Dualities of 3d gauge theories

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Outline

- Introduction
- 3d gauge theories from D3-branes and 5-branes
- Monopole operators
- The IR limit of N=4 QED with flavors

Motivation

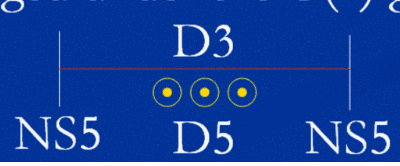
- Renewed interest in 3d Chern-Simons-matter theories now that some AdS duals are known
- Interesting interplay between dualities of 4d gauge theories and their 3d cousins
- A new duality of N=4 QED
- Learn more about monopole operators in CSM

From 4d to 3d

- A very fruitful way to construct interesting three dimensional theories is to begin with N = 4 SYM and compactify on a circle or an interval.
- There will be boundary conditions, and domain walls.
- For the abelian theory, Dirichlet = D5 brane, and Neumann = NS5 brane.
- Flow to the IR to obtain a 3d theory.

Hanany-Witten engineering

- Consider a D3 brane stretched between a pair of parallel NS5 branes. The boundary condition of ending on the NS5 brane breaks the supersymmetry on the D3 brane to N=4 in the effective 2+1 dimensions.
- Intersecting with N_f D5 branes at right angles to the NS5 branes results in N_f hypermultplets charged under the U(1) gauge theory on the D3.



IR limit of N=4 QED with flavors

- This is an abelian gauge theory in three dimensions with N_f massless charged hypermultiplets.
- For N_f=1, there is no Higgs branch, and the theory becomes free in the IR.
- Otherwise, one obtains a strongly coupled fixed point, located at the intersection point of the Coulomb and Higgs branches.

[Hanany Witten, Intiligator Seiberg, …]

Moduli space of 2+1 QED

- Naively, the Coulomb branch is a $\mathbb{C}^2$ parameterized by the vector multiplet, however there is a quantum correction that modifies it to $\mathbb{C}^2/\mathbb{Z}_{N_f}$
- The Higgs branch cannot be corrected in this N=4 theory, since the gauge coupling can be promoted to a vector superfield, decoupling it from the hypermultiplet moduli space.
- Thus one obtains the classical result $T^*\mathbb{CP}^{N_f-1}$

BPS boundary conditions for N=4 SYM with $\theta \neq 0$.

- So far I have been discussing the case $\tau = \frac{2\pi i}{g_{YM}^2}$
- By applying an SL(2,Z) transformation, there must be N = 4 configurations with NS5 branes and (1,1)5 branes. In general a different subgroup of supersymmetry will be unbroken.
- For NS5 and (1,k) 5 branes to preserve the same susy, one needs $\tau = k(1+e^{2i\psi})$
- The NS5 b.c. is $F_{3i} = \frac{\tan\psi}{2}\epsilon_{ijk}F^{jk},\ Y_i = 0,\ D_3X_i = 0$

[Gaiotto-Witten]

Chern-Simons on the D3

- First derived by [Kitao, Ohta, Ohta; Bergman, Hanany, Karch, Kol]
- Easy argument: consider turning on IIB axion. Then there is a term $\int \theta F \wedge F$ on the stretched D3. Integrating it by parts gives a CS term of level $\pm\theta$ on each boundary.
- If a D3 ends on a (1,k) 5, perform an SL(2,Z) to shift it to an NS5. This turns on the axion.
- Now if the D3 ends on NS5 and (1,k) 5, there will be a relative level k CS term.

Chern-Simons-matter theory

- We first consider the case with N=2 susy. It consists of a vector multiplet in the adjoint of the gauge group, and chiral multiplets in representations R_i

$$S_{CS}^{\mathcal{N}=2} = \frac{k}{4\pi} \int (A \wedge dA + \frac{2}{3} A^3 - \bar{\chi}\chi + 2D\sigma)$$

- The kinetic term for the chiral multiplets includes couplings $-\bar{\phi}_i \sigma^2 \phi_i - \bar{\psi}_i \sigma \psi_i$
- There is the usual D term $\bar{\phi}_i D \phi_i$

We integrate out D, σ , and χ

$$S^{\mathcal{N}=2} = \int \frac{k}{4\pi}(A\wedge dA + \frac{2}{3}A^3) + D_\mu\bar{\phi}_i D^\mu \phi_i + i\bar{\psi}_i\gamma^\mu D_\mu \psi_i$$

$$-\frac{16\pi^2}{k^2}(\bar{\phi}_i T^a_{R_i}\phi_i)(\bar{\phi}_j T^b_{R_j}\phi_j)(\bar{\phi}_k T^a_{R_k} T^b_{R_k}\phi_k) - \frac{4\pi}{k}(\bar{\phi}_i T^a_{R_i}\phi_i)(\bar{\psi}_j T^a_{R_j}\psi_j)$$

$$-\frac{8\pi}{k}(\bar{\psi}_i T^a_{R_i}\phi_i)(\bar{\phi}_j T^a_{R_j}\psi_j).$$

Note that this action has classically marginal couplings. It is has been argued that it does not renormalize, up to shift of k, and so is a CFT.

