On Non-equilibrium Phase Transitions in a Driven-Dissipative Polariton Fluid

Marzena Szymańska

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Acknowledgements

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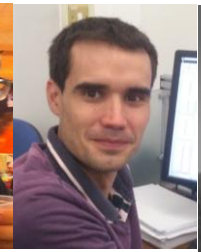
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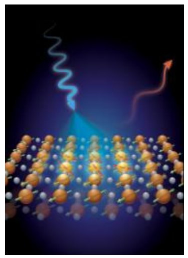
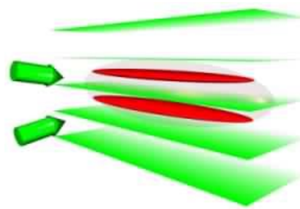
Motivations

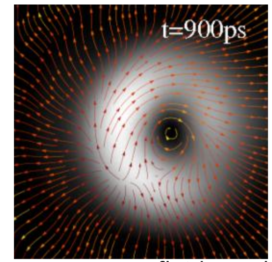
What is the nature of a phase transition between ordered and disorder phases in 2D non-equilibrium, driven, dissipative system?



Spins in high-Tc superconductors
Dean *et al*, *Nature Mat* (2012)



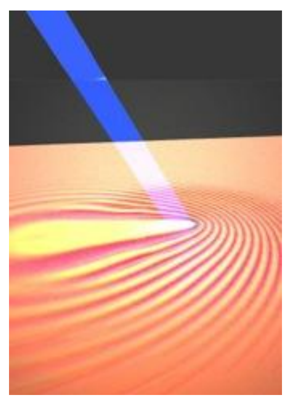
2D quantum fluids - atoms
Hadzibabic *et al.*, *Nature* (2006)



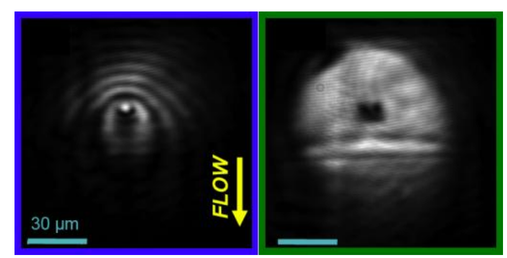
2D quantum fluids - polaritons
Sanvitto et al., *Nature Phys.* (2010)

Posed in the context of normal to superfluid phase transition

Helium



Atoms
Dalibard *et al. Nature Phys.* (2012)



Polaritons
Amo *et al. Nature Phys.* (2009)

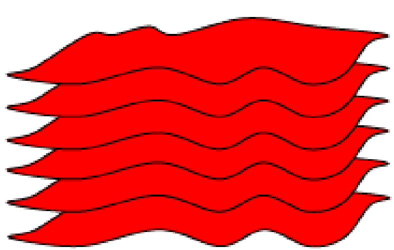
Bosons in 2D – Equilibrium System

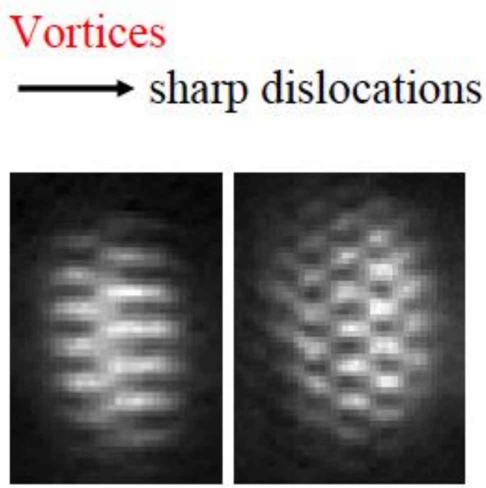
Normal to superfluid – BEC, BKT or ?

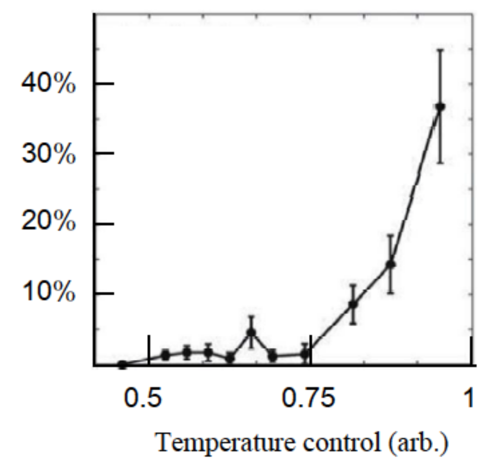
In 2D BEC possible only in trapped and non-interacting systems
Otherwise…

BKT transition

Confirmed in atomic gases – in harmonic trap and with interactions weaker then between other bosons i.e. polaritons







Z. Hadzibabic *et al.*, Nature **441**, 1118 (2006)

Equilibrium 2D Interacting System

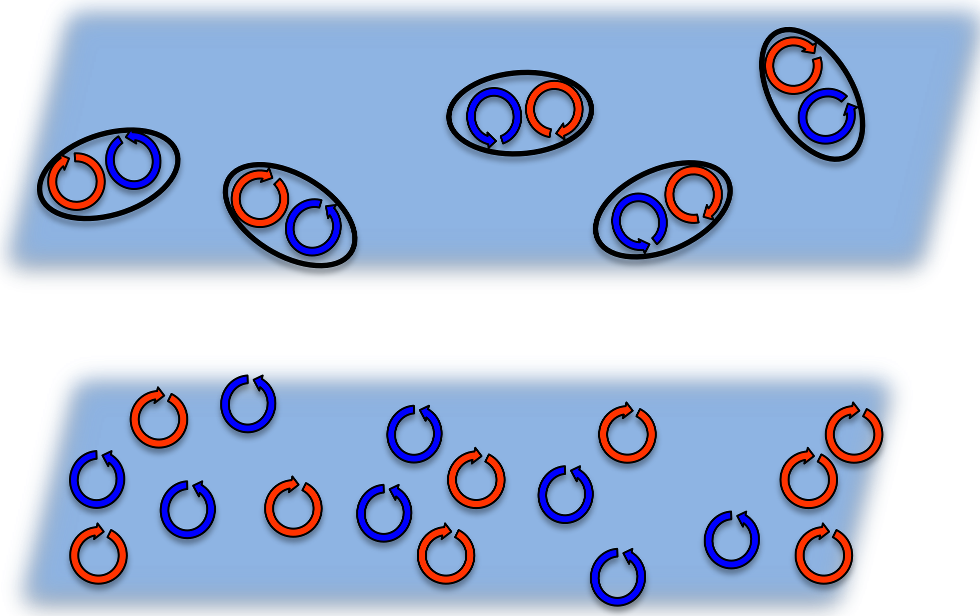
2D equilibrium superfluid below the BKT transition

$$g_1(r) = \langle \psi^\dagger(\mathbf{r})\psi(0)\rangle \propto \left(\frac{r}{r_0}\right)^{-a_p}$$

➢ Power Law decay of correlations

$$a_p = \frac{mk_B T}{2\pi\hbar^2 n_S} < \frac{1}{4}$$

➢ Upper bound on the exponent

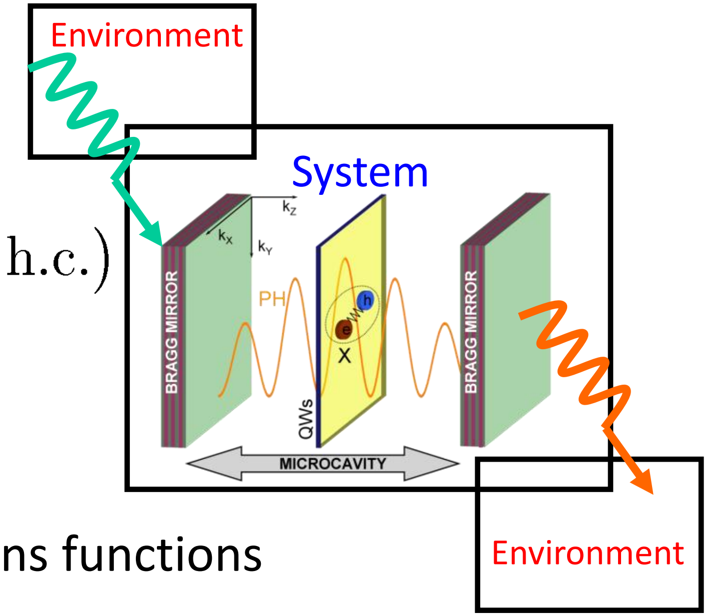


Non-equilibrium Systems

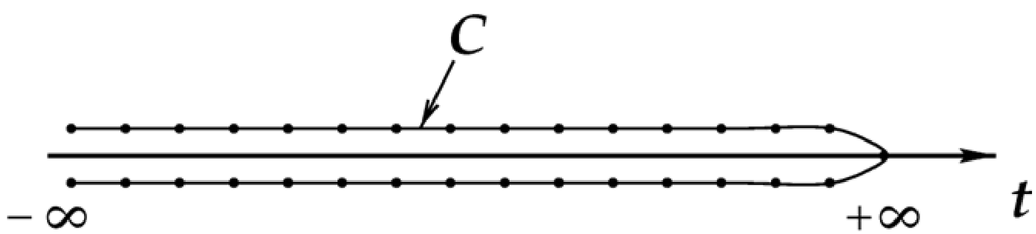
Hamiltonian

$$H = H_{\text{sys}} + H_{\text{sys,bath}} + H_{\text{bath}}$$

$$H_{\text{sys}} = \sum_{\mathbf{p}} \omega_{\mathbf{p}} \psi^{\dagger}_{\mathbf{p}} \psi_{\mathbf{p}} + \sum_{\alpha,\mathbf{p}} \left(g_{\alpha,\mathbf{p}} \psi_{\mathbf{p}} \phi^{\dagger}_{\alpha} + \text{h.c.} \right) + H_{\text{exc}}[\phi_{\alpha}, \phi^{\dagger}_{\alpha}]$$



Method: Non-equilibrium path integrals and Greens functions



Steady state $\quad \psi(t) = \psi e^{-i\mu_S t}$

Fluctuations

$$[D^R - D^A](t,t') = -i \left\langle \left[\psi(t), \psi^{\dagger}(t') \right]_- \right\rangle$$

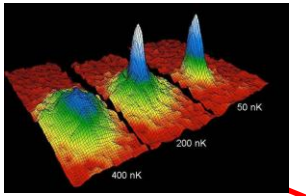
$$D^K(t,t') = -i \left\langle \left[\psi(t), \psi^{\dagger}(t') \right]_+ \right\rangle$$

