Warps in accretion discs

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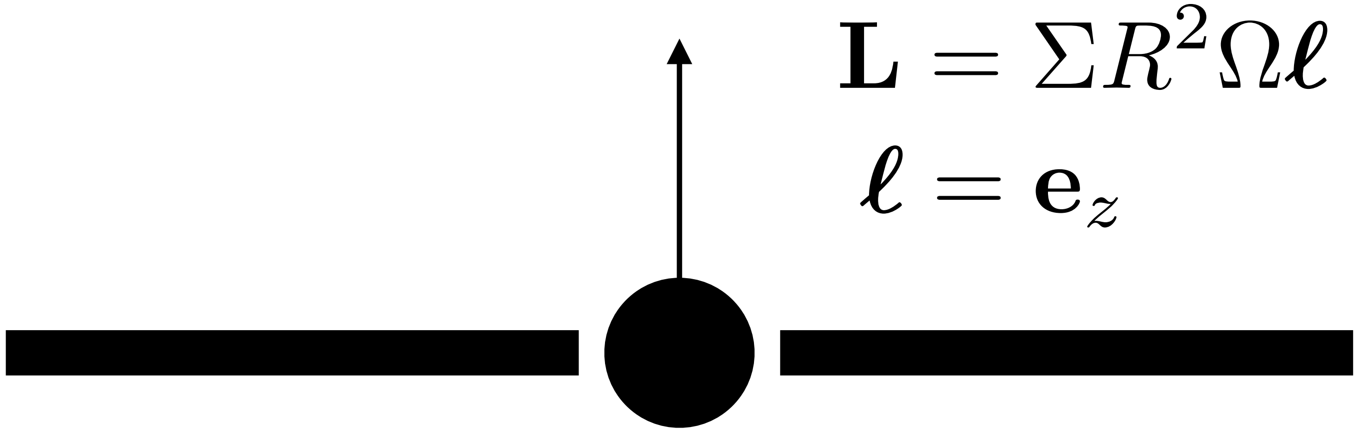
What is a warp?

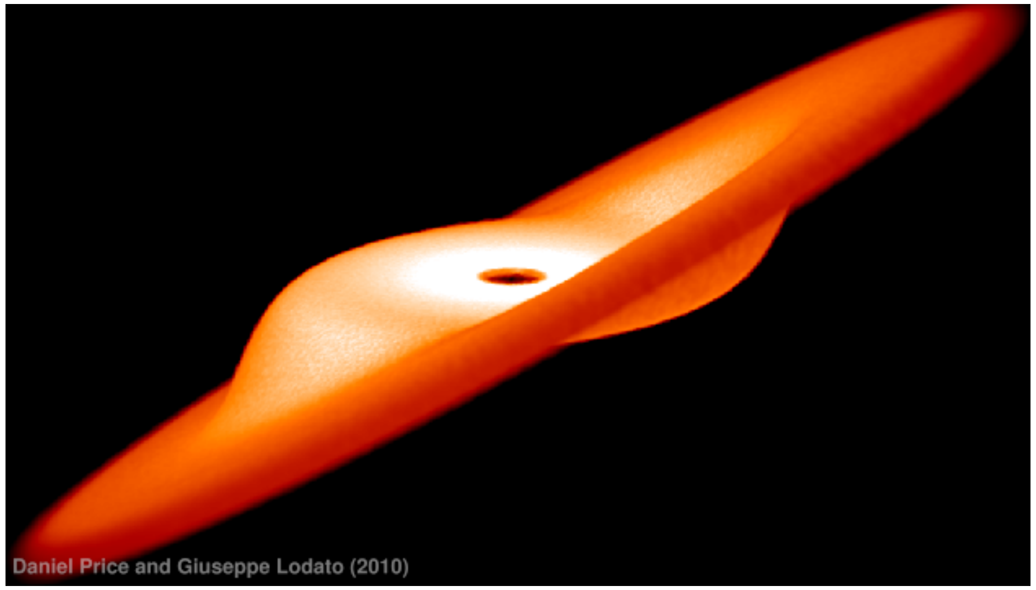
planar:



$$\mathbf{L} = \Sigma R^2 \Omega \boldsymbol{\ell}$$

$$\boldsymbol{\ell} = \mathbf{e}_z$$

warp:



Daniel Price and Giuseppe Lodato (2010)

$$\mathbf{L} = \Sigma R^2 \Omega \boldsymbol{\ell}$$

$$\boldsymbol{\ell}\left(R,t\right) = \left(\cos\gamma\sin\beta, \sin\gamma\sin\beta, \cos\beta\right)$$

Euler angles: $\gamma\left(R,t\right)$ disc twist

$\beta\left(R,t\right)$ disc tilt

Shear:

Rate of orbital shear: $\mathbf{S} = R\frac{\partial \boldsymbol{\Omega}}{\partial R} = R\frac{\mathrm{d}\Omega}{\mathrm{d}R}\boldsymbol{\ell} + R\Omega\frac{\partial \boldsymbol{\ell}}{\partial R}$

Suggests two effective viscosities:

1. Usual planar viscosity $\nu_1 = \alpha_1 c_{\mathrm{s}} H$

   [radial communication of component of ang. mom. **perpendicular** to local orbital plane]

2. Effective "vertical" viscosity $\nu_2 = \alpha_2 c_{\mathrm{s}} H$

   [radial communication of component of ang. mom. **parallel** to local orbital plane]

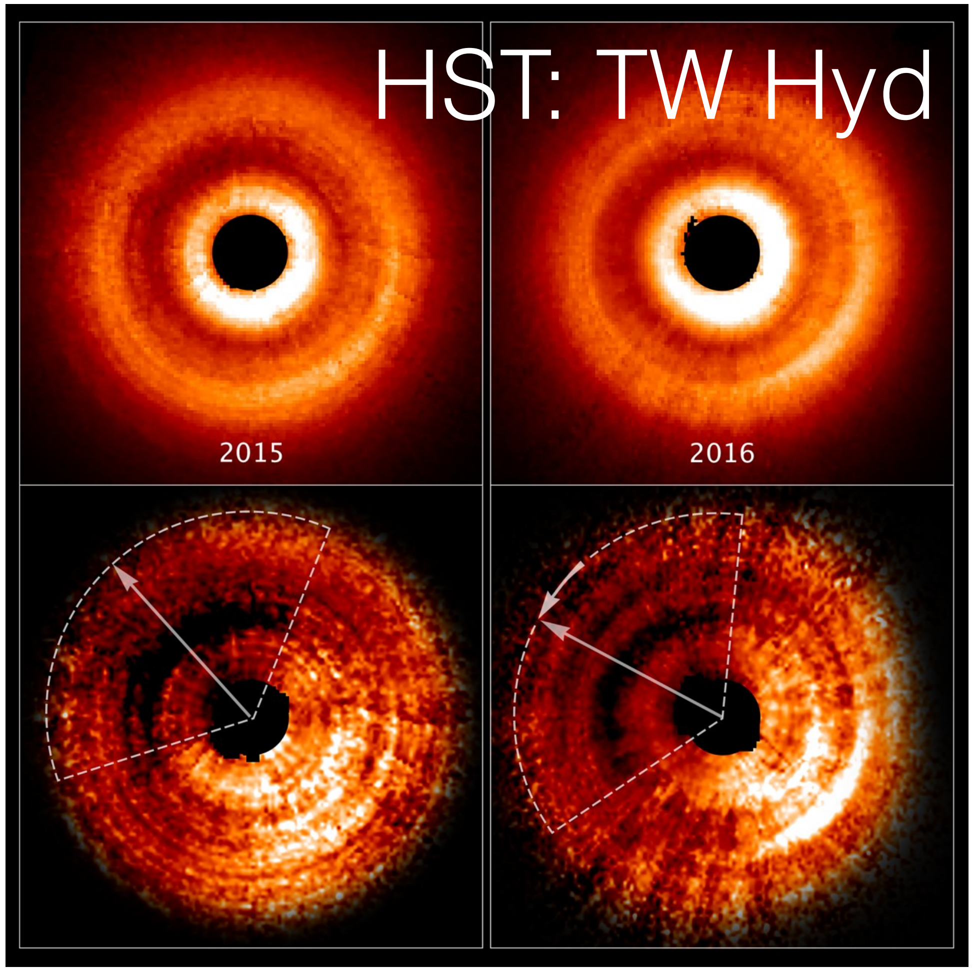
Timescales: rich, complex dynamics

$t_{\rm dyn} = \dfrac{1}{\Omega}$	dynamical time
$t_{\rm s} = \dfrac{R}{c_{\rm s}}$	sound/wave crossing time
$t_{\nu_1} = \dfrac{R^2}{\nu_1}$	accretion timescale
$t_{\nu_2} = \dfrac{R^2}{\nu_2}$	warp diffusion timescale
$t_{\rm prec} = \dfrac{1}{\Omega_{\rm p}}$	precession timescale

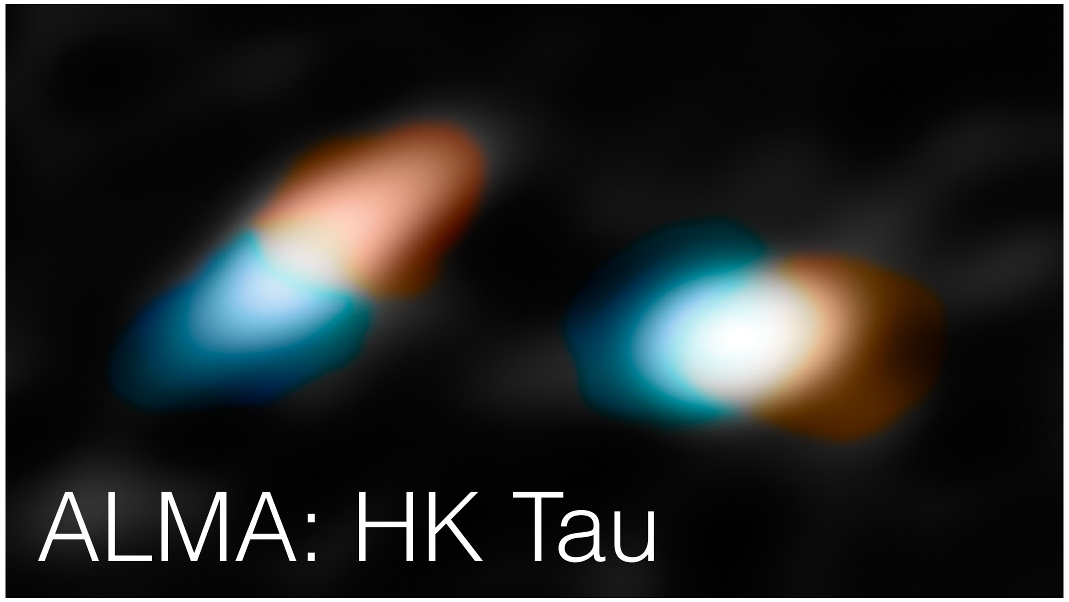
etc.

