26 August 2010

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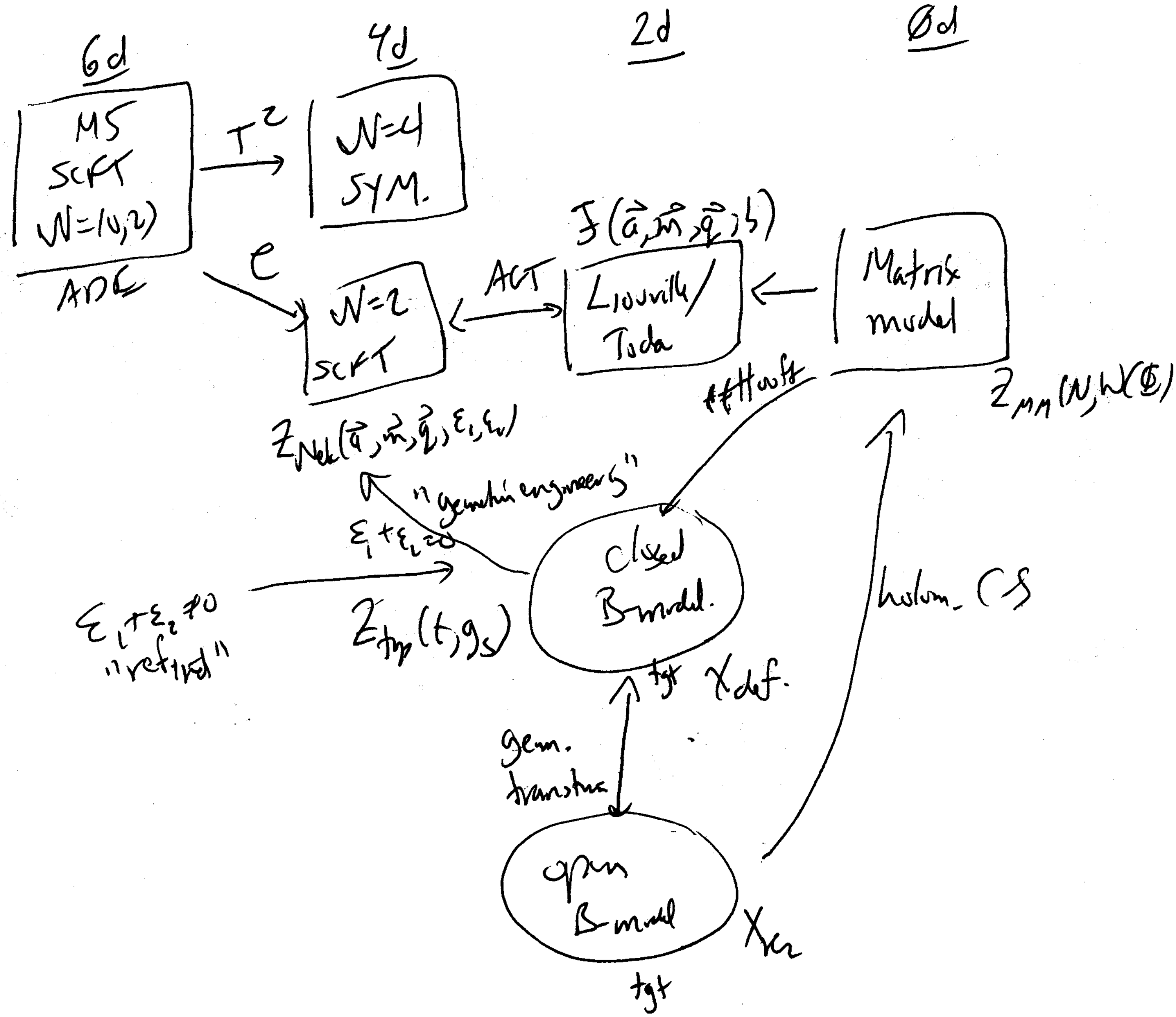
Topological strings, matrix model, supersymmetric gauge theories and Liouville conformal blocks



Dijkgraaf-Vafa '09.

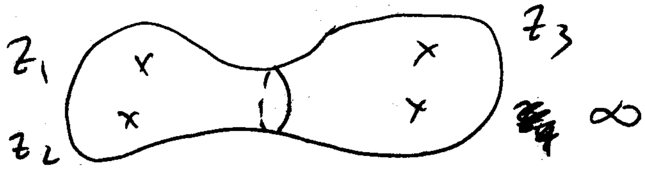
r = number of matrices
$= A_r$

$\mathcal{C} = \mathcal{C}_{g,n}$

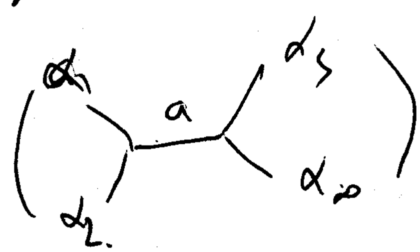
most of the talk: $\mathcal{C} = \mathcal{C}_{0,n}$, A_1.

