Tracing the Dark Matter Sheet in Phase Space

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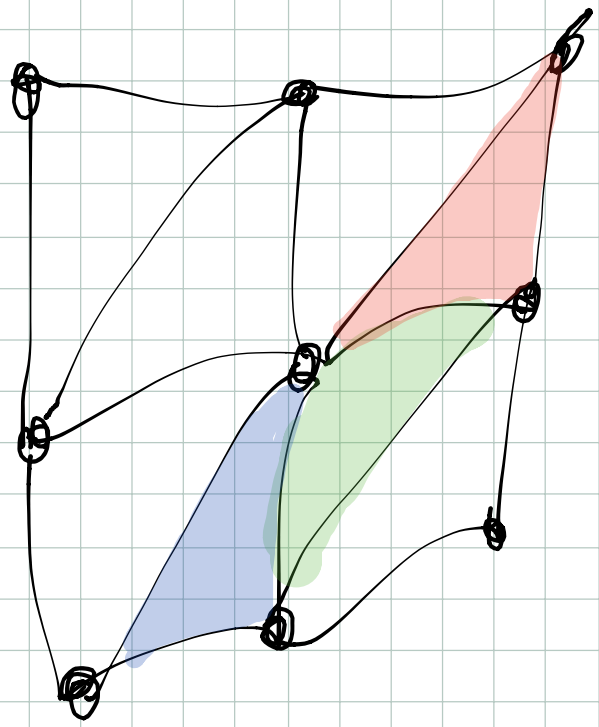


Outline:

- Dark Matter Sheet?
- Cosmological N-Body Simulations
- Phase Space
- Projecting to configuration space
- How useful is this?
- Collisionless fluids
- New ways to simulate DM & collisionless plasmas.

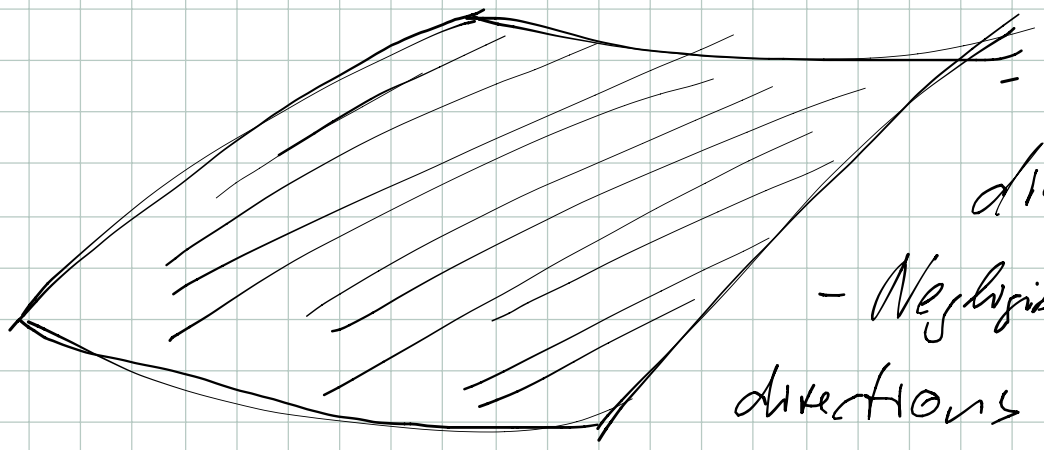
The Dark Matter Sheet ?

COOL WIMPS

Dark Matter is commonly hypothesized to originate within minutes after the BIG BANG. If it were moving relativistically today, galaxies and other structures would not exist. We speak of COLD DARK MATTER.

Working Hypothesis:

- Weakly interacting massive particle (say ≈ 100 GeV).
- Very cold. Even keV particles would only have $\sim \frac{m}{s}$ speeds today.



- Almost perfectly uniformly distributed.
- Negligible extent along velocity directions in phase space.

The Dark Matter Sheet ?

Fluid

If dark matter was a microscopic particle, our Galaxy would contain $\approx 10^{67}$ of them assuming a mass of 100 GeV per particle. Potentially this could be much larger for sterile neutrino candidates or atomic dark matter.

$$N \gg N_* \gg N$$ we can't on a computer

Would like to solve the **Vlasov-Poisson** system:

$$\vec{\nabla}^2\phi = -4\pi G \rho \qquad \& \qquad \frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f + \vec{a}\cdot\vec{\nabla}_v f = 0$$

where $\vec{a} = -\vec{\nabla}\phi$

Here f is the distribution function in phase space,

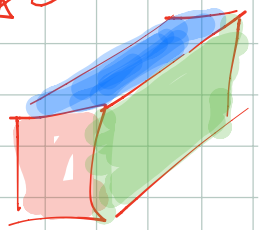
ϕ the gravitational potential.