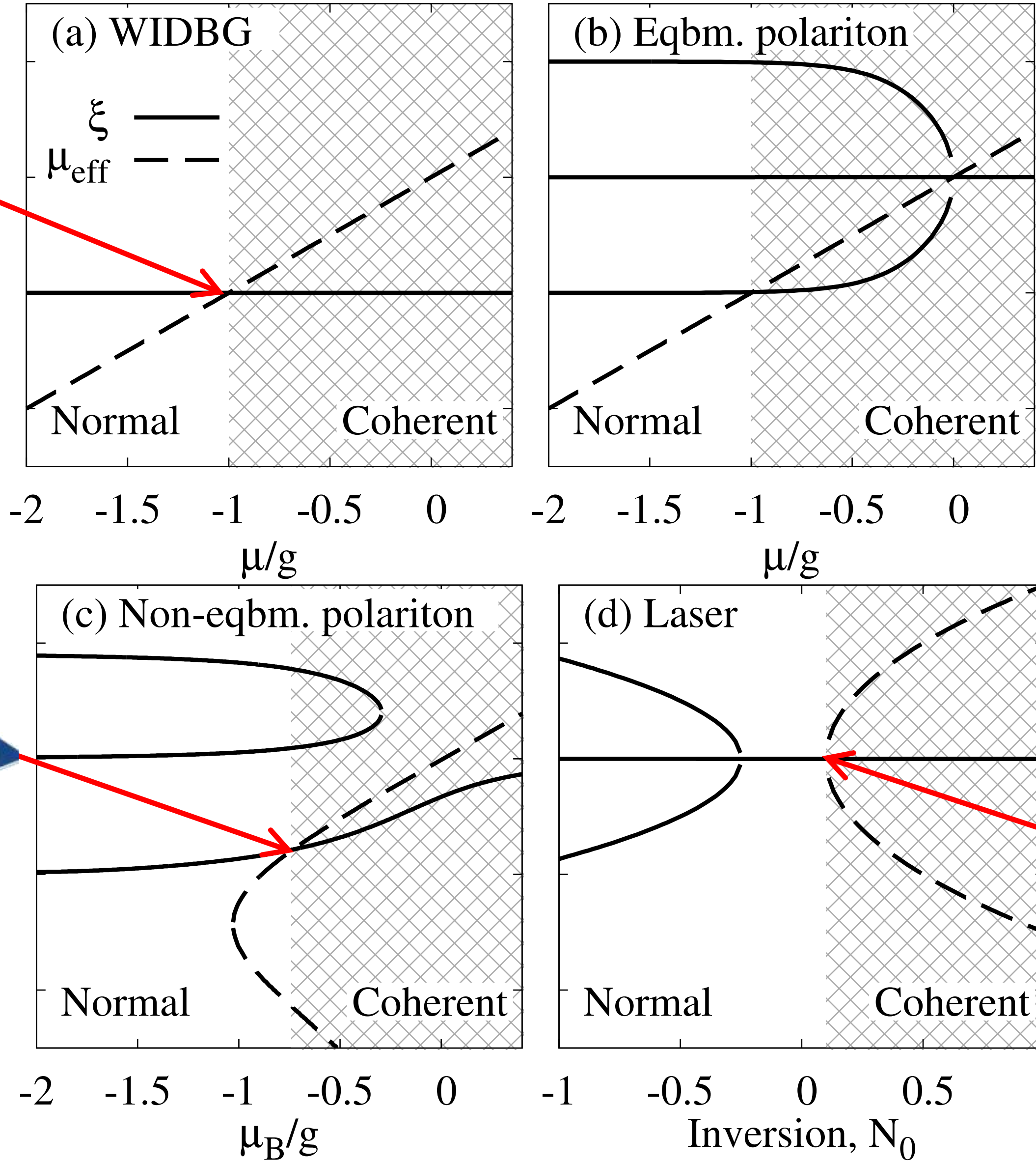
$$[D^R - D^A](\omega) = \text{DoS}(\omega)$$

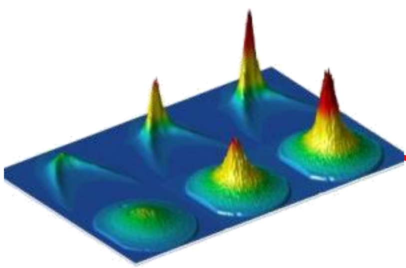
$$D^K(\omega) = (2n(\omega) + 1)\text{DoS}(\omega)$$

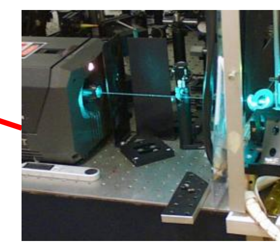
[Szymańska et al., *PRL* 2006; *PRB* 2007]

Non-equilibrium Condensation









[Szymanska et al., chapter ICP (2012)]

Fluctuations

In the normal state it is enough to expand to second order

$$\psi = \psi_0 + \delta\psi$$

Now we must treat phase fluctuations better

$$\psi = \sqrt{\rho_0 + \pi}\, e^{i\phi}$$

Photon causal Green's function (luminescence) with phase fluctuations to all orders but gradients of phase and amplitude to second order

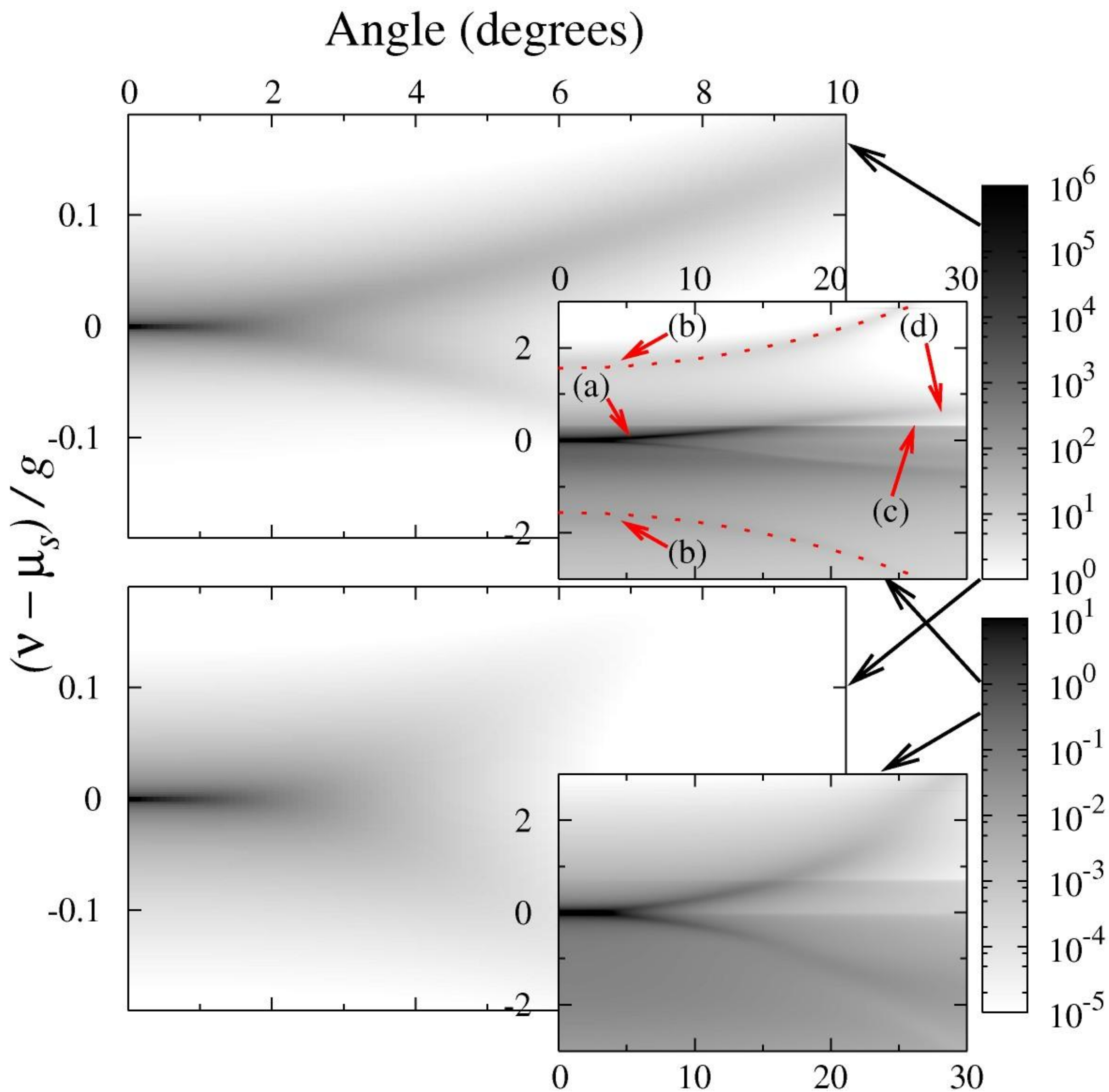
$$\begin{aligned} iD^{<}_{\psi^\dagger\psi}(t,r) = \rho_0 \Bigg\{ & 1 + \frac{i}{2\rho_0}\left[iD^{<}_{\phi\pi}(t,r) - iD^{<}_{\pi\phi}(t,r)\right] \\ & - \frac{1}{4\rho_0^2}\left[iD^{<}_{\pi\pi}(0,0) - iD^{<}_{\pi\pi}(t,r)\right] \\ & + \frac{1}{8\rho_0^2}\left[iD^{<}_{\phi\pi}(0,0) + iD^{<}_{\pi\phi}(0,0) - iD^{<}_{\phi\pi}(t,r) - iD^{<}_{\pi\phi}(t,r)\right]^2 \\ & \Bigg\} \exp\left\{-\left[iD^{<}_{\phi\phi}(0,0) - iD^{<}_{\phi\phi}(t,r)\right]\right\} \end{aligned}$$

mean-field density

Phase-phase correlation function

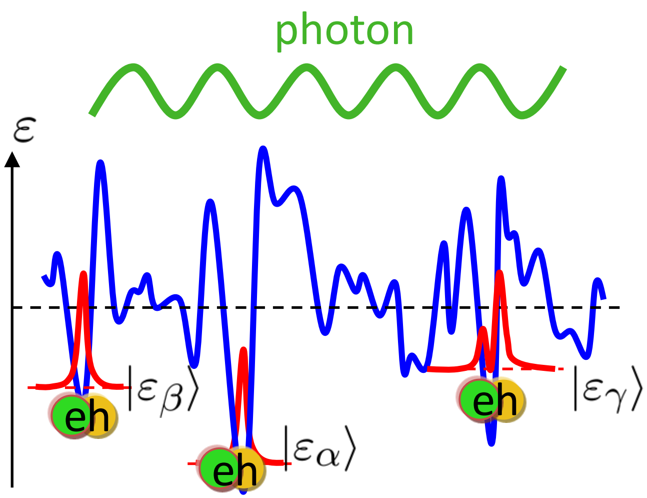
[Szymańska et al., *PRL* 2006; *PRB* 2007]

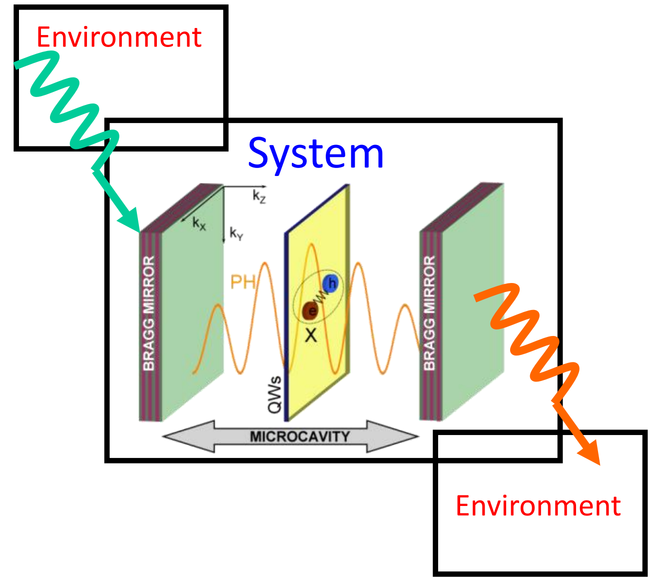
Luminescence – Ordered State



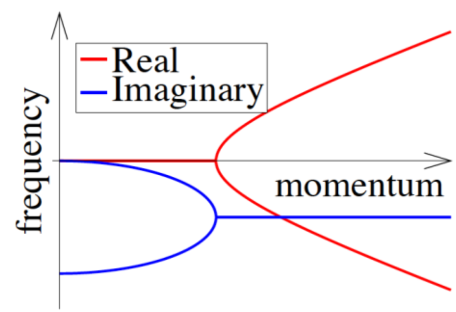
M. H. Szymanska, et al Phys. Rev. Lett. **96**, 230602 (2006)

