Current Sheets, Dust, and Planets in Protoplanetary Discs

Colin M^{c}Nally, QMUL
www.colinmcnally.ca

c.mcnally@qmul.ac.uk

I: Currents sheets and energy dissipation in MRI-driven turbulence

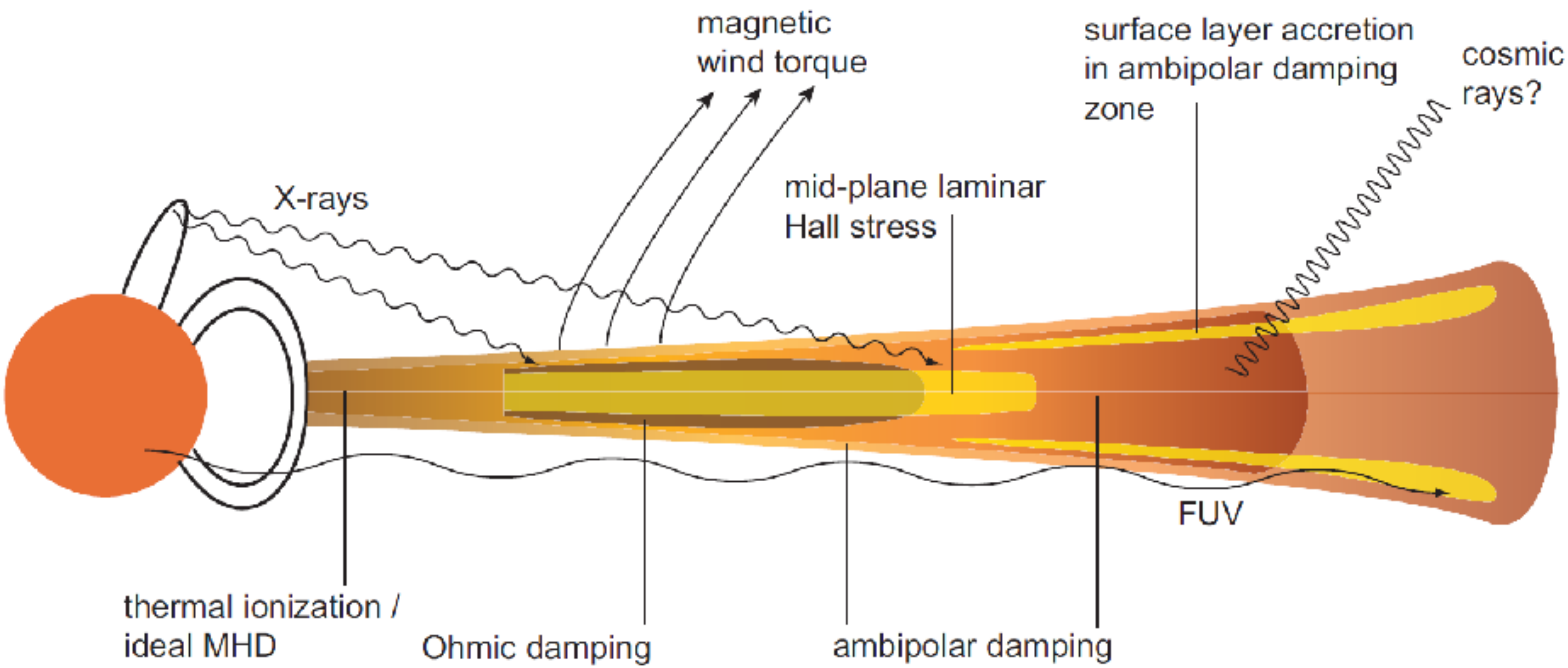
II: Dust settling and photophoresis

II: Planet migration in Hall/Ohmic dead zones

Part I: Current Sheets aren’t Shocking

CM, Alexander Hubbard (AMNH), Chao-Chin Yang (Lund), Mordecai-Mark Mac Low (AMNH)

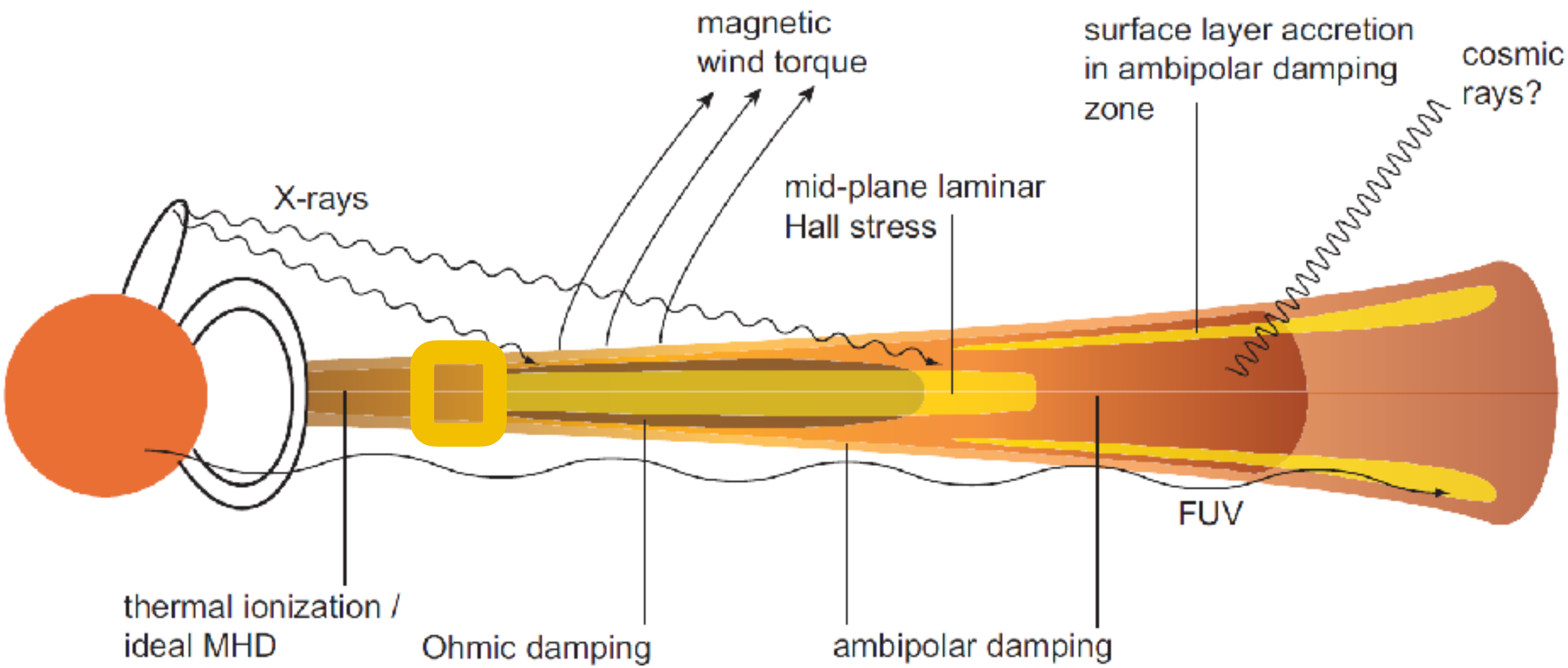
Protoplanetary Disk Structure



See KITP archive for Disks17 talks by:
G. Lesur, X.-N. Bai, J. Simon

Image: Phil Armitage, 2013

Protoplanetary Disk Structure



See KITP archive for Disks17 talks by:
G. Lesur, X.-N. Bai, J. Simon

Image: Phil Armitage, 2013

An Experiment with Current Sheets

Ask the simplest question:

Unstratified, local shearing box

Compressible Ohmic MHD w/ energy equation

Optically Thick (radiative diffusion)

Constant thermal relaxation time (large scale energy sink)

Net vertical field

Constant Ohmic resistivity (initial Elsasser number Λ = 0.5)

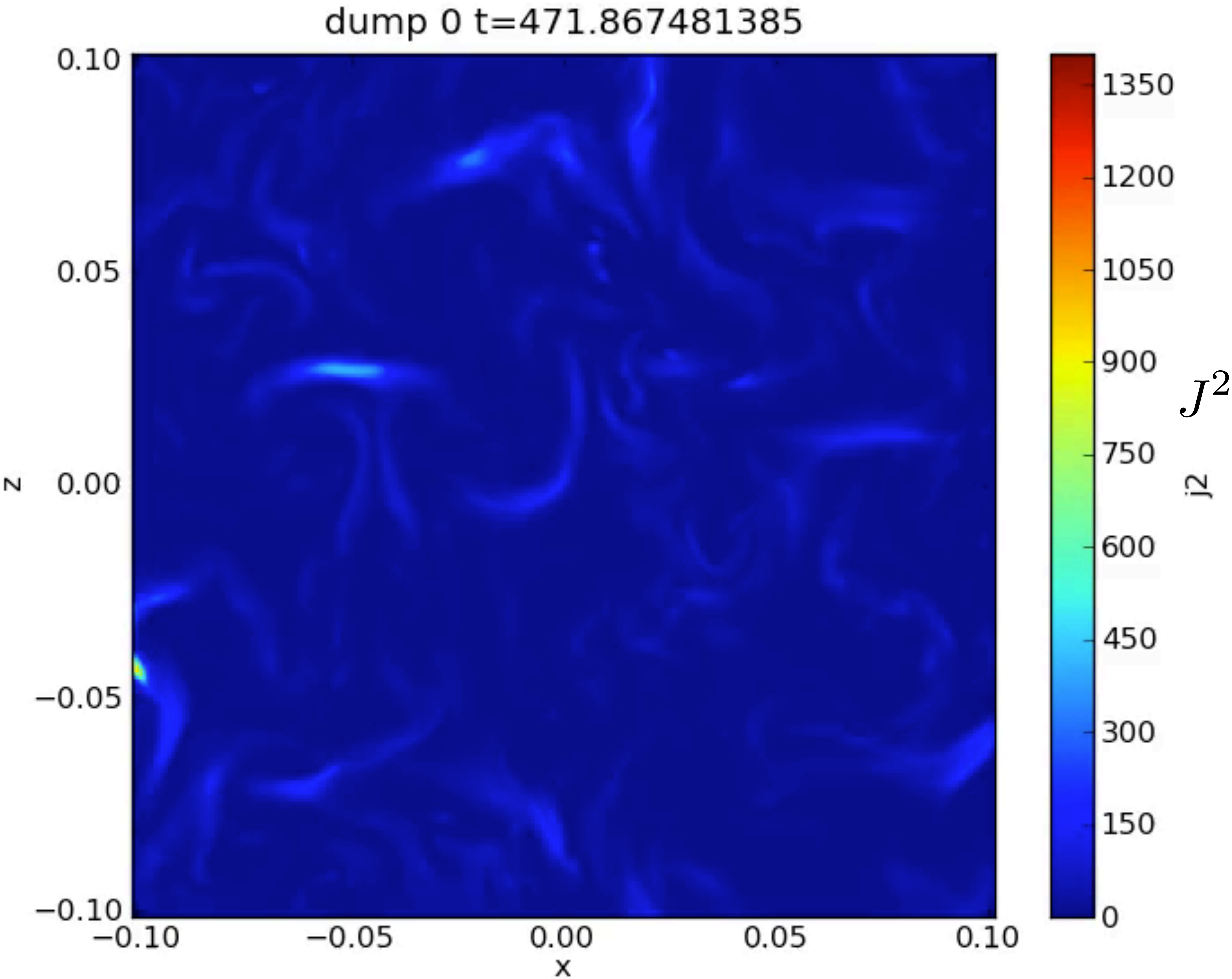
Magnetic Prandtl number Pm = Rm / Re = $\nu/\eta \ll 1$

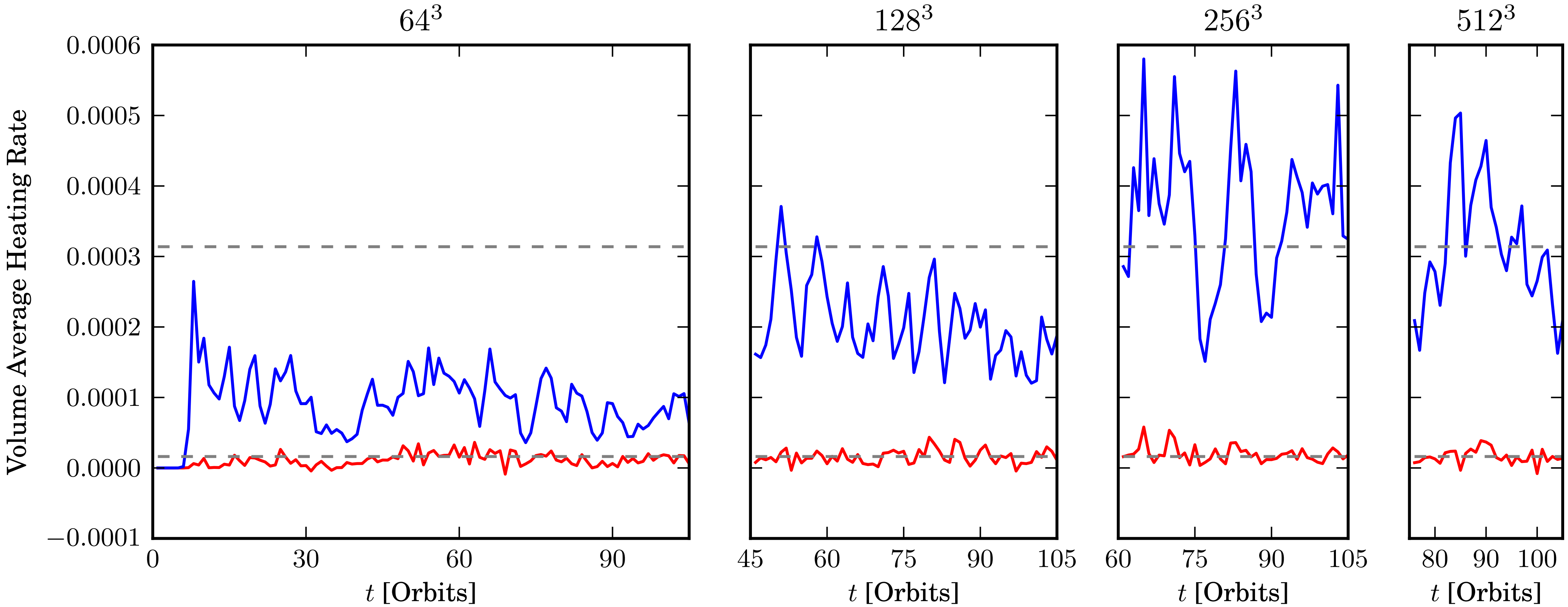
Pencil Code (FD, order 6th space, 3rd time)

Use lots of resolution:

