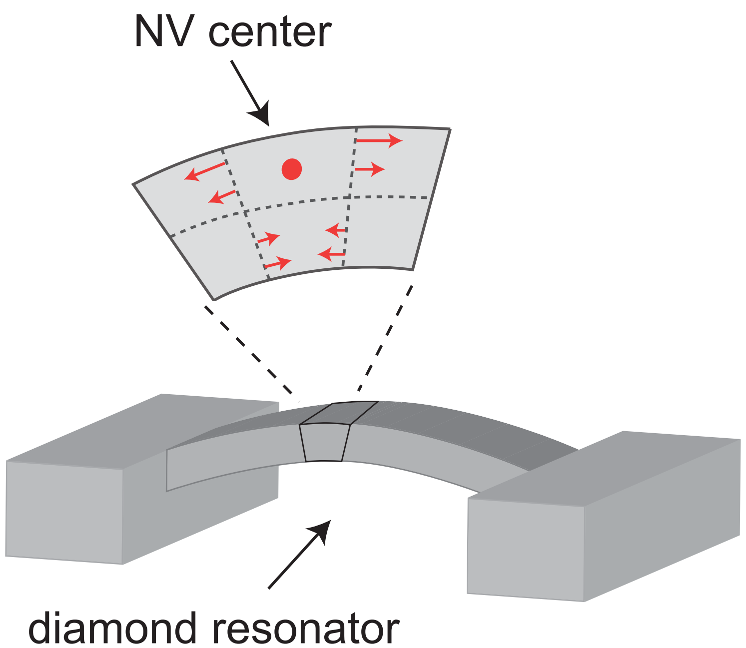
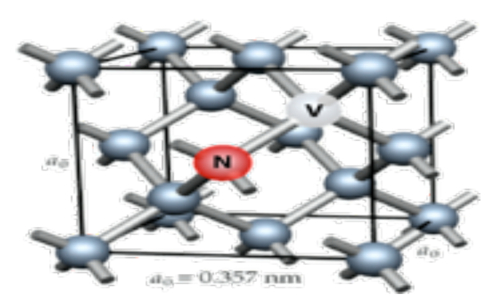
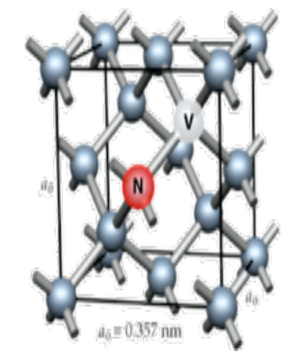


static / oscillating strain:





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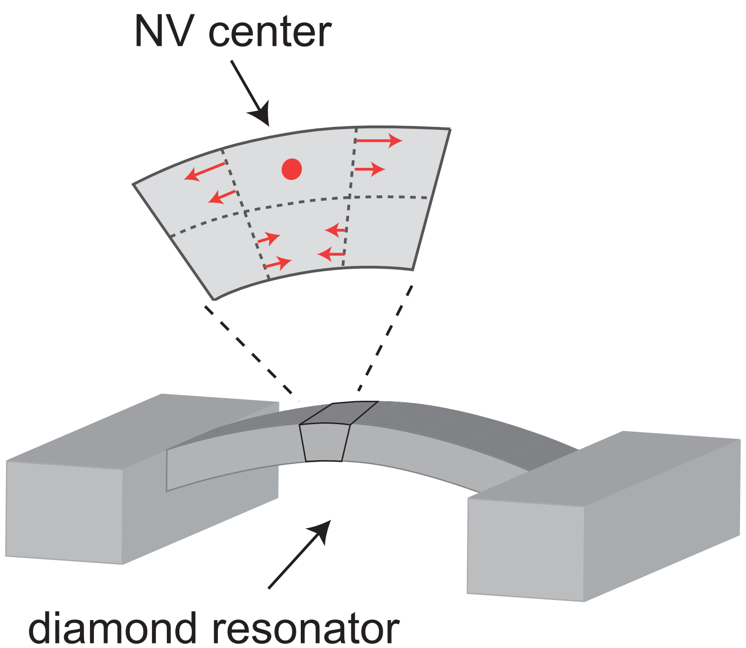

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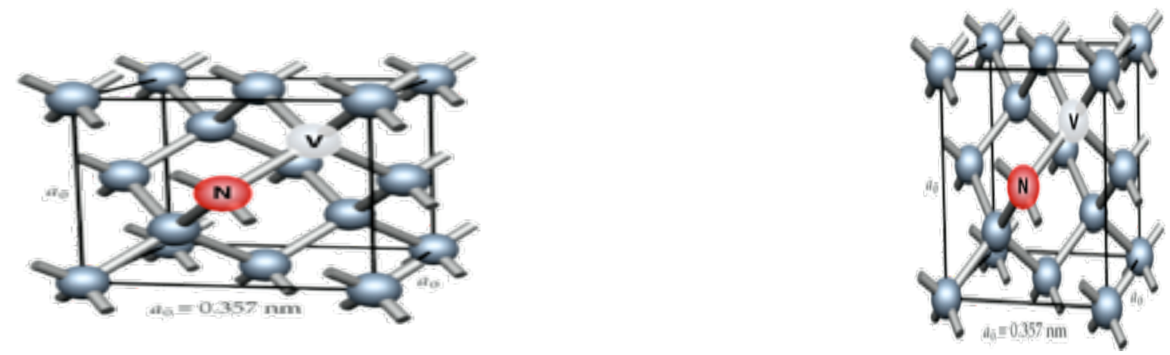
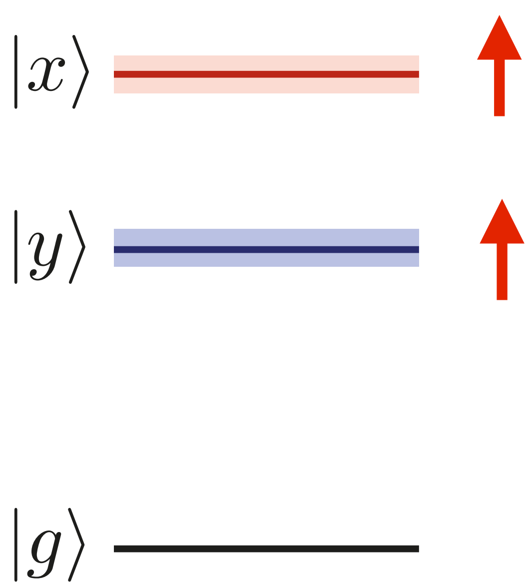
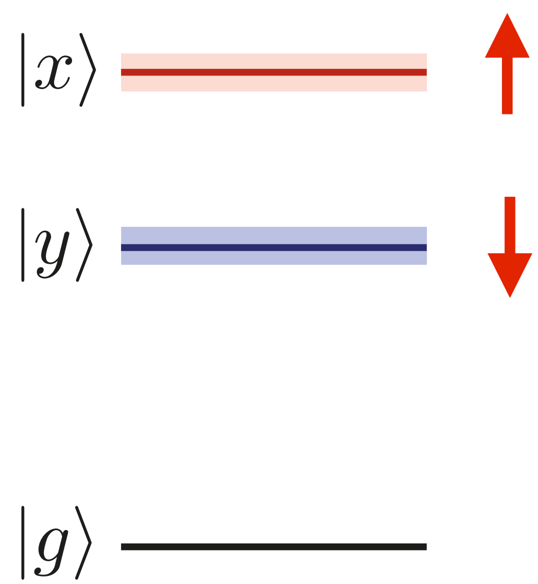
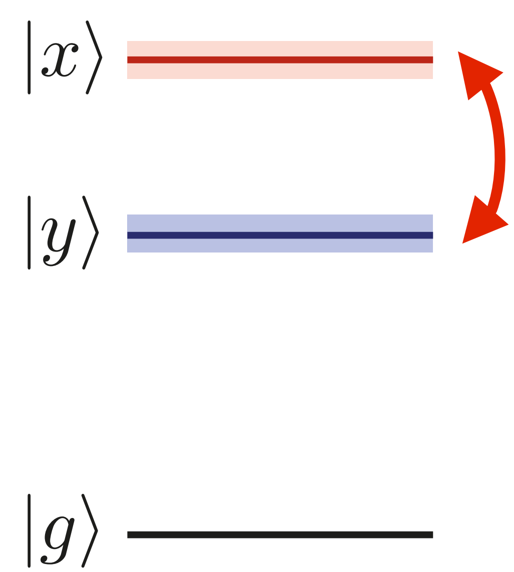




$$\Sigma_0 = |x\rangle\langle x| + |y\rangle\langle y|$$



$$\Sigma_{||} = |x\rangle\langle x| - |y\rangle\langle y|$$



$$\Sigma_\perp = |x\rangle\langle y| + |y\rangle\langle x|$$

*K.-M. Fu et al. PRL **103**, 256404 (2009); T. Abtew et al., PRL **107**, 146403 (2011).*

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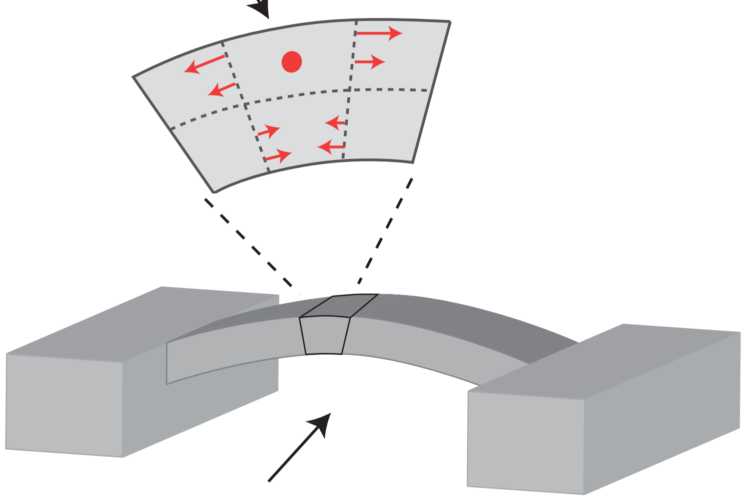

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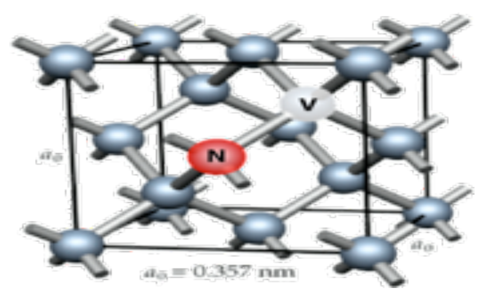
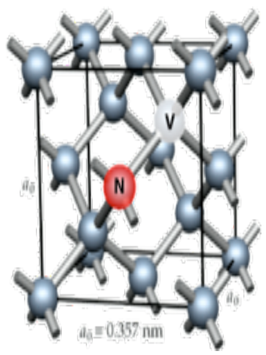
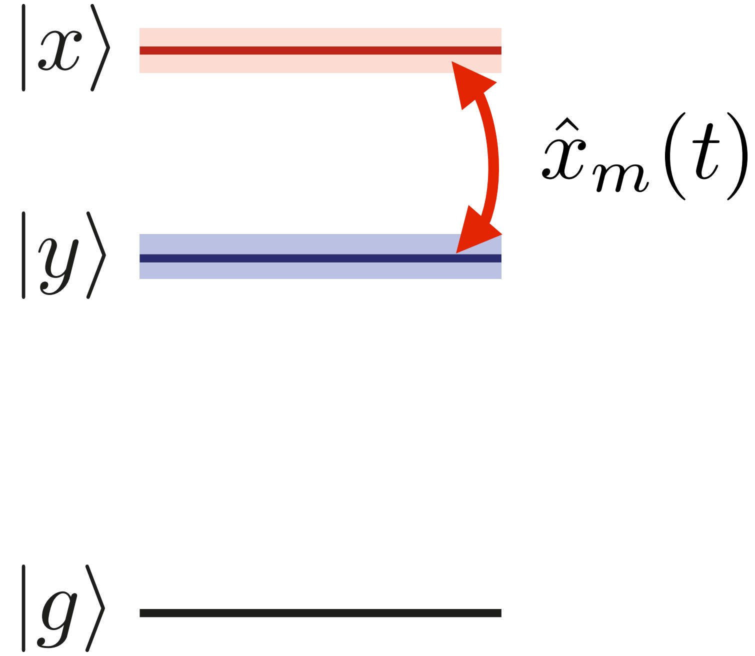






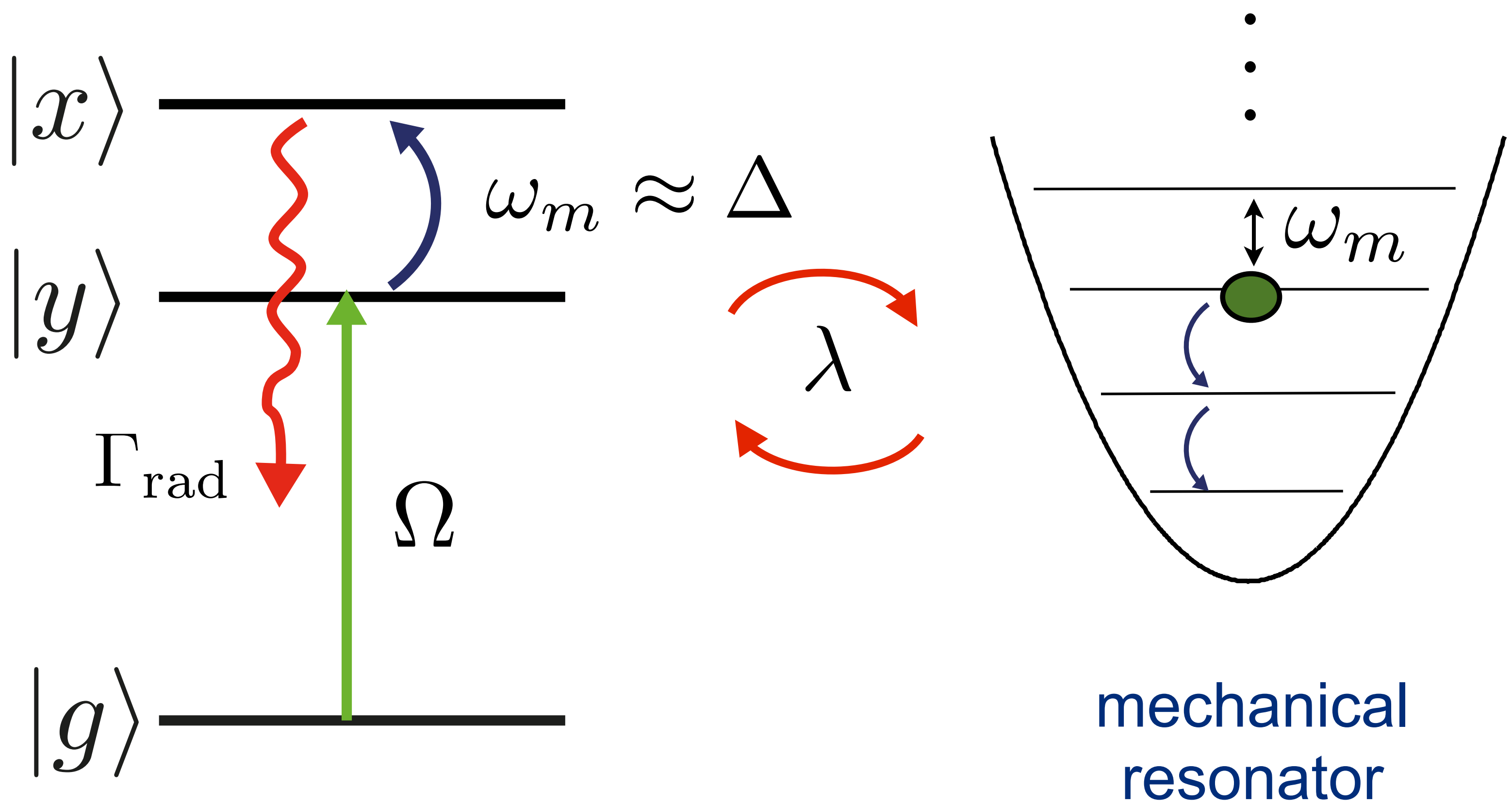
- **NV-phonon interaction:**

$$H_{\rm int} = \lambda(a + a^\dagger)(|x\rangle\langle y| + |y\rangle\langle x|)$$

- **single phonon coupling:**

$$\lambda \sim 1 - 10\ \mathrm{MHz} \lesssim \Gamma_{\rm rad}$$

Laser cooling: general idea



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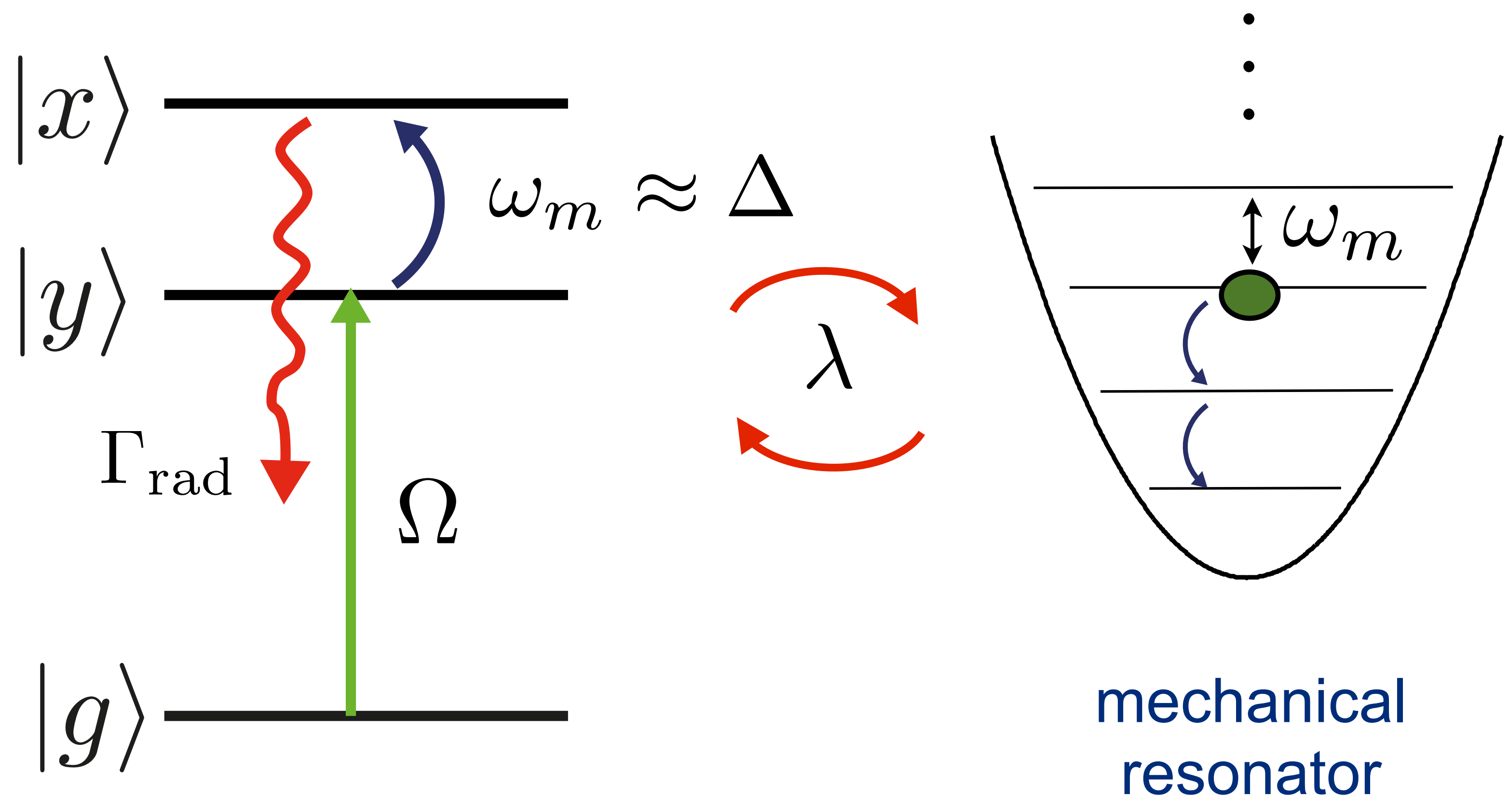

☒ **effective cooling rate:** $$\Gamma \approx \frac{\lambda^2}{\Gamma_{\rm rad}}$$

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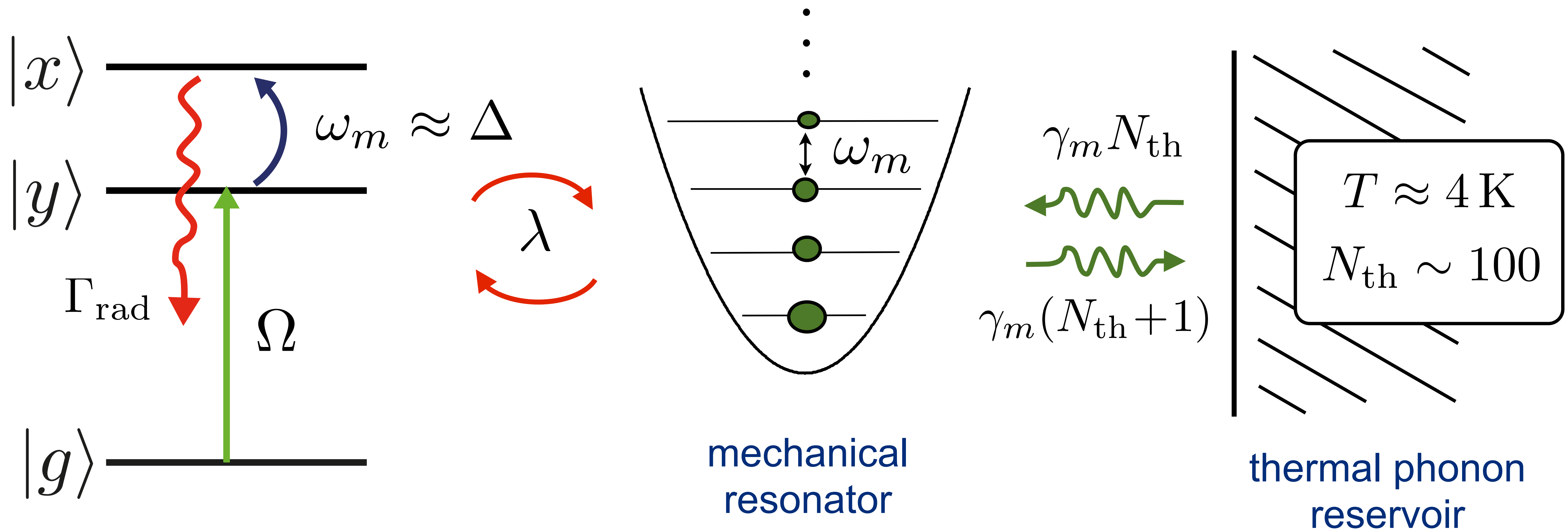
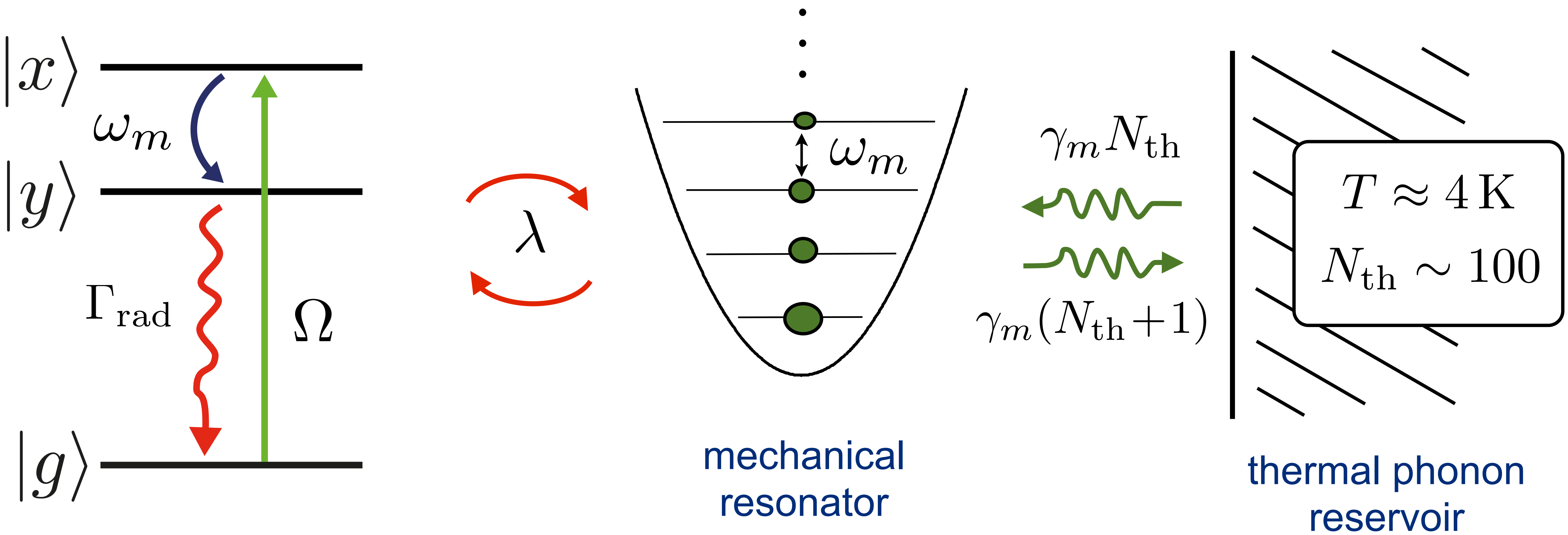


☒ **effective cooling rate:** $\Gamma \approx \dfrac{\lambda^2}{\Gamma_{\rm rad}}$

$$\langle a^\dagger a \rangle \simeq \frac{\gamma_m}{\gamma_m + \Gamma} \times N_{\rm th} \quad \longrightarrow \quad \mathcal{O}(1)$$

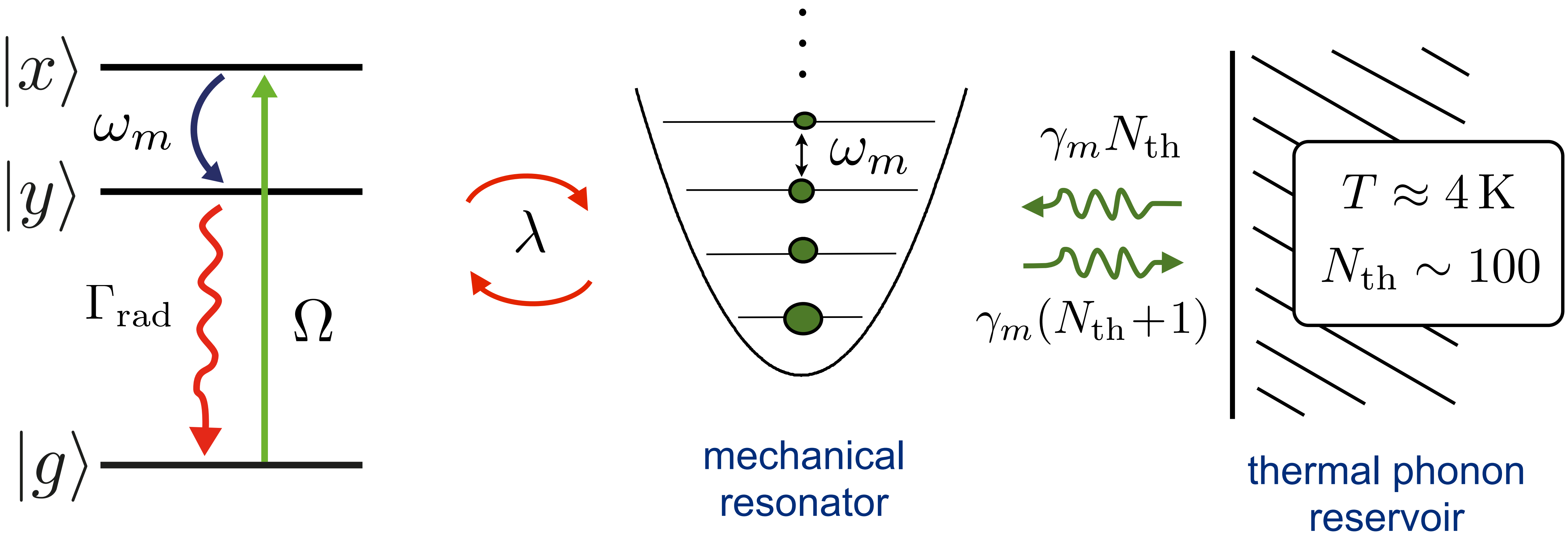
Phonon lasing



☒ **effective heating rate:** $\Gamma_{\rm h} = -\Gamma < 0$

$$\langle a^\dagger a \rangle \simeq \frac{\gamma_m}{\gamma_m - \Gamma} \times N_{\rm th} \longrightarrow \infty$$

Phonon lasing



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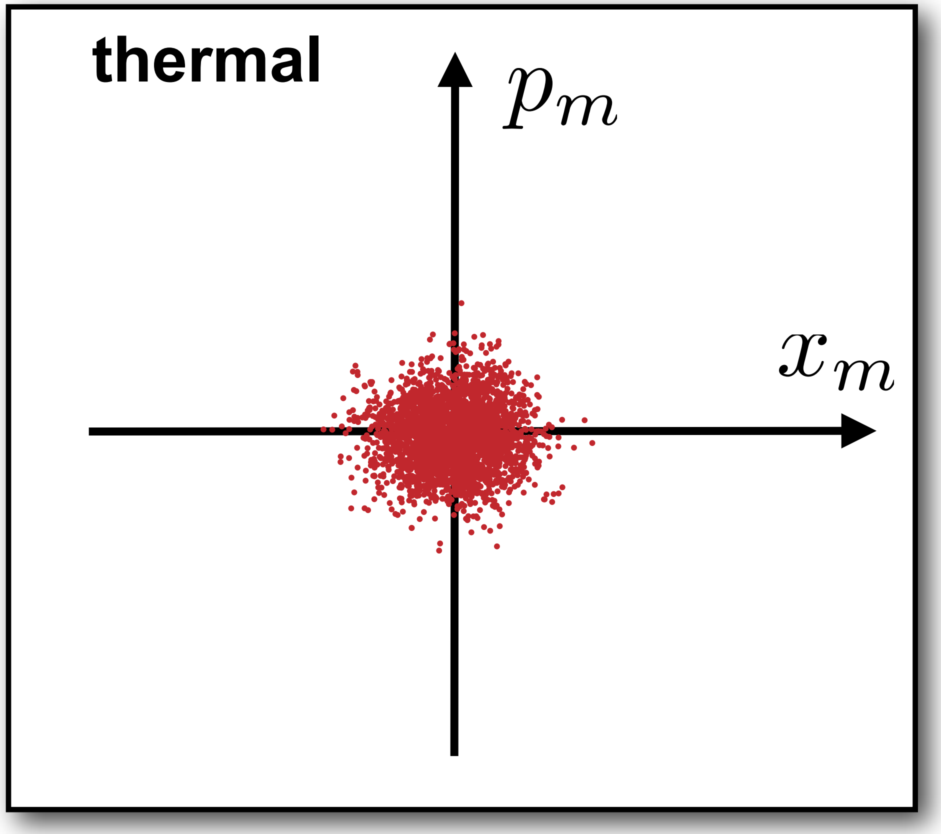
☒**saturation!!!**

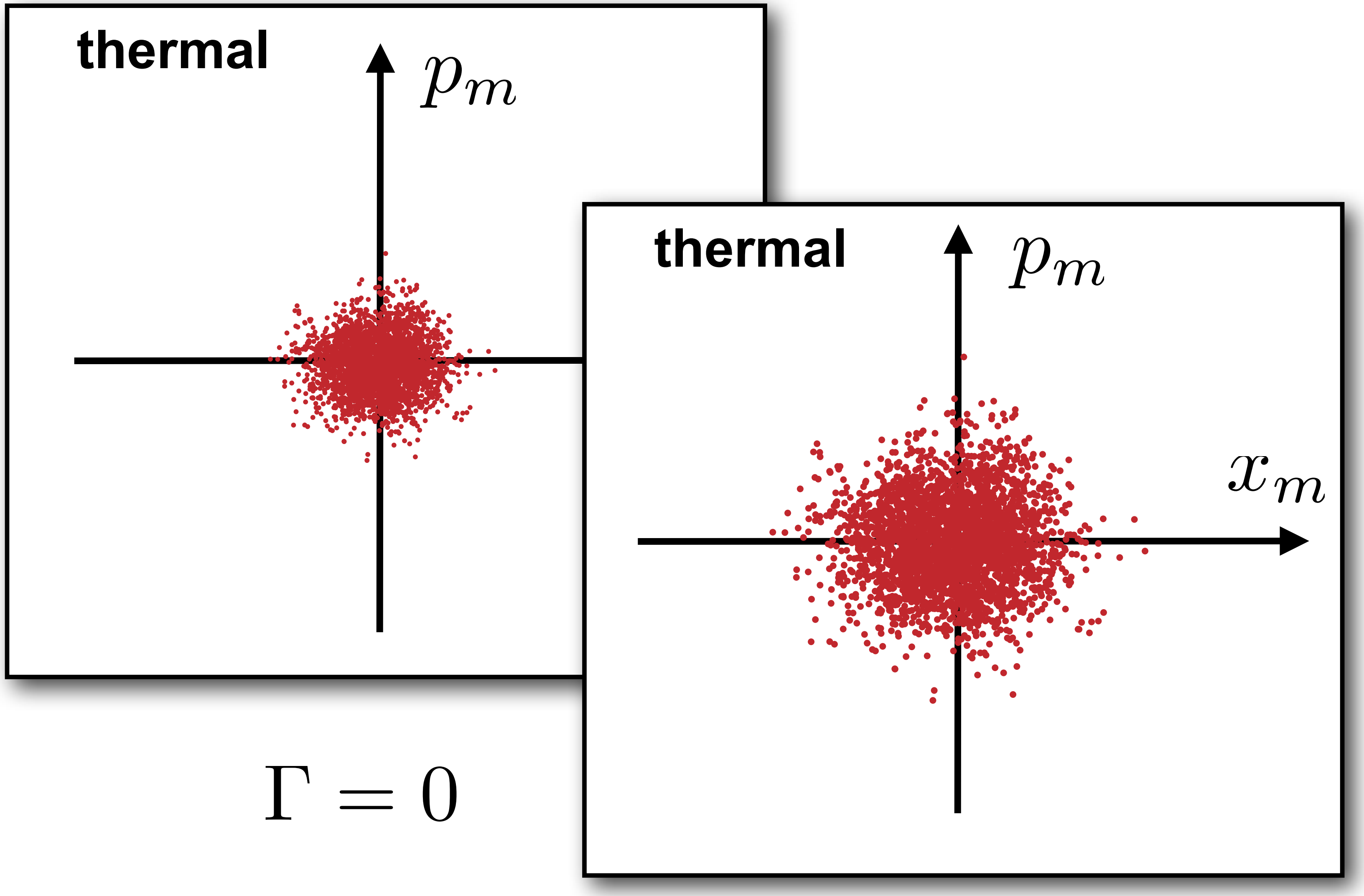
$$\langle a^\dagger a \rangle \simeq \frac{\gamma_m}{\gamma_m - \Gamma} \times N_{\rm th} \longrightarrow \infty$$

