13 August 2010
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Gauge Theory Observables and 2d CFT

Goal Explain how correlators in 2d CFT

$\updownarrow$

$\langle \mathcal{O}(p) \rangle_{4d\ \mathcal{N}=2}$ 4d $\mathcal{N}=2$ gauge theories in S^4

- Loop operators in 2d CFT $\Rightarrow$ $\begin{cases} \text{- Wilson-'tHooft loop operators} \\ \text{- Domain wall operators} \end{cases}$

4d QFT on $M = \mathbb{R}^4$ or S^4, observables

- local operators $\mathcal{O}(p)$, p = point, $\mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}$
- loop operators " p = curve in M

$\begin{cases} \text{- electric operators: Wilson loops} \quad \mathrm{Tr}_R\, P\exp\left(\oint_p iA + \phi\right) \\ \qquad\qquad R = \text{reps of } G. \\ \text{- magnetic operators: 'tHooft loops} \end{cases}$

- dyonic operators $(\mu_e, \mu_m)/W$

- Surface operators
- domain wall operators

Techniques

1) path integral / perturbative
2) AdS/CFT correspondence
   - Large N
3) SUSY/Localization: $\int [D\varphi] e^{-S[\varphi]} \xrightarrow{\text{reduction}}$ Reduced model

   a) $\mathcal{N}=1$ SUSY gauge theories $\longleftrightarrow$ Matrix integral (DV)
      W_{eff}

   b) Pestun: Z and $\langle W_R \rangle$ of 4d $\mathcal{N}=2$ gauge theories on S^4

$$Z = \int_{a \in \mathfrak{g}} [da] |Z_N|^2$$

$$Z_N = \text{Nekrasov partition function} = Z_{cl} \cdot Z_{1\text{-loop}} \cdot Z_{inst}$$

$$Z_{cl} = e^{2\pi i \tau \langle a, a \rangle}$$

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

AGT correspondence

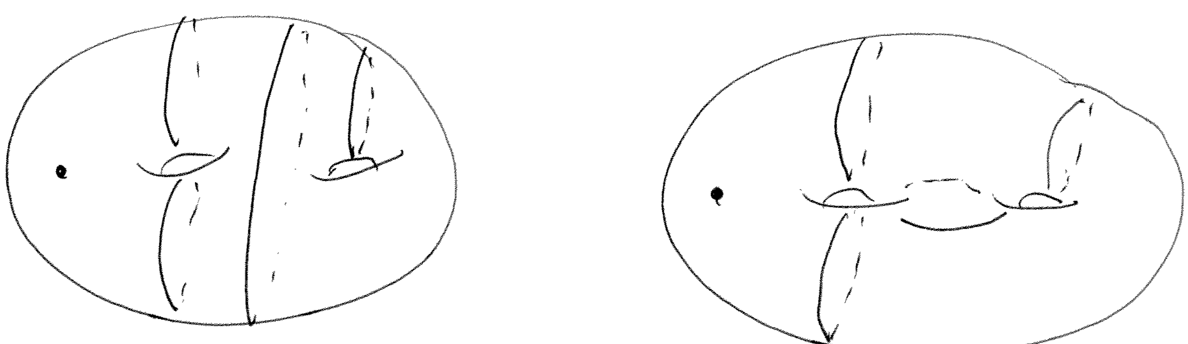
$Z_{\mathcal{N}=2}$ = correlation function of primary operators in Toda ~~$\mathfrak{g}$~~ CFT

<u>Goal</u>: Use 2d CFT to compute something new, <u>new observables</u> (invariant under Q_{Pestun}).

6d (0,2) on $C_{g,n}$ $\rightarrow$ 4d $\mathcal{N}=2$ theory $\mathcal{T}_{g,n}$

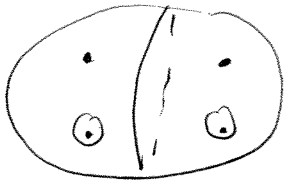
S-duality transformation

Different sewings of $C_{g,n}$ from trinions and tubes.



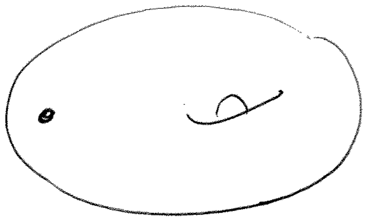
S-duality frame of $\mathcal{T}_{g,n}$ $\longleftrightarrow$ Sewing of $C_{g,n}$

4d $\mathcal{N}=2$ gauge theories

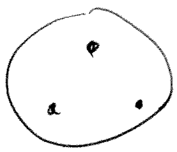


$C_{0,4}$ 4d $\mathcal{N}=2$ with $2N$ massive fundamental hypermultiplets

$= \text{SQCD}$



$C_{1,1}$ $\mathcal{N}=2^*$: $\mathcal{N}=2$ vector with a massive adjoint hypermultiplet.