N = rank of Φ.

<u>Ex</u>



q = cross-ratio

$\mathbb{F}$


$b^2 = -\varepsilon_1/\varepsilon_2$.

$\Delta_\alpha = \alpha(Q-\alpha)$, $Q = b + b^{-1}$

$$Z_{MM} = \int (d\Phi)_\beta \, e^{-\frac{1}{g_s} W(\Phi)}$$

$\beta = \varepsilon_1/\varepsilon_2$, $g_s^2 = -\varepsilon_1 \varepsilon_2$.

$$W(x) = g_s \sum_{I=1}^{n-1=3} \mu_I \log(x - z_I)$$

$\mu_I = -2b\alpha_I$

$$Q - Nb = \sum_{I=1}^{n-1} \alpha_I + \alpha_\infty$$

$W(x)$

$$\sum_{I=1}^{n-2} N_I = N.$$

$\to (n-3)$ independent "filling fraction"

$\updownarrow$

$(n-3)$ a's.

$$\hat{\mathcal{C}} \xrightarrow{r+1} \mathcal{C}_{g,n}$$

$\mathcal{C} \simeq \Sigma_{SW}$ (4d)

= "spectral curve" (0d)

$\subset X_{dof}$

$N \to \infty$

$\lambda_{SW} \sim \partial\varphi$

Geometry of Matrix Model

$\beta = 1$, $\varepsilon_1 + \varepsilon_2 = 0$

$$Z_{MM} = \int d\Phi \, e^{-\frac{1}{g_s} W(\Phi)}$$

$$= \int \left(\prod_{i=1}^{N} d\lambda_i \, e^{-\frac{1}{g_s} W(\lambda_i)} \right) \cdot \Delta^2 \qquad \Delta = \prod_{i<j} (\lambda_i - \lambda_j)$$

$\lambda_i \in \mathbb{C}$

classical

$$0 = \frac{1}{g_s} W(\lambda) - 2 \sum_{i \neq j} \frac{1}{\lambda_j - \lambda_i}$$

resolvent

$$\omega(x) = \frac{1}{N} \mathrm{Tr}\left(\frac{1}{\Phi - x} \right) \qquad \langle \mathrm{Tr}\, \Phi^k \rangle$$

't Hooft limit $\quad t = g_s N$, $N \to \infty$ with t fixed.

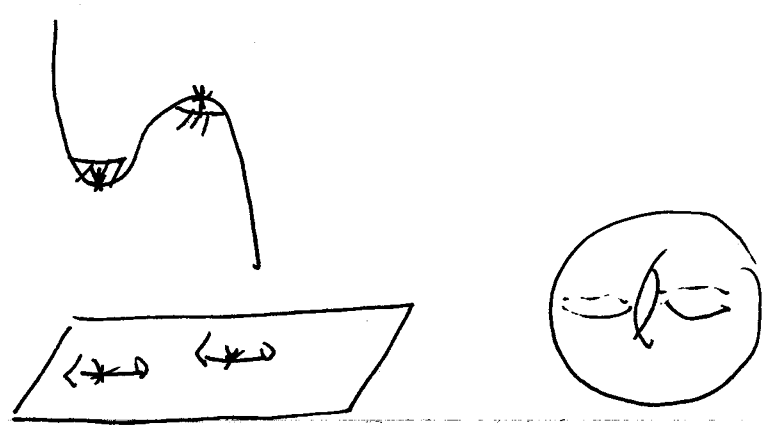
eom $\boxed{y(x)^2 = (W'(x))^2 - f(x)}$ $\quad \hat{e}$

spectral curve.

$y(x) \longleftrightarrow W(x), \omega(x)$

① $f(x)$ depends on λ-distribution

$t \to 0$, $\hat{e}$ singular



Conf. sym. of MM

[Kostov]

loop eqn : $\Phi \to \Phi + \varepsilon \Phi^n$ $\quad \langle \cdots \rangle_{MM}$

$$0 = \oint \frac{dx}{x-x'} \langle \tilde{T}(x') \rangle$$

$$\tilde{T}(x) = \frac{1}{2} \partial\varphi(x) \partial\varphi(x')$$

$$\langle \partial\varphi(x) \rangle = y(x)$$

$\tilde{T}(x)$ satisfies Vir $c=1$

φ : chiral boson

$$J_3 = \partial\phi$$

$$J_\pm = :e^{\pm i\varphi}: \qquad SU(2)$$

$$Z_{MM} = \langle N | e^{\oint \frac{U}{g_s} \partial\phi} e^{Q_+} | 0 \rangle$$

Fock-space of chiral boson with Liouville-like interaction

$$Q_+ = \int J_+ \, dx \quad \text{"screening charge op."}$$

$$Q_+ \leftrightarrow Q_- \, , \; b \leftrightarrow b^{-1}$$

$\langle N | = \langle 0 | e^{N\hat{q}}$ "N-charge coherent state"

projects on $\overbrace{Q_+ \cdots Q_+}^{N}$

$S_{Liou} \supset \int e^{b\varphi} d^2x$ $\int e^{i\varphi} dx$

$b = i,\ Q = 0 \rightarrow c = 1$

A_r
$(r+1)$ $\psi^{(1)}, \psi^{(2)}$

$\downarrow$

$U(r+1)$

$T = \partial\phi\partial\phi \rightarrow Vir$

$\updownarrow$

$T = \partial\varphi\partial\varphi \rightarrow Vir$

$\langle \partial\varphi(x) \rangle = y(x)$

$N \rightarrow \infty$

$\lambda_{SW} = \partial\varphi$

<u>generalisations</u>

① A_r.