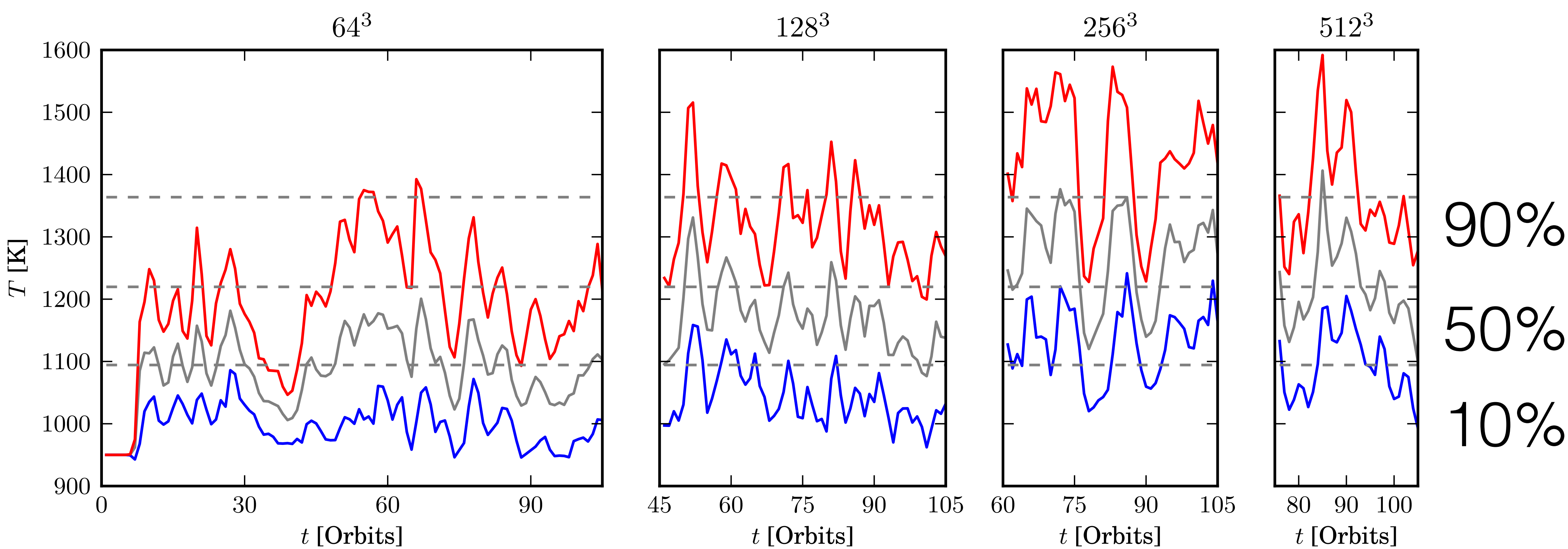
Remesh from 64^3 to 512^3

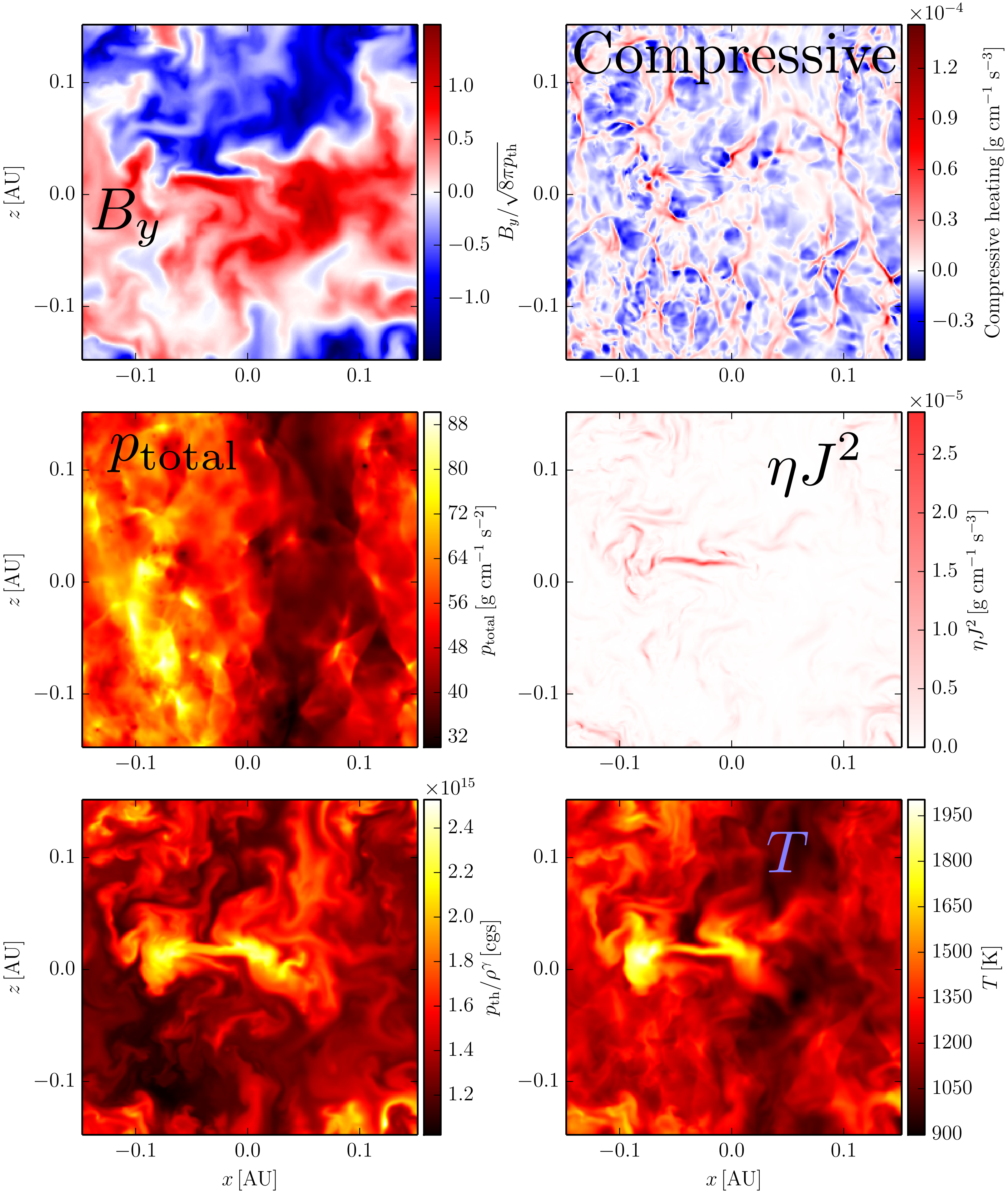
What does magnetic dissipation produce?


dump 0 t=471.867481385
0.10
0.05
0.00
−0.05
−0.10
z
−0.10
−0.05
0.00
0.05
0.10
x
1350
1200
1050
900
750
600
450
300
150
0
j2
J^2



Magnetic Heating dominates Compressive Heating



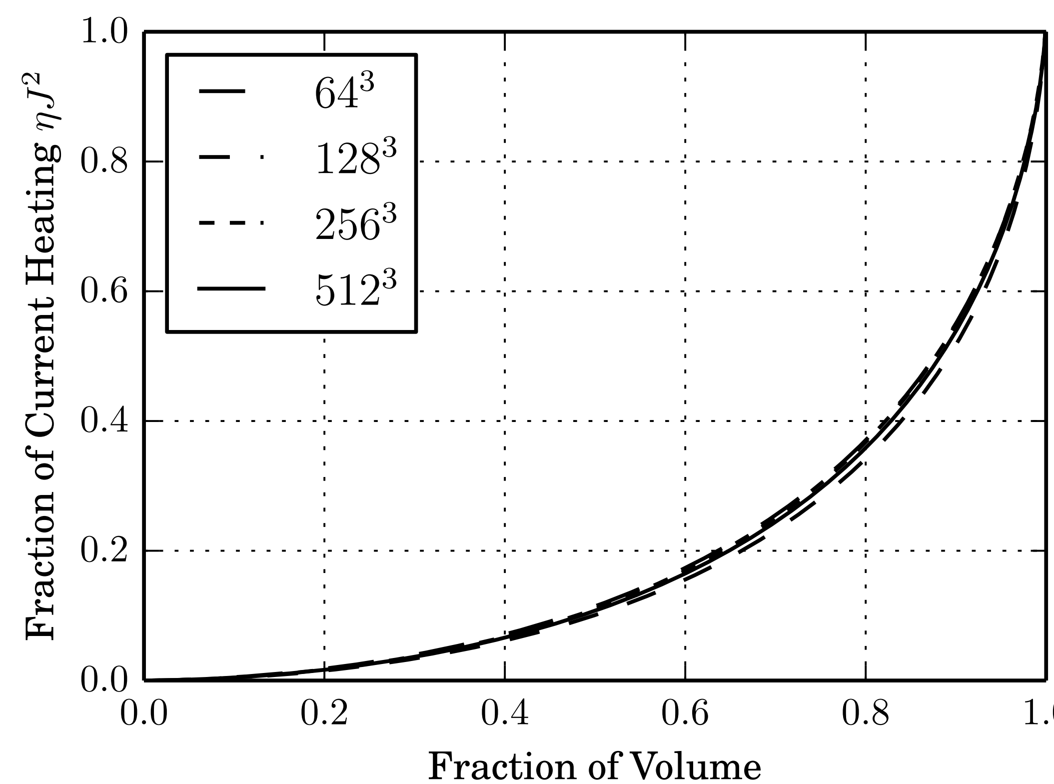


Hottest regions are current sheets

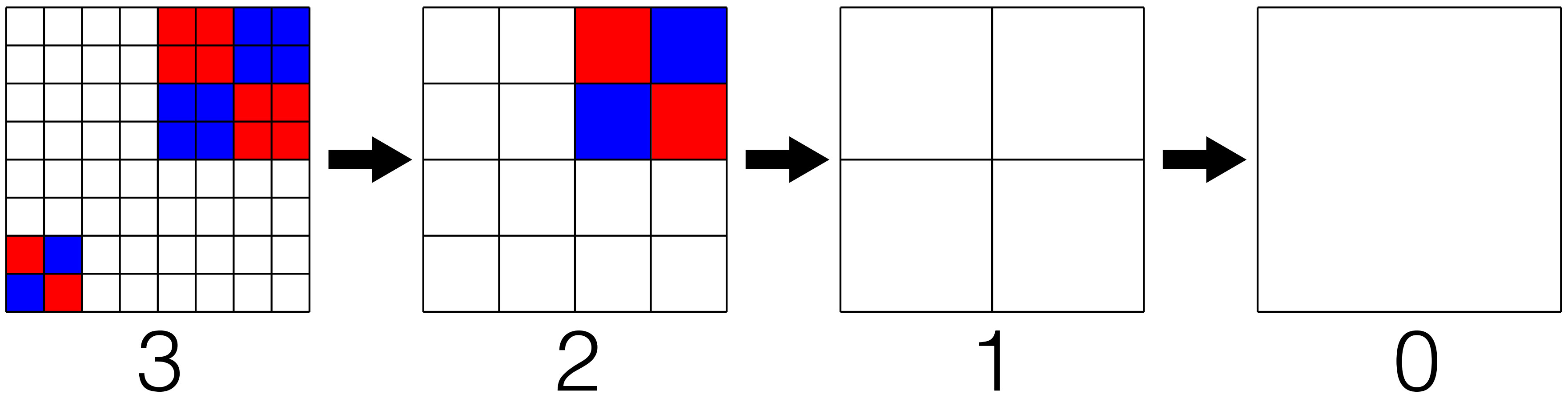
Largest current sheet occurs where dominantly azimuthal field reverses

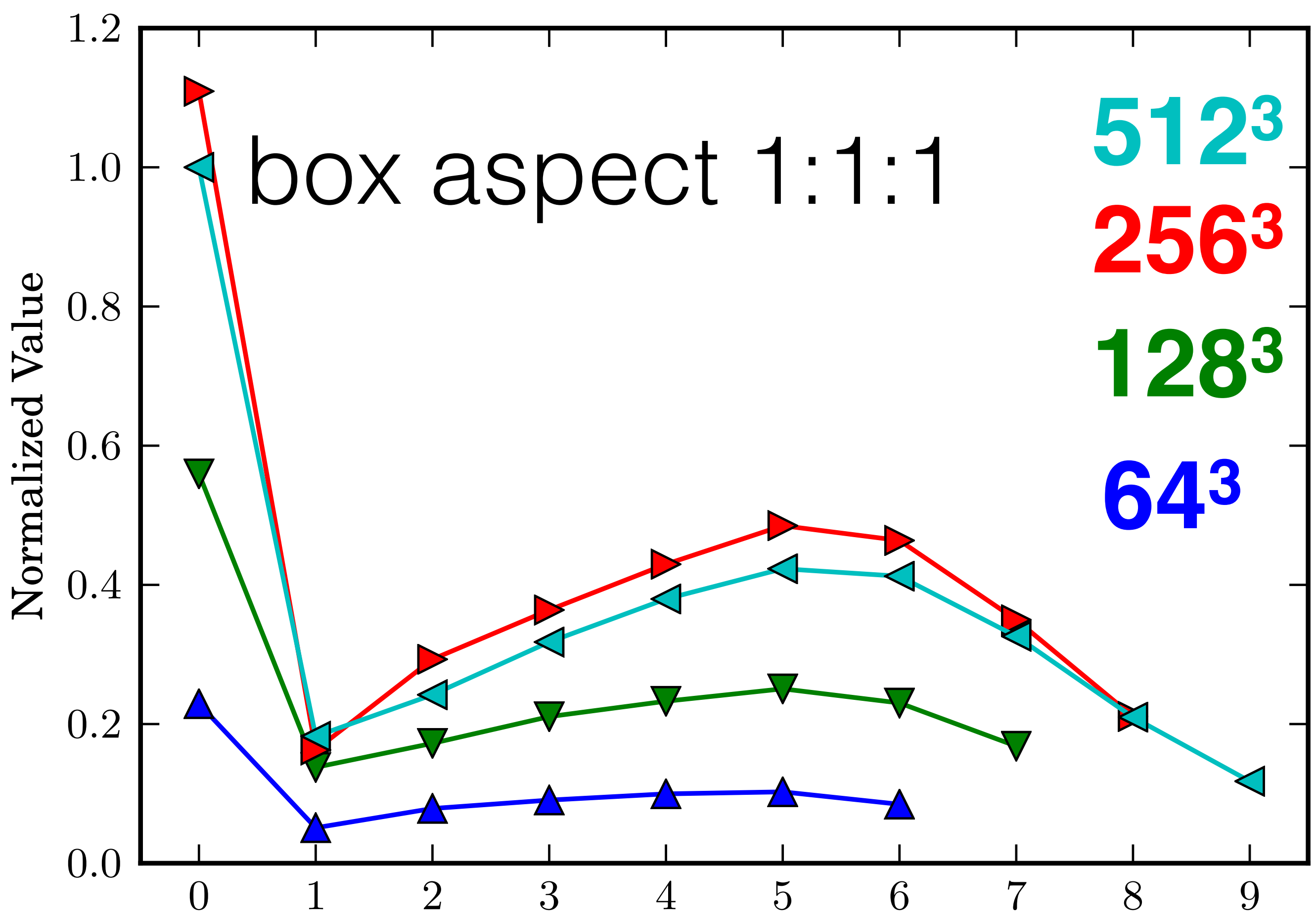
Compressive heating largely reversed by expansion