[ $= T_N$ (from A_{N-1} $(2,0)$)]

$$Z_{\mathcal{N}=2} = \langle V_{M_1} \cdots V_{M_n} \rangle^{\text{Toda}}_{C_{g,n}} \qquad (b=1)$$

2d Toda CFT (G) Let $r = \text{rank}(G)$, e_i = simple roots

Locally $\phi = \sum \phi_i e_i$

$$S = \int d^2z \langle \partial\phi, \bar\partial\phi \rangle + \mu \sum_{i=1}^{r} e^{b\langle e_i, \phi\rangle}$$

- $b^2 = \hbar$ of Toda
- W_N-symmetry generated by $W^{(2)}, \dots, W^{(N)}$

– Representation of W_N-Toda:

$$V_\mu = e^{\langle \mu, \phi \rangle} \qquad \mu = \text{"Toda momentum"}$$

μ / Weyl group

Kac determinant

– non-degenerate $\quad \mu = Q\rho + m$

$Q = b + \frac{1}{b}, \quad \rho = \text{Weyl vector}$

– semi-degenerate

matches punctures in $C_{g,n}$

– degenerate reps: $\quad \mu = -b\lambda_1 - \frac{1}{b}\lambda_2$

λ_1, λ_2 are highest weight vectors of reps R_1, R_2 of G

$\Rightarrow$ Need more data to complete bootstrap

$L_{-1} \to \partial$

$W_{-1} \to ?$

Bootstrap can be completed

$\updownarrow$

$T_{g,n}$ has a Lagrangian description

$T_{g,n} \leftarrow$ M5-branes on $C_{g,n}$

M5 + M2 brane

M5 ∩ M5 (4) $\longleftrightarrow$ codim 2 defect of (0,2) theory

M5 ∩ M2 (2) $\Leftarrow$ surface op of (0,2) theory

		1	2	3	4	5	6	7	8	
N	M5	1	2	3	4	5	6			
	M5	1	2	3		5		7	8	(curve in $C_{g,n}$) $\Rightarrow$ domain wall
	M2	1				5		7		$\Rightarrow$ loop operator

($\underbrace{5\ 6}_{C_{g,n}}$)

Loop operators in Toda CFT

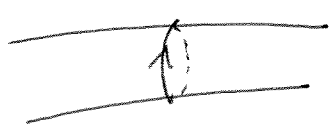
- semiclassical description: integrable structure

Toda CFT $\Leftarrow$ flat connection $\mathcal{L}$, $F = d\mathcal{L} + \mathcal{L}\wedge\mathcal{L} = 0$

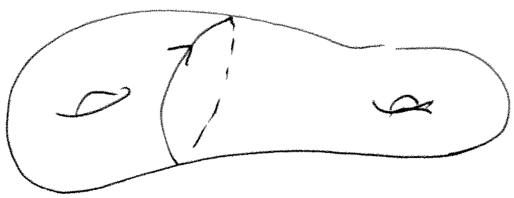
$$\Uparrow$$

$$(d+\mathcal{L})\psi = 0$$

$$U = P\exp\left(\oint_p \mathcal{L}\right)$$

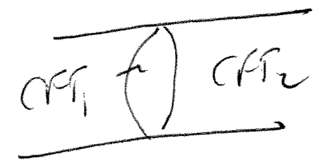
$U =$  $\quad \psi(\sigma+2\pi) = U\psi(\sigma)$

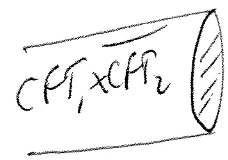
Topological defect / Verlinde loop operators