Observations:

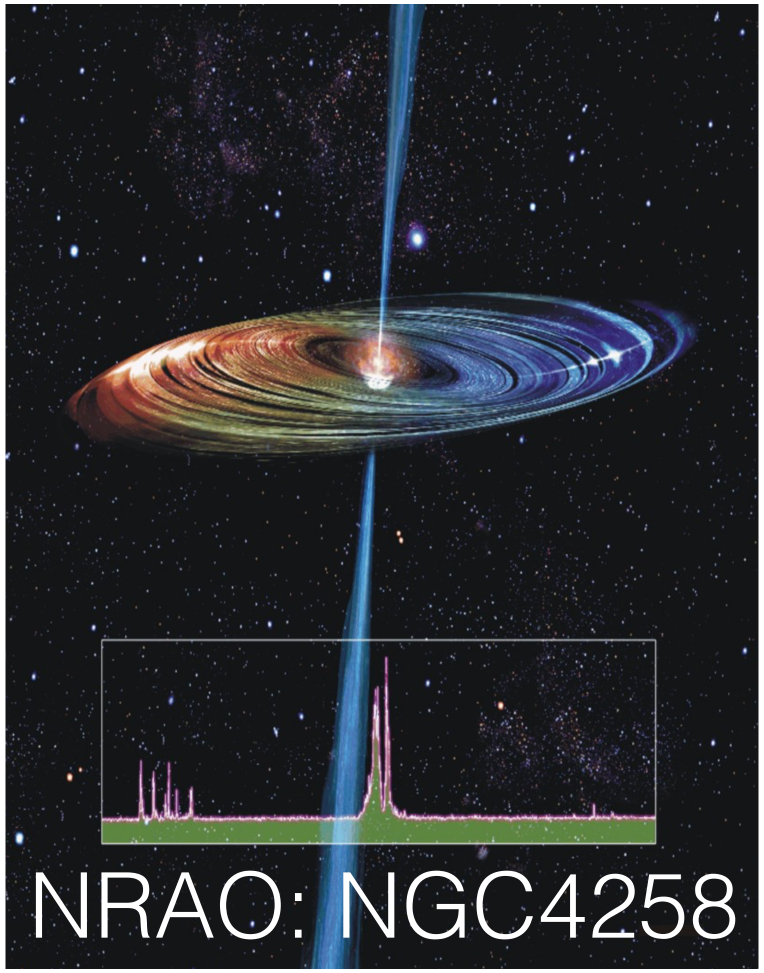
- protoplanetary discs
- water maser emission in AGN
- long X-ray periods in LMXBs e.g. Her X-1
- QPOs (?) e.g. relativistic precession model
- etc…



HST: TW Hyd



ALMA: HK Tau



NRAO: NGC4258

Misaligned discs are warped by precession:

- Lense-Thirring precession (Bardeen & Petterson 1975)
- tides from a companion (Papaloizou & Terquem 1995)
- magnetic warping (Lai 1999)

Aligned discs are warped by instabilities:

- radiation warping (Pringle 1996,1997)
- winds (Schandl & Meyer 1994)
- resonant tides (Lubow 1992, Lubow & Ogilvie 2000)

How is the warp propagated?

Before PP83 it was assumed that:

$$\nu_1 = \nu_2$$

but…

PP83 showed fluid effects are important

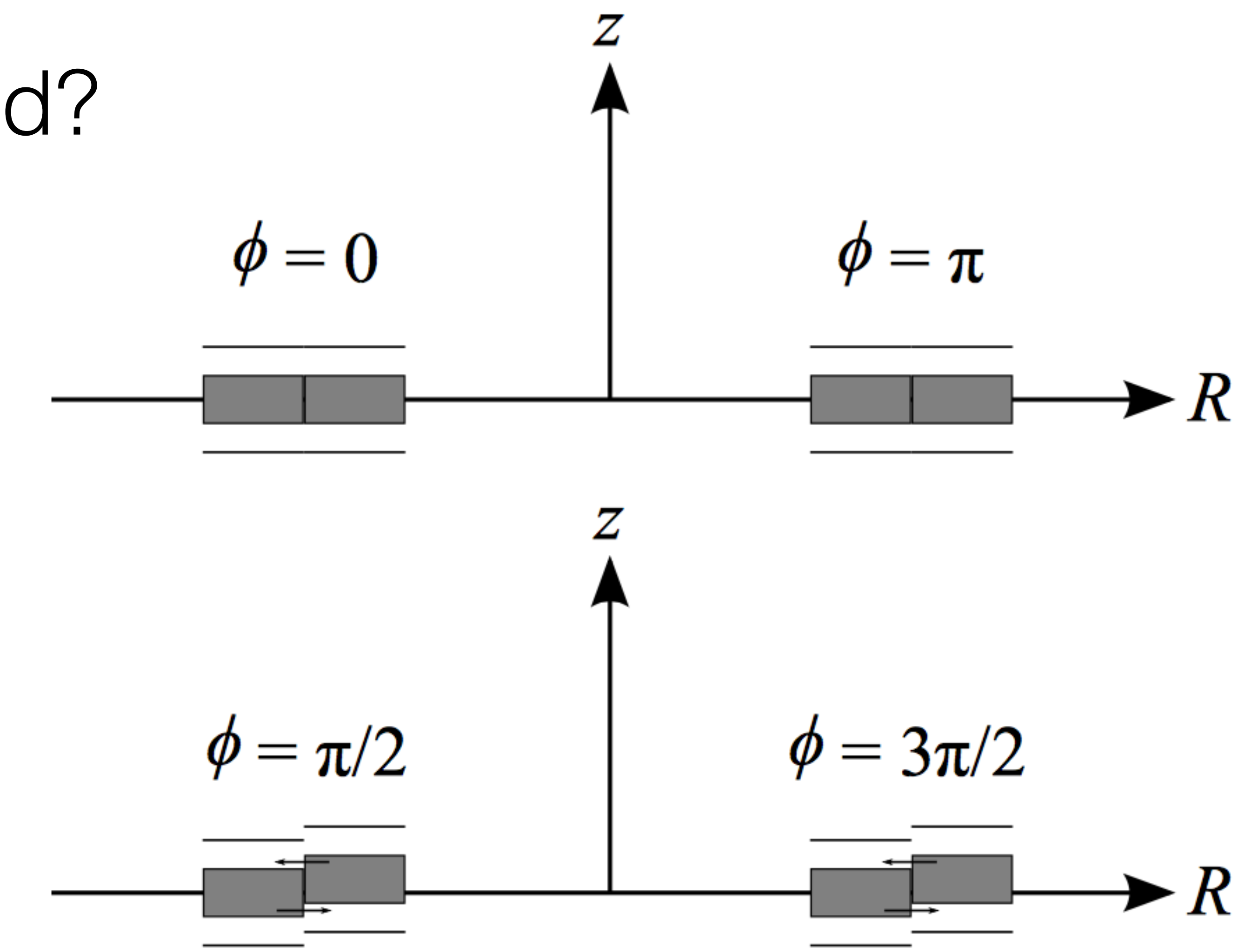
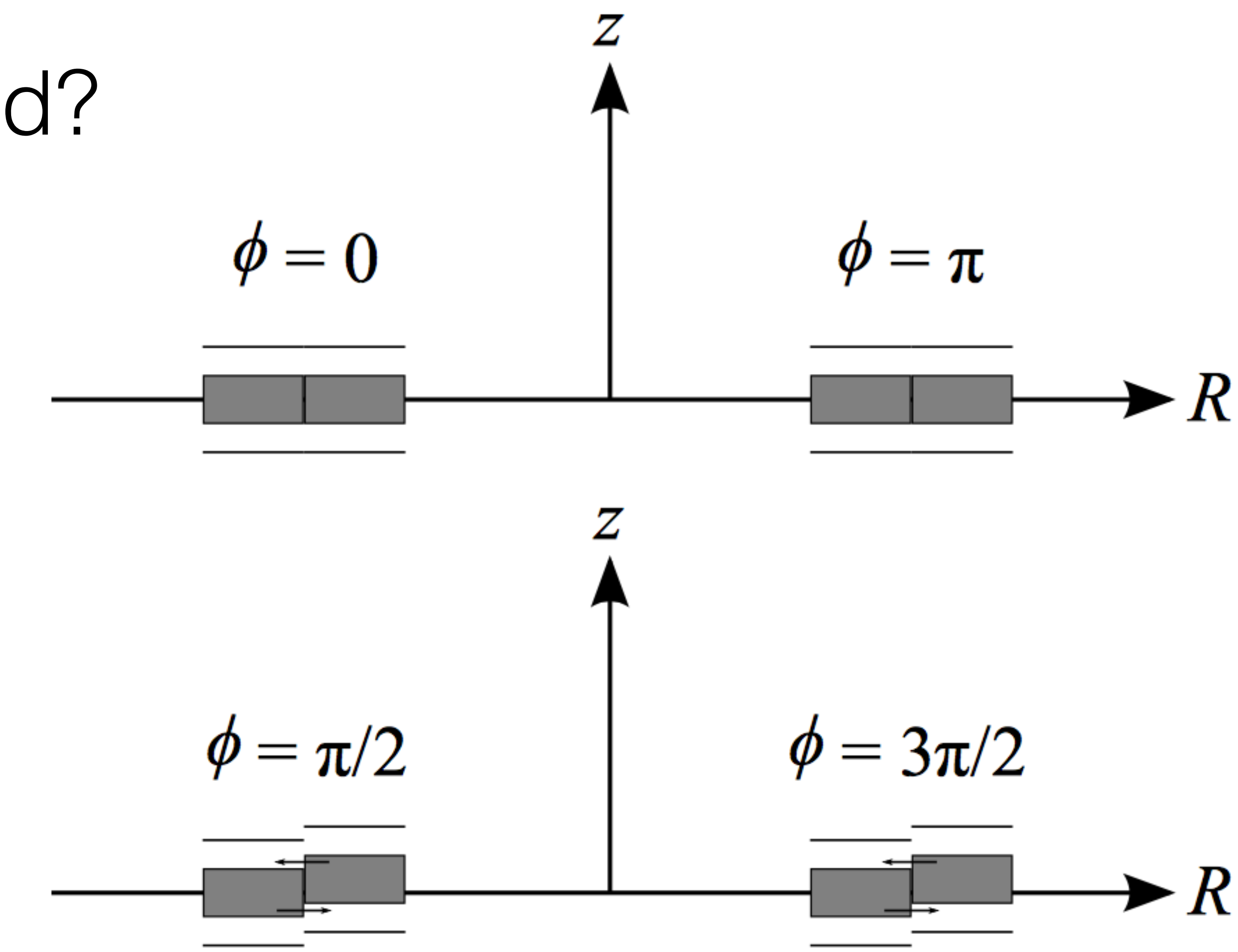


