Handling astrophysical uncertainties on direct detection experiments

Anne Green
University of Nottingham

- Astrophysical uncertainties
  - i) observations
  - ii) simulations
- Consequences
- Strategies
  - i) integrate out
  - ii) marginalize over
- Parameterising the speed distribution

Introduction

Differential event rate for elastic scattering:
(assuming spin-independent coupling and $f_p=f_n$)

$$\frac{\mathrm{d}R}{\mathrm{d}E} = \frac{\sigma_{\mathrm{p}}\rho_0}{\mu_{\mathrm{p},\chi}^2 m_\chi} A^2 F^2(E) \int_{v_{\min}}^{\infty} \frac{f(v)}{v}\,\mathrm{d}v \qquad v_{\min} = \left(\frac{E(m_A+m_\chi)^2}{2m_A {m_\chi}^2}\right)^{1/2}$$

Particle physics parameters:

WIMP mass and cross-section, $m_\chi \quad \sigma_{\mathrm{p}}$

Astrophysical input:

local DM density and speed distribution $\rho_0 \quad f(v)$

Realisation that uncertainties in f(v) will affect signals goes right the way back to the early direct detection papers in the 1980s (e.g. Drukier, Freese & Spergel).

Speed integral: $$g(v_{\min}) = \int_{v_{\min}}^{\infty} \frac{f(v)}{v}\,\mathrm{d}v \qquad \frac{\mathrm{d}R}{\mathrm{d}E} \propto g(v_{\min})$$

Experimental constraints on σ-m_χ plane usually calculated using 'standard halo model':

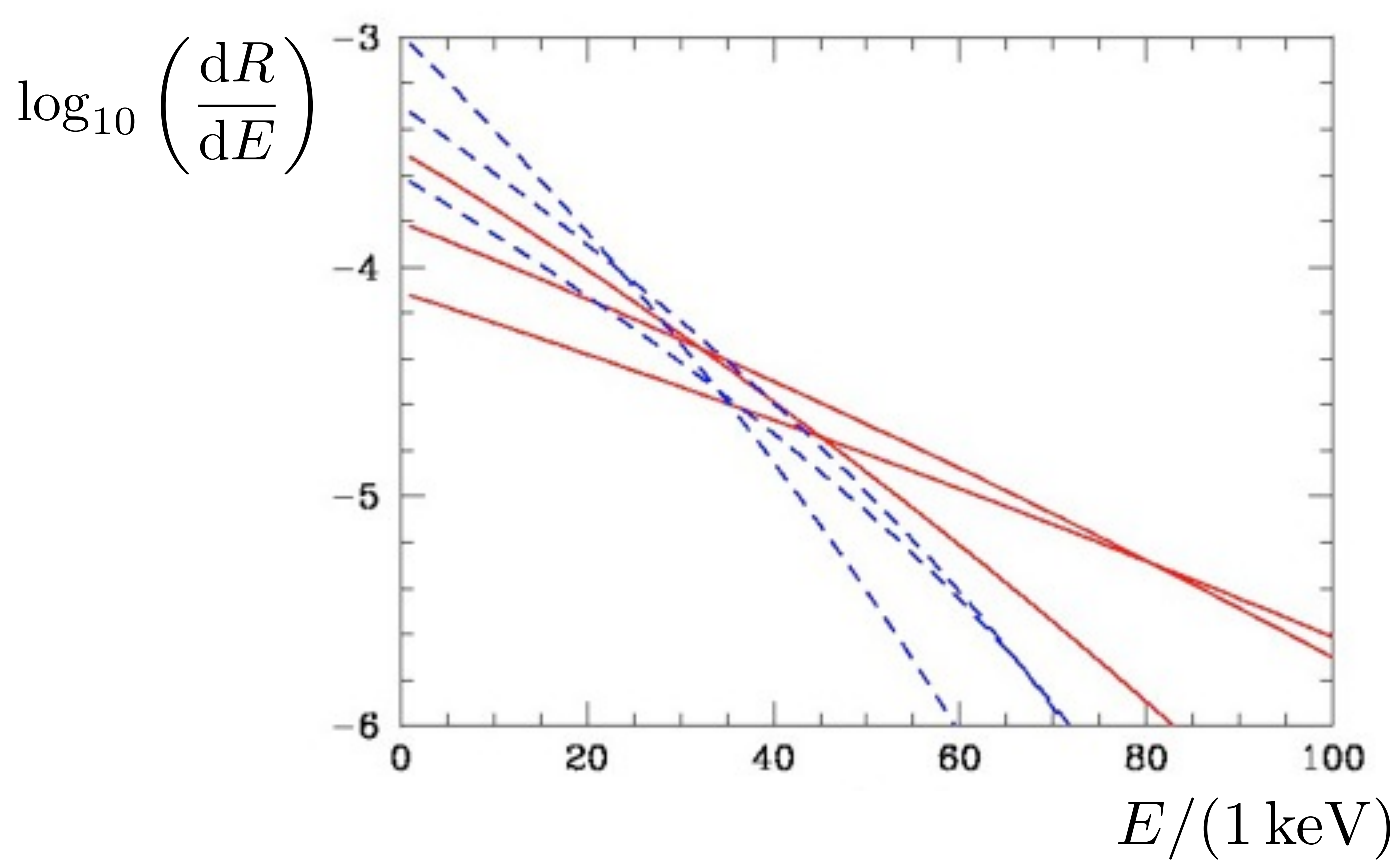
isotropic, isothermal sphere, with Maxwell-Boltzmann speed distribution

$$f(\mathbf{v}) \propto \exp\left(-\frac{3|\mathbf{v}|^2}{2\sigma^2}\right) \qquad \sigma = \sqrt{\frac{3}{2}} v_c$$

with v_c=220 km s^{-1} and local density ρ_0=0.3 GeV cm^{-3}

In this case (very roughly, ignoring escape speed & Earth's orbit)

$$g(v_{\rm min}) \propto \exp\left(-\frac{E}{E_{\rm R}}\right) \qquad E_{\rm R} = \frac{2\mu_{A\chi}^2 v_c^2}{m_A} \quad \begin{array}{ll} \propto m_\chi^2 & m_\chi \ll m_A \\ \sim \text{const} & m_\chi \gg m_A \end{array}$$



Differential event rate:
Ge and Xe m_χ = 50, 100, 200 GeV

Astrophysical uncertainties i) observations

Local density:

General approach: use multiple data sets (rotation curve, velocity dispersions of halo stars, local surface mass density, total mass...) and model for the MW (luminous components and halo).

For a 'fixed' halo density profile can get high precision determination:

e.g. Widrow et al. cuspy halos: $\rho_0 = (0.3 \pm 0.05)\,\mathrm{GeV\,cm^{-3}}$

Catena & Ullio NFW & Einasto profiles: $\rho_0 = (0.39 \pm 0.03)\,\mathrm{GeV\,cm^{-3}}$

With a range of profiles get a spread of values:

e.g. Weber and de Boer $\rho_0 = (0.2 - 0.4)\,\mathrm{GeV\,cm^{-3}}$

Model independent/minimal assumption methods give larger errors:

e.g. Salucci et al. eqn of centrifugal eqm $\rho_0 = (0.43 \pm 0.11 \pm 0.10)\,\mathrm{GeV\,cm^{-3}}$

Garbari et al. solve Jeans-Poisson eqns $\rho_0 = 0.85^{+0.57}_{-0.50}\,\mathrm{GeV\,cm^{-3}}$

Pato et al. DM density in stellar disc of simulated halos is ~ 20% larger than the shell average determined by observations.

Work in progress with Fornasa: what comes out is strongly dependent on what goes in.

Summary: recent determinations have ~10% statistical errors, but systematic uncertainties from modelling are still significantly larger.

Local circular speed:

IAU/Kerr & Linden-Bell compilation of measurements: $v_{\rm c} = (220 \pm 20)\,{\rm km\,s^{-1}}$

Proper motion of Sgr A* Reid & Brunthaler and maser data Reid et al: $v_{\rm c} \sim (250 \pm 10)\,{\rm km\,s^{-1}}$

Bovy et al. if non-random phases of masers modelled only get weak constraint combined with Sgr A* & GD-1 stellar stream, assuming flat rotation curve: $v_{\rm c} = (236 \pm 11)\,{\rm km\,s^{-1}}$

McMillan & Binney allowing non-flat rotation curve: $v_{\rm c} = (200 - 280)\,{\rm km\,s^{-1}}$

Bovy et al. APOGEE data (l.o.s. v of 3000 stars): $v_{\rm c} = (218 \pm 6)\,{\rm km\,s^{-1}}$

$v_{\phi,\odot} = 242^{+10}_{-3}\,{\rm km\,s^{-1}}$ in agreement with proper motion of Sgr A* but > $v_{\rm c} + v_{\phi,\odot,{\rm LSR}}$
$v_{\phi,\odot,{\rm LSR}}$ larger or LSR orbit non-circular (due to large scale streaming motions)?

Modelling uncertainties larger than statistical uncertainties here too.

n.b. For the standard halo there's a one-to-one relationship between circular speed and velocity dispersion, $\sqrt{2}\sigma = v_{\rm c}$, but in general the relationship depends on the density profile and velocity anisotropy, β :

$$\frac{1}{\rho}\frac{{\rm d}(\rho\sigma_r^2)}{{\rm d}r} + 2\frac{\beta\sigma_r^2}{r} = -\frac{v_c^2}{r}$$

Also for non-standard halos peak velocity, v_0, isn't equal to circular speed.