Spatial and Temporal Coherence





- ✧ Dimensionality: 2D
- ✧ Modes: diffusive
- ✧ Occupation: non-thermal

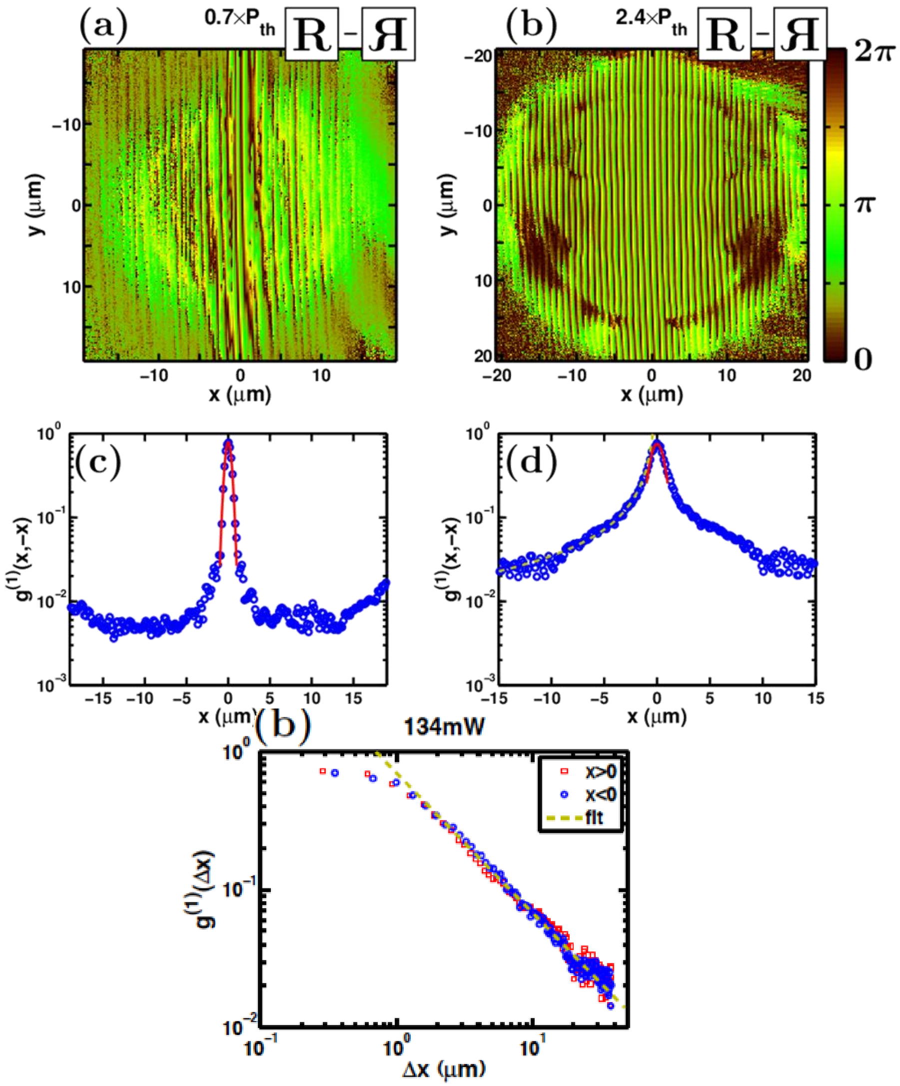


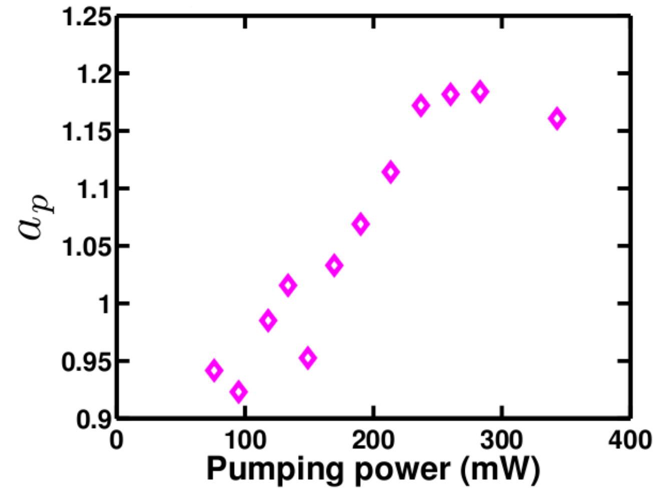
$$\langle \psi^\dagger(\mathbf{r},t)\psi(0,0)\rangle \simeq |\psi_0|^2 \exp\left[-a_p \begin{cases} \ln(r/r_0) & r\to\infty, t\simeq 0 \\ \frac{1}{2}\ln(c^2 t/\gamma_{\text{tot}} r_0^2) & r\simeq 0, t\to\infty \end{cases}\right]$$

$$a_p(\text{pump, decay, density})$$

[Szymańska et al., *PRL* 2006; *PRB* 2007]

Experimental observation of power law decay





$$g_1(r) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$

[Roumpos et al, *PNAS* 2012]

Exponent in a non-equilibrium 2D gas

$$g_1(r) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$

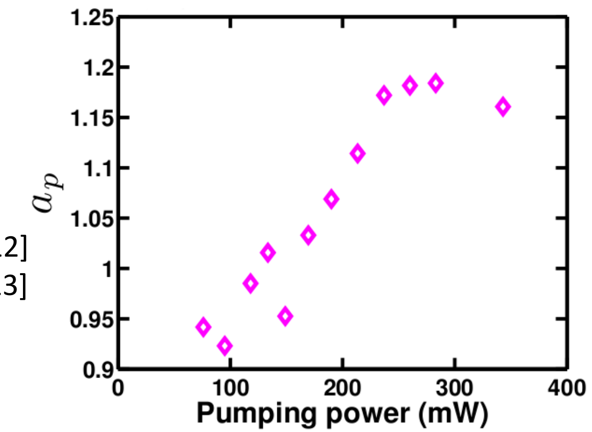
- ✧ Equilibrium closed system $\quad a_p = \frac{mk_BT}{2\pi\hbar^2 n_S} < \frac{1}{4}$

- ✧ Non-equilibrium driven system (diffusive modes)

  - ➢ thermalised $\quad a_p = \frac{mk_BT}{2\pi\hbar^2 n_S}$

  - ➢ non-thermalised $\quad a_p \propto \frac{\text{pumping noise}}{n_s}$

[Roumpos et al, *PNAS* 2012]
[Chiocchetta et al, EPL 2013]



- ✧ Experiment $\quad a_p \simeq 1.2$

- ✧ g1(r) and N(k) as in equilibrium with effective T

Experiment: faster decay possible then equilibrium upper limit

Theory: don't know, need to account for vortices and large fluctuations

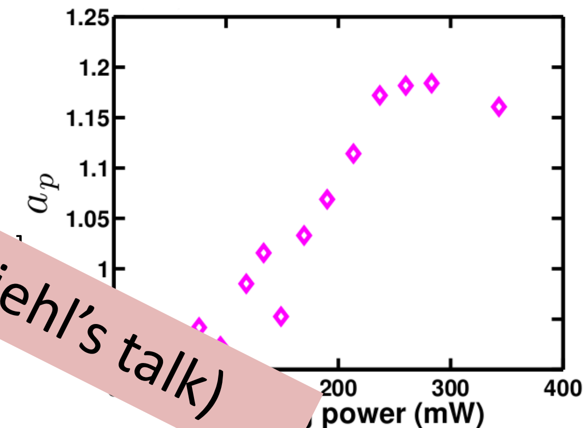
Exponent in a non-equilibrium 2D gas

$$g_1(r) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$

What about the KPZ physics? (Sebastian Diehl's talk)

- ✧ E… ium closed system $\quad a_p = \dfrac{mk_BT}{2\pi\hbar^2 n_S} < \dfrac{1}{4}$
- ✧ Non-equilibri… …n system (diffusive modes)
  - ➢ thermalised $\quad \dfrac{mk_BT}{\ldots\hbar^2 n_S}$
  - ➢ non-thermalised $\quad a_p \propto \dfrac{P\ldots}{\ldots}$ …oise

[Roumpos et …]
[Chiocchetta et al, …]



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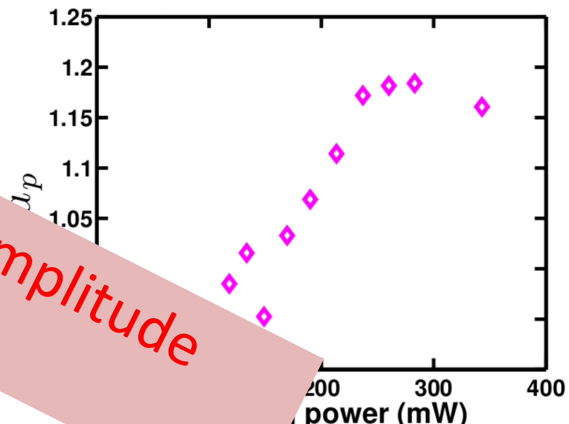
Exponent in a non-equilibrium 2D gas

$$g_1(r) \propto \left(\frac{r}{r_0}\right)^{-a_p}$$

Numerical approach to treat phase and amplitude fluctuations

- E... closed system $\quad a_p = \frac{mk_BT}{2\pi\hbar^2 n_S} < \frac{1}{4}$
- Non-equ... ystem (diffusive modes)
  - thermalised $\quad \frac{mk_BT}{\dots}$
  - non-thermalised $\quad a_p \propto \dots$

[Roum...
[Chiocchetta...



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✧ ... iffusive modes)

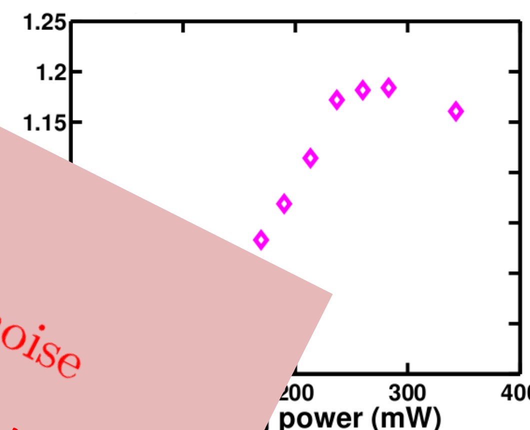
➢ therm...