Fig. 1 This figure (cf Fig. 10 of Lodato & Pringle 2007) illustrates the radial pressure gradient induced by a warp. The top and bottom panel show cross-sections of the same two neighbouring rings of gas, but at different azimuths ϕ. The shaded regions indicate the higher pressure around the local midplane, and the arrows show the resultant pressure gradient when the rings are misaligned. The azimuthal angle around a ring is measured in the direction of the flow from the descending node where $\phi = 0$. The tilted rings cross at the nodes, and so at these points are in aligned contact as usual. At all other azimuths the ring midplanes do not fully line up, causing a region of overpressure above or below the midplane. Each gas parcel feels an oscillating pressure gradient as it orbits in the warp.

Warp induces a radial pressure gradient which oscillates around the orbit

How is the warp propagated?



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Induces epicyclic motion: resonates with orbital forcing (near-Keplerian)

Strong torque + wave launched

Turbulence damps the wave propagation

Depending on strength of damping, evolution is either viscous/diffusive or wave-like

How is the warp propagated?

Assume (for now) alpha damping of wave: $t_{\rm damp} = \dfrac{1}{\alpha\Omega}$

(e.g. Lubow & Ogilvie 2000)

Warp waves propagate at speed $v_{\rm wave} \approx c_{\rm s}/2$

(e.g. Papaloizou & Lin 1995) $\implies t_{\rm wave} \approx \dfrac{R}{c_{\rm s}}$

Critical point where $t_{\rm damp} \approx t_{\rm wave}$

Solutions are diffusive for $t_{\rm damp} \ll t_{\rm wave} \implies \alpha \gg H/R$

Solutions are wave-like for $t_{\rm damp} \gg t_{\rm wave} \implies \alpha \ll H/R$

Restrictions:

- small scale turbulent alpha damping of vertical shear
  (e.g. no dominant large scale fields, or Alfvén waves)

- disc must be near-Keplerian
  (otherwise dispersive waves can dominate; Papaloizou & Lin 1994, Ogilvie 1999)

- shearing must be sub-sonic, i.e. linear warp amplitude
  (otherwise, e.g., parametric instability; Gammie, Goodman & Ogilvie, 2000)

Wavelike case:

- disc follows a wave equation with $v_{\rm wave} \approx c_s/2$

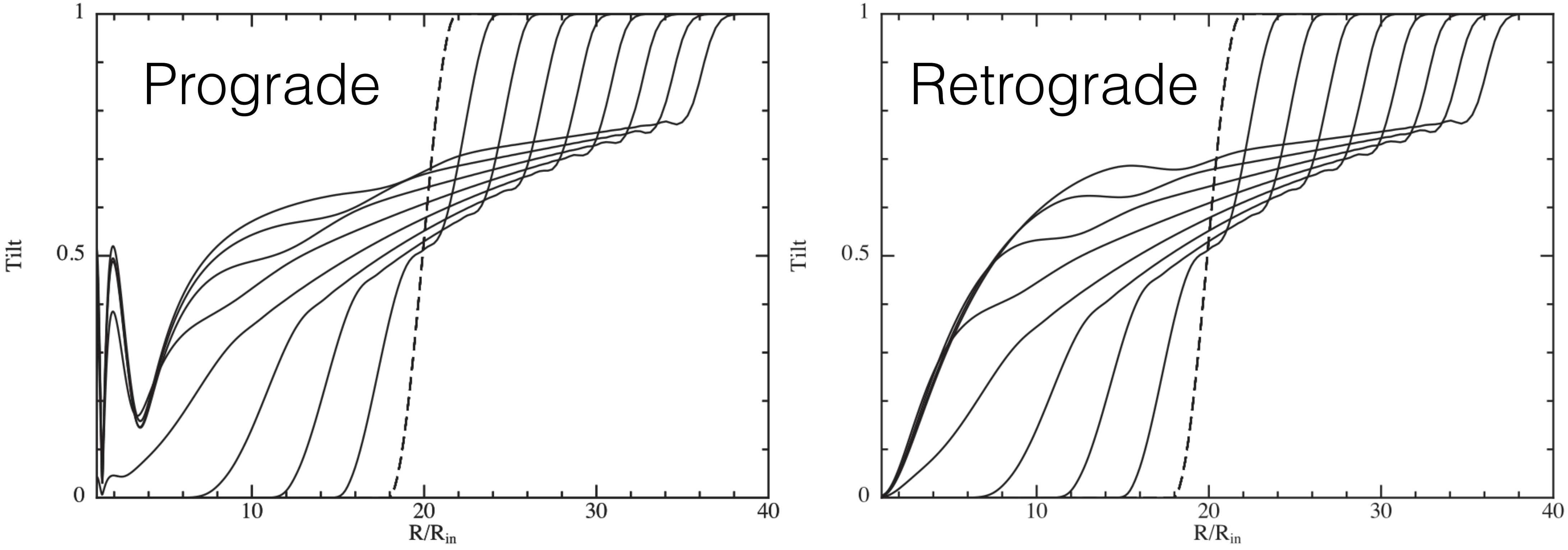
- no fully nonlinear alpha theory
  (see Ogilvie 2006 for weakly nonlinear)

- agreement between linear theory and hydro simulations (in the linear regime) (e.g. Larwood et al. 1997, Fragner & Nelson 2010, Nealon et al. 2015)

- for black hole discs can get “tilt oscillations”

Tilt oscillations:

- predicted analytically by Ivanov & Illarionov (1997)
- if nodal and apsidal precession have same sign, then disc solutions exhibit oscillating plane with radius

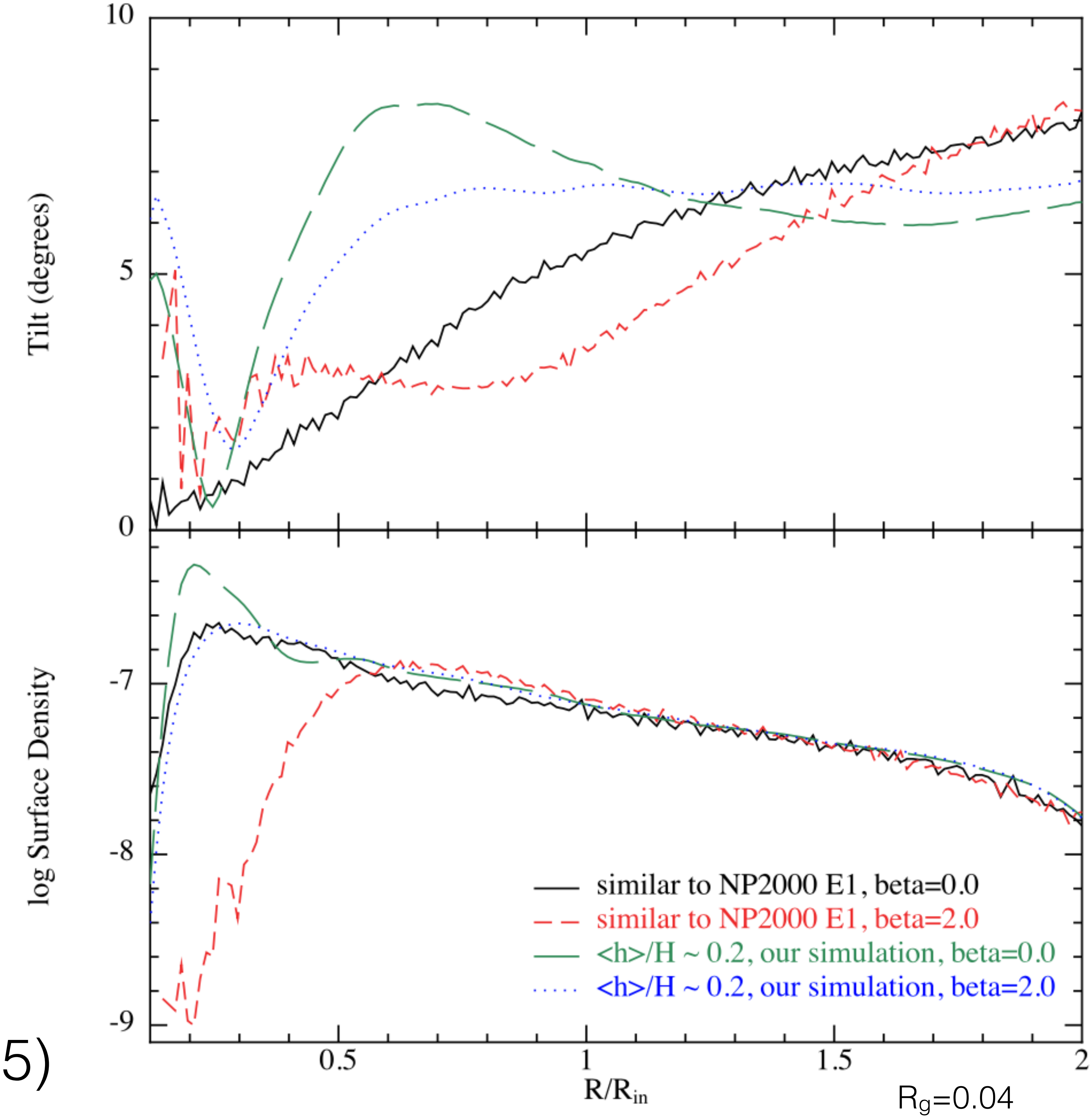


From Lubow, Ogilvie & Pringle (2002); Nealon et al. (2015)

Tilt oscillations: 3D simulations

- initially Nelson & Paploizou (2000): no oscillations (presumed enhanced dissipation by shocks?)