Local escape speed:

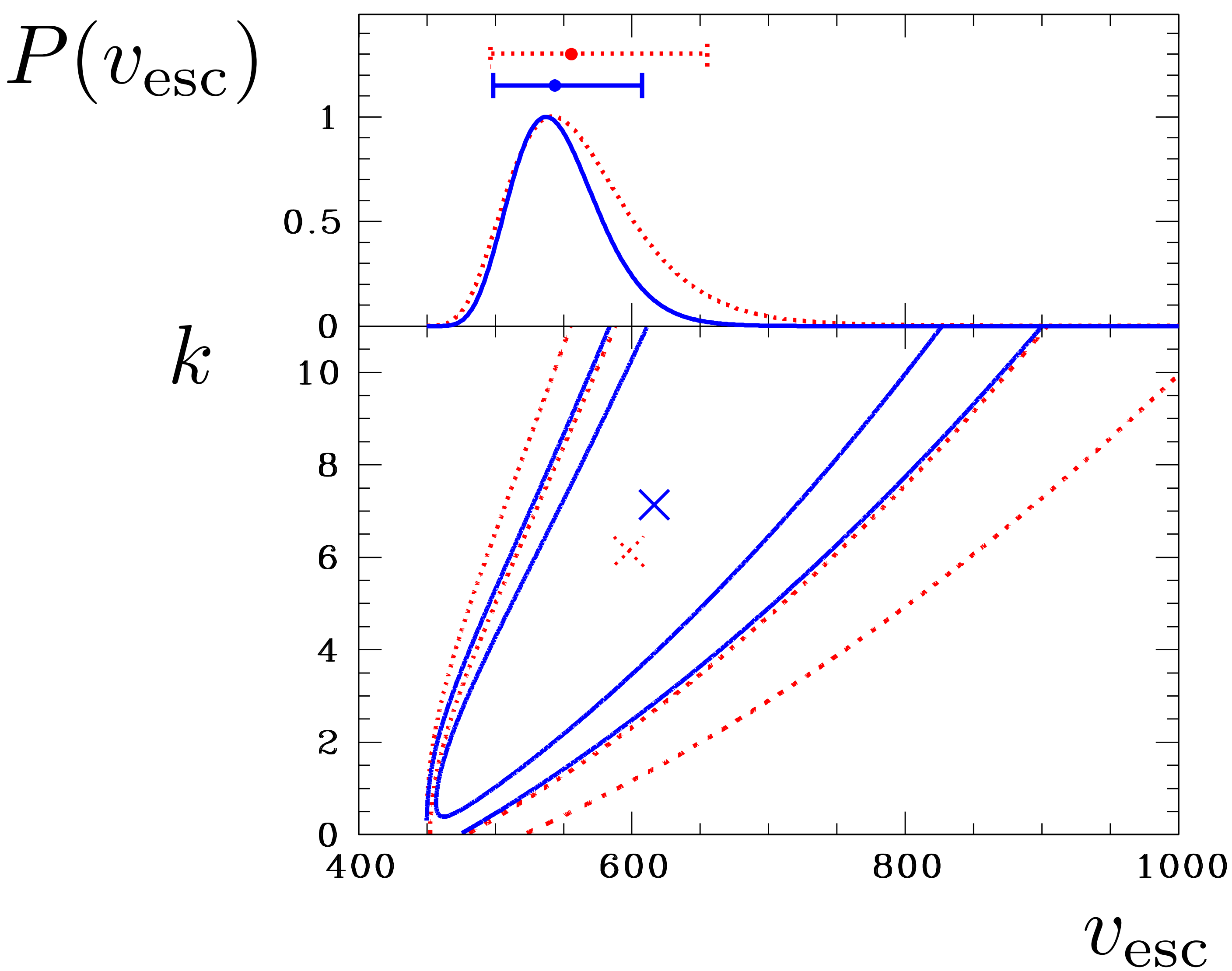
Smith et al: high velocity stars from the RAVE survey,

assume $f(|\mathbf{v}|) \propto (v_{\rm esc} - |\mathbf{v}|)^k$

with k in range 2.7 to 4.7 (motivated by numerical simulations):

$$498\,{\rm km\,s^{-1}} < v_{\rm esc} < 608\,{\rm km\,s^{-1}}$$

median likelihood: $v_{\rm esc} = 544\,{\rm km\,s^{-1}}$



all data

sub-sample of 16 high speed stars

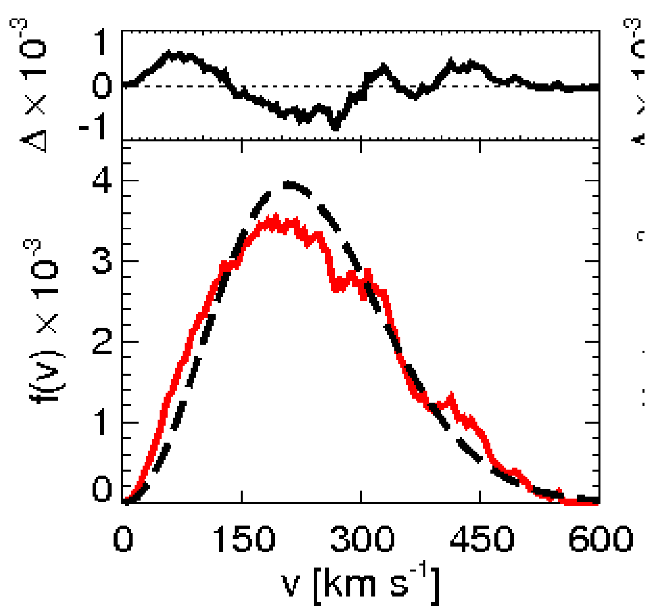
ii) simulations

Systematic deviations from multi-variate gaussian: more low speed particles, peak of distribution lower/flatter.

Features in tail of dist, ‘debris flows’, incompletely phased mixed material. Lisanti & Spergel; Kuhlen, Lisanti & Spergel

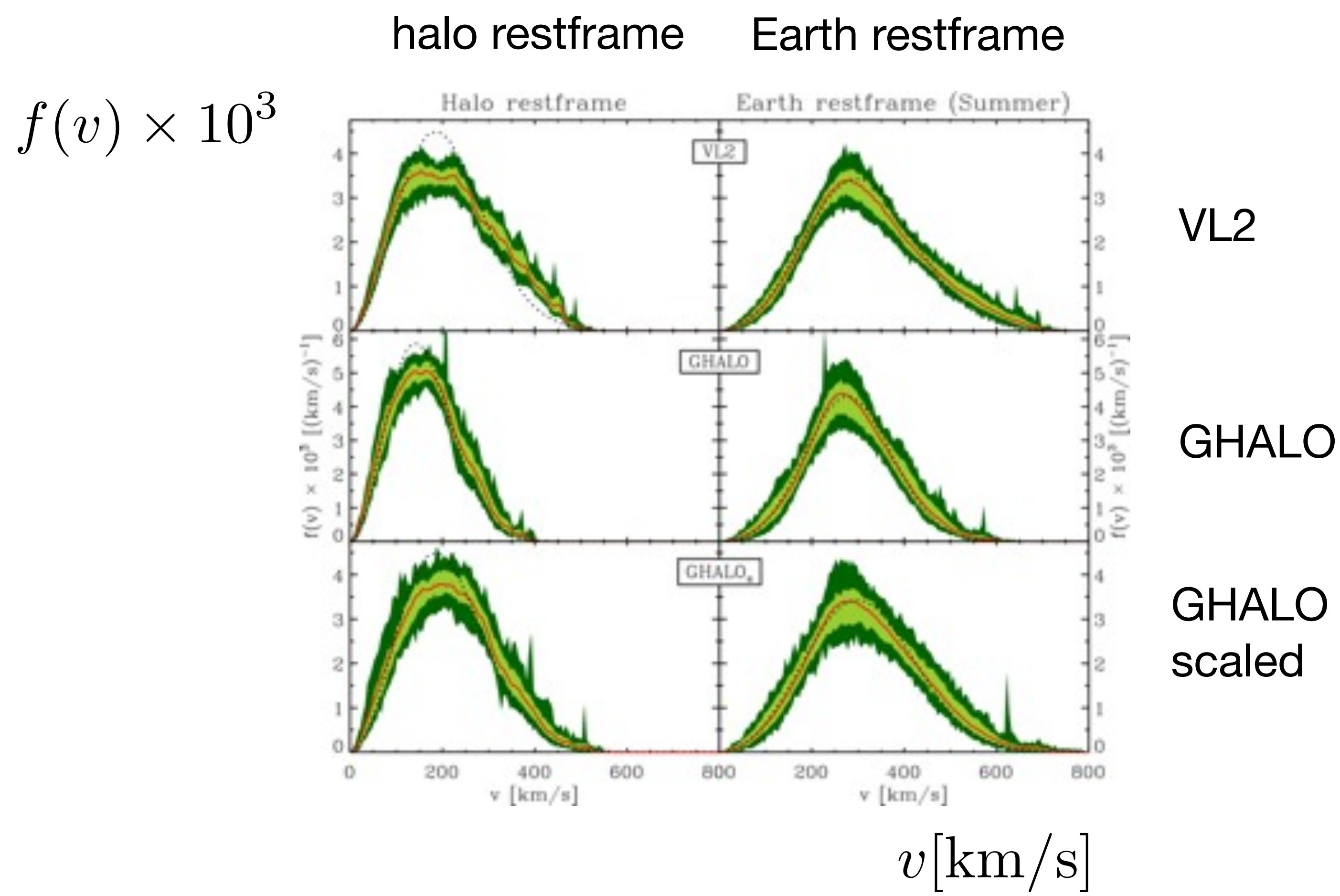
Deviations less pronounced in lab frame than Galactic rest frame.

Vogelsberger et al.



Aquarius simulation data,
best fit multi-variate Gaussian

Kuhlen et al.