An incompressible fluid in **six** dimensional phase space.

The complexity here is as bad as radiation transport in some respects.

The Dark Matter Sheet ?

3D MANIFOLD MOVING IN SIX DIMENSIONAL PHASE SPACE



Phase space volume is conserved

$$\Delta V \cdot (\Delta v)^3 = \text{const}$$

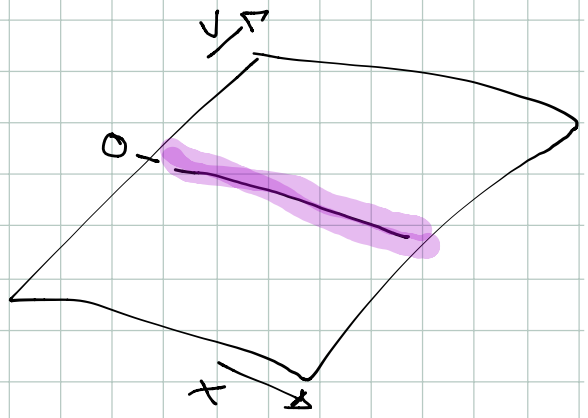
ΔV: spatial volume; $(\Delta v)^3$: volume in velocity space

redshift when $E \simeq 100\,\text{GeV}$: $1\,\text{eV}$ @ $z \sim 1000$

$100\,\text{GeV}$ @ $z \sim 10^{14}$

Matter density dropped by a factor $\sim 10^{42}$ since then.

$\Rightarrow$ YES. VERY COLD.