➢ non-thermalised

However model:

$$i\partial_t\psi = \left[-\frac{\nabla^2}{2m} + U|\psi|^2 + i\left(\gamma_{\rm net} - \Gamma|\psi|^2\right)\right]\psi + \text{noise}$$

has problems – we need a frequency dependent drive



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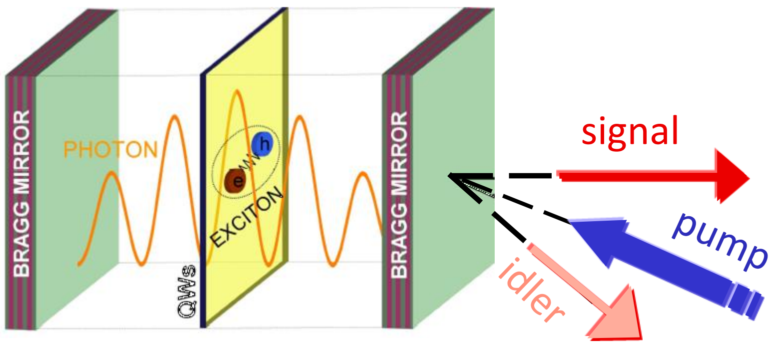
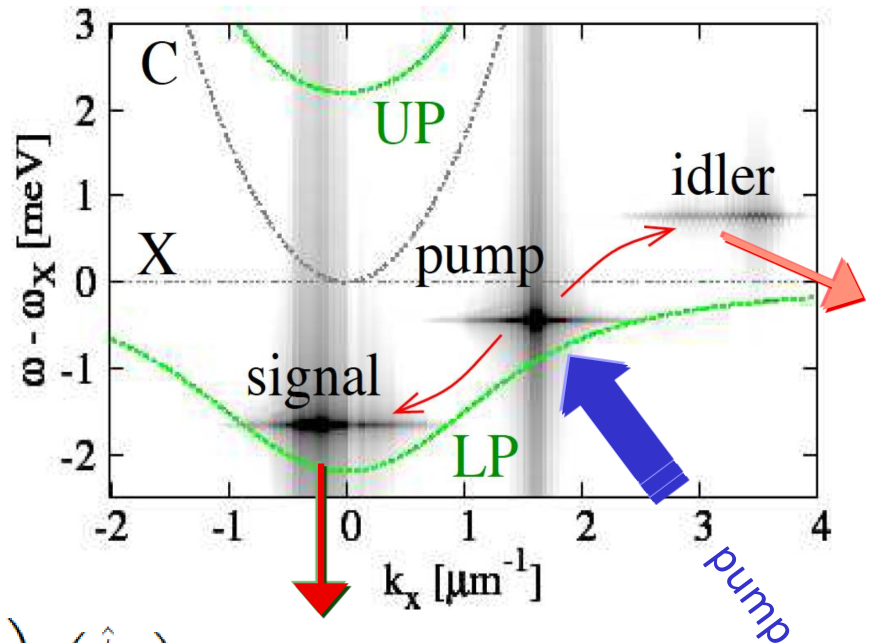
Microcavity Polaritons: OPO





$$\hat{H}_S = \int d\mathbf{r}\,(\hat{\psi}_X^\dagger \;\; \hat{\psi}_C^\dagger)\begin{pmatrix} \frac{-\nabla^2}{2m_X} + \frac{g_X}{2}|\hat{\psi}_X|^2 & \frac{\Omega_R}{2} \\ \frac{\Omega_R}{2} & \frac{-\nabla^2}{2m_C} \end{pmatrix}\begin{pmatrix} \hat{\psi}_X \\ \hat{\psi}_C \end{pmatrix}$$

$$\hat{H}_{SB} = \int d\mathbf{r}\left[F(\mathbf{r},t)\hat{\psi}_C^\dagger(\mathbf{r},t) + \mathrm{H.c.}\right]$$
$$+\sum_{\mathbf{k}}\sum_{l=X,C}\left\{\zeta_{\mathbf{k}}^l\left[\hat{\psi}_{l,\mathbf{k}}^\dagger(t)\hat{B}_{l,\mathbf{k}} + \mathrm{H.c.}\right] + \omega_{l,\mathbf{k}}\hat{B}_{l,\mathbf{k}}^\dagger\hat{B}_{l,\mathbf{k}}\right\}$$

✧ Non-thermal occupation

✧ Signal phase is completely free and idler phase locked to signal via pump

$$2\varphi_p = \varphi_s + \varphi_i$$

✧ Spontaneous U(1) symmetry breaking
gapless and diffusive Goldstone mode

[Wouters & Carusotto *PRA* (2007)]

Truncated Wigner for Polaritons

✧ Stochastic description

$$i\begin{pmatrix} d\psi_X \\ d\psi_C \end{pmatrix} = \left\{ \begin{pmatrix} \omega_X - i\kappa_X + g_X(|\psi_X|^2 - \frac{1}{\Delta V}) & \Omega_R/2 \\ \Omega_R/2 & \frac{\nabla^2}{2m_c} - i\kappa_C \end{pmatrix} \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} + \begin{pmatrix} 0 \\ F_p \end{pmatrix} \right\} dt + \begin{pmatrix} \sqrt{\kappa_X} dW_X \\ \sqrt{\kappa_C} dW_C \end{pmatrix}$$

$$F_p(\mathbf{r}, t) = \mathcal{F}_p(\mathrm{r}) e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$$

dW - Wienner noise delta correlated in space and time

Observables: MC averages over noise

✧ Derivation:

- Master equation
- Wigner representation of Bose field
  Ignore 3rd order derivative
- Map Fokker Planck to stochastic differential equation

[Carusotto at al PRB (2005)]

- From Keldysh action by ignoring the RG irrelevant terms

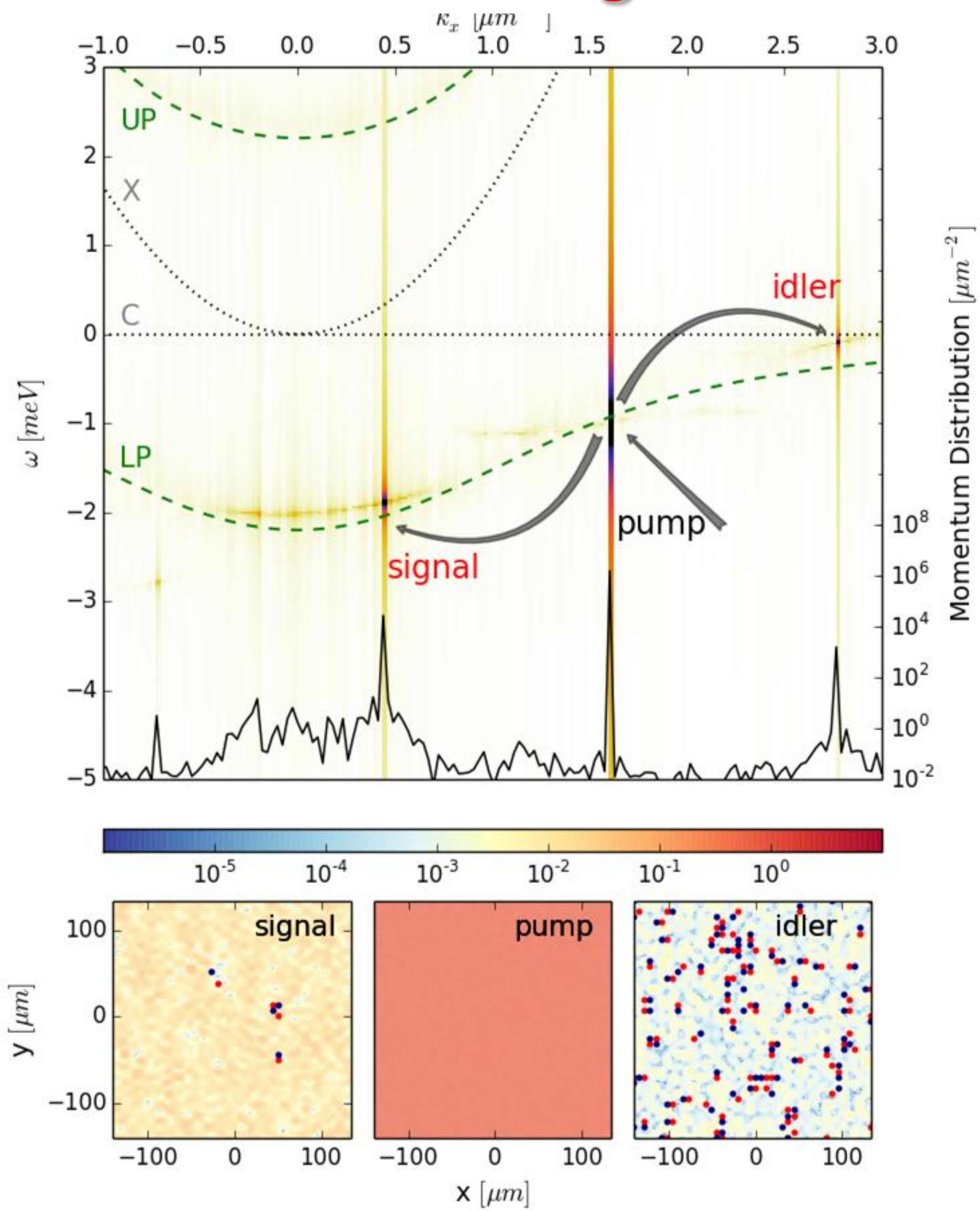
[Sieberer at al PRL (2013)]

✧ Advantages

- No few-mode approximation used
- Large fluctuations fully accounted
- Better for driven dissipative system then closed systems i.e. atomic gases

Truncation controlled by $\kappa_X \gg \frac{g_X}{\Delta V}$

Single Stochastic Path



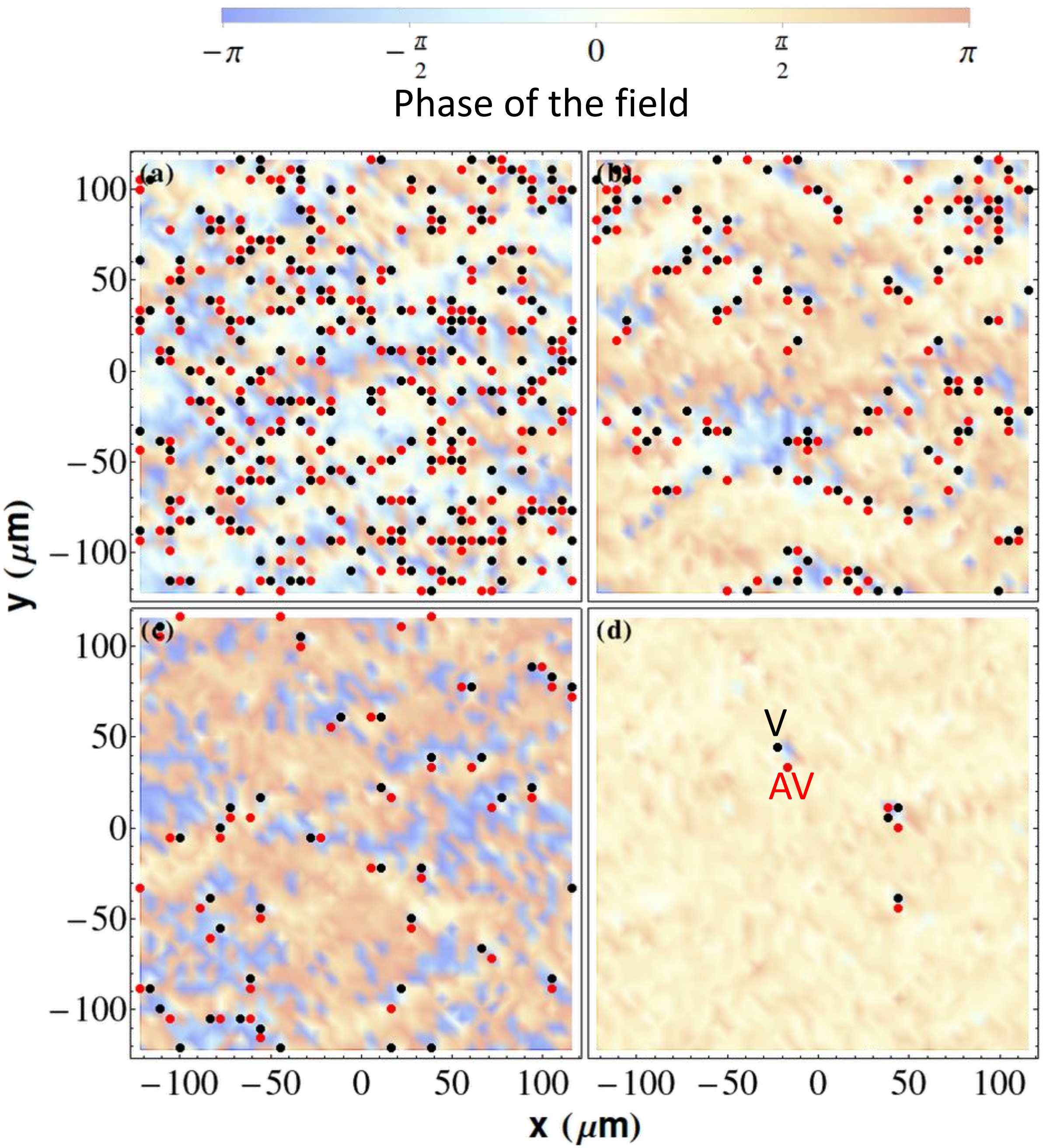
✧ "Condensate" at two momenta and energies: signal and idler