Purcell, Zentner & Wang DM component of Sagittarius leading stream may pass through the solar neighbourhood (as originally suggested by Freese, Gondolo & Newberg).

Lisanti et al.

For a double power-law density distribution $\rho(r) = \frac{\rho_s}{(r/r_s)^\alpha (1 + (r/r_s))^{\gamma-\alpha}}$

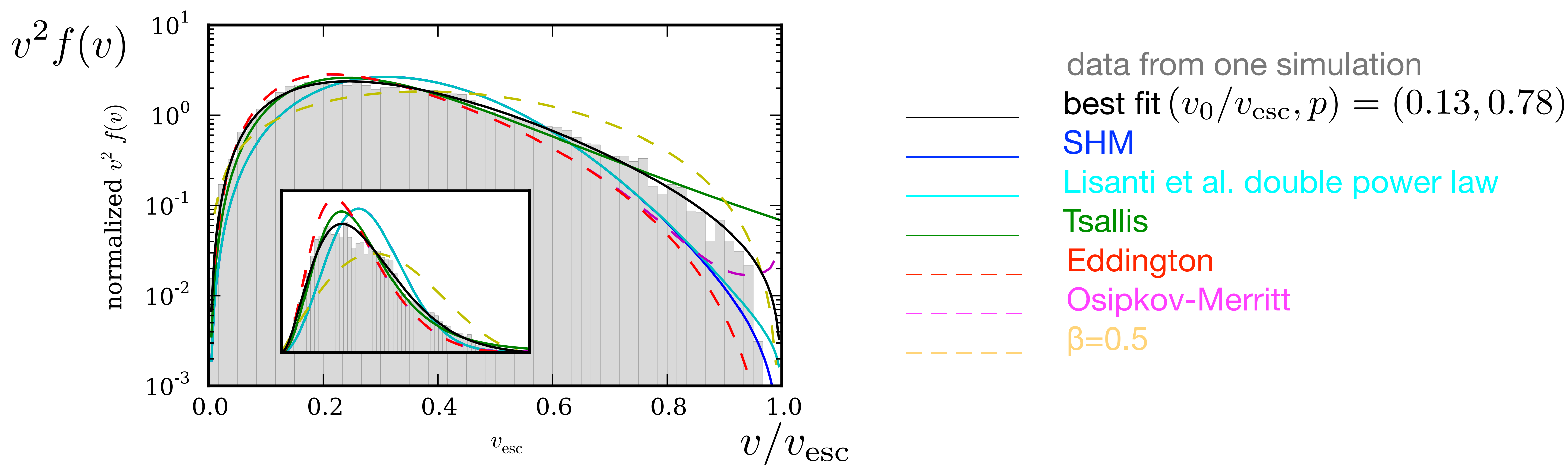
$$f(v) \propto \left[\exp\left(\frac{v_{esc}^2 - v^2}{k v_0^2} \right) - 1 \right]^k \Theta(v_{esc} - v)$$

is a good approx to numerical solutions of the Eddington equation (including a bulge and disk) and provides a better fit to the high speed tail of f(v) from simulations.

Mao et al.

Empirical function provides a better fit to simulation f(v) than previously considered fns.

$$f(\mathbf{v}) \propto \exp\left(-\frac{|\mathbf{v}|}{v_0} \right) (v_{esc}^2 - |\mathbf{v}|^2)^p \, \Theta(v_{esc} - |\mathbf{v}|)$$



Largest uncertainty comes from ratio of solar radius to scale radius.

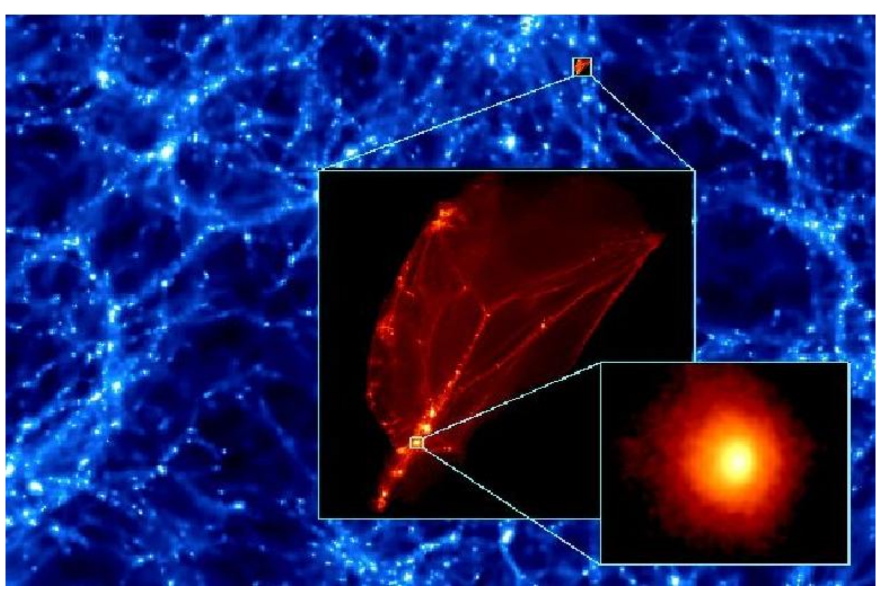
Caveats:

a) scales resolved by simulations are many orders of magnitude larger than those probed by direct detection experiments

~300 kpc

~30 kpc

~0.3 mpc



microhalo simulation

Diemand, Moore & Stadel

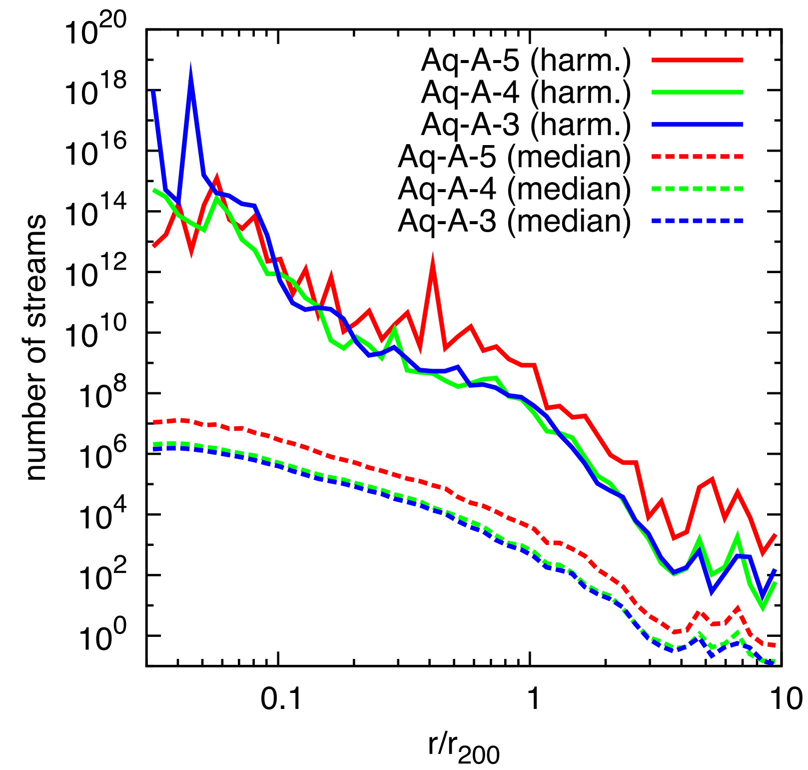
Resolution of best Milky Way simulations is many orders of magnitude larger than the mass of the first WIMP microhalos to form

fine structure in ultra-local DM velocity distribution?

Vogelsberger & White:

Follow the fine-grained phase-space distribution, in Aquarius simulations of Milky Way like halos.

From evolution of density deduce ultra-local DM distribution consists of a huge number of streams (but this assumes local density).



number of streams as a function of radius

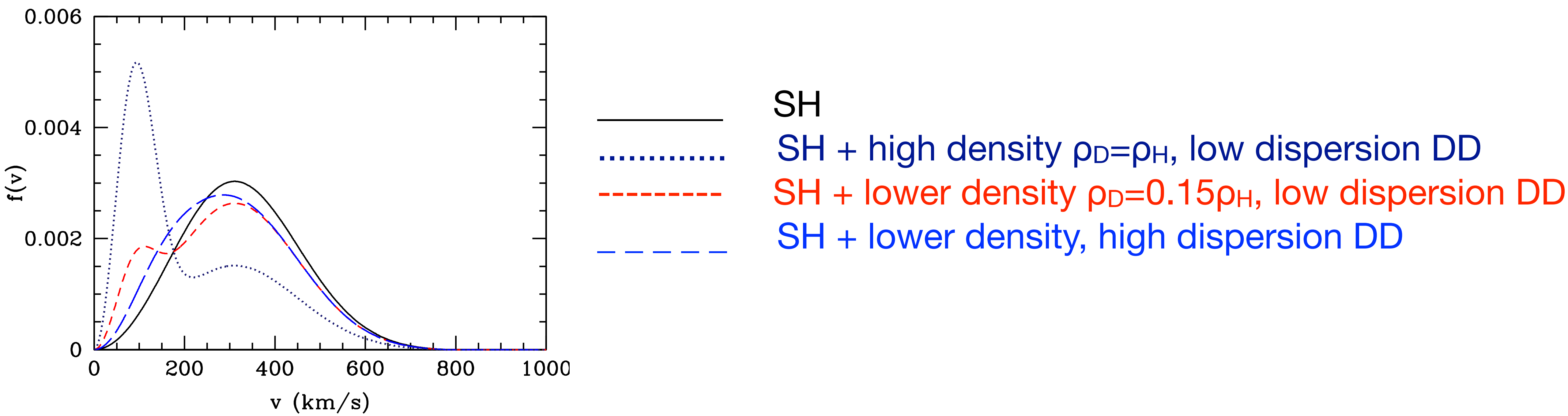
calculated using harmonic mean/median stream density

Schneider, Krauss & Moore: Simulate evolution of microhalos. Estimate tidal disruption and heating from encounters with stars, produces 10^2-10^4 streams in solar neighbourhood.

ii) effect of baryons on DM speed distribution?