Phonon lasing



$$\Gamma = 0$$

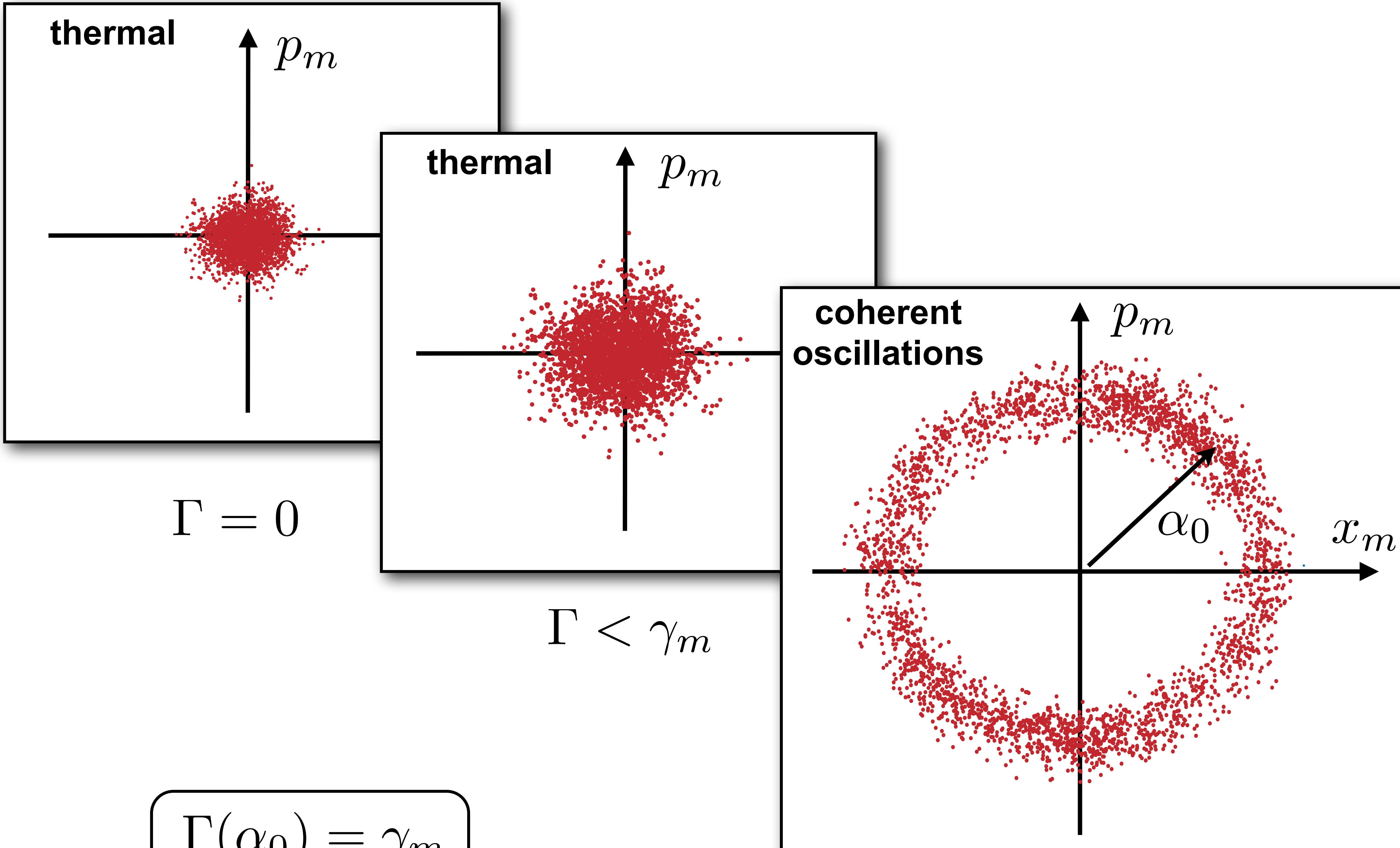
Phonon lasing



$$\Gamma = 0$$

$$\Gamma < \gamma_m$$

Phonon lasing



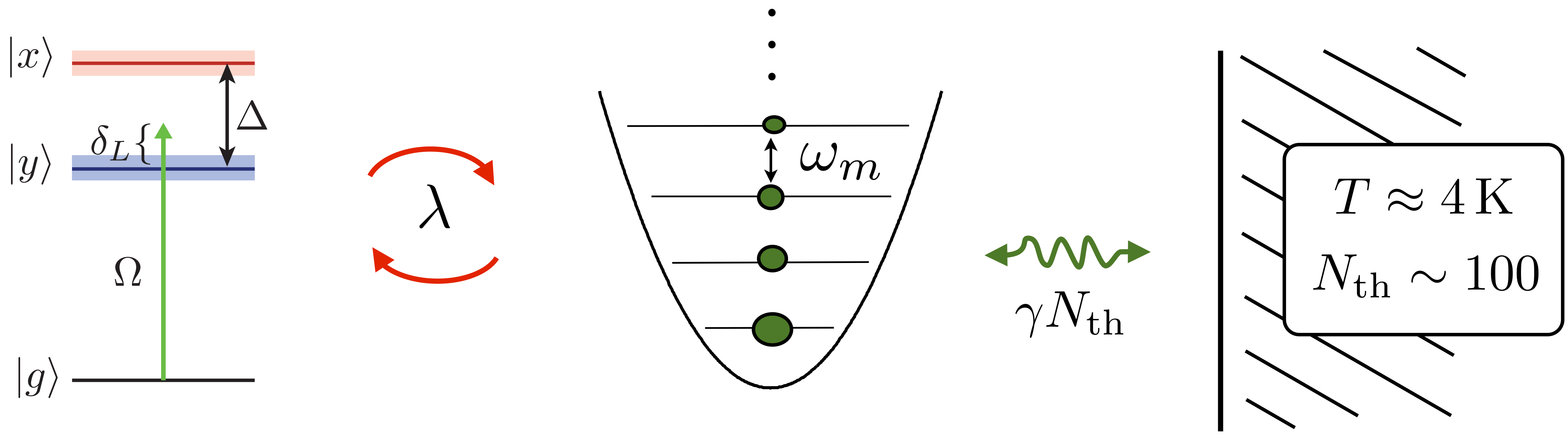
$\Gamma = 0$

$\Gamma < \gamma_m$

$\Gamma > \gamma_m$

$$\Gamma(\alpha_0) = \gamma_m$$

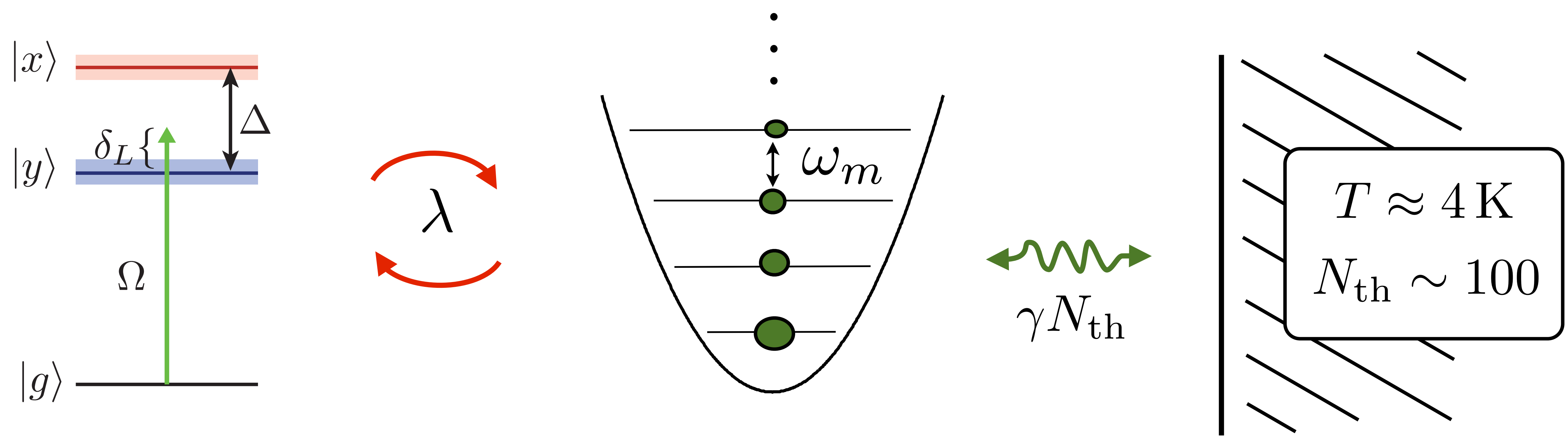
Summary: Phonon lasing/cooling



Effective resonator dynamics (rotating frame):

$$\dot{\alpha} = \Big(\pm\Gamma(\alpha) - \gamma \Big)\alpha + \sqrt{2\gamma N_{\rm th}}\,\xi(t)$$

Summary: Phonon lasing/cooling



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non-linear gain/loss:

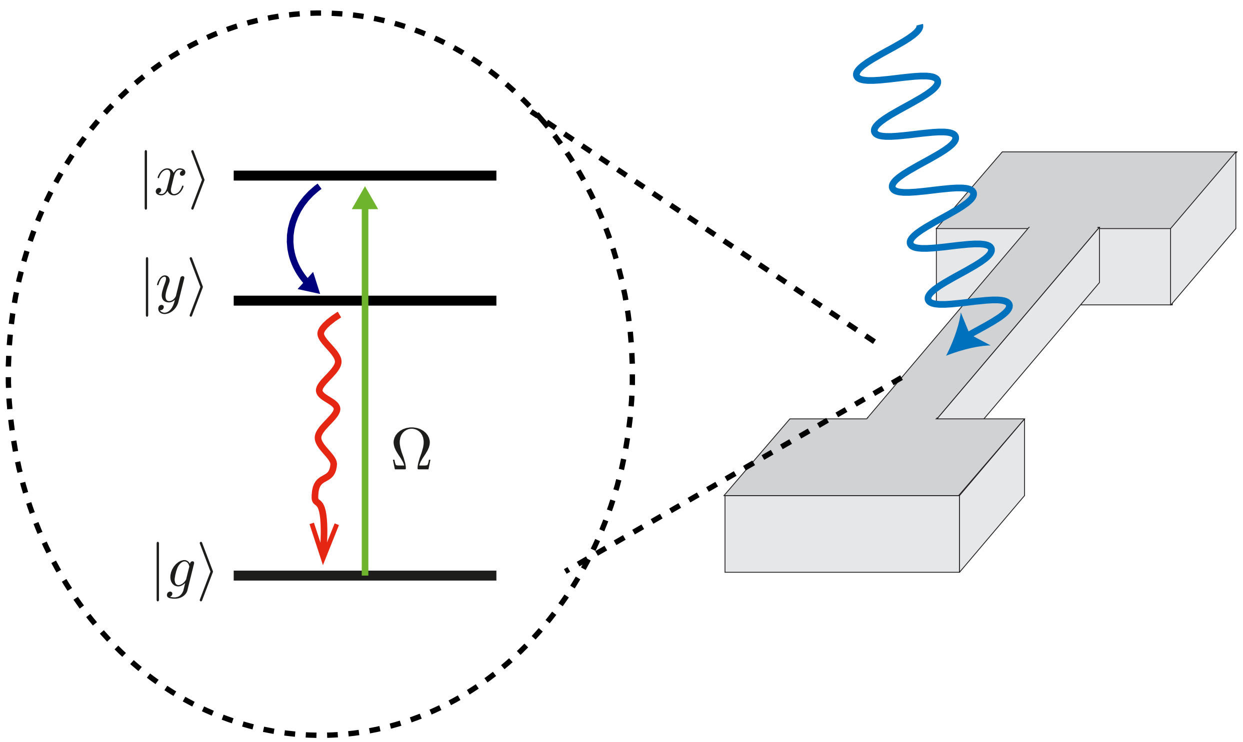
$$\Gamma(\alpha) = \frac{\Gamma}{(1 + |\alpha|^2/n_0)^2}$$

thermal noise

cutoff phonon number

Cooling AND Lasing

Cooling AND Lasing



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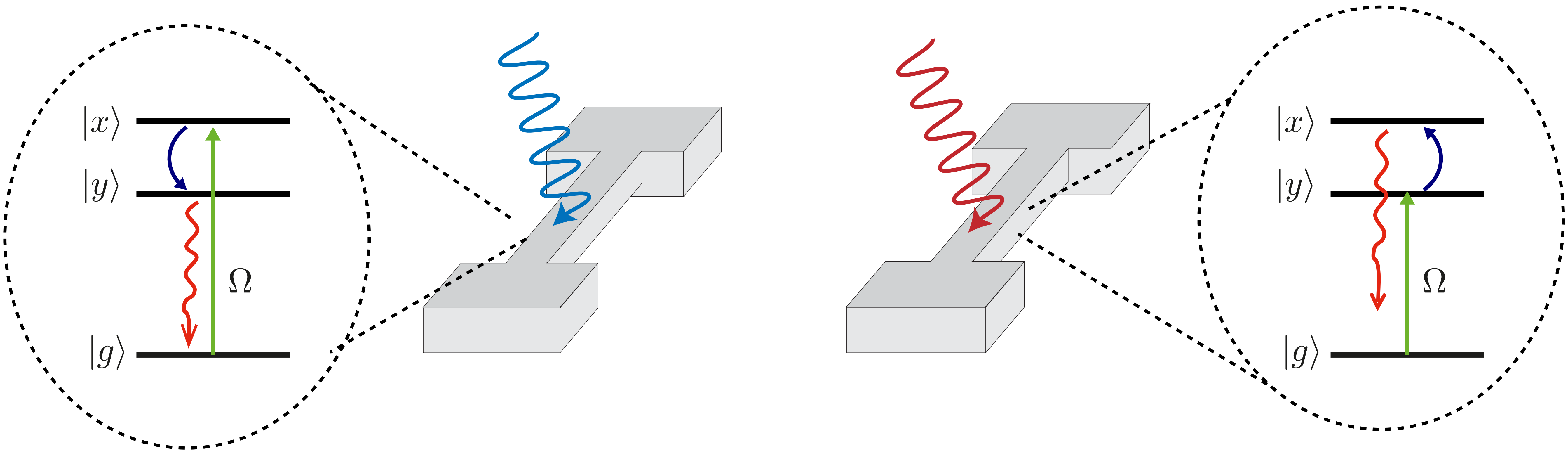

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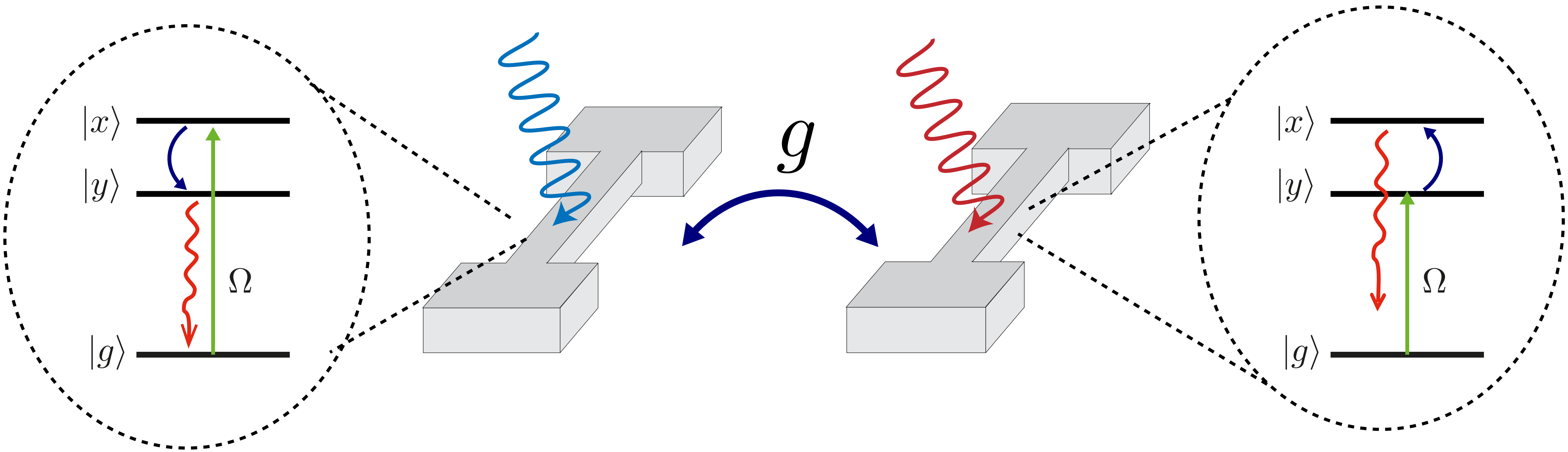

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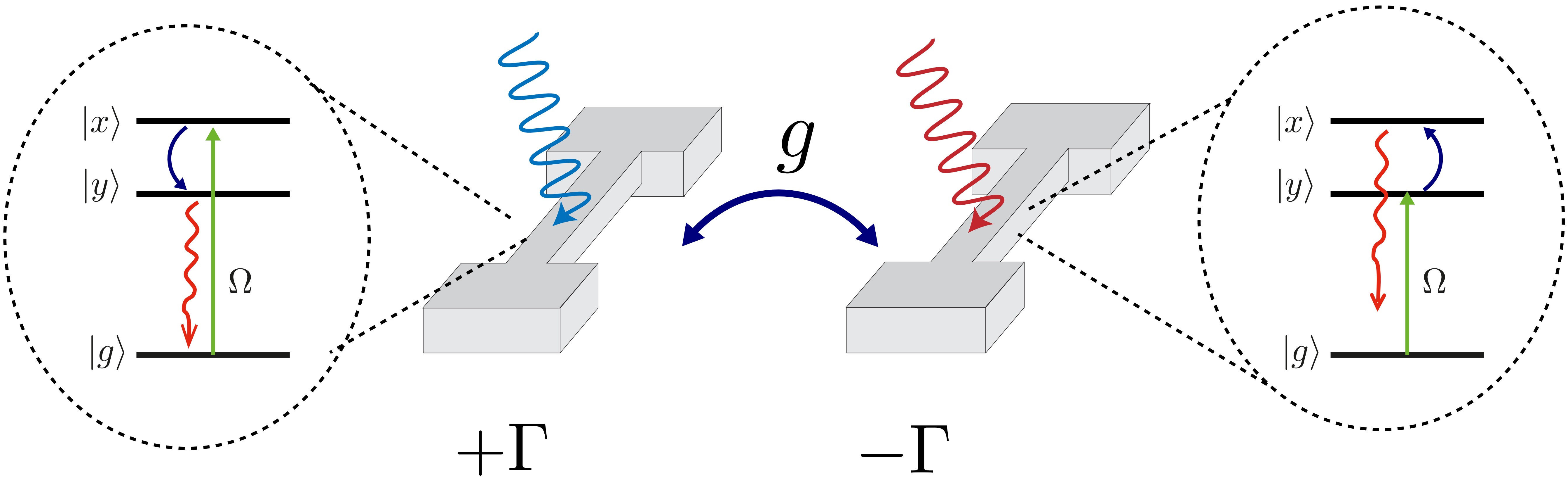
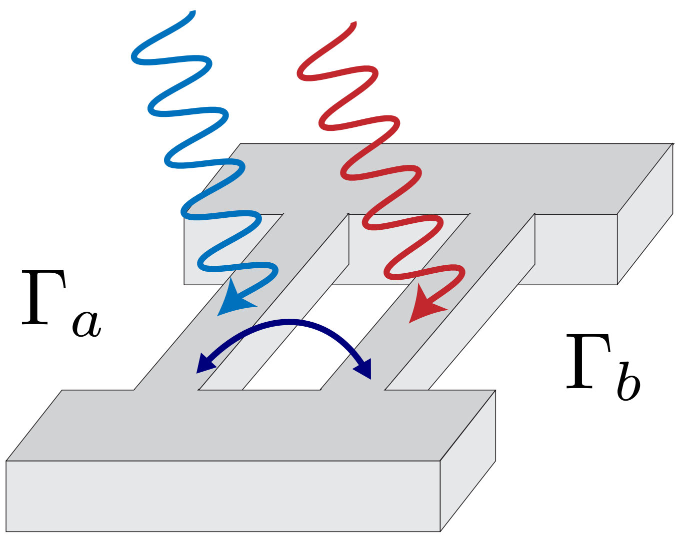


Gain = Loss ???

Non-Hermitian "Two-Level-System"



- ***Coupled modes (linear, rotating frame):***

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = -i \begin{pmatrix} -i\Gamma_a & g \\ g & -i\Gamma_b \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

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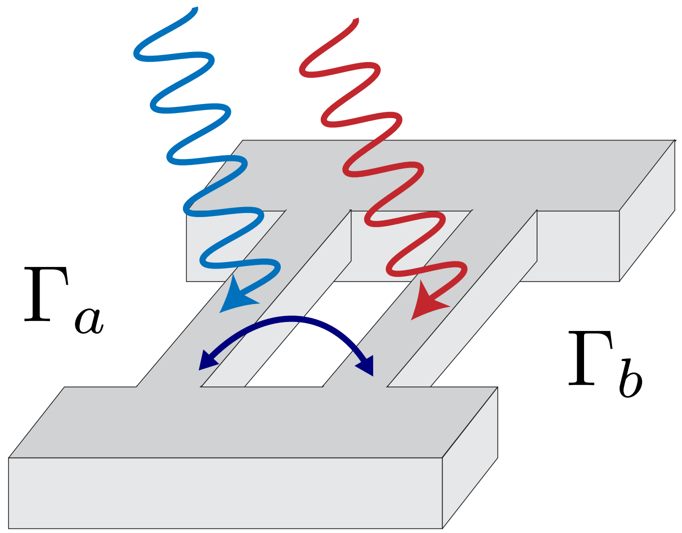

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non-Hermitian "Hamiltonian" !!

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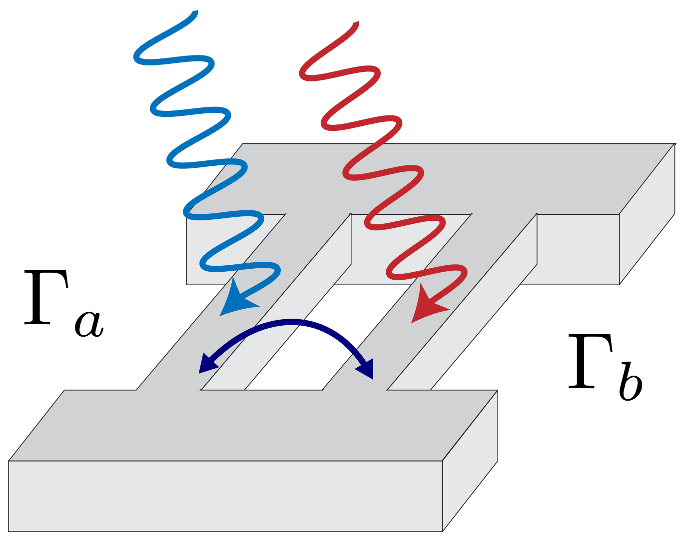

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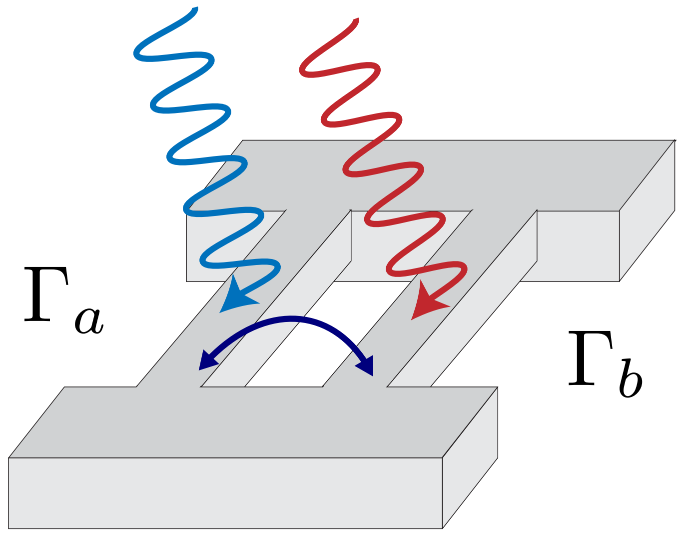
non-Hermitian "Hamiltonian" !!

$\mathcal{PT}$ - symmetric for $\Gamma_a = -\Gamma_b$

- Parity: $\mathcal{P} : (\alpha, \beta) \to (\beta, \alpha)$
- Time-reversal: $\mathcal{T} : i \to -i$

$$(\mathcal{PT})H(\mathcal{PT}) = H$$

PT-symmetry breaking



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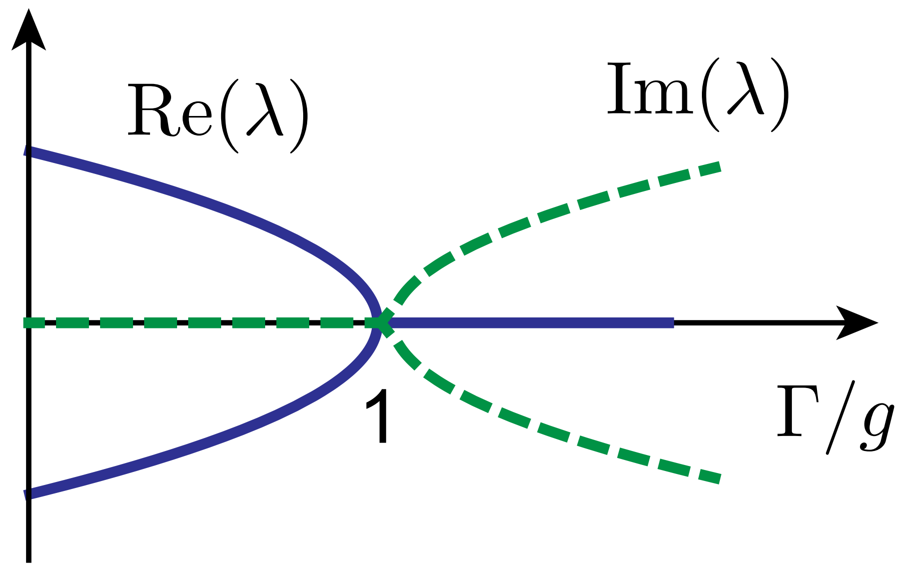

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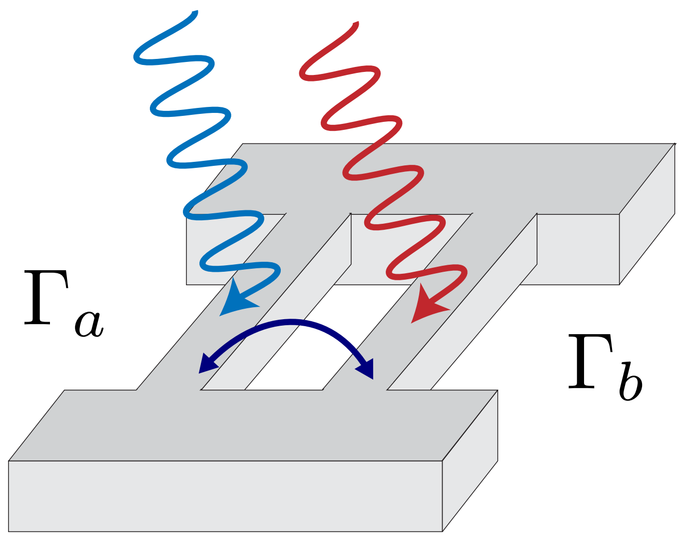

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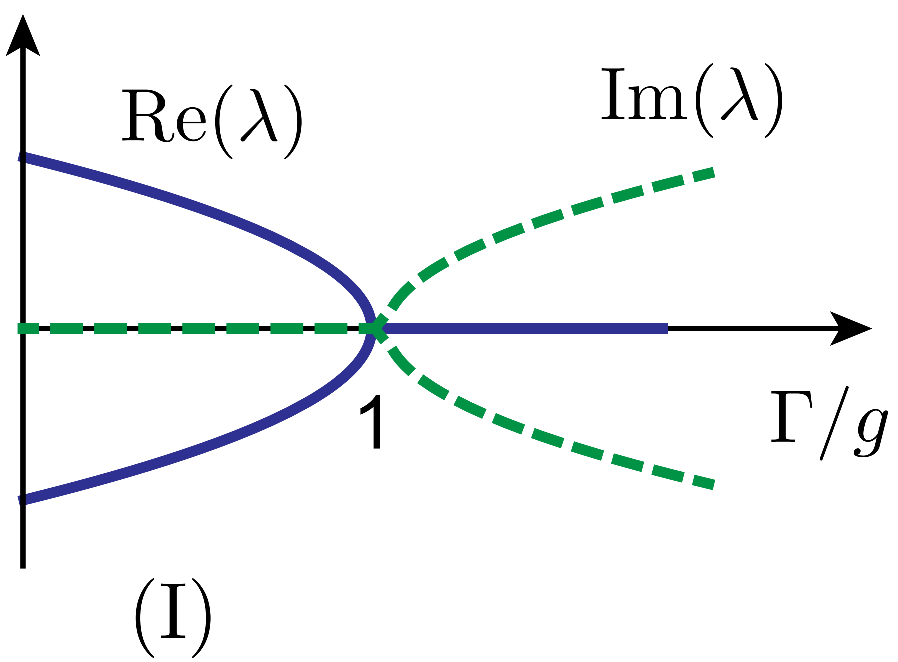
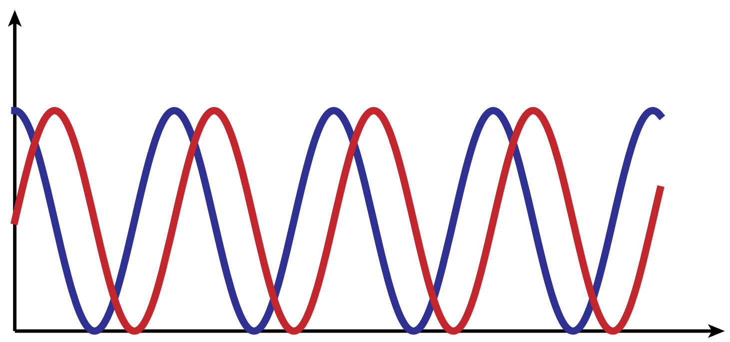


(I) purely oscillatory:



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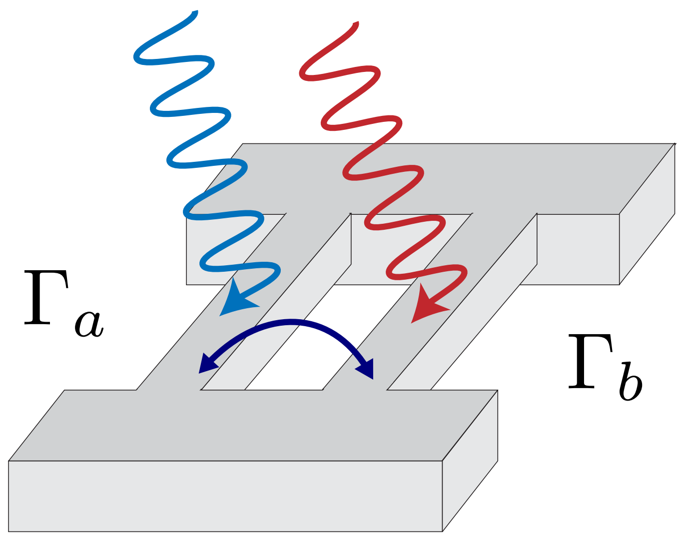

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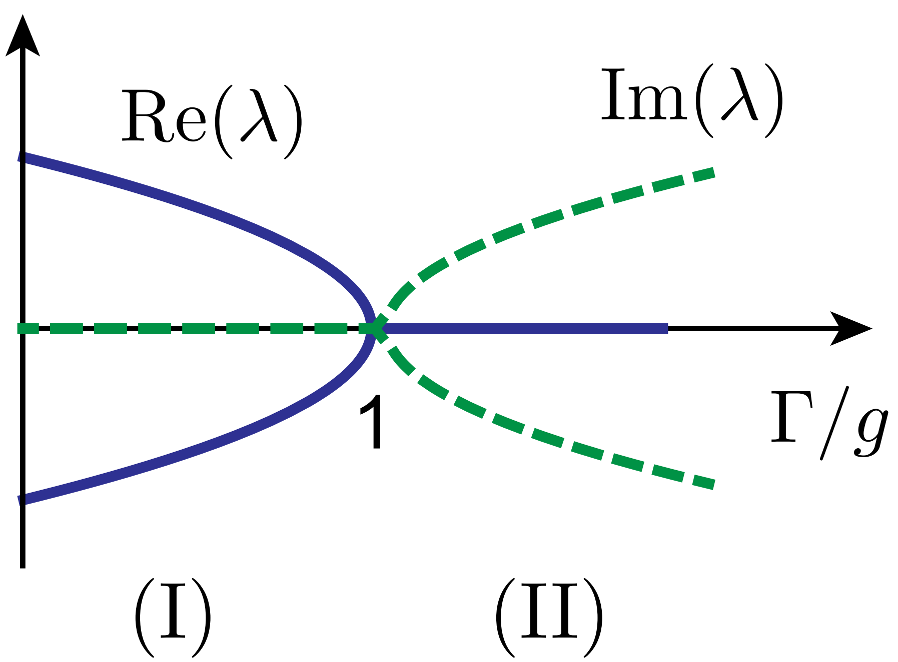

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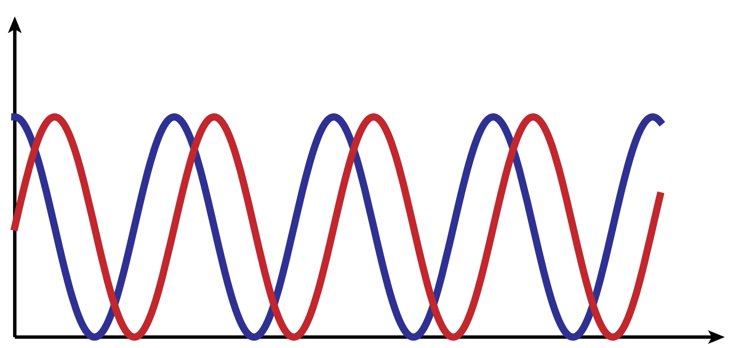
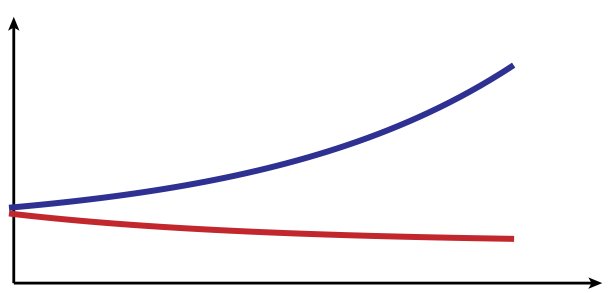


(II) gain/loss modes:



PT-symmetry breaking

Hamiltonian:

$$H = \begin{pmatrix} i\Gamma & g \\ g & -i\Gamma \end{pmatrix} \qquad \Rightarrow \qquad (\mathcal{PT})H(\mathcal{PT}) = H \qquad \forall\ \Gamma, g$$

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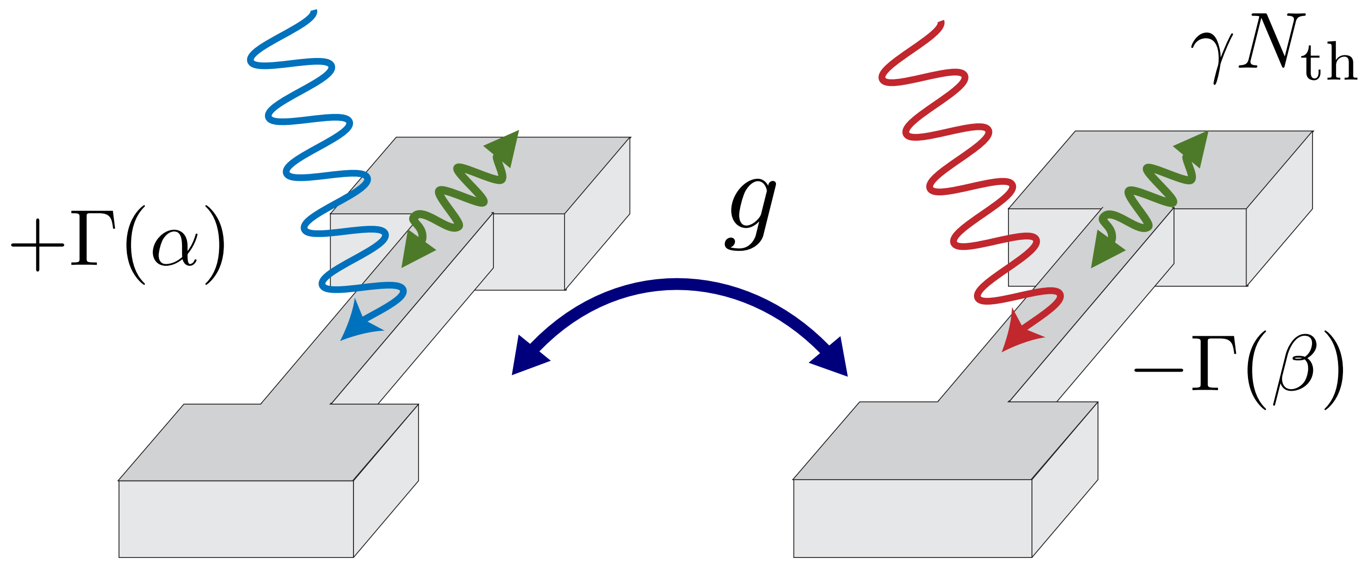
Eigenstates break symmetry of the Hamiltonian !

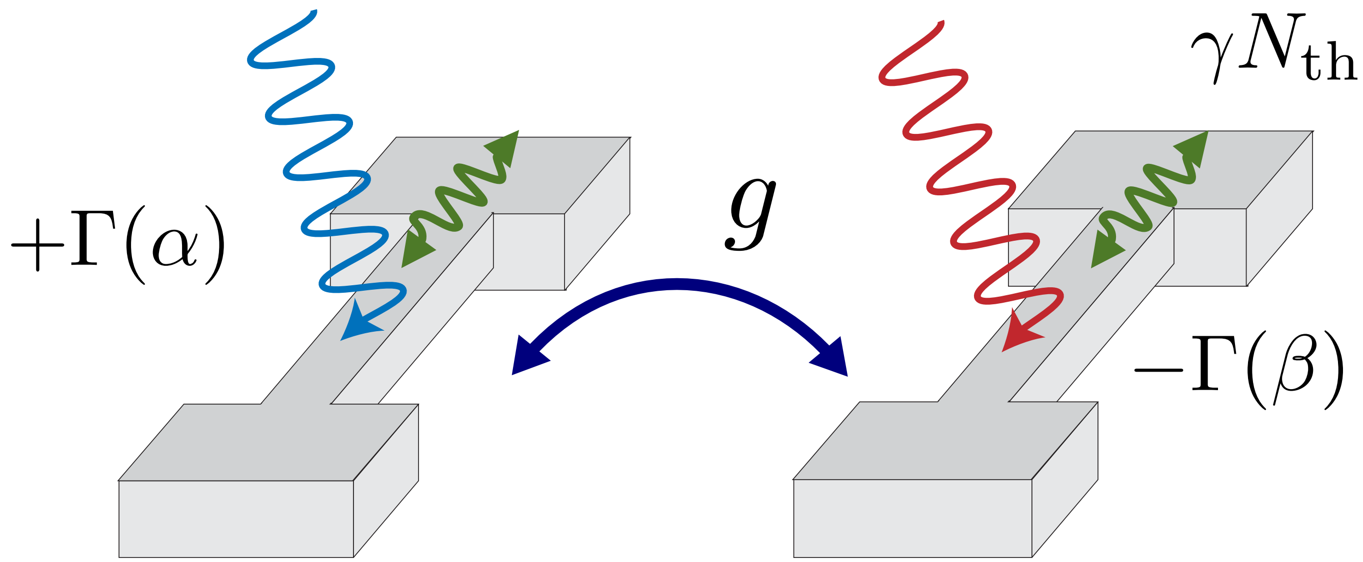
see e.g. C. M. Bender, Rep. Prog. Phys. ***70****, 947 (2007) and references therein.*

Stationary phases of PT-symm. systems



Steady state of real physical systems with PT-symmetry ?