$\tilde{C}$
$\downarrow^{r+1}$
C

r "collective fields" φ_a $\nearrow$ chiral A_r Toda

$\updownarrow$

$(r+1)$ chiral fermions $U(r+1)$

(2) β ensamble

$$\Delta^2 \to \Delta^{2\beta} = \prod (\lambda_I - \lambda_J)^{2b^2} \qquad Q_+ = \int e^{b\varphi} dx$$

loop eqn

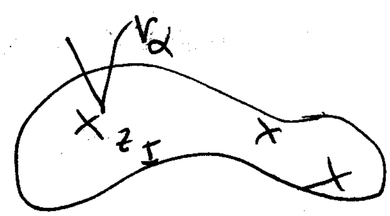
$$\tilde{T}(x) = \frac{1}{2}\partial\varphi\partial\varphi + Q\partial^2\varphi$$

Q: $\mathcal{H}^{\text{Liouv}}_{\text{conf blocks}} \overset{?}{\longleftrightarrow} \mathcal{H}^{MM}_{Z_{MM}(N,\, N(\Phi)),\ \text{saddle point}}$

Recall

$$Z_{MM} = \langle N | e^{\oint \frac{W}{g_s}\partial\varphi} e^{Q_+} | 0 \rangle$$

For Liouville: $V_\alpha = e^{\alpha\varphi}$



$\to$ "Penner-type" matrix model

$W, N \to$ external momenta

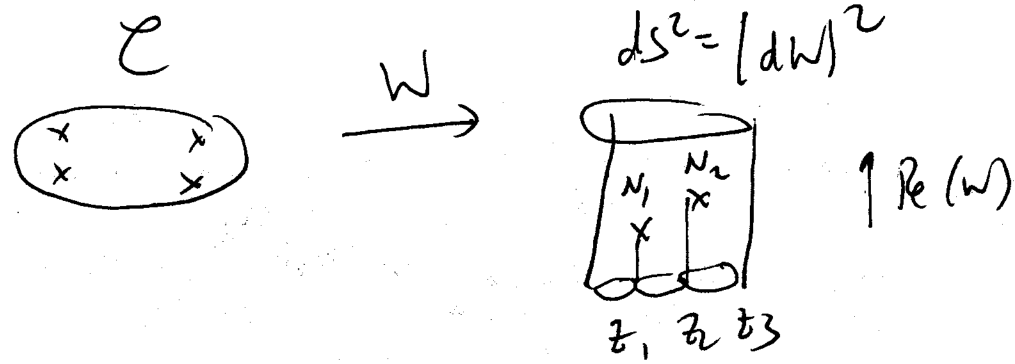
$\mu \overset{?}{\to}$ internal momenta

Recall

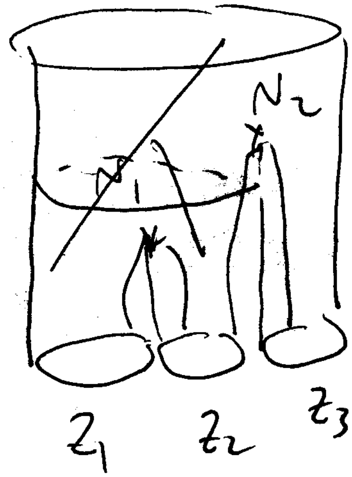
closed string diagram in light-cone gauge

quasi-conformal map $\quad (dW)^2 = \varphi_{xx} dx^2 \quad$ Strebel differential

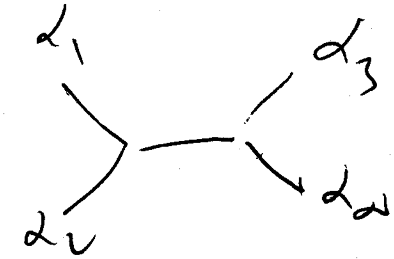
$ds^2 = |dW|^2$



$dW = 0$.

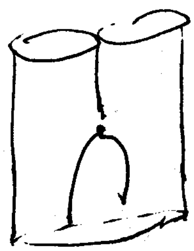


contour



e.g. 3-point

$\mu_1, \mu_2 > 0$



$$\underline{F_1}(\alpha_i)\, F_1(\alpha_i^*) = \underline{F_2}(\alpha_i)\, F_2(\alpha_i^*)$$

different holomorphic half

$\mathcal{C}$ invariant under monodromy,

not (Stokes phenomenon)