Sub-halos merging at $z<1$ preferentially dragged towards disc, where they're destroyed leading to the formation of a co-rotating dark disc. Read et al., Bruch et al., Ling et al.

Could have a significant effect if density is high and velocity dispersion low.



Properties of dark disc are uncertain.

Purcell, Bullock & Kaplinghat to be consistent with observed properties of thick disc, MW's merger history must be quiescent compared with typical ΛCDM merger histories, hence DD density must be relatively low, $<0.2\,\rho_H$. Also dispersion larger than stellar thick disk.

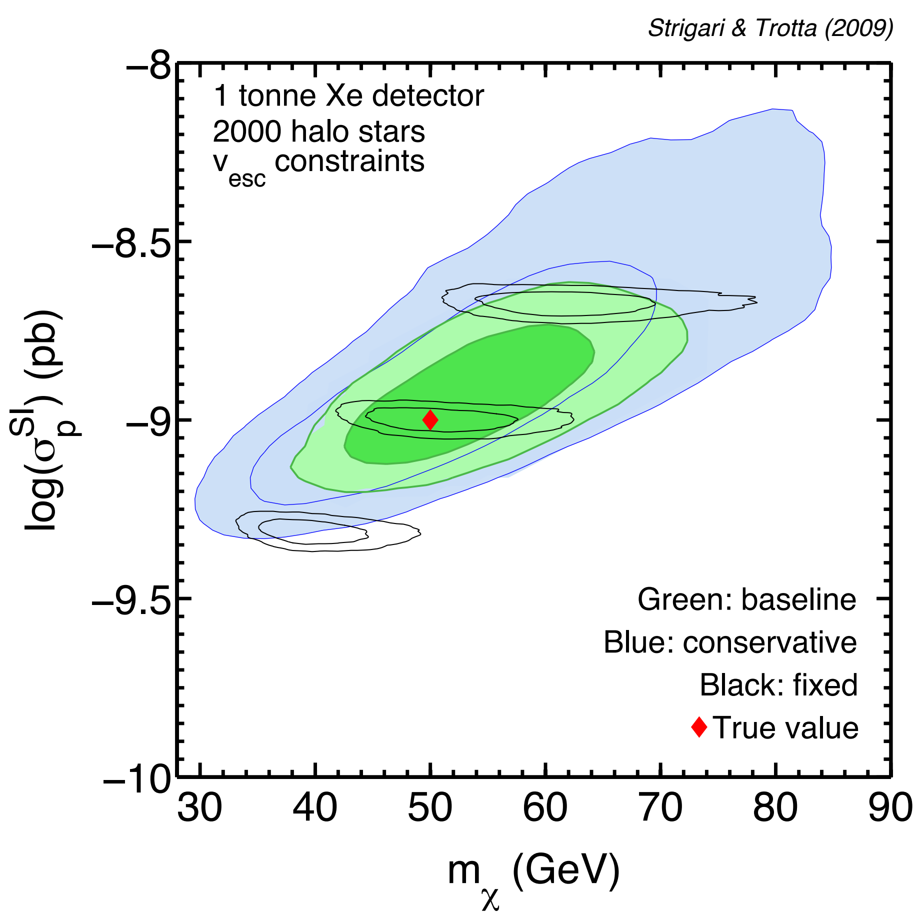
Bidin et al. measure surface density with 2-4 kpc of Galactic plane (using kinematics of thick disc stars), consistent with visible mass.

Consequences

Density:

Event rate proportional to product of σ and ρ, therefore uncertainties in ρ translate directly into uncertainties in σ, same for all DD experiments (but affects comparisons with e.g. collider constraints on σ).

Strigari & Trotta uncertainty leads to bias in determination of WIMP mass:



Circular speed (standard halo):

Shifts exclusion limits, similar, but not identical, effect for all experiments.

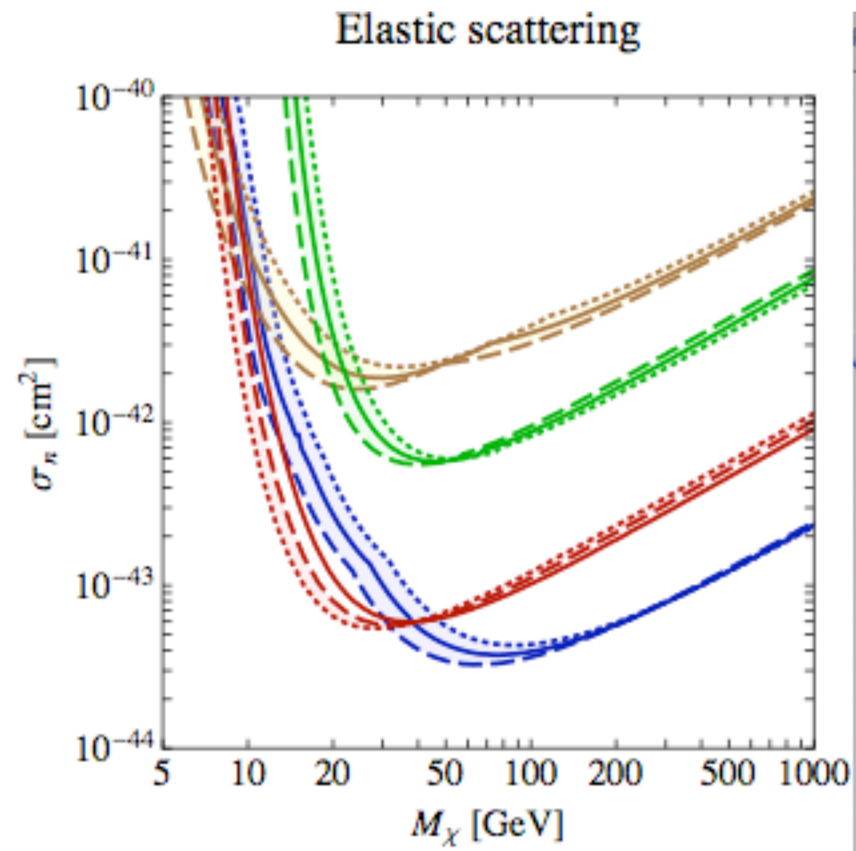
McCabe

.... v_c=195 km/s

_____ v_c=220 km/s

- - - v_c=255 km/s

(old)CDMSII Si, CDMSII Ge
CRESST, ZENON 10



Bias in future WIMP mass determination:

$$E_{\rm R} = \frac{2\mu_{A\chi}^2 v_c^2}{m_A}$$

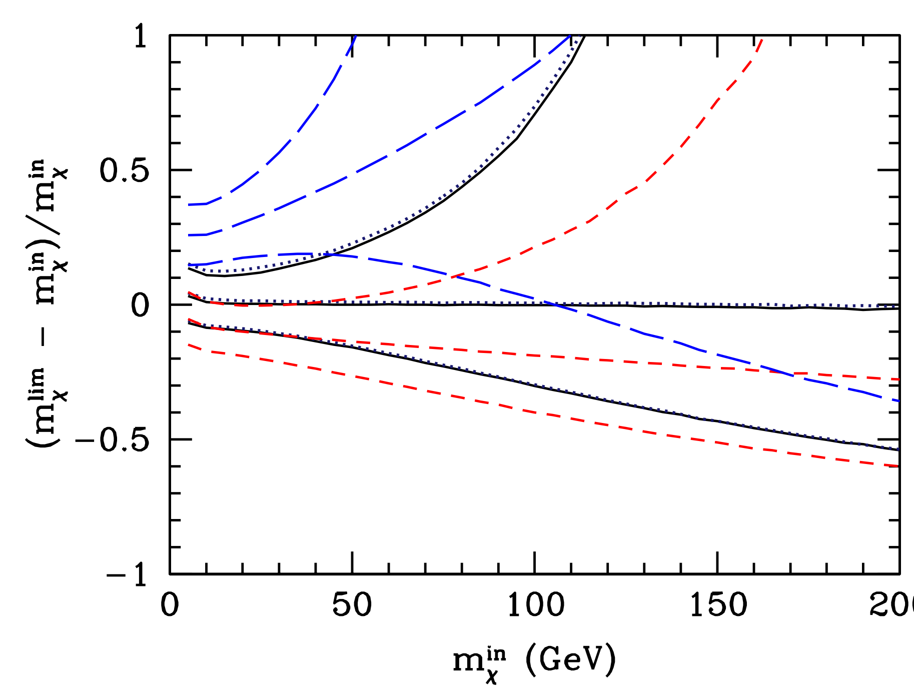
$$\frac{\Delta m_\chi}{m_\chi} = [1 + (m_\chi / m_{\rm A})] \frac{\Delta v_c}{v_c}$$

_____ v_c = 220 km/s

- - - - 200 km/s

_ _ _ _ 280 km/s

fractional mass limits from a simulated ideal Ge experiment, $\sigma = 10^{-8}$ pb