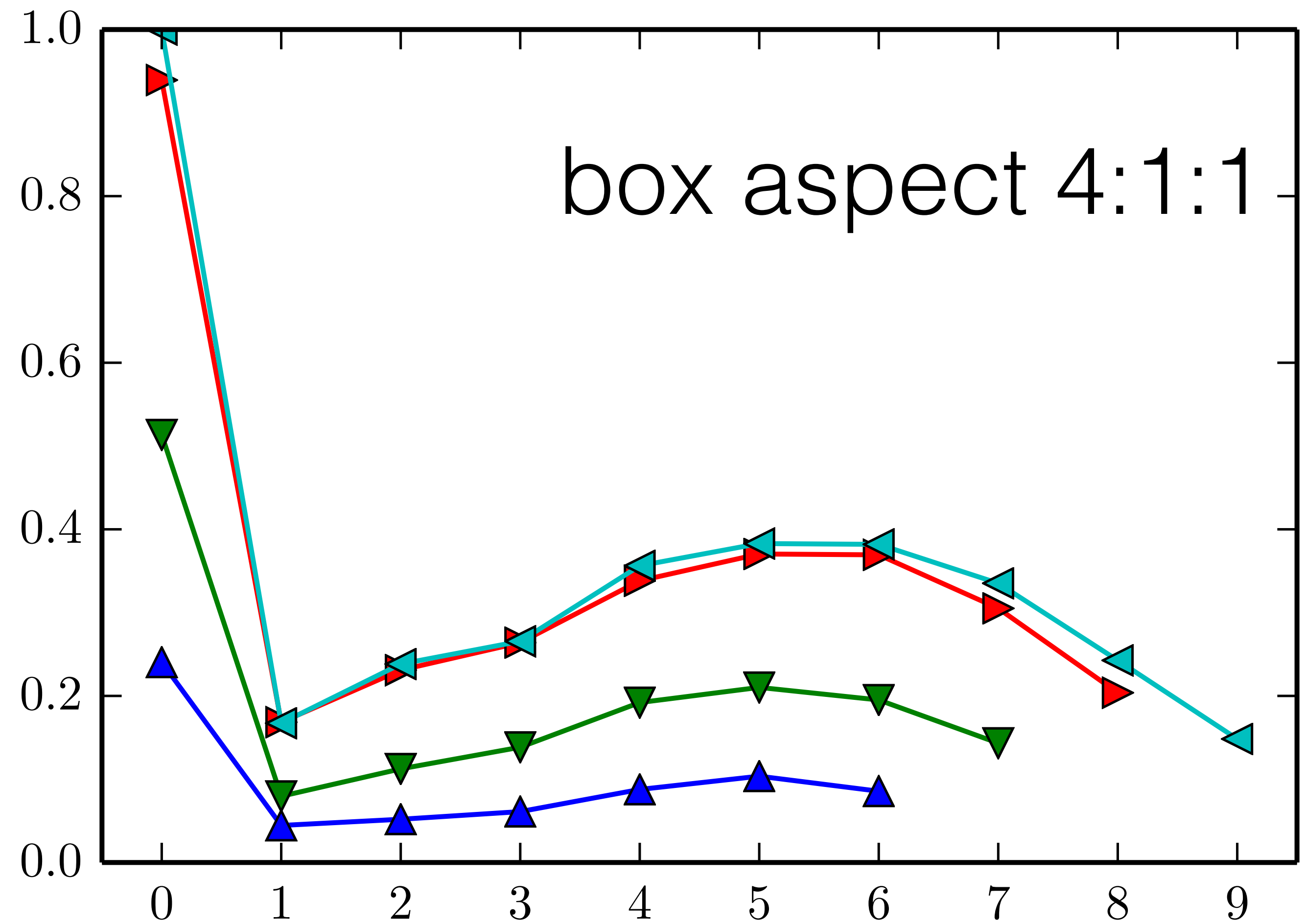
Current sheets to not stand out in total (thermal + magnetic) pressure



Multiresolution analysis of J^2 reveals convergence







Averaged Residual by Grid Level

Current sheets in Ohmic MRI

The Ohmic heating in MRI driven turbulence is very intermittent, even in the most resistive MRI-active regime

A few large current sheets dominate the generation of temperature fluctuations

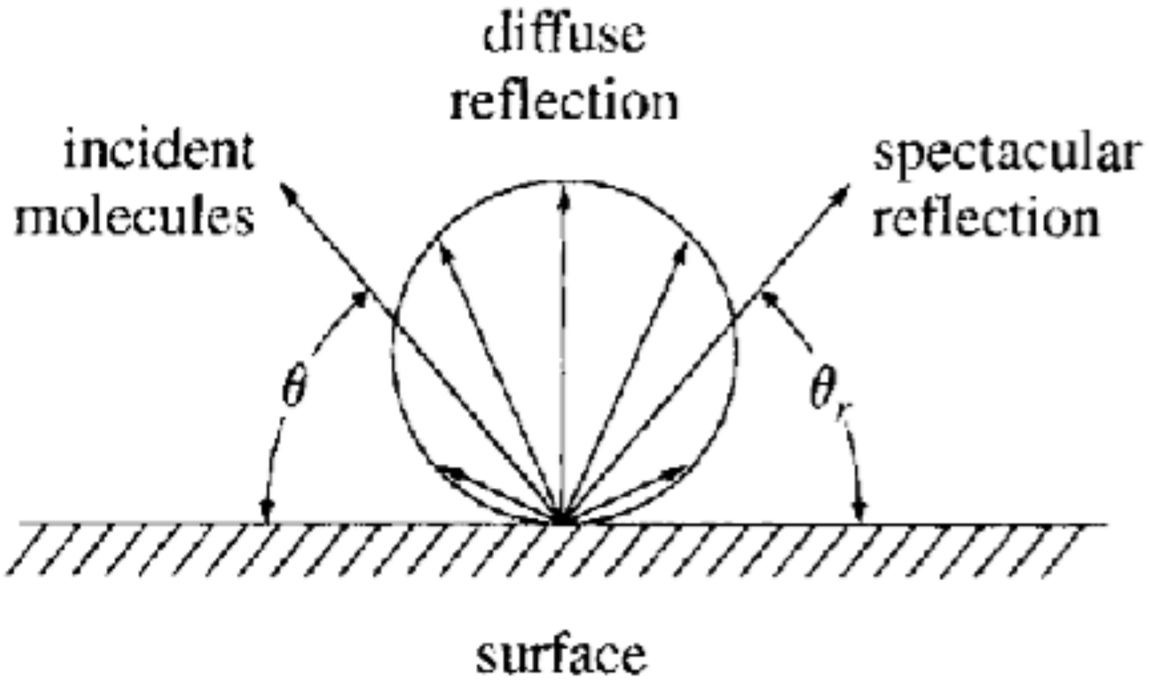
It is possible to resolve these features, and the temperature structure - required 50-100 zones/scale height

Temperature Fluctuations driven by Magnetorotational Instability in Protoplanetary Disks
C.P. McNally, A. Hubbard, C.-C. Yang, M.-M. Mac Low
2014, ApJ, 791, 1, 62

Part II: Photophoresis Is a Bit of a Drag

CM, Melissa McClure (ESO), Alexander Hubbard (AMNH)

Diffuse-Specular Reflection Model of Rarefied / Dilute Gas-Surface Interactions



Potter, J., “Rarefied Gas Dynamics”, in “Handbook of Fluid Dynamics and Fluid Machinery”, Wiley, 1996
or any other aerodynamics book on rarefied gas

Accommodation Coefficients

Energy

$$\alpha = (E_i - E_r)/(E_i - E_w)$$

E_i = energy flux incident on the wall
E_r = energy flux reemitted from wall
E_w = energy flux that would be reemitted if molecules were reemitted with a Maxwellian distribution corresponding to wall temperature.

Normal Momentum $\sigma' = (P_i - P_r)/(P_i - P_w)$

Tangential Momentum $\tau' = (\tau_i - \tau_r)/(\tau_i - \tau_w)$

Diffuse reflection with full accommodation $\alpha = \tau' = \sigma' = 1$
Specular reflection with no energy exchange $\alpha = \tau' = \sigma' = 0$

Forces on a Slowly Moving Sphere

C.-T. Wang, AIAA, 1972
Ivanov & Yanshin, Fluid Dyn. 1980
Volkov, Fluid Dyn. 2009
(among others)

Follows from integrating the molecule reflection over the surface, assuming Maxwell-Boltzmann statistics

Free-molecular flow i.e. $\mathrm{Kn} \gg 1$

Knudsen number: mean free path / flow length scale

The speed ratio matters! Here: $v_d \ll c_T$ $\quad c_T = \sqrt{2k_B T_g / \mu m_H}$

Dust velocity smaller than thermal velocity.

Epstein Drag (originally Epstein, 1924)

$$\mathbf{F}_D = -\left(\frac{8\sqrt{\pi}}{3} - \frac{4}{3}\sqrt{\pi}\tau' + \sqrt{\pi}(4+\pi)\sigma'\right)\rho_g c_T r_d^2 \mathbf{v}_d$$

Magnus Effect (lift)

$$\mathbf{F}_L = -\frac{1}{3}\pi r_d^3 \rho_g \tau' \omega \times \mathbf{v}_d$$

Drag Torque

$$\mathbf{T} = -\frac{4}{3}\sqrt{\pi}\rho_g c_T \tau' r_d^4 \omega$$

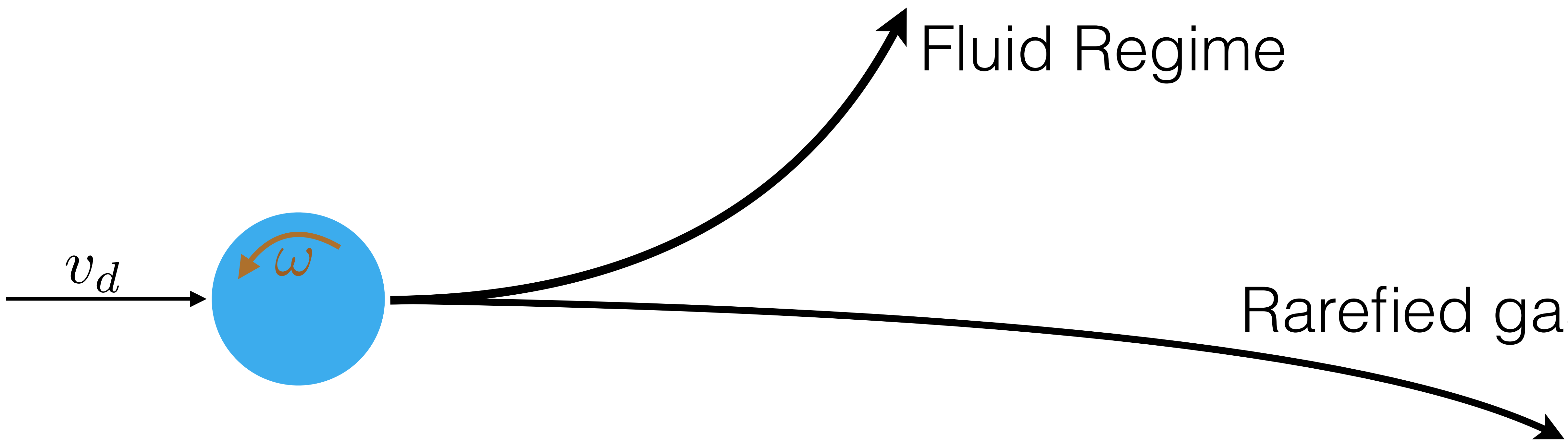
Spin-down and stopping times about the same

τ' probably should not be 0 in your calculations!

Aside - Surprises in the Magnus Effect

Fluid - Small Kn $\qquad \mathbf{F}_L = \pi r_d^3 \rho_g \omega \times \mathbf{v}_d$

Rarefied - Large Kn $\qquad \mathbf{F}_L = -\frac{1}{3} \pi r_d^3 \rho_g \tau' \omega \times \mathbf{v}_d$



Direction of lift force changes near Kn=1 !

C.-T. Wang, AIAA, 1972
Ivanov & Yanshin, Fluid Dyn. 1980
Volkov, Fluid Dyn. 2009
(among others)

(NB: Not accounted for in Forbes 2015, "Curveballs in PPDs")

Deep in a disc: Grains slow down until left in Brownian motion (thermal equilibrium)

E. Krugel, "The physics of interstellar dust", IOP, 2003

$\frac{1}{2}k_B T_g$ per degree of freedom

$$v_{\mathrm{brownian}} = \sqrt{\frac{9 k_B T_g}{4\pi\rho_d}} r_d^{-3/2} \qquad \omega_{\mathrm{brownian}} = \sqrt{\frac{45 k_B T_g}{\rho_d}} r_d^{-5/2}$$

Rotation periods: 1 mm — 10^4 s   1 micron — 10^{-3} s

disorder time ~ stopping time

Deep in a disk, does a small dust grain spin in thermal equilibrium with other dust grains, or the gas?

Order of magnitude, works out to the mass of gas collision v.s. dust collisions per unit time:

$$\Gamma \sim \frac{1}{\sqrt{\alpha_{\mathrm{ss}}\mathrm{St}}}\frac{\rho_g}{n m_d} \gg 1$$

$\alpha_{\mathrm{ss}} =$ turbulent alpha
St = "Stokes number" t_s/Ω
$\rho_g/n m_d =$ gas/dust ratio

this analysis: CM

What if our dust particle is locally hotter/colder than the gas?