✧ Vortices in signal and idler but ....

not in pump state

✧ No perfect locking: more vortices in idler as it is weaker

V-AV Pairs Proliferation and Binding



Low density:

Vortex/antivortex proliferation

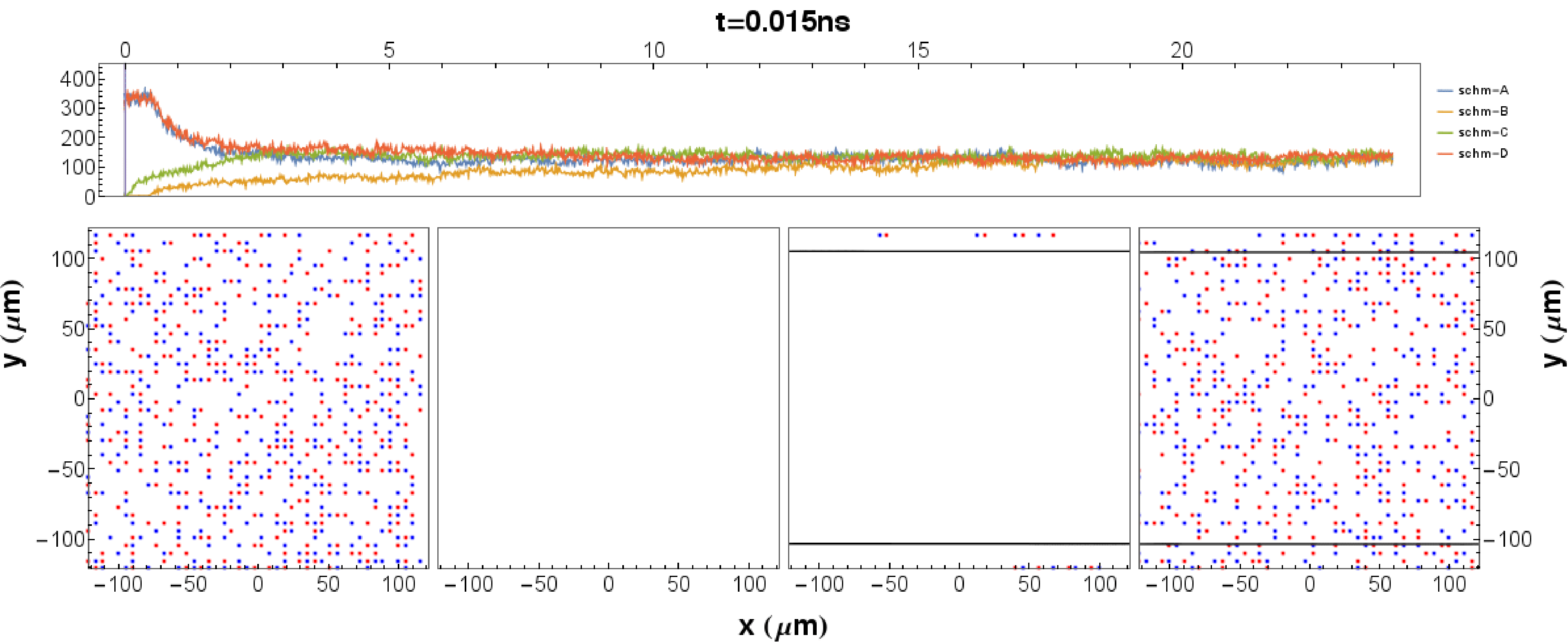
Medium density:

V/AV pairing

High density:

V/AV annihilation, no vortices

Initial Conditions



Vacuum noise

Mean-field “condensate”

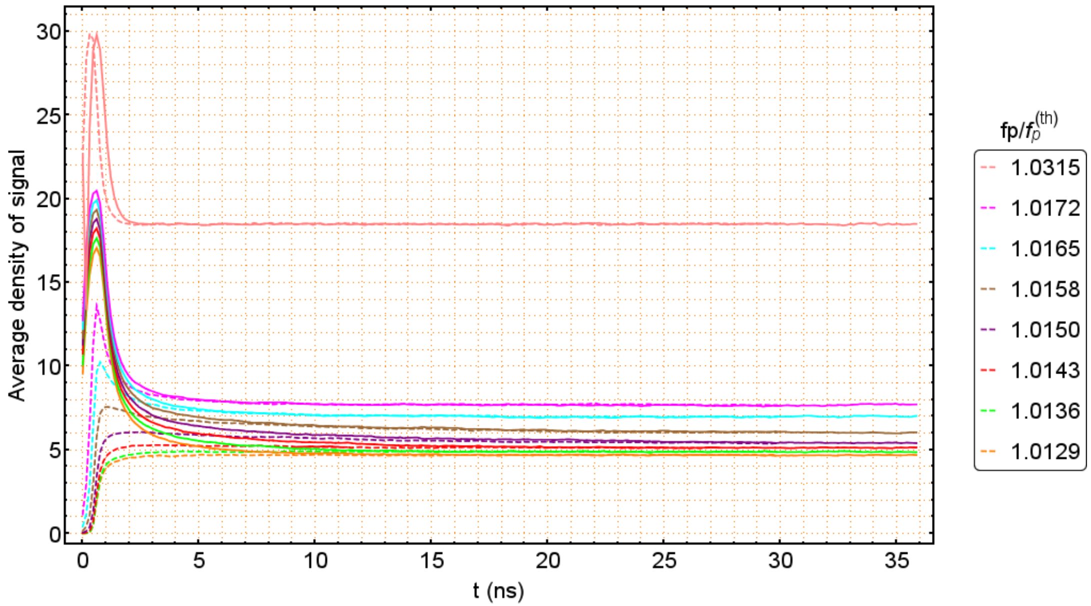
Vacuum noise with reservoir

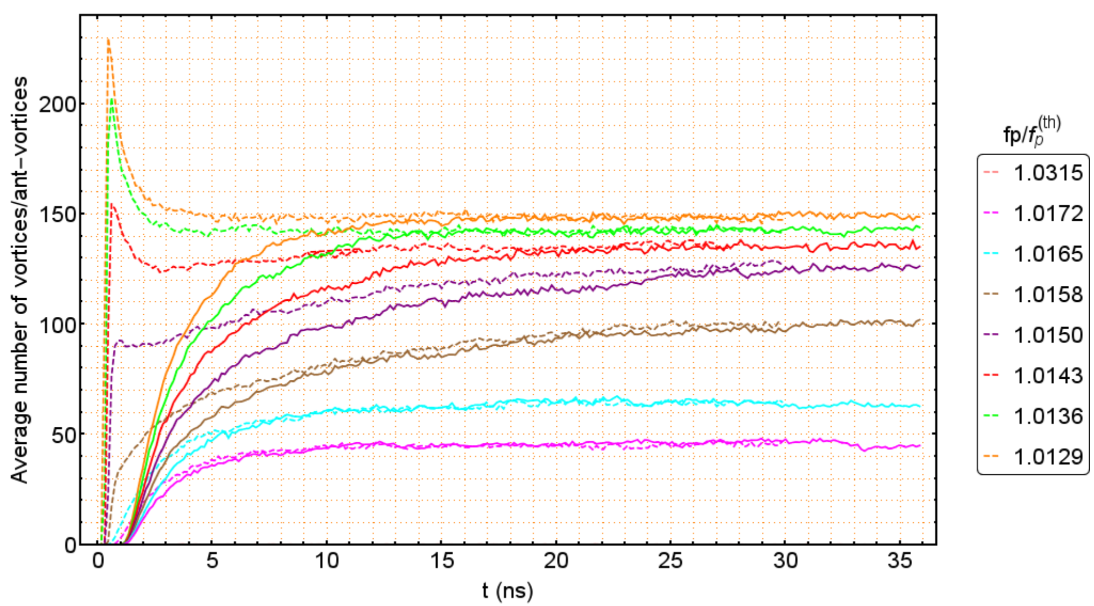
Mean-field “condensate” with reservoir

Very different initial conditions lead to the same steady-state

unique steady state solution

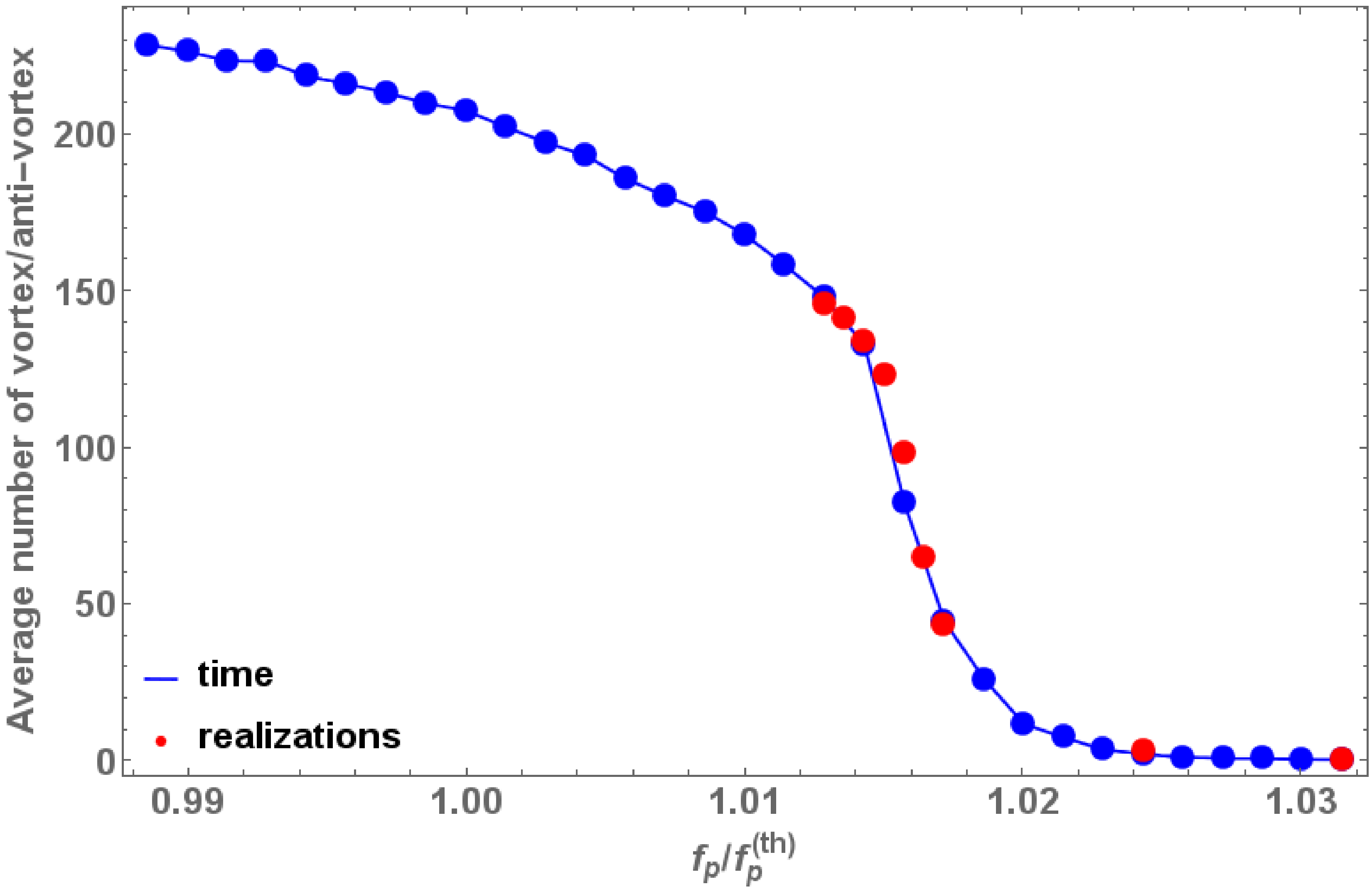
Reaching the Steady State – Stochastic Averages