Stationary phases of PT-symm. systems



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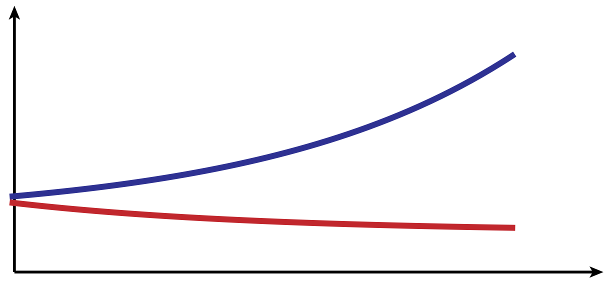

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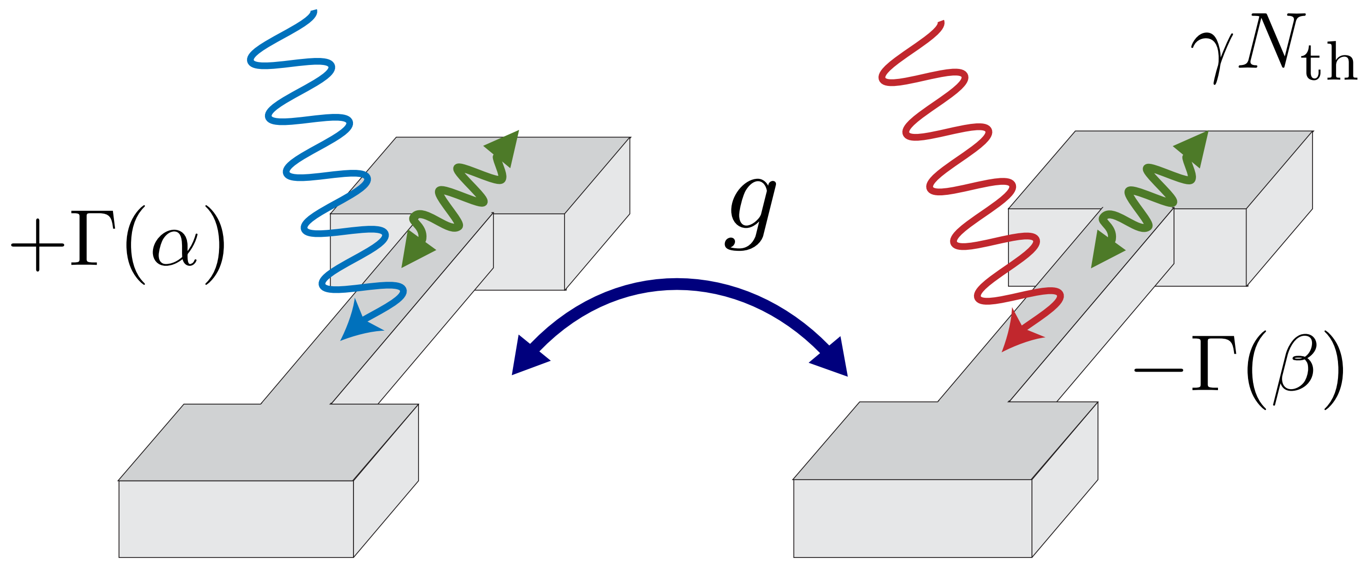

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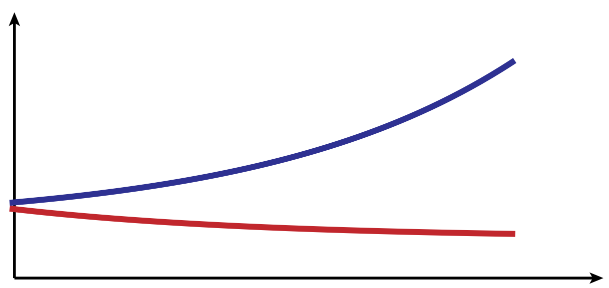
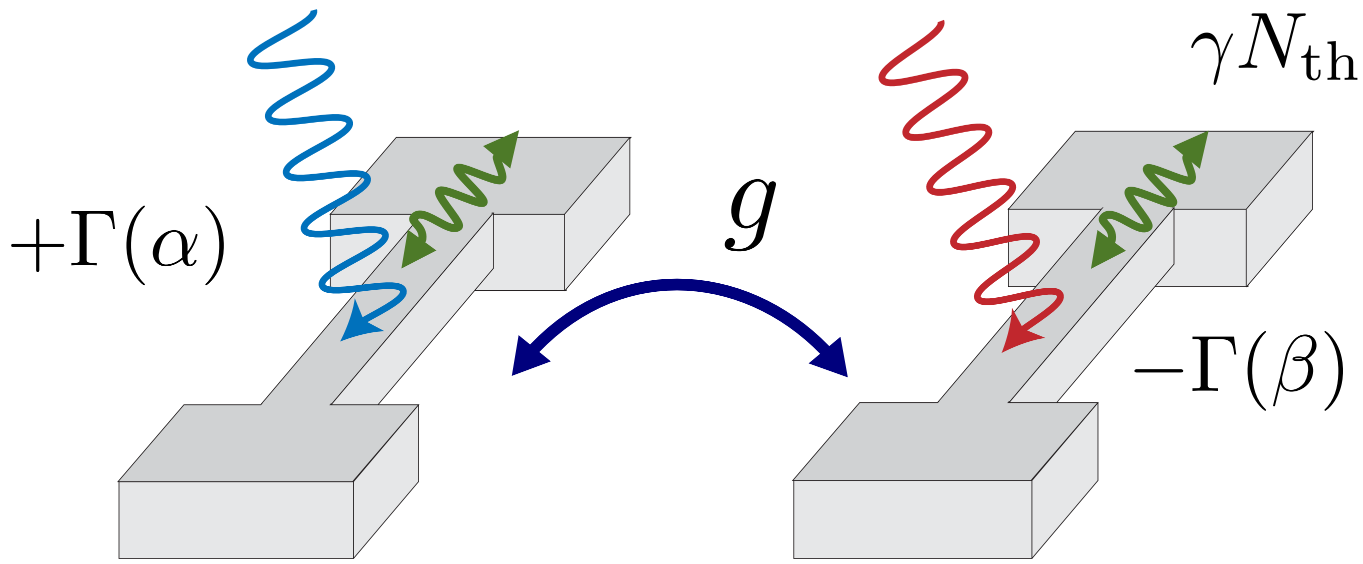


role of nonlinearities ?

Stationary phases of PT-symm. systems



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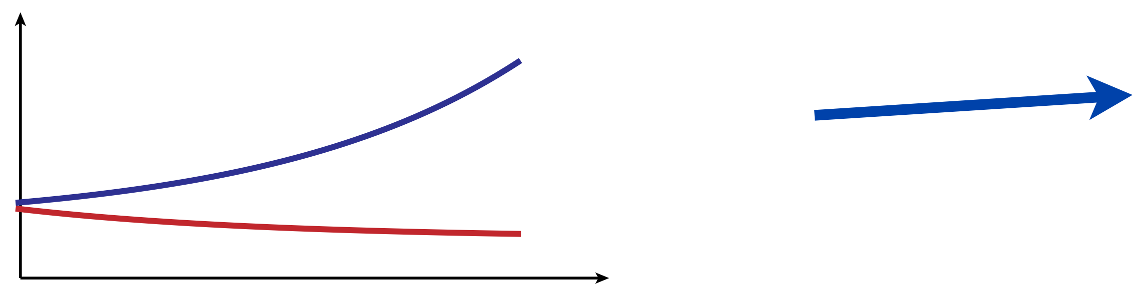

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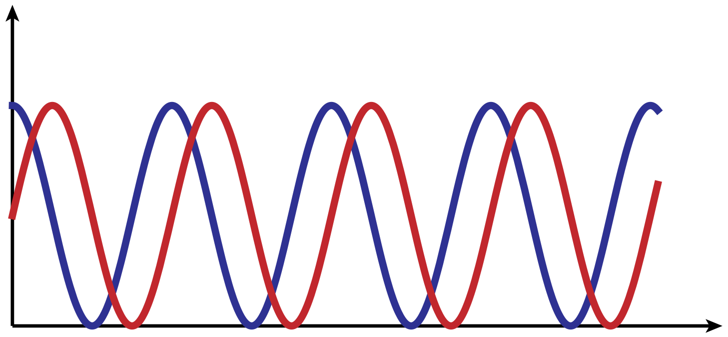
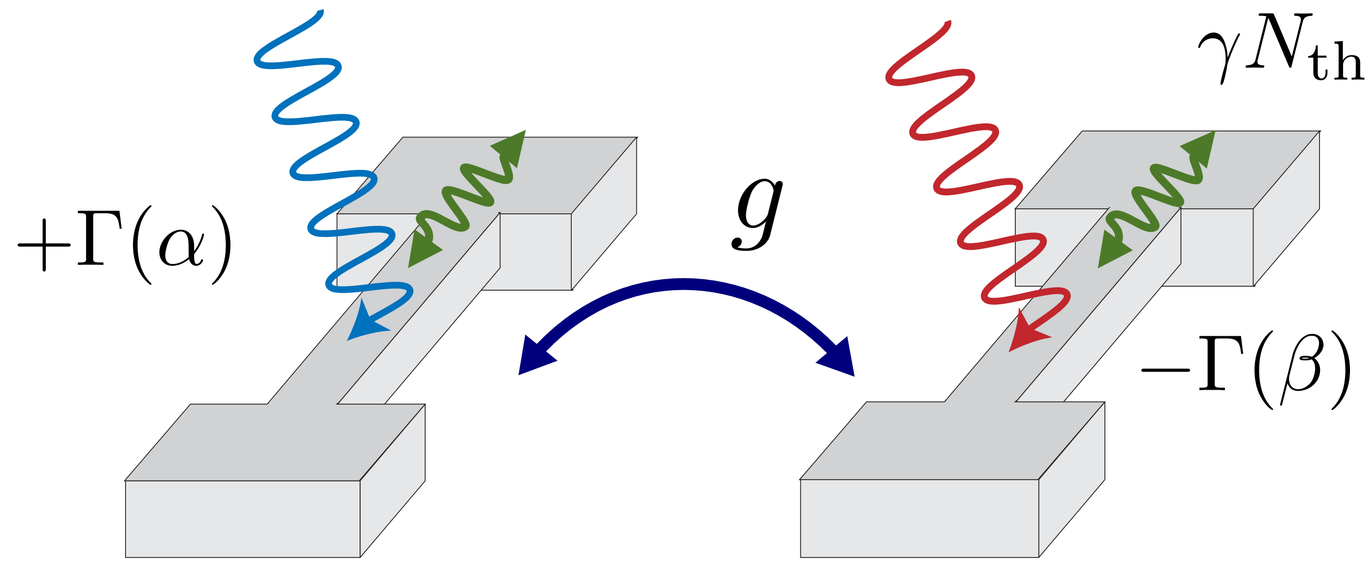


Stationary phases of PT-symm. systems



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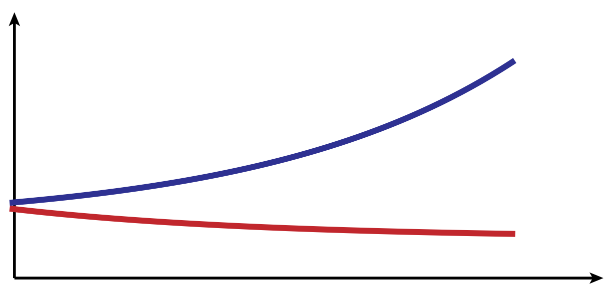

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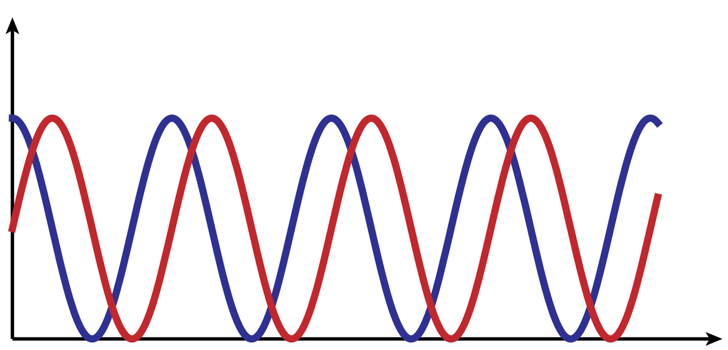
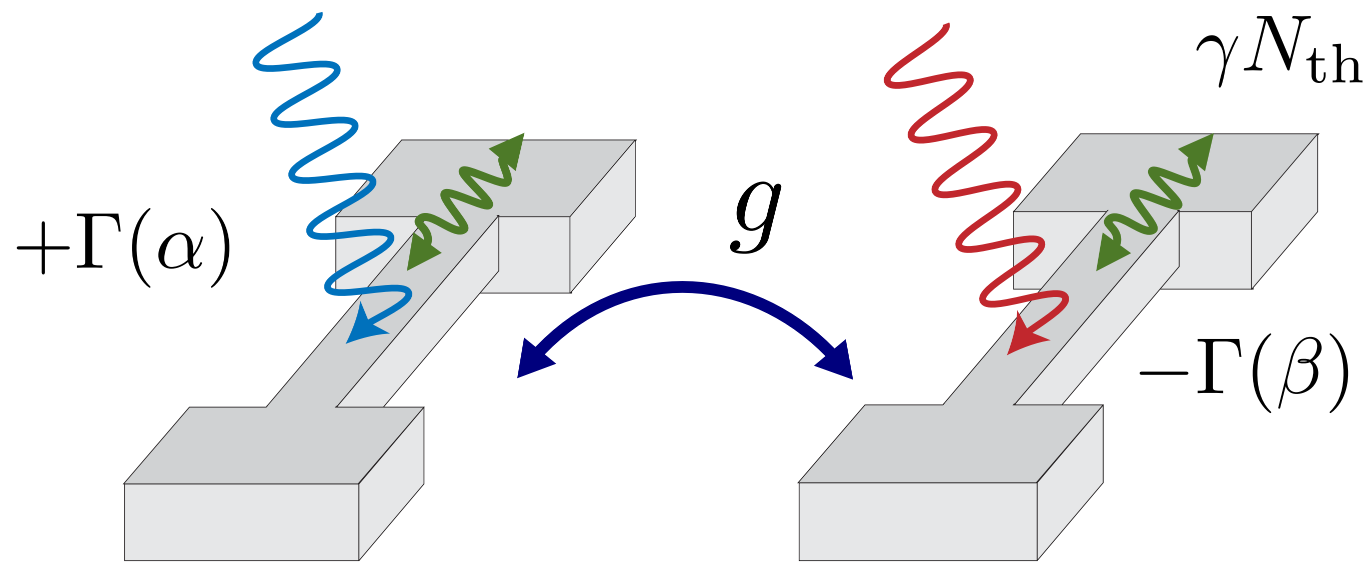


→ ***role of noise ?***

PT-symmetric (phonon) lasers



steady state ???

Equations of motion:

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = -i \begin{pmatrix} i\Gamma(\alpha) - i\gamma & g \\ g & -i\Gamma(\beta) - i\gamma \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \begin{pmatrix} \sqrt{2\gamma N_{\mathrm{th}}}\eta(t) \\ \sqrt{2\gamma N_{\mathrm{th}}}\xi(t) \end{pmatrix}$$

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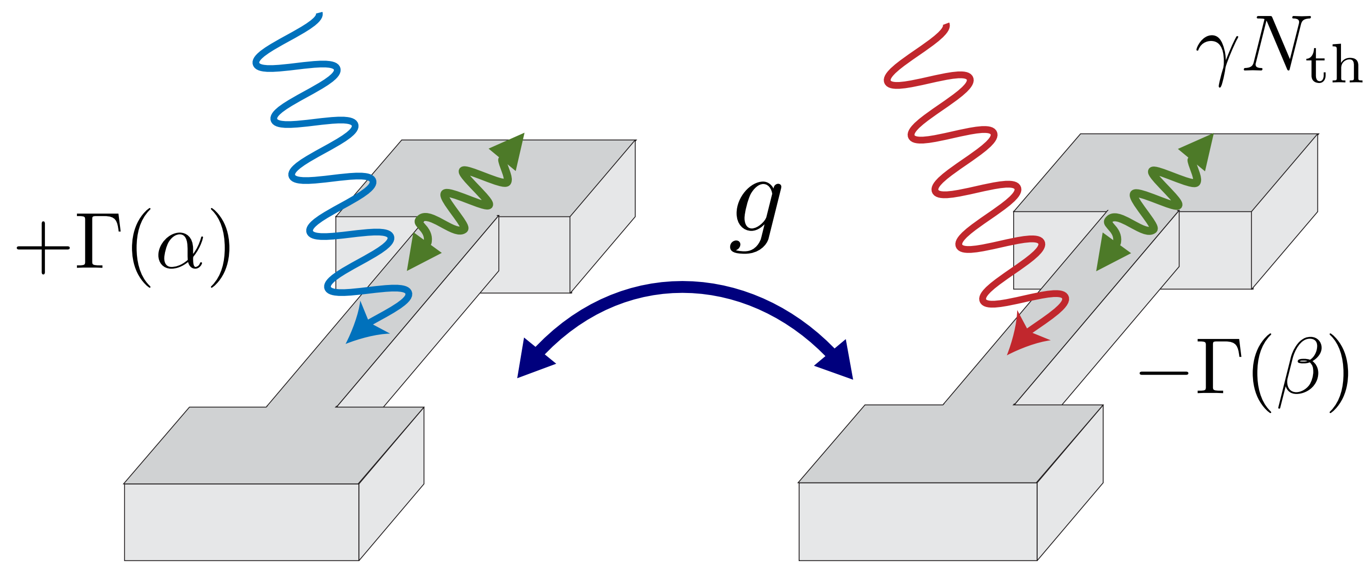

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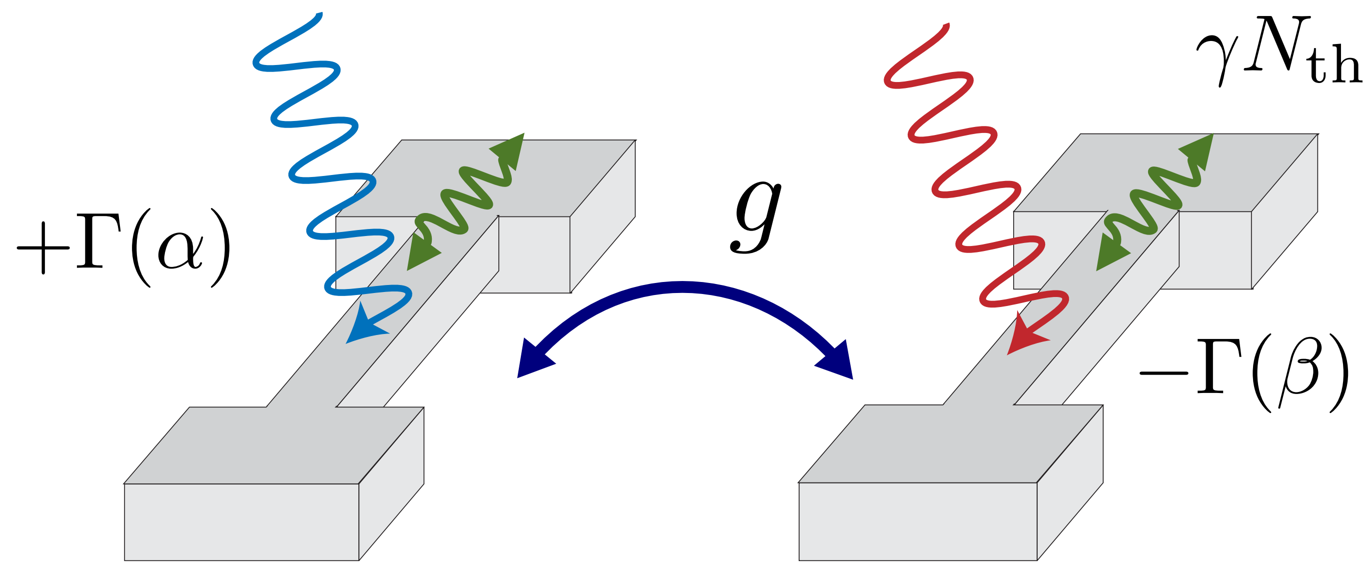
non-linear gain/loss: $$\Gamma(\alpha) = \frac{\Gamma}{(1+|\alpha|^2/n_0)^\nu}$$

cutoff parameter $1 \leq \nu \leq 2$

cutoff phonon number

PT-symmetric (phonon) lasers



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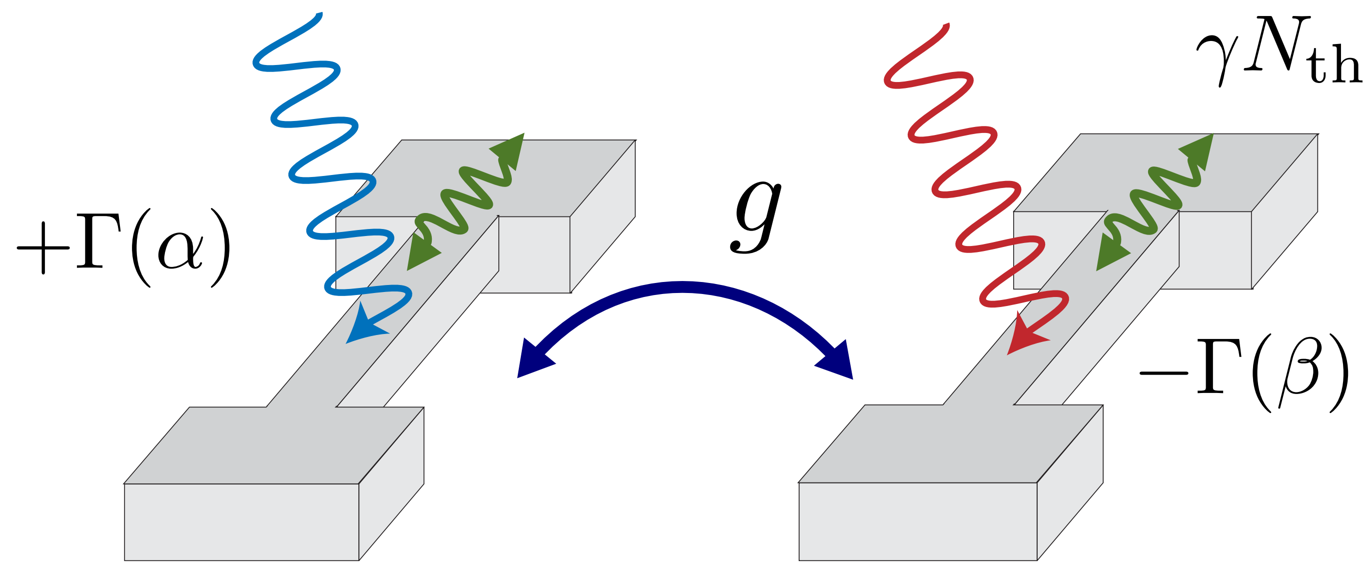
intrinsic damping: $\gamma \to 0^+$

PT-symmetric (phonon) lasers



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thermal noise !!

PT-symmetry breaking: noise → 0 limit

steady state amplitudes:

PT-symmetry breaking: noise → 0 limit

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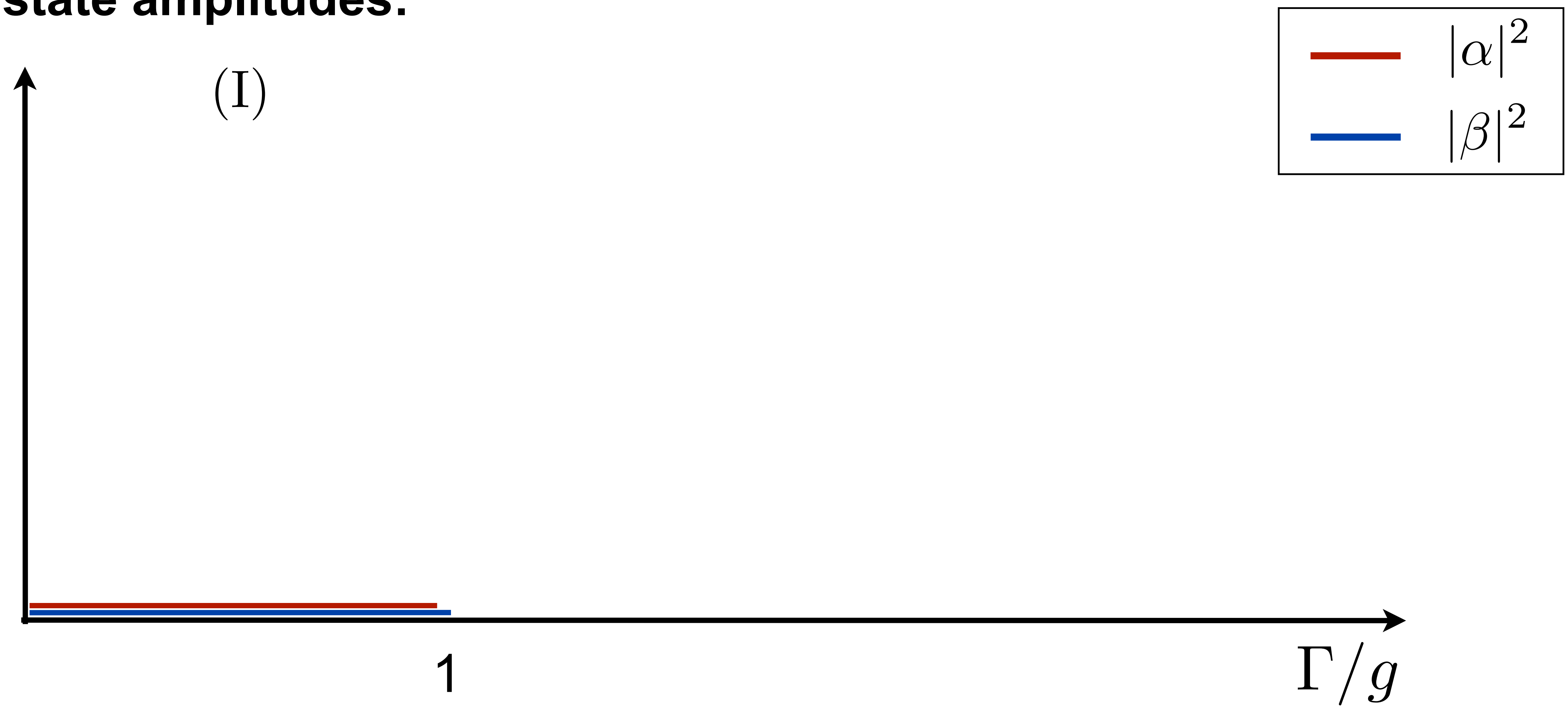

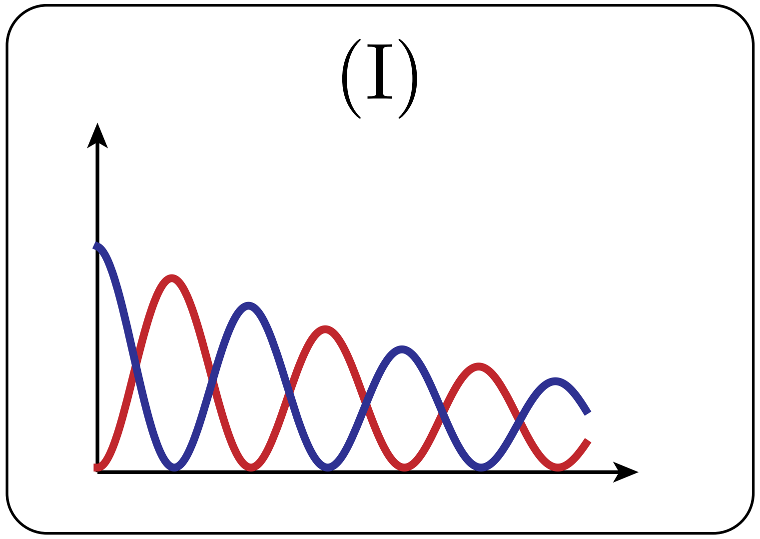

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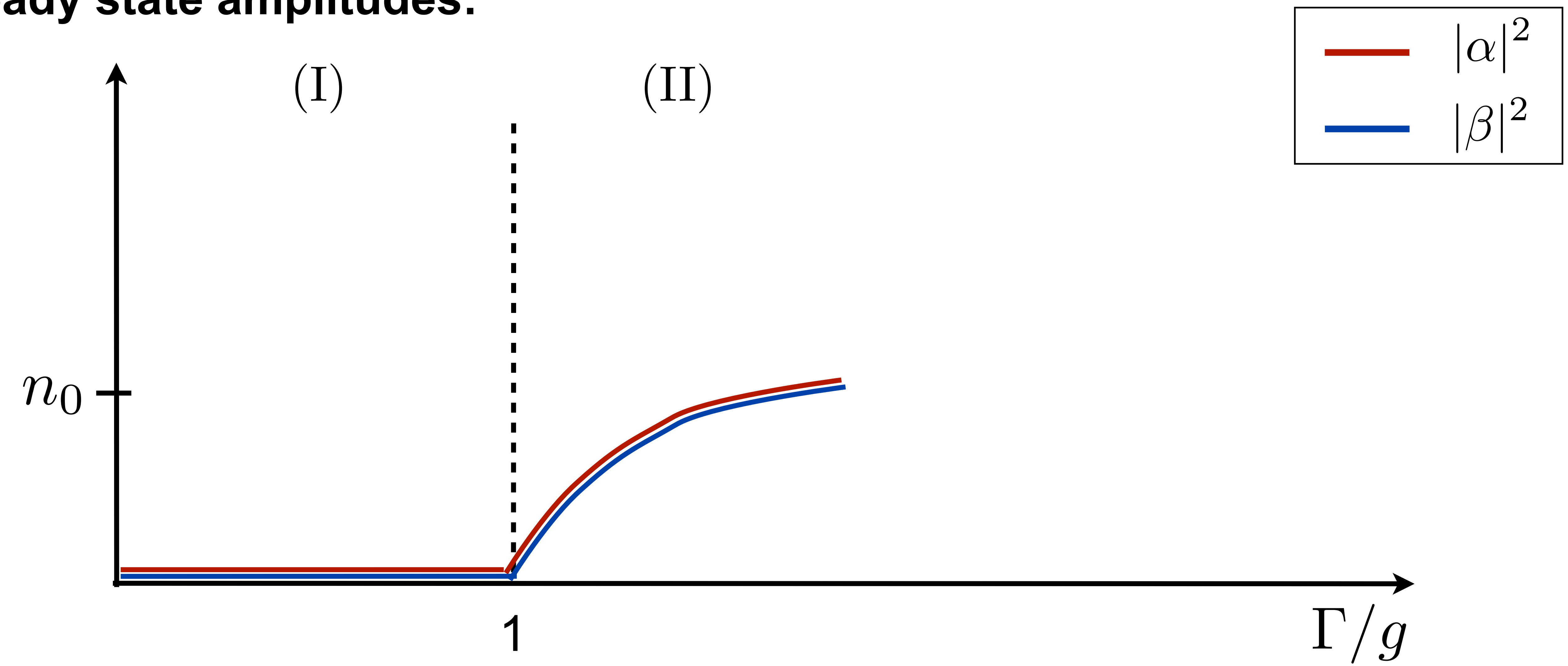

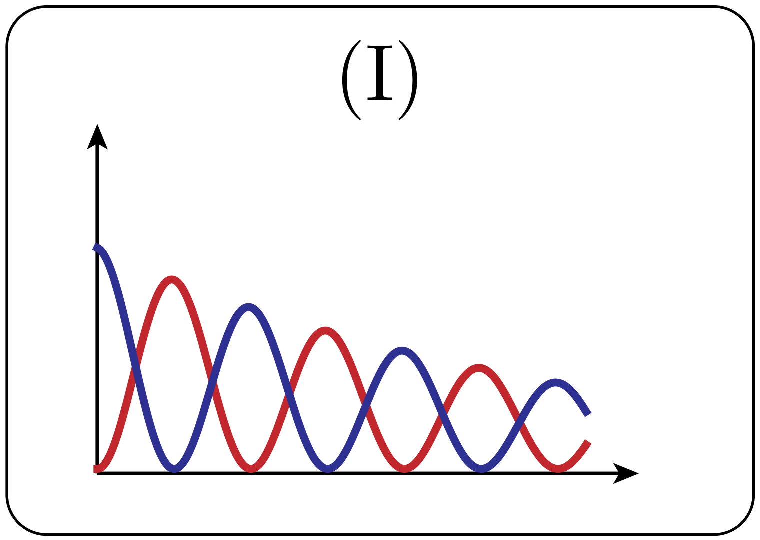

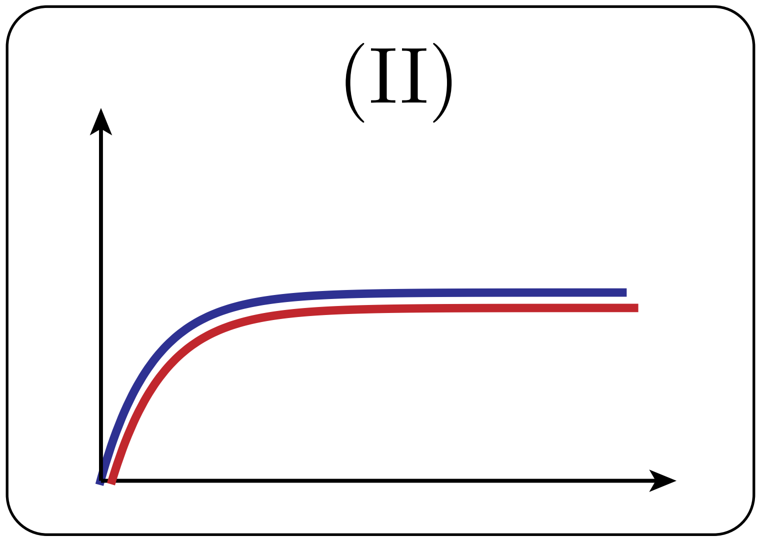

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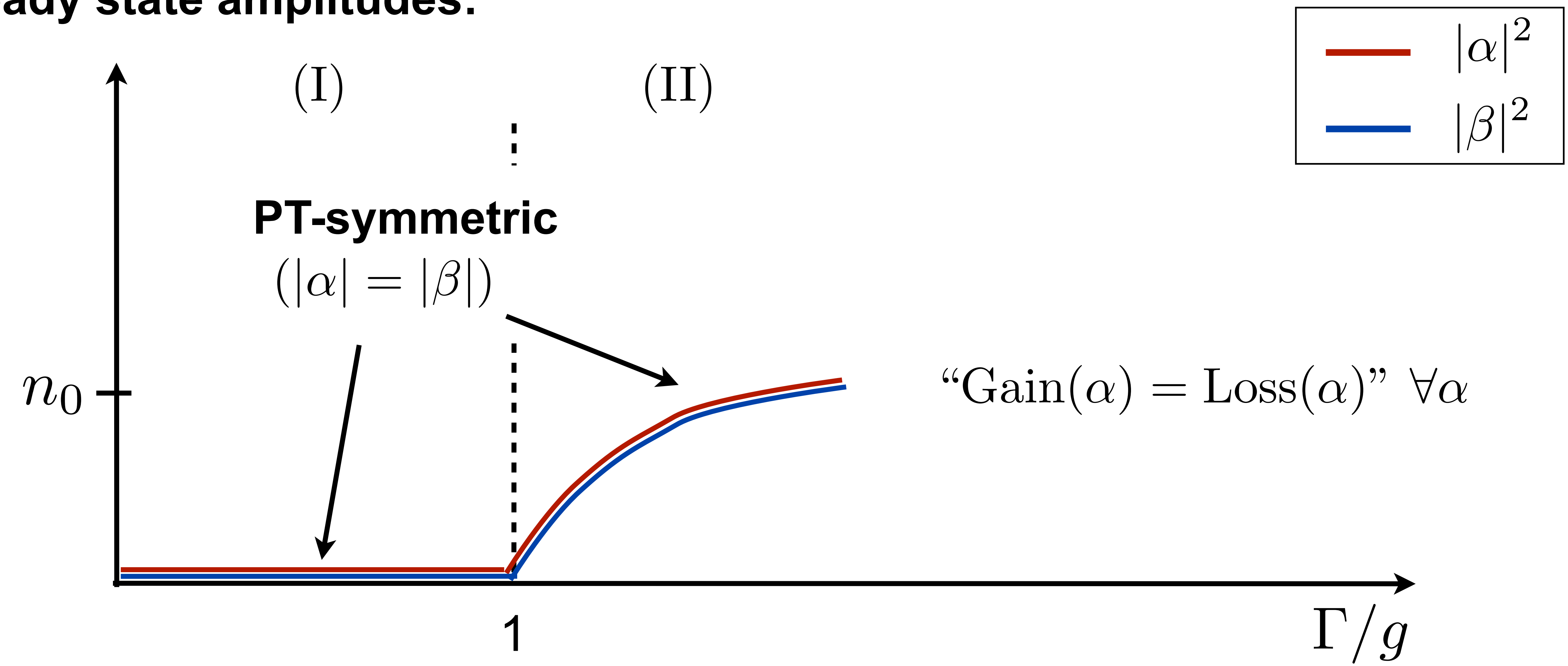

