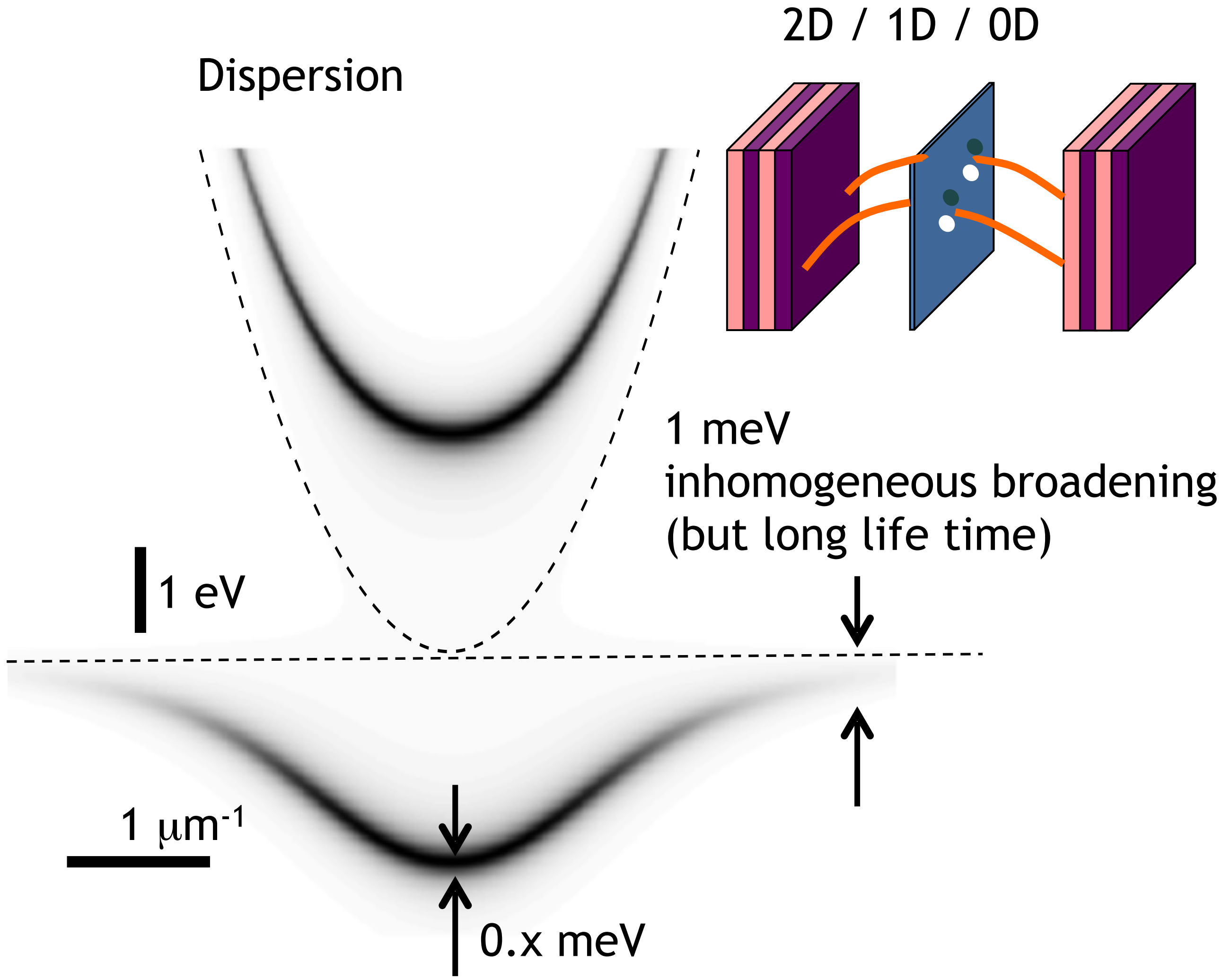
Coherence and superfluidity of nonequilibrium polariton quantum fluids

Michiel Wouters

Universiteit Antwerpen

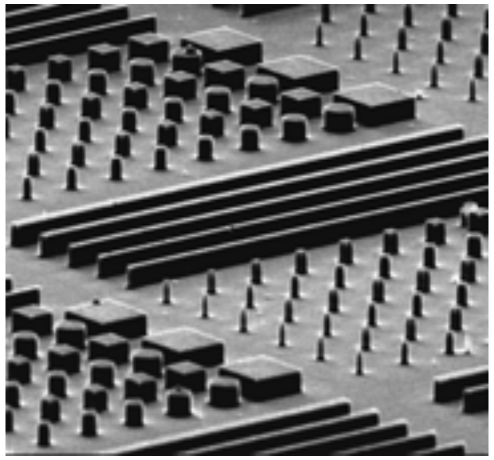
Linear polariton properties



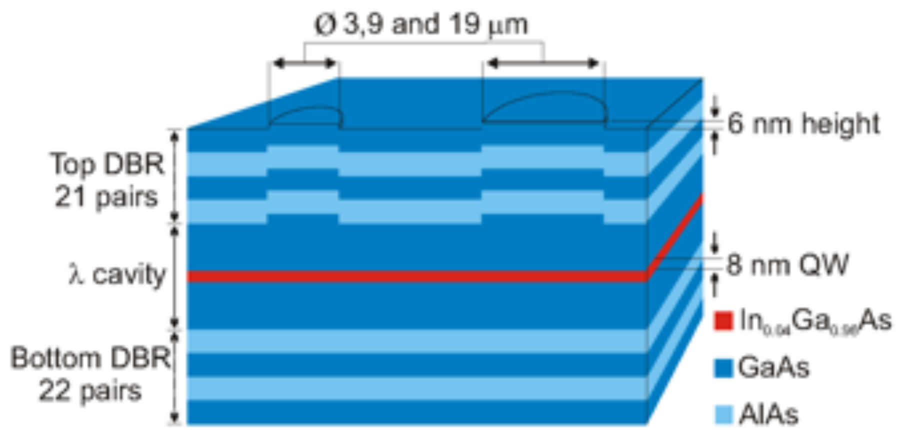
Polariton life time 1-100's ps

nonequilibrium

some structures



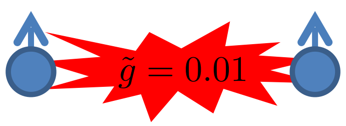
LPN Paris

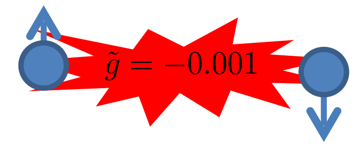


EPFL Lausanne

losses: diagnostic + new physics

Interaction properties





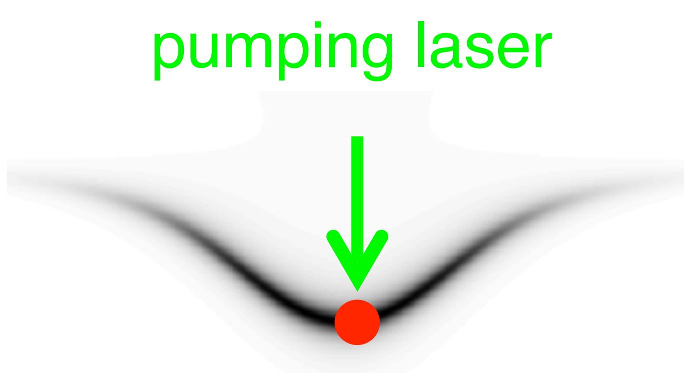
- Interactions due to excitonic component
- Short range
- Polarization dependent (bi-exciton Feshbach resonance)
- Weak interactions: $\tilde{g} = mg/\hbar^2 \ll 1$

  $\Rightarrow$ mean field theory mostly OK

N. Takemura et al., Nat. Phys./PRB 2014,
M. Vladimirova et al. PRB 2010.

Excitation schemes

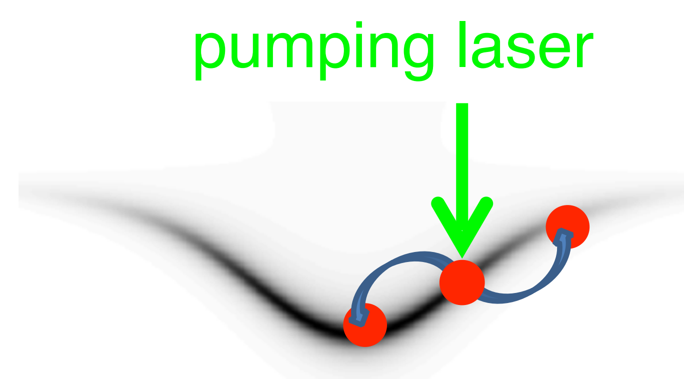
resonant excitation



pumping laser directly injects coherent polaritons

- ✓ clean
- ✓ flexible
- ✗ phase inherited by pumping laser
  → coherence trivial

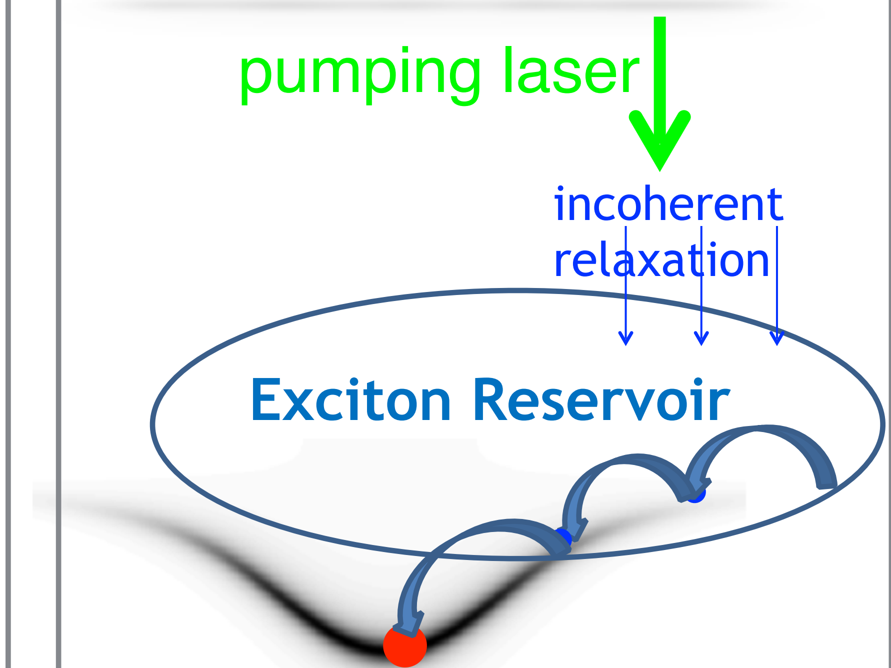
parametric excitation



one scattering process creates k=0 polaritons

- ✓ spontaneous coherence
- ✓ relatively clean
- ✗ very sensitive to laser parameters

nonresonant excitation



complicated relaxation process creates k=0 polaritons

- ✓ spontaneous coherence
- ✓ robust with respect to laser parameters
- ✗ dirty

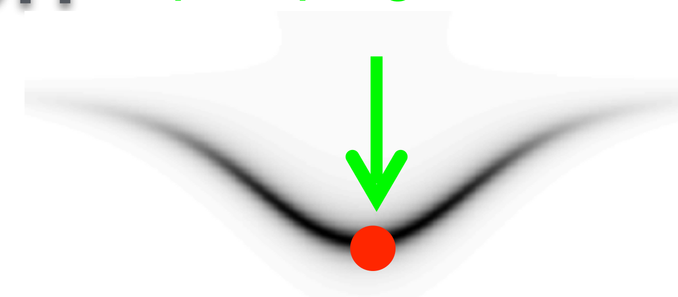
Outline

- generalized Gross-Pitaevskii equation
- coherence properties (including fluctuations)
- superfluid properties (including gauge field)

I. Carusotto and C. Ciuti, RMP 2013

Gross-Pitaevskii equation

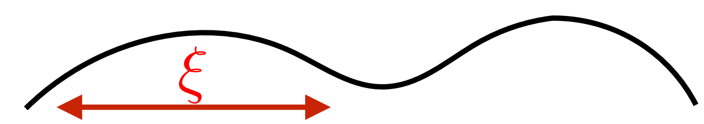
pumping laser



$$i\frac{\partial}{\partial t}\langle\hat{\psi}\rangle = \langle[\hat{\psi}, \hat{H}]\rangle$$

$$H = \int dx \left[\hat{\psi}^\dagger(x)\left(-\frac{\nabla^2}{2m} + V + \frac{g}{2}|\hat{\psi}(x)|^2\right)\hat{\psi}(x) + A_L(x,t)\hat{\psi}^\dagger(x) + A_L^*(x,t)\hat{\psi}(x)\right]$$

and the approximation $\langle\hat{\psi}^\dagger\hat{\psi}\hat{\psi}\rangle \approx |\langle\hat{\psi}\rangle|^2\langle\hat{\psi}\rangle$ which requires $|\langle\hat{\psi}\rangle|^2\xi^d \gg 1$



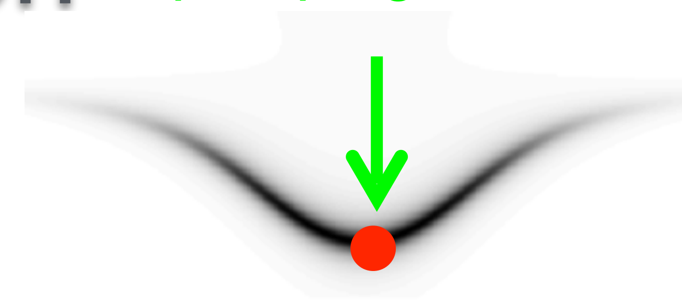
yields $i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g|\psi|^2\right)\psi + A_L$

coherent pumping can be taken into account as a Hamiltonian term

cf. c-field theory for atomic condensates [Svistunov, Gajda,…]

Gross-Pitaevskii equation

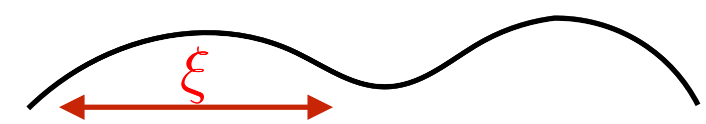
pumping laser



$$i\frac{\partial}{\partial t}\langle\hat{\psi}\rangle = \langle[\hat{\psi}, \hat{H}]\rangle$$

$$H = \int dx \left[\hat{\psi}^\dagger(x)\left(-\frac{\nabla^2}{2m} + V + \frac{g}{2}|\hat{\psi}(x)|^2\right)\hat{\psi}(x) + A_L(x,t)\hat{\psi}^\dagger(x) + A_L^*(x,t)\hat{\psi}(x)\right]$$

and the approximation $\langle\hat{\psi}^\dagger\hat{\psi}\hat{\psi}\rangle \approx |\langle\hat{\psi}\rangle|^2\langle\hat{\psi}\rangle$ which requires $|\langle\hat{\psi}\rangle|^2\xi^d \gg 1$



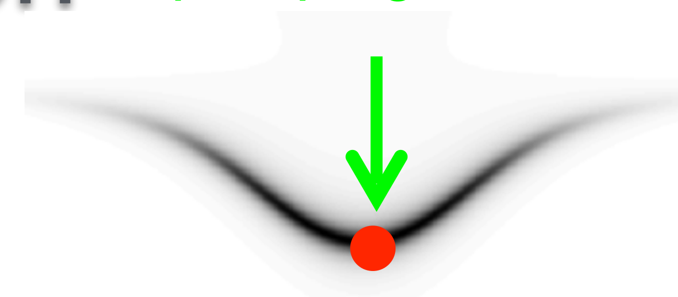
yields $i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g|\psi|^2\right)\psi + A_L - i\frac{\gamma}{2}\psi$

coherent pumping can be taken into account as a Hamiltonian term

cf. c-field theory for atomic condensates [Svistunov, Gajda,…]