Energy/Momentum accommodation means that molecules reflecting from the surface recoil faster/slower from a hotter/colder surface

$$\mathbf{F}_p = -\frac{1}{2}\oint_S p_g \left(1 + \sqrt{\frac{\tilde{T}}{T_g}}\right) d\mathbf{S}$$

$$\tilde{T} \equiv T_g + \alpha(T_s - T_g)$$

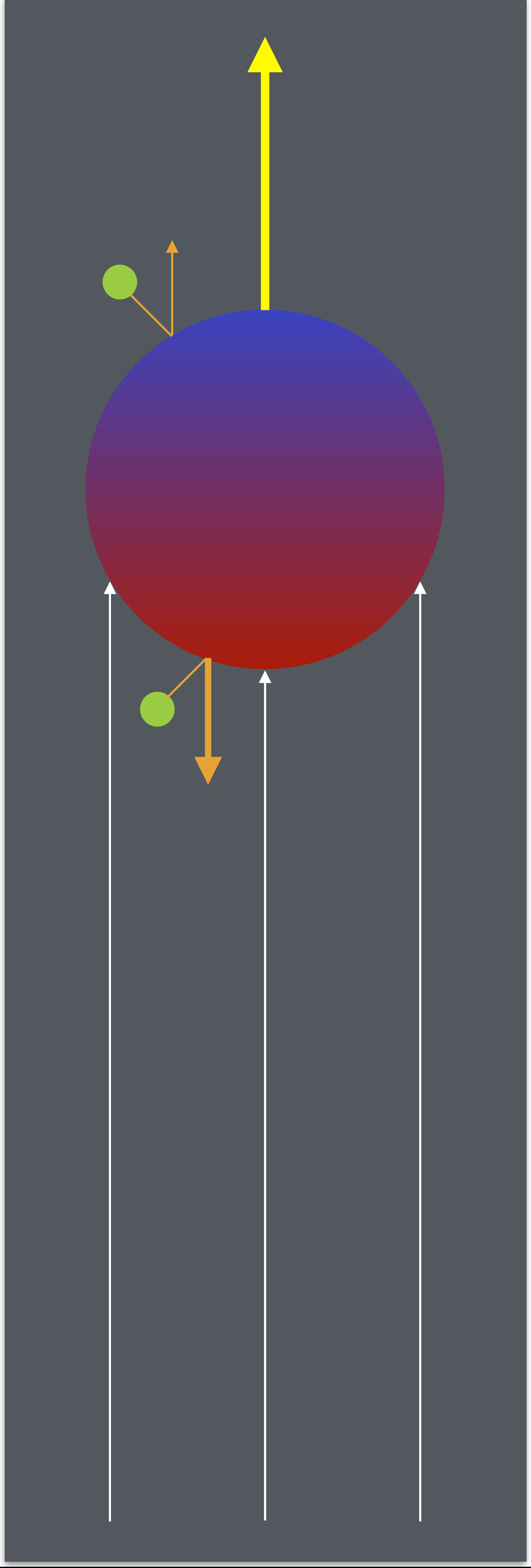
Rohatschek & Zulehner 1985
following Hidy & Brock 1970

The 'Epstein Drag' force on the particle is not zero, even when it is at rest!

This phenomena is 'photophoresis'

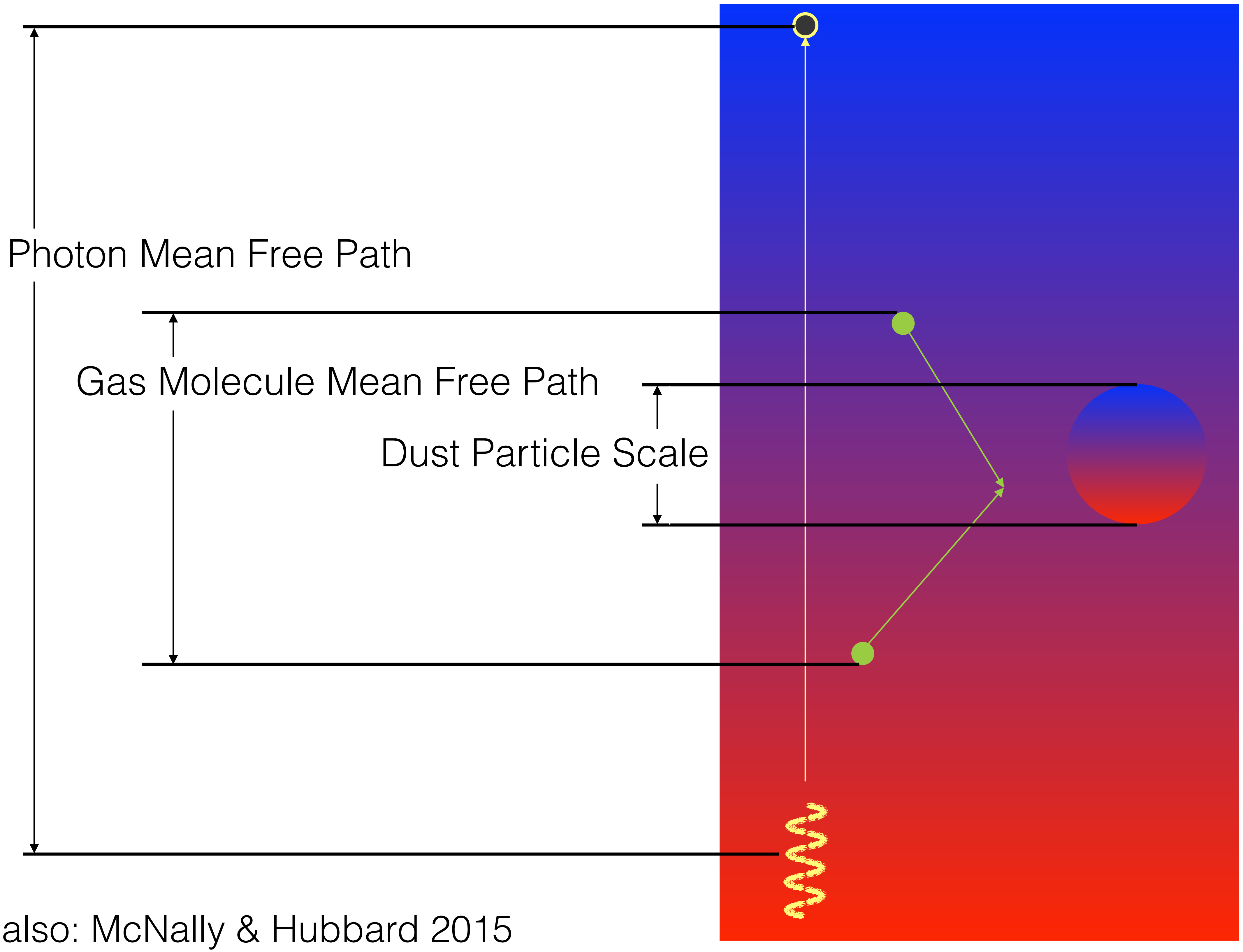
Ehrenhaft (1918)

(or, technically, ΔT_s-photophoresis in the free molecular flow regime)



Three Length Scales in a Dilute, Optically Thick Medium

Typical in a protoplanetary disc



see also: McNally & Hubbard 2015

How to Handle Photophoresis

Force in the large Knudsen number regime

$$\mathbf{F}_p = -\frac{1}{2}\oint_S p\left(1+\sqrt{\frac{\tilde{T}}{T_g}}\right)d\mathbf{S}$$

T_s surface temperature
T_g gas temperature

$$\tilde{T} \equiv T_g + \alpha(T_s - T_g)$$

Rohatscheck, H. 1995, J. Aerosol Sci., 26, 717

Solve heat transfer problem inside the particle

Laplace

$$\nabla^2 T = 0$$

B.C.

Radiation *Conduction* *Irradiation*

$$k\left.\frac{\partial T(r,\theta)}{\partial r}\right|_a + \sigma_{SB}T(a,\theta)^4 - I_c(\theta) = I_n(\theta)$$

Assume: small temperature difference $T \sim T_g$
mean temperature equal to local gas temperature

Photophoresis Force Formulas

Optically thin: many versions exist with various complications

Optically Thick - McNally & Hubbard 2015

Photophoresis Force

Temperature Gradient

Radiation From Particle

Conduction to Gas

$$\mathbf{F}_p \approx \frac{8\pi}{9}\frac{k_B}{\mu m_H}\alpha\frac{a^3}{k}\rho_g\sigma_{SB}T_g^4\Gamma\left(1+4\sigma_{SB}T_g^3\frac{a}{k}+\Upsilon T_g^{1/2}\frac{a}{k}\right)^{-1}\hat{\mathbf{e}_\mathbf{z}}$$

$$\Gamma \equiv -\frac{1}{\kappa_R\rho_g}\frac{\partial \ln T_g}{\partial z}$$

Nondimensional Temperature Gradient

a particle radius

k thermal conductivity

Two Stream RT— McNally & McClure 2017

With the form of the two-stream RT approximated intensity field:

$$I(z,\theta) = J(z) - \frac{3}{4\pi}F(z)\cos(\theta)$$

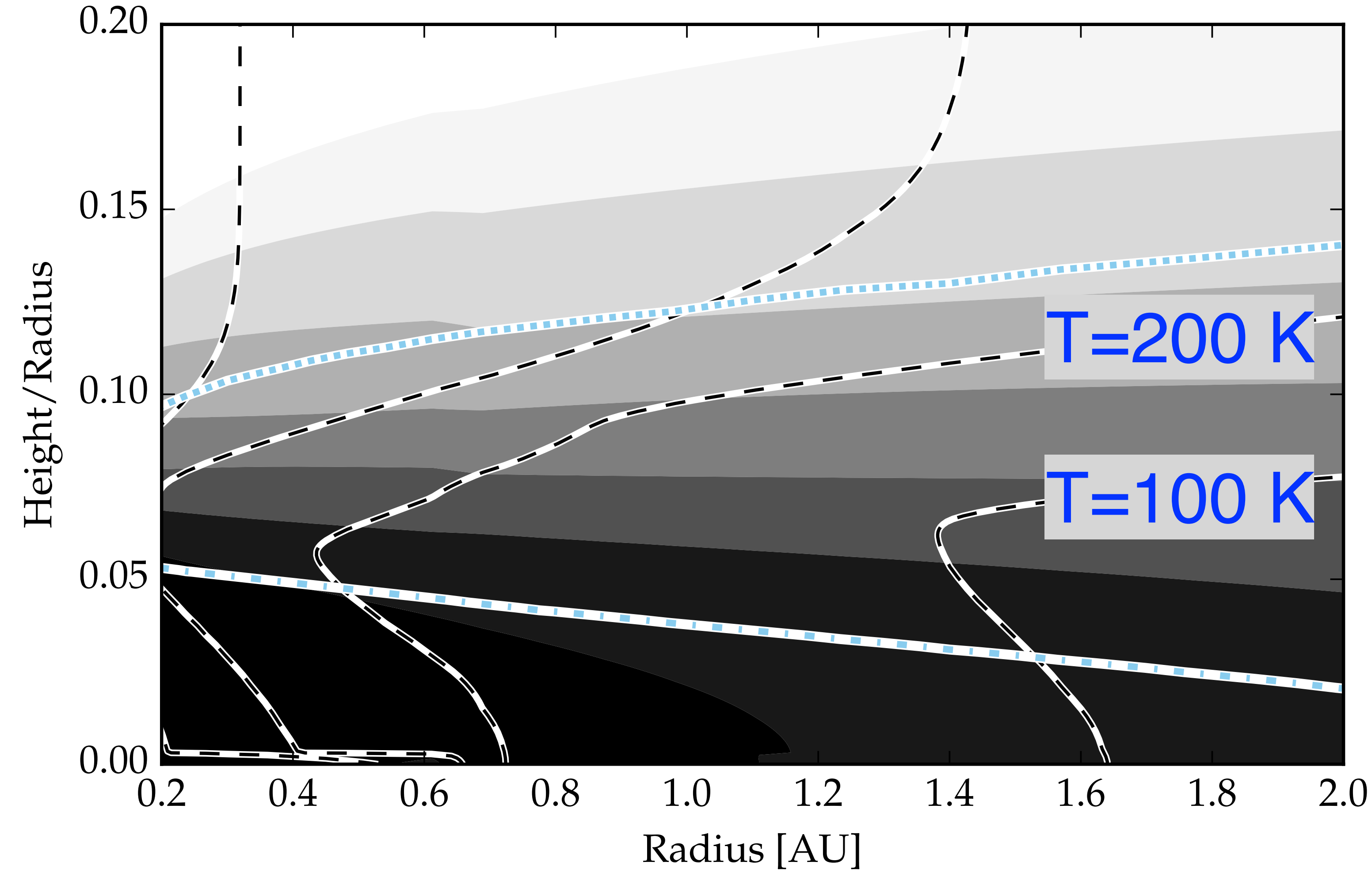
$$\mathbf{F}_p \approx \frac{\pi}{6}\alpha\frac{k_B\rho_g}{\mu m_H}\frac{a^3}{k}\boxed{F}\left(1+4\sigma_{SB}T_g^3\frac{a}{k}+\Upsilon T_g^{1/2}\frac{a}{k}\right)^{-1}\hat{\mathbf{e}_\mathbf{z}}$$

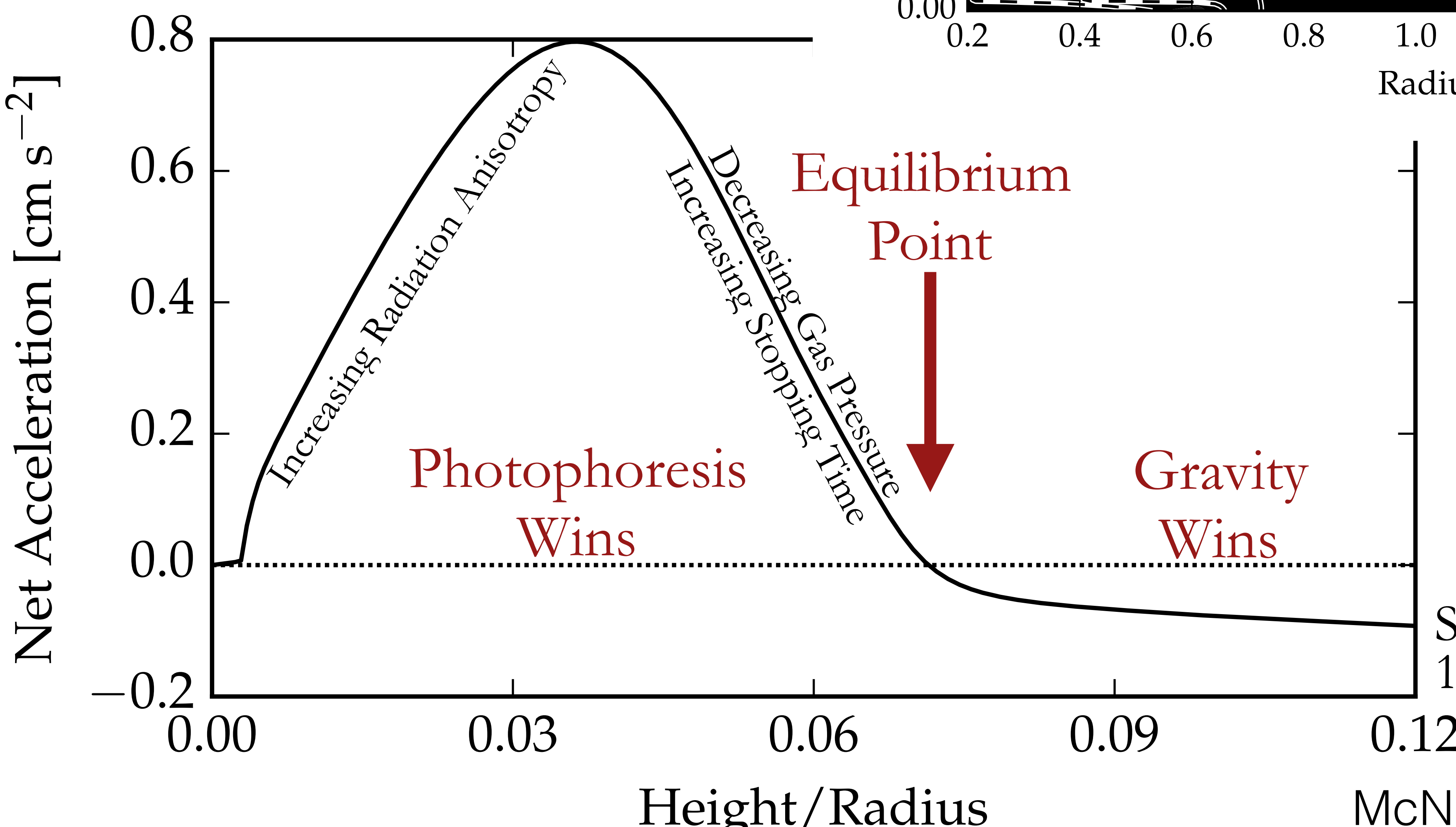
Where can Photophoresis matter?