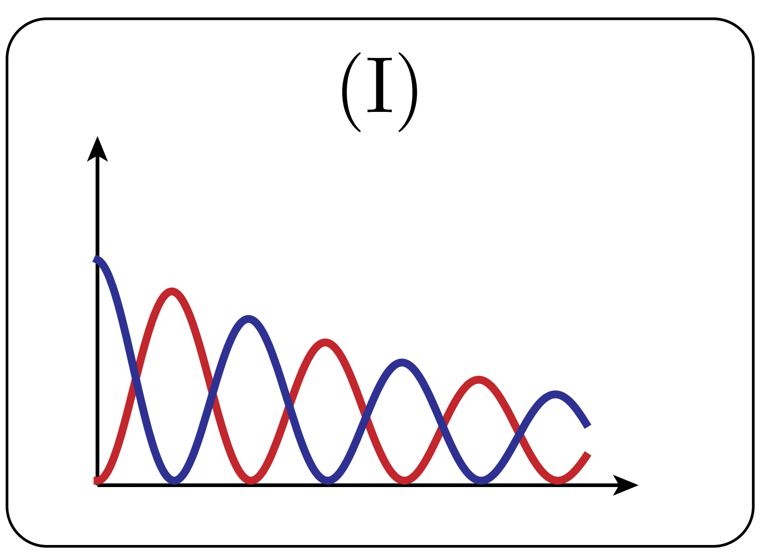

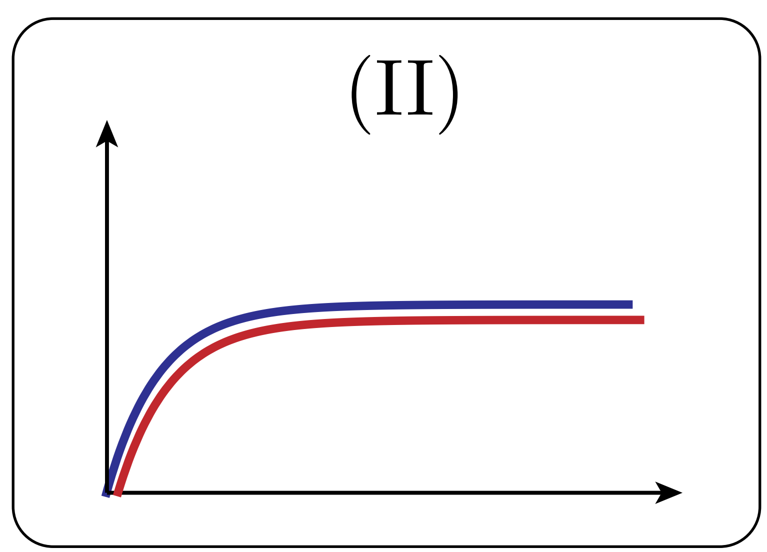

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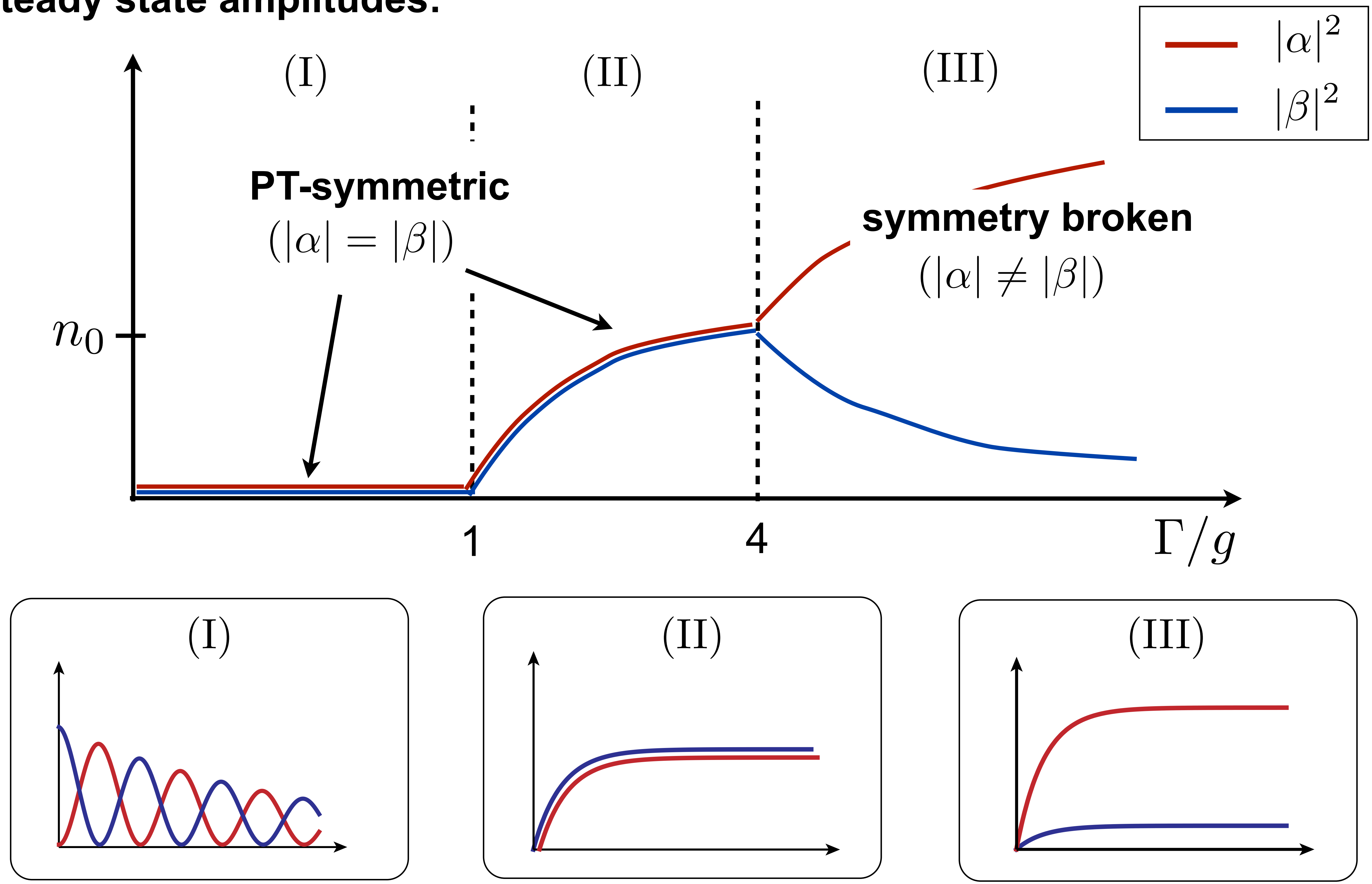

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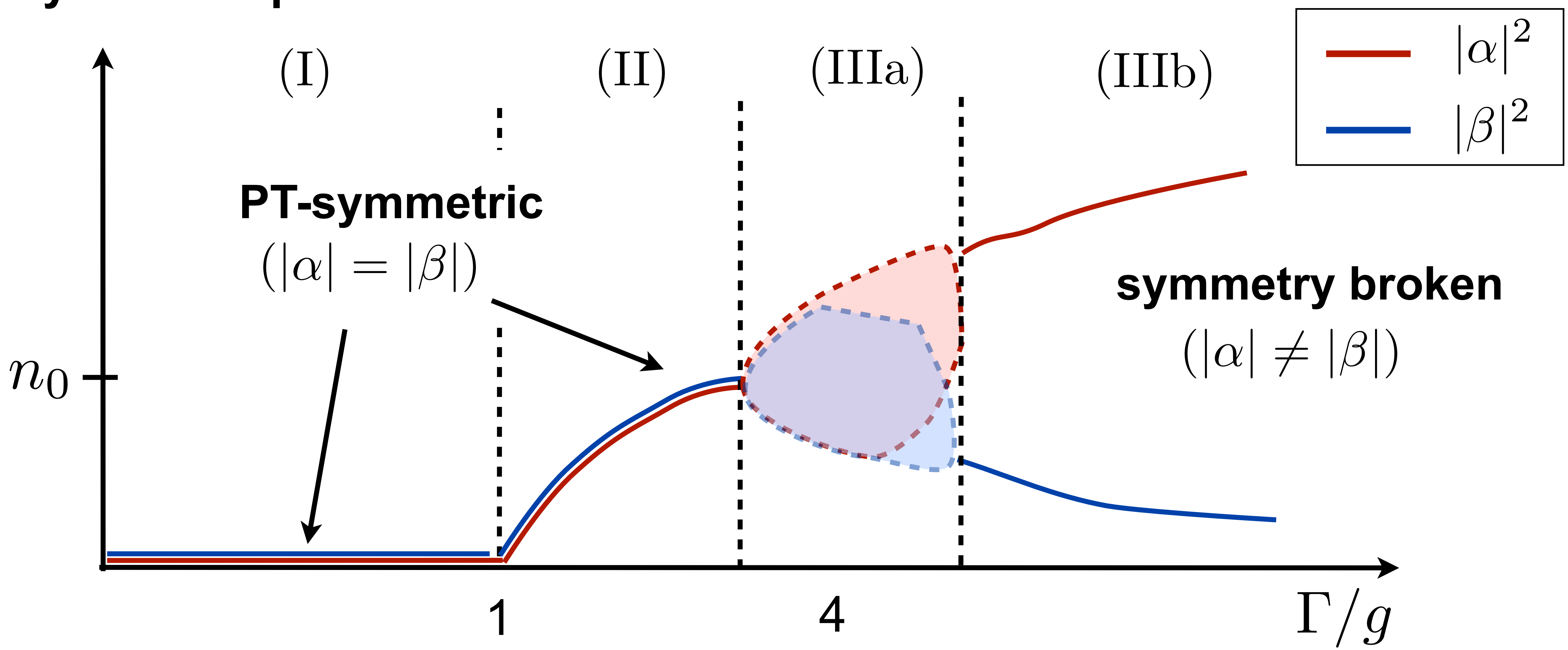
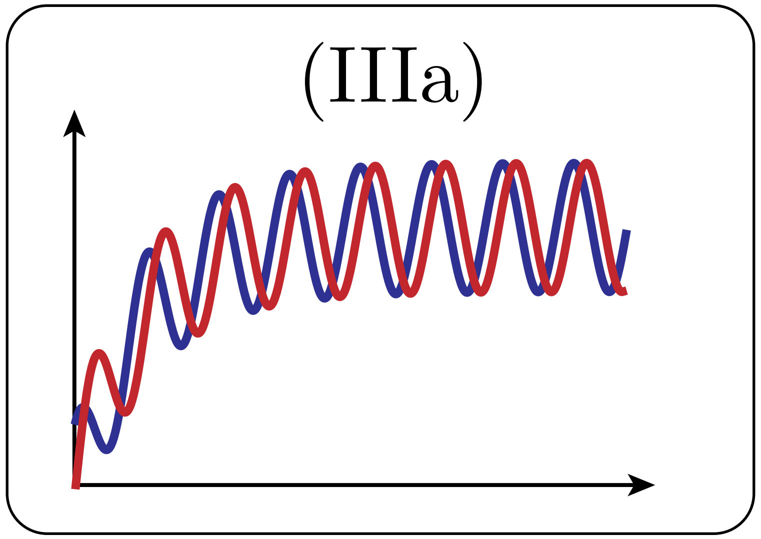


$$\Gamma(\alpha) = \frac{\Gamma}{(1 + |\alpha|^2/n_0)^{\nu}}$$

$$1 < \nu < 2$$

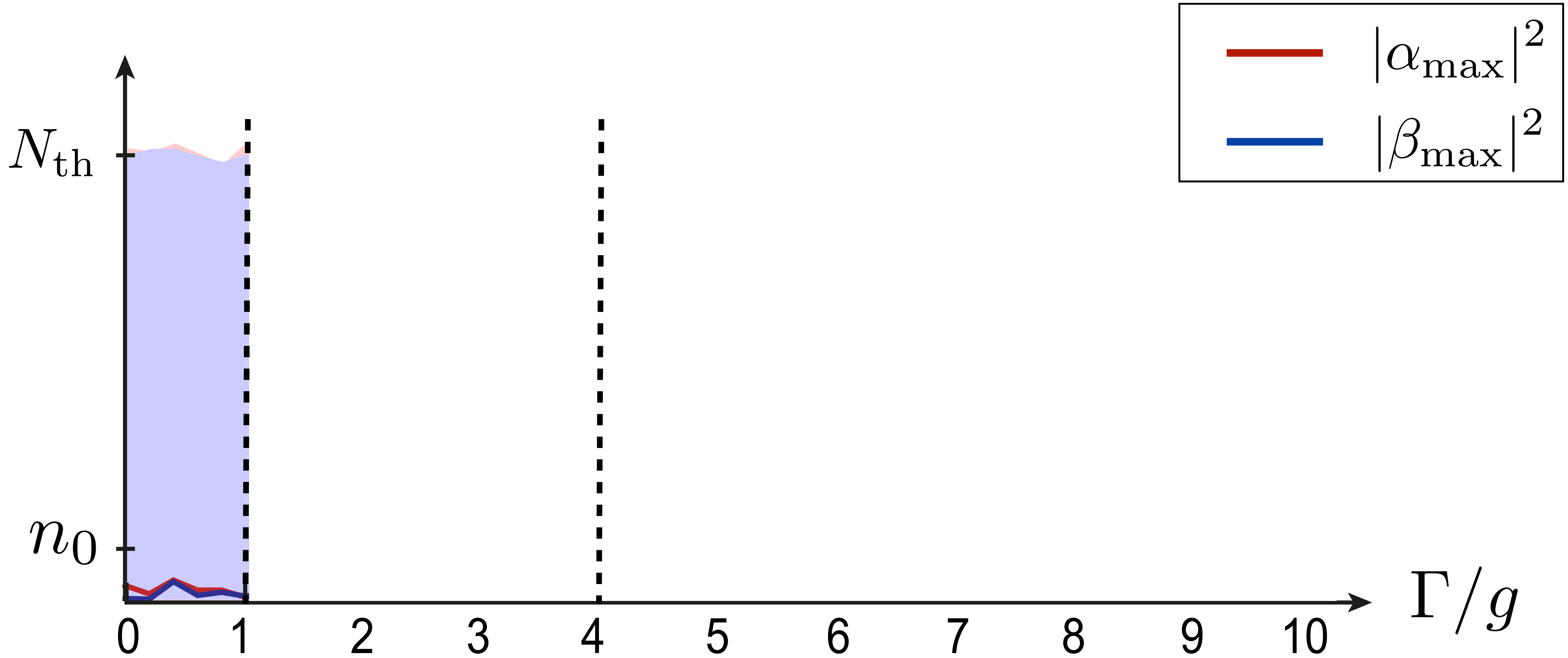


limit-cycle !

symmetry preserved on average !

PT-symmetry breaking: large noise limit

$N_{th} \gg n_0$



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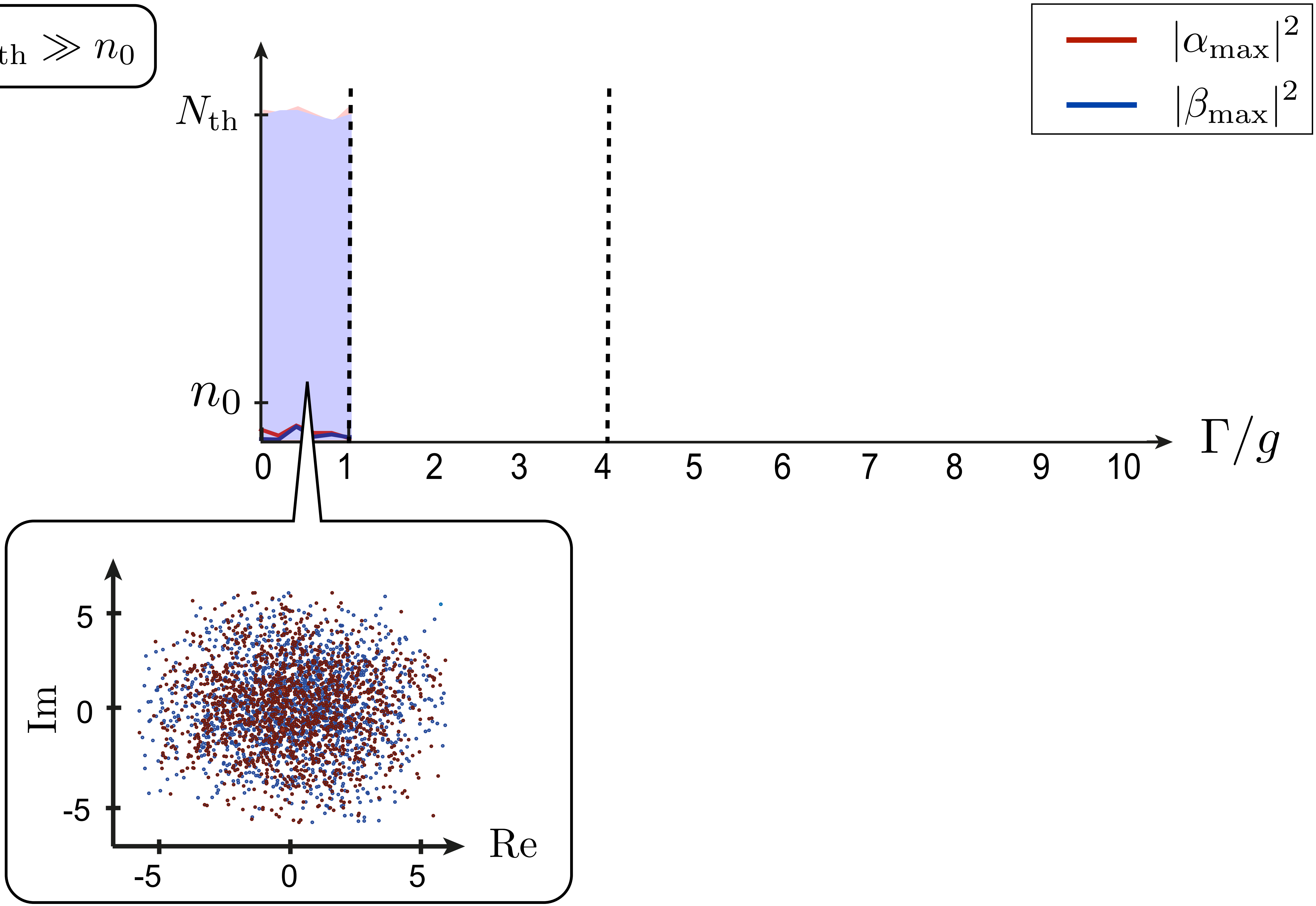

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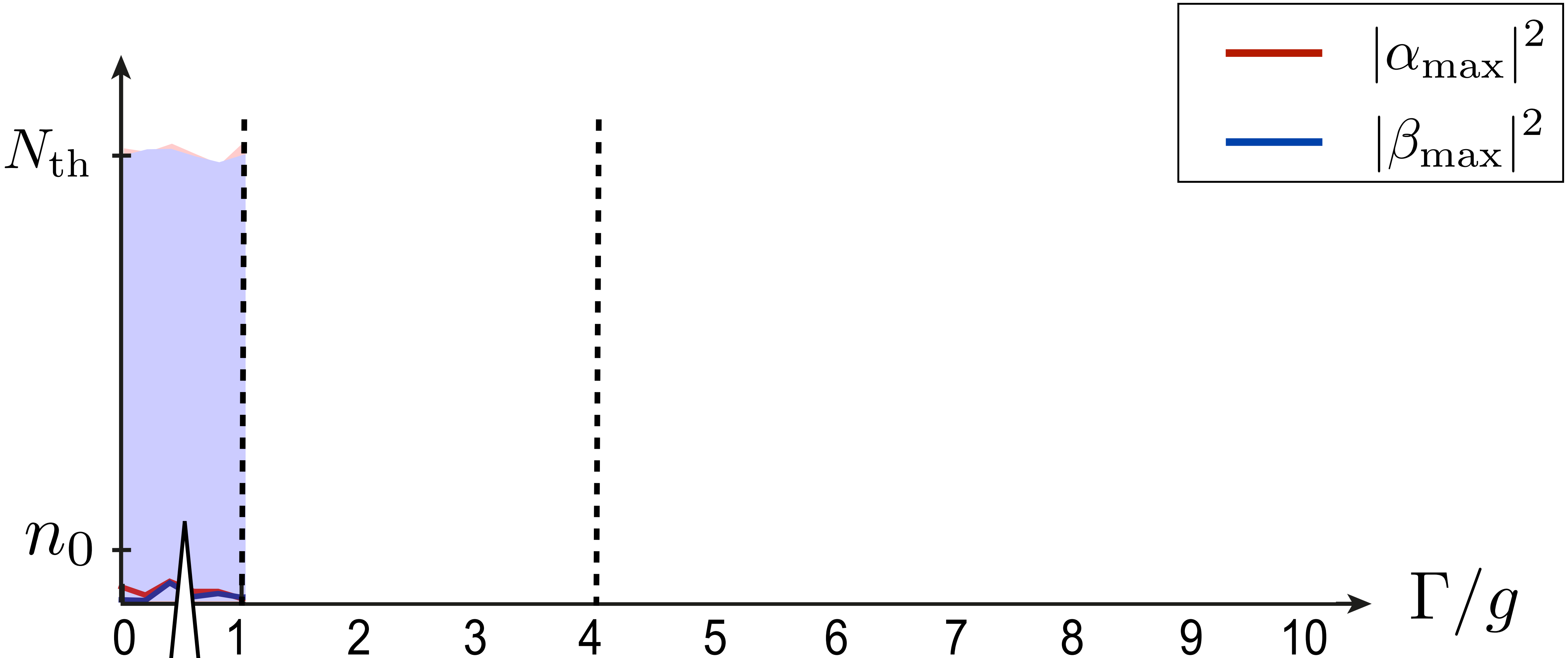

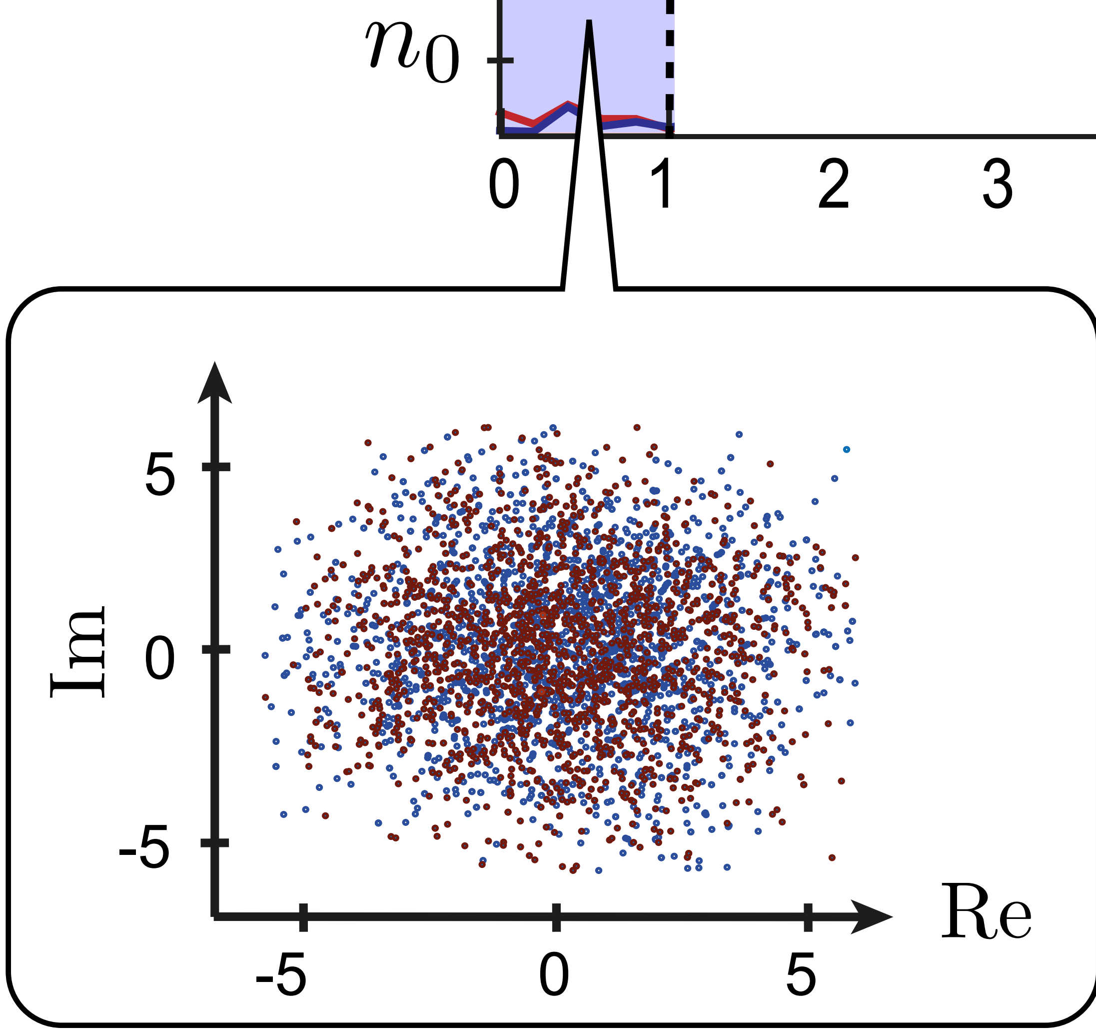

$$\langle \Gamma(\alpha) - \Gamma(\beta) \rangle_{ss} \approx 0$$

(PT-symmetric phase)

PT-symmetry breaking: large noise limit

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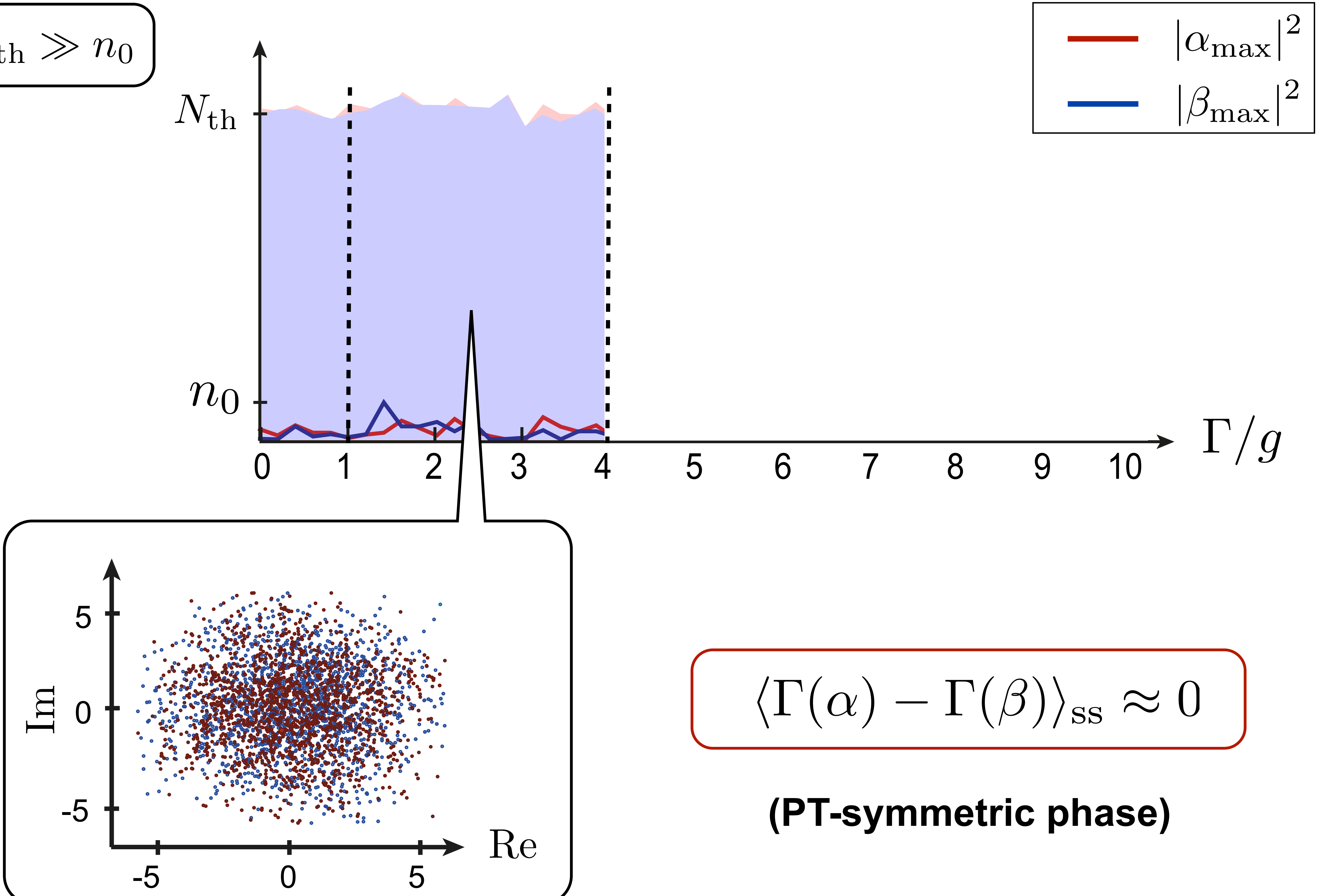

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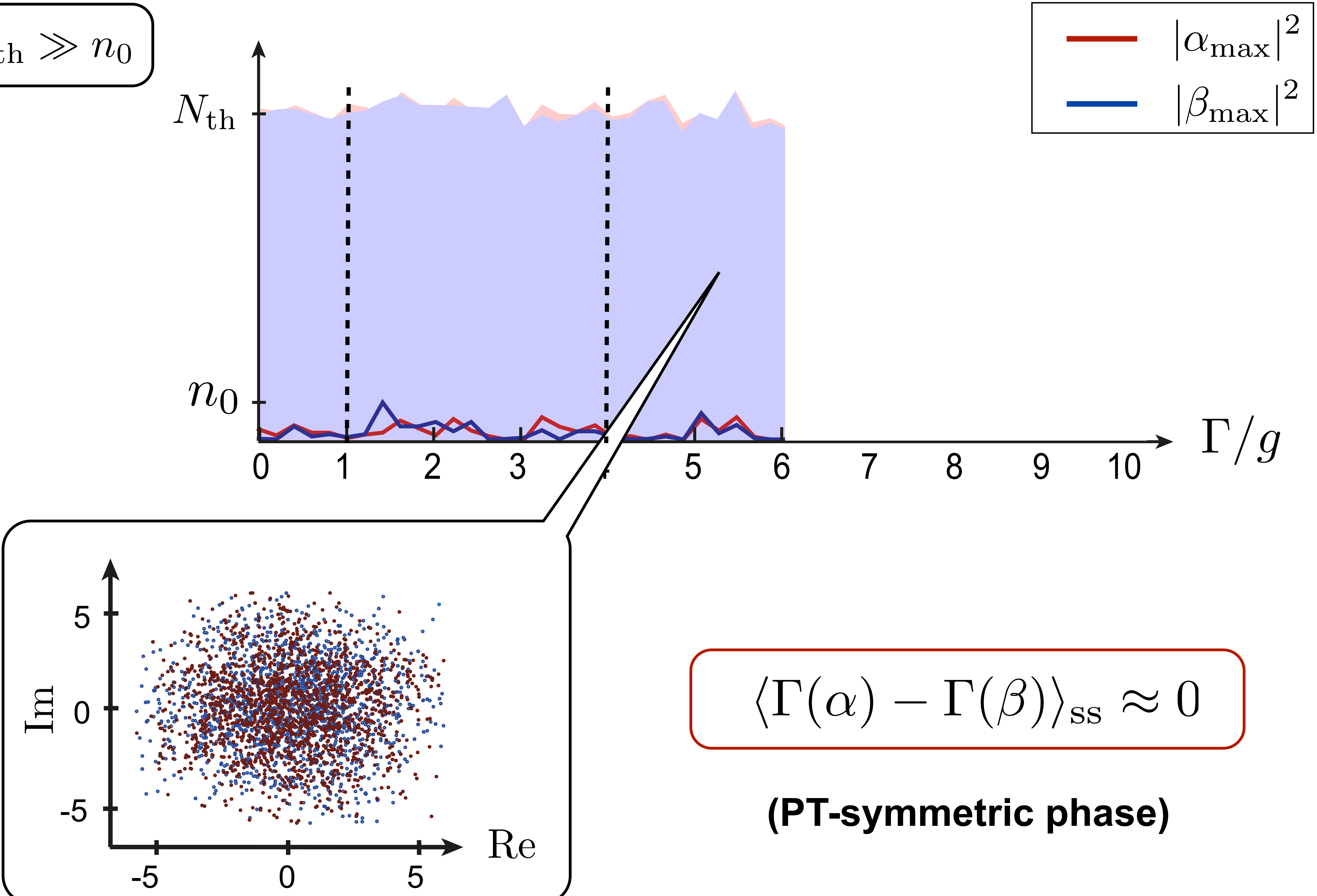

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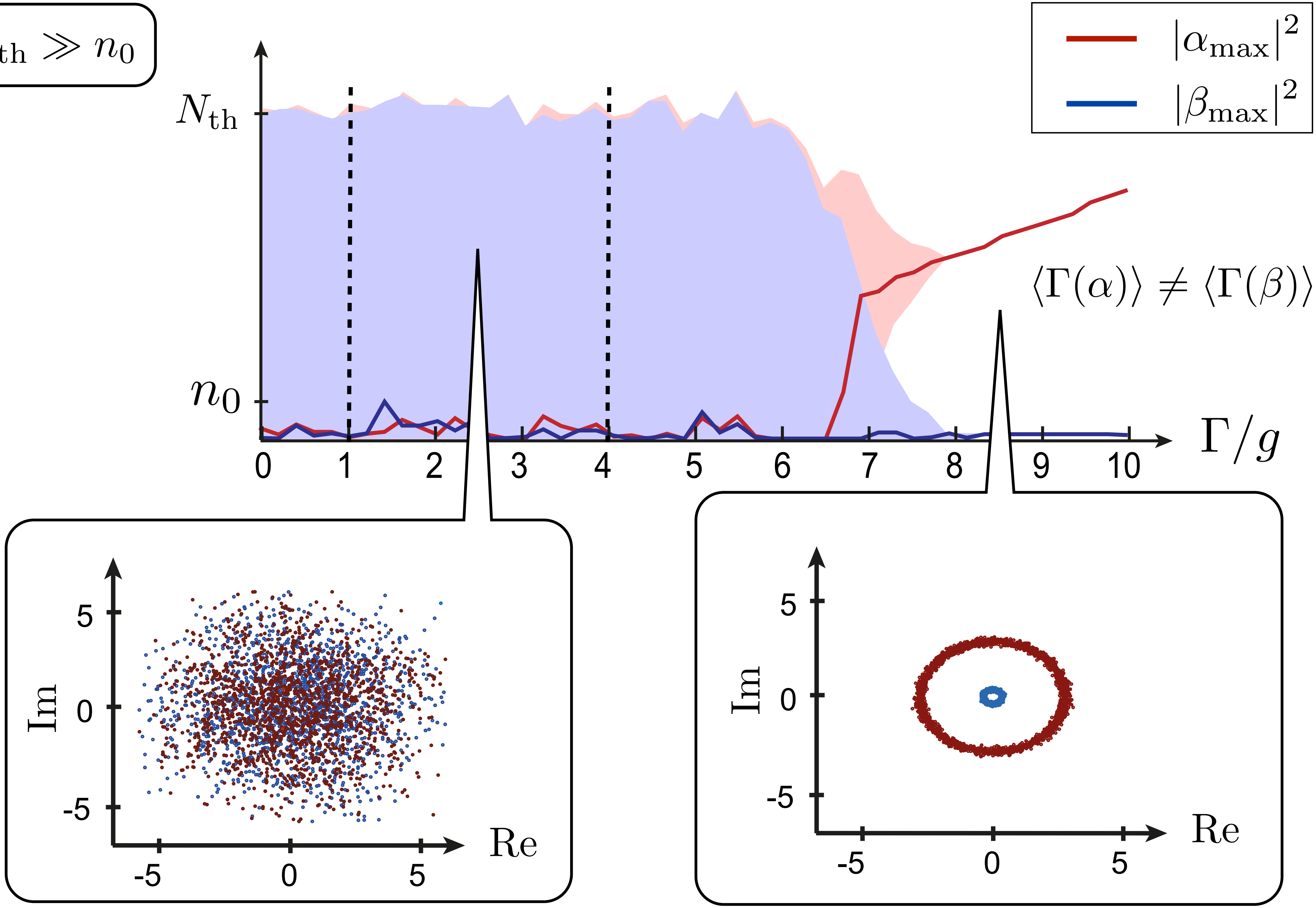
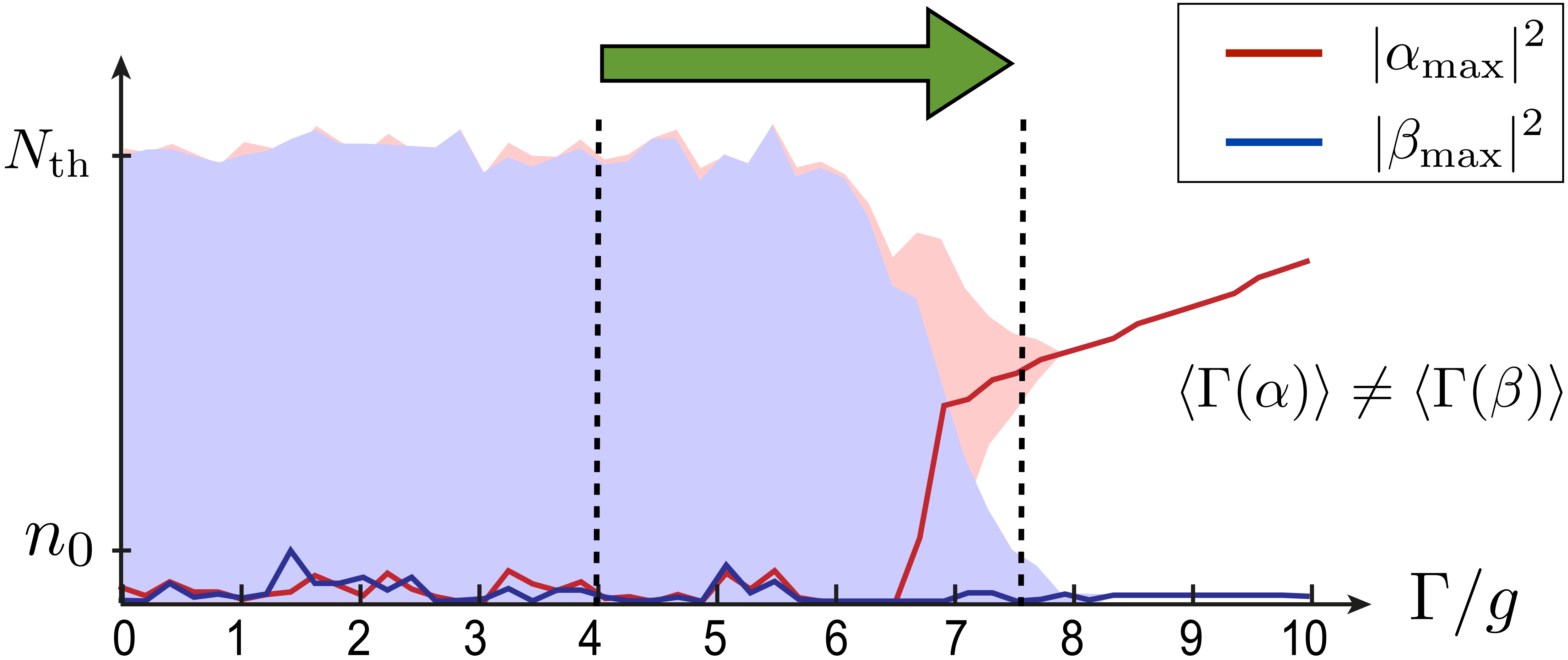


PT-symmetry breaking mechanism ?

$N_{\mathrm{th}} \gg n_0$



linear fluctuation analysis $\Rightarrow$ $$\left.\frac{\Gamma}{g}\right|_{\mathrm{c}} = 4$$

PT-symmetry breaking mechanism ?