Losses cannot be described by a Hamiltonian and require a master equation description

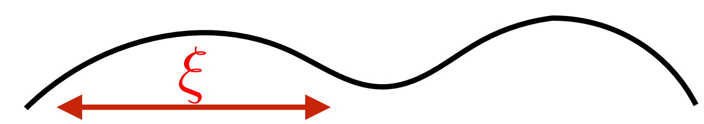
Gross-Pitaevskii equation



$$i\frac{\partial}{\partial t}\langle\hat{\psi}\rangle = \langle[\hat{\psi}, \hat{H}]\rangle$$

$$H = \int dx \left[\hat{\psi}^\dagger(x)\left(-\frac{\nabla^2}{2m} + V + \frac{g}{2}|\hat{\psi}(x)|^2\right)\hat{\psi}(x) + A_L(x,t)\hat{\psi}^\dagger(x) + A_L^*(x,t)\hat{\psi}(x)\right]$$

and the approximation $\langle\hat{\psi}^\dagger\hat{\psi}\hat{\psi}\rangle \approx |\langle\hat{\psi}\rangle|^2\langle\hat{\psi}\rangle$ which requires $|\langle\hat{\psi}\rangle|^2\xi^d \gg 1$



yields $i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g|\psi|^2\right)\psi + A_L - i\frac{\gamma}{2}\psi$

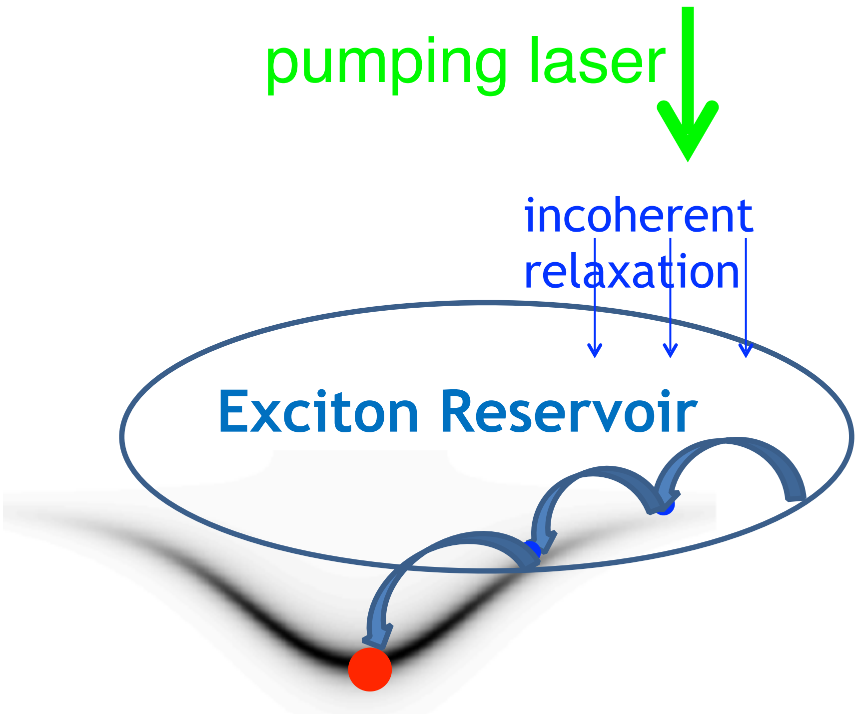
coherent pumping can be taken into account as a Hamiltonian term

cf. c-field theory for atomic condensates [Svistunov, Gajda,…]

Losses cannot be described by a Hamiltonian and require a master equation description

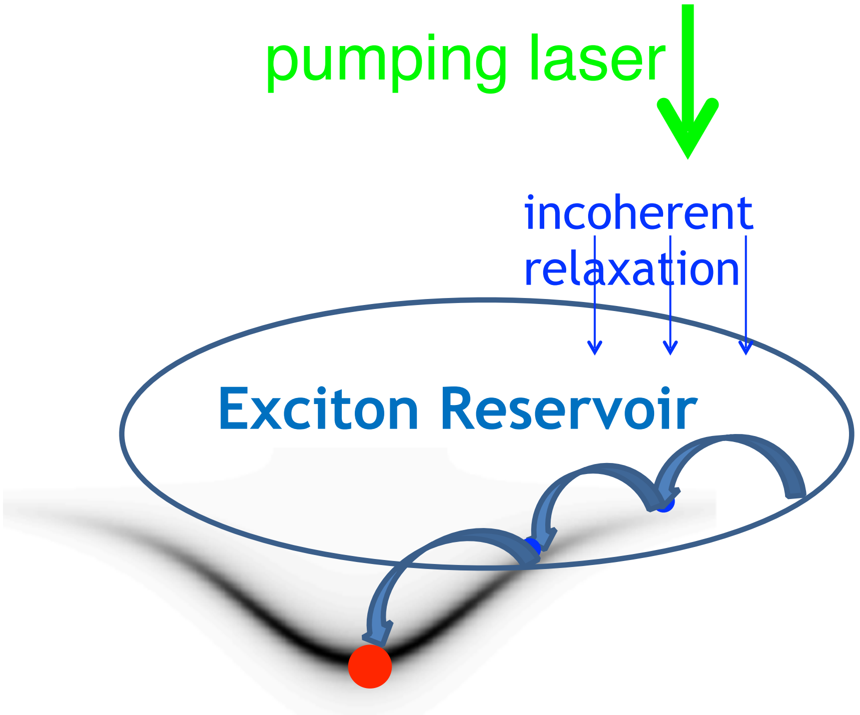
This equation describes almost all features seen in resonant pumping experiments

Phenomenological extension for incoherent excitation



$$i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g|\psi|^2 + \tilde{g}n_R\right)\psi + \frac{i}{2}\left(Rn_R - \gamma\right)\psi$$

Phenomenological extension for incoherent excitation

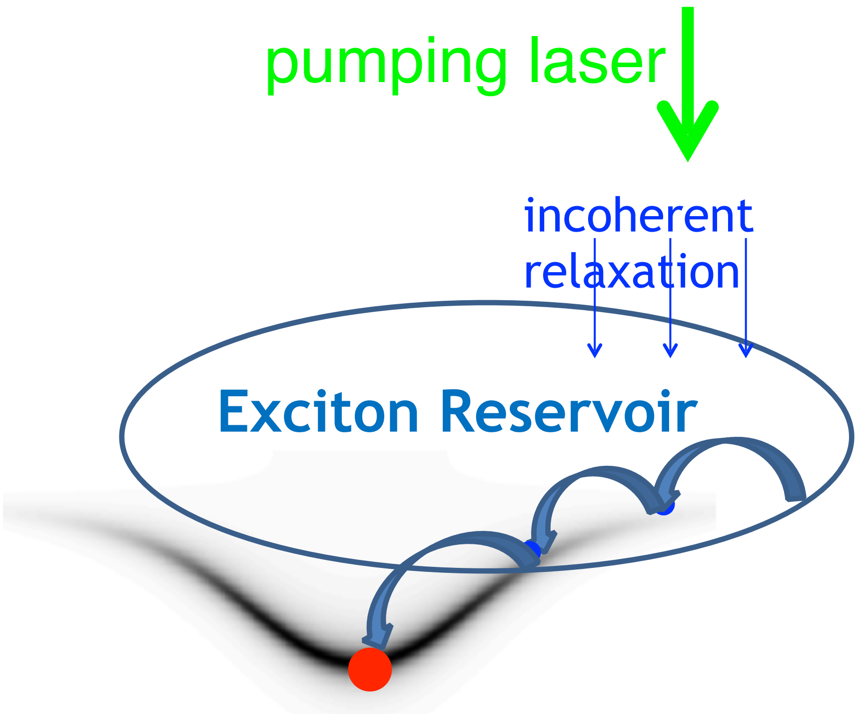


$$i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g|\psi|^2 + \tilde{g}n_R\right)\psi + \frac{i}{2}\left(Rn_R - \gamma\right)\psi$$

when $Rn_R < \gamma: \quad \psi = 0$ below threshold

$Rn_R > \gamma: \quad \psi \to \infty$ above threshold:
gain saturation needed

Phenomenological extension for incoherent excitation



$$i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g|\psi|^2 + \tilde{g}n_R\right)\psi + \frac{i}{2}\left(Rn_R - \gamma\right)\psi$$

when $Rn_R < \gamma : \quad \psi = 0$ below threshold

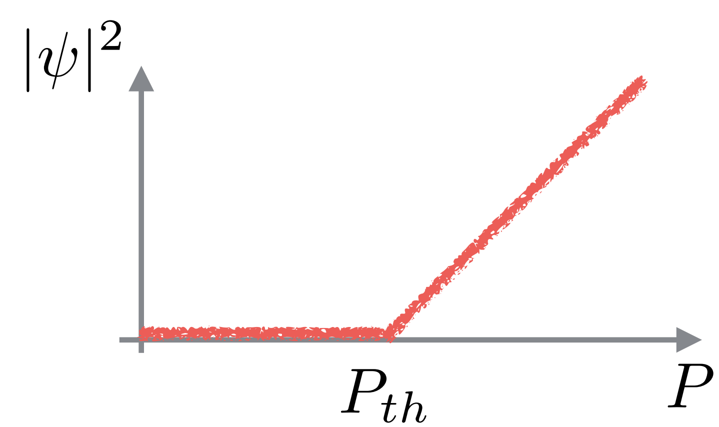
$Rn_R > \gamma : \quad \psi \to \infty$ above threshold: gain saturation needed

Rate equation for the reservoir

$$\frac{dn_R}{dt} = P - \gamma_R n_R - Rn_R|\psi|^2$$

threshold from $Rn_R = \gamma$ is $P_{th} = \gamma\gamma_R/R$

above threshold: $|\psi|^2 = (P - P_{th})/\gamma$



discontinuous derivative
because of mean field approximation

Adiabatic elimination of reservoir → cGLE

when the reservoir can follow instantaneously the polariton density

N. Bobrovska, M. Matuszewski, PRB 2015

$$\frac{dn_R}{dt} = P - \gamma_R n_R - R n_R |\psi|^2 = 0$$

we have a single equation for the polariton field

$$i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g|\psi|^2 + \frac{\tilde{g}P}{\gamma_R + R|\psi|^2} \right)\psi + \frac{i}{2}\left(\frac{RP}{\gamma_R + R|\psi|^2} - \gamma \right)\psi$$

= laser model with saturable gain

Adiabatic elimination of reservoir → cGLE

when the reservoir can follow instantaneously the polariton density

N. Bobrovska, M. Matuszewski, PRB 2015

$$\frac{dn_R}{dt} = P - \gamma_R n_R - R n_R |\psi|^2 = 0$$

we have a single equation for the polariton field

$$i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g|\psi|^2 + \frac{\tilde{g}P}{\gamma_R + R|\psi|^2}\right)\psi + \frac{i}{2}\left(\frac{RP}{\gamma_R + R|\psi|^2} - \gamma\right)\psi$$

= laser model with saturable gain

expand the gain nonlinearity:

$$i\frac{\partial}{\partial t}\psi = \left(-\frac{\nabla^2}{2m} + V + g_{eff}|\psi|^2\right)\psi + \frac{i}{2}\left(P_{eff} - \gamma - a|\psi|^2\right)\psi$$

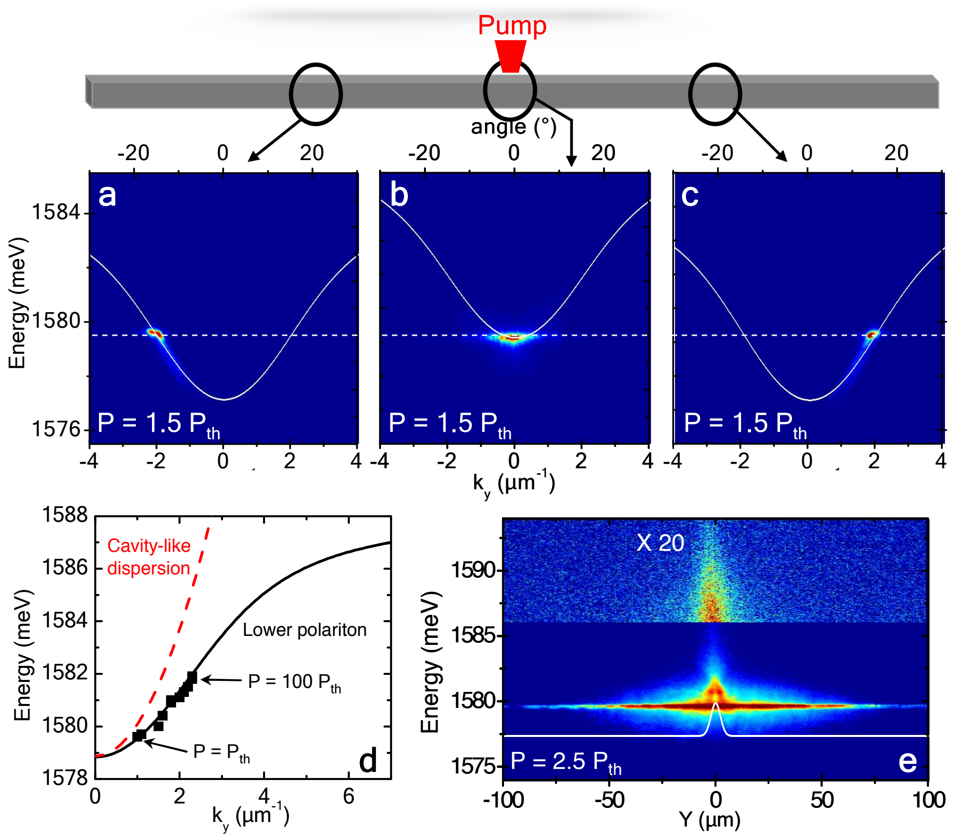
which is of the form of the complex Ginzburg Landau equation

general review: Aranson and Kramer RMP 2002

for polaritons: Keeling and Berlof PRL 2008

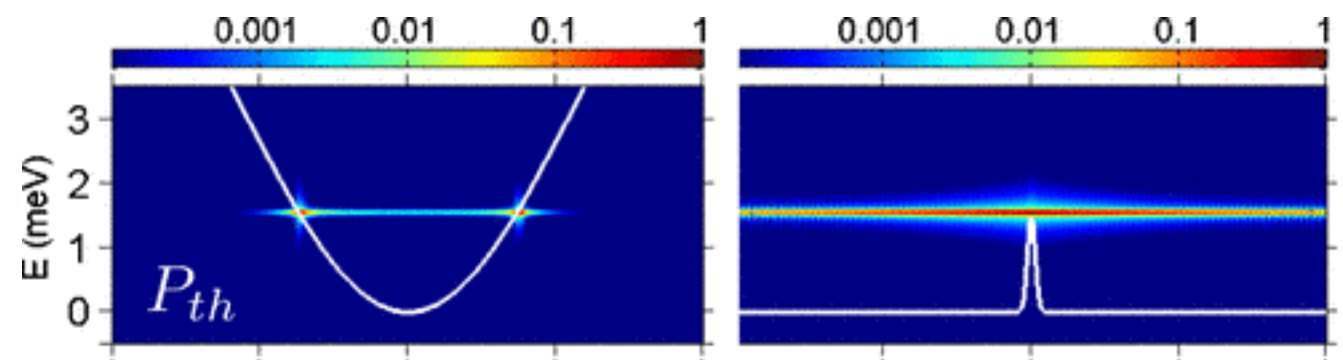
Flows in inhomogeneous polariton condensates