Inner part of a T Tauri disc

Disc's own outgoing thermal radiation drives photophoresis

This model: CI Tau from McClure et al. 2013, (DISCO code)





Solid silicate sphere, radius 100 microns, at R=0.78 AU.

McNally & McClure, ApJ 2017

Ways of modeling dust settling

Continuum

Using an advection-diffusion equation

dust fluid density; gas density; dust vertical drift velocity

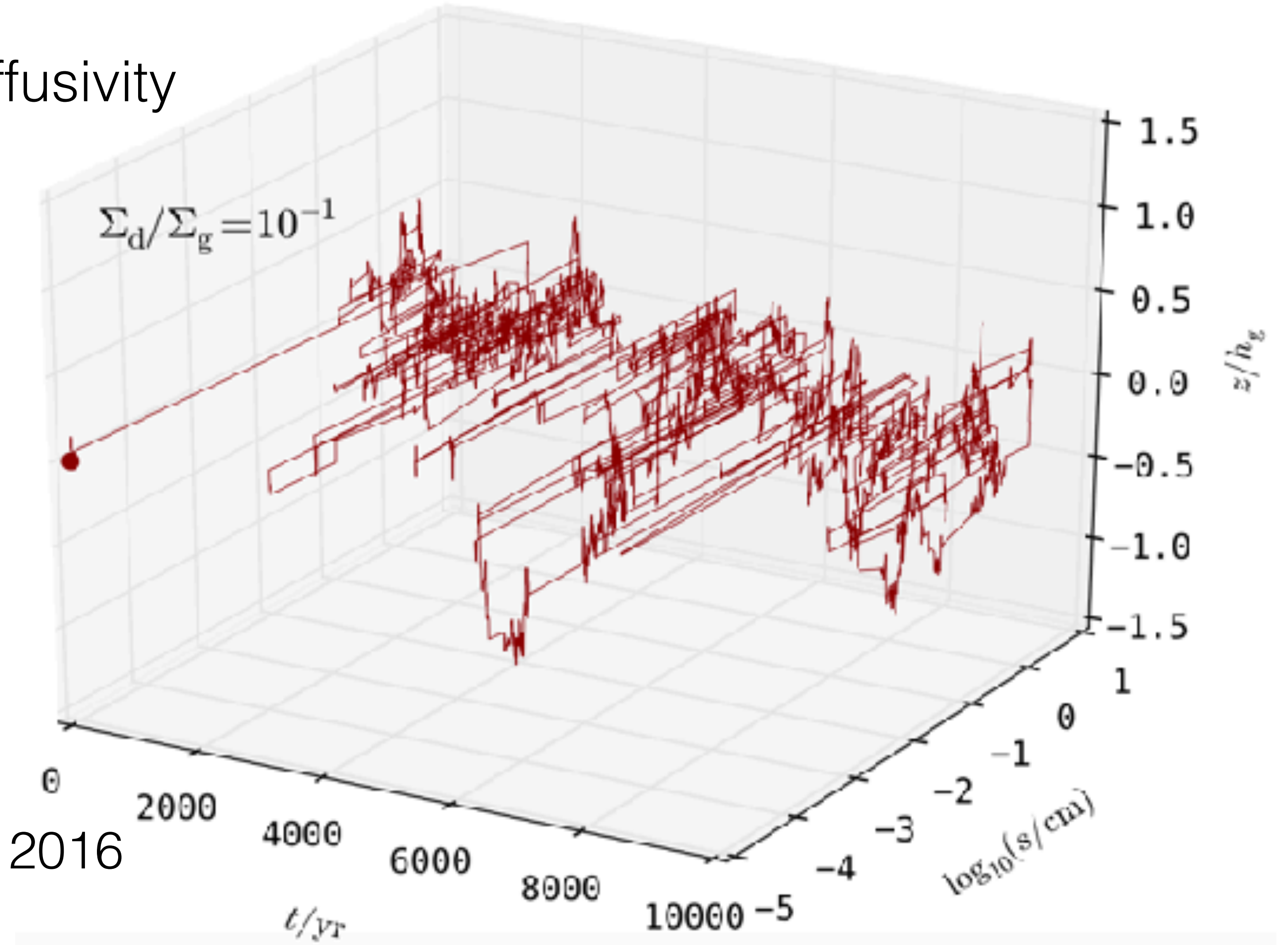
$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial z}\left[\rho_g D \frac{\partial}{\partial z}\left(\frac{n}{\rho_g}\right)\right] + \frac{\partial}{\partial z}(n v_d) = 0$$

dust vertical diffusivity

Monte-Carlo

Tracking individual particles

Easy to include complex effects
such as particle sizes and collisions



Ex: Size, position, and time
evolution from Krijt & Ciesla 2016

Dust Settling with Photophoresis

A Dullemond & Dominik 2004 style vertical settling calculation.

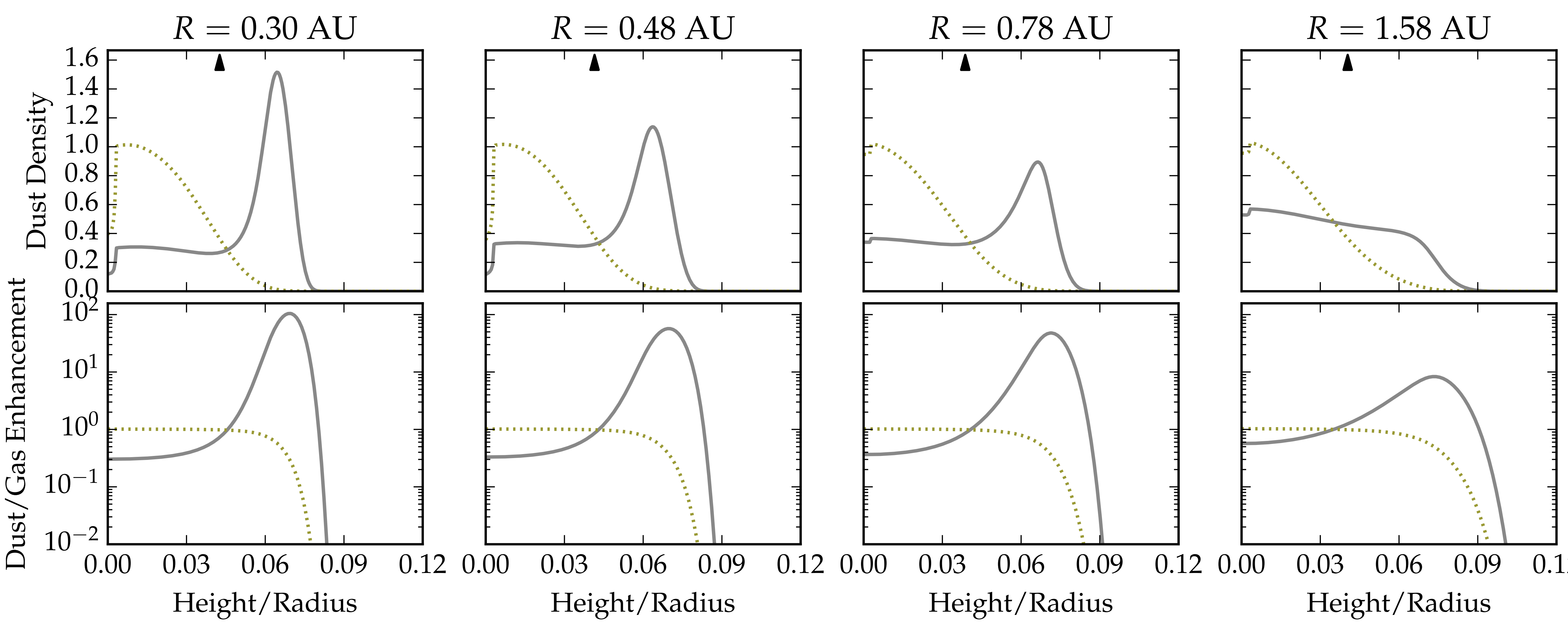
McNally & McClure, ApJ 2017

Solid silicate sphere, radius 100 microns, Schmidt number 1.5

Gravity + Turbulence

add Photophoresis

Traces midplane accretion heating

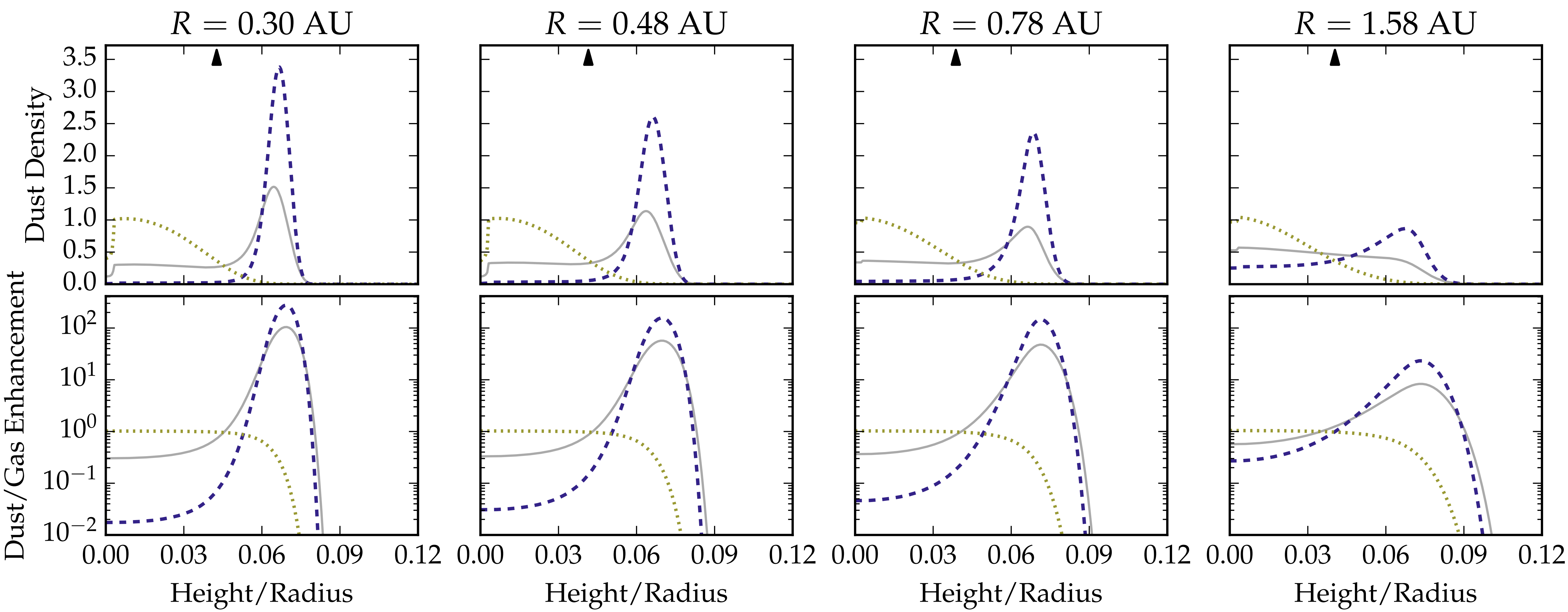


Dust/Gas mass ratio strongly enhanced in dust trap!