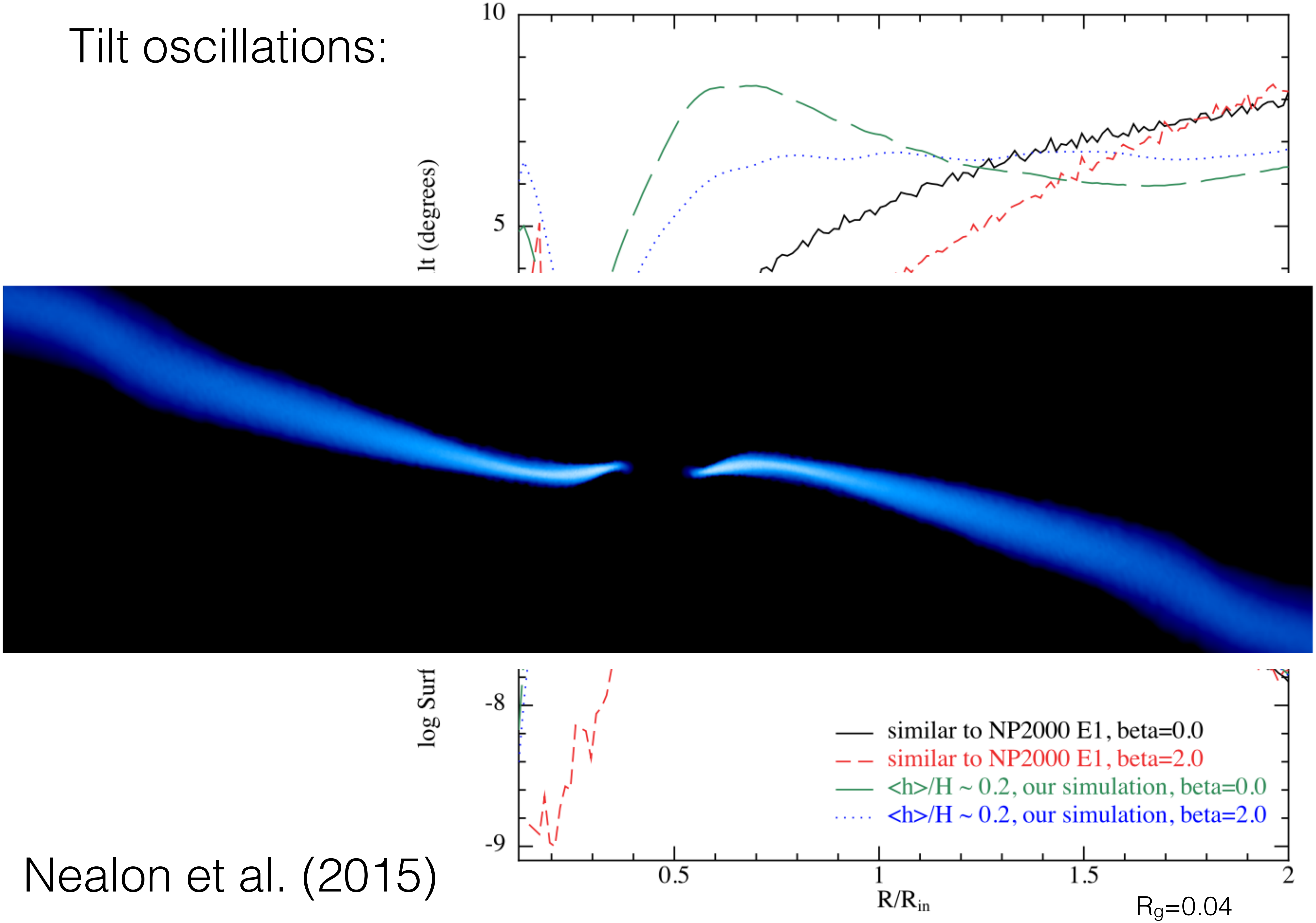
Tilt oscillations:



R_g=0.04

Nealon et al. (2015)

Tilt oscillations:



Nealon et al. (2015)

$R_g=0.04$

Much left to do…

- nonlinear waves, shocks
- evolution of tilt oscillations (jet direction?)
- wave damping and deposition of energy
- use protoplanetary disc observations to constrain models (KH15D, TW Hyd)
- wave-trapping, QPOs?

Diffusive case: large alpha damping of wavelike case

Usual planar viscosity $\nu_1 = \alpha_1 c_\mathrm{s} H$

Effective “vertical” viscosity $\nu_2 = \alpha_2 c_\mathrm{s} H$

Resonant response to radial pressure gradient:

$$\implies \nu_1 \neq \nu_2$$

Diffusive case:

If we assume alpha damps shear isotropically

[i.e. turbulence is small scale compared to H]

Then $\alpha_1 \approx \alpha$

$$\alpha_2 \approx \frac{1}{2\alpha}$$

(linear theory
Papaloizou & Pringle, 1983)

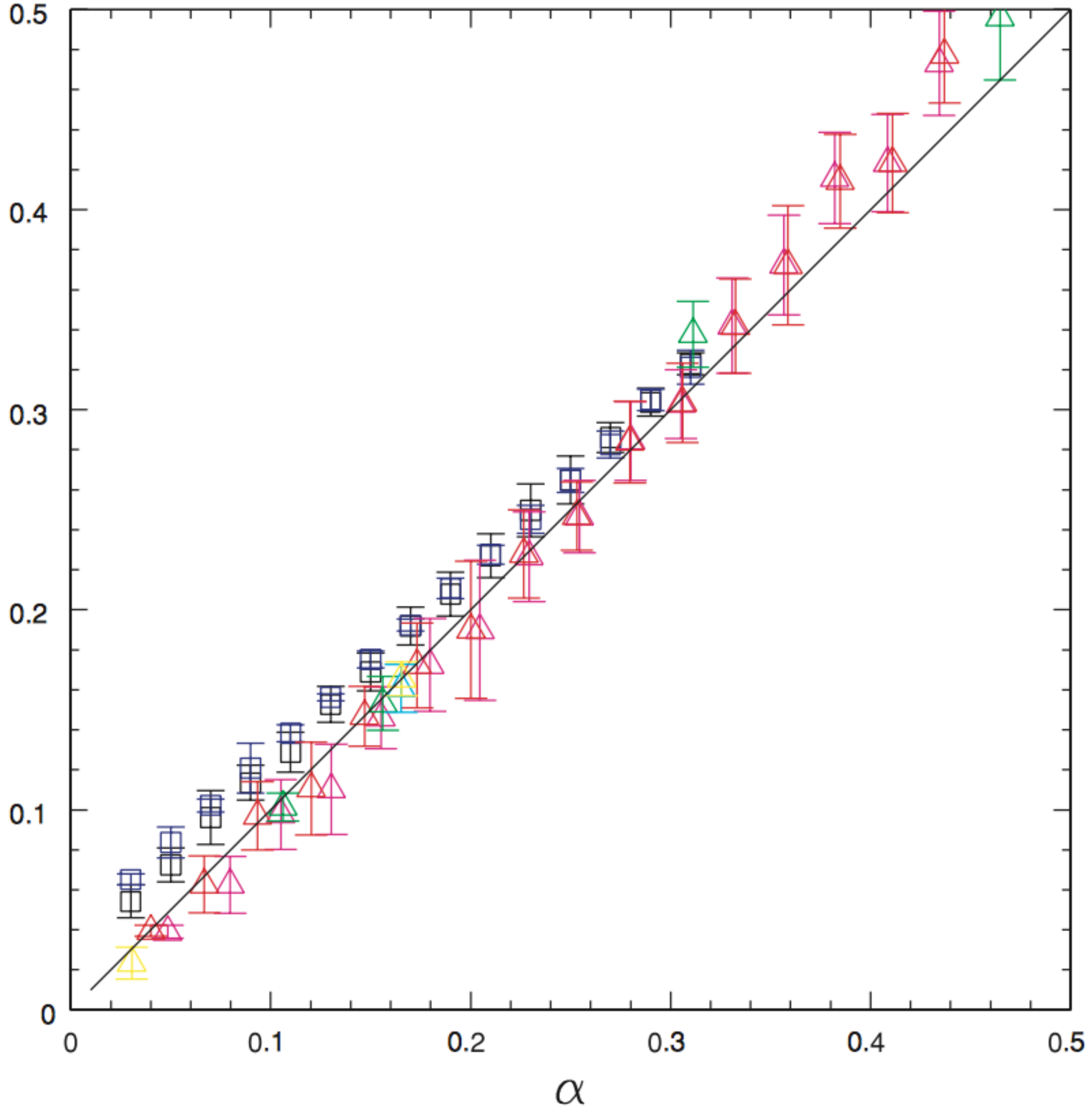
“Vertical” viscous torque is restricted by damping of epicyclic motions: inverse dependence on alpha.

“Isotropic viscosity” $\not\Longrightarrow$ “Isotropic effective viscosity”

Nonlinear equations+solutions for arbitrary alpha and warp amplitude given in Ogilvie (1999,2000).