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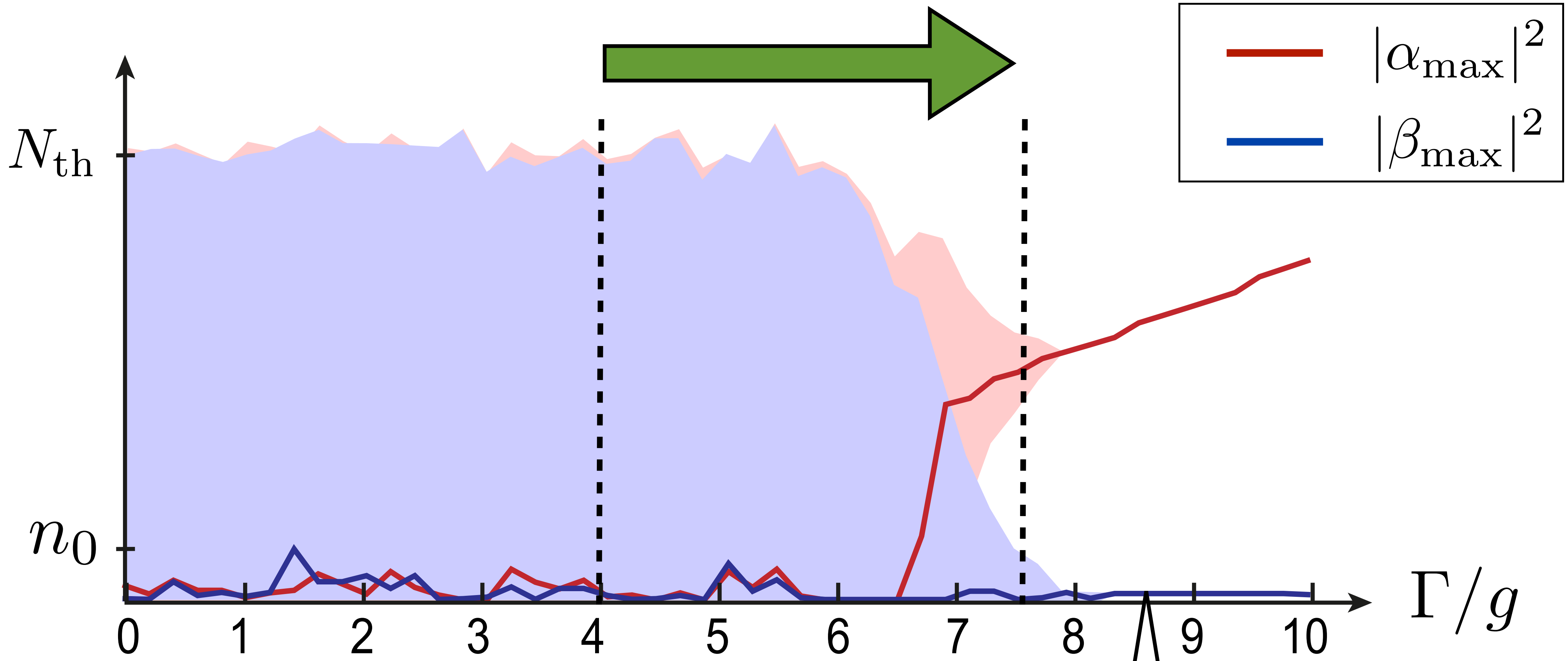
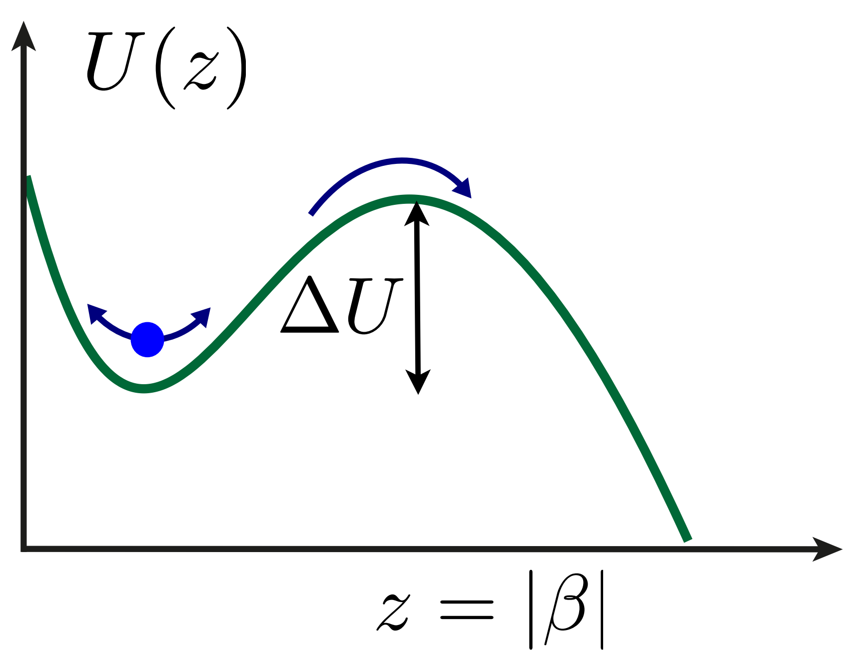


EOM for "loss mode" $z(t) = |\beta(t)|$ **:**

$$\partial_t z = -\partial_z U(z) + \sqrt{\gamma N_{\rm th}}\,\xi_z(t)$$



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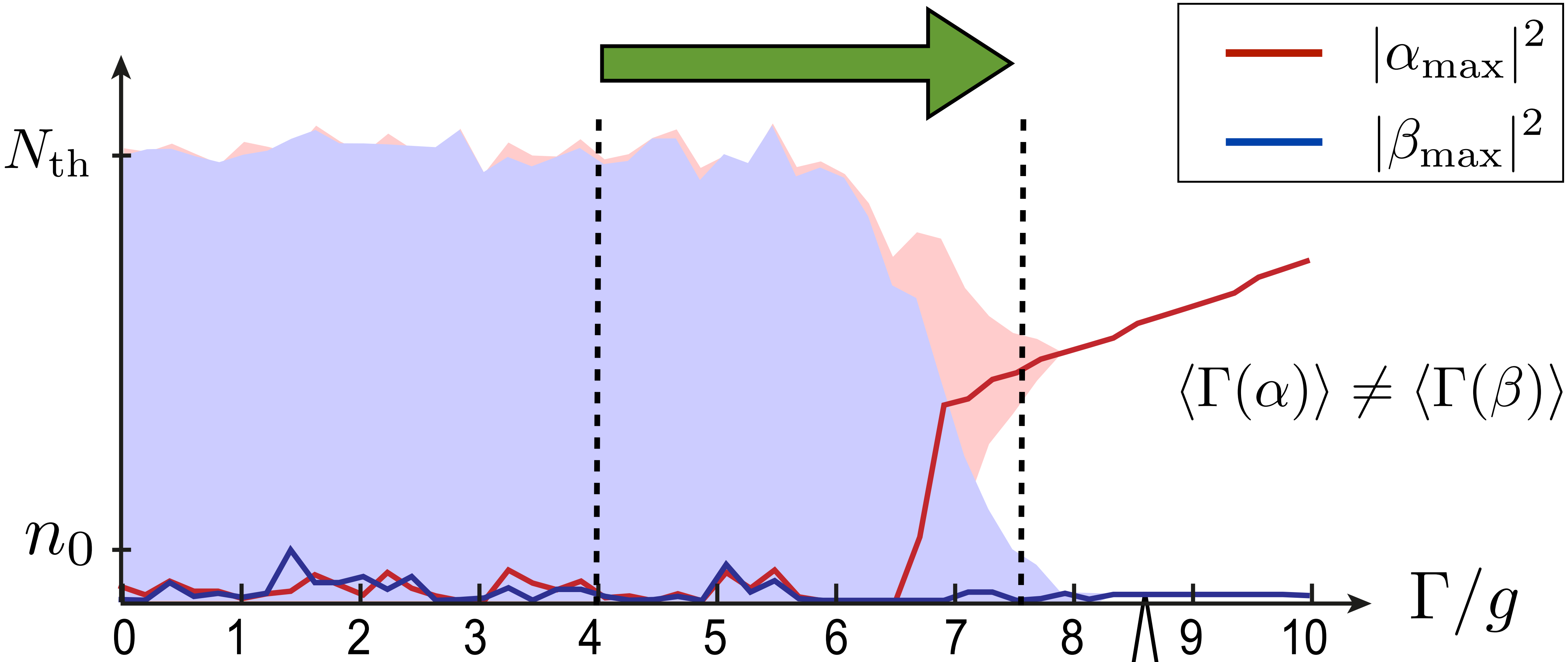
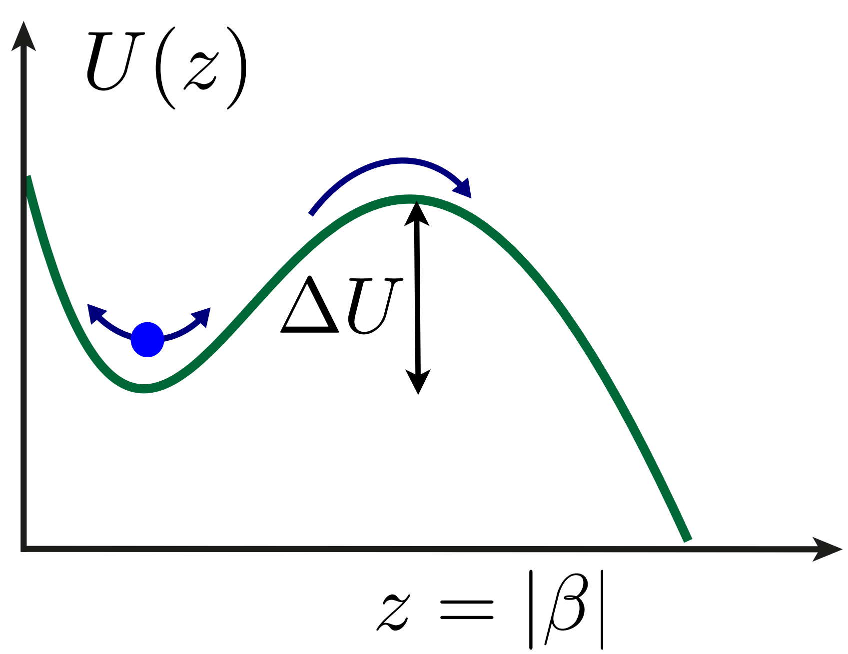


$\langle\Gamma(\alpha)\rangle \neq \langle\Gamma(\beta)\rangle$

thermally activated escape rate:

$$\Gamma_{\mathrm{esc}} \simeq \Gamma_0 e^{-\frac{\Delta U}{\gamma N_{\mathrm{th}}}}$$



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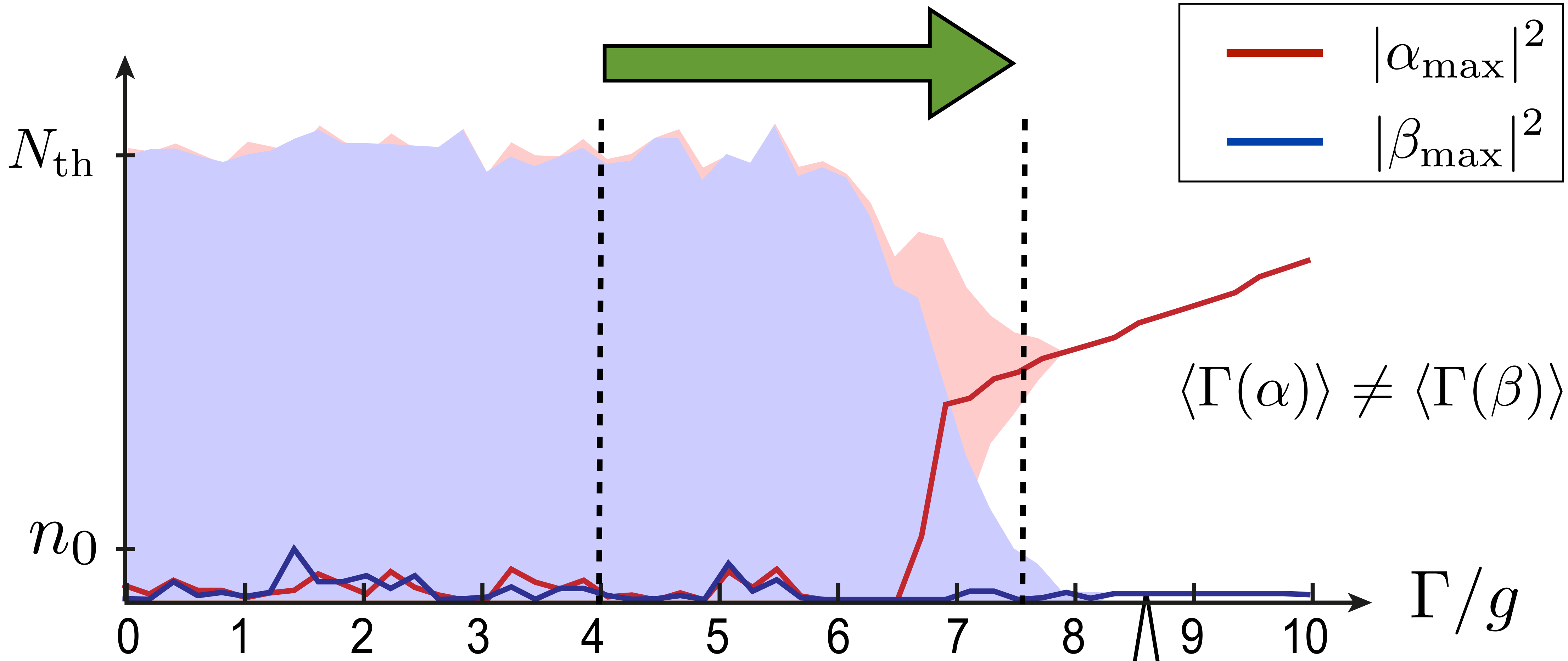
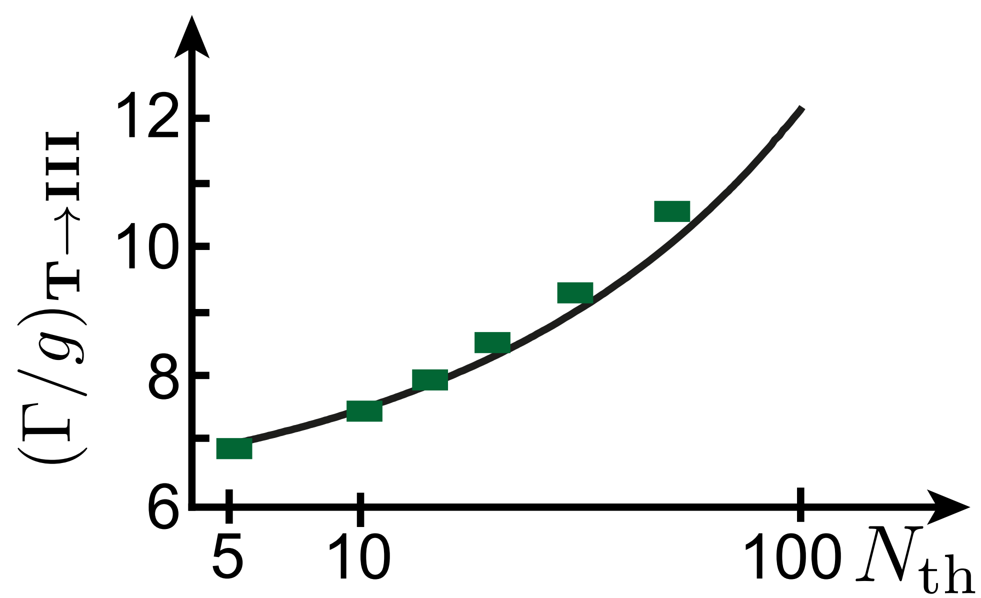
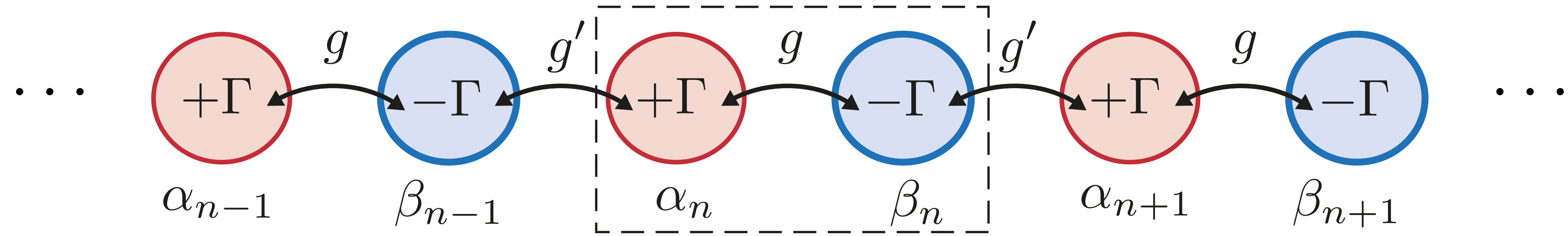


thermally activated escape rate:

$$\Gamma_{\rm esc} \simeq \Gamma_0 e^{-\frac{\Delta U}{\gamma N_{\rm th}}} \overset{!}{=} \gamma$$



PT-symmetric phonon-laser arrays



Equations of motion:

$$\begin{pmatrix} \dot{\alpha}_n \\ \dot{\beta}_n \end{pmatrix} = \begin{pmatrix} \Gamma_+(\alpha_n) & -ig \\ -ig & \Gamma_-(\beta_n) \end{pmatrix} \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix} - ig' \begin{pmatrix} \beta_{n-1} \\ \alpha_{n+1} \end{pmatrix} + \begin{pmatrix} F_{n,+}(t) \\ F_{n,-}(t) \end{pmatrix}$$

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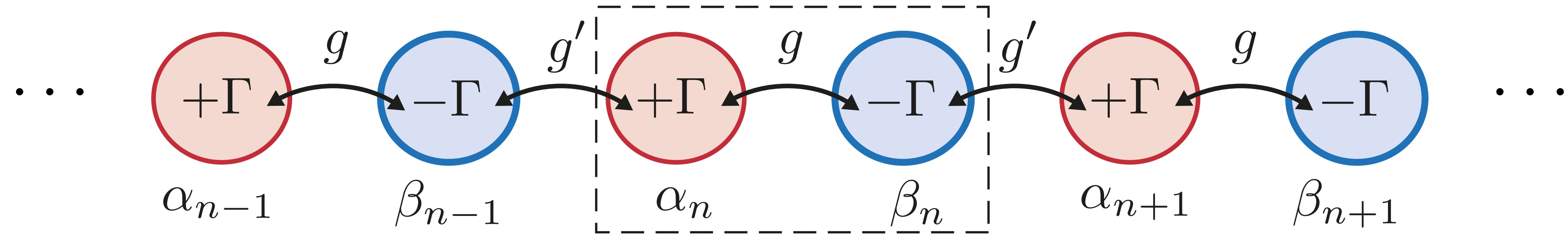

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Plane-wave ansatz:

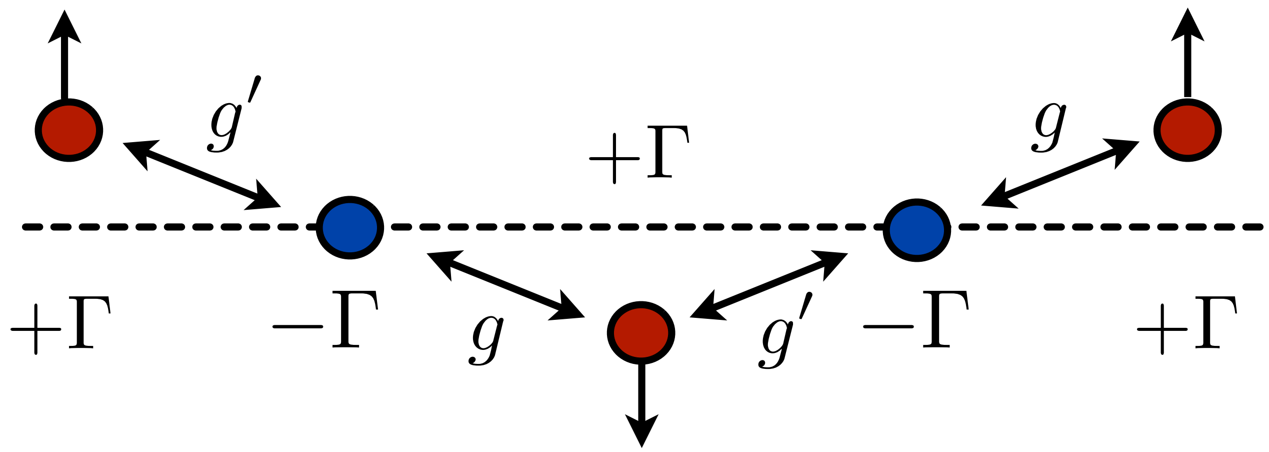
$$\left.\begin{aligned} \alpha_n &= A_k e^{ikn} \\ \beta_n &= B_k e^{ikn} \end{aligned}\right\}$$

two-mode problem:

$$g \mapsto g_k = g + g' e^{ik}$$

PT-symmetric phonon-laser arrays

Unstable mode: $k = \pi$



$$g_{k=\pi} = |g - g'|$$

PT-symmetric phonon-laser arrays

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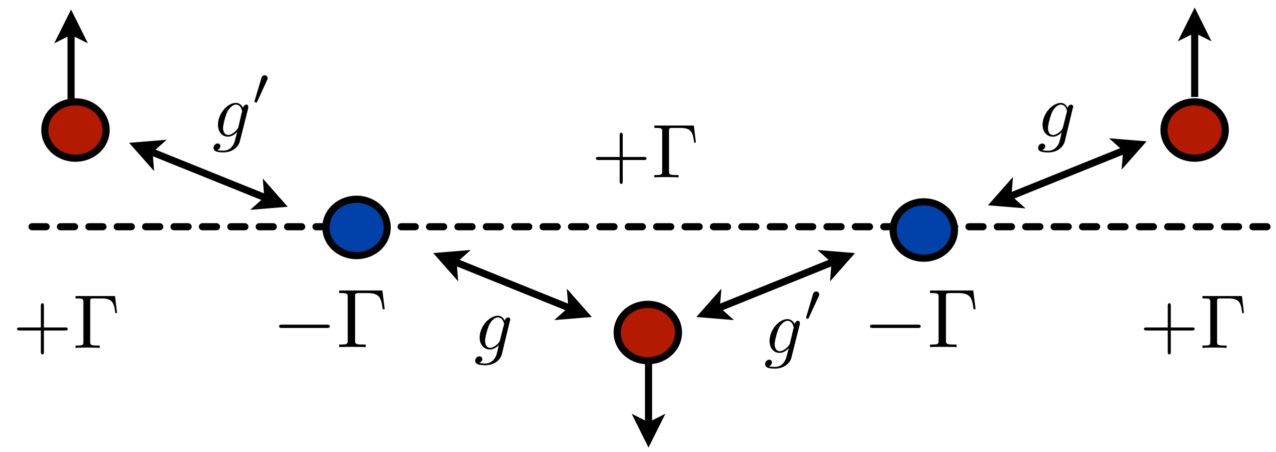

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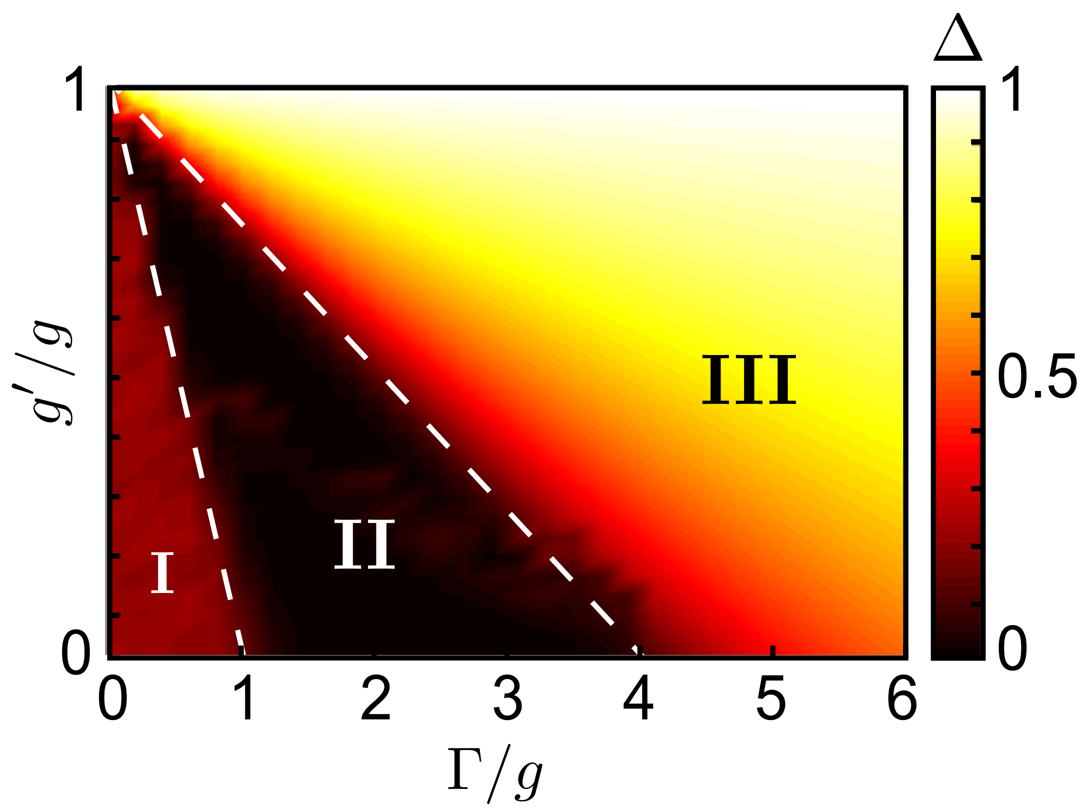
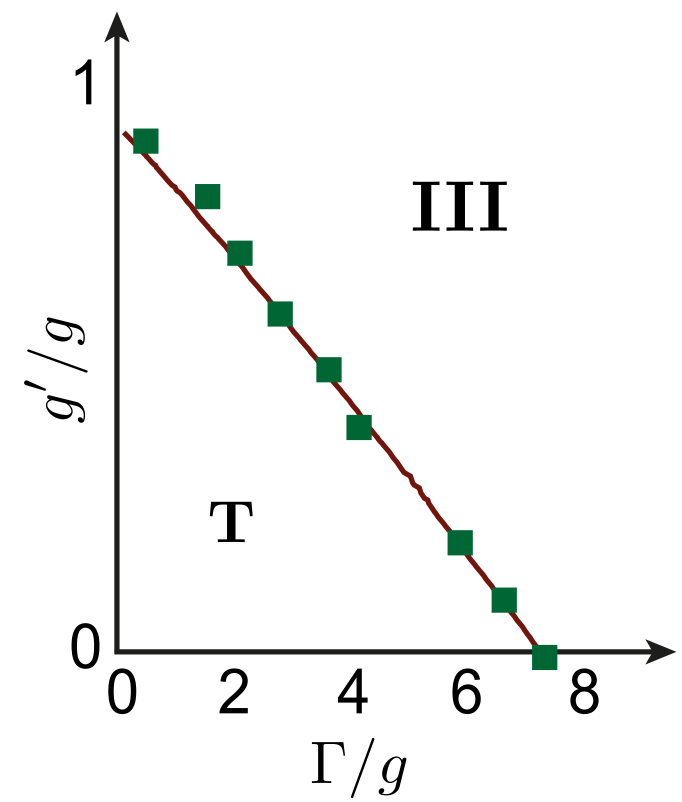
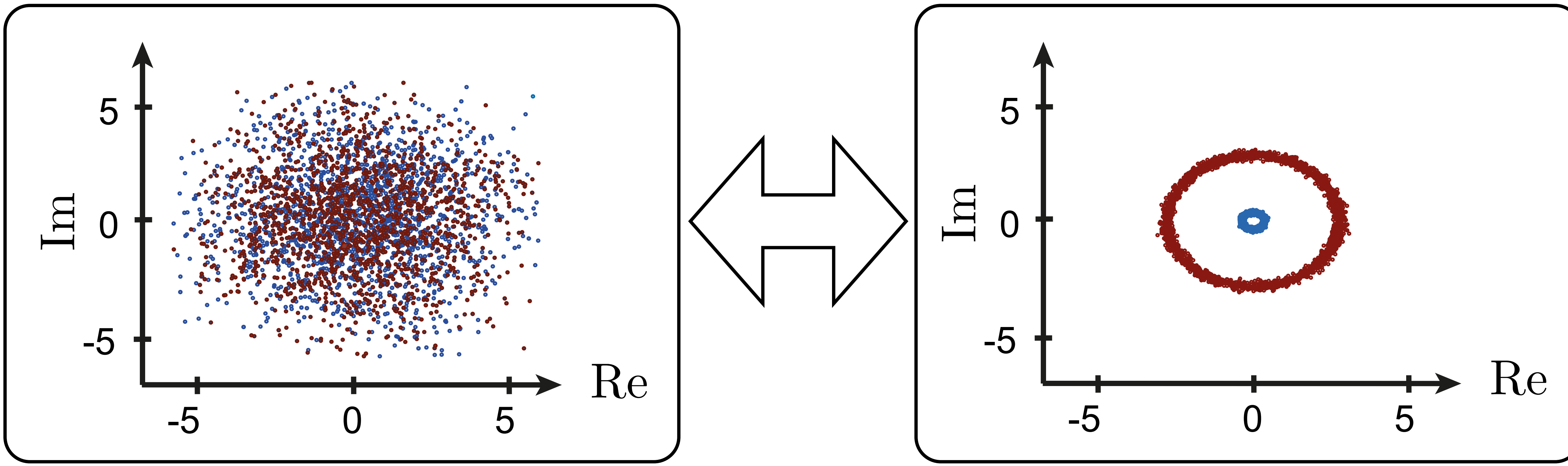




Summary & conclusions

PT-symmetry breaking in steady state:



$$\langle \mathrm{Gain} \rangle_{\mathrm{ss}} = \langle \mathrm{Loss} \rangle_{\mathrm{ss}}$$

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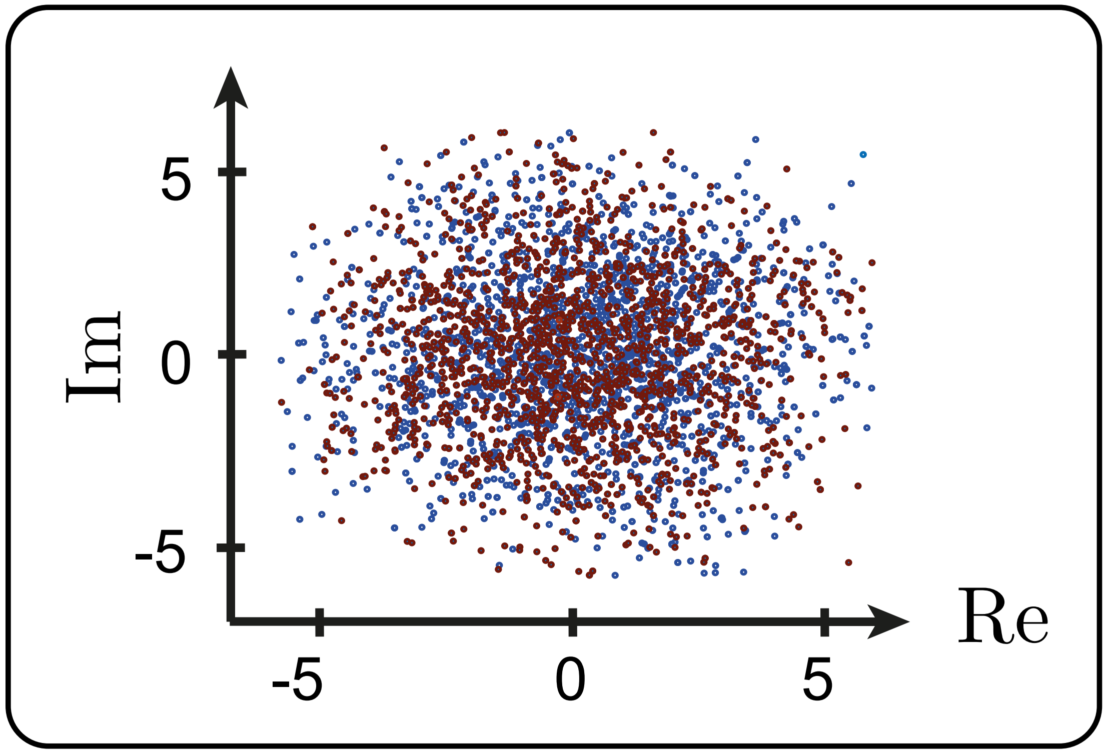
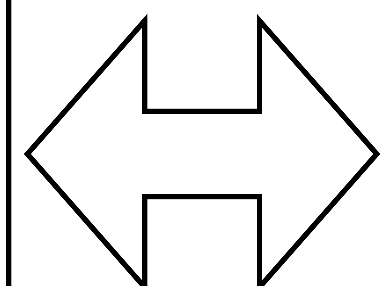
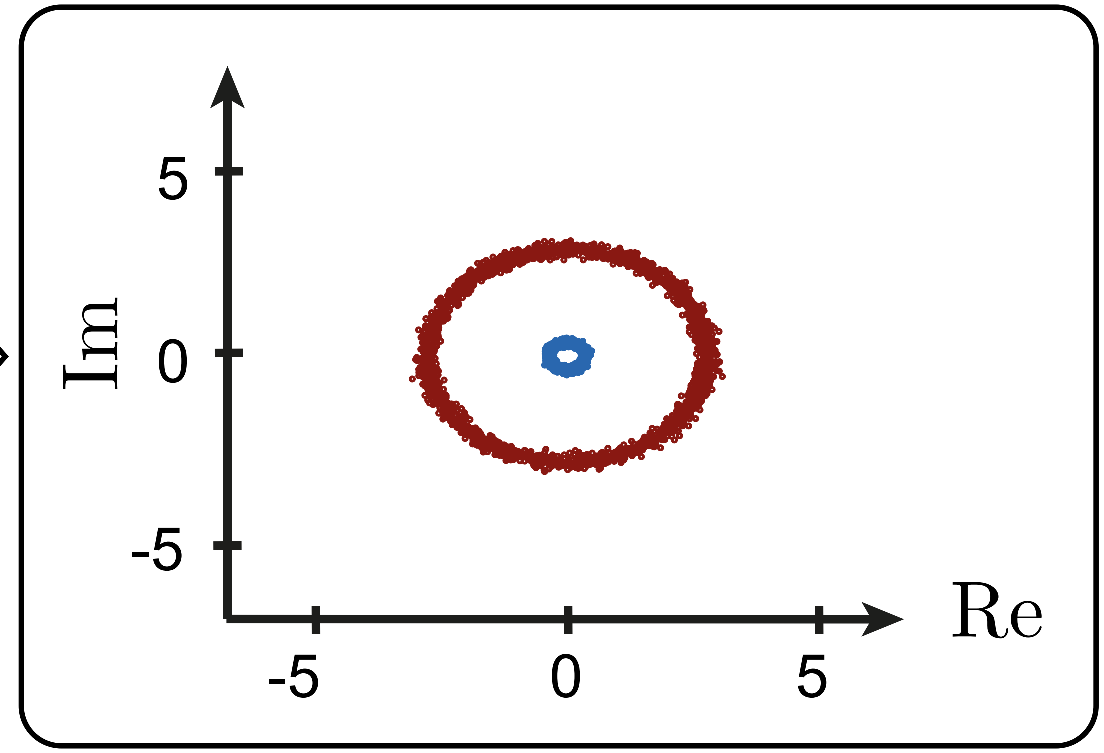
Summary & conclusions

PT-symmetry breaking in steady state:





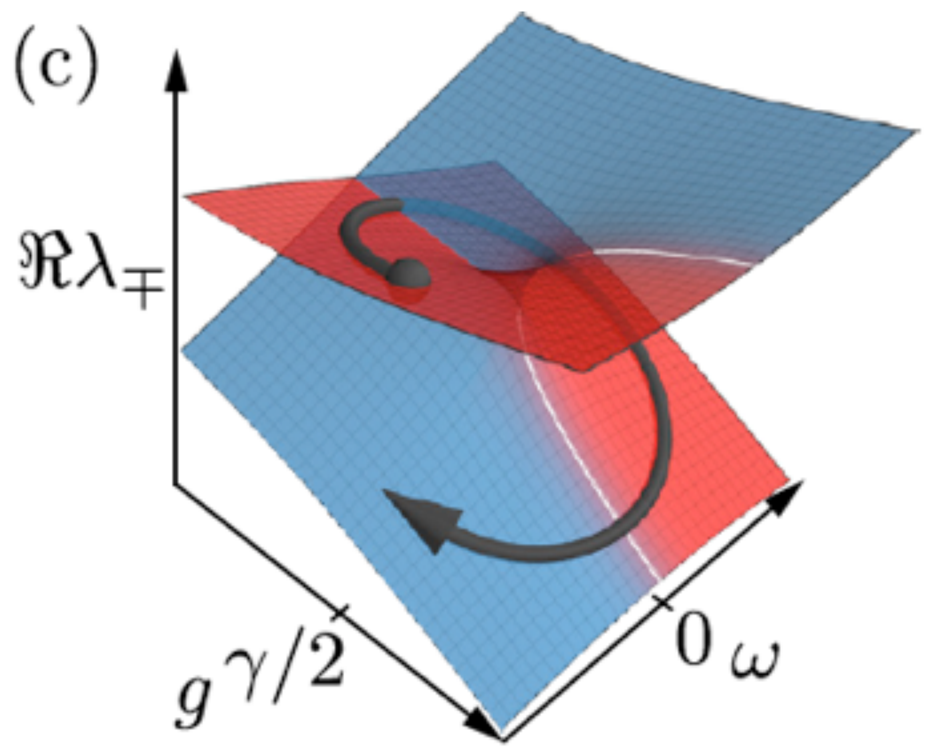


$$\langle \mathrm{Gain} \rangle_{\mathrm{ss}} = \langle \mathrm{Loss} \rangle_{\mathrm{ss}}$$

$$\langle \mathrm{Gain} \rangle_{\mathrm{ss}} \neq \langle \mathrm{Loss} \rangle_{\mathrm{ss}}$$

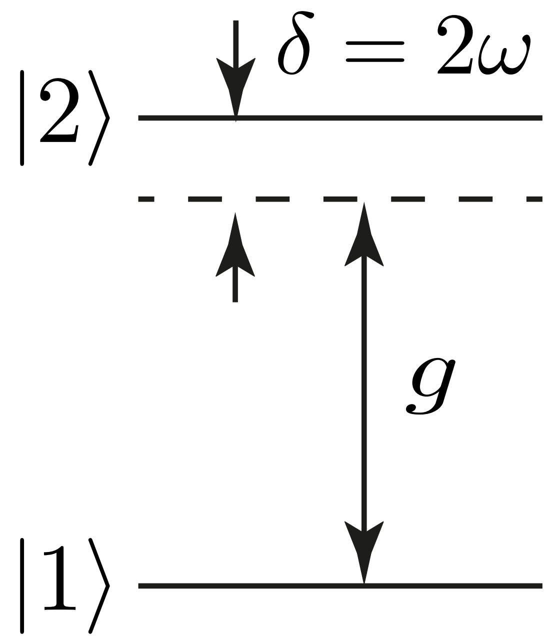
“equilibrium to non-equilibrium transition“ (??)