Schmidt number dependence

$$\frac{\partial n}{\partial z} - \frac{\partial}{\partial z}\left[\rho_g D \frac{\partial}{\partial z}\left(\frac{n}{\rho_g}\right)\right] + \frac{\partial}{\partial z}(n v_d) = 0$$

Diffusivity of dust $D = \frac{\nu}{\mathrm{Sc}}$ Viscosity (momentum diffusivity) Schmidt Number



Solid silicate sphere, radius 100 microns, Schmidt number 2.5

McNally & McClure, ApJ 2017

The Problem with the Schmidt Number

$\mathrm{Sc} \equiv \frac{\nu}{D}$ depends hugely on the nature of the non-ideal MHD and magnetic field geometry

Some measurements in MRI driven flows:

Carballido et al. 2005	11 (radial)
Johansen & Klahr 2005	1.27 to 1.60 (vertical) 0.79 to 0.90 (radial)
Turner et al 2006	1 to 2 (vertical)
Fromang & Paploizou 2006	2.8 (vertical)
Nelson & Gressel 2010	1.6 (radial)

Zhu, Stone & Bai 2015

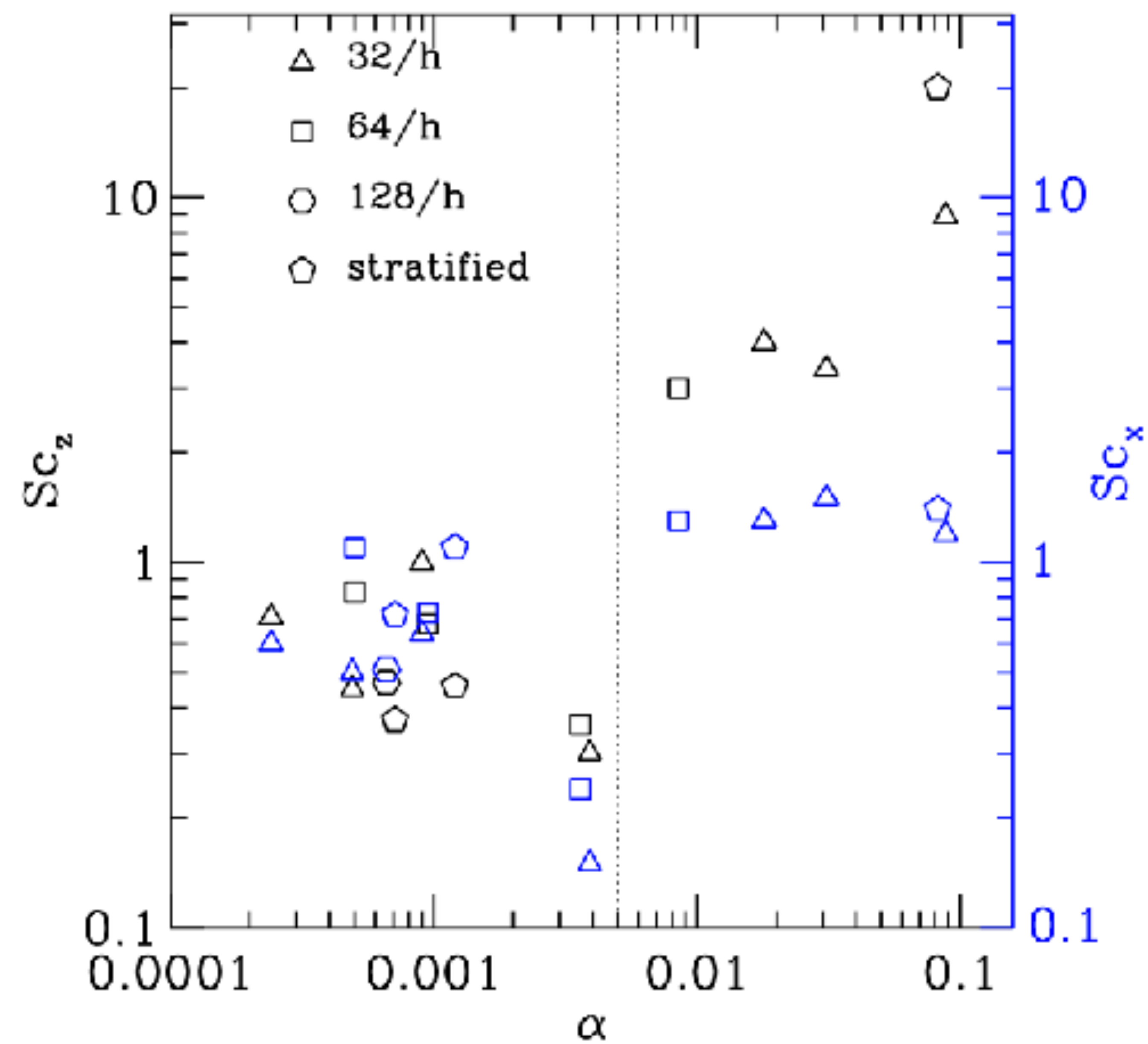


Figure 16. Sc_x (blue points) and Sc_z (black points) for all our shearing box simulations. The dotted line separates ideal MHD simulations (on the right) from MHD simulations with AD (on the left). Clearly, Sc_z is smaller than 1 for AD runs, while $\mathrm{Sc}_z \gtrsim 3$ for ideal MHD runs.

Deep in protoplanetary discs…

Aerodynamic forces in the free molecular flow regime can be derived by the integral of molecular collisions with surface

Momentum accommodation is probably not zero (due to diffuse reflections)

Grains have similar stopping and spin-down times

Dust is approximately in thermal equilibrium with the gas, not the other dust

In equilibrium with the gas, particles have brownian motion (spin and speed)

Photophoresis is just a variation on Epstein drag*, which results from differential heating and energy accommodation (*in the free molecular flow regime)

Photophoresis can form a trapped layer of dust in the inner regions of T Tauri discs (maybe others)

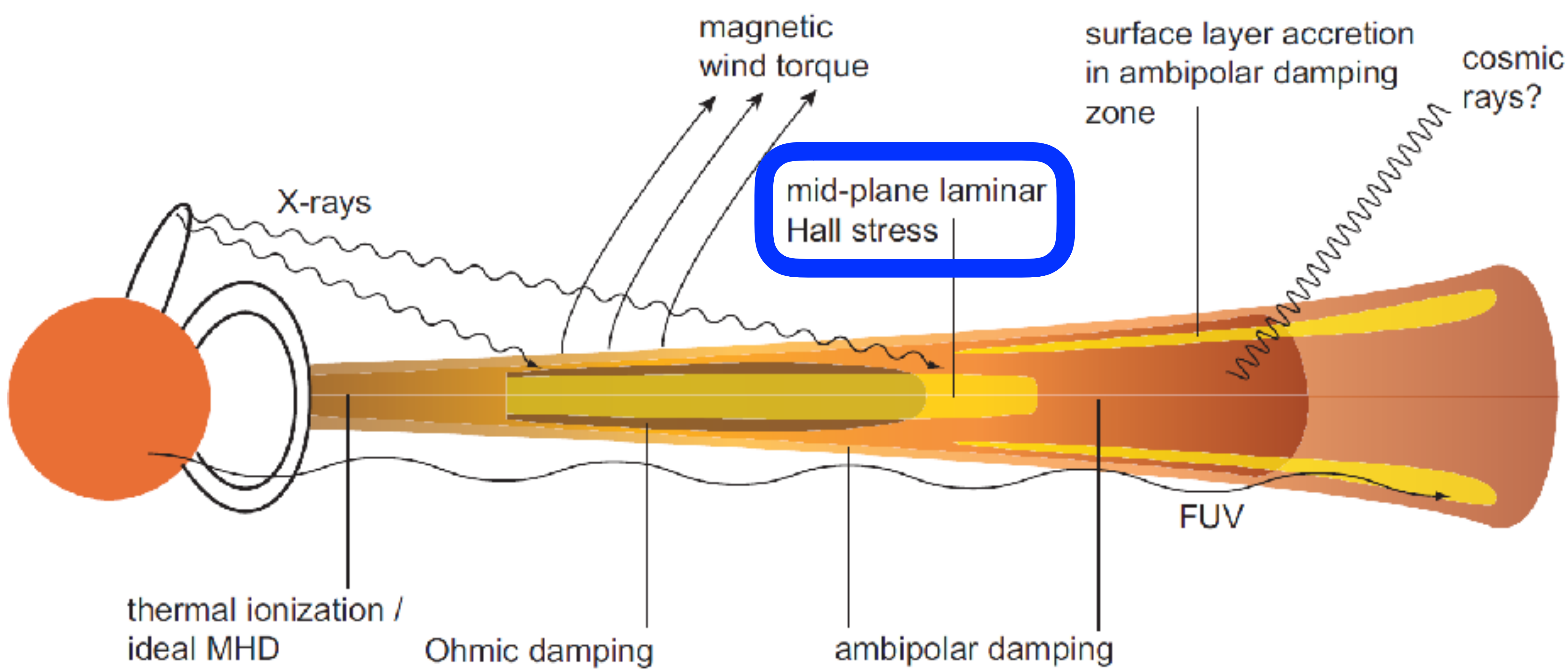
Photophoresis in a Dilute, Optically Thick Medium and Dust Motion in Protoplanetary Disks
C.P. McNally, A. Hubbard
2015, ApJ, 814, 37

Photophoretic Levitation and Trapping of Dust in the Inner Regions of Protoplanetary Disks
C.P. McNally, M.K. McClure
2017, ApJ, 834, 48

Part II: Low Mass Planet Migration in Hall-affected disks

CM, Sijme-Jan Paardekooper, Richard Nelson (QMUL),
Wladimir Lyra (CSUN)

Protoplanetary Disc Structure



If there is low viscosity in the Ohmic dead zone, corotation torques saturate, and Lindblad (wave) torques migration drives planetesimals into the star.

Image: Phil Armitage, 2013

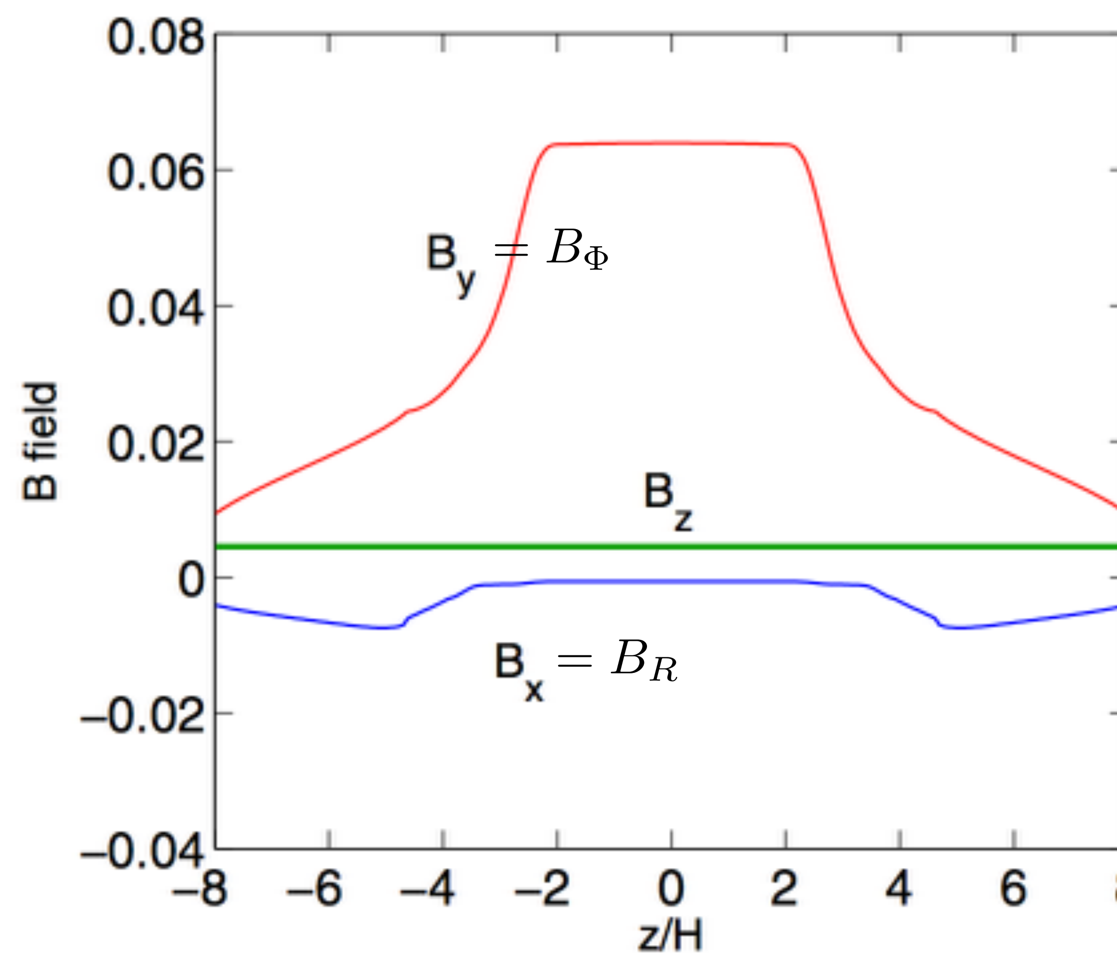
Origin of Dead-Zone Radial Maxwell Stress

Hall-shear instability in surface layers imposes B_r on dead zone midplane

Keplerian orbital shear winds up azimuthal field

Radial Maxwell stress due to spiral field

$$T^{\mathrm{Max}}_{R\Phi} = -B_R B_\Phi$$



Bai & Stone 2013, Fig 5

2D Model Structure: Approximate Equilibrium

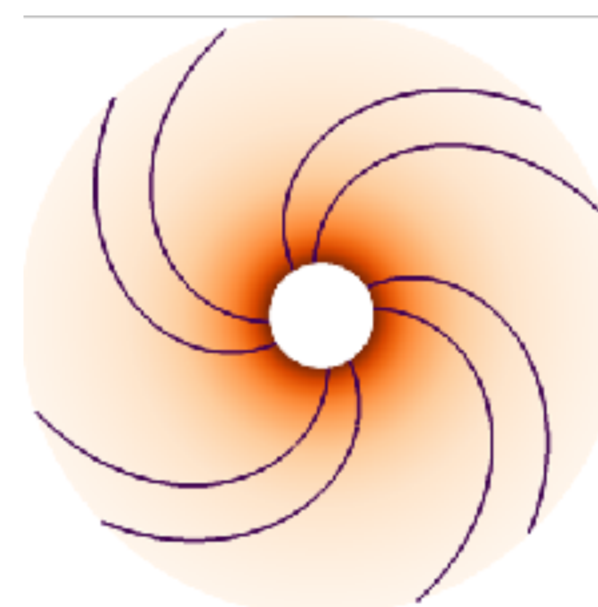
Impose a radial field in 2D radial-azimuthal

Assume Keplerian rotation (thin disk), balance shear-stretching with Ohmic diffusion

Choose a constant accretion rate $\rightarrow \eta \propto r^0$

$$B_r = B_0 \left(\frac{r}{r_0}\right)^{-1}; \qquad B_\phi = -2B_0\Omega_0 r_0^2 \frac{\mu_0}{\eta_0}\left(\frac{r}{r_0}\right)^{-1/2}$$

Accretion flow $\rho v_r = -\dfrac{2B_0^2 r_0^2}{\eta_0 r}$



Critical Parameter

$$\chi = \frac{\text{Flushing Timescale}}{\text{Libration Timescale}}$$

similar to a_c parameter of Masset & Papaloizou 2003 in runaway migration

$$a_c = \frac{\text{migration across corotation region}}{\text{Libration Time}}$$

closely related to the dynamical corotation torque on a migrating planet (Paardekooper 2014)