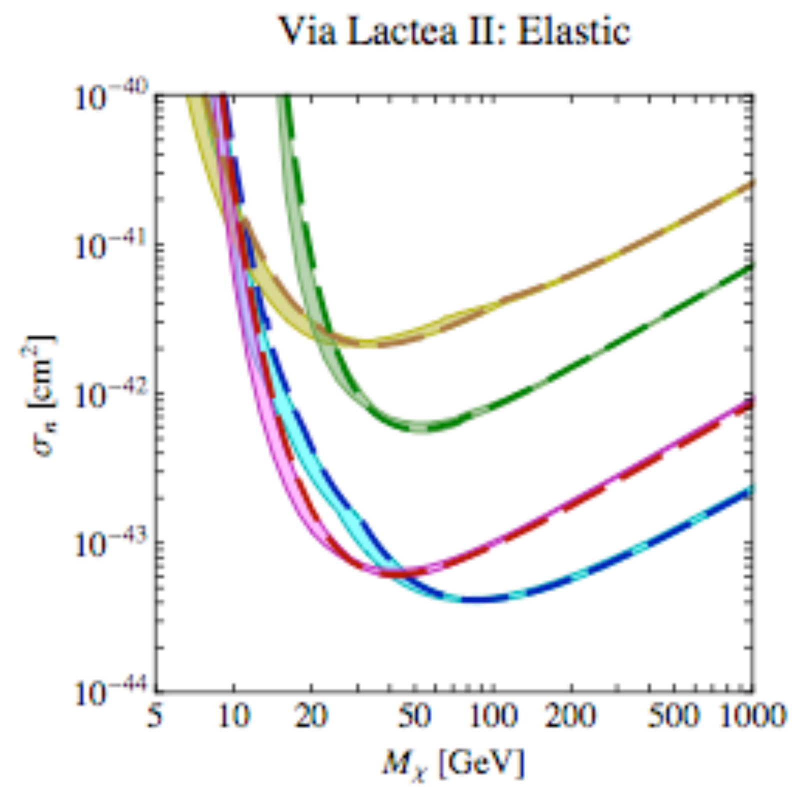
Shape of velocity distribution

Differential event rate is proportional to integral over speed distribution so exclusions limits are relatively insensitive to exact shape of velocity distribution:

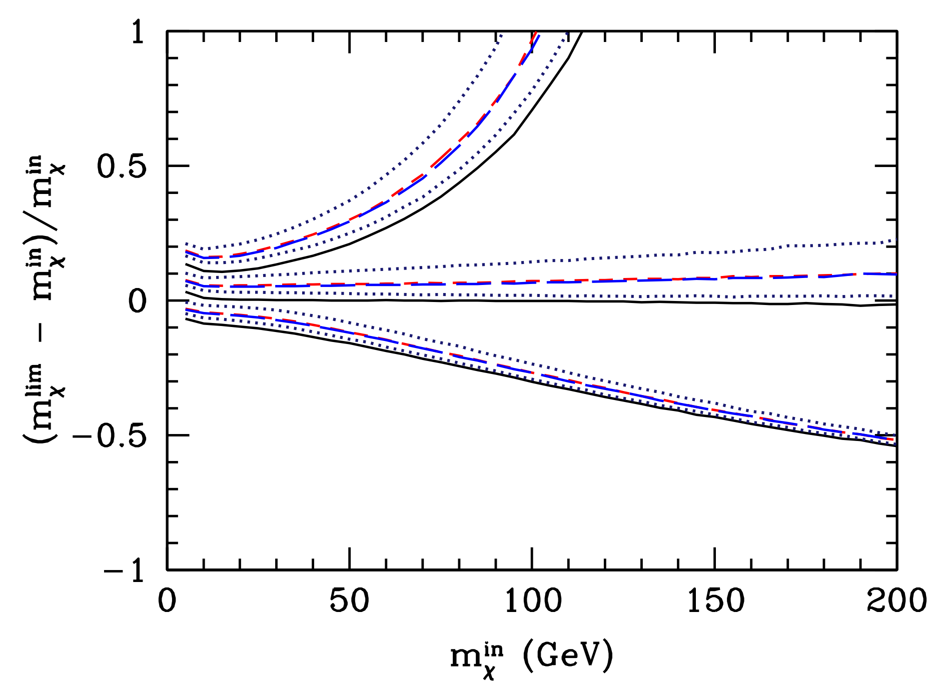
(smallish) change in shape/stochastic uncertainty in exclusion limits.

McCabe

(old)CDMSII Si, CDMSII Ge
CRESST, XENON 10

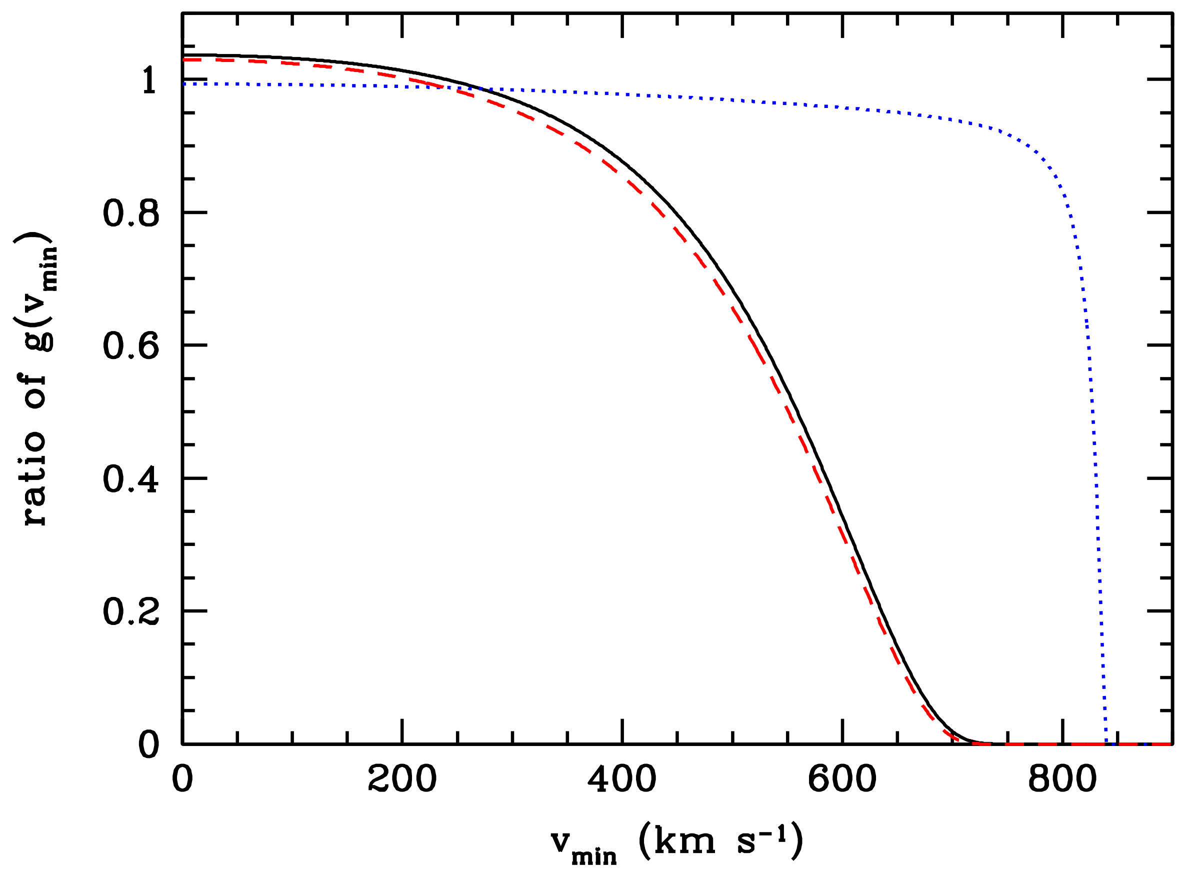


2-5% bias in future WIMP mass determination.



Escape speed (& shape of high v tail)

Can have significant effect on event rates/exclusion limits for light WIMPs:



Ratio of speed integral to that of Maxwellian with sharp cut-off at $v_{\rm esc} = 608 \, {\rm km \, s^{-1}}$:

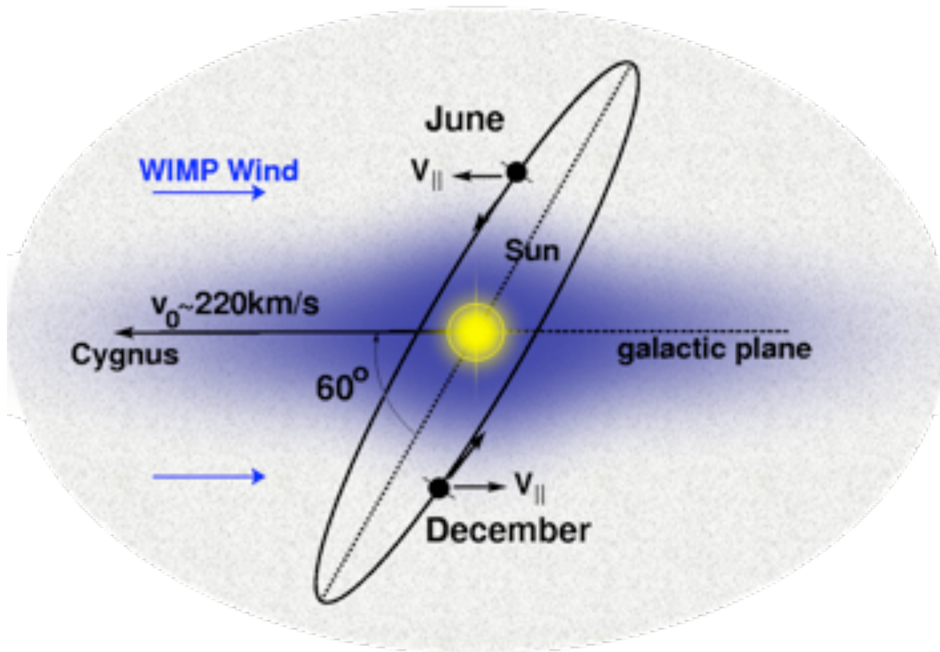
same f(v) neglecting Earth's orbit

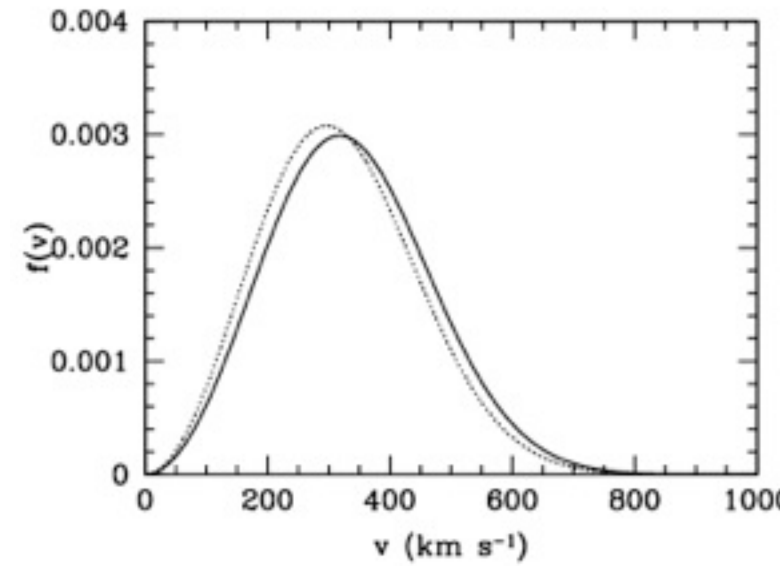
Lisanti et al. k=1.5 $v_{\rm esc} = 498 \, {\rm km \, s^{-1}}$

Lisanti et al. neglecting Earth's orbit

Dark disc

Could have a significant effect on mass determination and annual modulation, if density sufficiently high and/or velocity dispersion low.





Maxwellian speed dist.

detector rest frame (summer and winter)

Annual modulation

(arises from Earth's motion w.r.t. Galactic rest frame)

Phase, and amplitude, sensitive to detailed shape of speed distribution.

Direction dependence

(arises from Sun's motion w.r.t. Galactic rest frame)

Rear-front directional asymmetry is robust, but peak recoil direction of high energy recoils can change. Kuhlen et al.

Strategies i) integrate out

Fox, Liu & Weiner

Compare experiments in $g(v_{min})$ space:

$$g(v_{\rm min}) = \int_{v_{\rm min}}^{\infty} \frac{f(v)}{v}\, {\rm d}v \qquad\qquad v_{\rm min} = \left(\frac{E(m_A + m_\chi)^2}{2 m_A m_\chi^2} \right)^{1/2}$$

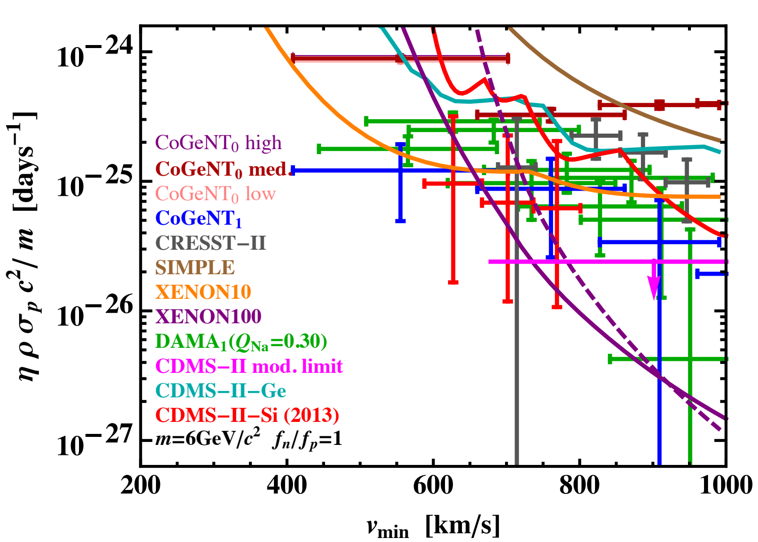
v_{min} values probed by each experiment depend on, unknown, WIMP mass, therefore need to do comparison for each mass of interest.

Can incorporate experimental energy resolution and efficiency Gondolo & Gelmini, and also annual modulation signals. Frandsen et al.; Herrero-Garcia, Schwetz & Zupan.

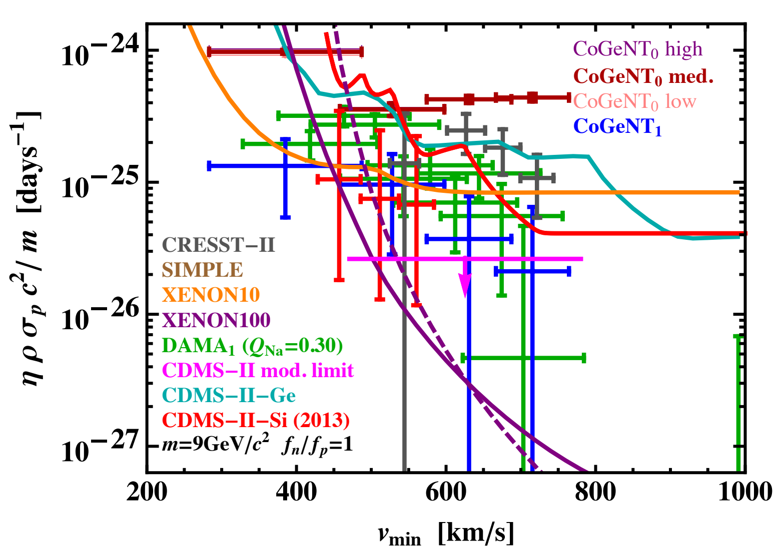
Extremely powerful for checking consistency of signals and exclusion limits. Frandsen et al.; Del Nobile, Gelmini, Gondolo & Huh.