Volcano effect



E. Wertz et al. Nature Physics 2010

is reproduced numerically with any of the generalised GPEs



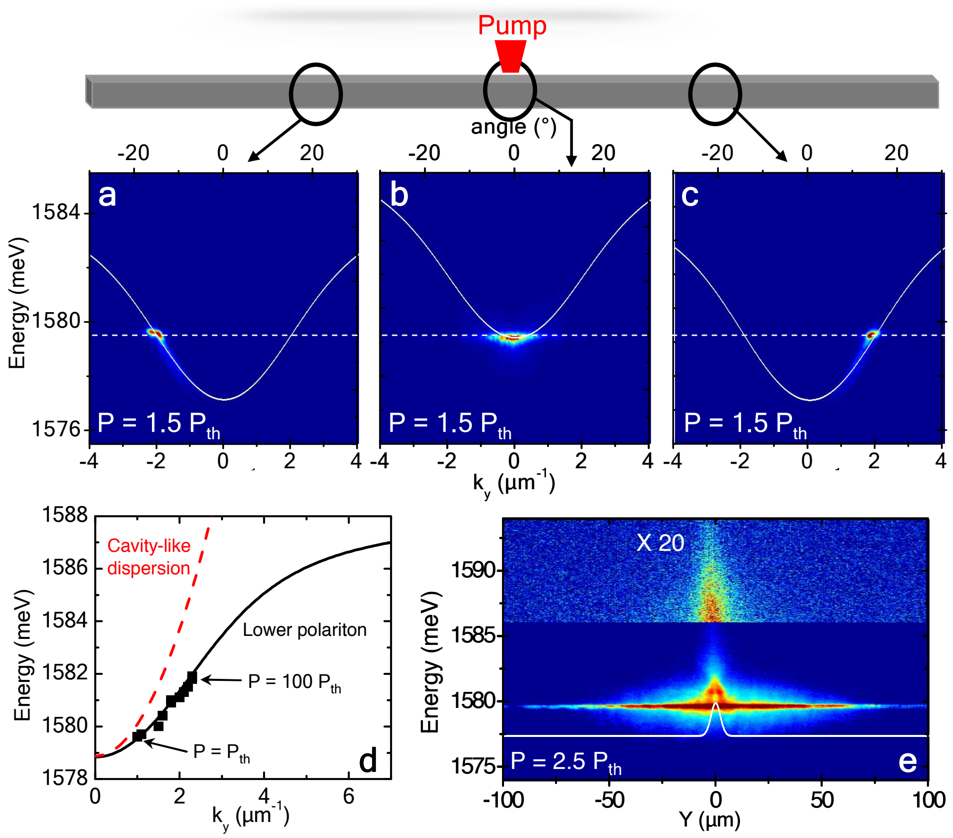
Liew, Wouters & Savona, PRB 2010
Wouters, Carusotto, Ciuti, PRB 2008

as a simple consequence of ‘energy conservation’ $\psi(x,t) = e^{-i\omega_c t}\sqrt{n(x)}e^{i\theta(x)}$

requires that $\omega = \frac{1}{2m}(\nabla\theta)^2 + gn + gn_R + V$ is constant

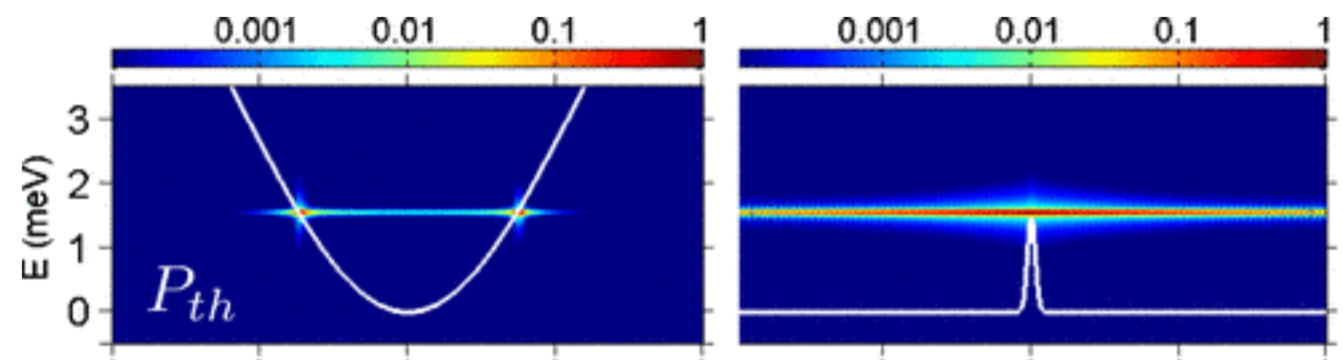
Flows in inhomogeneous polariton condensates

Volcano effect



E. Wertz et al. Nature Physics 2010

is reproduced numerically with any of the generalised GPEs



Liew, Wouters & Savona, PRB 2010
Wouters, Carusotto, Ciuti, PRB 2008

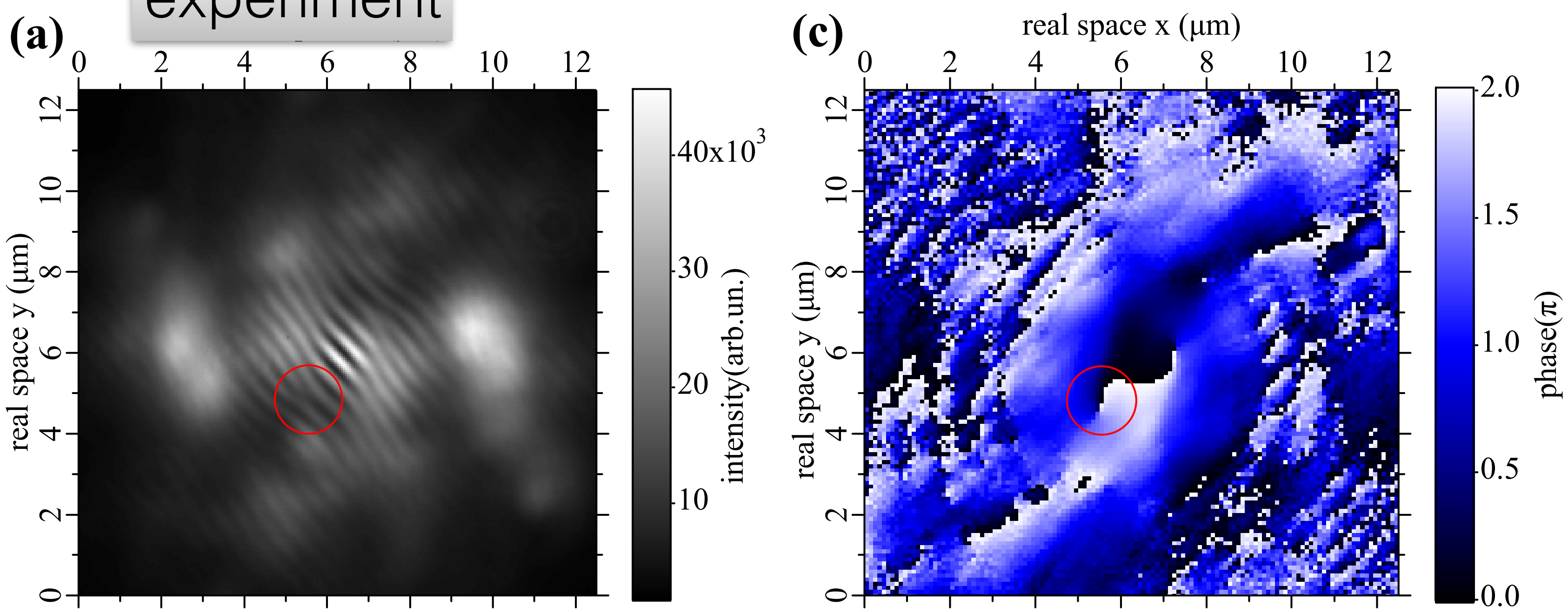
as a simple consequence of 'energy conservation' $\psi(x,t) = e^{-i\omega_c t}\sqrt{n(x)}e^{i\theta(x)}$

requires that $\omega = \frac{1}{2m}(\nabla\theta)^2 + gn + gn_R + V$ is constant

flows in the steady state are due to broken time-reversal invariance (driving+dissipation)
no free energy minimisation

or in 2D with disorder: vortices

experiment



theory

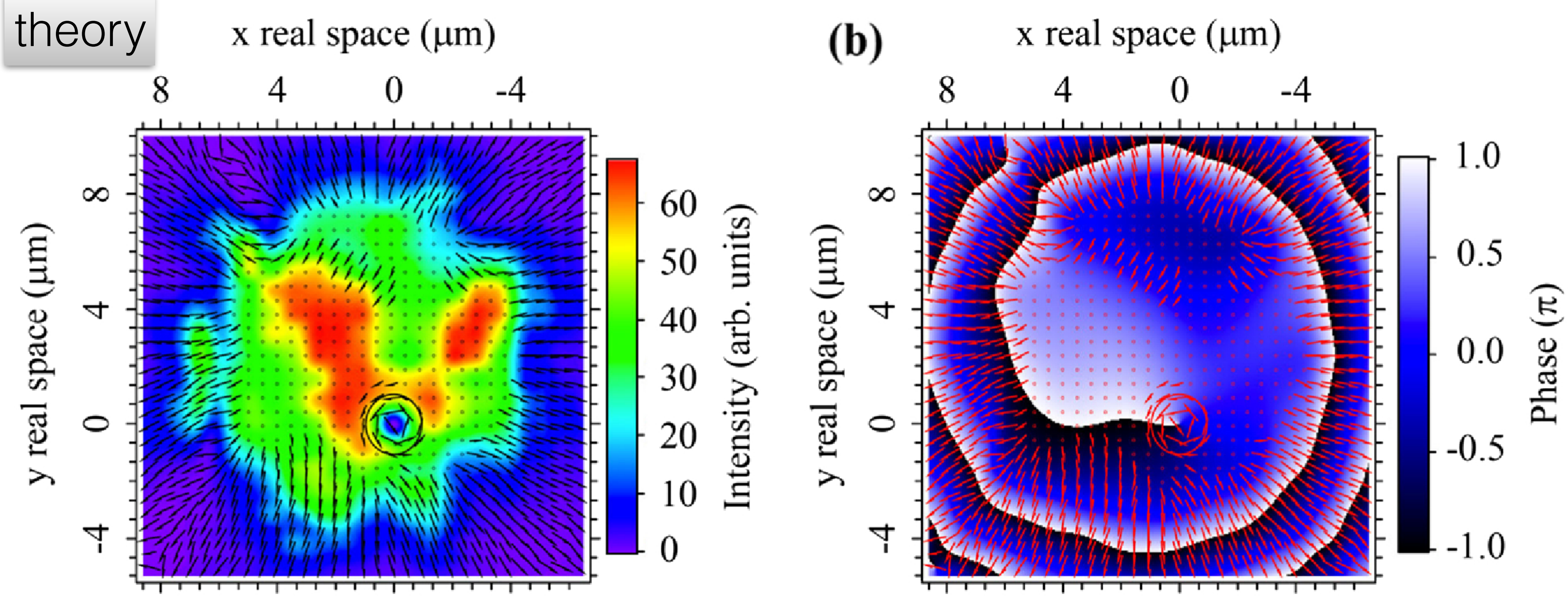
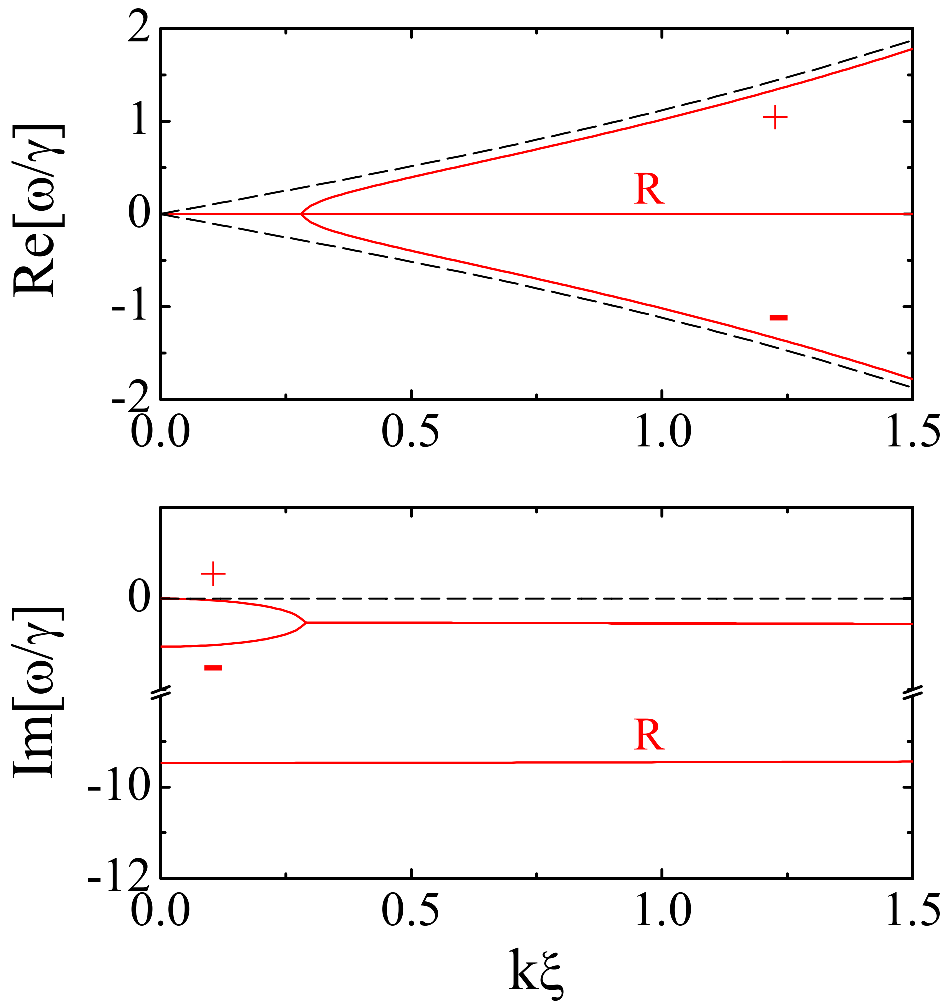
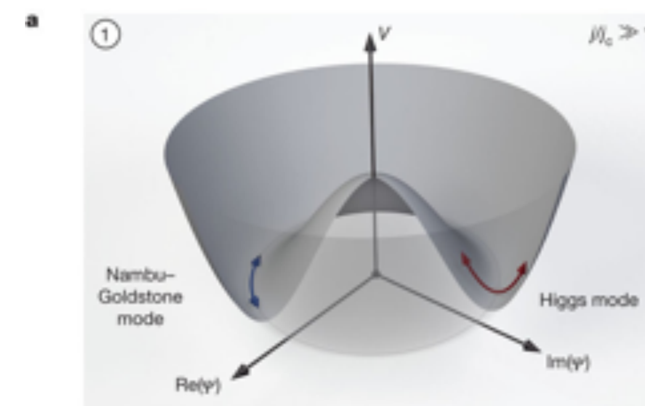


Fig. 3.