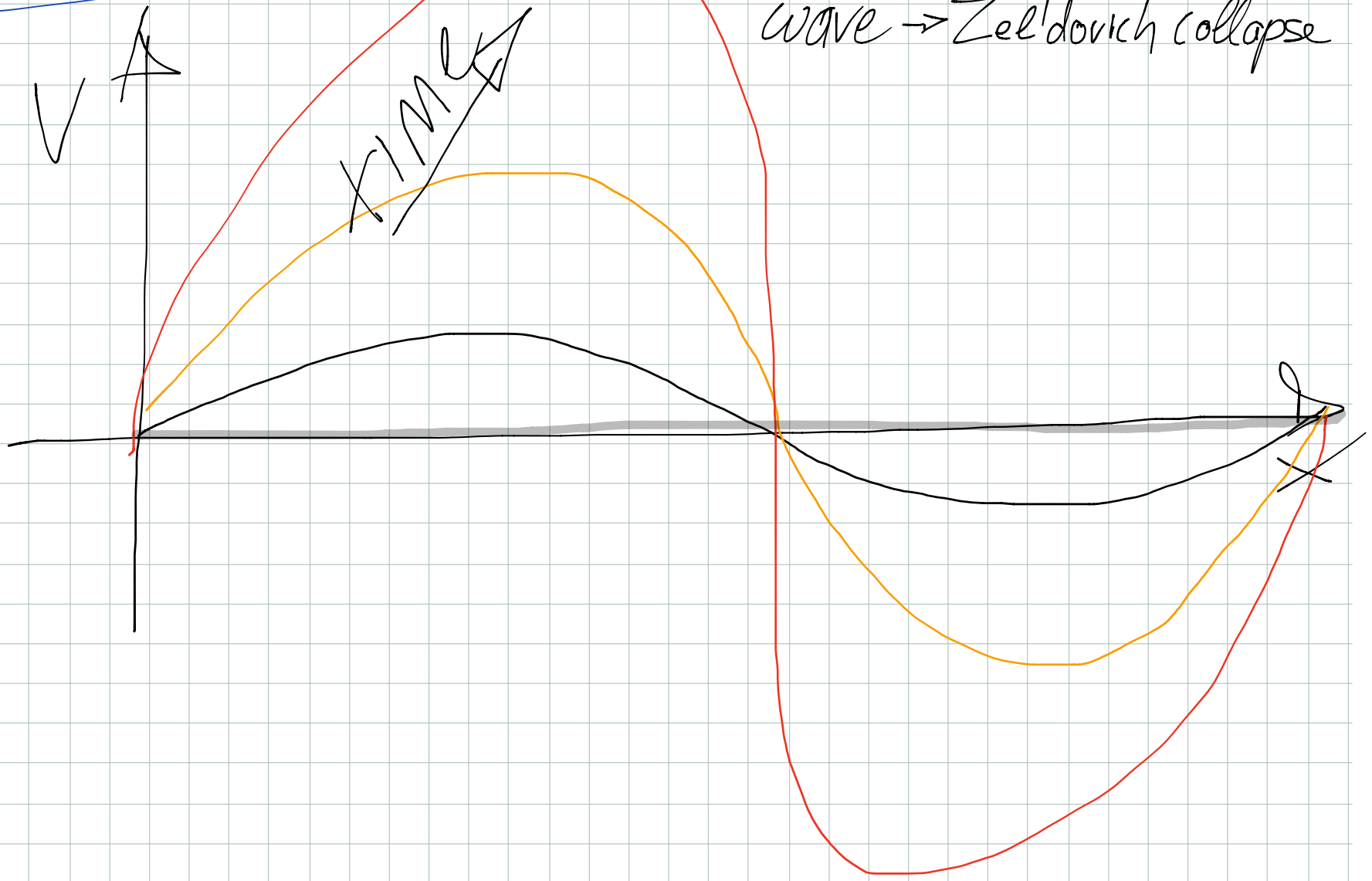
Tiny initial peculiar velocities

$\Rightarrow$ distribution function $f(\vec{x},\vec{v}) = f_0\,\delta(\vec{v})$

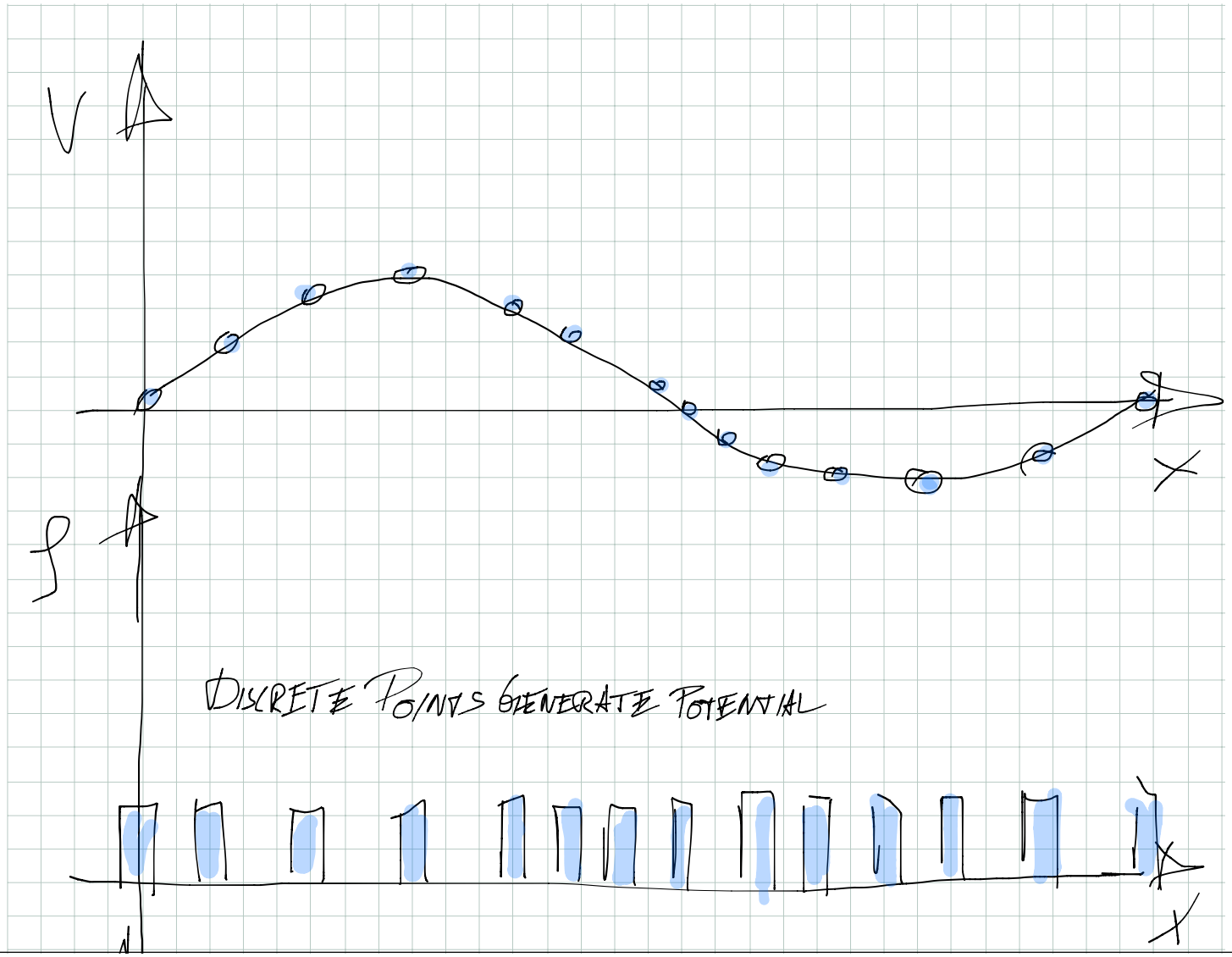