Quasi-adiabatic encircling of exceptional points



T. Milburn, J. Doppler, C. Holmes, S. Portolan, S. Rotter, PR, arXiv:1410.1882

Adiabatic Theorem

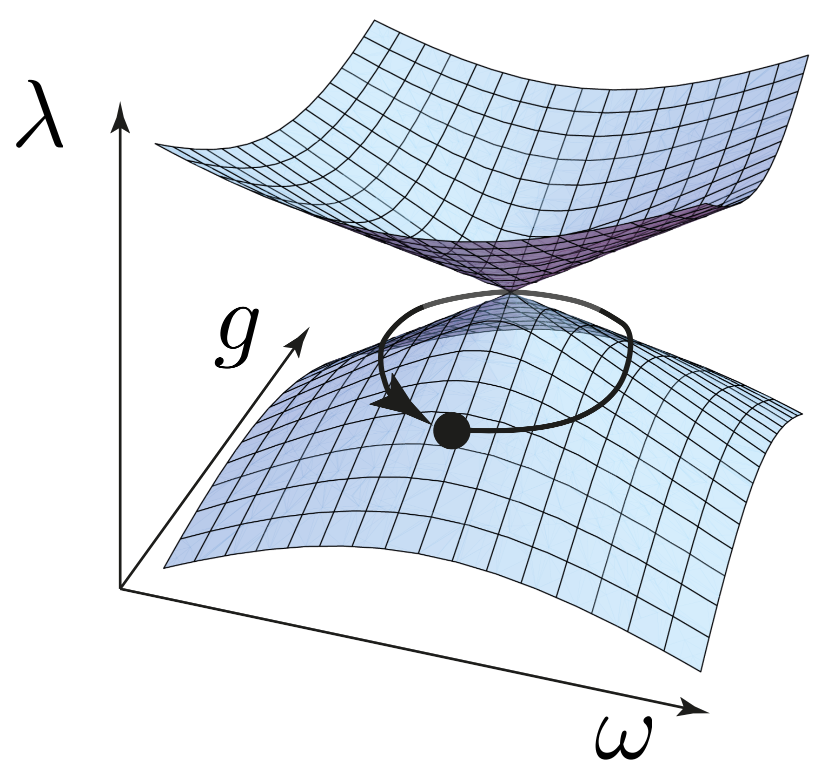


$$|\psi\rangle = \alpha_1|1\rangle + \alpha_2|2\rangle$$

$$\begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{pmatrix} = -i \begin{pmatrix} -\omega & g \\ g & \omega \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

Hamiltonian

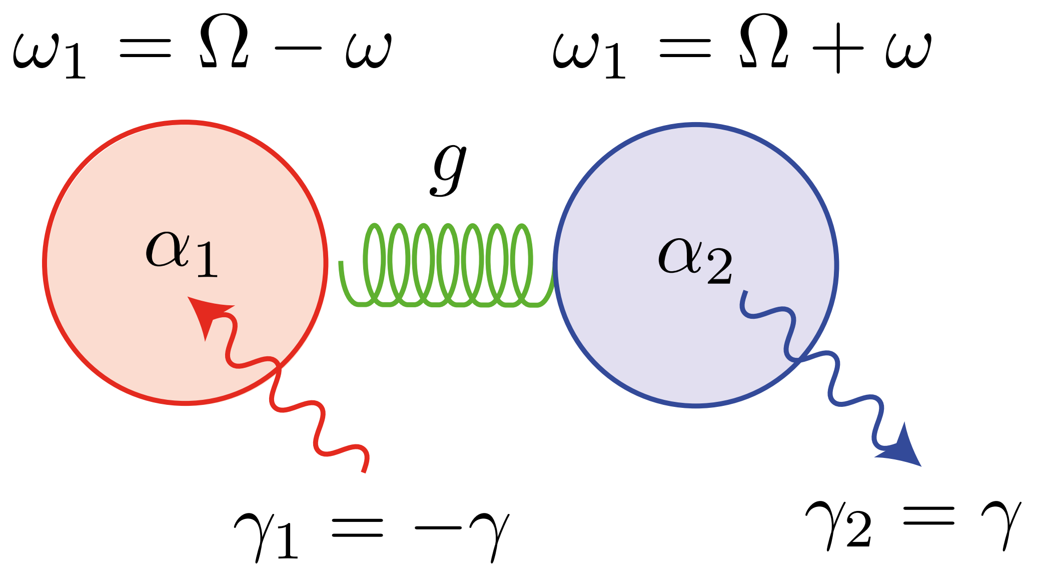
$$\lambda_\pm = \pm\sqrt{\omega^2 + g^2}$$



"adiabatic"

$$\varepsilon := \frac{1}{|\lambda_+ - \lambda_-|T} \ll 1$$

Non-Hermitian "Two-Level-System"



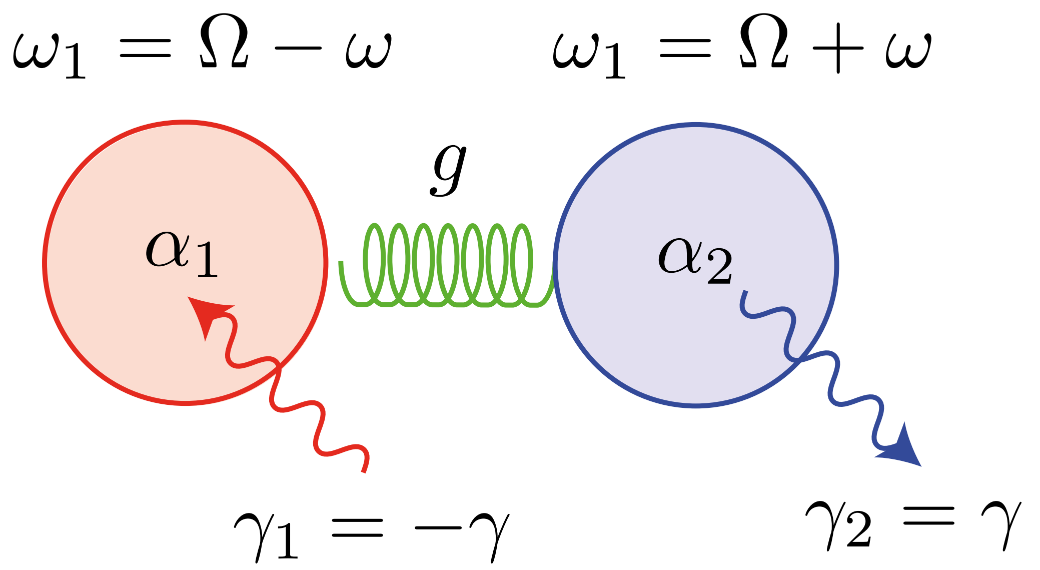
$$\begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{pmatrix} = -i \begin{pmatrix} -\omega - i\frac{\gamma}{2} & g \\ g & \omega + i\frac{\gamma}{2} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

non-Hermitian

$$g, \omega, \gamma \qquad \longrightarrow \qquad g(t), \omega(t), \gamma(t)$$

Adiabaticity in non-Hermitian systems ?

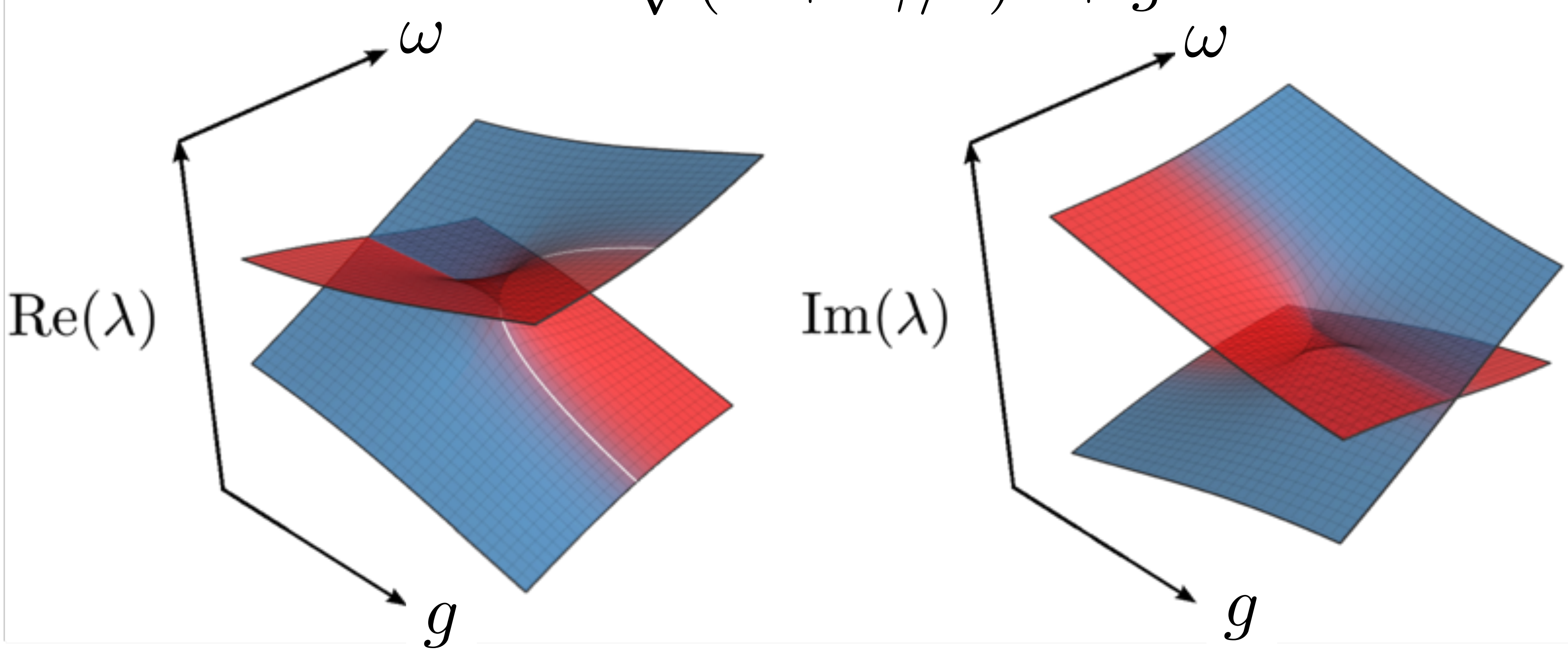
Non-Hermitian "Two-Level-System"



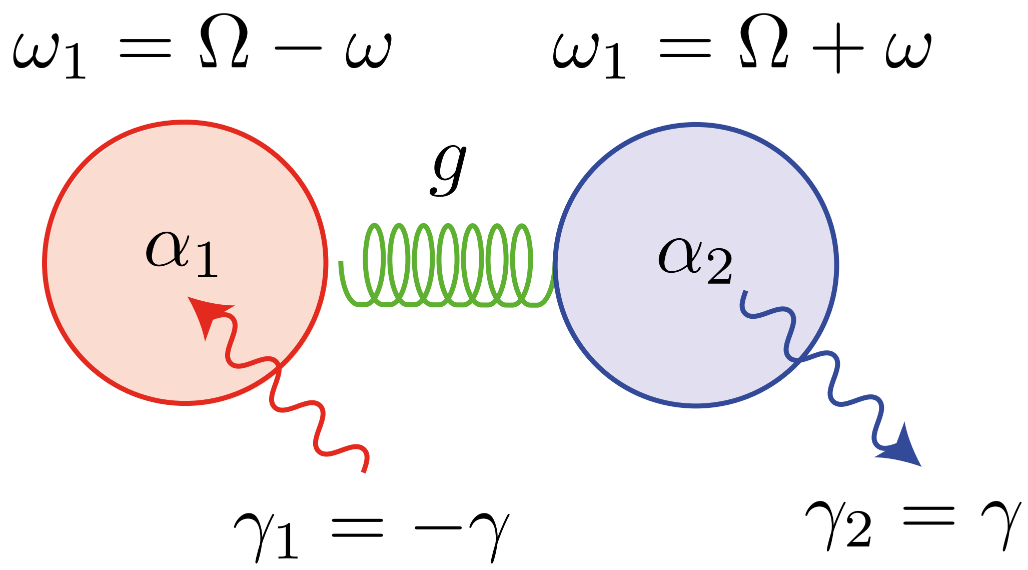
$$\begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{pmatrix} = -i \begin{pmatrix} -\omega - i\frac{\gamma}{2} & g \\ g & \omega + i\frac{\gamma}{2} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

non-Hermitian

$$\lambda = \pm\sqrt{(\omega + i\gamma/2)^2 + g^2}$$



Non-Hermitian "Two-Level-System"



$$\begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{pmatrix} = -i \begin{pmatrix} -\omega - i\frac{\gamma}{2} & g \\ g & \omega + i\frac{\gamma}{2} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

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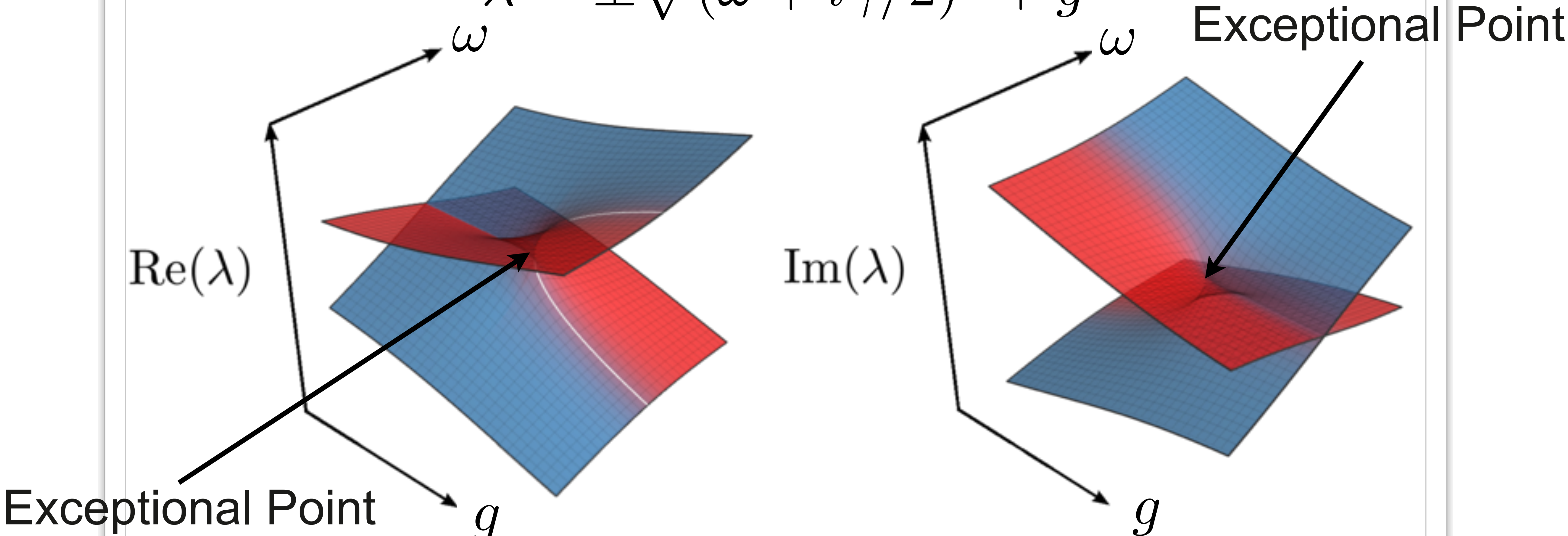
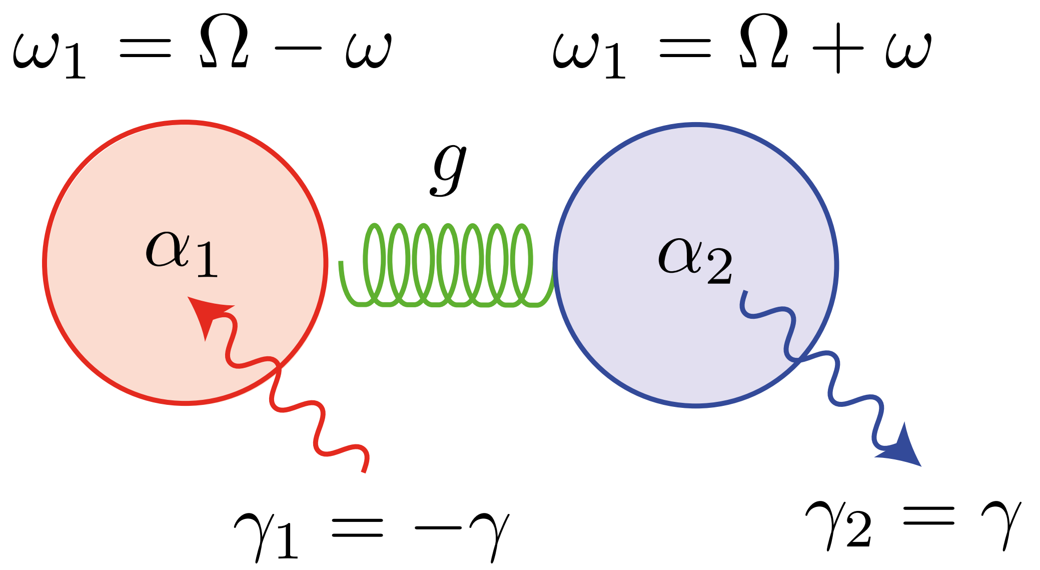


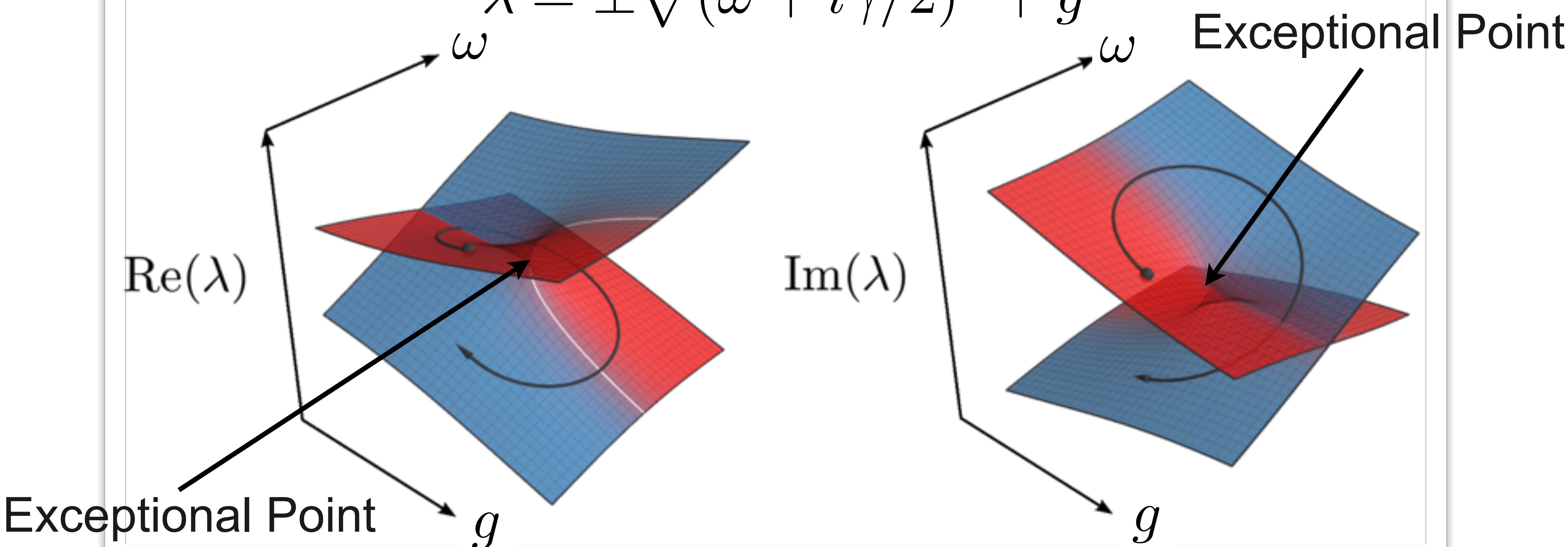
Non-Hermitian "Two-Level-System"



$$\begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{pmatrix} = -i \begin{pmatrix} -\omega - i\frac{\gamma}{2} & g \\ g & \omega + i\frac{\gamma}{2} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

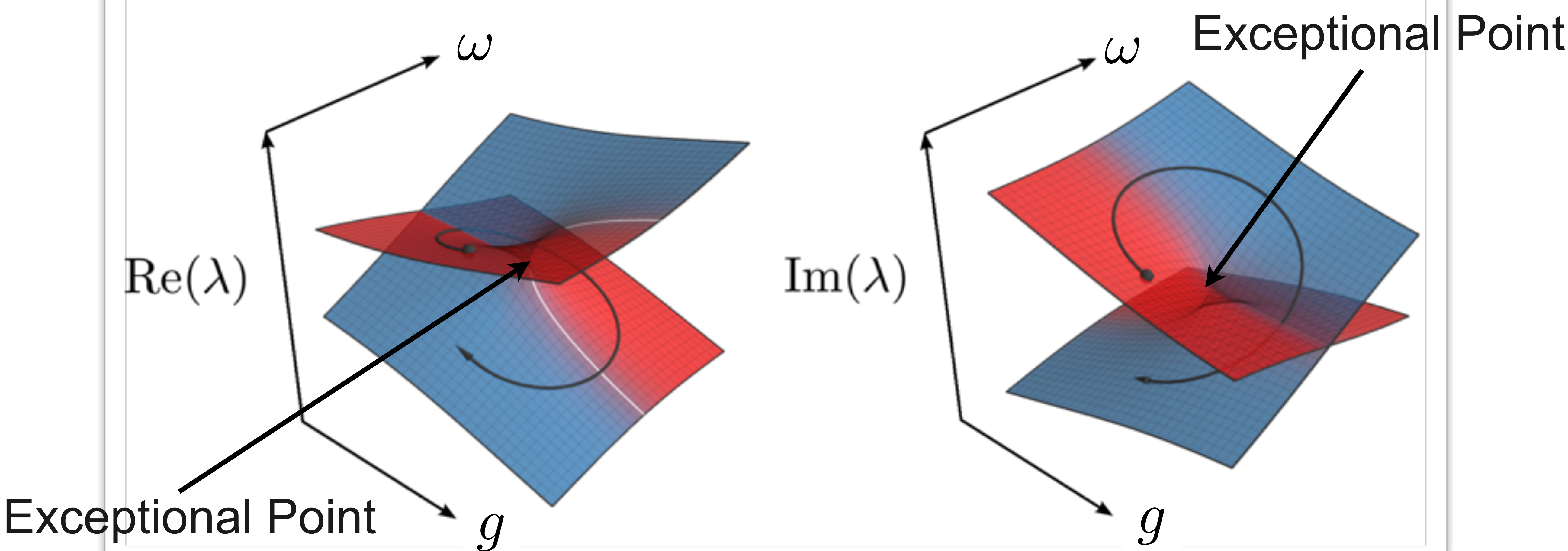
non-Hermitian

$$\lambda = \pm\sqrt{(\omega + i\gamma/2)^2 + g^2}$$



Non-Hermitian "Two-Level-System"

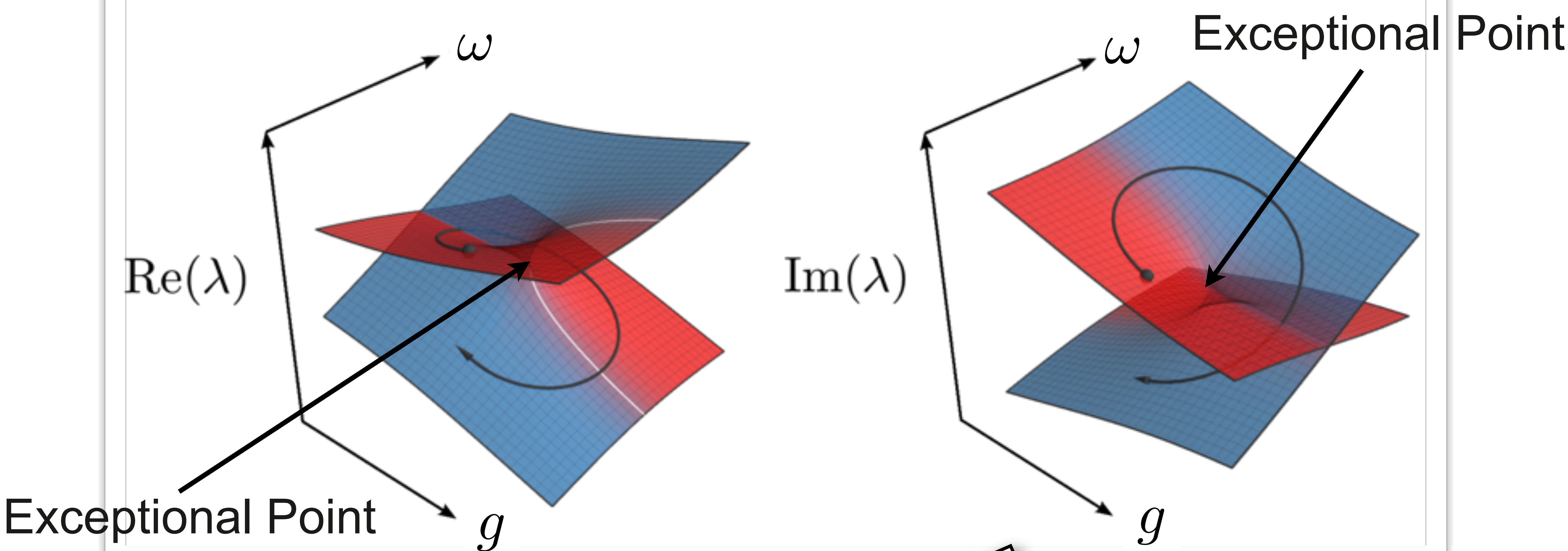
$$\varepsilon := \frac{1}{|\lambda_+ - \lambda_-|T} \ll 1$$



Can we observe an adiabatic state flip in a non-Hermitian system ?

Non-Hermitian "Two-Level-System"

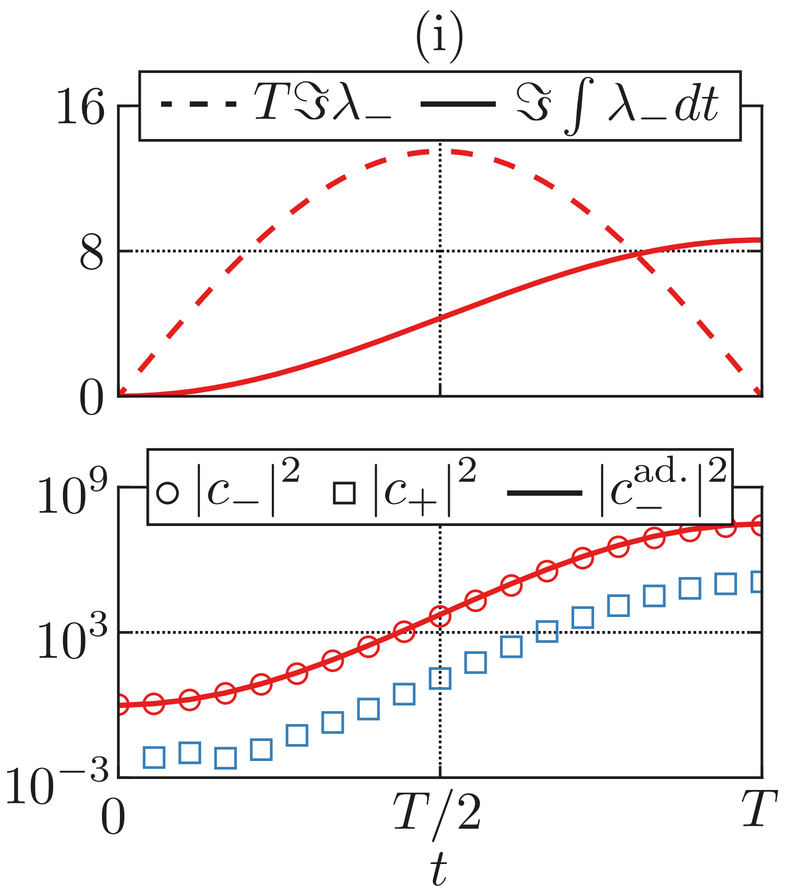
$$\varepsilon := \frac{1}{|\lambda_+ - \lambda_-|T} \ll 1$$

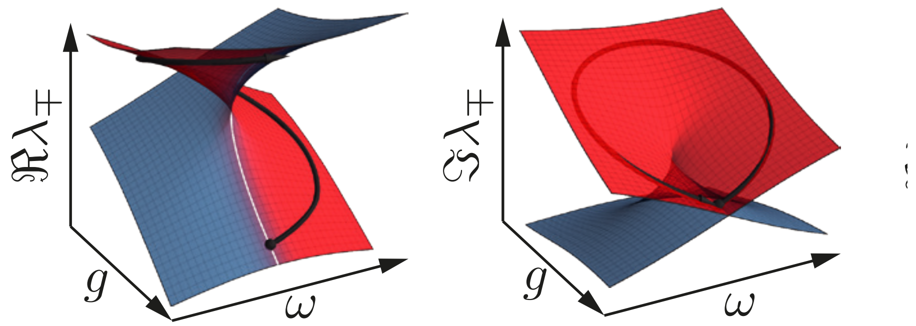


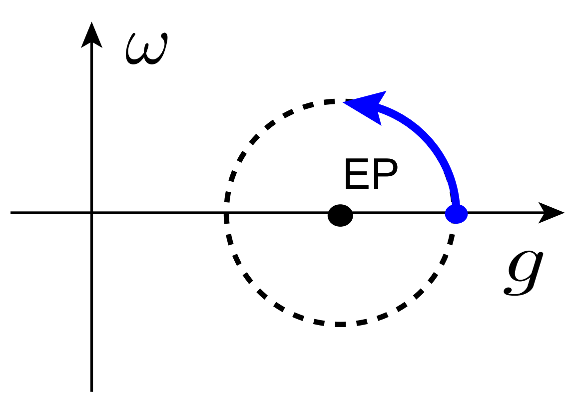
Can we observ abatic state flip in a mitian system ?

Almost!

Quasi-adiabatic non-Hermitian dynamics







$$\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = c_-(t)\vec{v}_-(t) + c_+(t)\vec{v}_+(t)$$

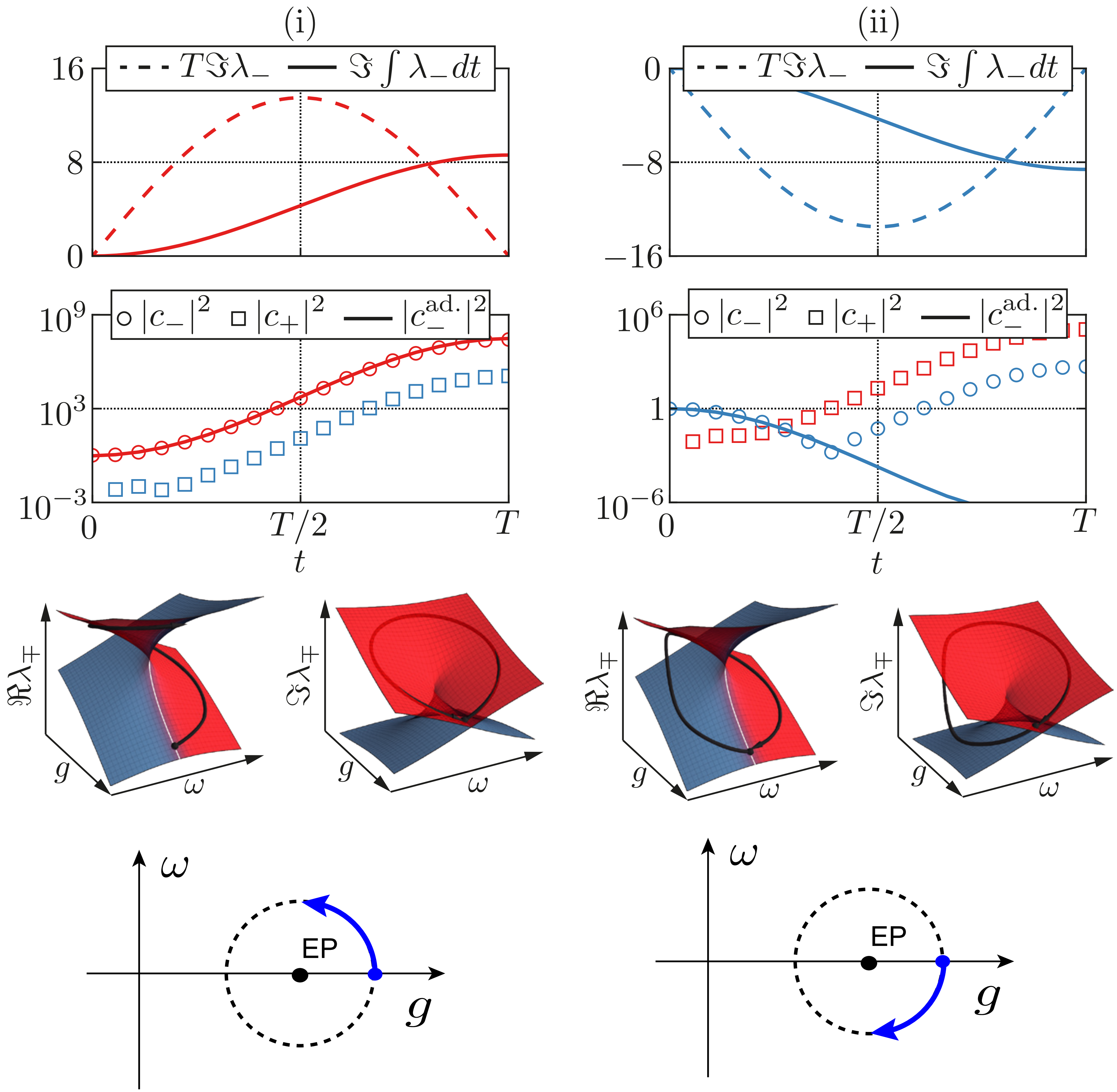
instantaneous eigenvectors

$$c_-\big|_{t=0} = 1 \qquad c_+\big|_{t=0} = 0$$

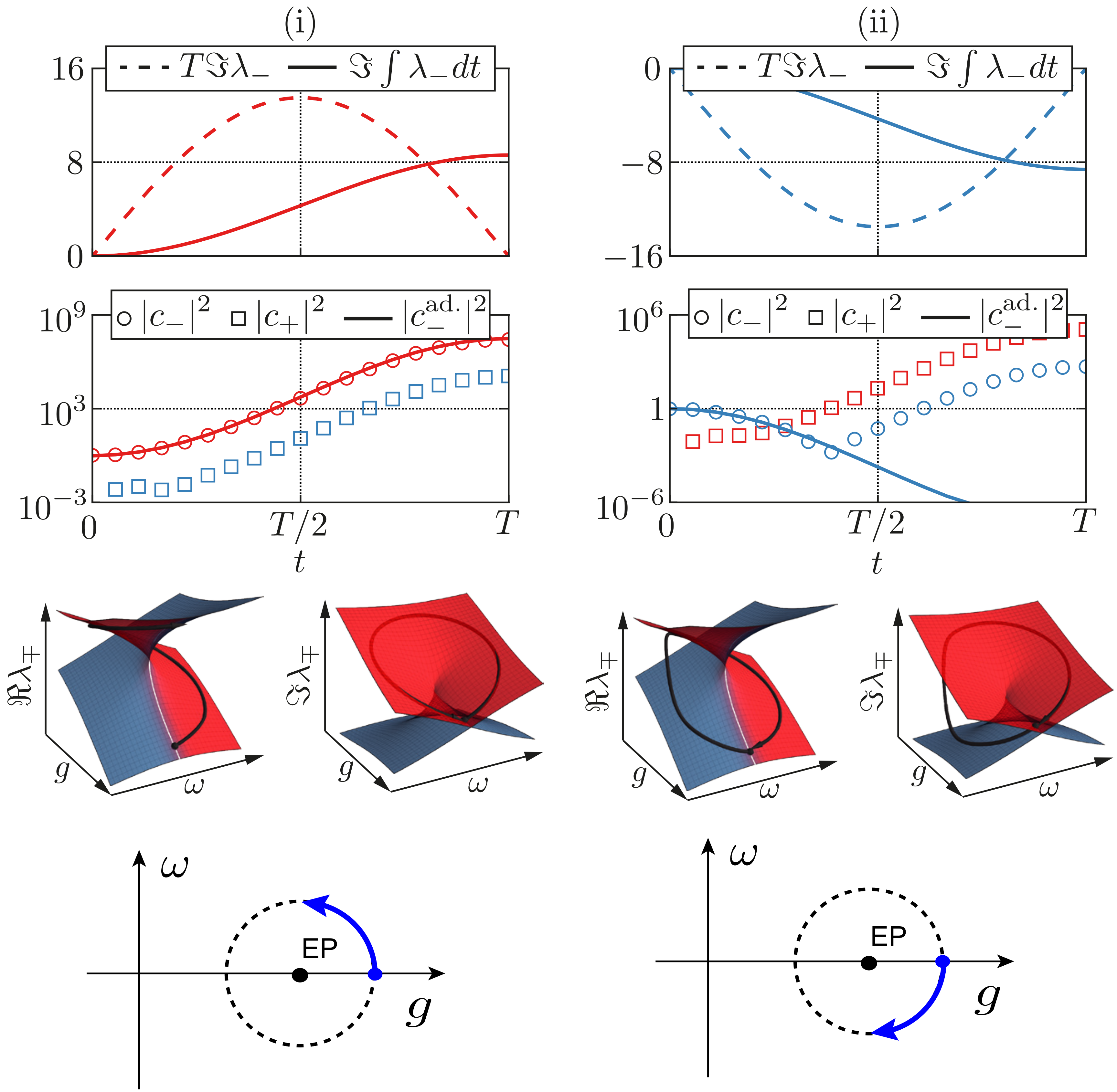
$$\varepsilon := \frac{1}{|\lambda_+ - \lambda_-|T} \ll 1$$