Nonlinear fluid solutions for a warp: Ogilvie (1999)

code test: Lodato & Price (2010)

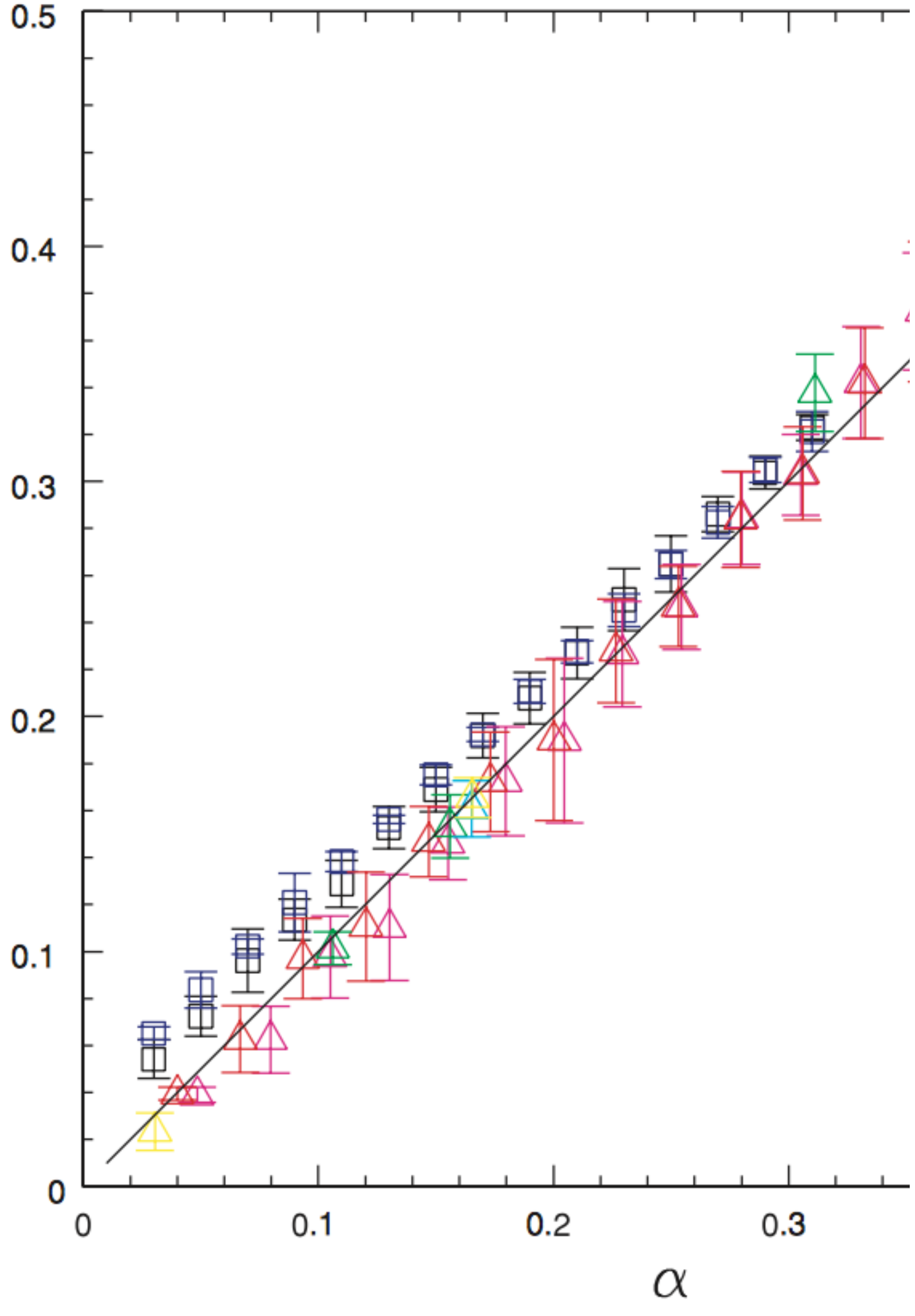


Measured alpha

Input alpha

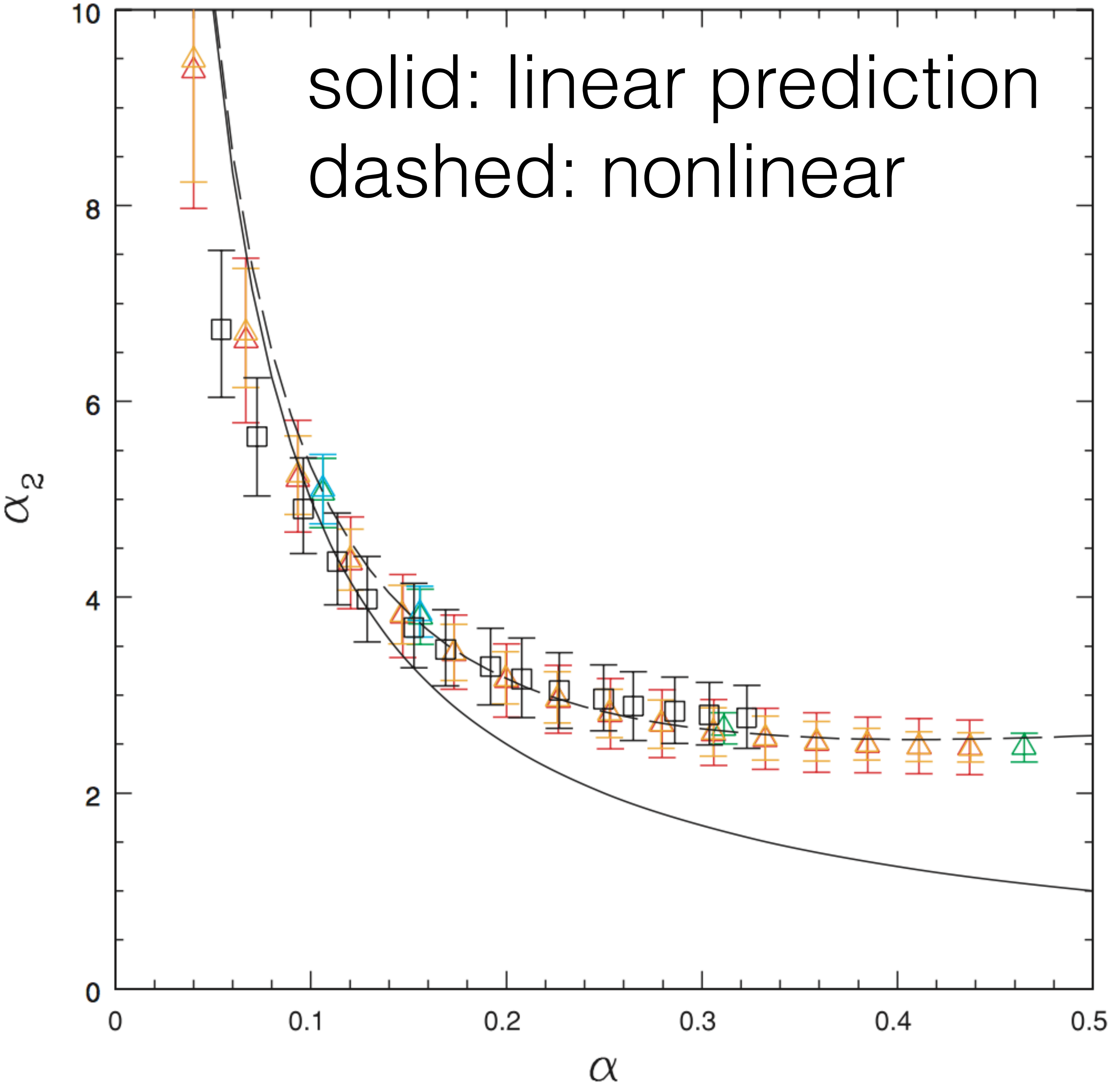
Nonlinear fluid solutions for a warp: Ogilvie (1999)

code test: Lodato & Price (2010)



Measured alpha

Input alpha



solid: linear prediction
dashed: nonlinear

Can we compare “isotropic alpha” with MHD?

Main (only?) numerical investigation:

Torkelsson et al. (2000)

- shearing box, MHD
- induced epicyclic motion
- decay consistent with isotropy $\alpha_{\rm h} \sim \alpha_{\rm v} \sim 0.01$

But, shearing box, low-alpha, one study, etc.

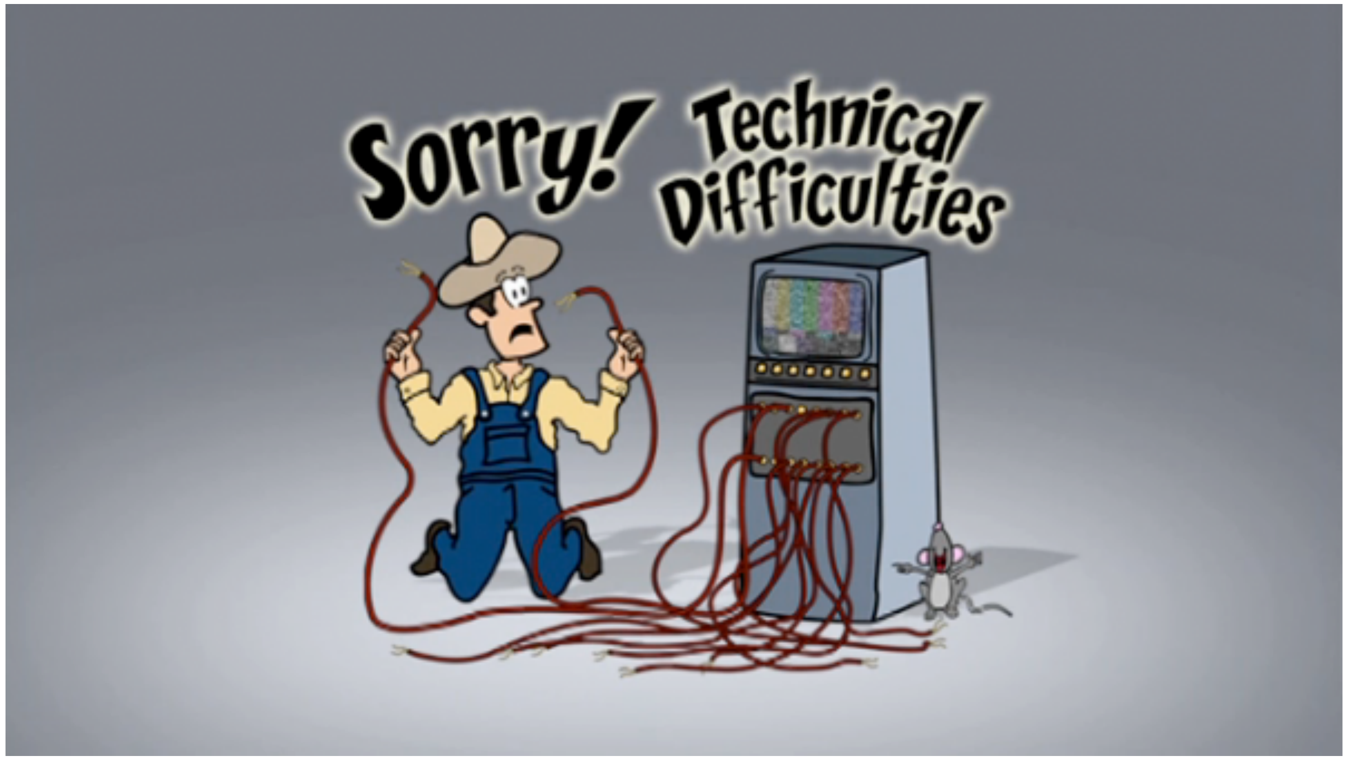
Can we compare “isotropic alpha” with MHD?