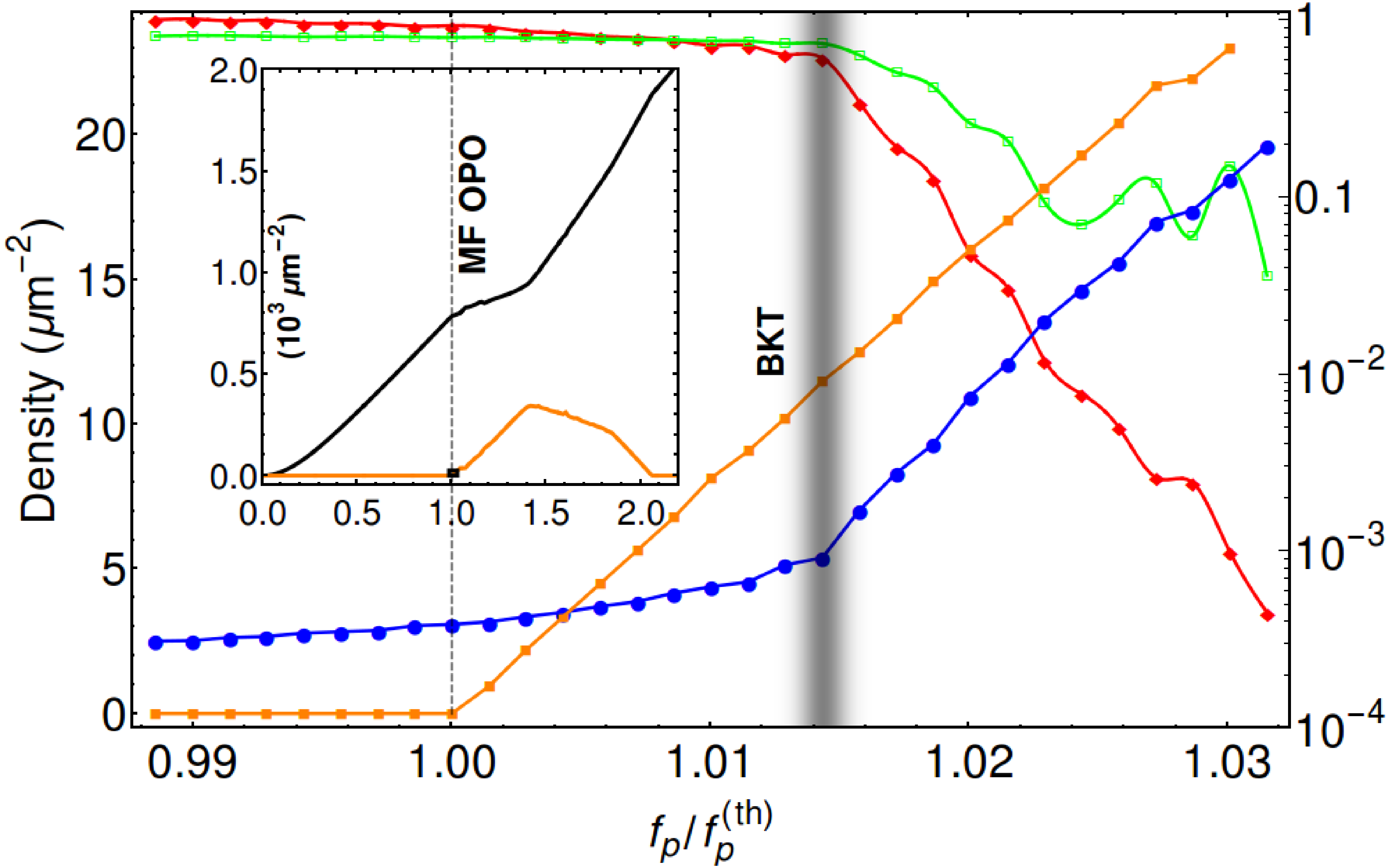
Density of signal polaritons and number of vortices very well converged in time

Stochastic Averages



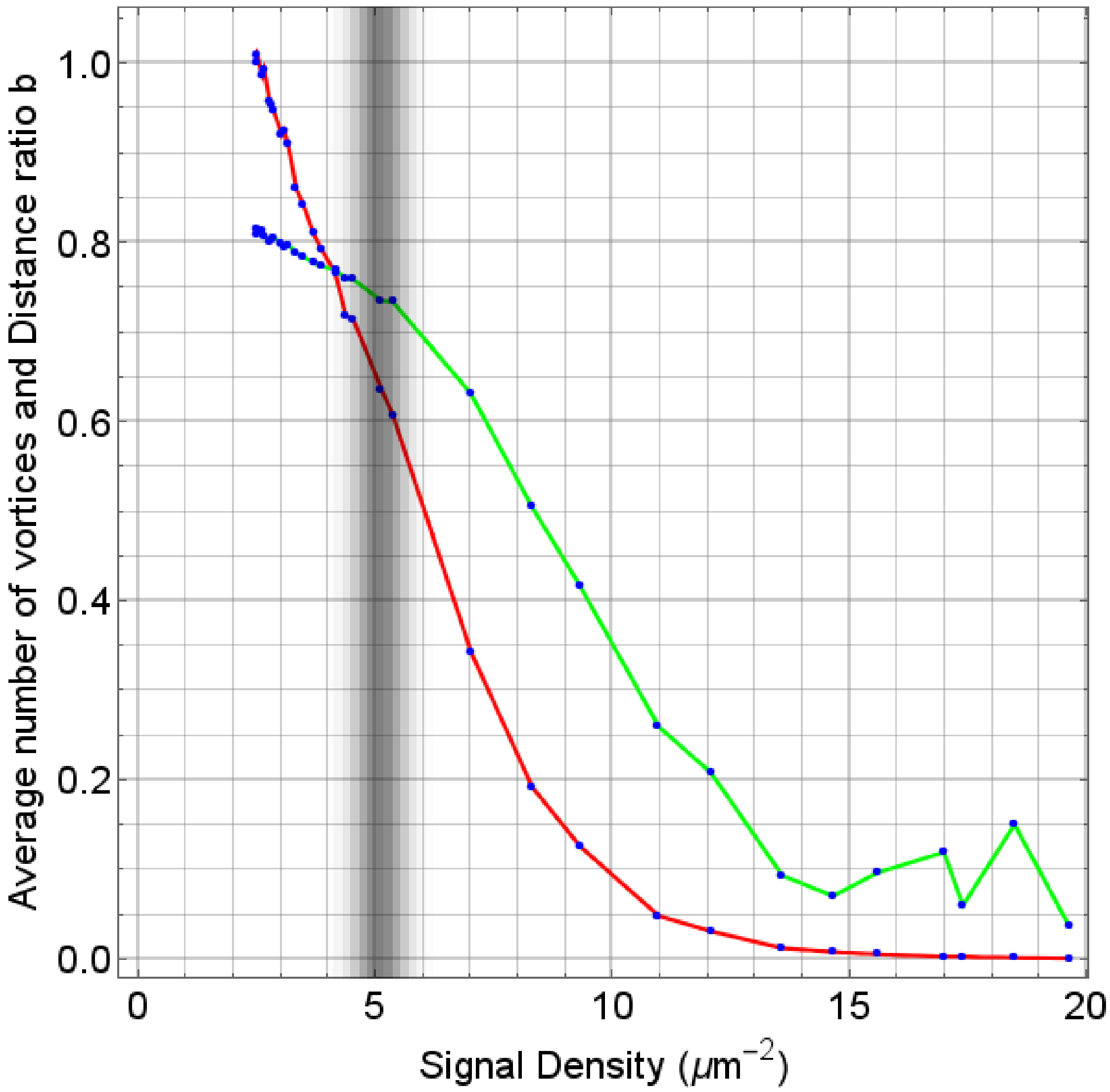
Averages over realisations = averages over different time snapshots

Phase Diagram



Note: the transition region VERY narrow in pump powers

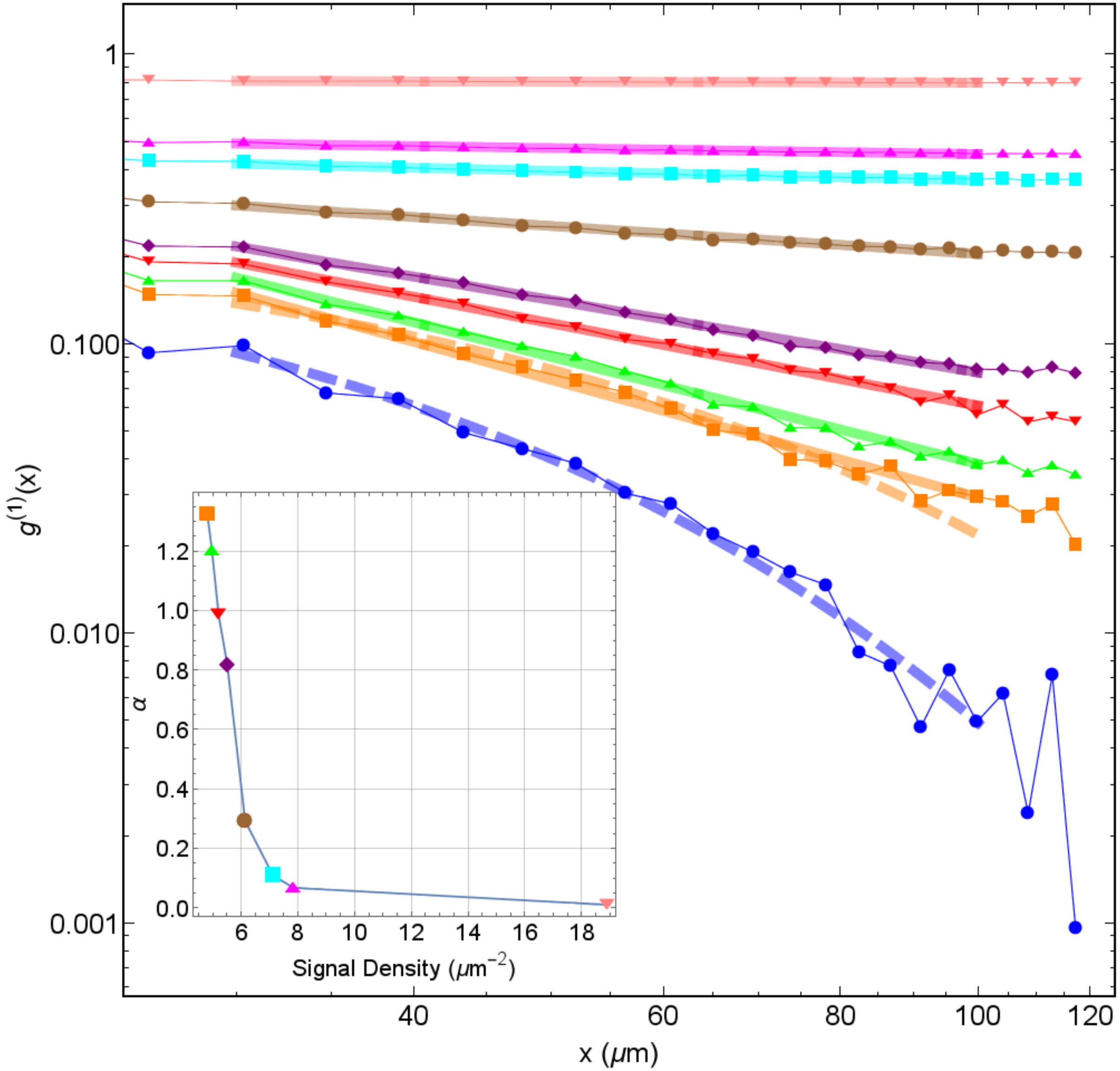
Phase Diagram



Not so narrow in particle density

First Order Spatial Coherence

$$g_s^{(1)}(\mathbf{r}) = \frac{\langle \psi_s^*(\mathbf{r}+\mathbf{R})\psi_s(\mathbf{R})\rangle}{\sqrt{\langle \psi_s^*(\mathbf{r})\psi_s(\mathbf{r})\rangle\langle \psi_s^*(\mathbf{r}+\mathbf{R})\psi_s(\mathbf{r}+\mathbf{R})\rangle}}$$