is single valued at every $\vec{x}$

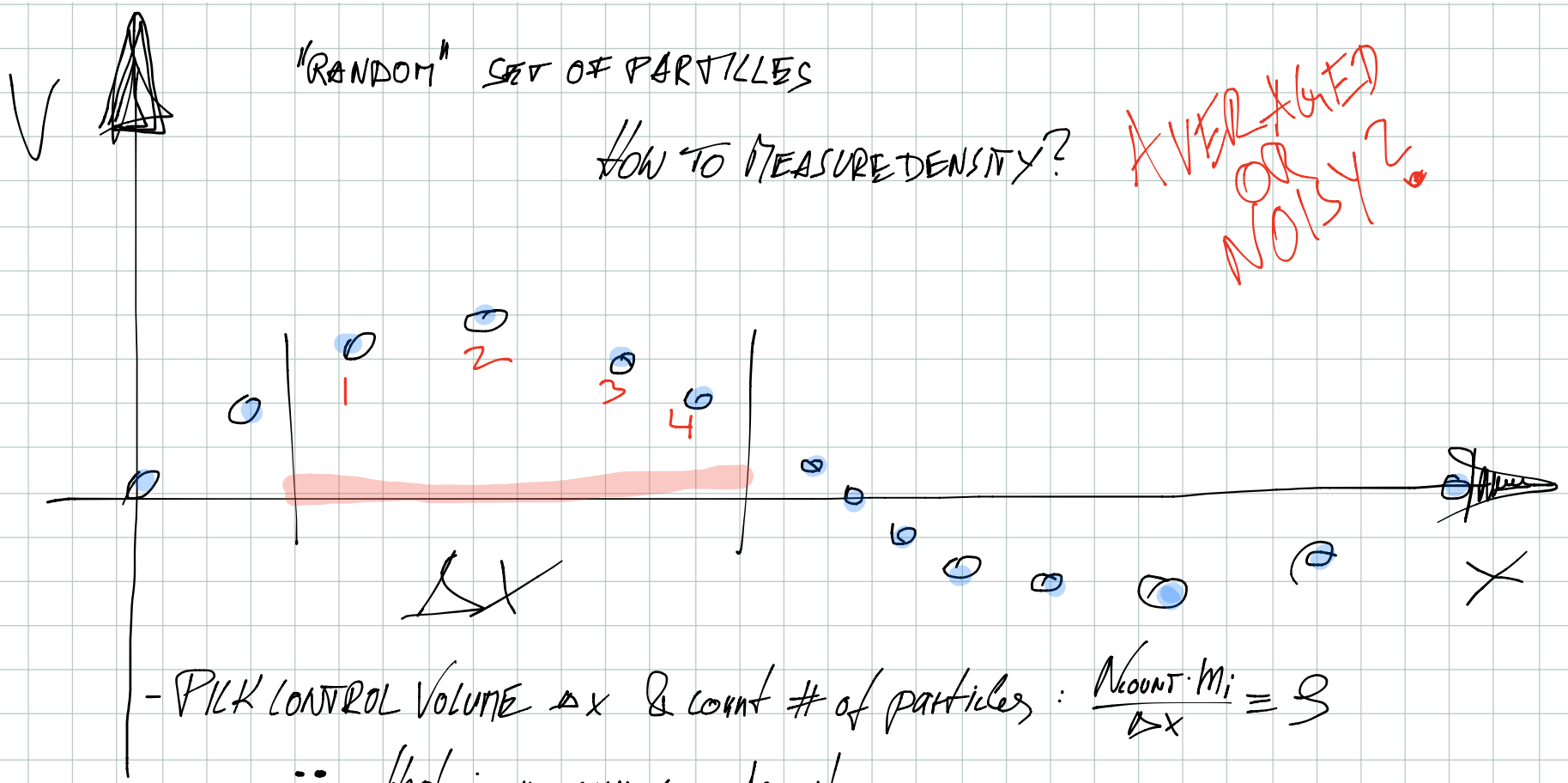
δ: DIRAC DELTA

PHASE SPACE



Steepening of initial SINE wave → Zeel'dovich collapse