Goldstone mode above threshold

$$\psi = \psi + \delta\psi$$

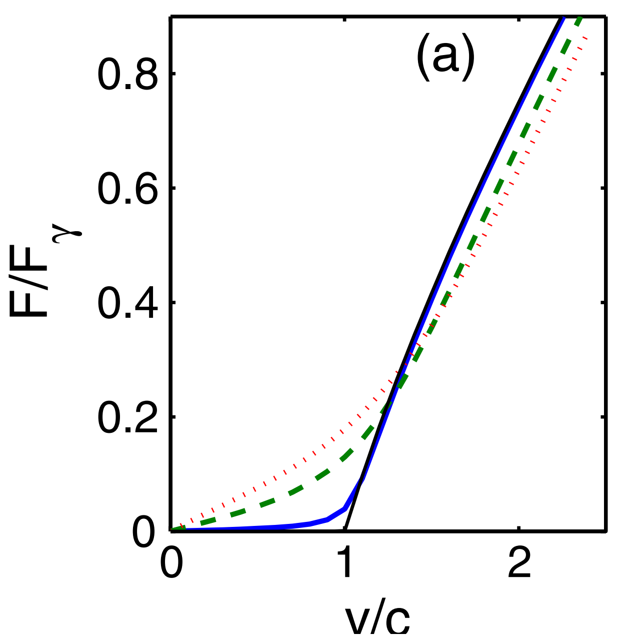




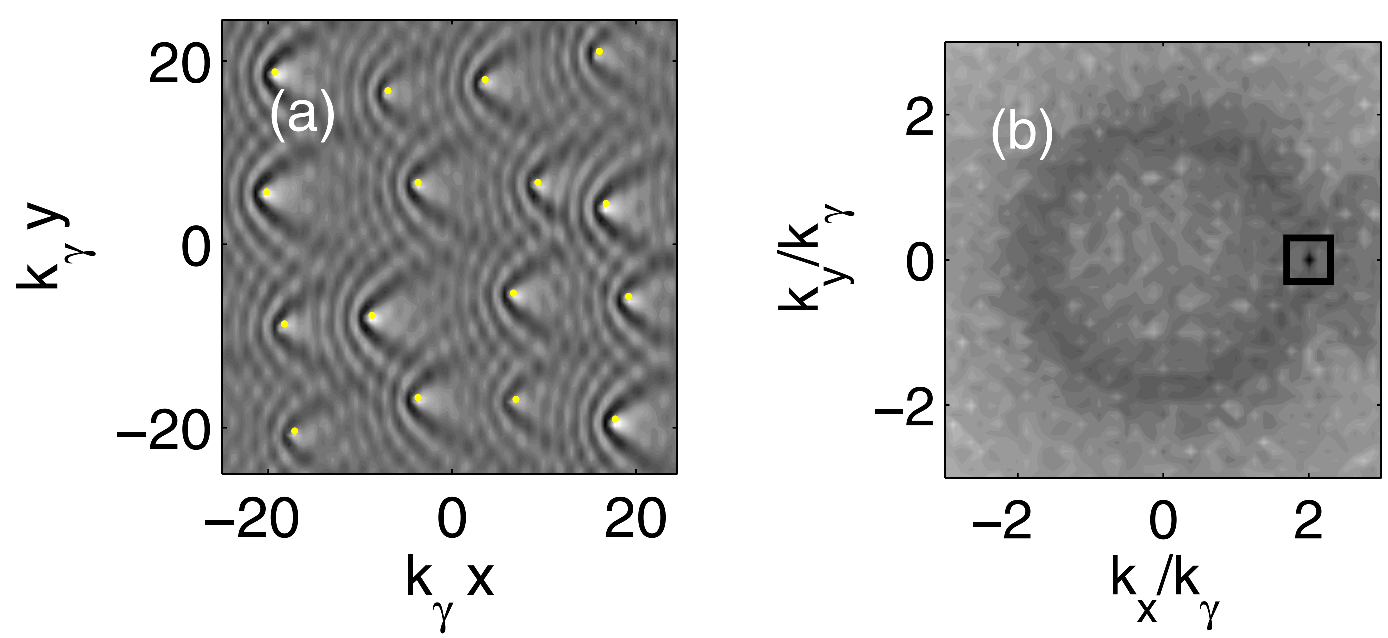
- there is always a zero frequency (real and imaginary part) mode due to the spontaneous U(1) symmetry breaking.
- the low energy part becomes diffusive due to dissipation

Drag force under incoherent excitation

smoothed threshold for increasing dissipation



… but drag does not prohibit persistent superflows



Wouters and Carusotto PRL 2010.

coherence under cw nonresonant excitation

GPE + noise

Gross-Pitaevskii equation: mean field classical physics

? effect of (quantum) fluctuations out of equilibrium

Truncated Wigner approximation:
add a noise term to Gross-Pitaevskii, that is proportional to losses

$$i\frac{\partial}{\partial t}\psi = \left[-\frac{\nabla^2}{2m} + g|\psi|^2 + i\left(\frac{P}{1+|\psi|^2/n_s} - \gamma\right)\right]\psi + \xi$$

white Gaussian noise

[Carusotto and Ciuti 2005, MW and Savona 2009]

cw excitation

noisy GGPE

$$i\frac{\partial}{\partial t}\psi = \left[-\frac{\nabla^2}{2m} + g|\psi|^2 + i\left(\frac{P}{1+|\psi|^2/n_s} - \gamma\right)\right]\psi + \xi$$

density-phase representation

$$\psi = \sqrt{n}e^{i\theta}$$

far above threshold: phase fluctuation only $\langle e^{i[\theta(x)-\theta(x')]}\rangle = e^{-\frac{1}{2}\langle[\theta(x)-\theta(x')]^2\rangle}$

$$\frac{\partial\theta}{\partial t} = -\eta\left(-\frac{\mu}{m}\nabla^2 + \frac{1}{4m^2}\nabla^4\right)\theta - \frac{1}{2m}(\nabla\theta)^2 + \sqrt{\frac{D}{n_0}}\xi$$

from gain saturation

$\eta = (1 - \frac{n_s}{n_0})\gamma^{-1}$

$[\omega_{\mathrm{Bogoliubov}}(-i\nabla)]^2$

superfluid kinetic energy

$\frac{1}{2}mv_s^2$

Schawlow-Townes phase noise

Bogoliubov approximation

For weak and slow phase fluctuations, neglect nonlinear term

$$\frac{\partial\theta}{\partial t} = -\eta\left(-\frac{\mu}{m}\nabla^2 + \frac{1}{4m^2}\nabla^4\right)\theta - \frac{1}{2m}(\nabla\theta)^2 + \sqrt{\frac{D}{n_0}}\xi$$

$$\langle|\theta(k)|^2\rangle = \frac{D/(2\eta n)}{\mu[k^2/2m + (k^2/2m)^2]} \sim \frac{\text{noise}}{k^2}$$

$\langle\psi^\dagger(x)\psi(x')\rangle \sim \exp(-|x-x'|/\ell_c)$ exponential decay, as in equilibrium

$\ell_c = \frac{4\hbar^2 n}{Dm}\eta\mu$ cf. equilibrium $\ell_c = \frac{2\hbar^2 n}{k_B T m}$

in 2D: Bogoliubov theory predicts power law decay

to thermalise or not to thermalise

Small momenta

Bose-Einstein distribution/classical field theory: $n_k \propto \frac{T}{k^2}$ (thermal equilibrium)

stochastic models for polariton condensation: $n_k \propto \frac{\text{noise}}{k^2}$ (diffusive Goldstone mode)

to thermalise or not to thermalise

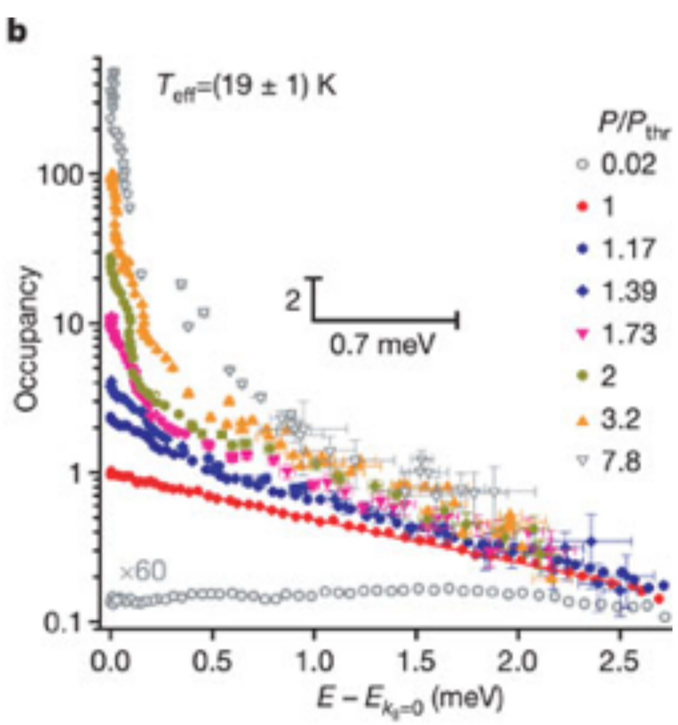
Small momenta

Bose-Einstein distribution/classical field theory: $n_k \propto \frac{T}{k^2}$ (thermal equilibrium)

stochastic models for polariton condensation: $n_k \propto \frac{\text{noise}}{k^2}$ (diffusive Goldstone mode)

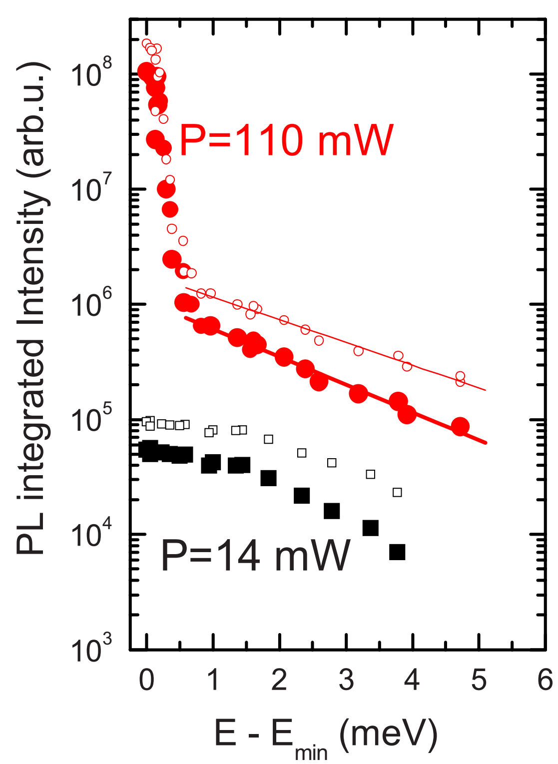
Large momenta

polariton Boltzmann tail



Kasprzak et al. 2006

photon Boltzmann tail



Bajoni et al. 2007

rate equation

$$\frac{dn_k}{dt} = R_{in}(k)(n_k + 1) - R_{out}(k)n_k$$

steady state

$$n_k = \frac{1}{\frac{R_{out}(k)}{R_{in}(k)} - 1}$$

eq. reservoir: ‘Kennard-Stepanov’

$$\frac{R_{out}(k)}{R_{in}(k)} = Ae^{-E_k/T}$$

cf. Jonathan’s talk

what with weak interactions?

$$\ell_c = \frac{4\hbar^2 n}{Dm}\eta\mu$$

What for $\mu \to 0$?? (equilibrium: fragmentation of the condensate)

Can a laser be spatially coherent without photon-photon interactions?

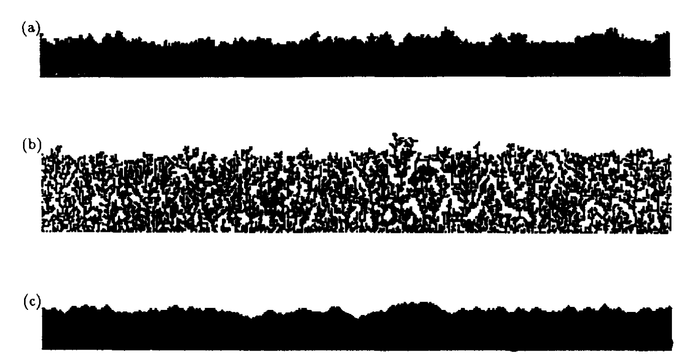
Kardar-Parisi-Zhang physics

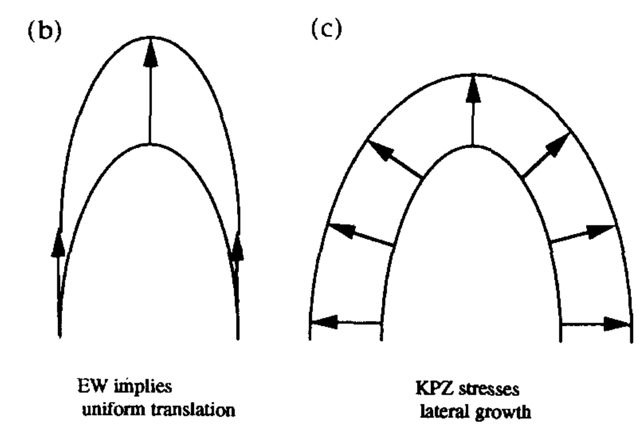
Without fourth order derivative: KPZ equation

$$\frac{\partial\theta}{\partial t} = -\eta\left(-\frac{\mu}{m}\nabla^2 + \frac{1}{4m^2}\nabla^4\right)\theta - \frac{1}{2m}\left(\nabla\theta\right)^2 + \sqrt{\frac{D}{n_0}}\xi$$

- nice review: T. Halpin-Healy, Y.-C. Zhang, Phys. Rept. 1995

- originally derived in crystal growth: Kardar, Parisi, Zhang, PRL 1986





- For atomic Bose quantum fluids [Kulkarni et al. PRA 2013, Arzamasovs et al. arxiv: 1309.2647]

- For polaritons, see also: E. Altman et al., arxiv:1311.0876, L. Sieberer et al. arXiv: 1412.5579.