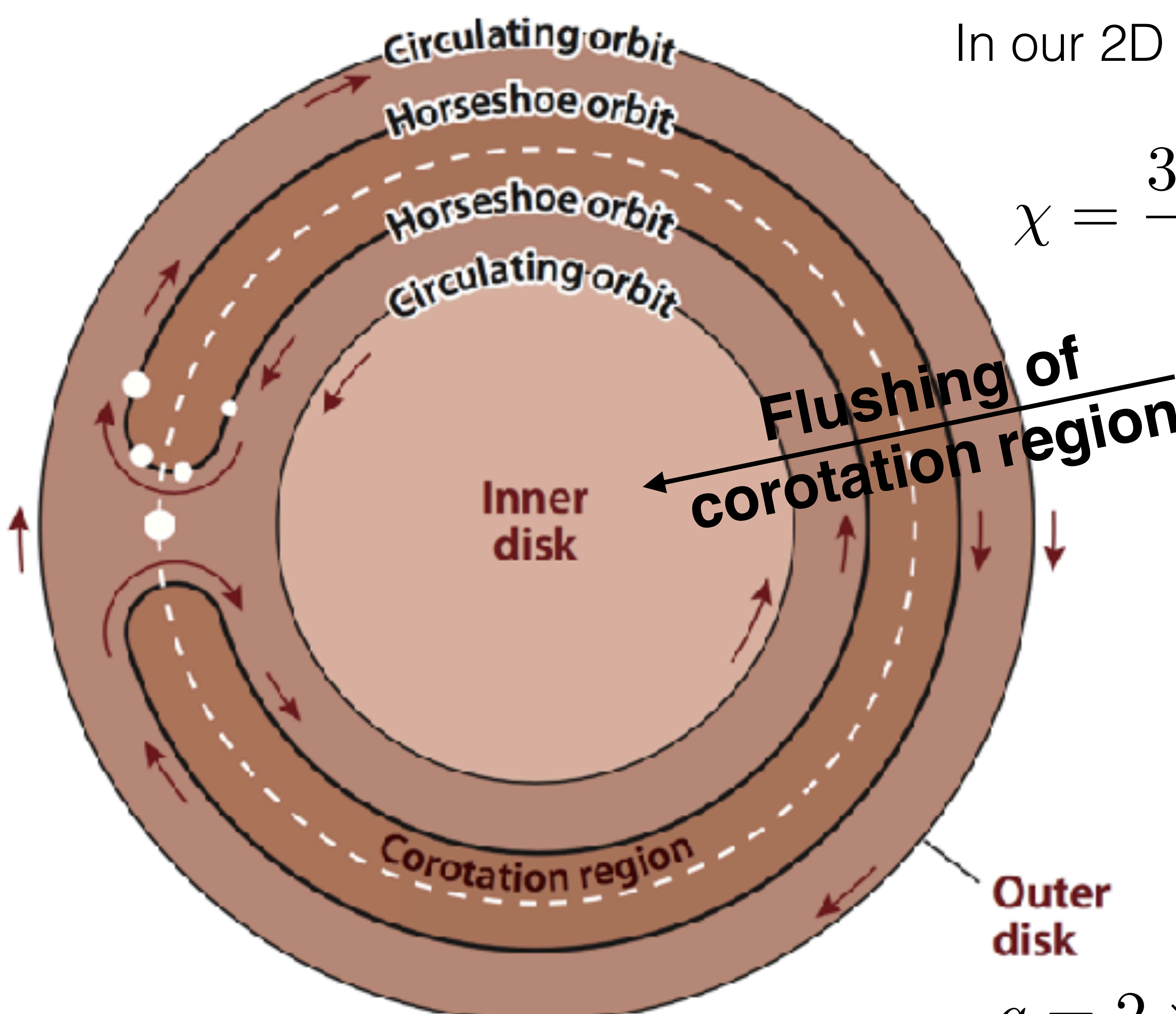


image: Kley & Nelson 2012

In our 2D model:

$$\chi = \frac{3(1.2^2)}{8\pi}\Omega_0\beta_0\frac{1+4\Omega_0^2 r_0^4(\mu_0/\eta_0)^2}{2c_s^2}\frac{\eta_0}{\mu_0}\frac{q}{h}$$

Planet Mass (q)

Diffusivity (η_0)

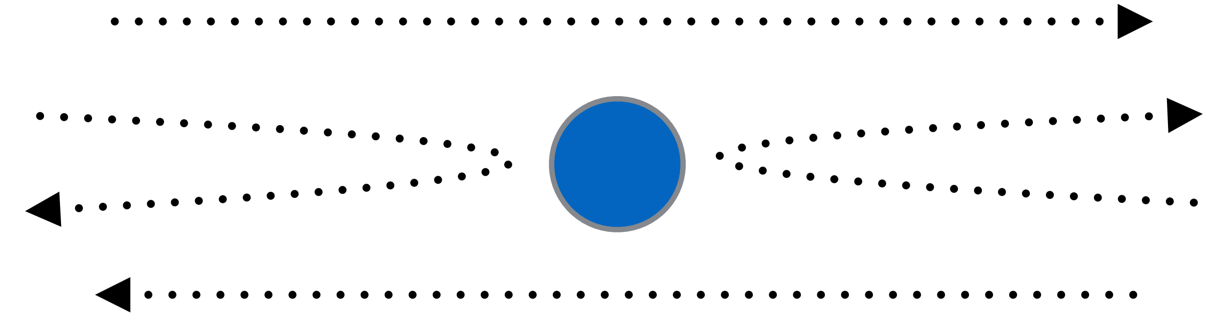
Scale Height (h)

Constraints: $\beta > 1$

$$\frac{\eta_{\min}\chi}{\mu_0} = 2\Omega_0 r_0^2$$

$$q = 2\times 10^{-5},\ c_s = h = 0.05 \ \rightarrow \beta_{\max} = 18.1805$$

Timescales



The planet does not significantly alter the magnetic field structure of the disk:

$$\tau_{\rm diff} \ll \tau_{\rm U\text{-}turn}$$

What's the critical diffusivity for this timescale relation?

$$\tau_{\rm diff} \sim \tau_{\rm U\text{-}turn} \sim h\tau_{\rm lib}$$

$$(\eta/\mu_0)_{\rm crit} \sim 3\times 10^{-5}\frac{(q/2\times 10^{-5})^{3/2}}{(h/0.05)^{5/2}}\Omega_0 r_0^2$$

What's the critical diffusivity for an Ohmic dead zone?

$$\Lambda \equiv \frac{v_A^2}{(\eta/\mu_0)\Omega_0}$$

$$(\eta/\mu_0)_{\rm crit} \sim \frac{c_s^2}{\beta\Omega_0} \sim 10^{-8}\left(\frac{(c_s/0.05)^2}{(\beta/10^5)}\right)\Omega_0 r_0^2$$