Comparing hydro+alpha with MHD: difficult…

- different physics e.g. GR or post-Newtonian
- different numerics e.g. grid or SPH

Not clear how to interpret any potential differences

No clear differences
to date(?)



Can we compare “isotropic alpha” with MHD?

But we try: Nealon et al. (2016)

Try to match Krolik & Hawley (2016) parameters

See what happens…


F. S. Church

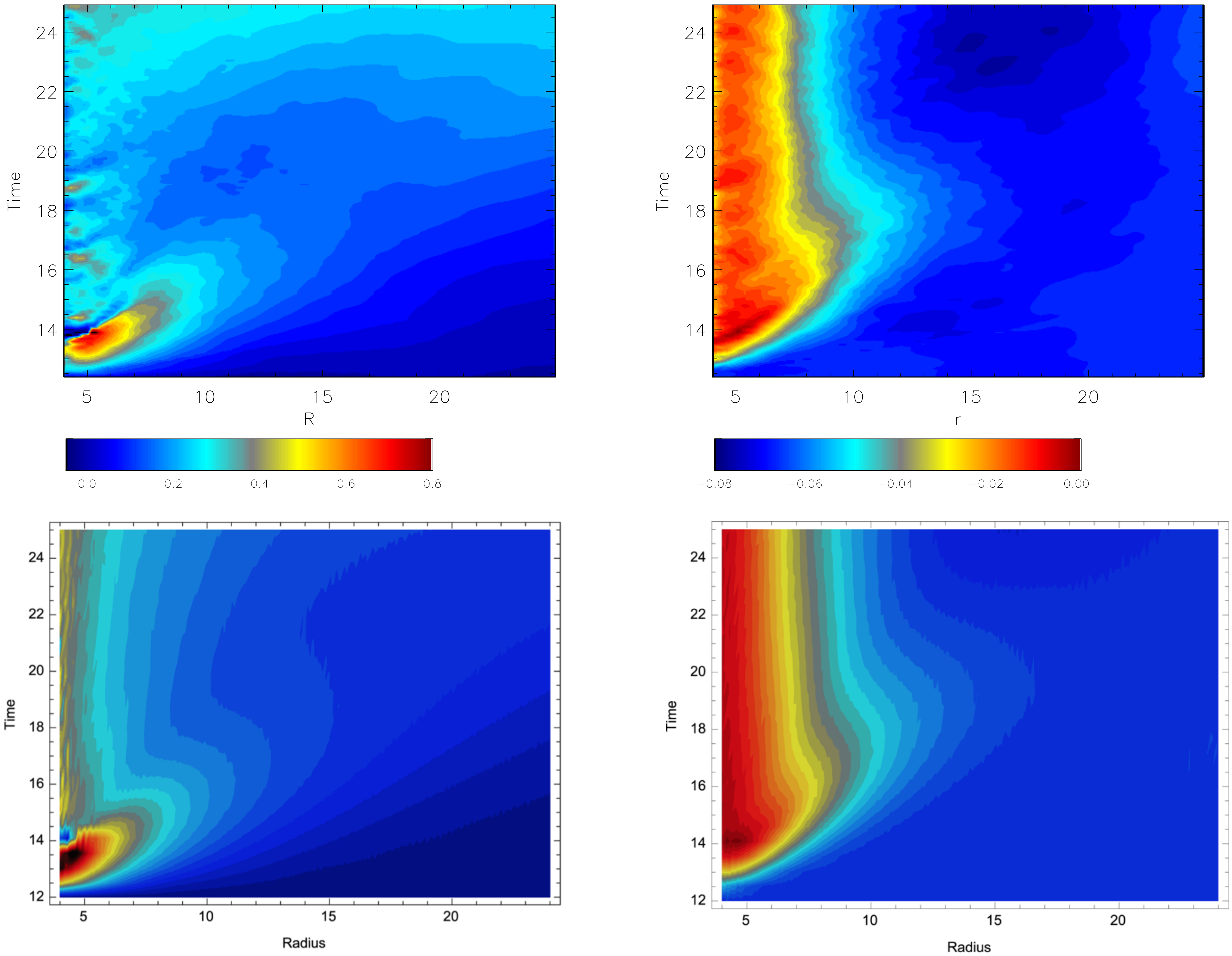
[Note they take a=1.05, we take a=1.0, but we try to match radial range, surface density, H/R, alpha(R) etc.]

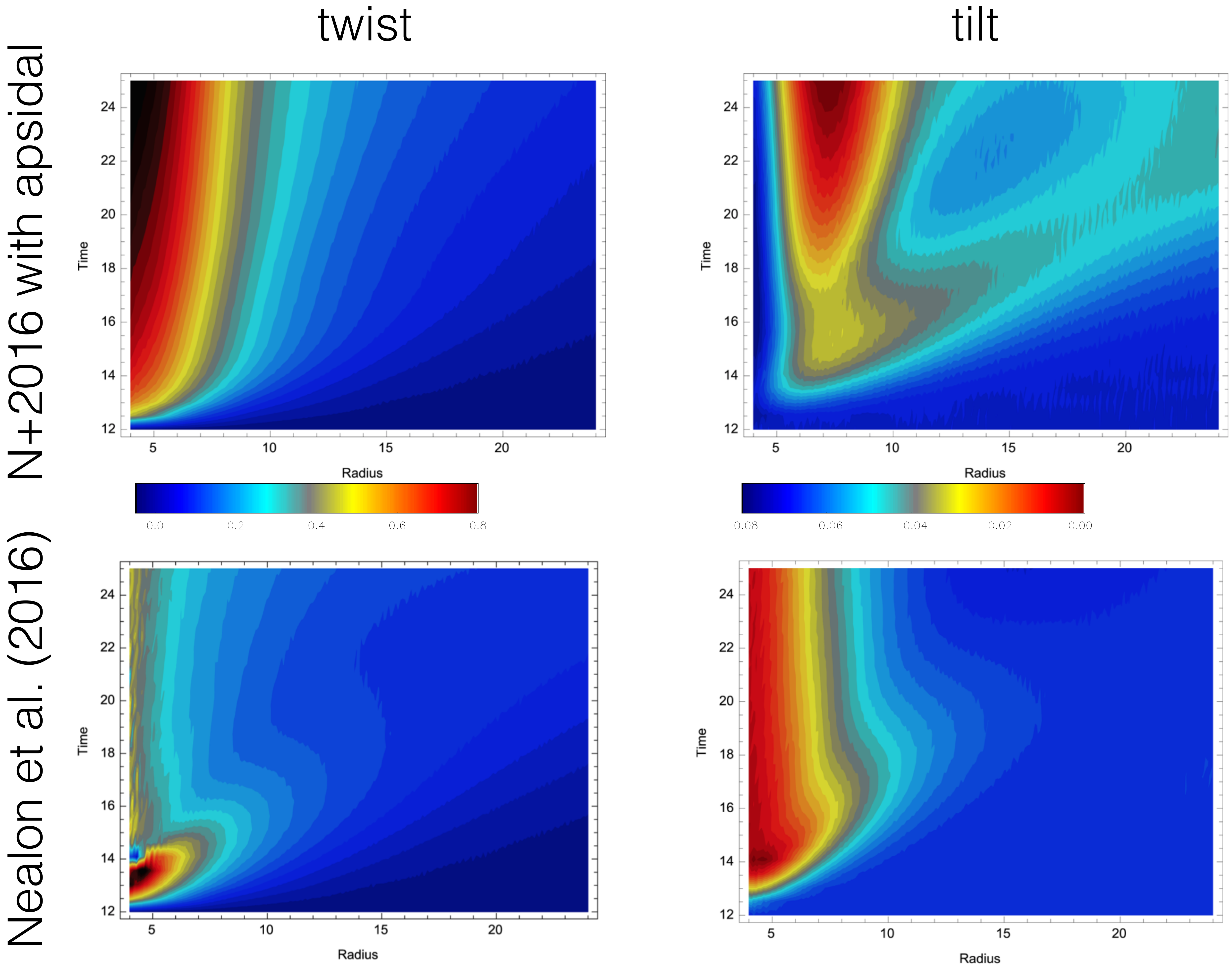
twist

tilt

KH2015

Nealon et al. (2016)





twist

tilt

N+2016 with apsidal

Nealon et al. (2016)

Conclusion: LOTS MORE TO DO!