Kuramoto-Sivashinski equation

Without noise

$$\frac{\partial \theta}{\partial t} = -\eta\left(-\frac{\mu}{m}\nabla^2 + \frac{1}{4m^2}\nabla^4\right)\theta - \frac{1}{2m}\left(\nabla\theta\right)^2 + \sqrt{\frac{D}{n_0}}\xi$$

originally introduced without noise term to describe

- reaction-diffusion systems [Y. Kuramoto, T. Tsuzuki, Progr. Theoret. Phys. , 1977]
- flame front propagation [G. Sivashinsky, Acta Astron. , 1977]

shows chaotic dynamics for attractive interactions that is in KPZ universality class

Without second order derivative

$$\frac{\partial \theta}{\partial t} = -\eta \left(-\frac{\mu}{m} \nabla^2 + \frac{1}{4m^2} \nabla^4 \right) \theta - \frac{1}{2m} \left(\nabla \theta \right)^2 + \sqrt{\frac{D}{n_0}} \xi$$

… not much studied

known to be in the KPZ universality class (Ueno et al. PRE 2005)
⇒ exponential decay of spatial coherence

We find with dimensional analysis/numerics:

$$\ell_c = 2\, \eta^{1/7} \left(\frac{\hbar^2}{2m} \right)^{6/7} \left(\frac{n_0}{D} \right)^{5/7}$$

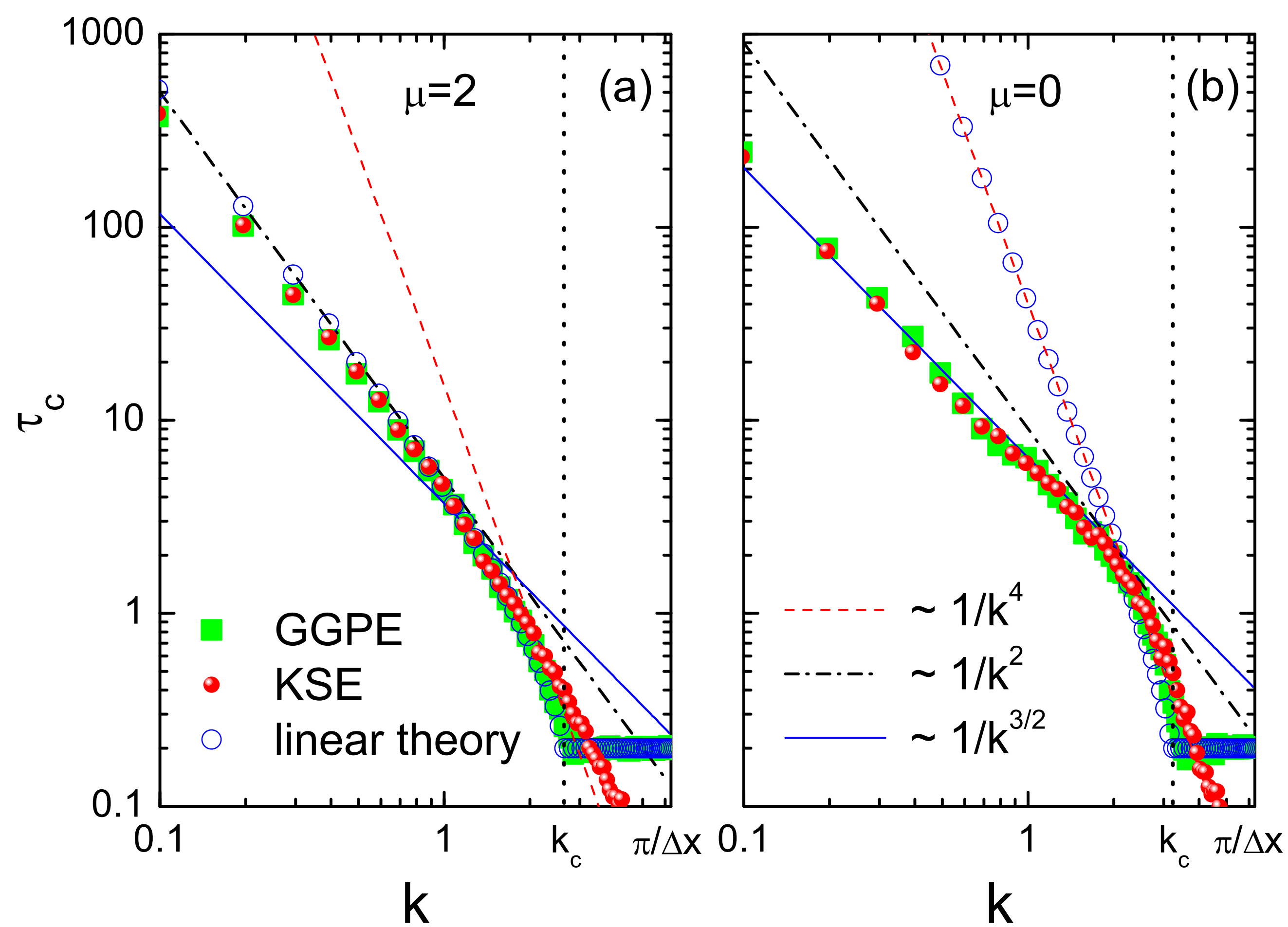
Temporal coherence

KPZ scaling: $\langle\psi^\dagger(x,t)\psi(x',t')\rangle \sim \exp(-|x-x'|/\ell_c) \times f\left(\frac{|t-t'|}{|x-x'|^{3/2}}\right)$

characteristic KPZ scaling of the coherence time as $\tau_c(k) \sim k^{-3/2}$

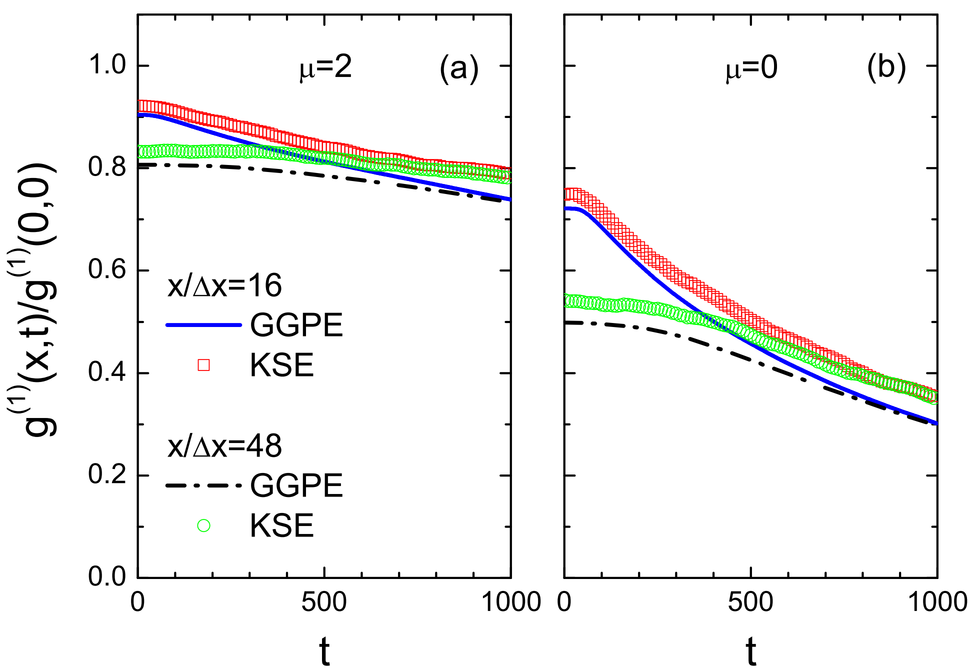
different scaling from Bogoliubov theory $\tau_c(k) \sim k^{-2}$

GGPE simulations:
KPZ scaling easier to reach for weaker interactions

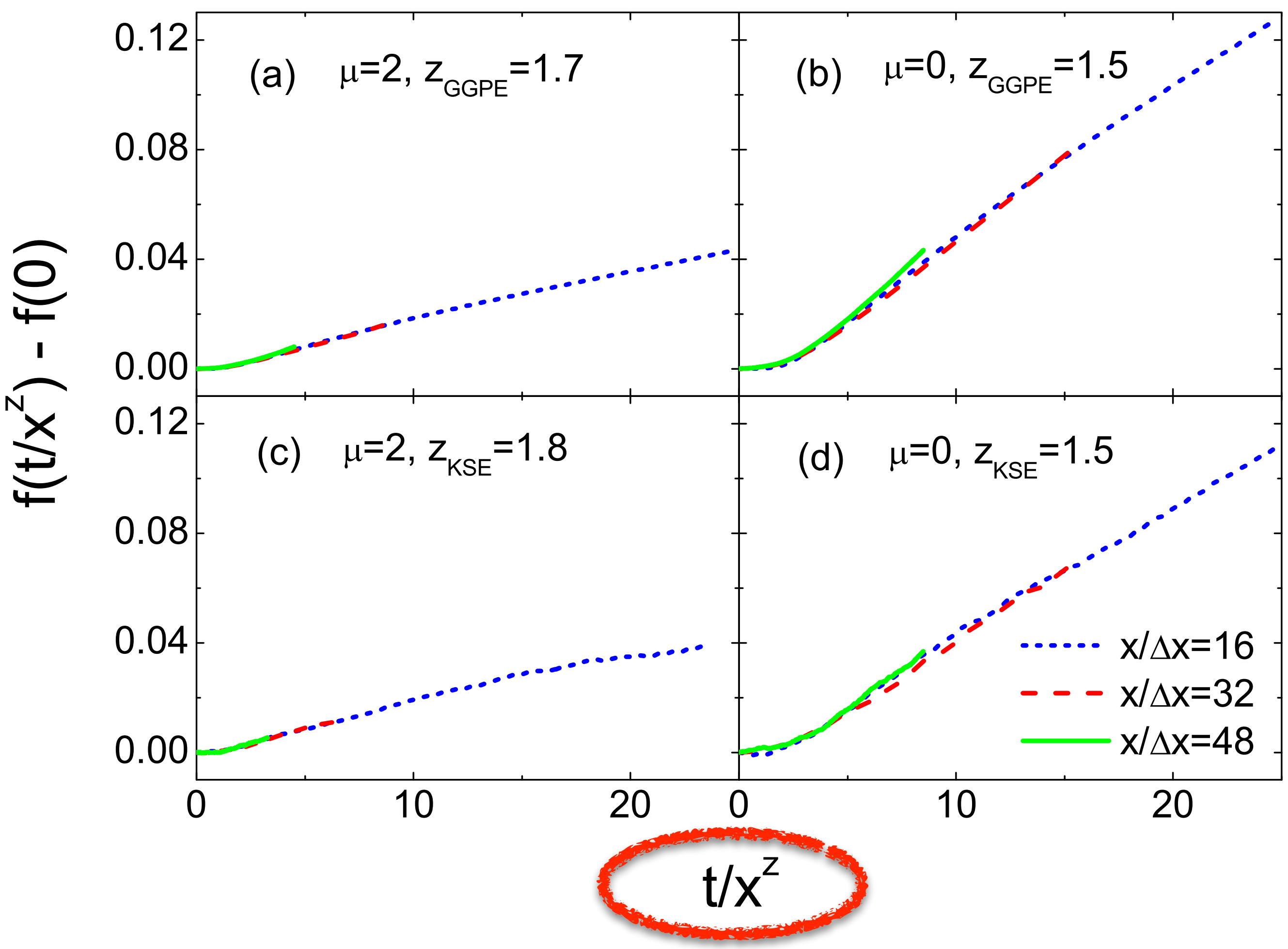


The scaling function

temporal coherence at fixed points



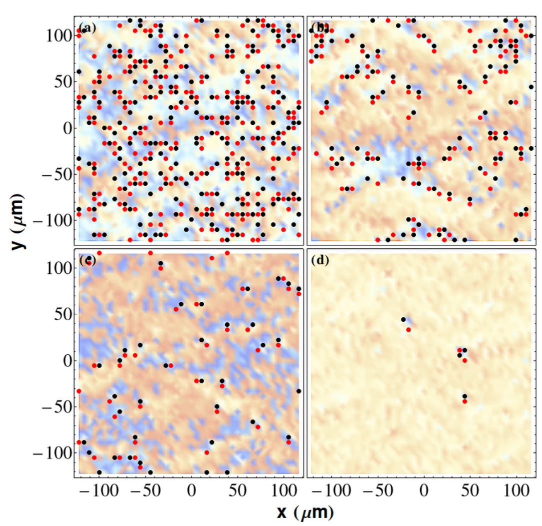
$$f(t/x^z) = C(x,t)x^{-2\chi} = -2\ln\left[g^{(1)}(x,t)\right]x^{-2\chi}$$

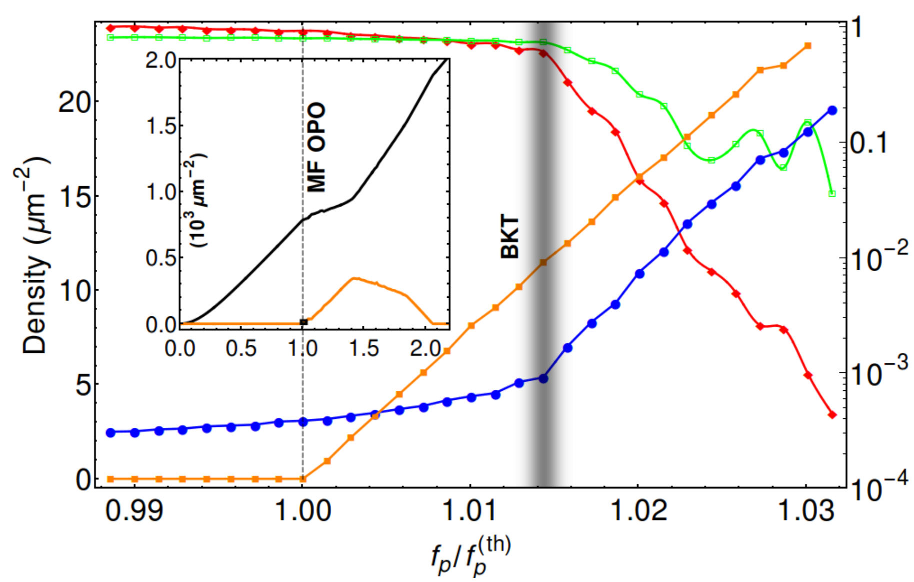


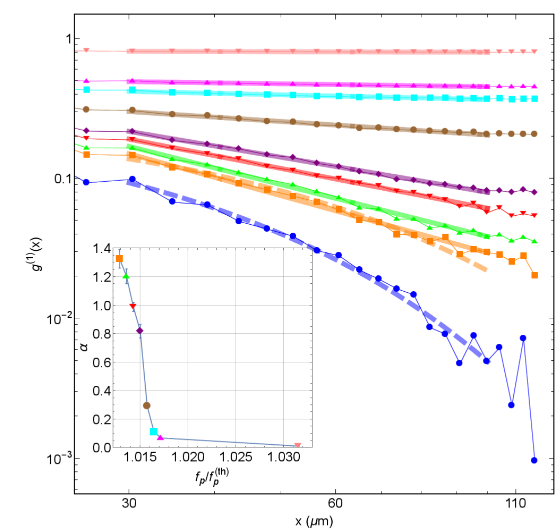
determine z such that rescaled temporal coherences at different positions collapse.