Normalised $g(v_{min})$ versus v_{min} Del Nobile, Gelmini, Gondolo & Huh

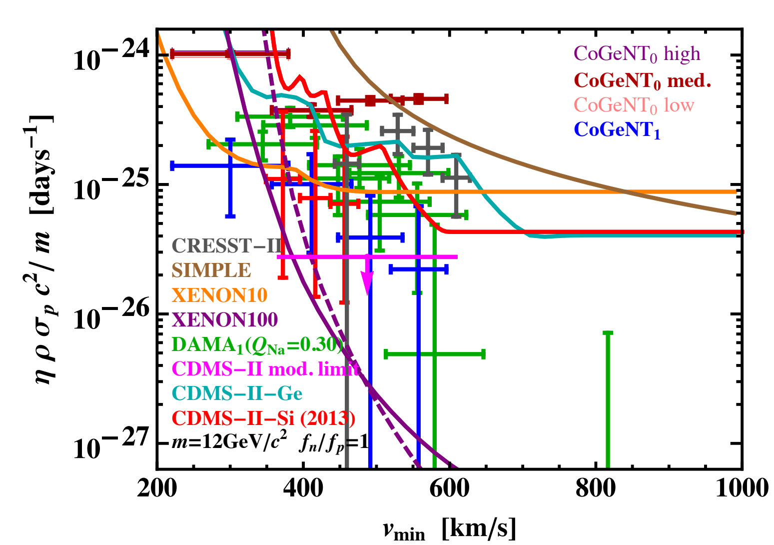
$m_\chi = 6\,{\rm GeV}$



$m_\chi = 9\,{\rm GeV}$



$m_\chi = 12\,{\rm GeV}$



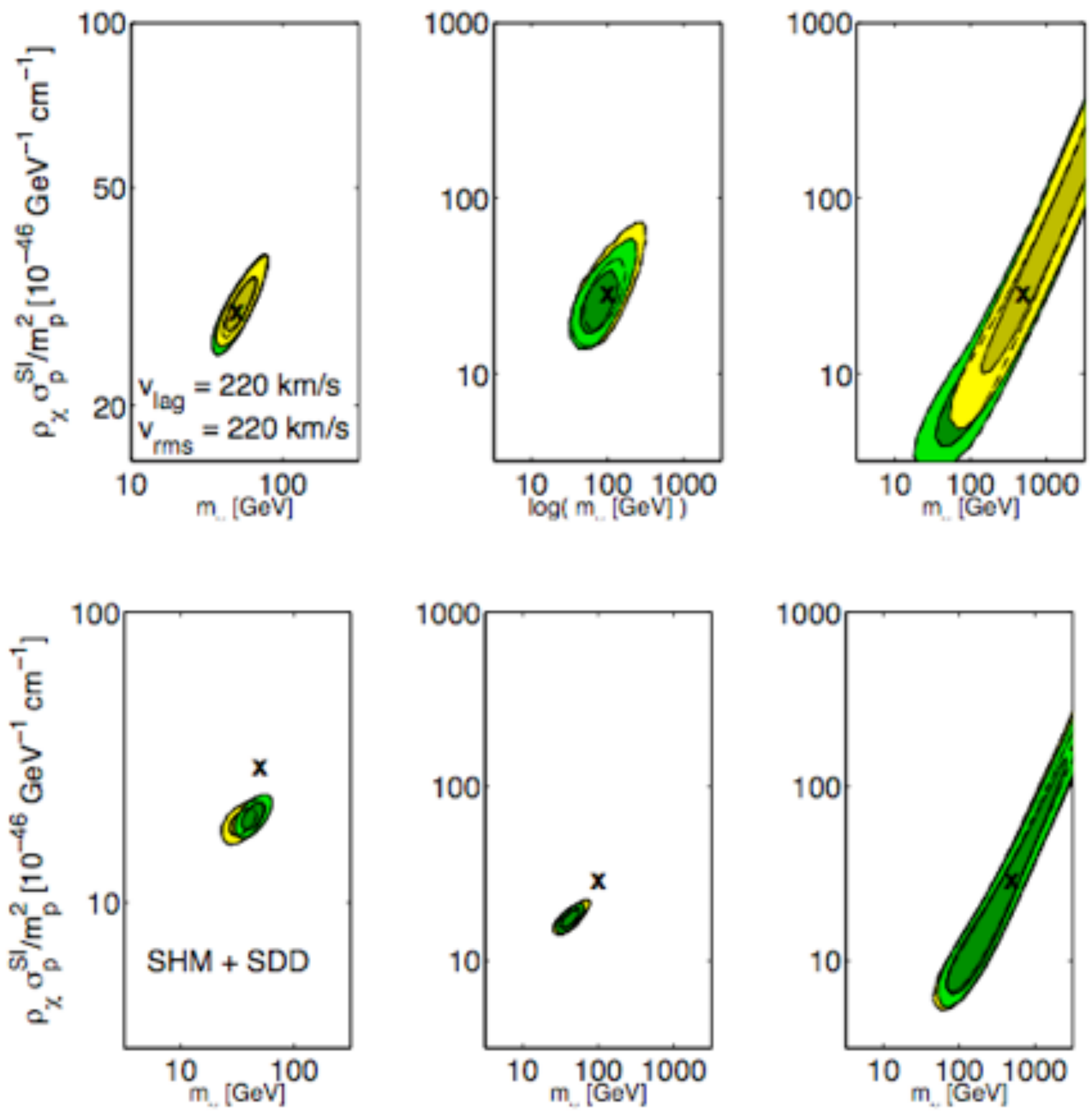
Strategies ii) marginalize over

Parameterize f(v) and/or Milky Way model and marginalize over these parameters, possibly including astrophysical data too e.g. stellar kinematics.
Strigari & Trotta; Peter x2; Pato et al. x2; Lee & Peter; Billard, Meyet & Santos; Alves, Hedri & Wacker; Kavanagh & Green x2; Friedland & Shoemaker

If actual shape of f(v) is similar to assumed shape this works well, but if not can get significant biases:

$m_\chi = 50\,\mathrm{GeV}$ $\quad 100\,\mathrm{GeV}$ $\quad 500\,\mathrm{GeV}$

$$D = \frac{\rho_0 \sigma_{\mathrm{p}}}{m_\chi^2}$$



m_χ

Peter Simulated data from future tonne scale Xe, Ar & Ge expts, analysed assuming standard halo model (allowing v_{lag} & v_{rms} to vary).

Standard halo model in

Standard halo model + dark disc in

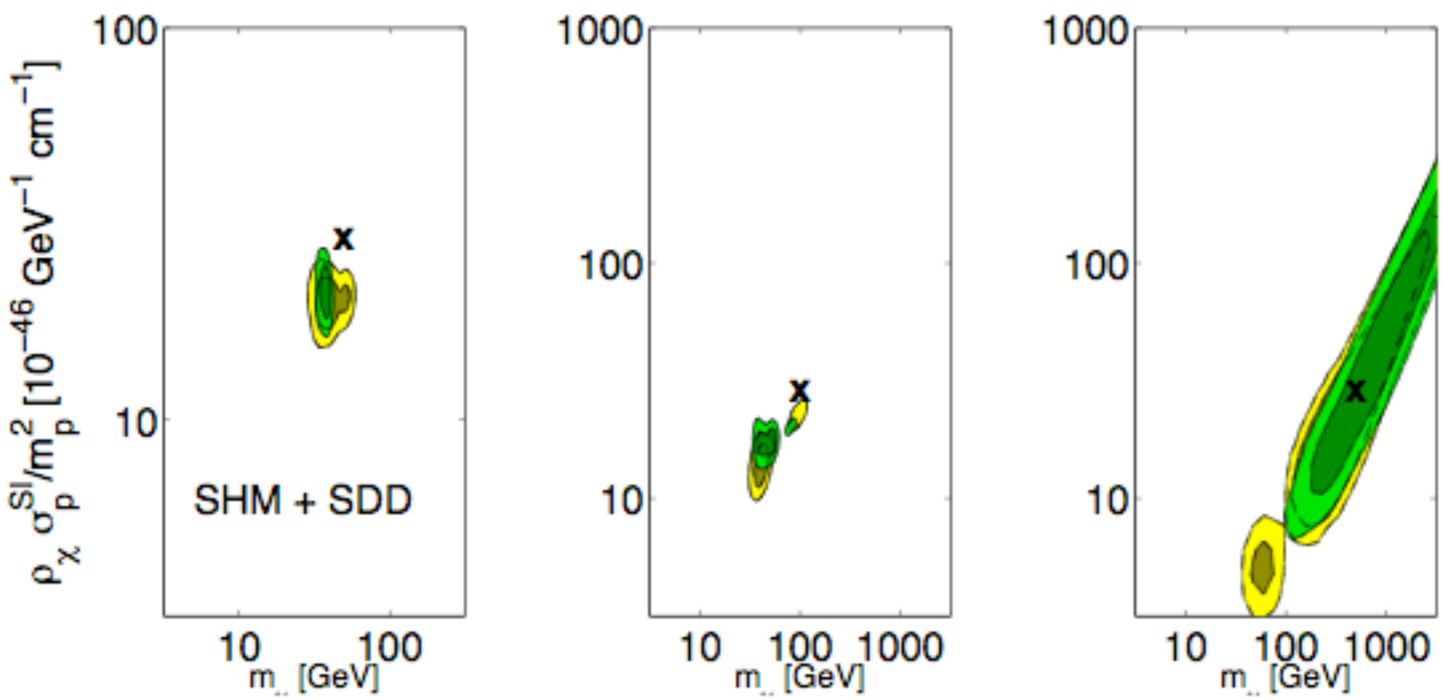
Parameterizing speed distribution

With a single experiment can't say anything about the WIMP mass without making assumptions about f(v) (recoil energies depend on speeds and mass).

But with multiple experiments can break this degeneracy. Drees & Shan; Peter

Peter Use empirical parameterization of f(v), and constrain its parameters along with mass & cross-section.

First approach: piece-wise constant in bins



Standard halo model + dark disc in

Better than assuming wrong f(v), but m_χ & σ both biased (experiments can't probe all of lowest speed bin ⟶ low σ).

Kavanagh & Green 12

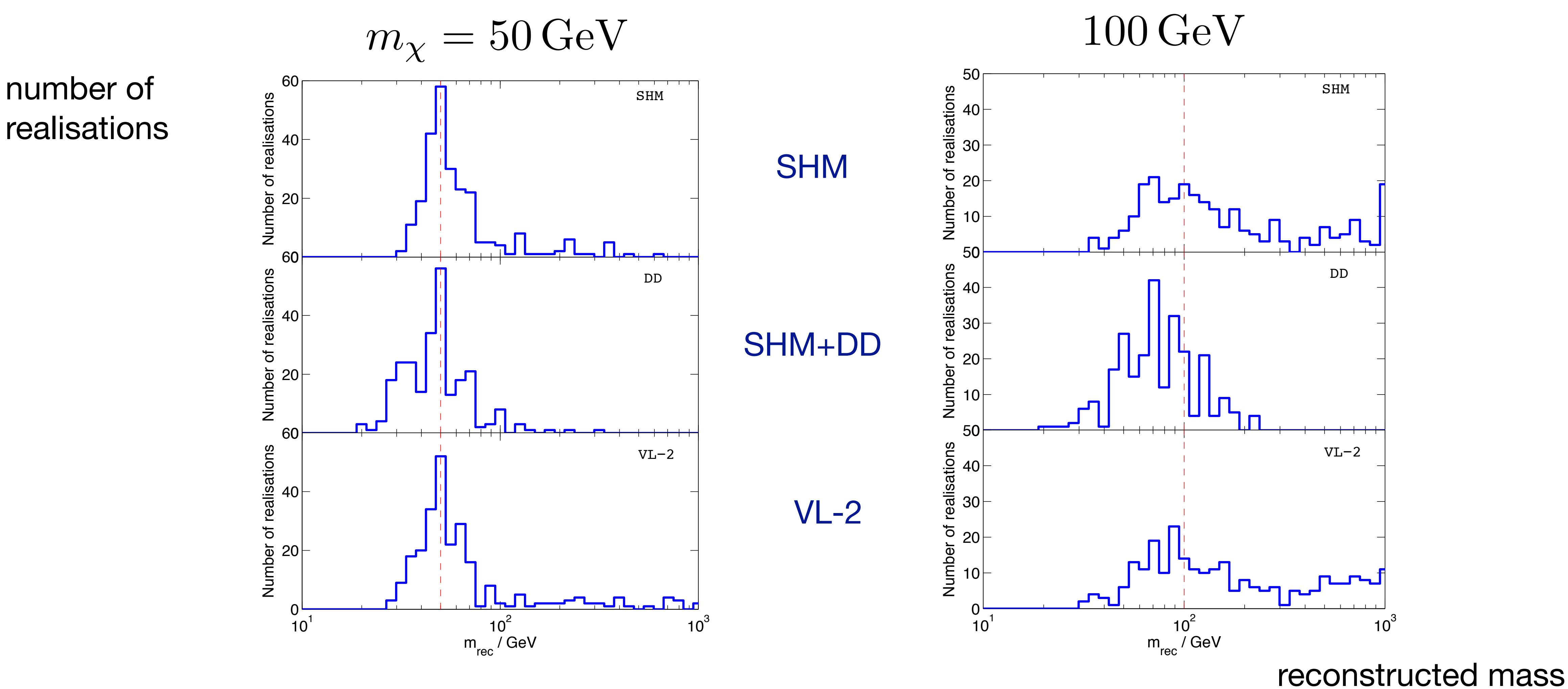
With fixed speed bins get better fit if more bins probed, can achieve this by reducing $m_\chi \longrightarrow$ low m_χ.