“adiabatic”

Quasi-adiabatic non-Hermitian dynamics



Quasi-adiabatic non-Hermitian dynamics



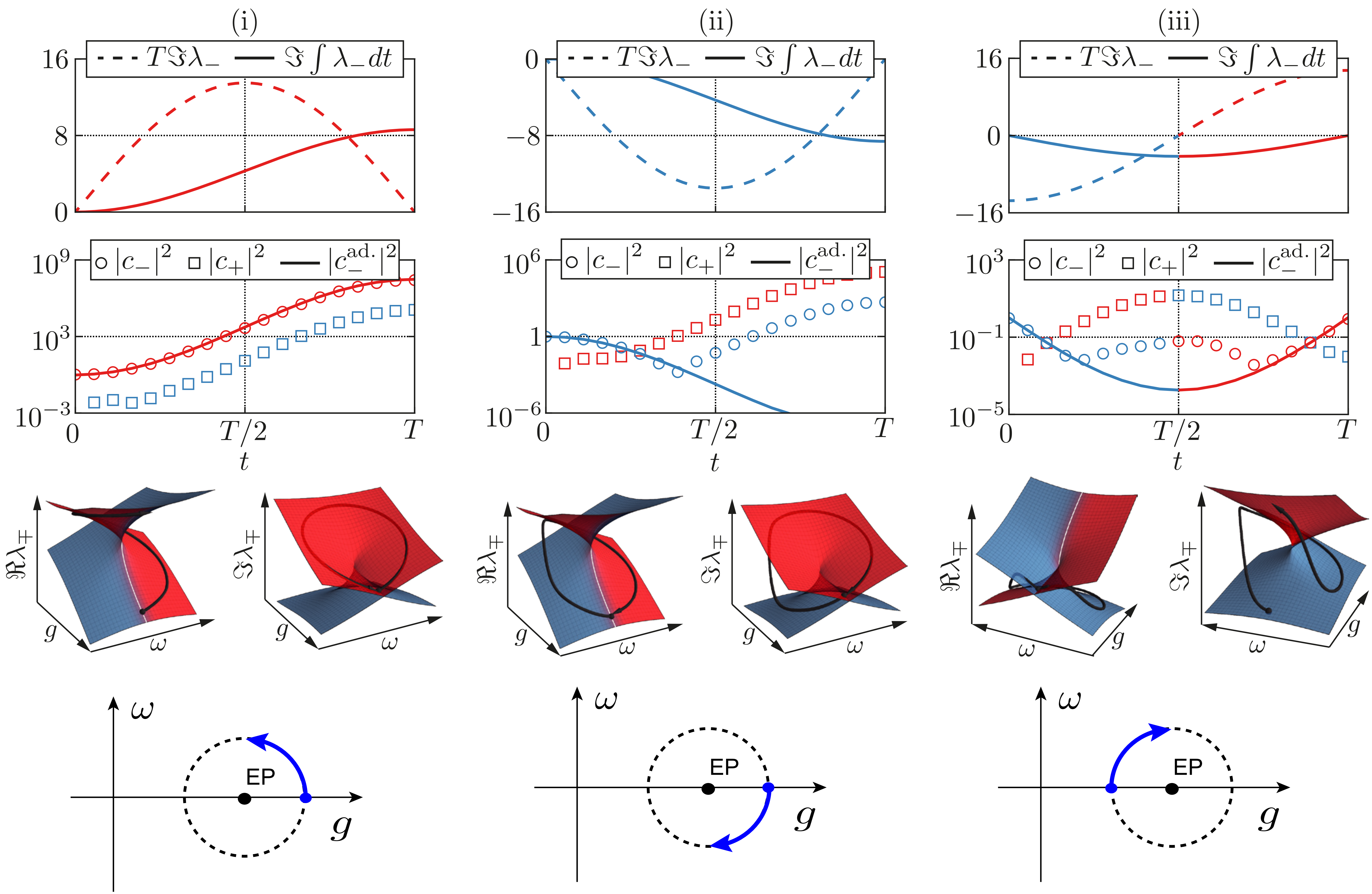
break down of the adiabatic theorem

and

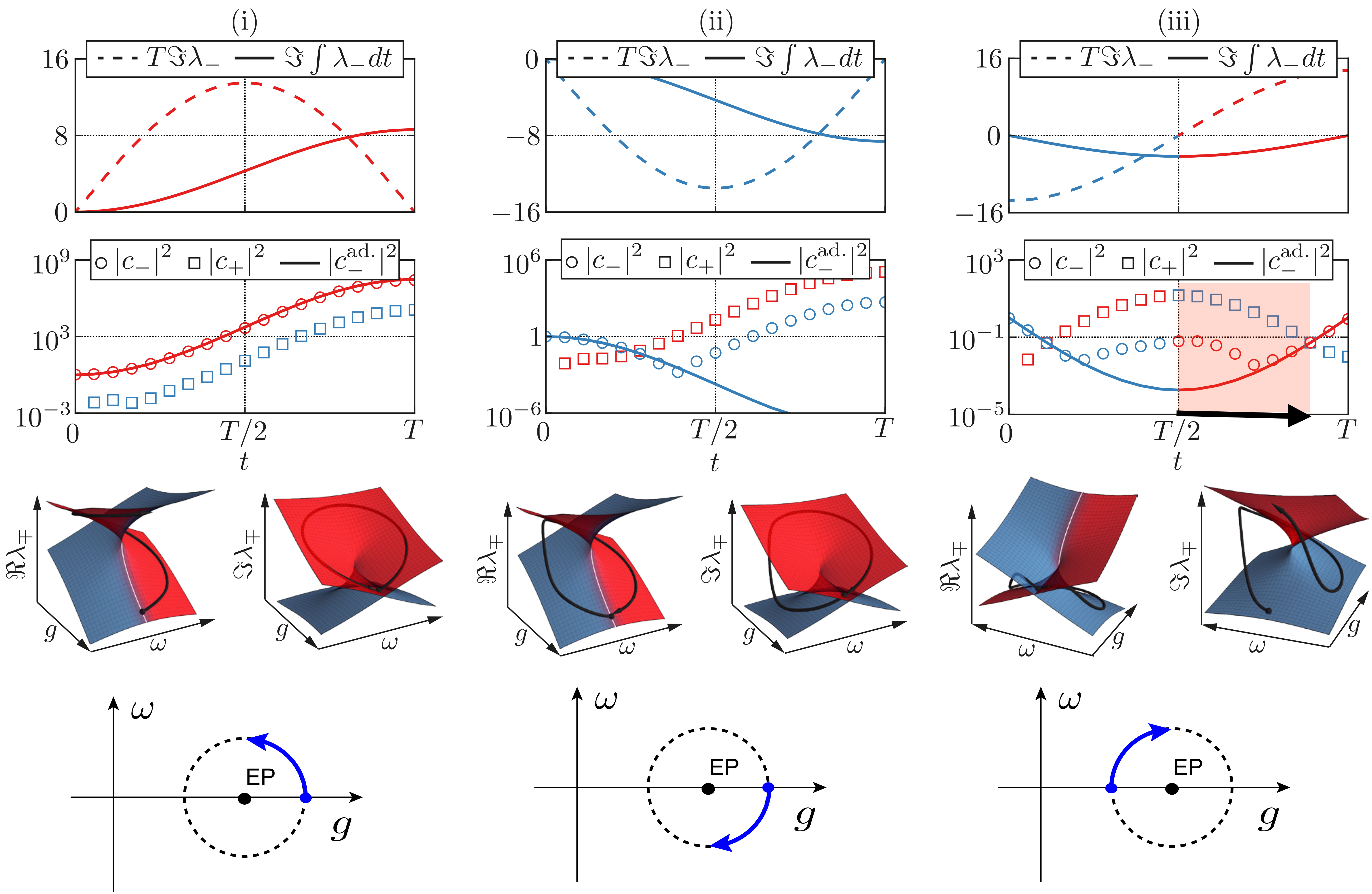
"chiral behavior"

Uzdin, Mailybaev, Moiseyev, J. Phys. A: Math. Theor. **44**, 435302 (2011)

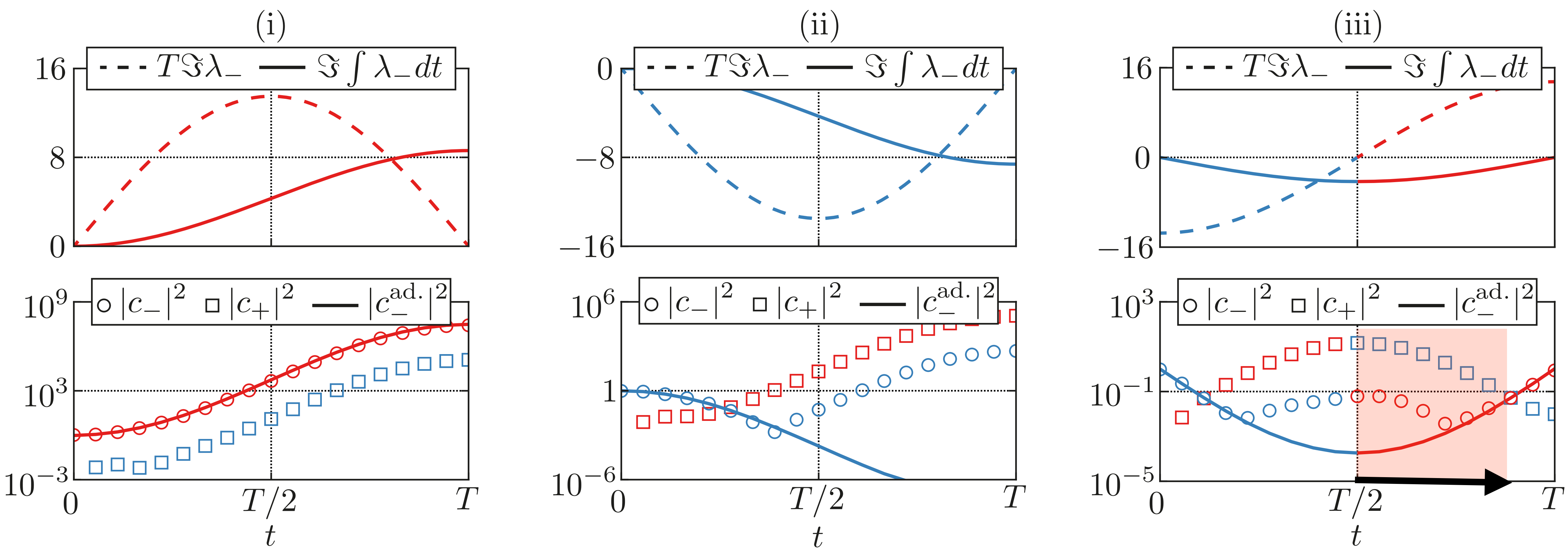
Quasi-adiabatic non-Hermitian dynamics



Quasi-adiabatic non-Hermitian dynamics



Quasi-adiabatic non-Hermitian dynamics



Phenomenology:

i) State flips due to 4π periodicity around exceptional points !

ii) Break-down of adiabaticity due to gain/loss !

iii) “Piece-wise adiabatic” dynamics, beyond instability points !

Quasi-adiabatic dynamics

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = c_-(t)\vec{v}_-(t) + c_+(t)\vec{v}_+(t)$$

instantaneous eigenvectors

$$\Rightarrow \qquad \begin{pmatrix} \dot{c}_- \\ \dot{c}_+ \end{pmatrix} = -i \begin{pmatrix} -\lambda(t) & -f(t) \\ f(t) & \lambda(t) \end{pmatrix} \begin{pmatrix} c_- \\ c_+ \end{pmatrix}$$

Quasi-adiabatic dynamics

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = c_-(t)\vec{v}_-(t) + c_+(t)\vec{v}_+(t)$$

instantaneous eigenvectors

$$\Rightarrow \qquad \begin{pmatrix} \dot{c}_- \\ \dot{c}_+ \end{pmatrix} = -i \begin{pmatrix} -\lambda(t) & -f(t) \\ f(t) & \lambda(t) \end{pmatrix} \begin{pmatrix} c_- \\ c_+ \end{pmatrix}$$

non-adiabatic couplings

$$\varepsilon(t) := \left| \frac{f(t)}{2\lambda(t)} \right| \ll 1$$

adiabaticity parameter

Non-adiabatic transition amplitudes

$$c_-(0) = 1$$
$$c_+(0) = 0$$

$$R(t) := \frac{c_+(t)}{c_-(t)}$$

Non-adiabatic transition amplitudes

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$$R(t) := \frac{c_+(t)}{c_-(t)}$$

$R(t) \ll 1$ — **system in adiabatic eigenstate**

$R(t) \gg 1$ — **transition to non-adiabatic eigenstate**

Non-adiabatic transition amplitudes

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$$c_+(0) = 0$$

$$R(t) := \frac{c_+(t)}{c_-(t)}$$

$R(t) \ll 1$ — **system in adiabatic eigenstate**

$R(t) \gg 1$ — **transition to non-adiabatic eigenstate**

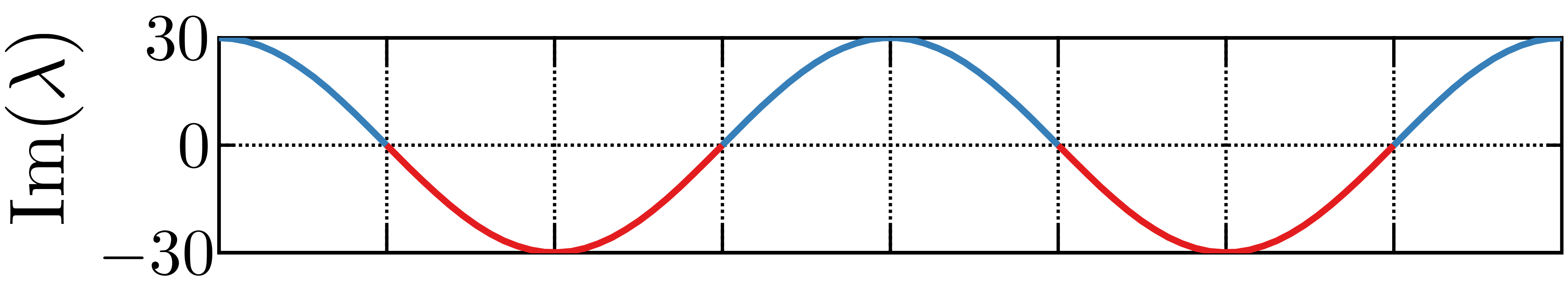
Dynamics [1]:

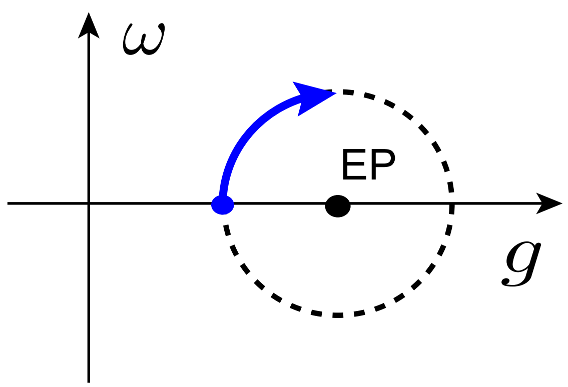
$$\frac{dR}{dt} = -2i\lambda(t)R - if(t)(1+R^2)$$

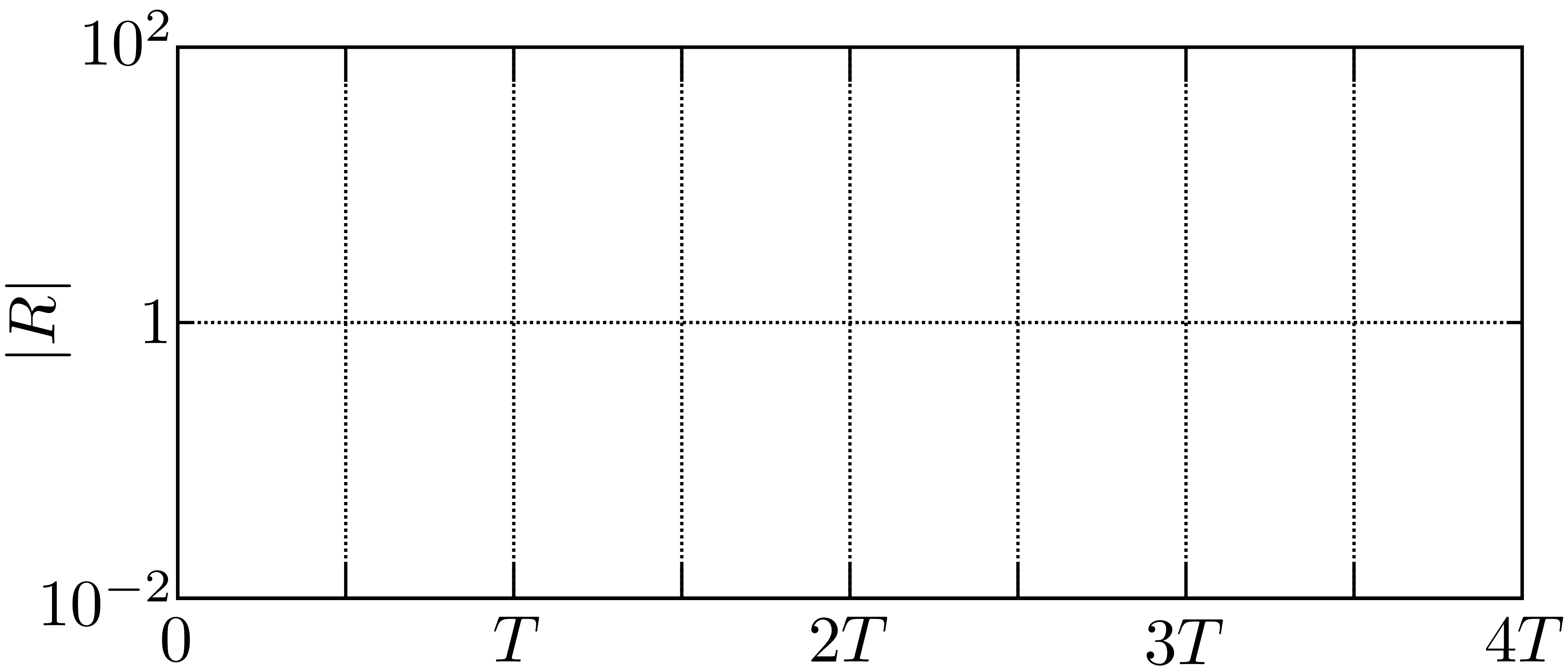
[1] see also M. V. Berry, R. Uzdin, J. Phys. A: Math. Theor. ***44****,435303 (2011)*
M. V. Berry, J. Opt. ***13****, 115701 (2011).*

Non-adiabatic transition amplitudes

$$\frac{dR}{dt} = -2i\lambda(t)R - if(t)(1+R^2)$$

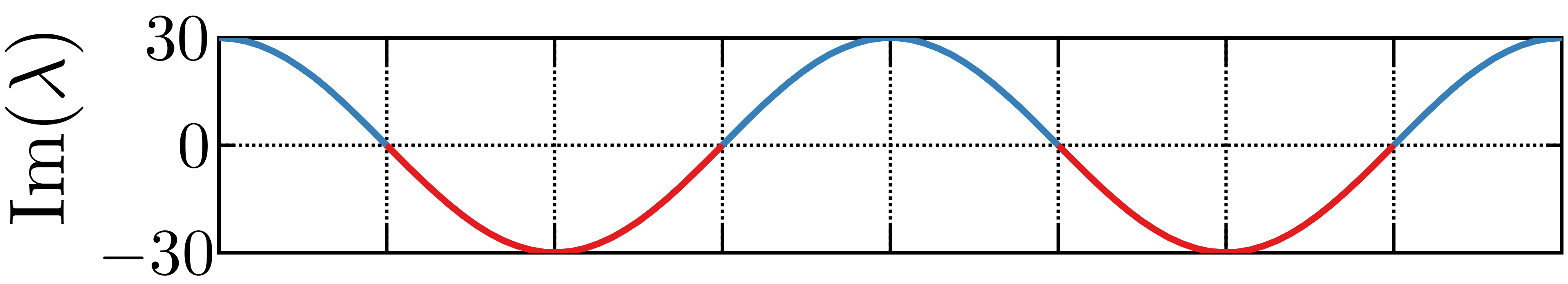


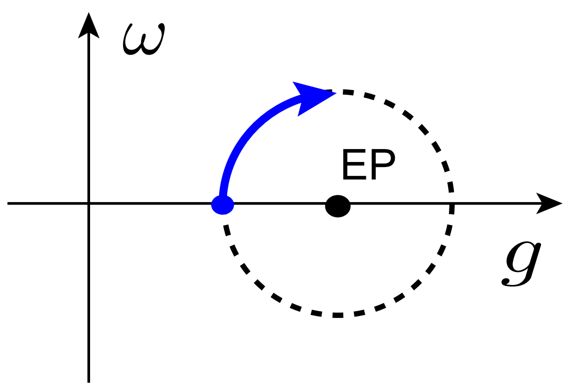


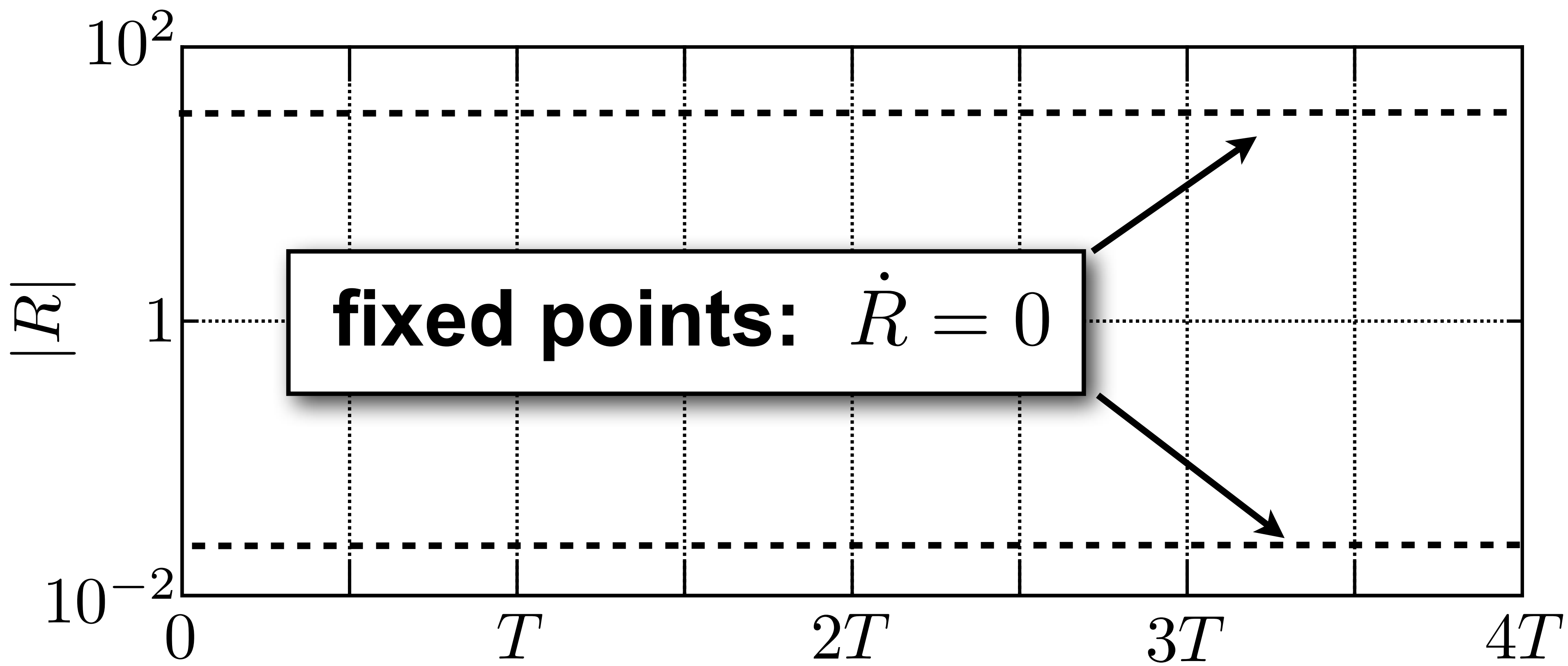


Non-adiabatic transition amplitudes

$$\frac{dR}{dt} = -2i\lambda(t)R - if(t)(1+R^2)$$







$\sim \varepsilon^{-1} \to \infty$

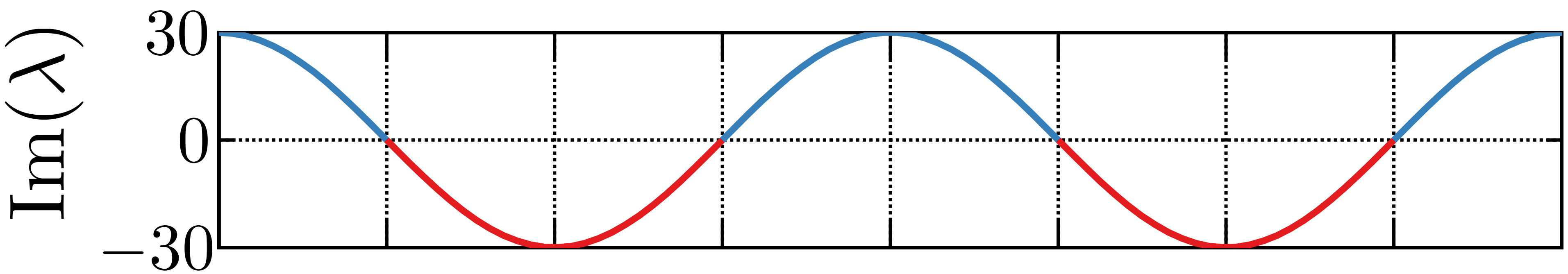
(non-adiabatic)

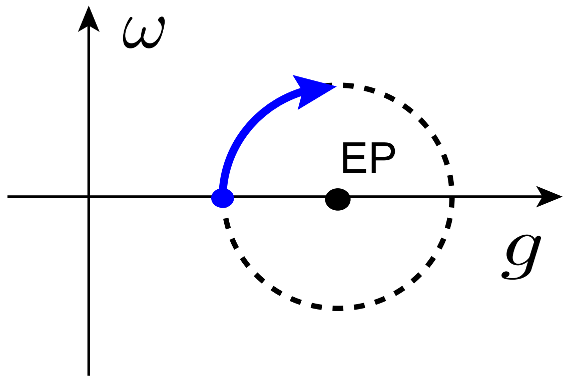
$\sim \varepsilon \to 0$

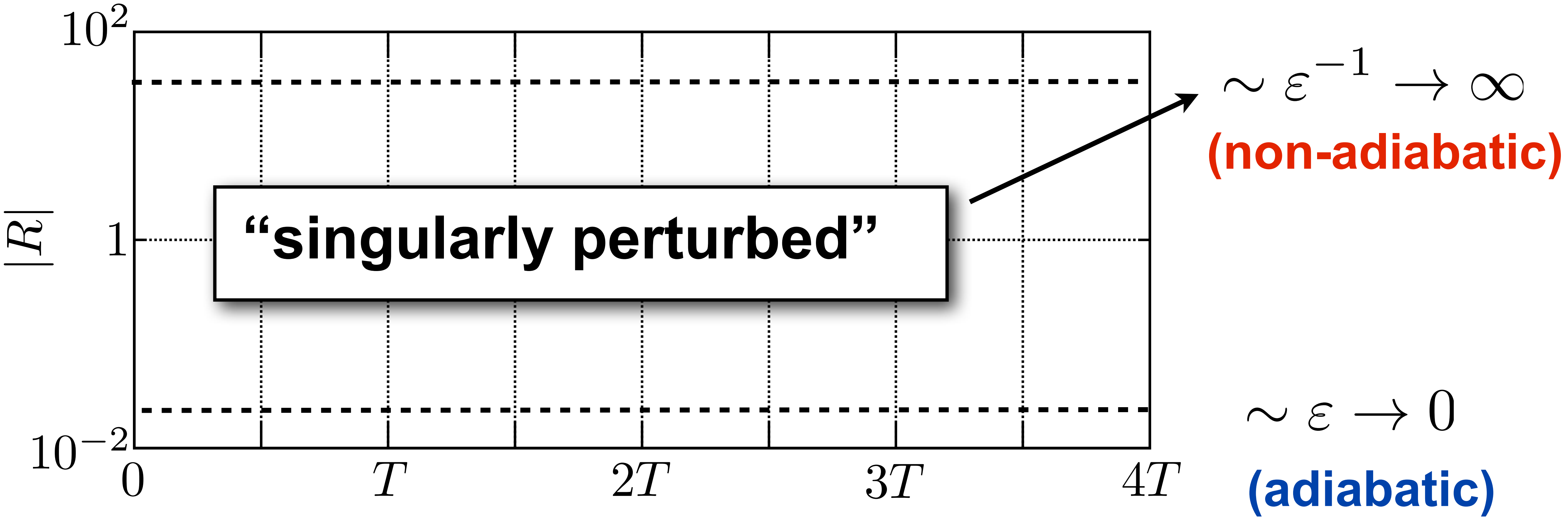
(adiabatic)

Non-adiabatic transition amplitudes

$$\frac{dR}{dt} = -2i\lambda(t)R - if(t)(1+R^2)$$

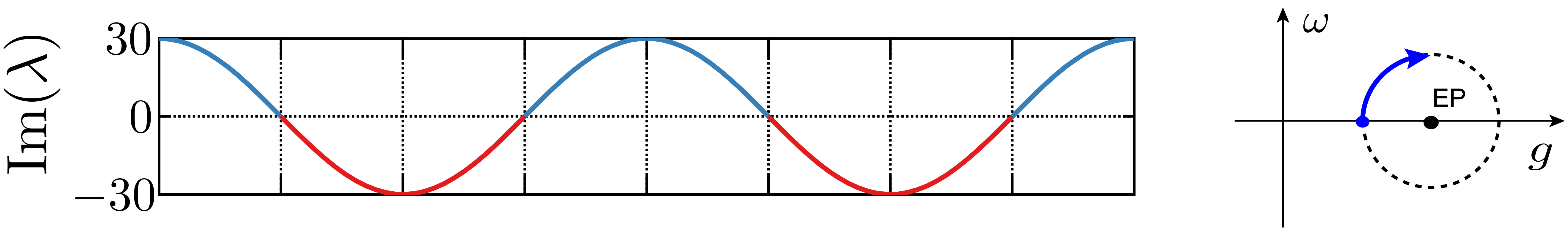


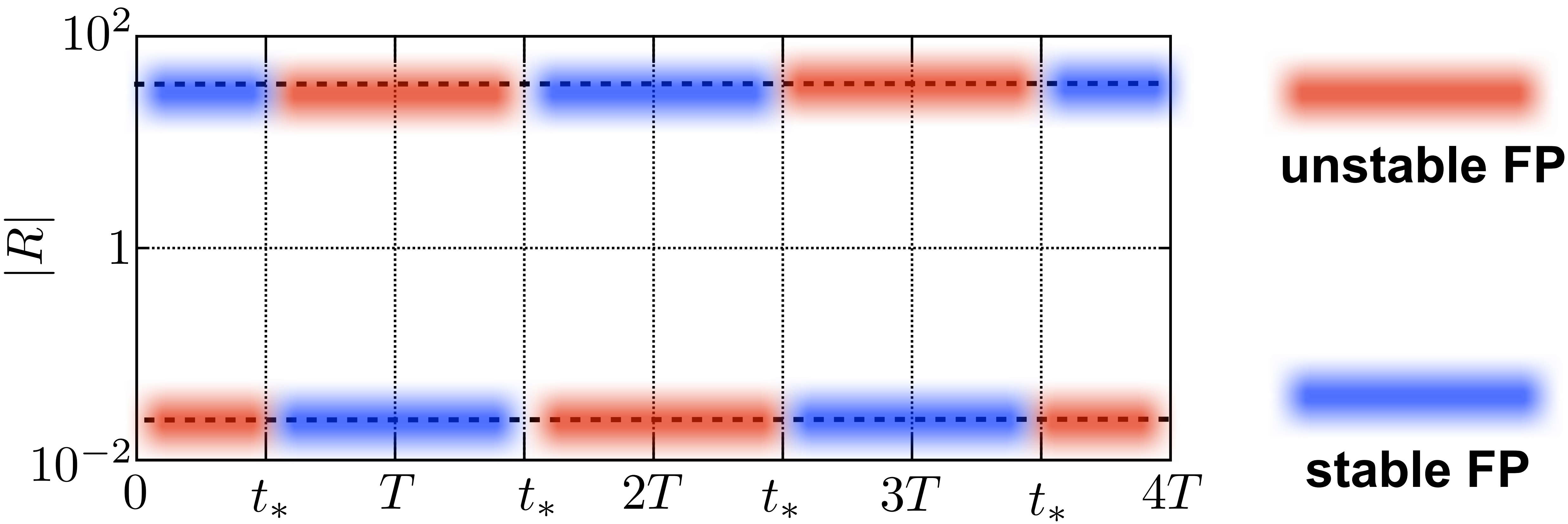




Non-adiabatic transition amplitudes

$$\frac{dR}{dt} = -2i\lambda(t)R - if(t)(1 + R^2)$$





Non-adiabatic transition amplitudes

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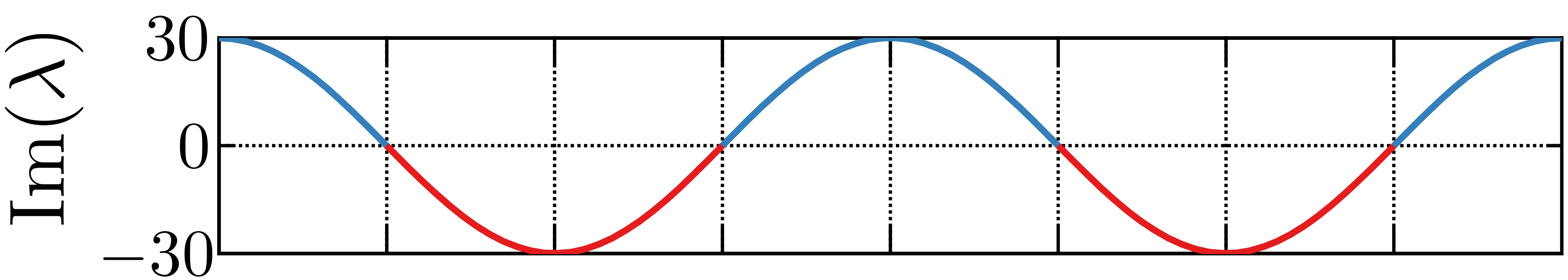
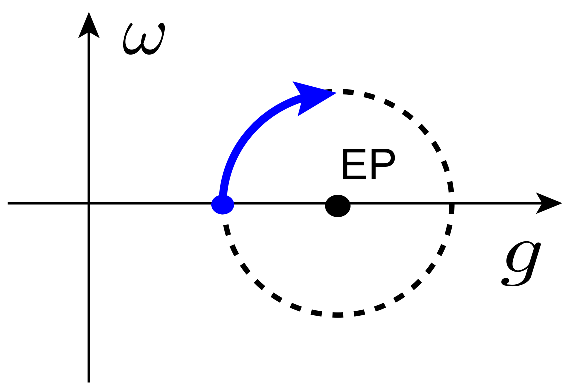
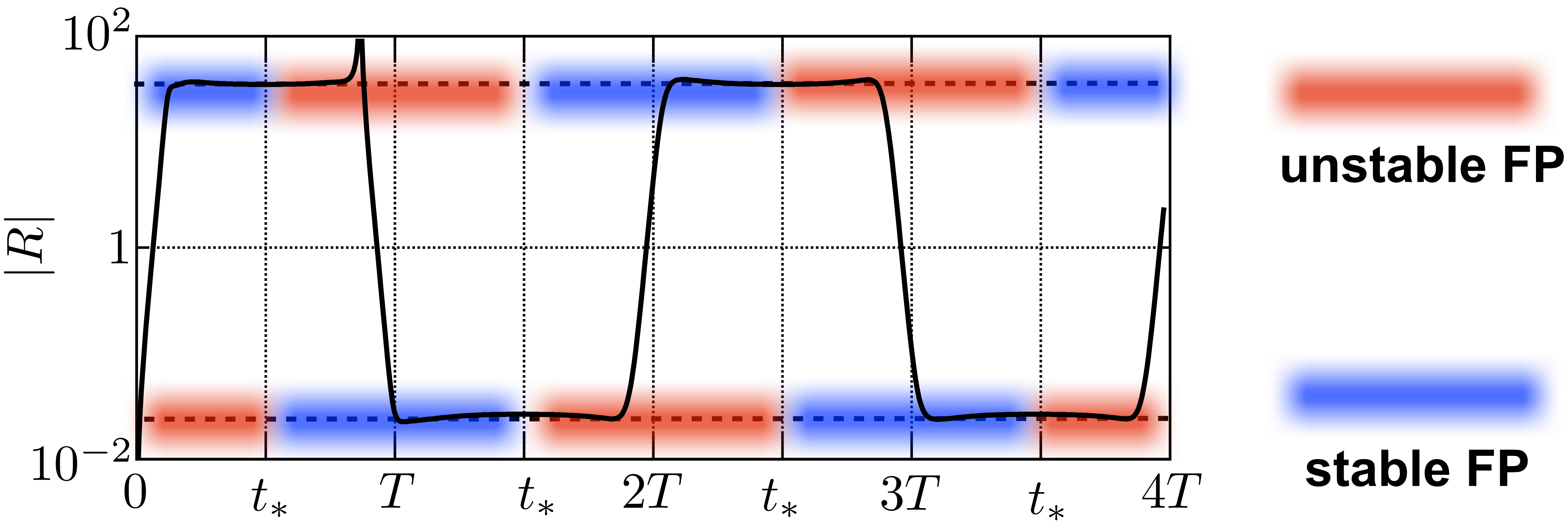
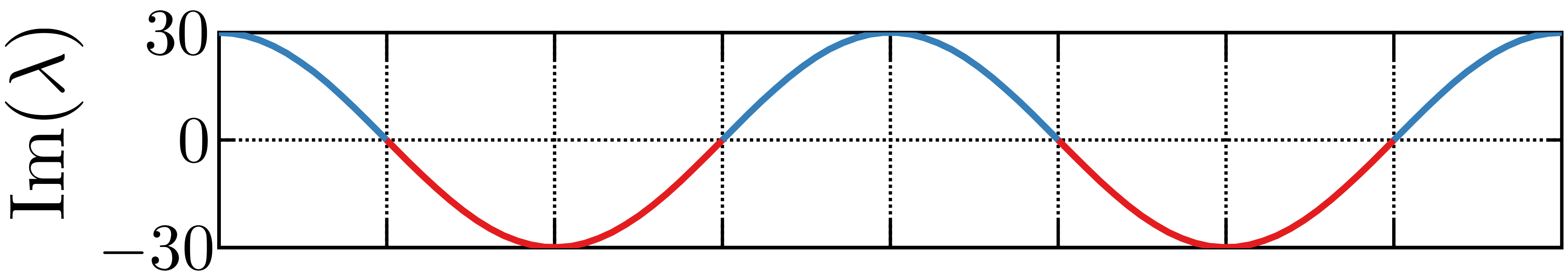