2D beyond phase fluctuations: BKT physics

Extensive numerical simulations have been performed in the OPO case



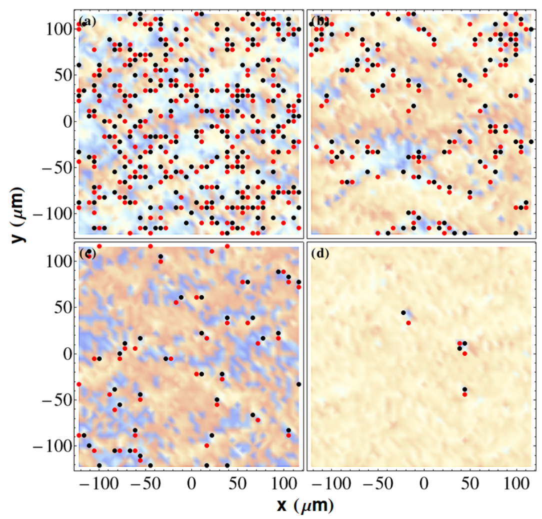


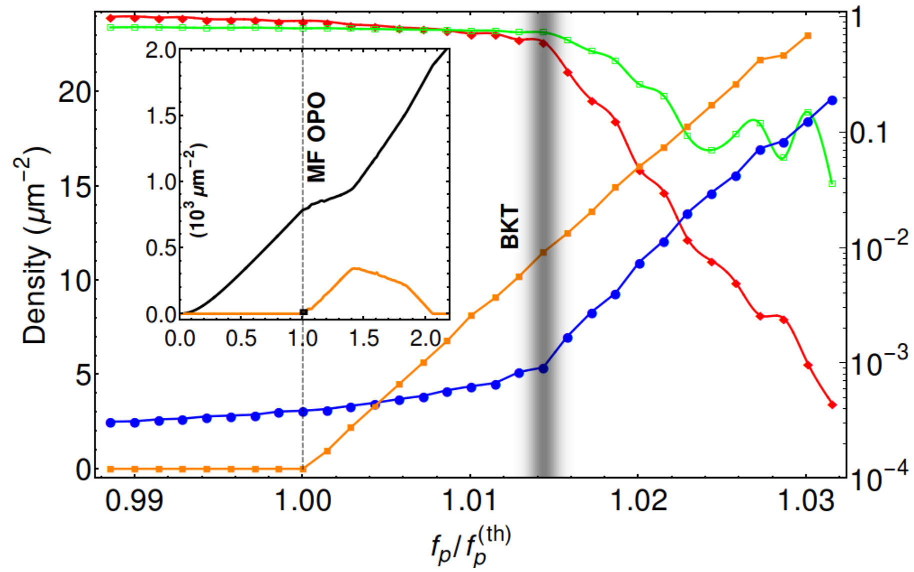


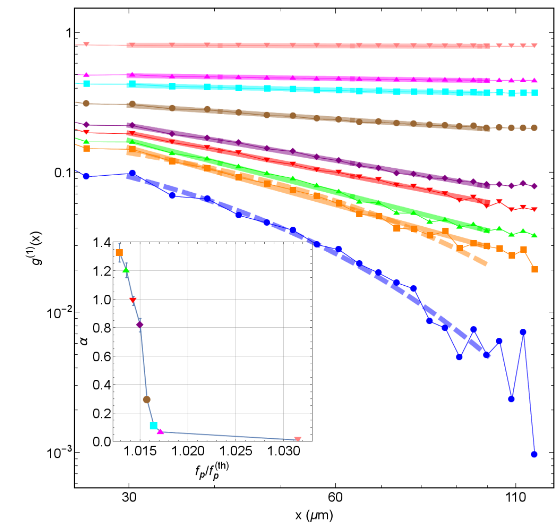
Dagvadorj et al. Phys. Rev. X 5, 041028 (2015).

2D beyond phase fluctuations: BKT physics

Extensive numerical simulations have been performed in the OPO case





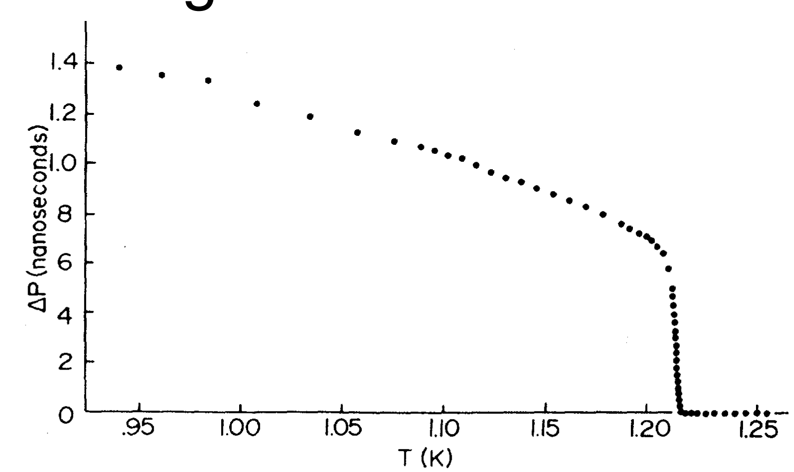


Dagvadorj et al. Phys. Rev. X 5, 041028 (2015).

Equilibrium BKT:
universal jump in the superfluid density at the transition

$$\rho_s = \frac{2m^2}{\pi} T_{BKT}$$

change in moment of inertia



D. J. Bishop and J. D. Reppy Phys. Rev. Lett. **40**, 1727 (1978).

Superfluid fraction out of equilibrium

Superfluid fraction

twisted boundary condition $E(\theta) - E(\theta = 0) = f_s \frac{N}{2m} (\nabla\theta)^2$

(only the superfluid part responds to a small phase twist)

$\frac{E}{N} = \mu =$ frequency in $\psi(x,t) = e^{-i\mu t}\ \psi(x,0)$

can also be used out of equilibrium:
Janot et al. 2013

For a homogeneous equilibrium Bose gas:

$$\mu = gn + \frac{1}{2m}(\nabla\theta)^2 \quad \Rightarrow \quad f_s = 1$$

with an external potential: *f_s can be <1*

P.C. Hohenberg and P.C. Martin, Ann. Phys. N.Y. 34, 291 (1965) .

E. H. Lieb, R. Seiringer, and J. Yngvason, Phys. Rev. B 66, 134529 (2002)

A. Janot, T. Hyart, P. R. Eastham, and B. Rosenow, Phys. Rev. Lett. 111, 230403 (2013).

Superfluid fraction

twisted boundary condition $E(\theta) - E(\theta = 0) = f_s \frac{N}{2m}(\nabla\theta)^2$

(only the superfluid part responds to a small phase twist)

$\frac{E}{N} = \mu =$ frequency in $\psi(x,t) = e^{-i\mu t}\,\psi(x,0)$

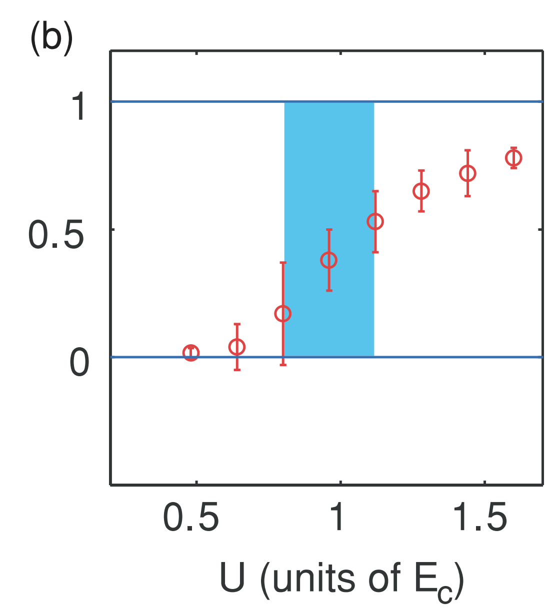
can also be used out of equilibrium:
Janot et al. 2013

For a homogeneous equilibrium Bose gas:

$$\mu = gn + \frac{1}{2m}(\nabla\theta)^2 \quad \Rightarrow \quad f_s = 1$$

with an external potential: *f_s can be <1*

Bose gas in disorder potential: mean field version of superfluid-Bose glass phase transition



Luca Fontanesi, Michiel Wouters, and Vincenzo Savona, Phys. Rev. A **81**, 053603 (2010).

P.C. Hohenberg and P.C. Martin, Ann. Phys. N.Y. 34, 291 (1965) .

E. H. Lieb, R. Seiringer, and J. Yngvason, Phys. Rev. B 66, 134529 (2002)

A. Janot, T. Hyart, P. R. Eastham, and B. Rosenow, Phys. Rev. Lett. 111, 230403 (2013).

Normal fraction

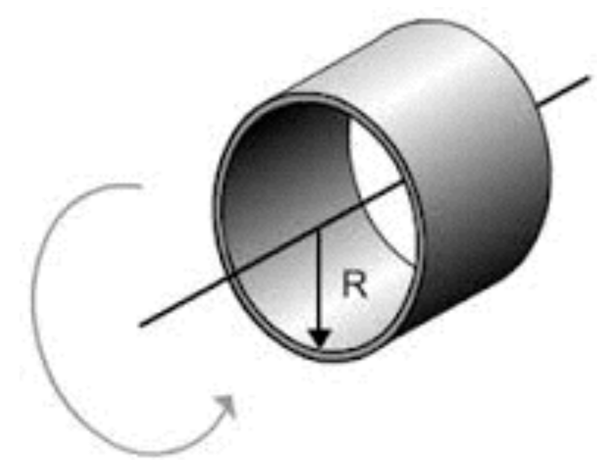
Twisted boundary condition is equivalent to vector potential $A = (\theta/L)\mathbf{e}_x$
or a slow rotation of the system

The kinetic energy becomes $(-i\nabla - A\mathbf{e}_x)^2$

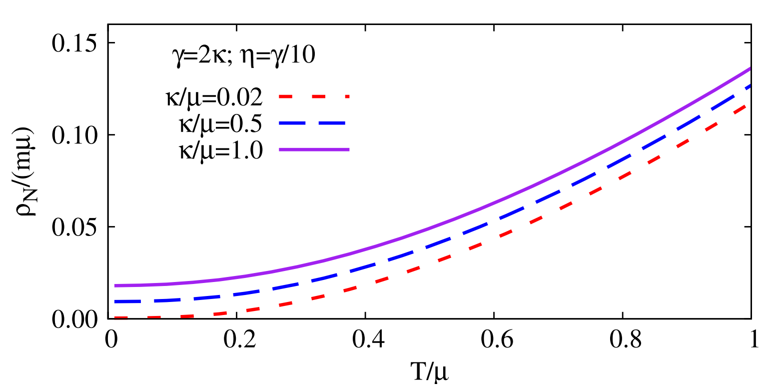
For vanishing rotation speed, the superfluid part cannot move because of phase quantisation (Hess-Fairbank effect)

We compute the current of a rotating cylindrical shell and define the normal fraction as $f_n = \frac{\langle j_x \rangle}{\langle n \rangle A}$

