23 August 2010
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Gauge, Hitchin, Liouville : Part II

$d = 3g-3+n \rightsquigarrow$ Quant of $\mathcal{M}_H(C)$

$\rightsquigarrow$ Liouville theory

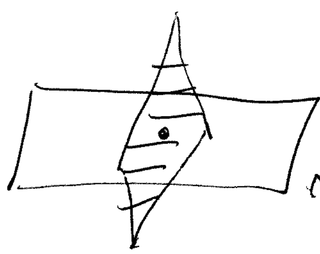
$d=0$ [Nekrasov-Witten] $\leftrightsquigarrow$ q-Teichmüller

© Concluding remarks

1) Relation to gauge — Need surface operators!

(i) AGGTV

(ii) AT



$x_2 + i x_3 = r e^{i\theta}$

$A_r \sim \alpha_r d\theta \qquad r = 1, \to 3g-3+n.$

$\alpha_r \in \mathbb{T}.$

Observation (AGGTV):

complexified α's naturally live on Jacobian Variety of SW-curve $\quad v^2 - \operatorname{tr}\theta^2(y) = 0.$

Σ
$\downarrow$
C

Abel-map: $\mathrm{Jac} \rightsquigarrow \Sigma^{(d)}$ $(w_1, \ldots, w_d)$

$w_k \rightsquigarrow$ Insertion of a deg field

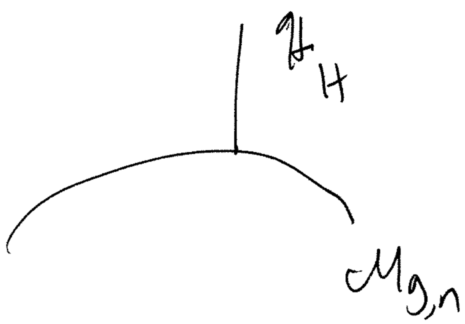
Generalized Nekrasov partition function:

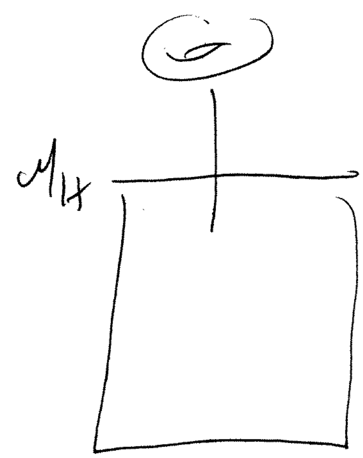
$$Z(a, \varepsilon_1, \varepsilon_2, m, q, w)$$

Conjecture: Z solves $\mathcal{D}_k^{BPZ} Z = 0$.

Pol. (w, x) dependent on $\mathcal{C}$

$\Psi(w, z)$ ← complex str. of $\mathcal{C}$





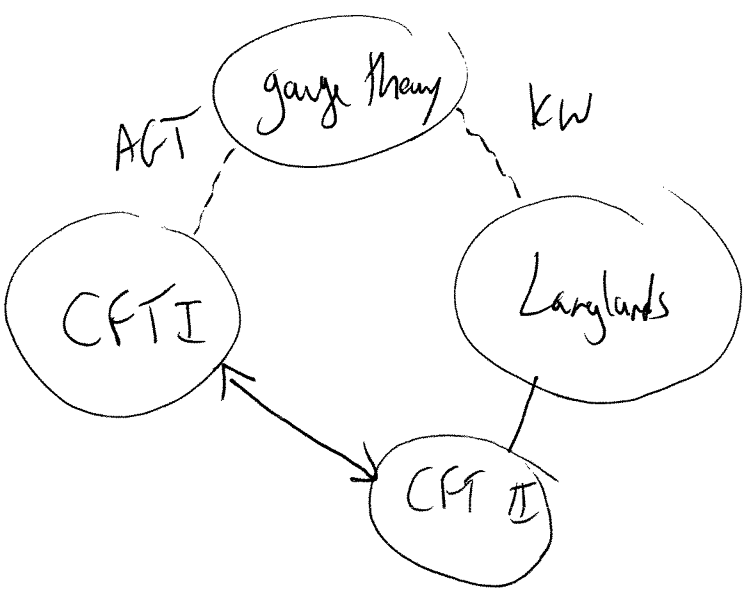
$$b^2 \frac{\partial}{\partial z_r} \Psi(\ldots) = H_r^w \Psi(w, z)$$

larger picture

$$\epsilon_2 \partial_\mu + M(\mu)$$

$$\begin{tikzcd}
 & (M_H, I) \text{ (int. syst.)} \arrow[dl, "\epsilon_2"'] \arrow[dr, "\epsilon_1"] & \\
(M_H, J) \text{ isomonodromic defs} \arrow[dr, "\epsilon_1"'] & & \hbar\text{-Hitchin (geom. Langlands) (}\mathcal{D}\text{-modules)} \arrow[dl, "\epsilon_2"] \\
 & \left(\begin{array}{c}\text{Liouville} \\ \text{theory}\end{array}\right)_{\epsilon_1, \epsilon_2} &
\end{tikzcd}$$

So to complete the triangle



local systems

$\begin{pmatrix} \text{Liouville} \\ \text{theory} \end{pmatrix}_{\varepsilon_2 \varepsilon_1}$

$$\mathcal{F}(a, b, m, q) \equiv \mathcal{F}(a, b^{-1}, m, q)$$

$$c = 1 + 6(b + b^{-1})^2$$