Power-law (solid)

exponential (dashed)

Exponent can exceed 1/4

First Order Spatial Coherence

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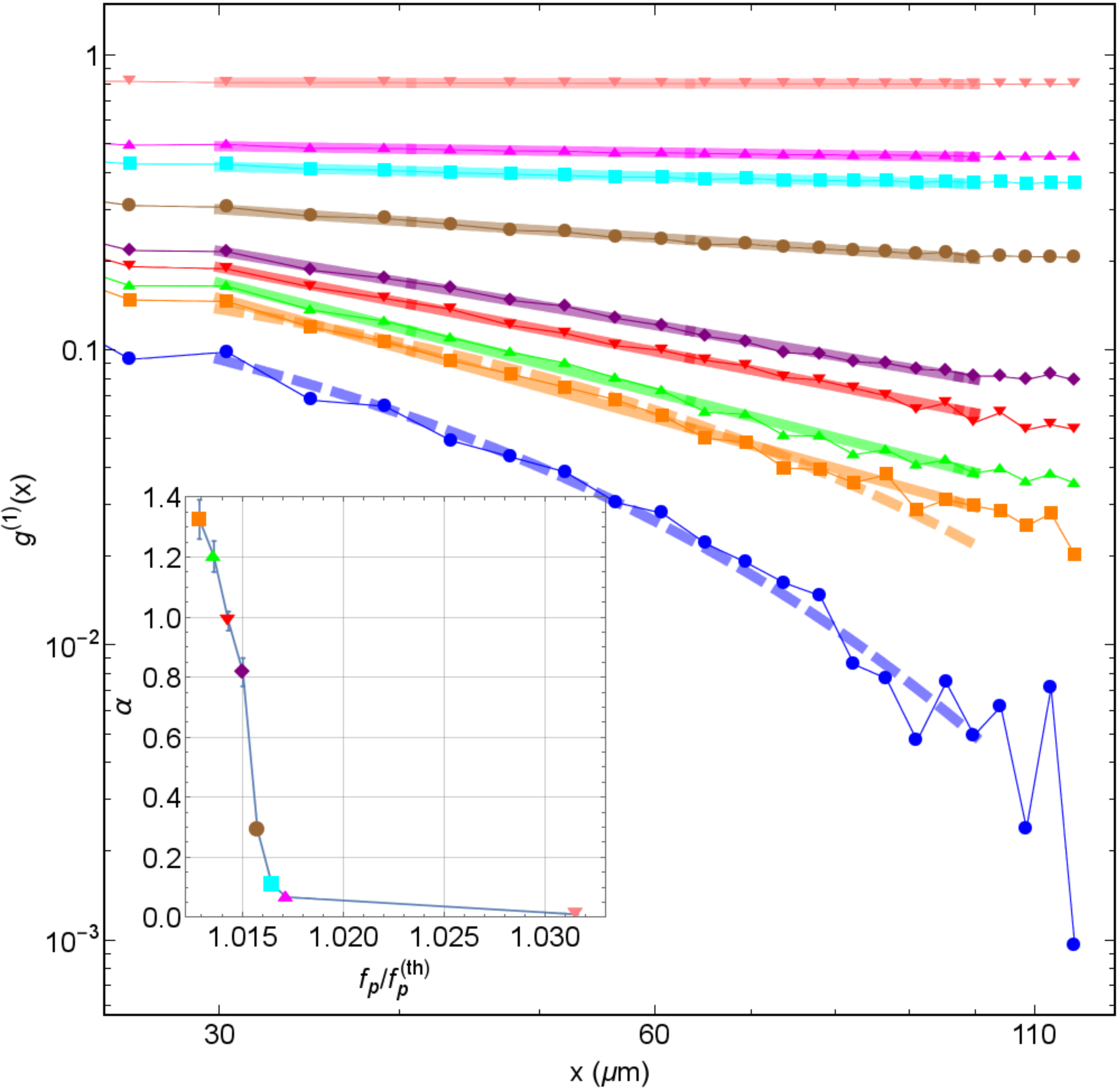


Power-law (solid)

exponential (dashed)

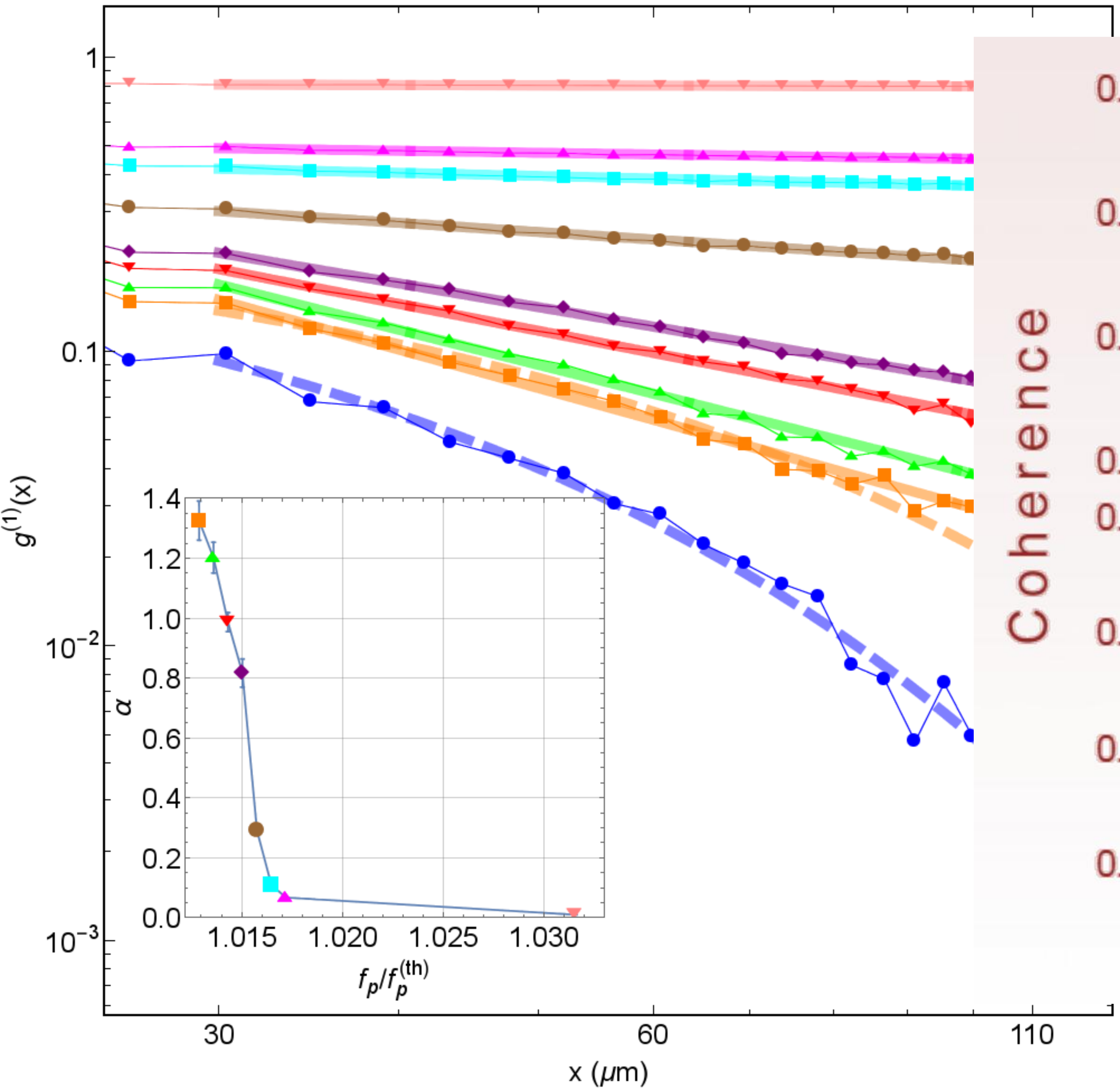
Exponent can exceed ¼

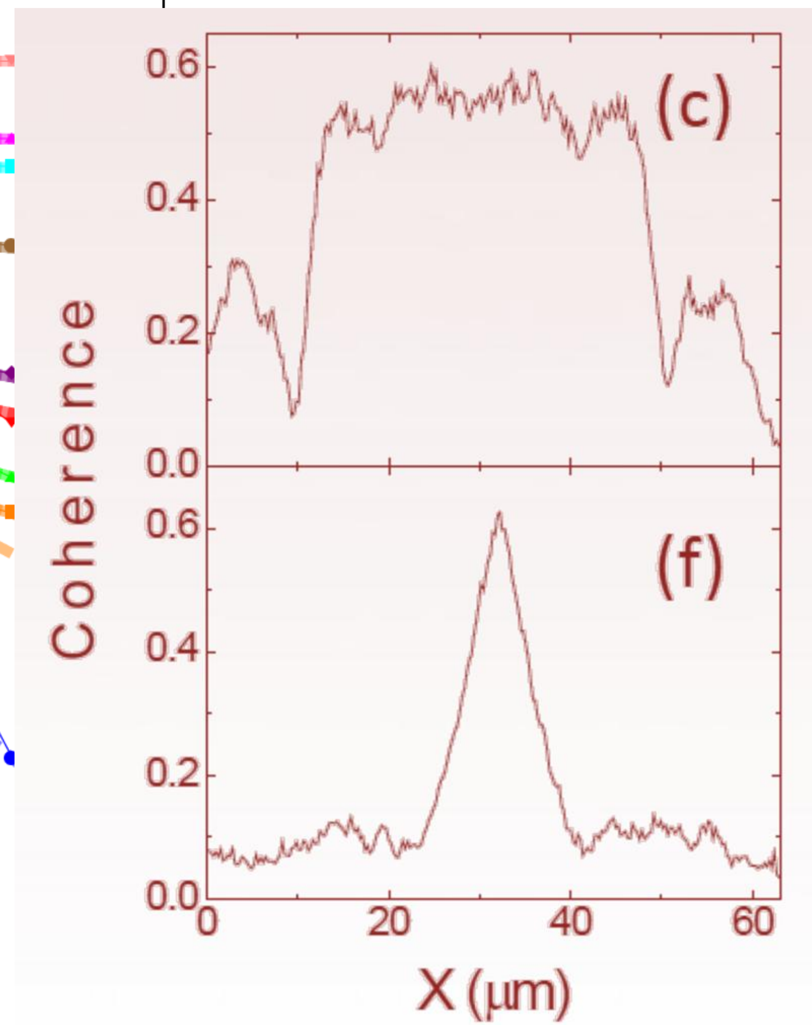
=

External drive excites collective fluctuations preferentially over topological defects