A. J. Leggett, Rev. Mod. Phys. 71, 318 (1999).



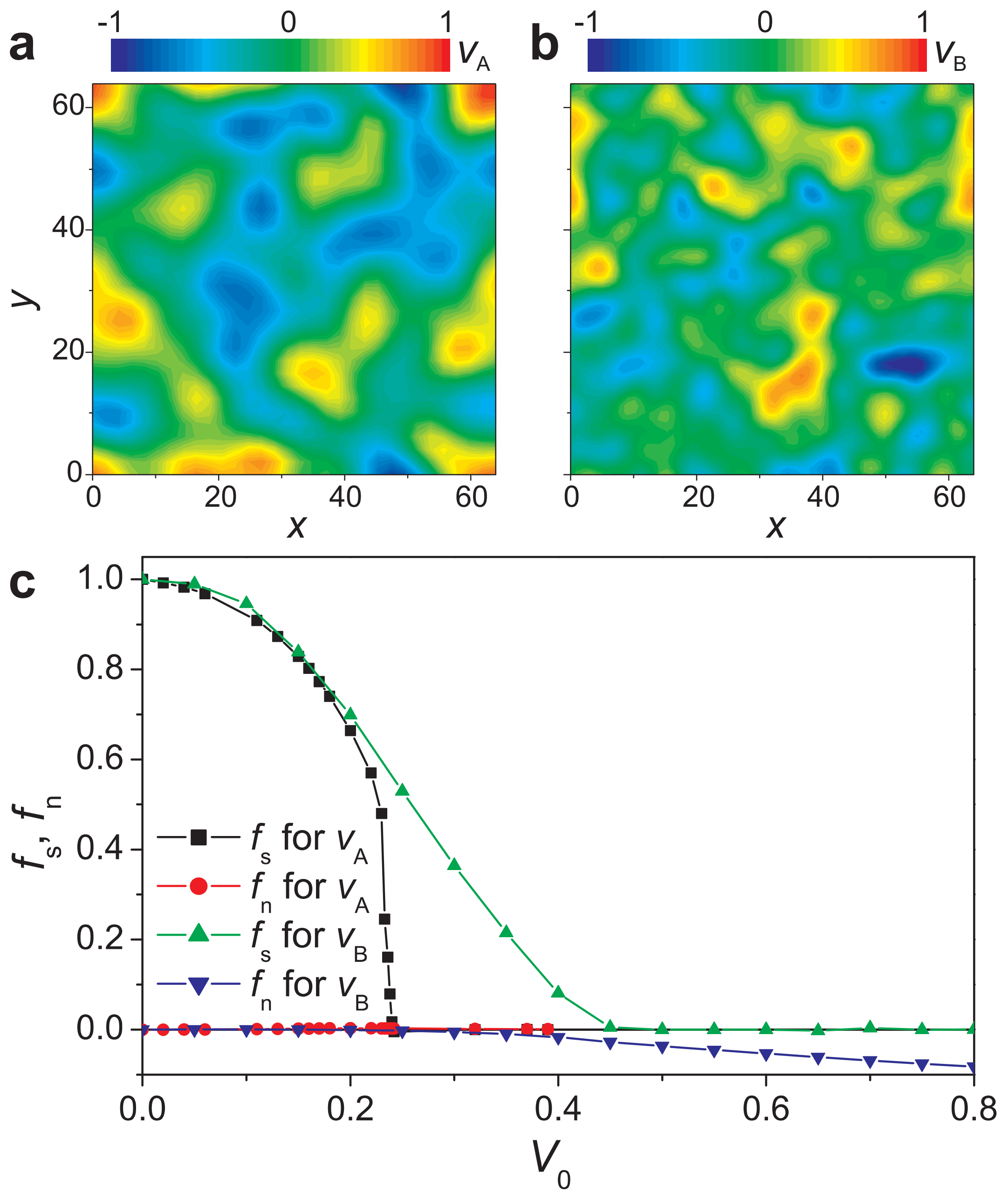
cf. Jonathan Keeling's work



J. Keeling, Phys. Rev. Lett. 107, 080402 (2011).

Numerical results

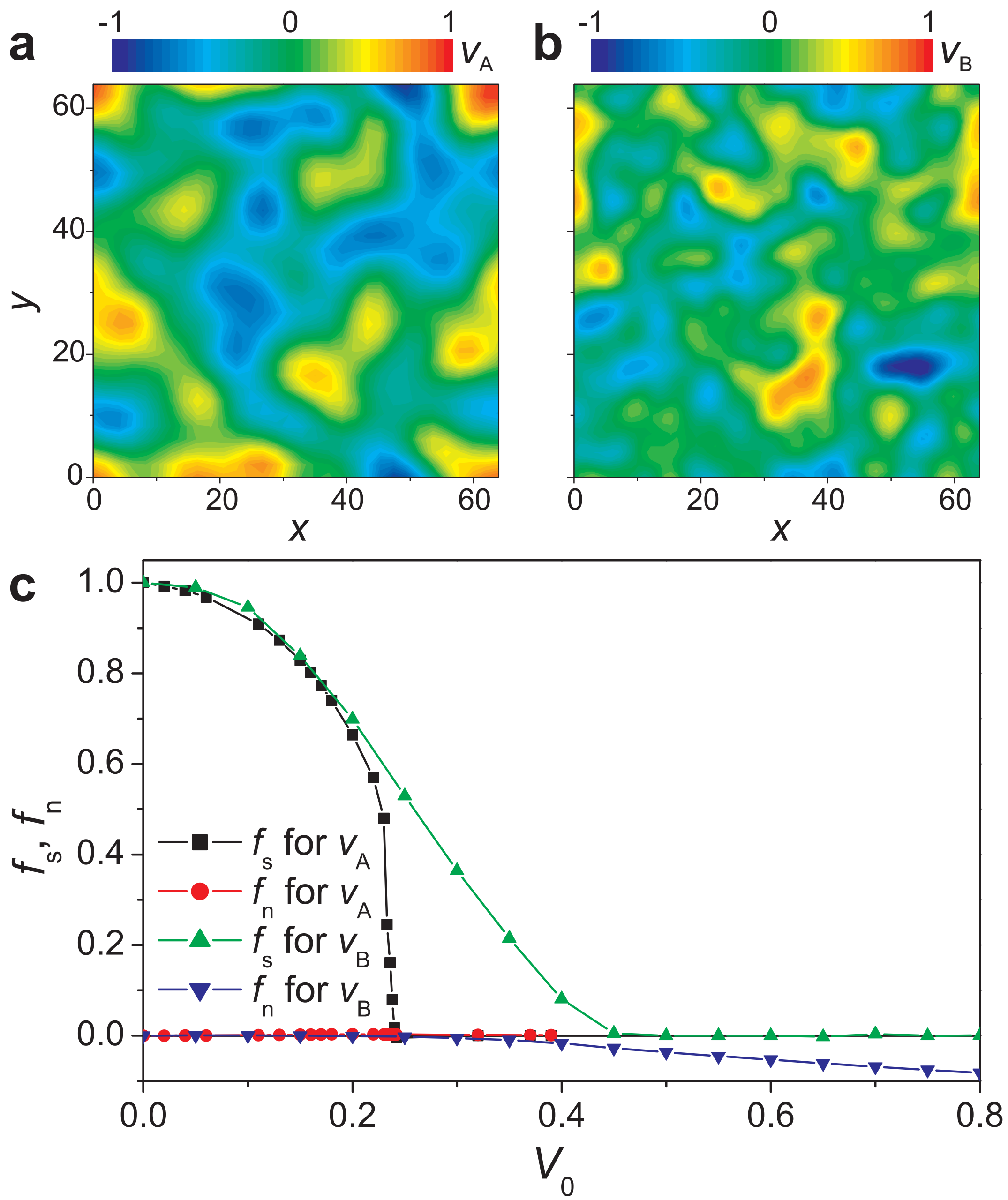
Disorder



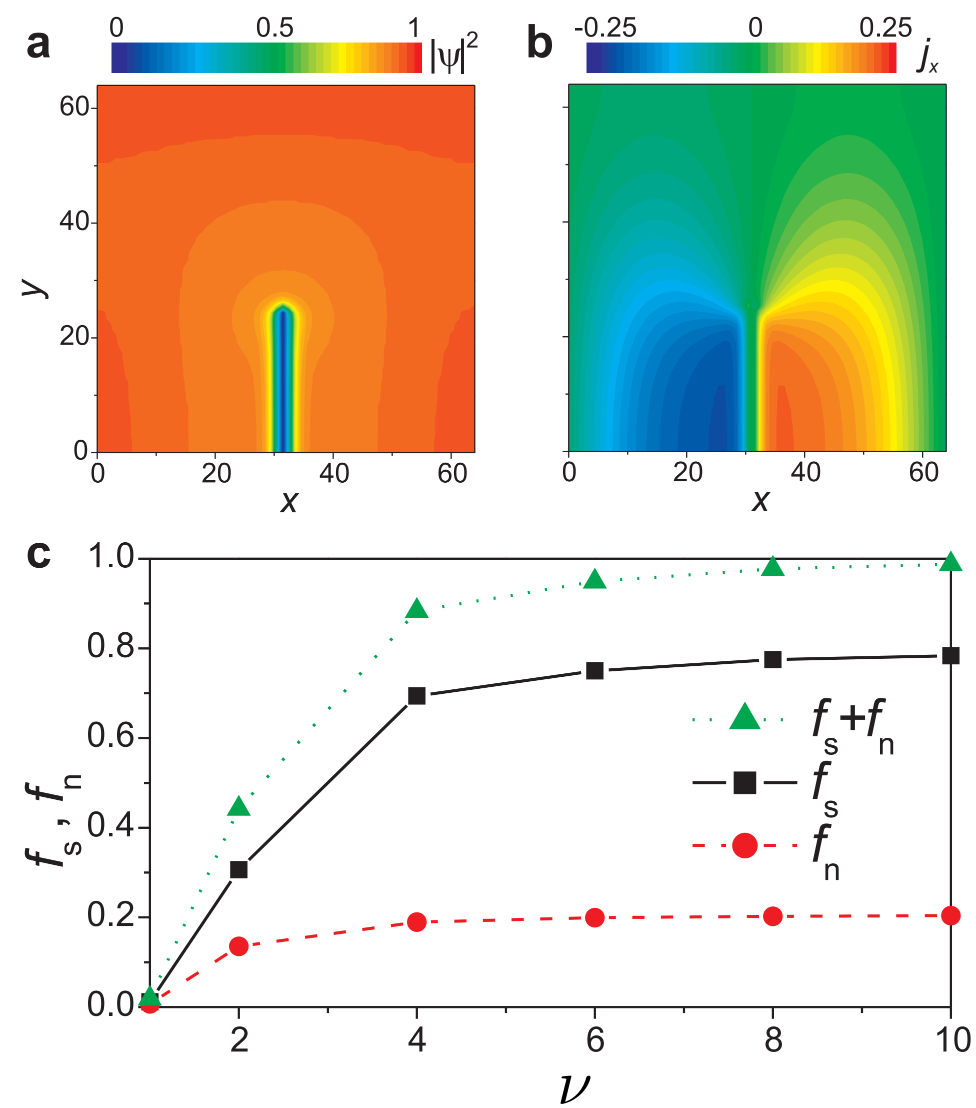
cf. A. Janot, T. Hyart, P. R. Eastham, and B. Rosenow, Phys. Rev. Lett. 111, 230403 (2013).

Numerical results

Disorder



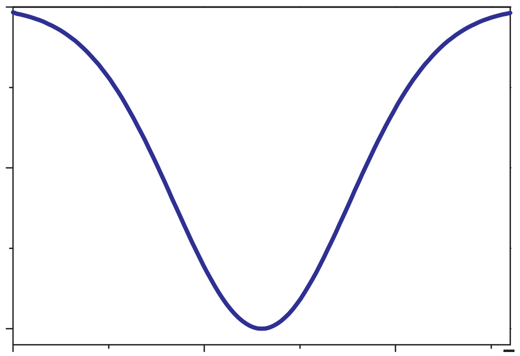
Regular cut

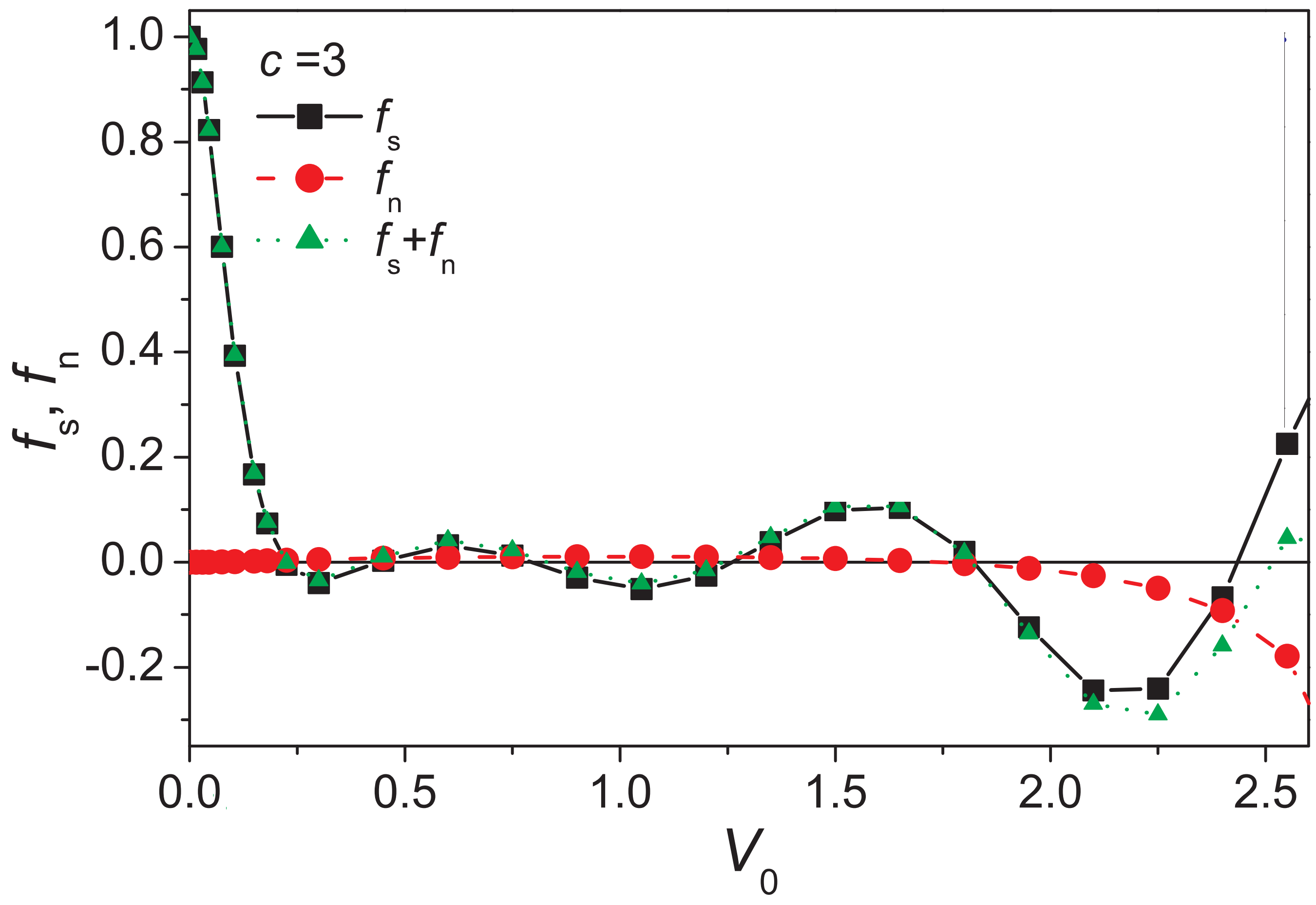


toward equilibrium →

cf. A. Janot, T. Hyart, P. R. Eastham, and B. Rosenow, Phys. Rev. Lett. 111, 230403 (2013).

Potential dip





Nonequilibrium strongly affects the reaction of a condensate to (weak) gauge fields.

Our interpretation: due to the currents in the stationary state without gauge field

Conclusions

- Nonequilibrium condensation invites us to revisit the phyiscs of BEC/superfluids

- Coherence properties determined by KPZ nonequilibrium physics

- Response to Gauge field very different from equilibrium GPE (→ implications for BKT?)

Acknowledgements

Mathias Van Regemortel
Selma Koghee
Dries Sels
Maarten Baeten
Vladimir Gladilin
Kai Ji
Onur Umucalilar
Daniele De Bernardis

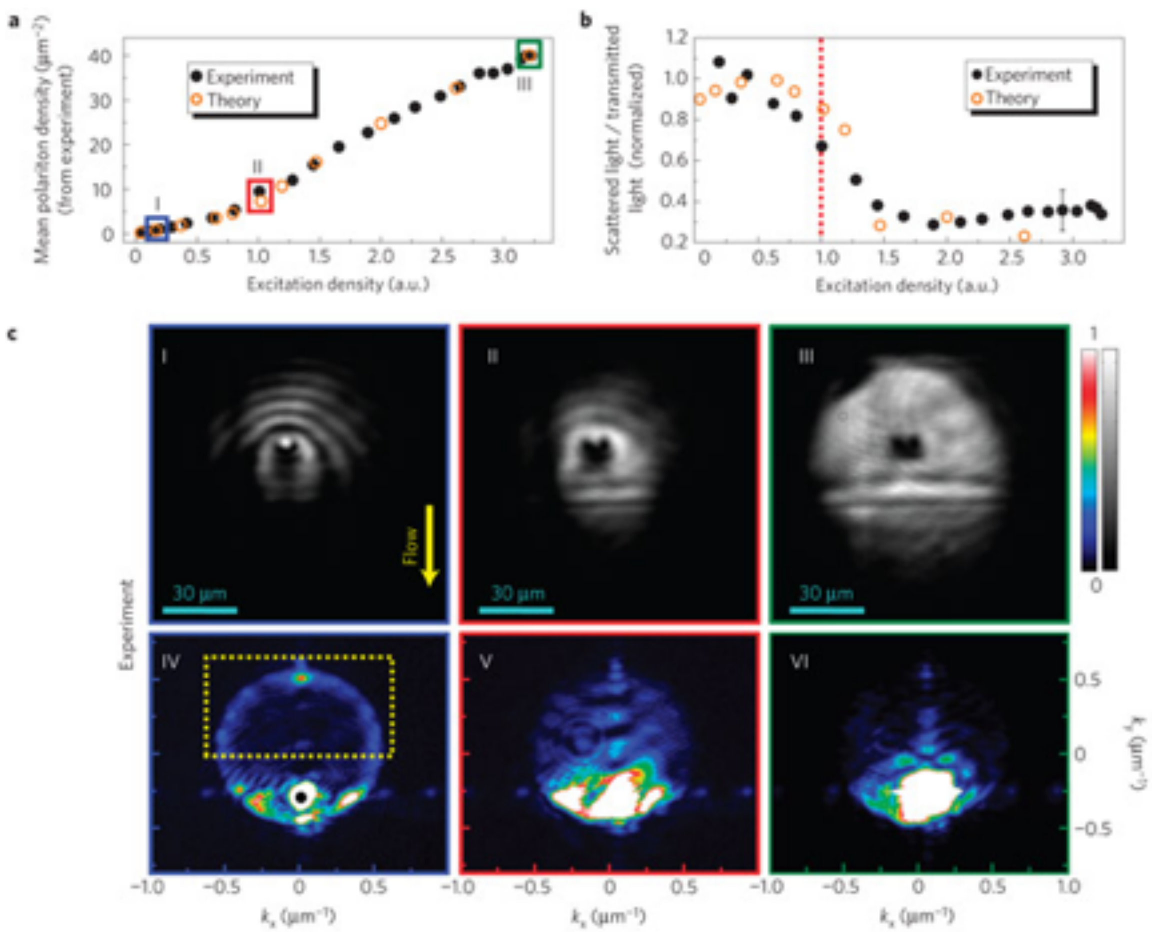
Iacopo Carusotto

Vincenzo Savona

Benoit Deveaud & co

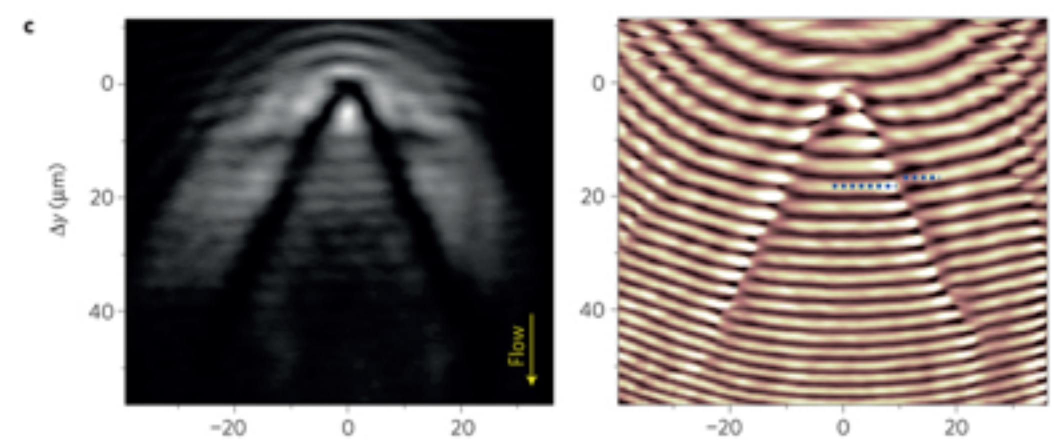
Resonant excitation experiments

superfluidity:
scattering on defect suppressed



Amo et al. Nat. Phys. 2009

quantum hydrodynamics:
soliton emission in wake of defect

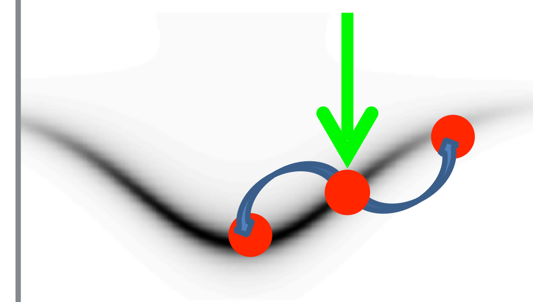


no pump at soliton location
(phase freedom)