[Gaiotto Yin]

N=3 CS-matter

- To obtain a more supersymmetric theory, begin with N=4 YM-matter. Then add the CS term, breaking to N=3.
- Thus we add a chiral multiplet φ ,with no kintetic term in the adjoint, and the matter chiral multiplets, $\Phi_i, \tilde{\Phi}_i$ must come in pairs.
- There is a superpotential, $W = -\frac{k}{8\pi}Tr(\varphi^2)$ from the CS term.

- Integrating out φ one obtains the same action as before, but with a superpotential:

$$W = 4\pi (k^{-1})_{ab} (\tilde{\Phi}_i T^a_{R_i} \Phi_i)(\tilde{\Phi}_j T^a_{R_j} \Phi_j)$$

- These N=3 theories are completely rigid, and hence superconformal. It is impossible to have more supersymmetry in a YM-CS-matter theory, but we shall see that for particular choices of gauge groups and matter representations, the pure CSM can have enhanced supersymmetry.

  [Schwarz; Gaiotto Yin]

Simple example

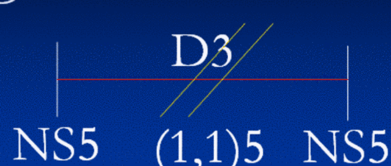
- Consider a U(1) x U(1) CSM theory, with a BF-like Chern-Simons coupling $\frac{2k}{4\pi} \int a \wedge db$
- Take a pair of matter hypers $(X, \tilde{X}),\ (Y, \tilde{Y})$ in the fundamental of the first and second U(1).
- In this theory the supersymmetry is enhanced to N=4; one can check that the boson-fermion coupling is invariant under a separate SU(2) acting on each fundamental hyper.

$$W = \frac{8\pi}{k} (X\tilde{X})(Y\tilde{Y})$$

RG Flow

- The Chern-Simons terms dominate the rg flow.
- At long distances, the Yang-Mills coupling diverges, and its ordinary kinetic term just disappears.
- But the Lagrangian remains nonsingular due to the CS term.
- Morally this is due to conformality in the 3+1 theory. There is no other renormalization.

SL(2,Z) generalized mirror symmetry



- Apply STS transformation to turn D5 branes into (1,1)5 branes. If N_f < 4, move them to get a CSM quiver with level +1 and -1.
- Moving the (1,1) 5 branes so they alternate with the NS5 branes gives CSM theories with one hyper bifundamental between each node.

$$U(1)_1 \times U(1)_{-1} \times U(1)_1$$

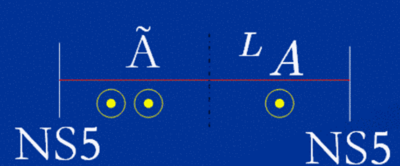
The CSM model

- In the three node theory, integrating out the overall trivially acting Chern-Simons gauge field results in precisely our simple example, with k=1.
- In general, consider the same theory with N_f-1 fields $(X_i, \tilde{X}_i)$, and the same $(Y, \tilde{Y})$.
- This also results from integrating out in the N_f=3 case.

$$W = \frac{8\pi}{k} Y\tilde{Y} \sum_{i=1}^{N_f - 1} X_i \tilde{X}_i$$

Piecewise s-duality

- Cut the D3 brane, and glue with $\int_{3d} \tilde{A} \wedge *F_A$
- Note that the theory on the right is the self mirror boundary condition. Dualize that segment.
- This results in the Chern-Simons-matter theory above, with BF like coupling $\int \tilde{A} \wedge d\, {}^L A$



[Kapustin-Strassler]

Quantum correction to hypermultiplet moduli space

- In Yang-Mills theories with eight supercharges, the moduli space of hypermultiplets is normally not corrected, since one can promote the coupling to a vector superfield, which decouples from the hypers.
- In CSM theories, no such argument exists for the CS level. However, corrections to the metric must respect the hyperkahler structure.
- We will find a correction of this type.

Monopole operators

- Reducing a four dimensional magnetic monopole to three dimensions results in a vortex configuration.
- Alternatively, reduce to obtain a magnetic instanton with $\int_{S^2} F \neq 0$
- In 3d CFT, the operator that creates the vortex corresponds by state-operator correspondence to that state in radial quantization.