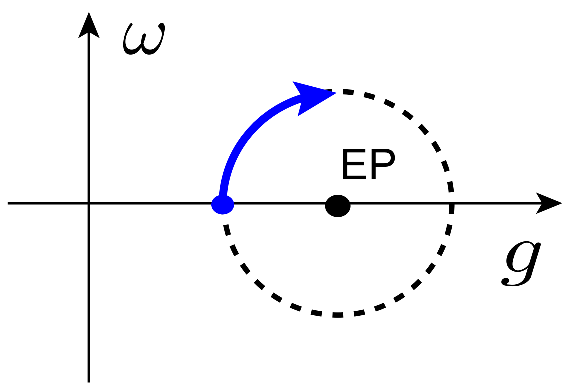


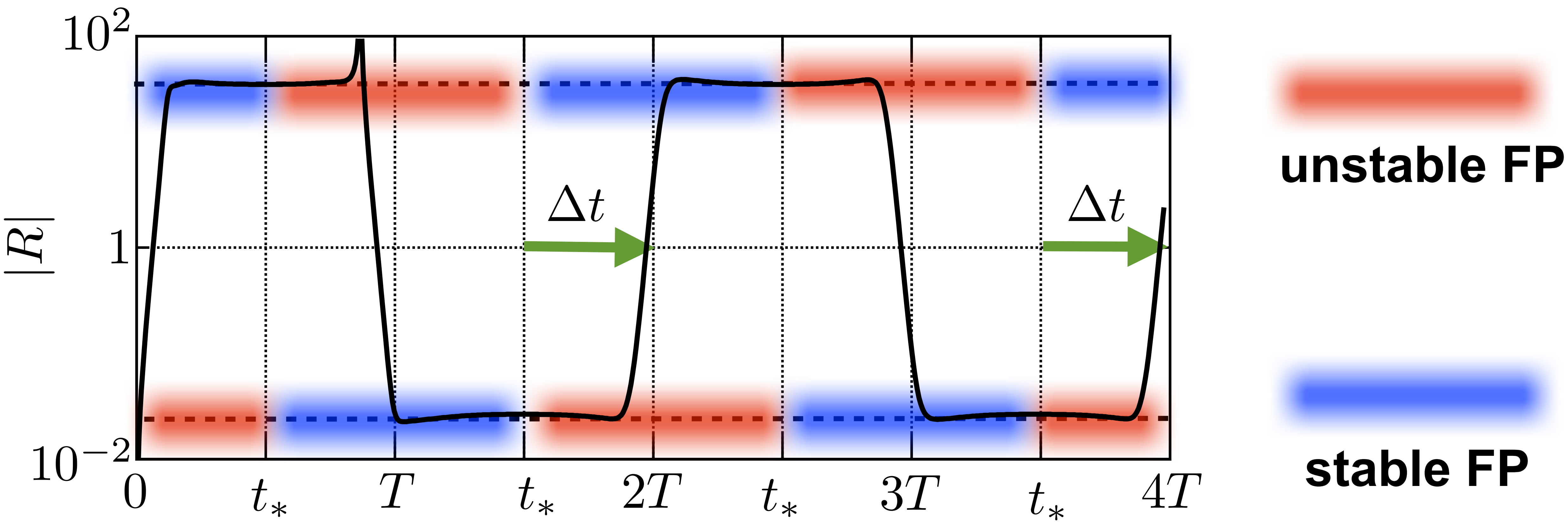


Non-adiabatic transition amplitudes

$$\frac{dR}{dt} = -2i\lambda(t)R - if(t)(1+R^2)$$







Non-adiabatic transition amplitudes

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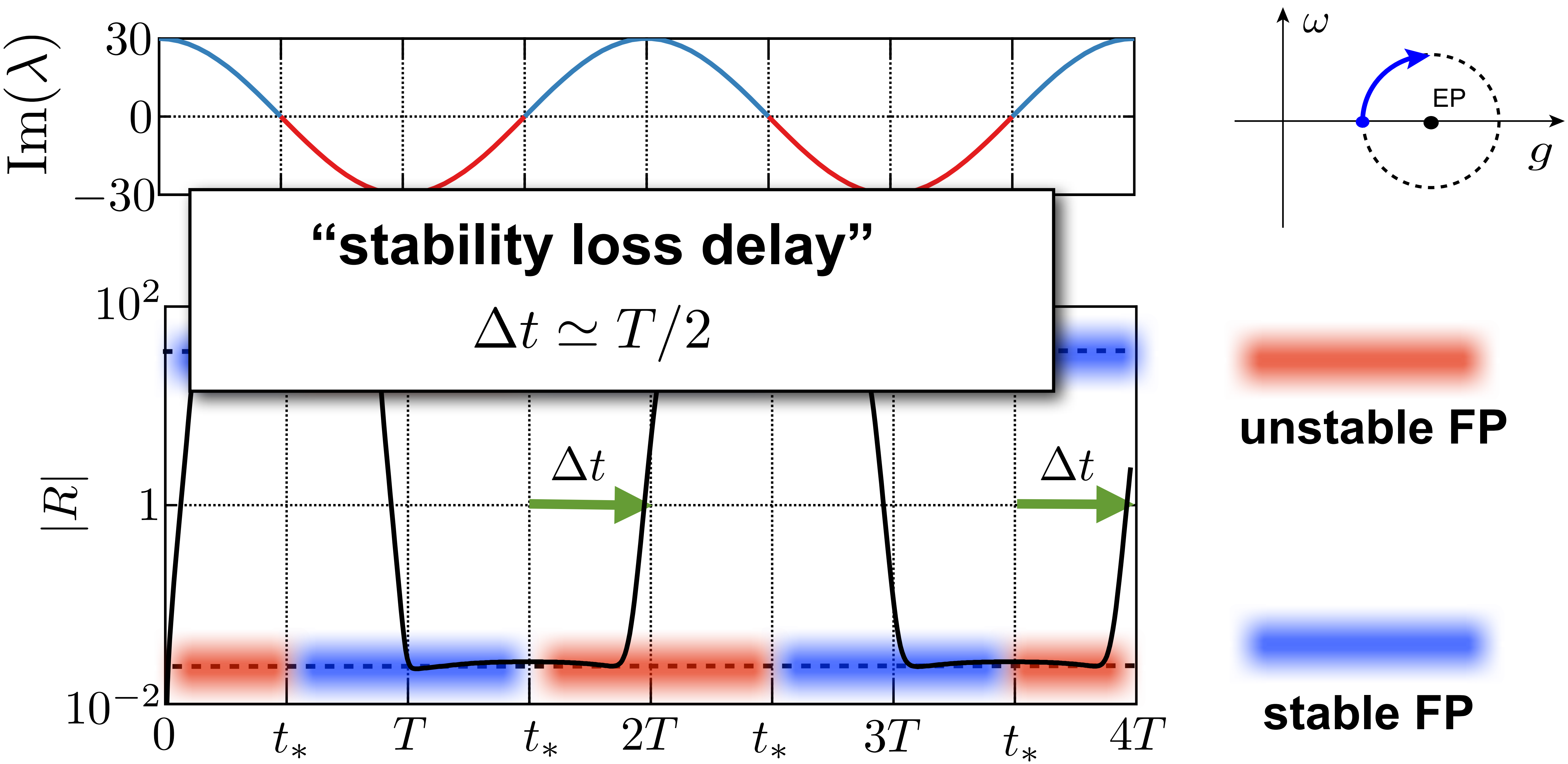
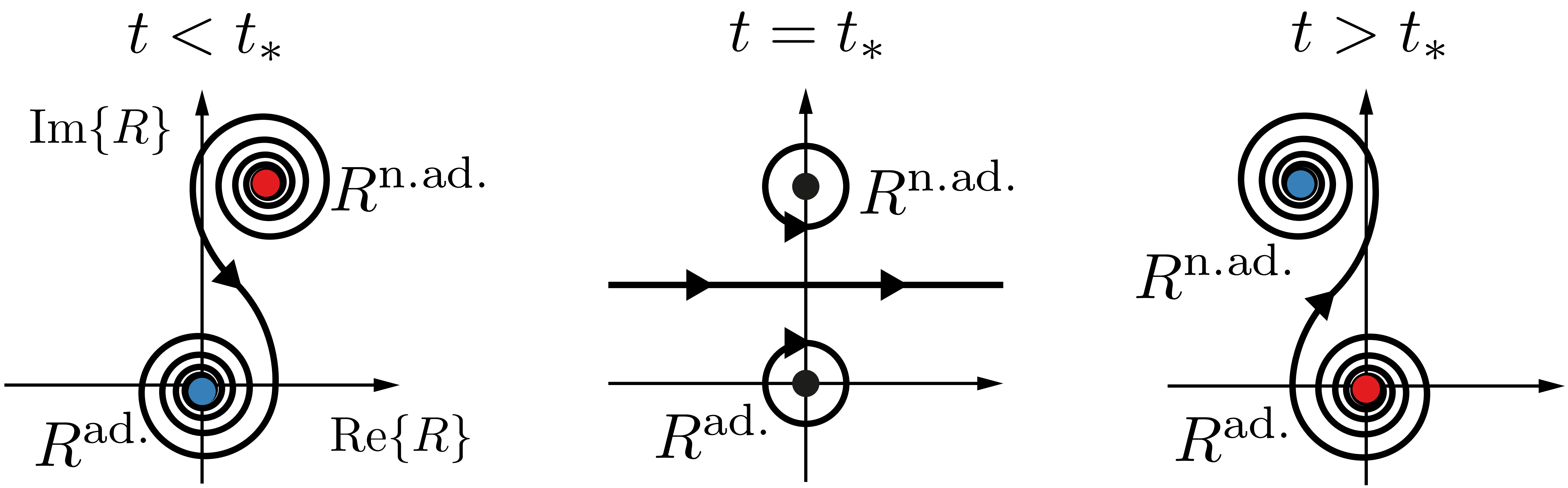
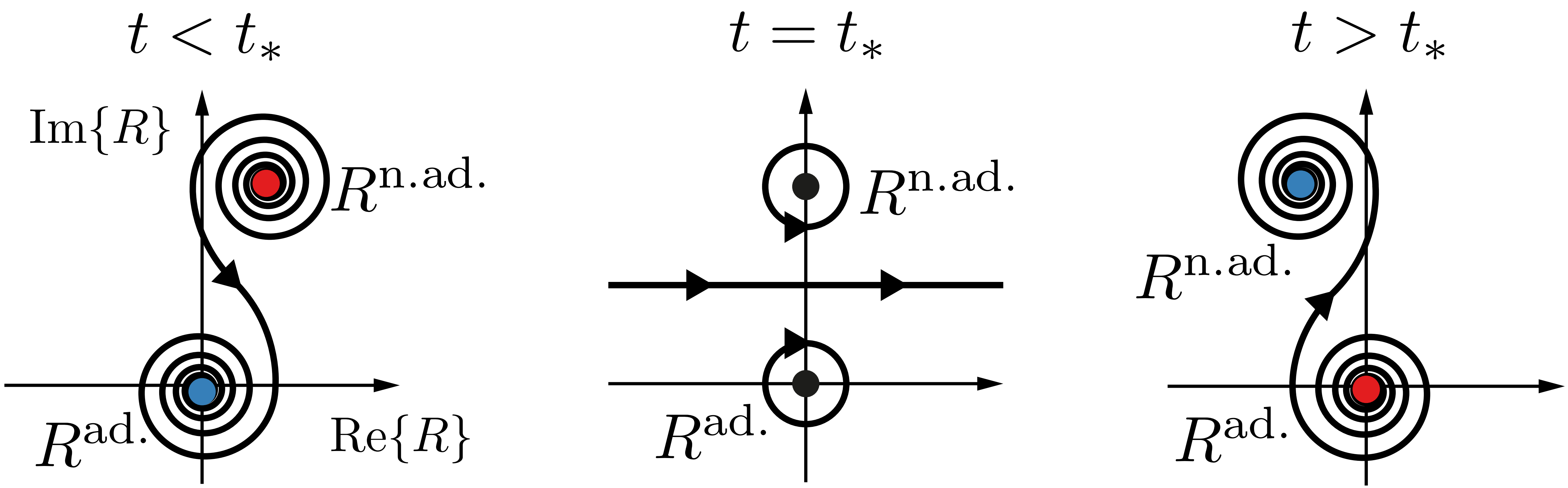
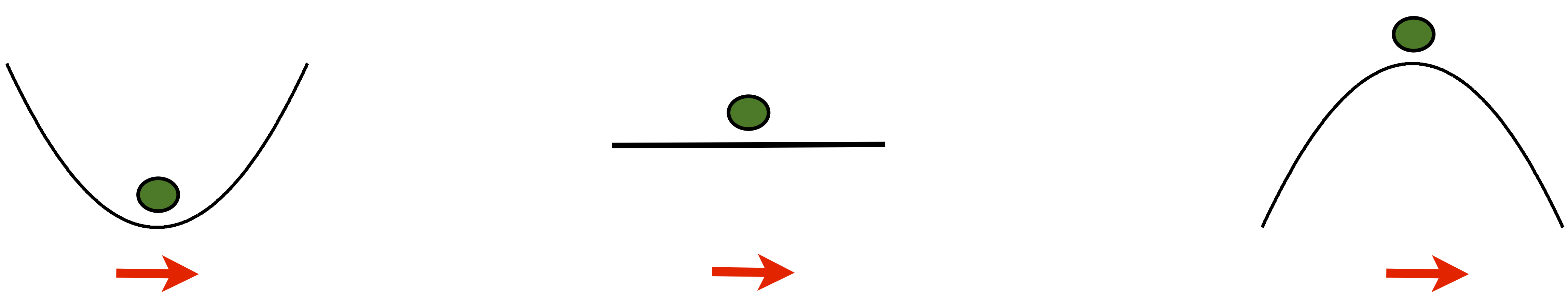


Stability loss delay

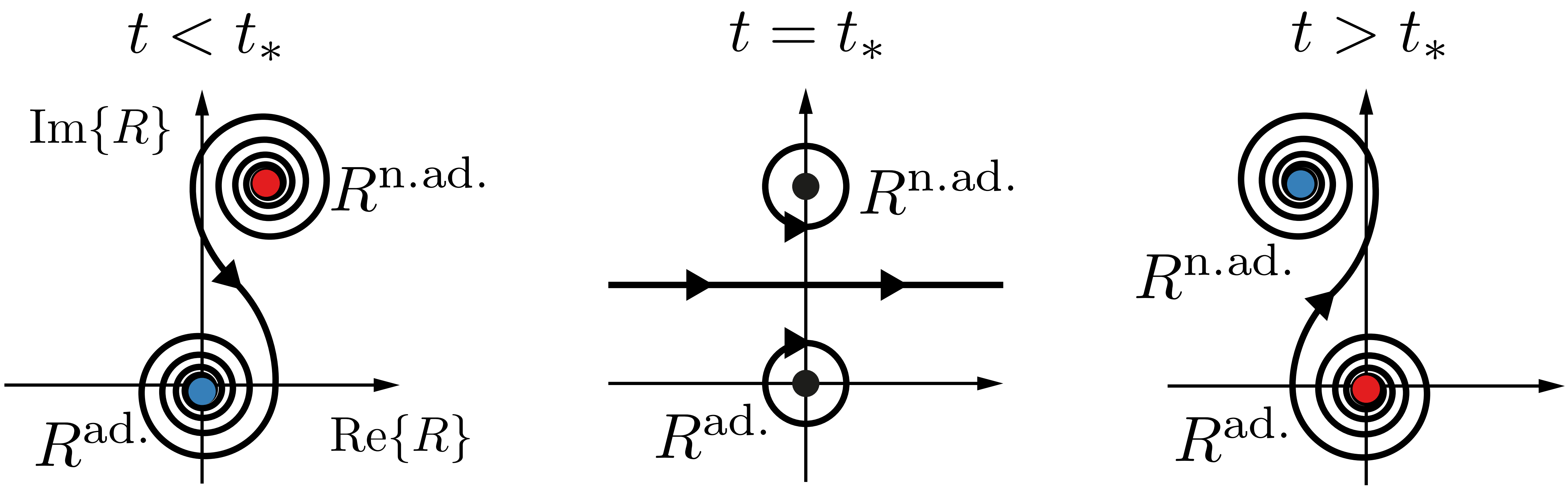


Stability loss delay





Stability loss delay



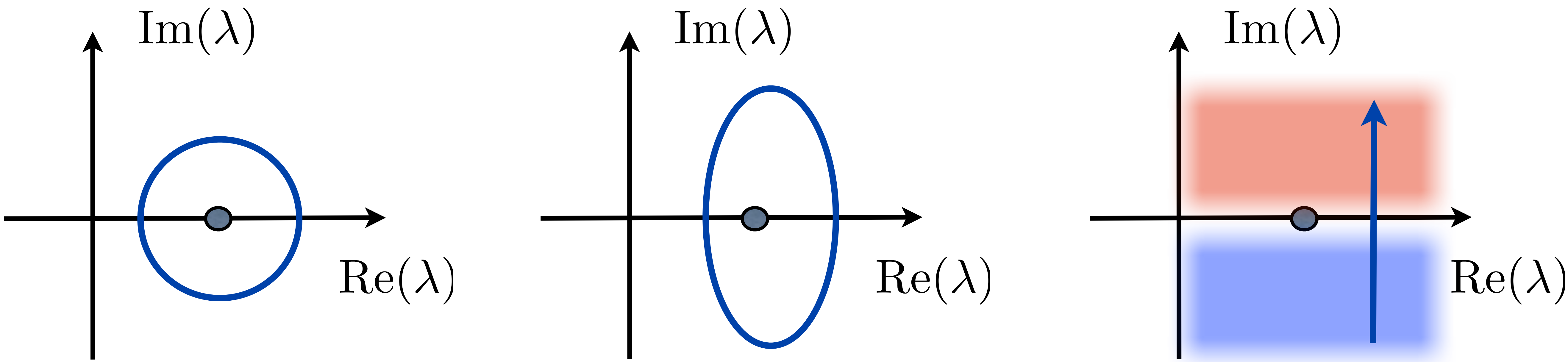
switching dynamics:

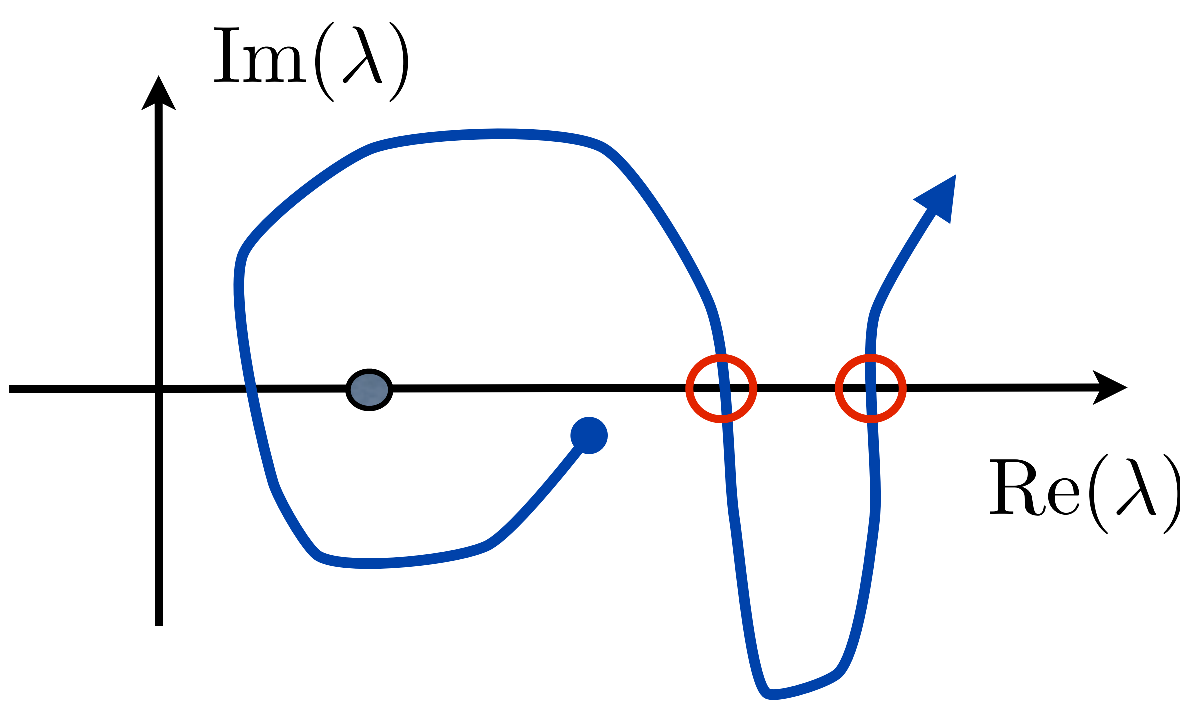
$$R(t) \simeq -\varepsilon e^{i\pi t/T} - 2i\varepsilon^2 e^{2i\pi t/T} + \dots$$
$$-\frac{\pi}{2}\,\mathrm{sgn}\,(t - t_*)\, e^{\frac{i}{2\varepsilon} e^{-i\pi t/T}}$$

non-converging series

non-analytic in ε

Applications





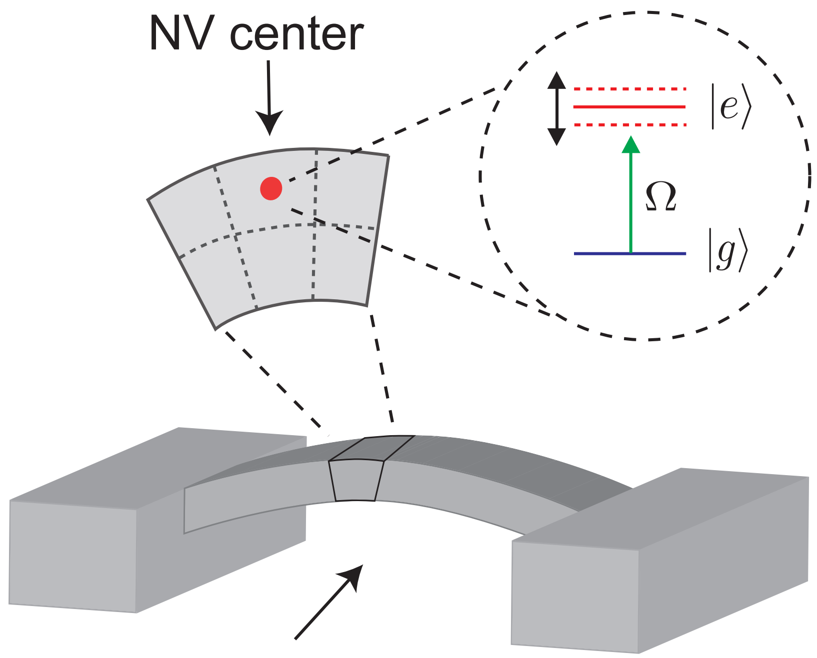
i) Find ad. and non-ad. fixed points.

ii) Find stability crossings points.

iii) Calculate switching delay times.

Summary & conclusions

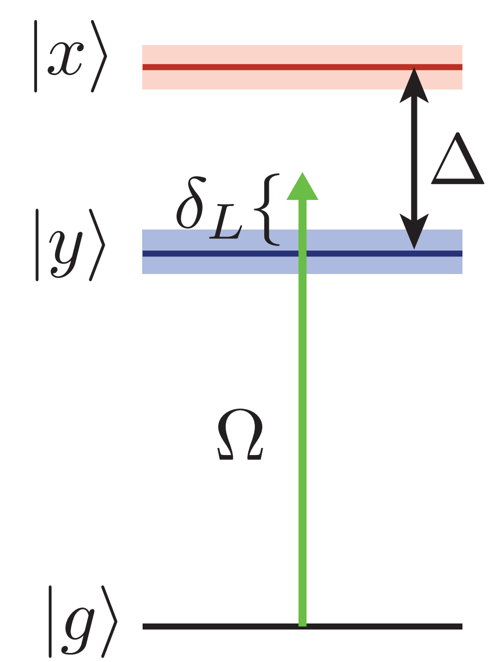
Strain coupling in diamond: Summary



- *Strain coupling in diamond:*

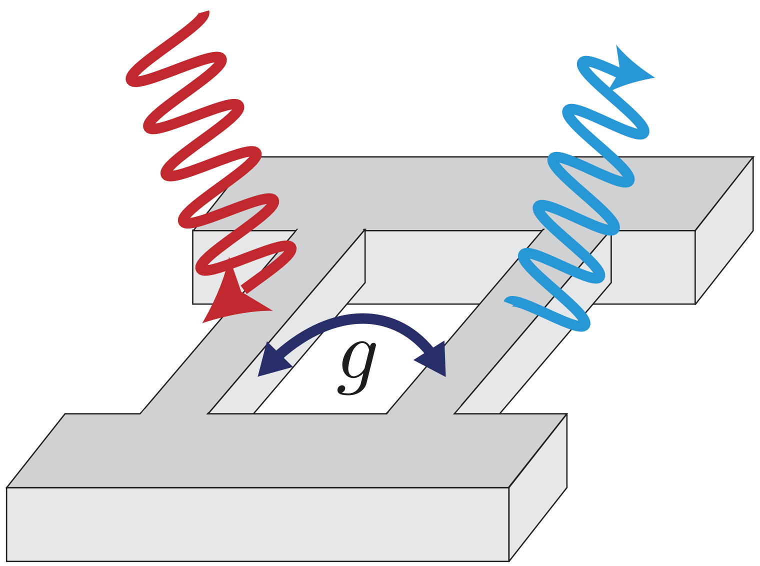
$$\lambda \sim \Gamma_{\mathrm{rad}} \gg \gamma_m$$

- *Resonantly enhanced NV-phonon interactions.*
- *Ground state cooling & lasing schemes.*
- *Probing NV-phonon interactions !*

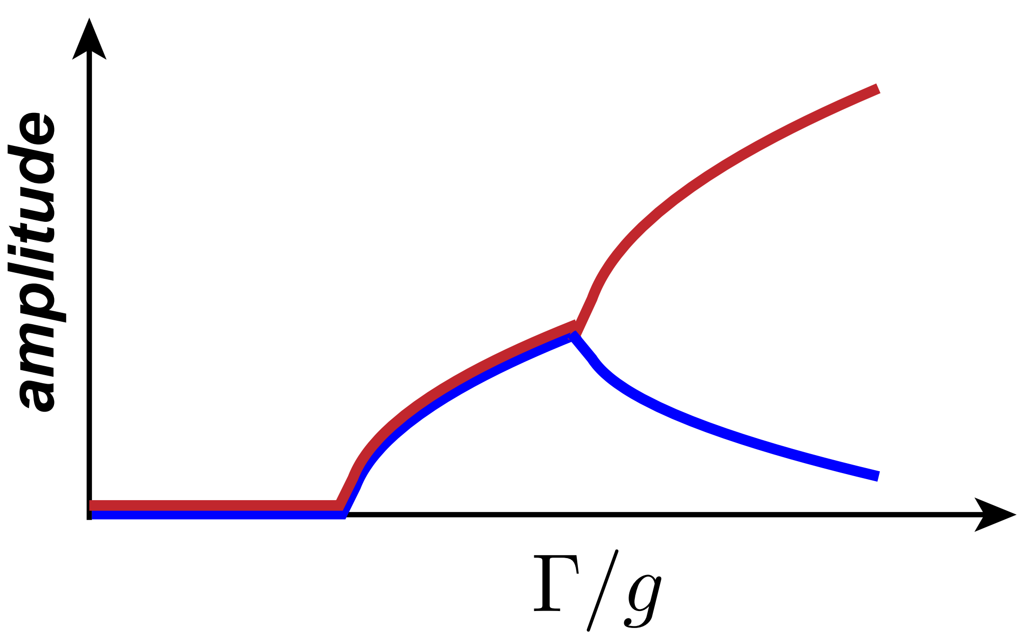


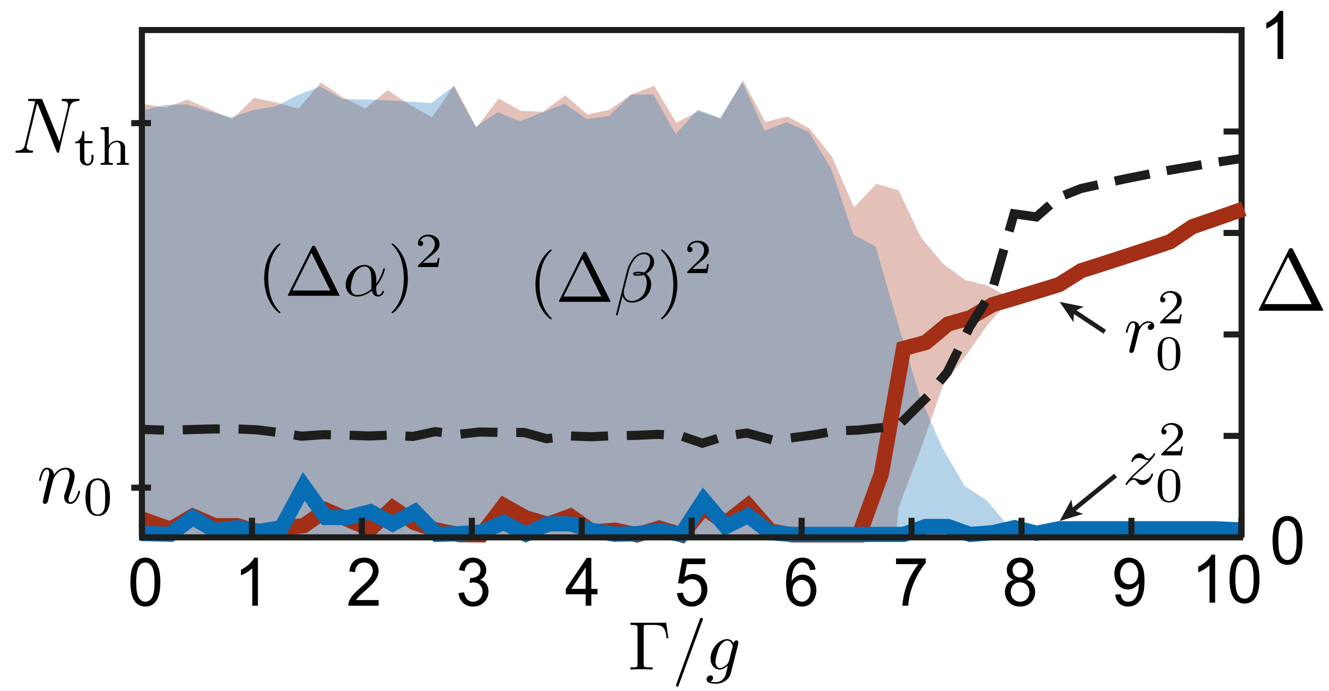
*K. Kepesidis, S. Bennett, S. Portolan, M. Lukin, PR, PRB **88**, 064105 (2013)*

PT-symmetry breaking in the steady-state



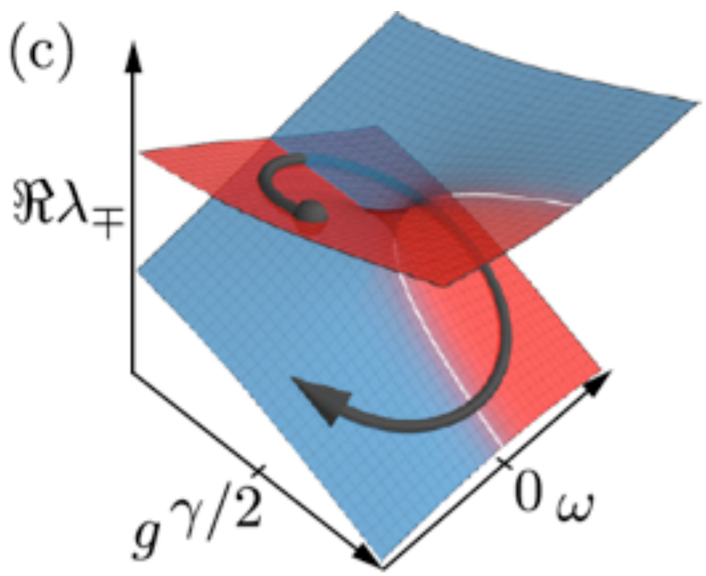
- Non-Hermitian / PT-symmetric phonon systems.
- Unconventional PT-symmetry breaking & lasing transitions.



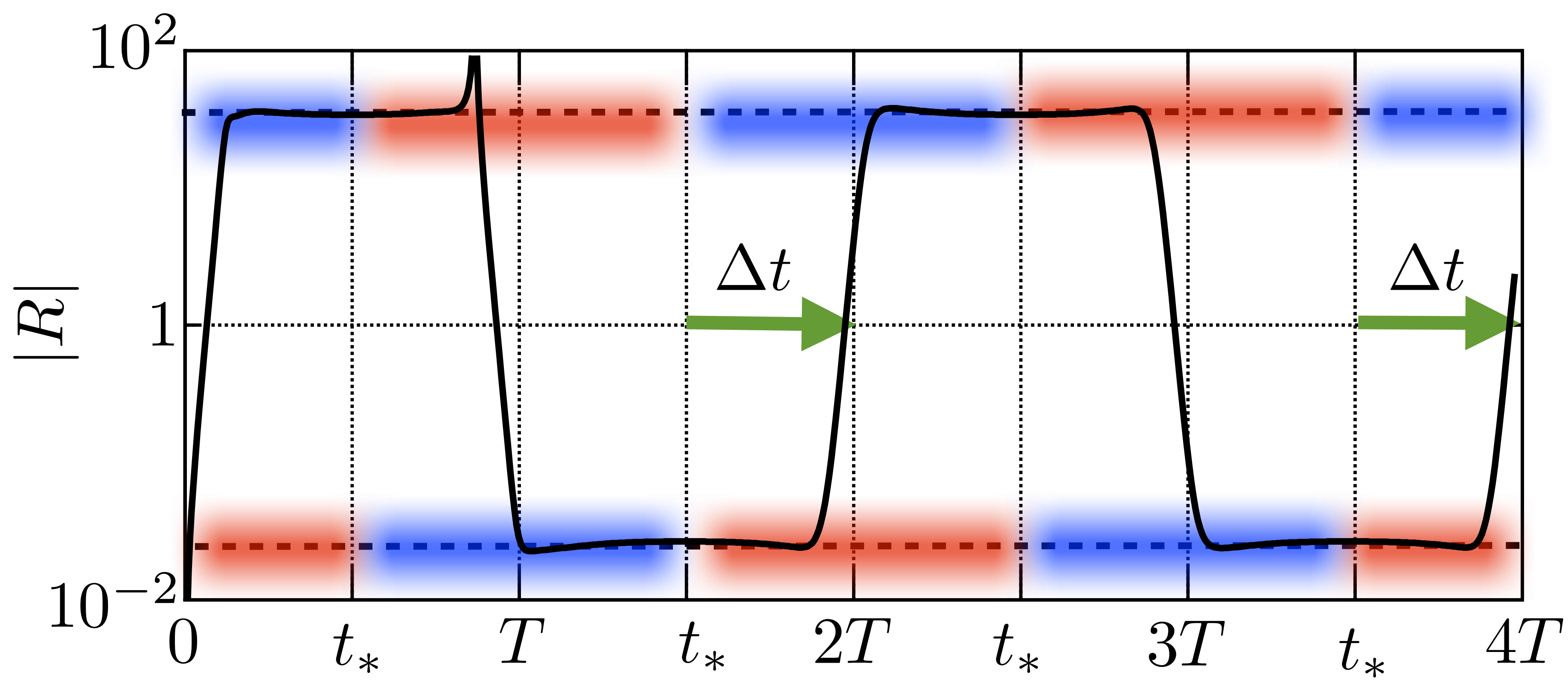


Kepesidis, T. Milburn, K. Makris, S. Rotter, PR, arXiv:1508.00594

Exceptional Points and Quasi-Adiabaticity



- EP & quasi-adiabaticity in non-Hermitian system.



T. Milburn, J. Doppler, C. Holmes, S. Portolan, S. Rotter, PR, arXiv:1410.1882

Thank you !