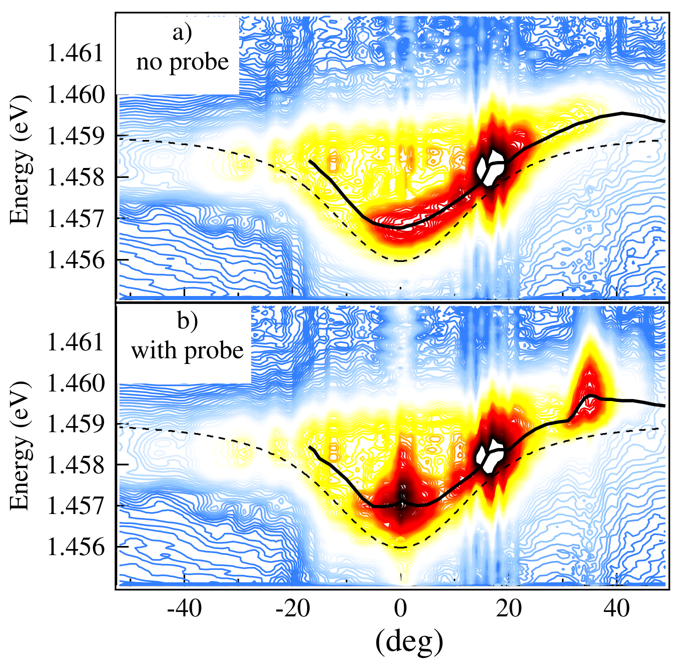
Amo et al. Science 2011

Parametric oscillation experiments

pumping laser

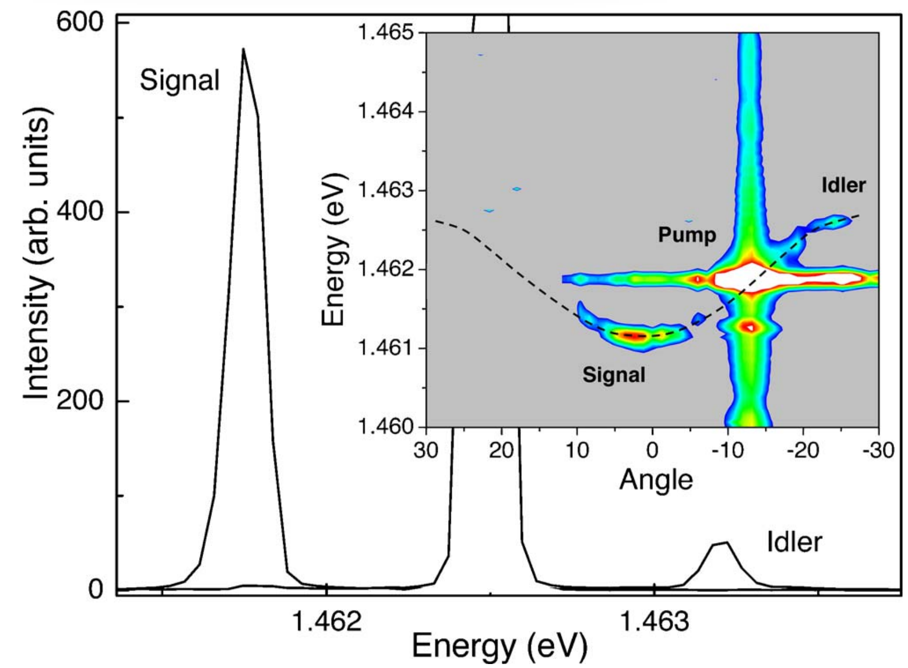


Parametric amplification



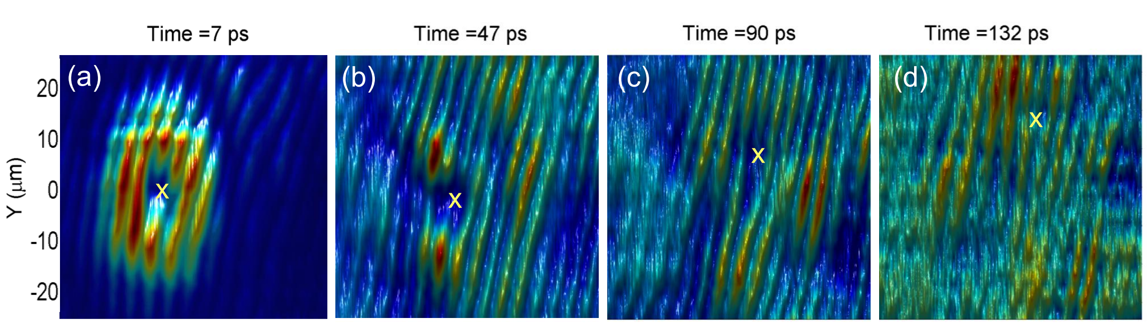
Savvidis et al. PRL 2000

Parametric oscillation



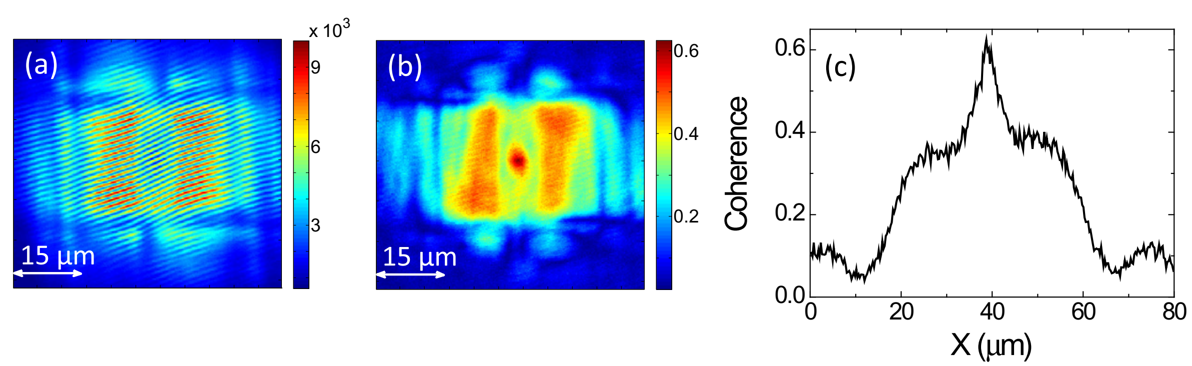
Sanvitto et al. PRB 2006

Triggered vortices



D. Sanvitto et al. Nat. Pays. 2011

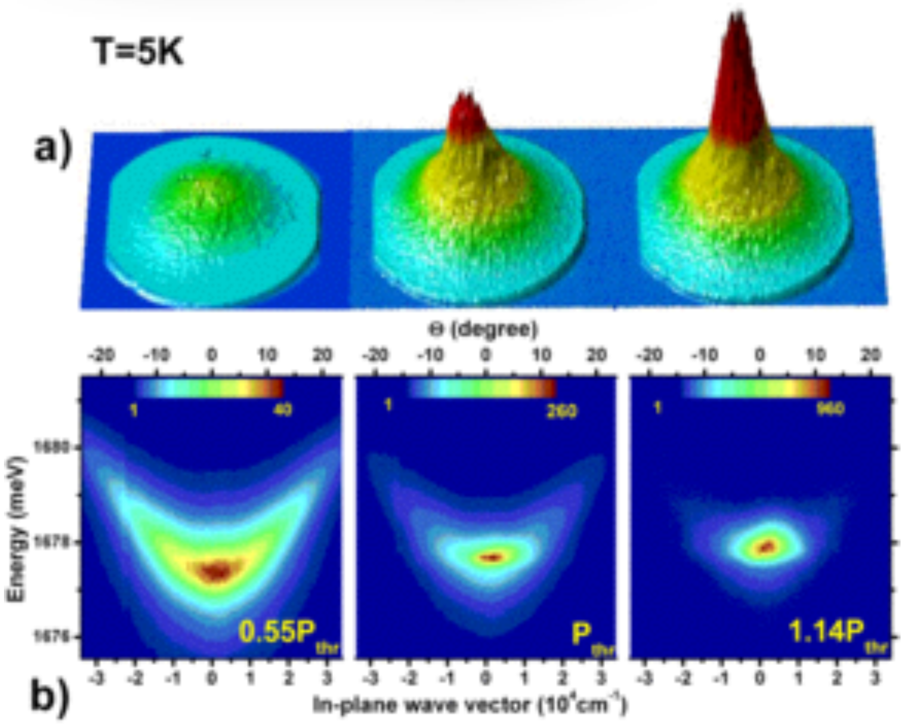
Coherence measurements



R. Spano et al., NJP 2012

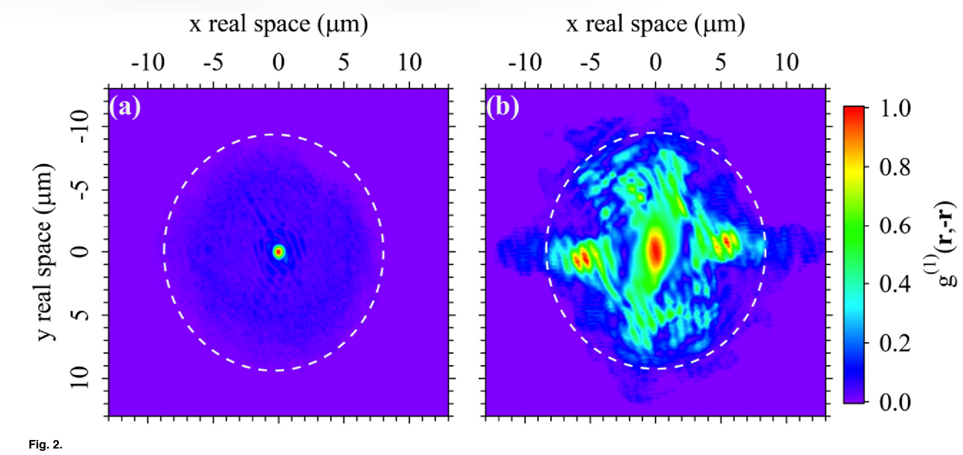
Nonresonant pumping experiments

momentum space

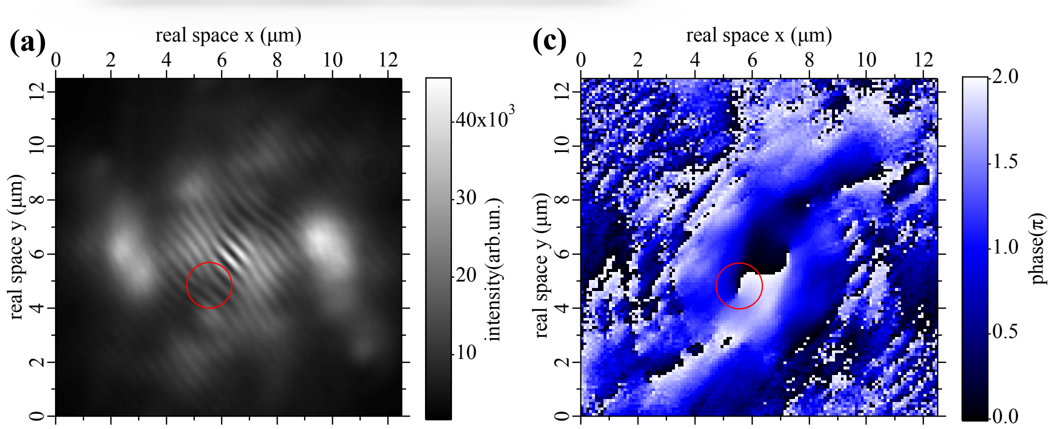


Kasprzak et al. Nature 2006

real space coherence $\langle \psi^\dagger(x)\psi(-x)\rangle$

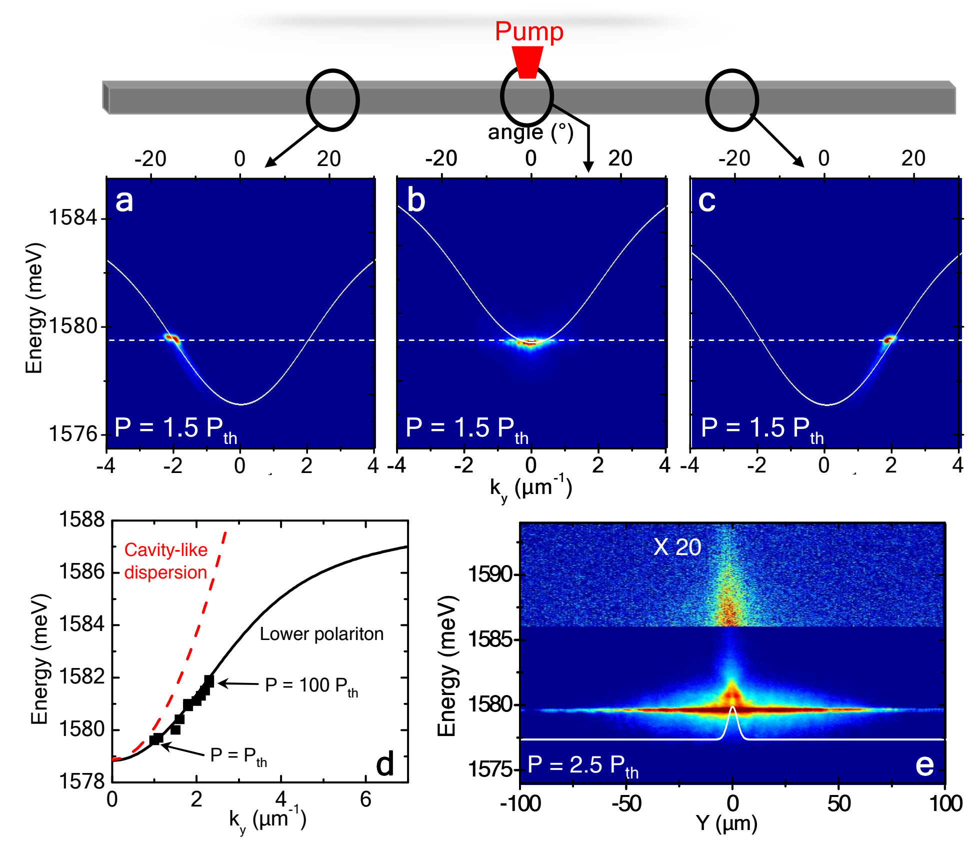


quantised vortices



Lagoudakis et al. Nature Physics 2008

Volcano effect



E. Wertz et al. Nature Physics 2010