Branches of CSM moduli space

- In Chern-Simons theories, there is no Coulomb branch, as the vector multiplets are all effectively massive.
- N=3 supersymmetry protects the dimensions of ordinary chiral operators formed from the matter fields; the quantum correction depends on the existence of monopole operators.
- Rich structure of branches distinguished by the spectrum of allowed monopoles.

Classical moduli space

- The superpotential is dictated by N=3 supersymmetry to be $W = \frac{8\pi}{k}(X\tilde{X})(Y\tilde{Y})$
- Thus there are two branches, $\mathcal{M}_a$ and $\mathcal{M}_b$, on which the respective U(1) is unbroken.
- Naively, one would quotient by the nontrivially acting U(1), but would leave 3d, so can't be.
- $\frac{2k}{4\pi}\int a \wedge db$ is only invariant under a Z_k .

$$\mathcal{M}_a = \mathcal{M}_b = \mathbb{C}^2/\mathbb{Z}_k$$

Extra massless fields at origin

- The two branches intersect at the origin, where there are extra massless fields. In particular, on $\mathcal{M}_a$ which is parameterized by $(Y, \tilde{Y})$ the X fields have a mass

$$\frac{4\pi}{k}\bar{Y}_{(a}Y_{b)} = \frac{4\pi}{k}\left(\frac{1}{2}(|Y|^2 - |\tilde{Y}|^2), Y\tilde{Y}, \bar{Y}\bar{\tilde{Y}}\right) \equiv \vec{m}_X$$

- We will see that integrating out these fields changes the singularity at the origin.

Mukhi-Papageorgakis effect

- Forget about the $(X, \tilde{X})$ mutliplet for a moment. Going onto the moduli space by turning on $(Y, \tilde{Y})$ gives a mass to the broken gauge field, b.
- Integrating out b gives Yang-Mills kinetics to the unbroken gauge field, a! It can then be dualized to a scalar, φ_Y which transforms under the U(1) in the same way as the phase of the hyper Y, but with charge k. The Z_k arises by gauge fixing.

Correction to the hyperkahler metric

- As familiar from the Coulomb branch of N=4 2+1 gauge theories, integrating out a charged massive hypermultiplet at 1 loop gives rise to a term

$$\int \frac{1}{8\pi|\vec{m}|}(\partial_\mu\vec{m}\cdot\partial^\mu\vec{m} - |da|^2) + \epsilon^{\mu\nu\rho}\epsilon^{ijk}\partial_i\left(\frac{1}{8\pi|\vec{m}|}\right)a_\mu\partial_\nu m_j\partial_\rho m_k$$

$$= \int \frac{1}{8\pi|\vec{m}|}(\partial_\mu\vec{m}\cdot\partial^\mu\vec{m} - |da|^2) + (*da)^\mu\omega_i\partial_\mu m_i$$

- Note that this already introduces a Yang-Mills term for the gauge field a.

- Before integrating out the broken gauge field b, we dualize a, treating F_a as the fundamental variable .

$$\int |D_\mu Y|^2 + \frac{1}{8\pi|\vec{m}|}(\partial_\mu\vec{m}\cdot\partial^\mu\vec{m} - |\tilde{F}_a|^2) + \int \tilde{F}_a\wedge\left(d\varphi_Y + \omega_i dm_i + \frac{k}{2\pi}b\right)$$

- Integrating out F_a leads to

$$\int |D_\mu Y|^2 + \frac{1}{8\pi|\vec{m}|}\partial_\mu\vec{m}\cdot\partial^\mu\vec{m} + 2\pi|\vec{m}|\left(\partial_\mu\varphi_Y + \omega_i\partial_\mu m_i + \frac{k}{2\pi}b_\mu\right)^2$$

$$Y \to e^{i\Lambda}Y, \quad b_\mu \to b_\mu + \partial_\mu\Lambda, \quad \varphi_Y \to \varphi_Y - \frac{k}{2\pi}\Lambda$$

- The U(1)_b acts on the space of$Y$, Y, and φ_Y. The metric is nontrivial due to the quantum correction as seen.

$$\mathcal{M}_a = \mathbb{C}^2/\mathbb{Z}_{k+1}$$

Monopoles in the chiral ring

- There are monopole operators in YM-CS-matter theories, which we follow to the IR CSM.
- In radial quantization, it is a classical background with magnetic flux $\int_{S^2} F_a = 2\pi n$, and constant scalar, $\sigma = n/2$. Of course, in the CSM limit, $\sigma_a = 1/k\ (|Y|^2 - |\tilde{Y}|^2)$
- It is crucial that Y is not charged under a.
- Call this monopole operator T.