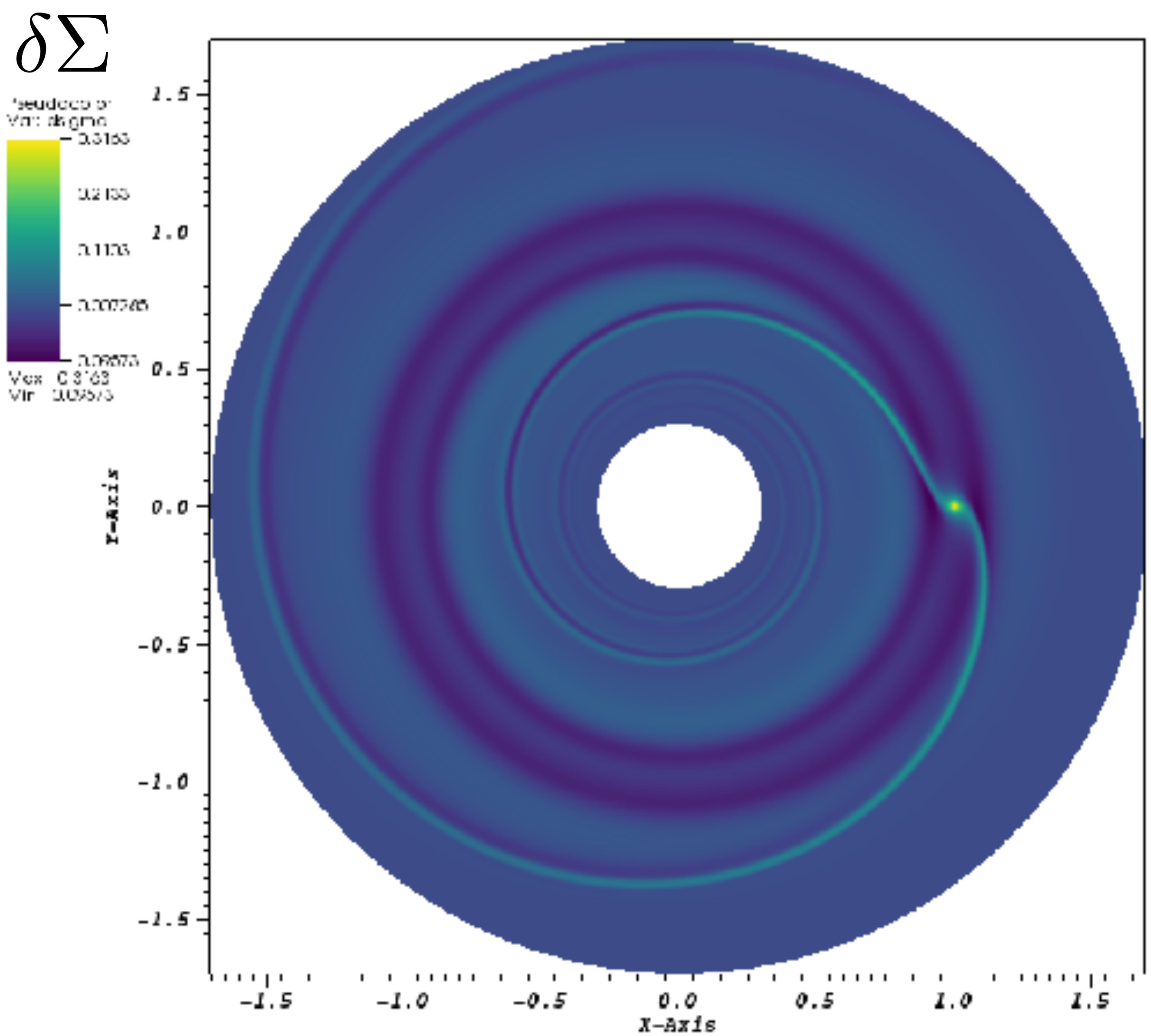
Add the Magnetically Driven Inflow

No Inflow

No Magnetic Field

$\chi = \infty$

Only Lindblad (wake) torque

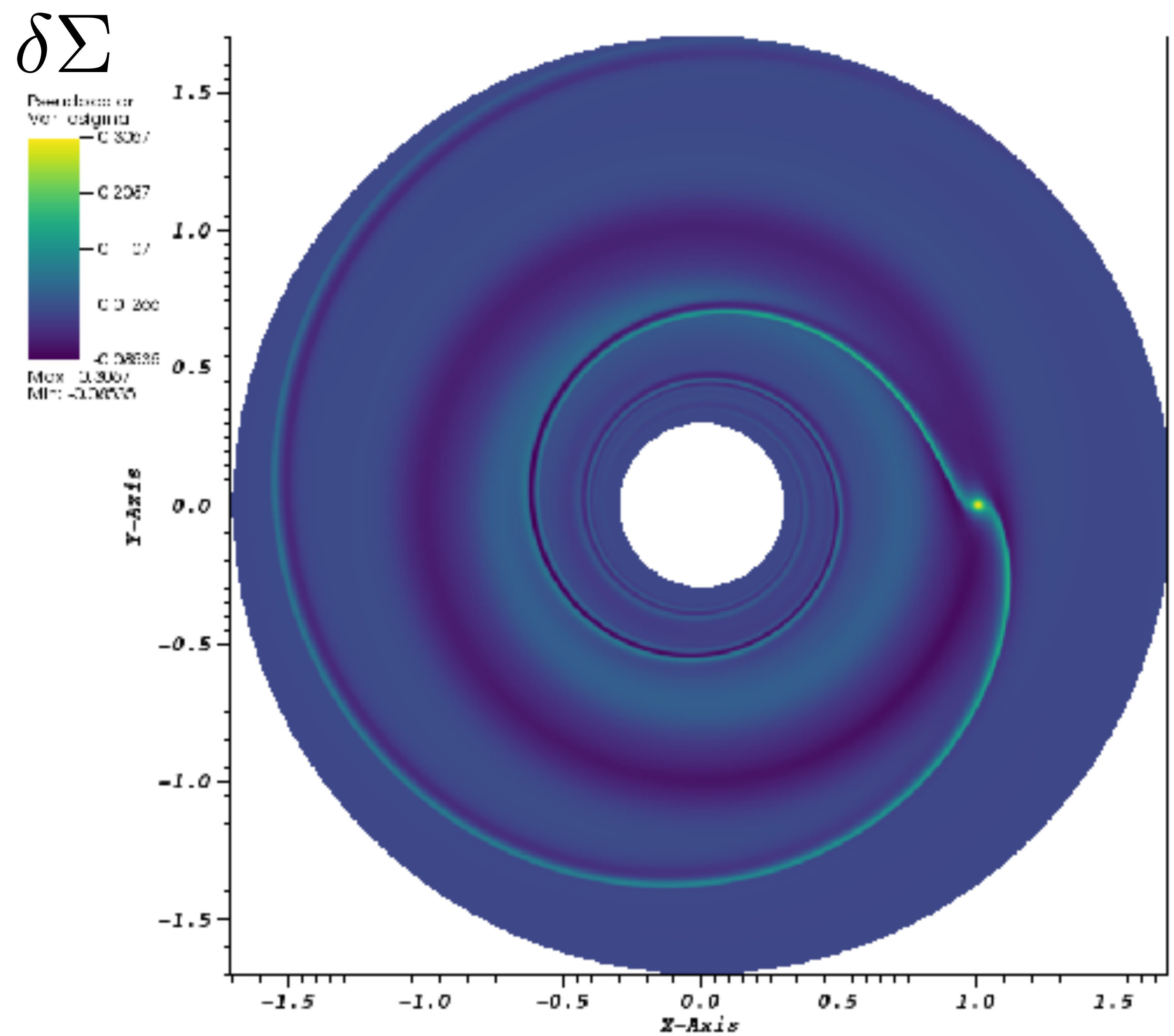


Inflow

Laminar Spiral Field

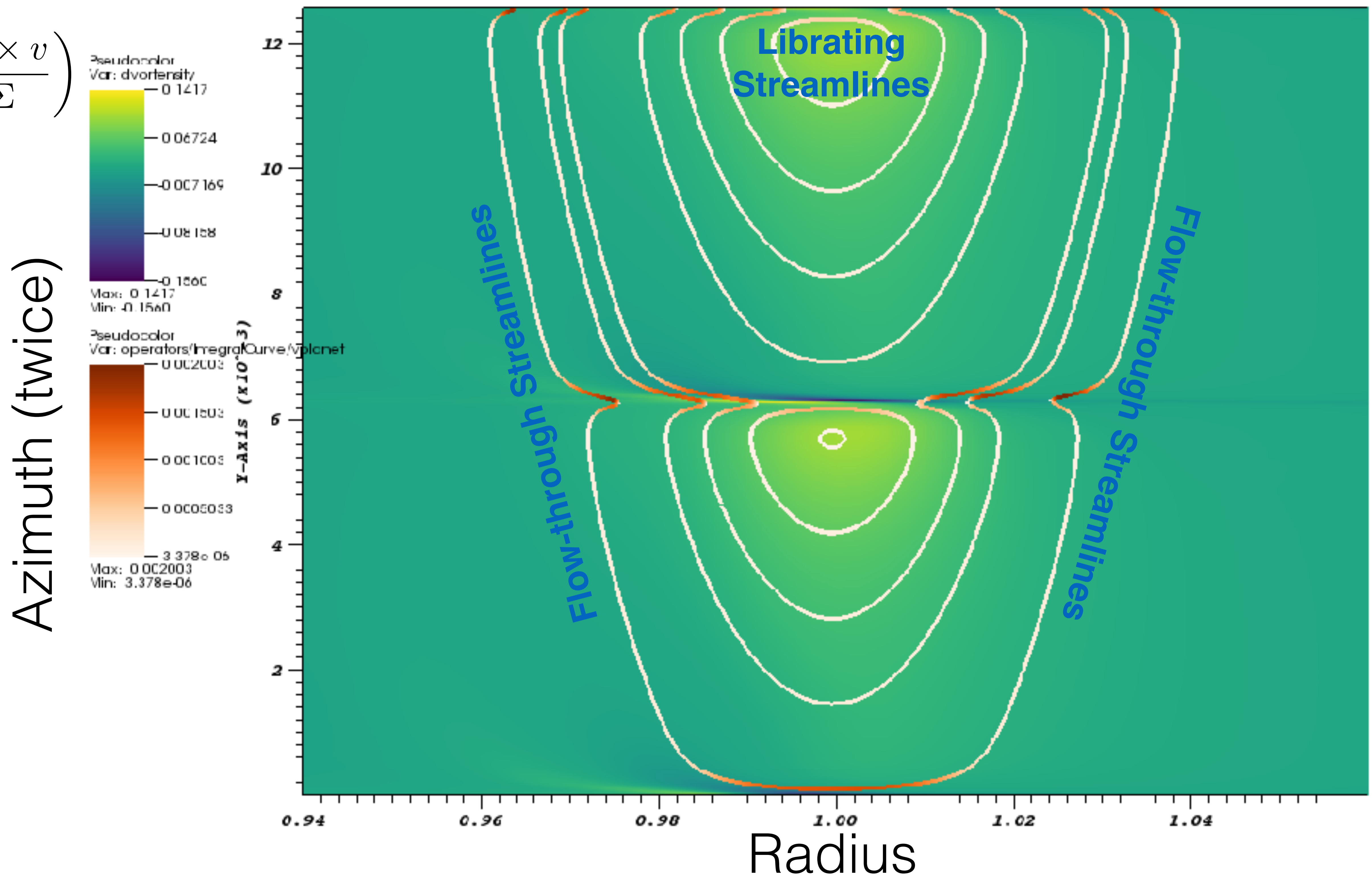
$\chi = 1$

Linblad (wake) torque + corotation torque



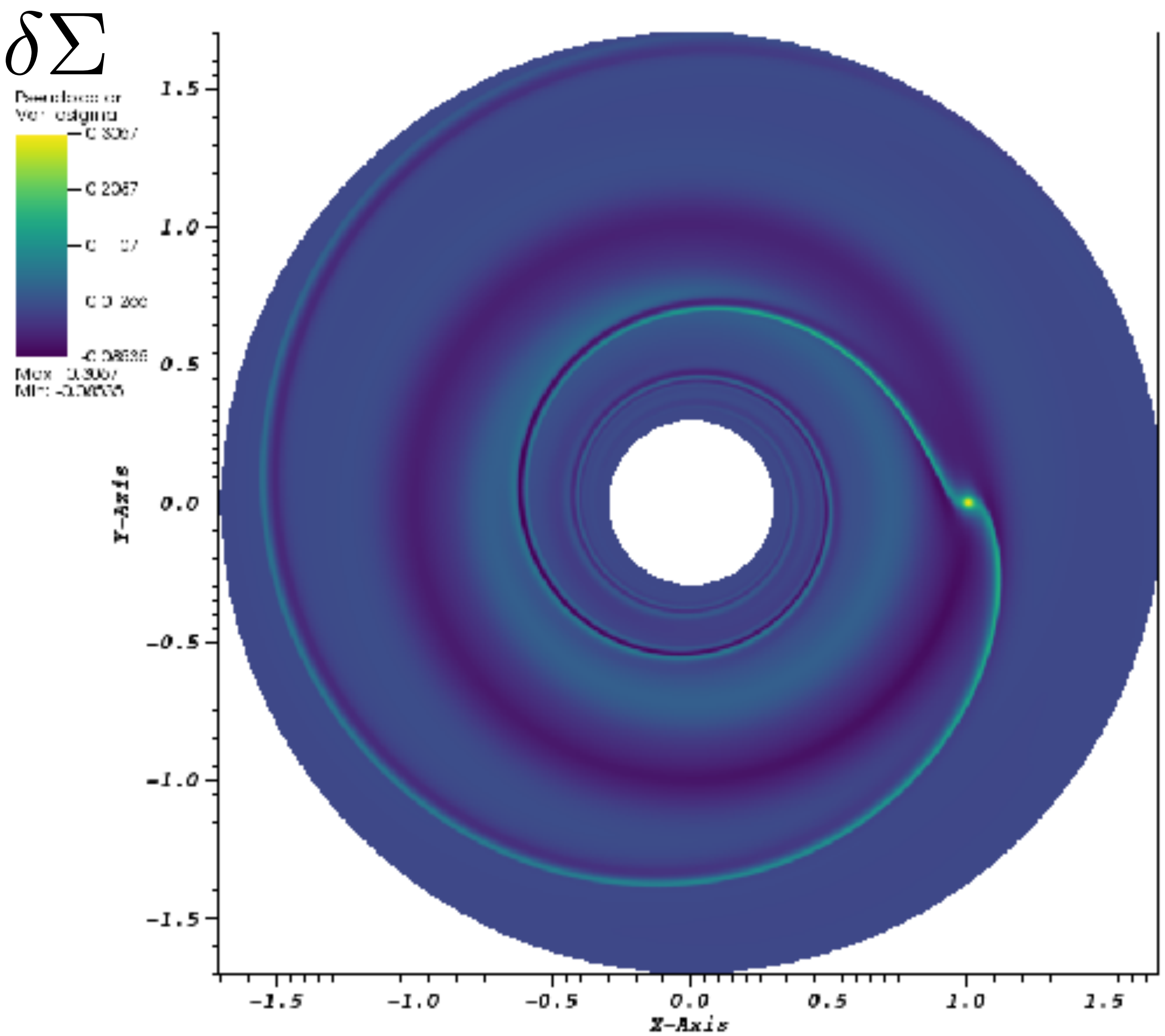
Streamlines and Vortensity Perturbation: Inflow

$$\delta\left(\frac{\nabla \times v}{\Sigma}\right)$$



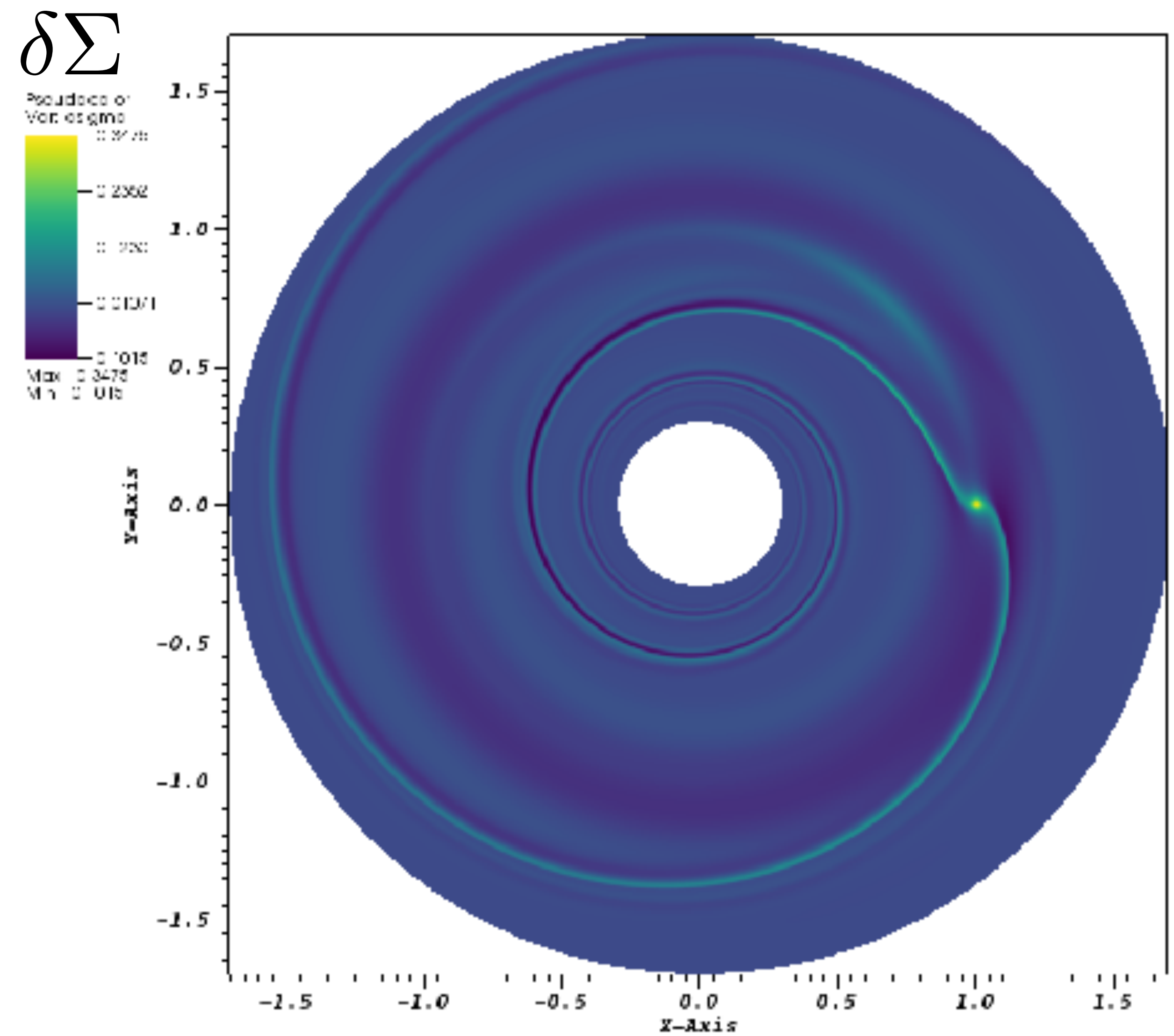
Changing the direction

Inflow torque



Density minimum
behind planet

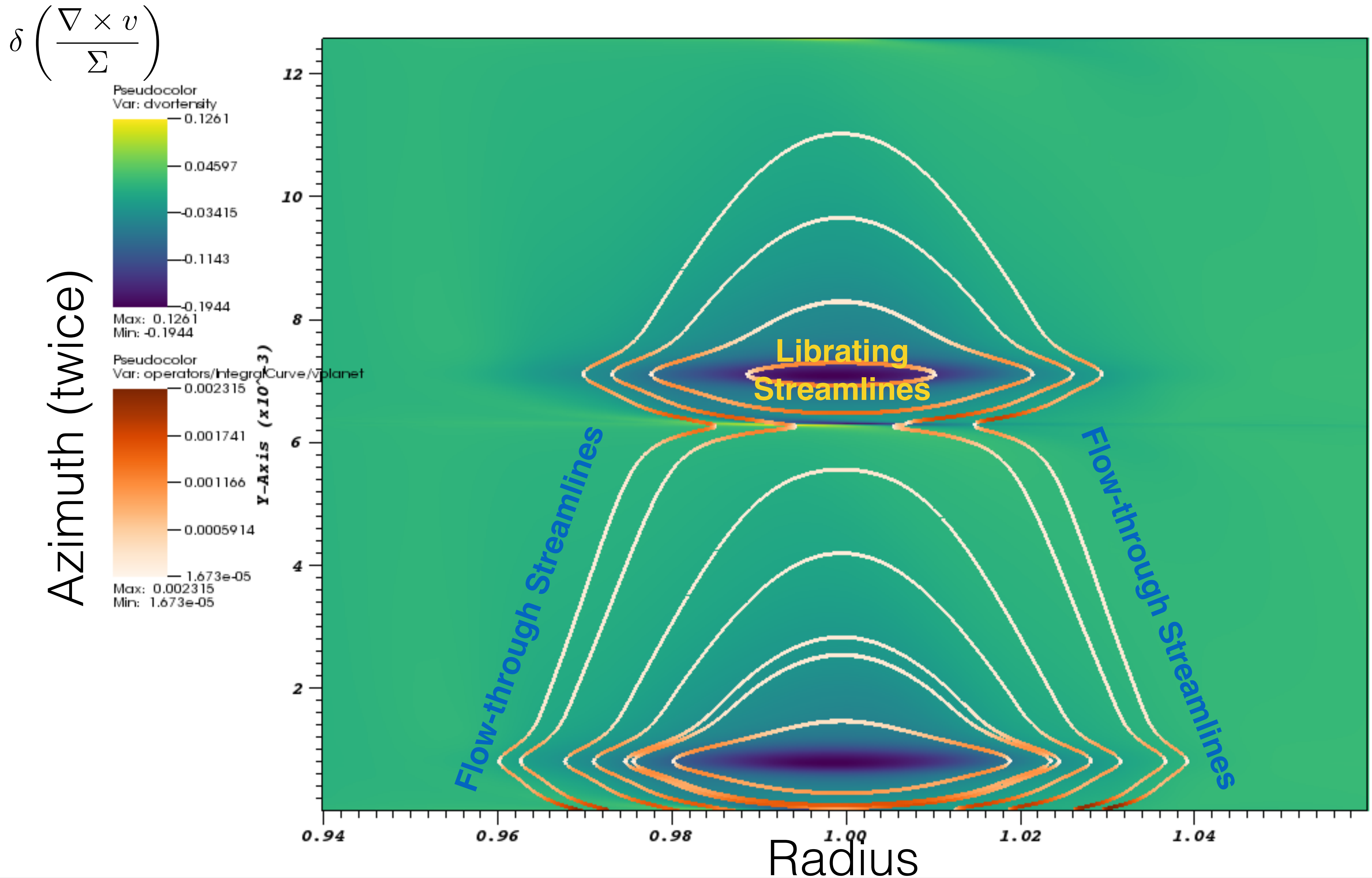
Outflow torque



Density maximum
in front of planet

Both configurations have a positive corotation torque

Changing the direction: Outflow



Conclusions on Magnetized Dead Zone Planet Migration

Laminar field magnetic field stress in the dead zone can alter the corotation torque

Critical parameter $\chi = \frac{\text{Flushing Timescale}}{\text{Libration Timescale}}$

Magnetic field largely passive, simply provides an inflow/outflow torque to the gas

Type I corotation torque on planet depends on to the disk gas-planet radial motion, always outward

McNally, Paardekooper, Lyra, Nelson, in prep