- MHD makes small change from alpha
- getting precession physics right makes large change

Is this the final answer? NO

- capturing large scale MHD structures needed
- highly parameter dependent

  (previous sim: strong precession, wavelike, nonlinear dissipation etc)
- need a controlled experiment to explore differences in models

Extreme warping:

Wavelike: misalignment communicated by pressure waves

Diffusive: misalignment communicated by “vertical” viscosity

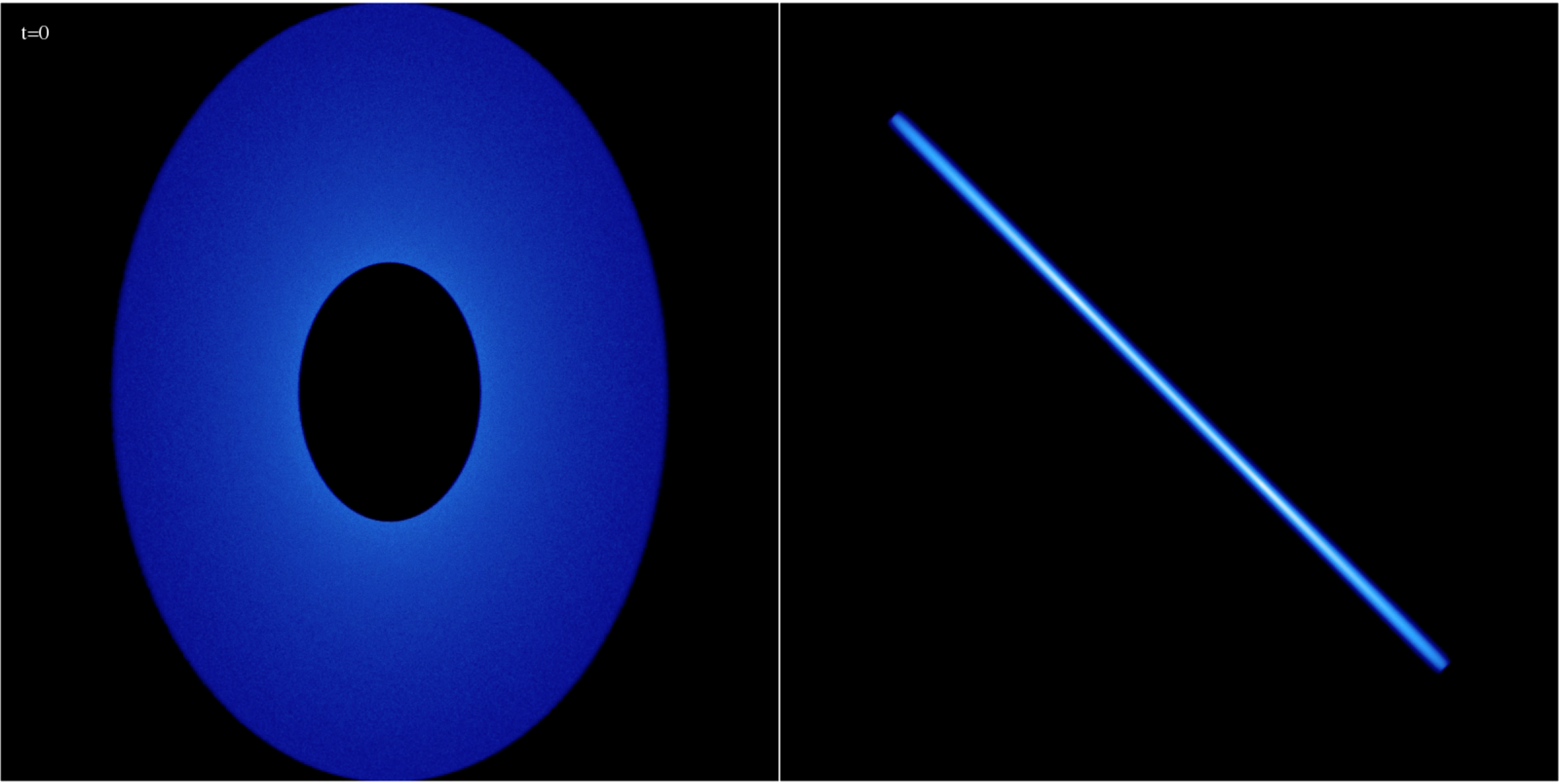
In both cases, the precession time may be faster…

Disc “breaks”:

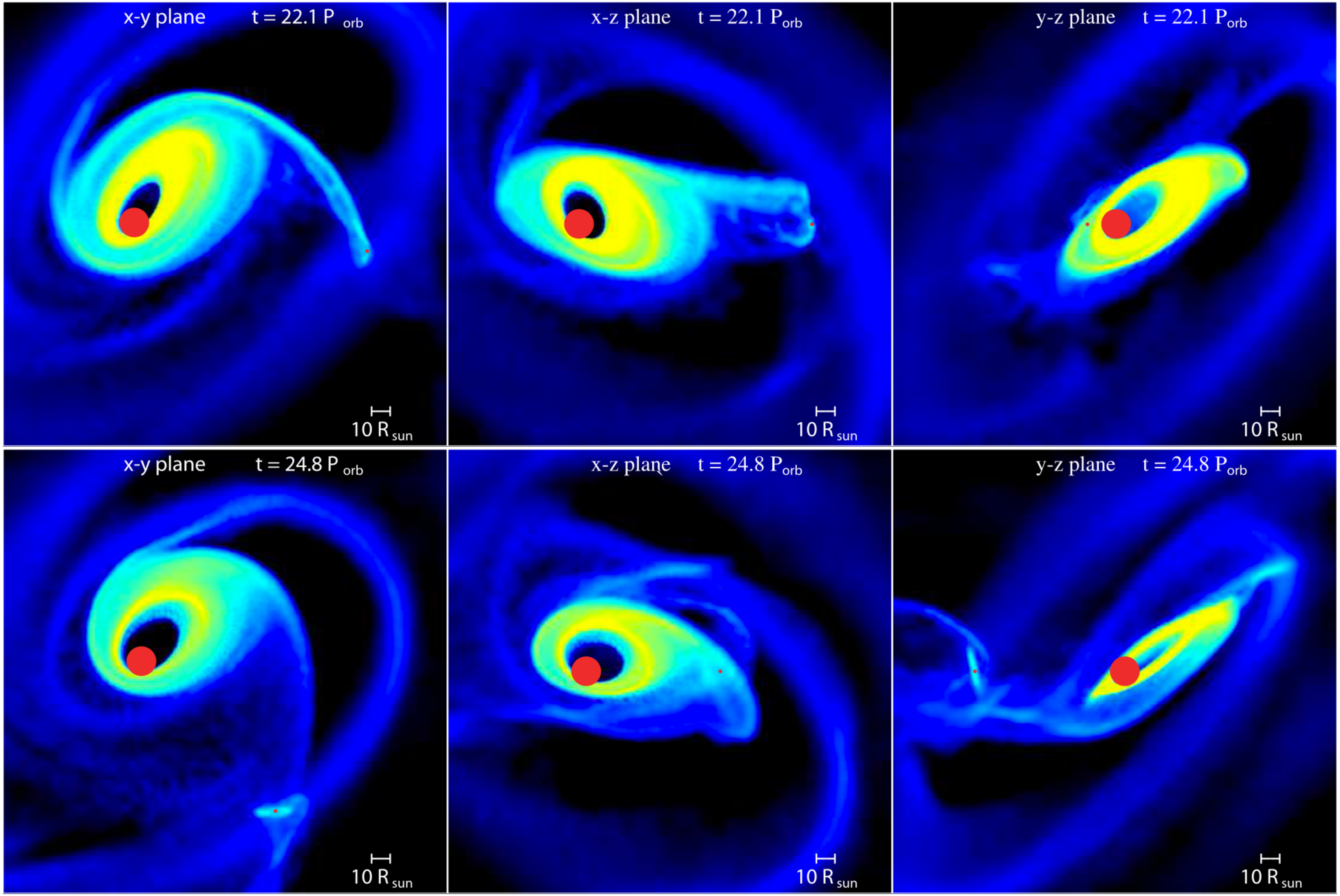
predominantly SPH
no MHD (yet)

Larwood et al. (1996)
Larwood & Papaloizou (1997)
Lodato & Price (2010)
Fragner & Nelson (2010)
Nixon & King (2012)
Nixon et al. (2012,2013)
Dogan et al. (2015)
Nealon et al. (2015,2016)
etc.

Lense-Thirring disc tearing



Be Star/X-ray binaries: Martin et al. (2014)