First Order Spatial Coherence

$$g_s^{(1)}(\mathbf{r}) = \frac{\langle \psi_s^*(\mathbf{r}+\mathbf{R})\psi_s(\mathbf{R})\rangle}{\sqrt{\langle \psi_s^*(\mathbf{r})\psi_s(\mathbf{r})\rangle\langle \psi_s^*(\mathbf{r}+\mathbf{R})\psi_s(\mathbf{r}+\mathbf{R})\rangle}}$$

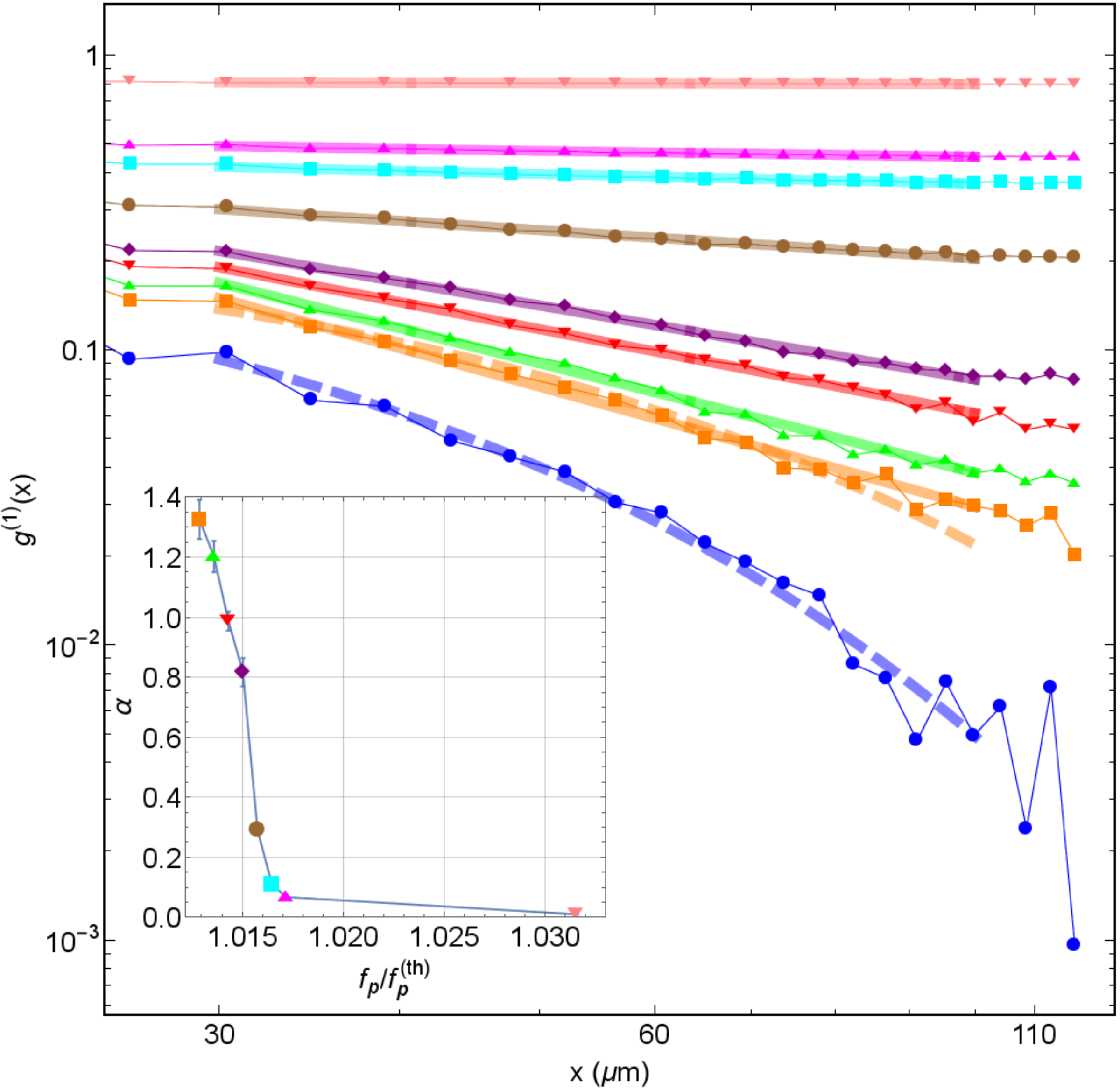




R. Spano et al. (2011)

First Order Spatial Coherence

$$g_s^{(1)}(\mathbf{r}) = \frac{\langle \psi_s^*(\mathbf{r}+\mathbf{R})\psi_s(\mathbf{R})\rangle}{\sqrt{\langle \psi_s^*(\mathbf{r})\psi_s(\mathbf{r})\rangle\langle \psi_s^*(\mathbf{r}+\mathbf{R})\psi_s(\mathbf{r}+\mathbf{R})\rangle}}$$

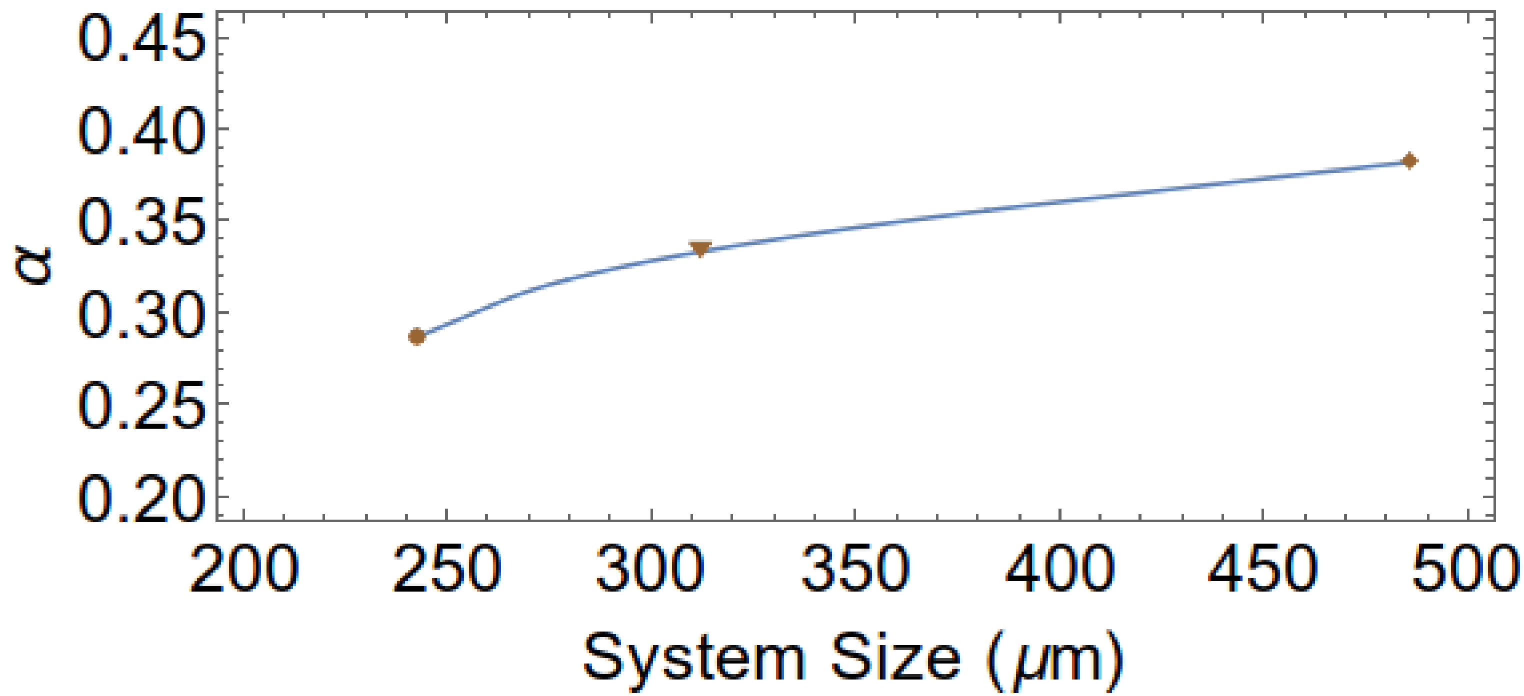


To observe:

Larger systems and closer to the phase transition

Increasing the System Size

$$g_s^{(1)}(\mathbf{r}) = \frac{\langle \psi_s^*(\mathbf{r}+\mathbf{R})\psi_s(\mathbf{R})\rangle}{\sqrt{\langle \psi_s^*(\mathbf{r})\psi_s(\mathbf{r})\rangle\langle \psi_s^*(\mathbf{r}+\mathbf{R})\psi_s(\mathbf{r}+\mathbf{R})\rangle}}$$

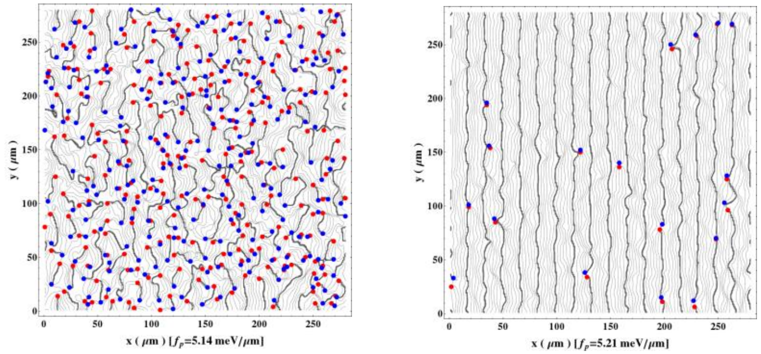


α converges clearly to a value larger then 1/4

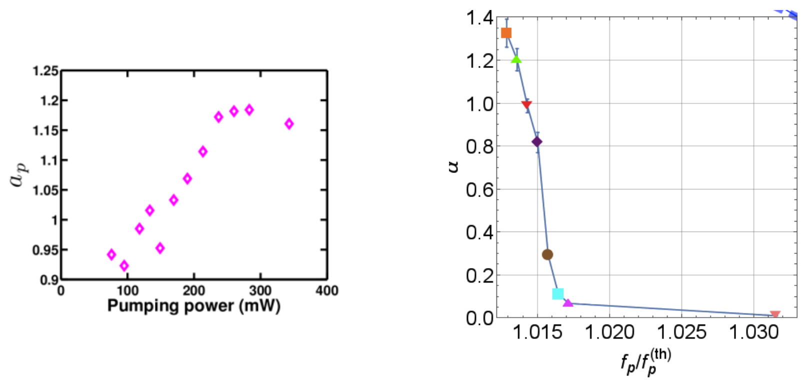
Conclusions

arXiv:1412.7361, PRX (to appear)

✧ Phase transition in driven 2D systems of the BKT type



✧ The exponent of the decay of correlations can exceed equilibrium upper limit – ordered phase more robust or more disordered



✧ Phase transition between normal and coherent phases in polariton system is of BKT and not BEC type