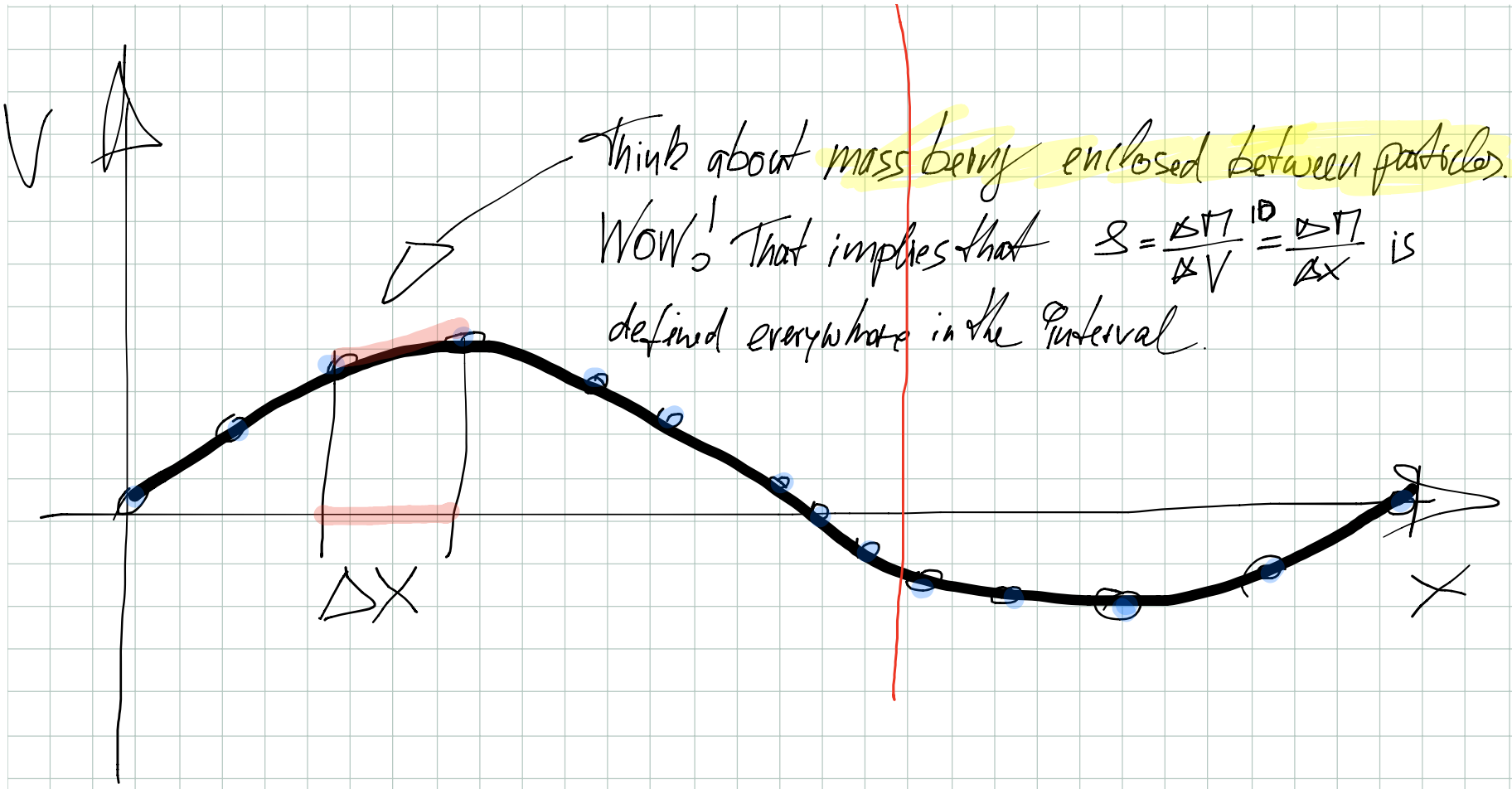

V
X
DISCRETE POINTS GENERATE POTENTIAL
X



- PICK CONTROL VOLUME Δx & count # of particles: $\frac{N_{COUNT} \cdot m_i}{\Delta x} \equiv \rho$