Solution: parameterize the reduced WIMP-nucleus momentum $\mathbf{p}_{\rm N} = \mu_{\chi,{\rm N}} \mathbf{v}$

minimum accessible momentum for each experiment is independent of the WIMP mass: $p_{\rm min} = \sqrt{m_{\rm N} E/2}$

parameterize momentum distribution over range of momenta accessible by experiments.

Reduced bias and better statistical coverage:



Coverage statistics for SHM:

coverage	speed	momentum
68%	36 ± 3 %	71 ± 3 %
95%	63 ± 3 %	92 ± 2 %

High mass tail: limitation of method or statistical limitation?
(c.f. Strege et al. ‘bad reconstructions’, flat spectrum due to Poisson fluctuations)

For SHM+DD still get bias in WIMP mass (and undercoverage). Distribution function varies rapidly at low speeds, not well parameterized by constant bins.

Kavanagh & Green 13

Want parameterization without fixed scales, and with ability to accommodate features in speed distribution.

Since f(v) ≥ 0, parameterize log of f(v) in shifted Legendre polynomials:

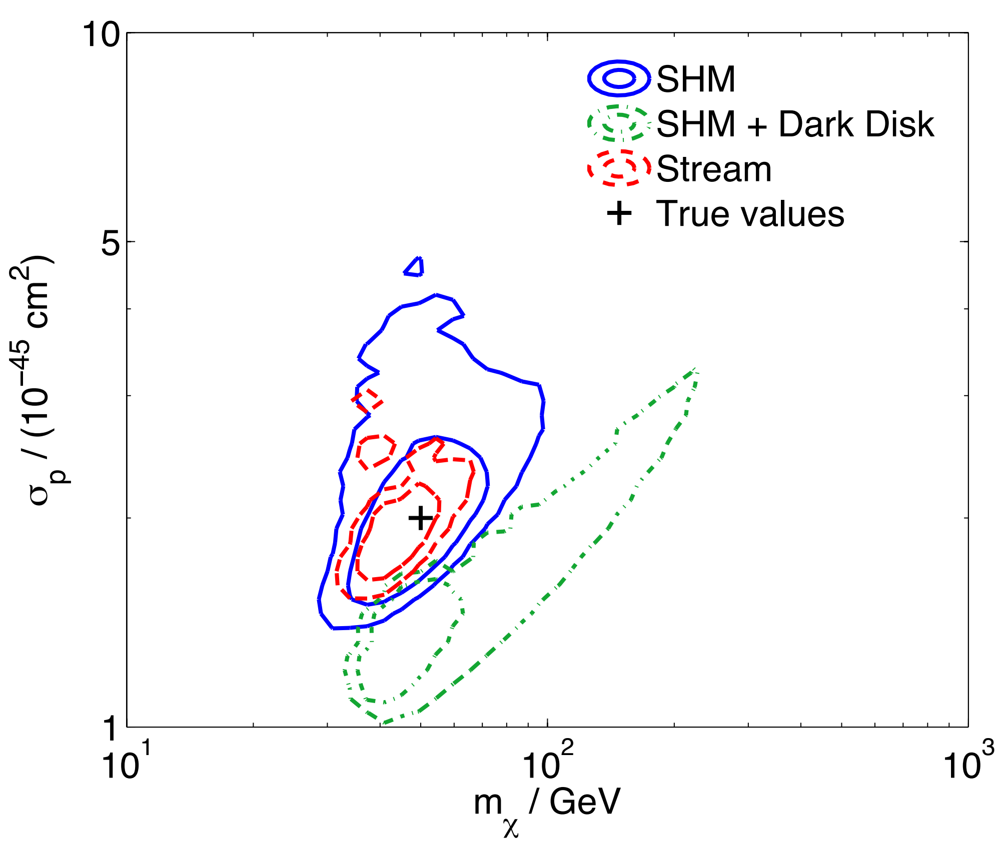
$$f(v) \propto \exp\left\{ -\sum_{k=0}^{N} a_k \bar{P}_k(v/v_{\mathrm{max}}) \right\}$$

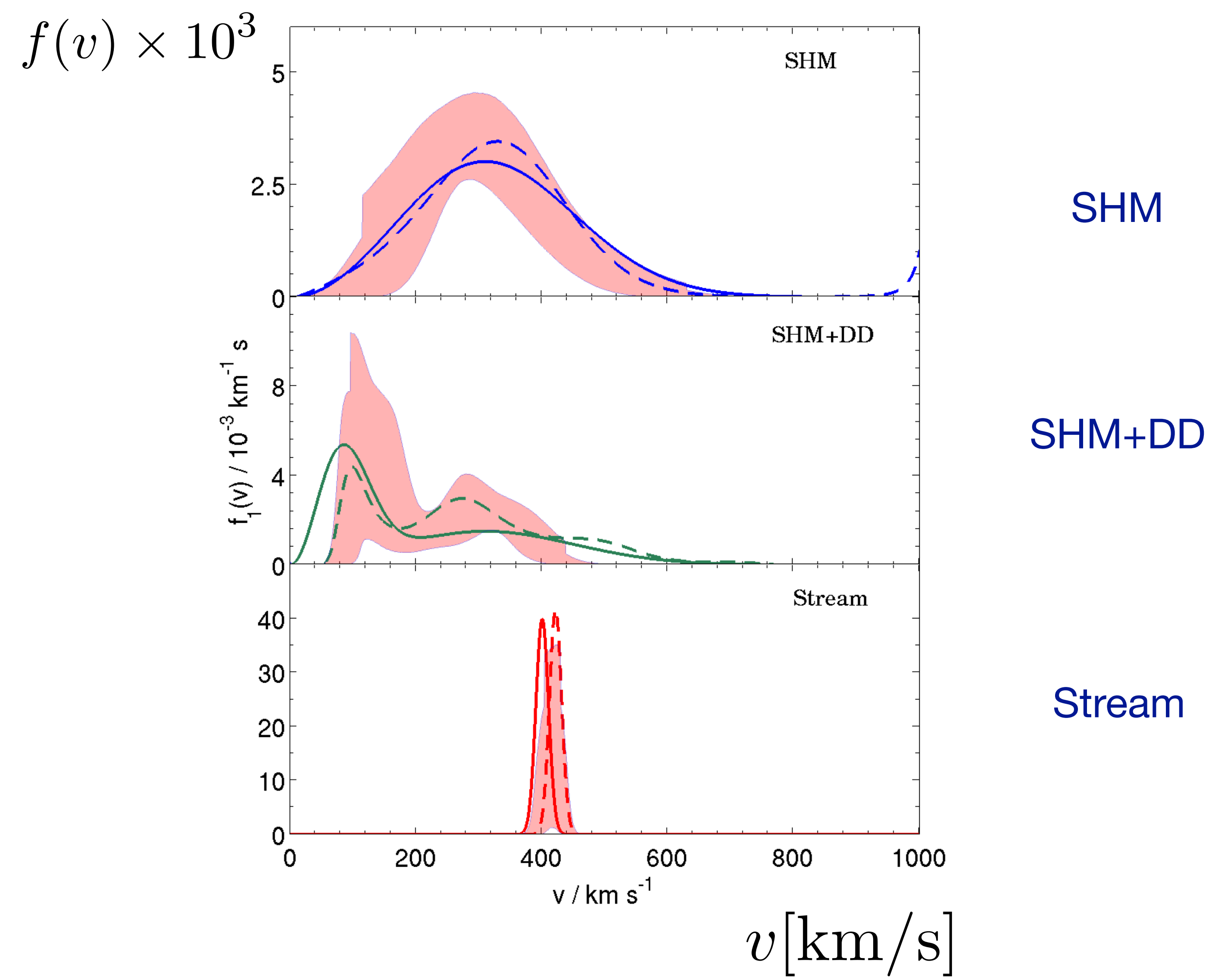
Shifted argument $2(v/v_{\mathrm{max}}) - 1$ ranges from -1 to +1 so small changes in coefficients a_k lead to small changes in f(v).

By varying N can accommodate features in f(v), and since polynomials are orthogonal earlier coefficients won't change dramatically.

Alves, Hedri & Wacker used shifted Legendre polynomials for f(ε) when studying reconstruction of f(v) using directional data.

Gives good reconstruction of WIMP mass even for extreme input f(v) (stream or dark disc), and allows f(v) to be reconstructed:





σ_p is underestimated since can't probe f(v) below lowest v_{min} threshold.

Summary

- Direct detection energy spectrum depends on the local dark matter density, ρ_0, and velocity distribution, f(v):

  local DM density → normalisation of event rate, and hence σ
  velocity dispersion → characteristic scale of energy spectrum and hence m_χ
  shape of WIMP velocity distribution → event rate for light WIMPs and amplitude and phase of annual modulation signal

- Determinations of ρ_0 and v_c have ~10% statistical errors, but systematic errors are larger.

- Can assess compatibility of signals/exclusion limits in speed integral, $g(v_{min})$, space ('integrating out the astrophysics').

- Parameterizing f(v)/Milky Way model and marginalizing (+ astrophysical data) works well **if** actual shape of f(v) is close to assumed shape

- Or use a suitable empirical parameterization (e.g. shifted Legendre polynomials), and probe f(v) too.