[Borokhov Kapustin Wu]

CS induced charge of T

- The Chern-Simons term induces a charge for the operator T we have just defined. Writing $\frac{2k}{4\pi} \int a \wedge db = kn \int_{radius} b$ in the monopole background, it is a particle of charge n k under U(1)_b
- Equivalently, in radial quantization, the Gauss' law constraint is modified, and some matter field zero modes must be turned on.

Anomalous dimension

- The dimension of the monopole operator will be the sum of the two contributions

$$Q_0 = \frac{1}{2}\Big(\sum_{i \in hyper} - \sum_{i \in vector}\Big)|q_i|$$

and the dimension of the scalar fields used in the dressing.

- This was calculated in Borokhov-Kapustin-Wu by quantizing the matter fields in the monopole background with constant $\sigma = n/2$

- The result was that the spectrum of fermions from the hypermultiplets became asymmetric,

$$E_p^+ = |n|/2 + p, \quad E_p^- = -|n|/2 - p, \quad E_0 = +|n|/2$$

- The spectrum of scalars was found to be symmetric. Thus only the fermions contributed to the vacuum energy, which is exactly the anomalous dimension of the operator.
- We will include the CS terms simply noting this operator is charged under the gauge group.
- This is sensible since the matter fields needed to "dress" the operator are neutral under the magnetic U(1). Needed for it to be in chiral ring

General (N=3) picture

- The moduli space equations $(k^{-1})^{ij}\mu_i^\alpha T_j^{ab} q_b^A = 0$ are in terms of the moment maps $\mu_i^\alpha = T_i^{ab} q_a^A q_b^B \Gamma_{AB}^\alpha$
- The subgroup generated by $k^{-1}\mu$ is unbroken, and $(k^{-1})^{ij}\mu_i^\alpha \mu_j^\beta = 0$ implies this is a null direction in the CS form, thus there are monopole operators with those fluxes.
- The CS term is not invariant under constant gauge transformations in such backgrounds $\frac{k}{4\pi}\Lambda \int_{S^2} F$
  Only quotient by the kernel of $e^{iS_{CS}} : G \to U(1)$

Our example

- We have monopoles $T_a,\ \tilde{T}_a,\ T_b, \tilde{T}_b$, the first two on the branch $\mathcal{M}_a$, and the latter pair on $\mathcal{M}_b$
- Each has one hypermultiplet charged under the associated U(1), so it gets a dimension ½
- The CS induced charge of T is (0,k) under the U(1) x U(1) gauge group.
- The chiral operators on $\mathcal{M}_a$ are $Y\tilde{Y},\ T\tilde{Y}^k,\ \tilde{T}Y^k$
  exactly as expected for $\mathcal{M}_a = \mathbb{C}^2/\mathbb{Z}_{k+1}$

  $z_1 z_2,\ z_1^{k+1},\ z_2^{k+1}$

Agreement in the chiral ring

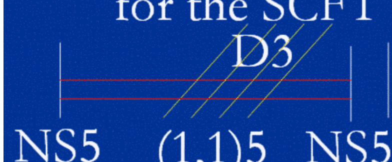
- On the branch where X's are set to zero, the chiral operators are $Y\tilde{Y}$, $T_a\tilde{Y}$, $\tilde{T}_a Y$ with dimensions 1, N_f/2, N_f/2. This is exactly what ones expects for $\mathbb{C}^2/\mathbb{Z}_{N_f}$
- On the other branch, the operators are formed out of gauge invariant combinations of

  $X_i,\ \tilde{X}_i,\ T_b,\ \text{and}\ \tilde{T}_b$

  all 1 + (N_f-1) of these have dimension ½ and charge ± 1 under U(1)_a. Agrees perfectly!

Non-abelian generalization

- It is easy to generalize the SL(2,Z) transformation and fivebrane moves we applied to QED with three or less flavors to the U(N) theory. But one is really interested in 3d theories with $N_f \geq 2N_c$
- This can be done by introducing extra NS5 branes. Complicated result, but get a Lagrangian for the SCFT that is classically conformal.



$U(1)_1 \times U(2)_{-1} \times U(2)_1 \times U(2)_{-1} \times U(1)_1 \times U(2)_{-1}$

Summary

- We studying the quantum corrections to the moduli spaces of Chern-Simons-matter CFTs both directly and using the chiral ring.
- This allowed us to identify an interesting CFT dual to the IR limit of QED with flavors.
- Used more of the full SL(2,Z), and saw that it reproduced some results of piecewise abelian duality.