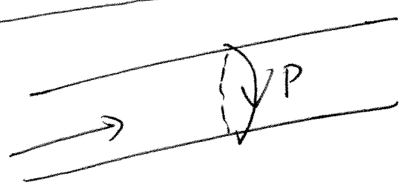
$T_1 = T_2$

$\overline{T}_1 = \overline{T}_2$



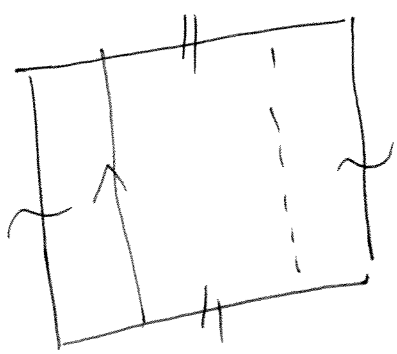


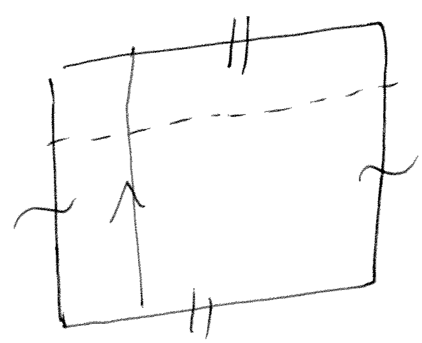
$\mathcal{O}_\mu(p)$



$$\mathcal{H} = \bigoplus_\alpha R_\alpha \otimes R_{\alpha^*}$$

$$\mathcal{O}_\mu(p) = D_{\mu\alpha}\, \mathbb{1}_{\alpha\otimes\alpha^*}$$





$$\mathcal{O}_\mu(p) = \frac{S_{\mu\alpha}}{S_{1\alpha}}\, 1_{\alpha\otimes\alpha^*}$$

$S_{\mu\alpha}$ = modular matrix

$$\chi_\mu(-1/\tau) = S_{\mu\alpha}\,\chi_\alpha(\tau)$$

• μ to be a degenerate representation

$$\frac{S_{\mu\alpha}}{S_{1\alpha}} = \mathrm{Tr}_R\, e^{2\pi a}$$

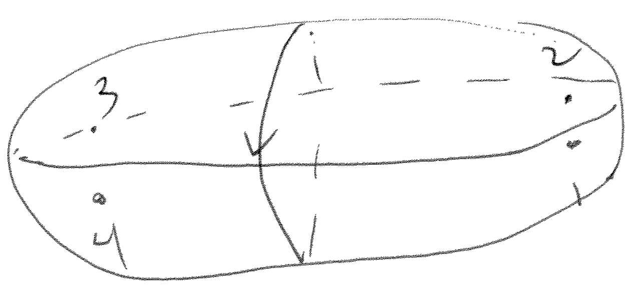
$$\langle \mathcal{O}_\mu(p)\rangle^{\text{Toda}}_{C_{g,n}} = \langle W_R\rangle_{N=2} = \int [da]\, \mathrm{Tr}_R\, e^{2\pi a} |Z|^2$$

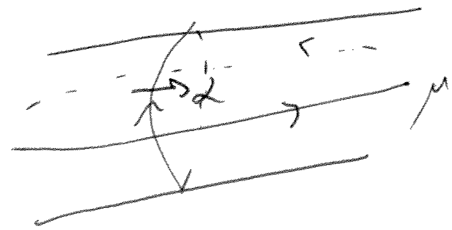
$G = SU(2)$.

classify charges of loop operators

$\updownarrow$ 1-1

classification of non-self-intersecting curves on $C_{g,n}$





$$O_\mu(p)\, \mathcal{F}_\alpha = \int d\alpha' \; \underbrace{O(\alpha,\alpha')}_{\text{kernel}} \mathcal{F}_{\alpha'}$$

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1. $O_\mu(p)$ μ is a degenerate representation
p is a non-self-intersecting curve.

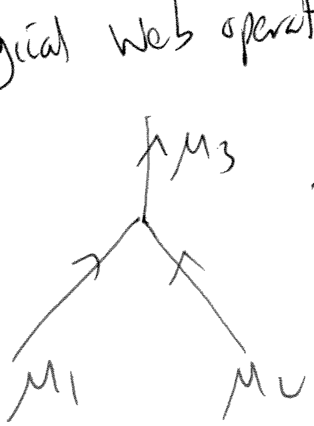
$SU(2)$: describes all loop operators

$SU(N>2)$: loop operators in the <u>duality orbit</u> of any Wilson loop

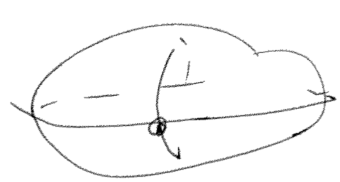
<u>example</u> $\mathcal{N}=2^*$ (μ_e, μ_m) $SL(2,\mathbb{Z})$

- $\mu_e \parallel \mu_m \longrightarrow$ Wilson loop
- $\mu_e \nparallel \mu_m \not\longrightarrow O_\mu(p)$

– Topological web operators

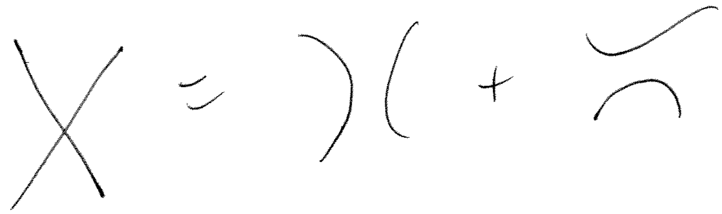


Toda



$N_{\mu_1\mu_2}^{\mu_3}$

SU(2)



SU(N>2)