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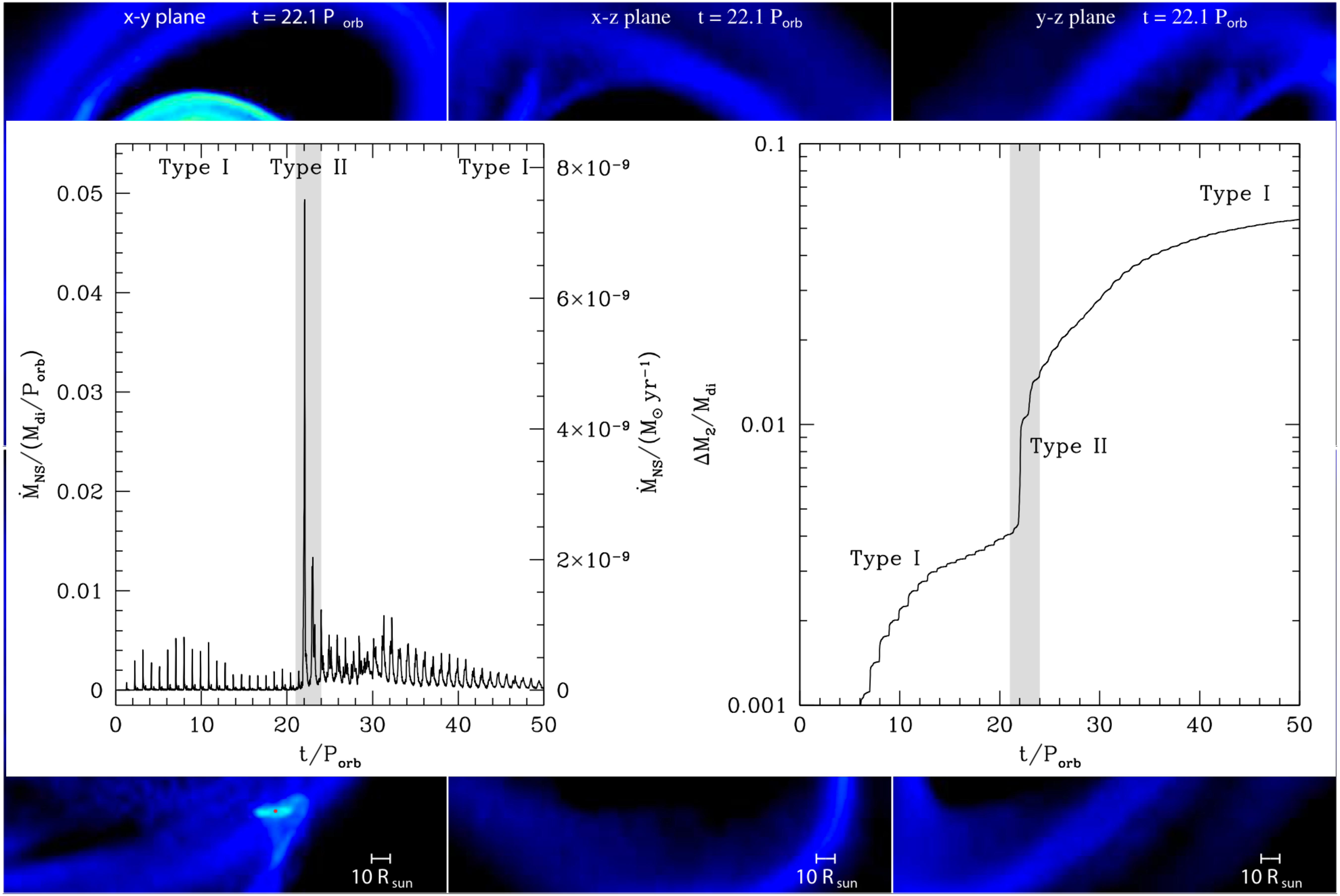
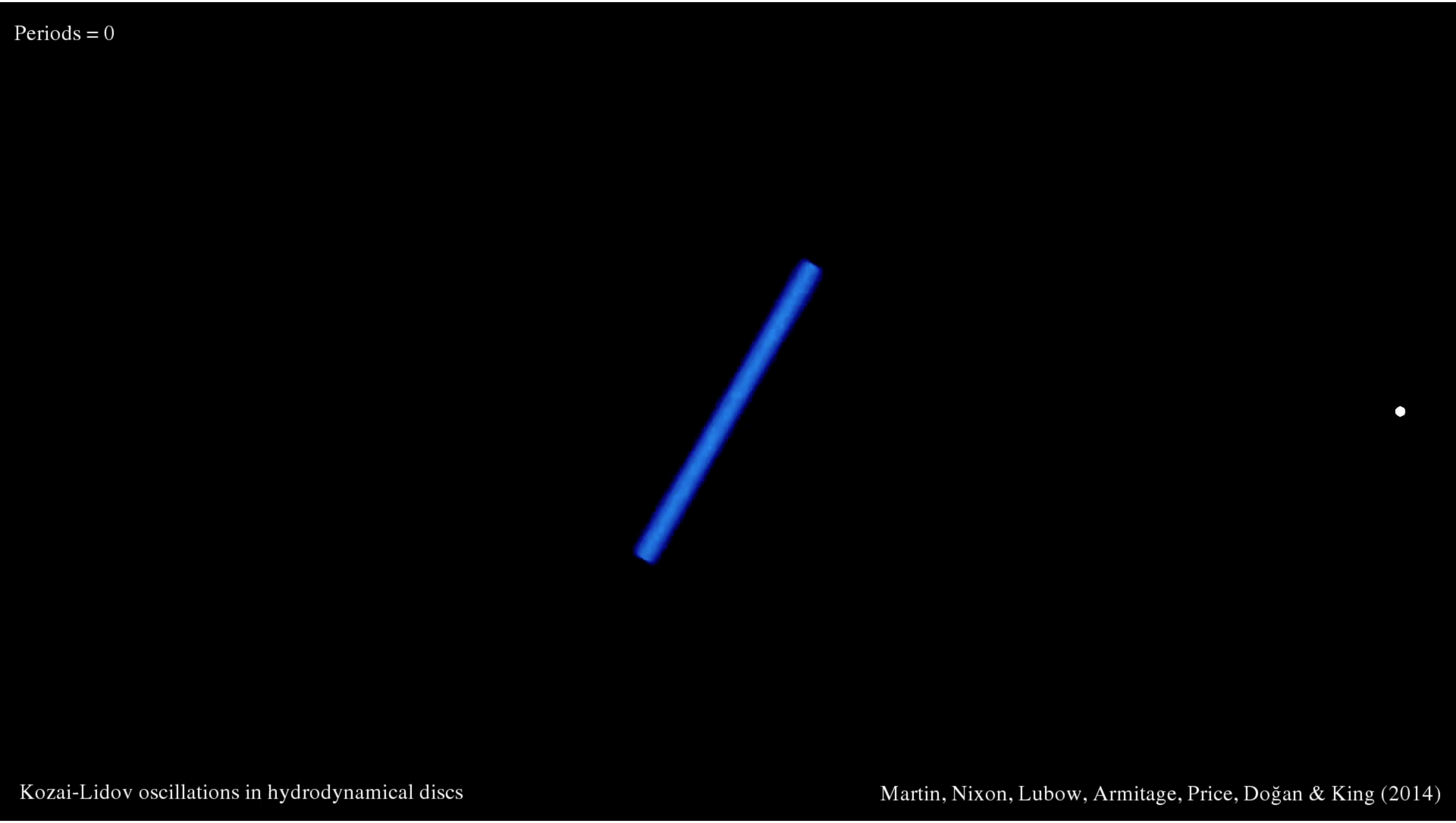


Kozai-Lidov cycles in an accretion disc:



Kozai-Lidov oscillations in hydrodynamical discs

Martin, Nixon, Lubow, Armitage, Price, Doğan & King (2014)

Kozai-Lidov cycles in an accretion disc:

Periods = 0

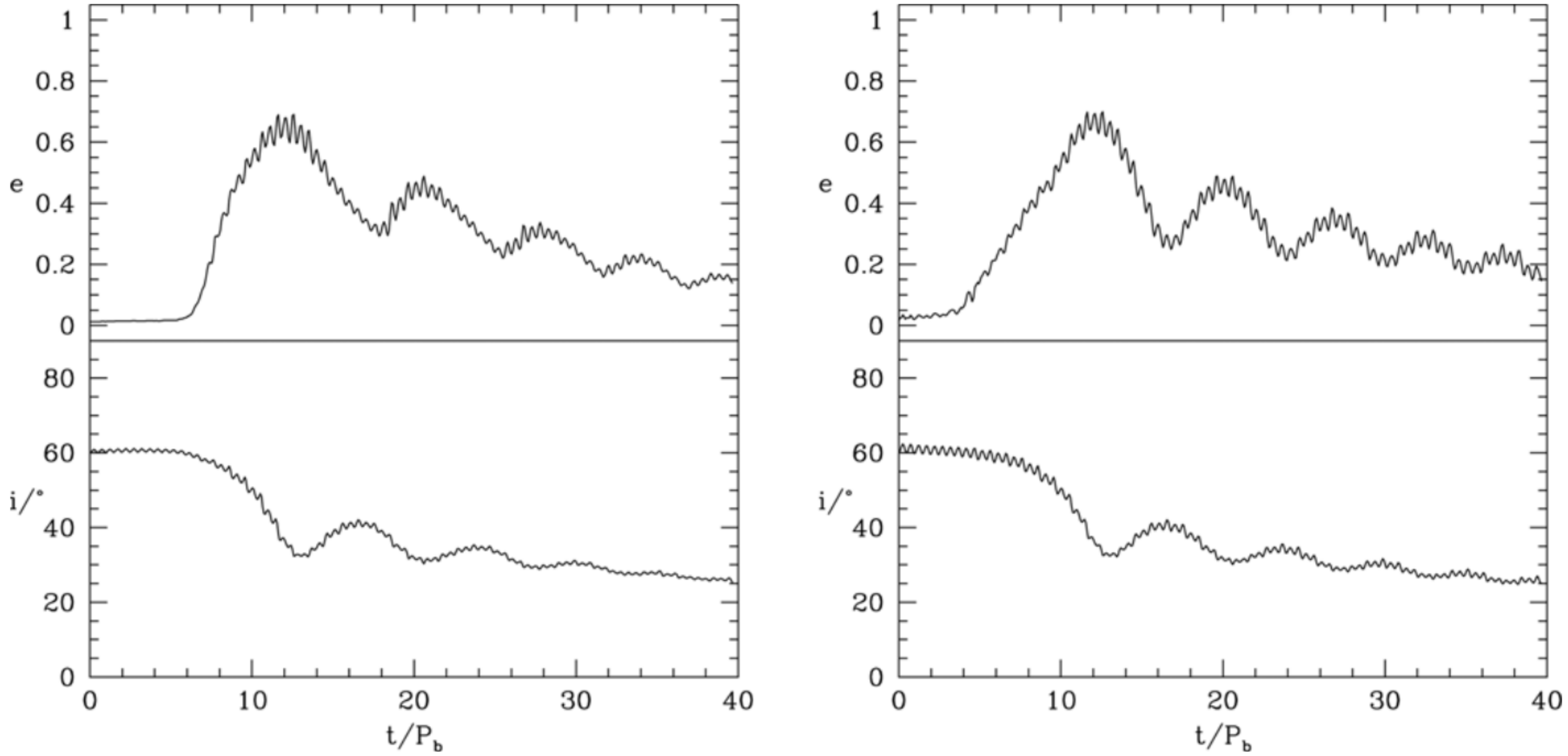


Figure 3. Eccentricity and inclination evolution of the disk at a radius $d = 0.1\,a$ (left) and $d = 0.2\,a$ (right) from the primary.

Kozai-Lidov oscillations in hydrodynamical discs

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Recap:

LOTS TO DO…!