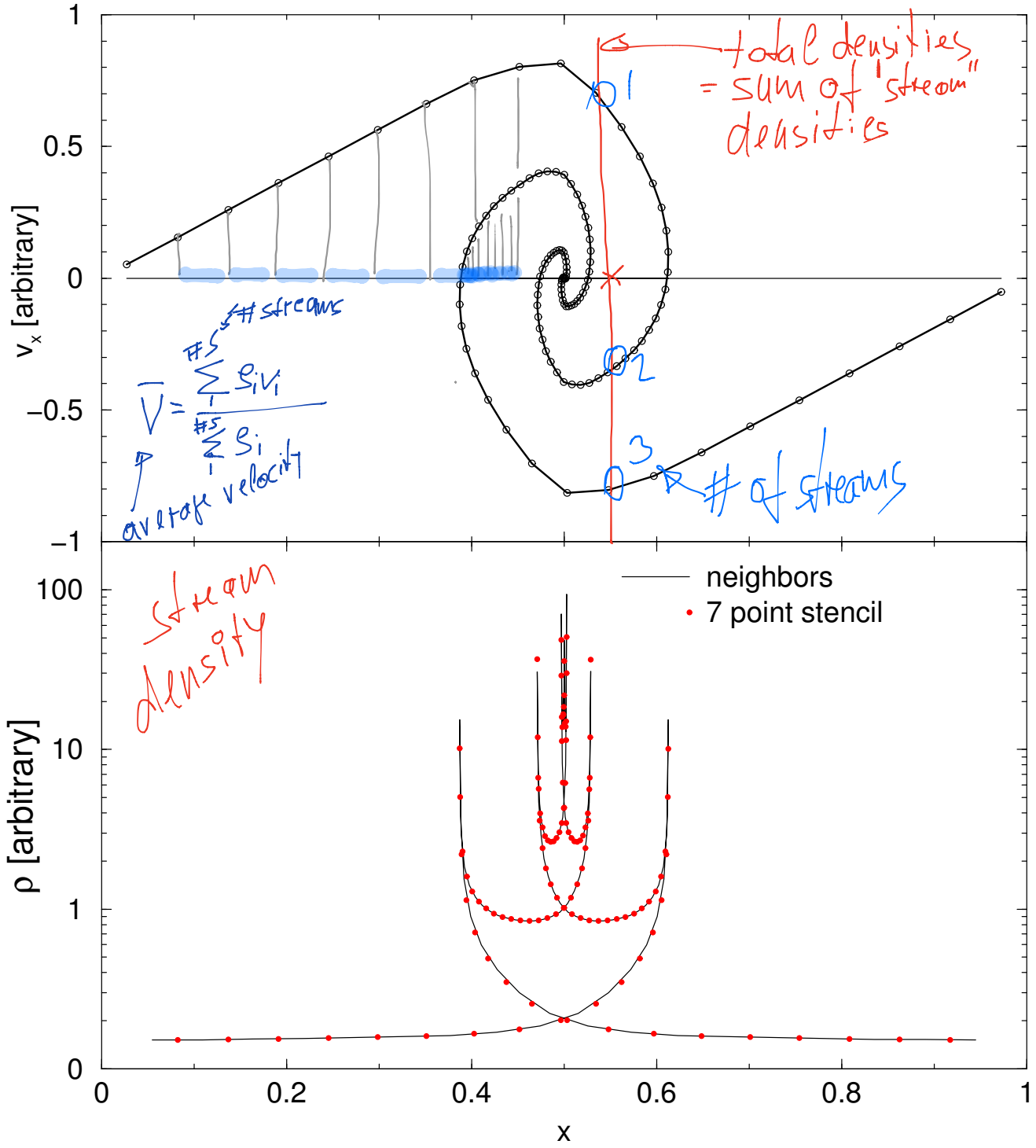
∴ that is an average density.

- Adaptive Kernel smoothing? Still an average...
- VORONOI? Assign every particle minimal volume around it. Not an average but potentially noisy...



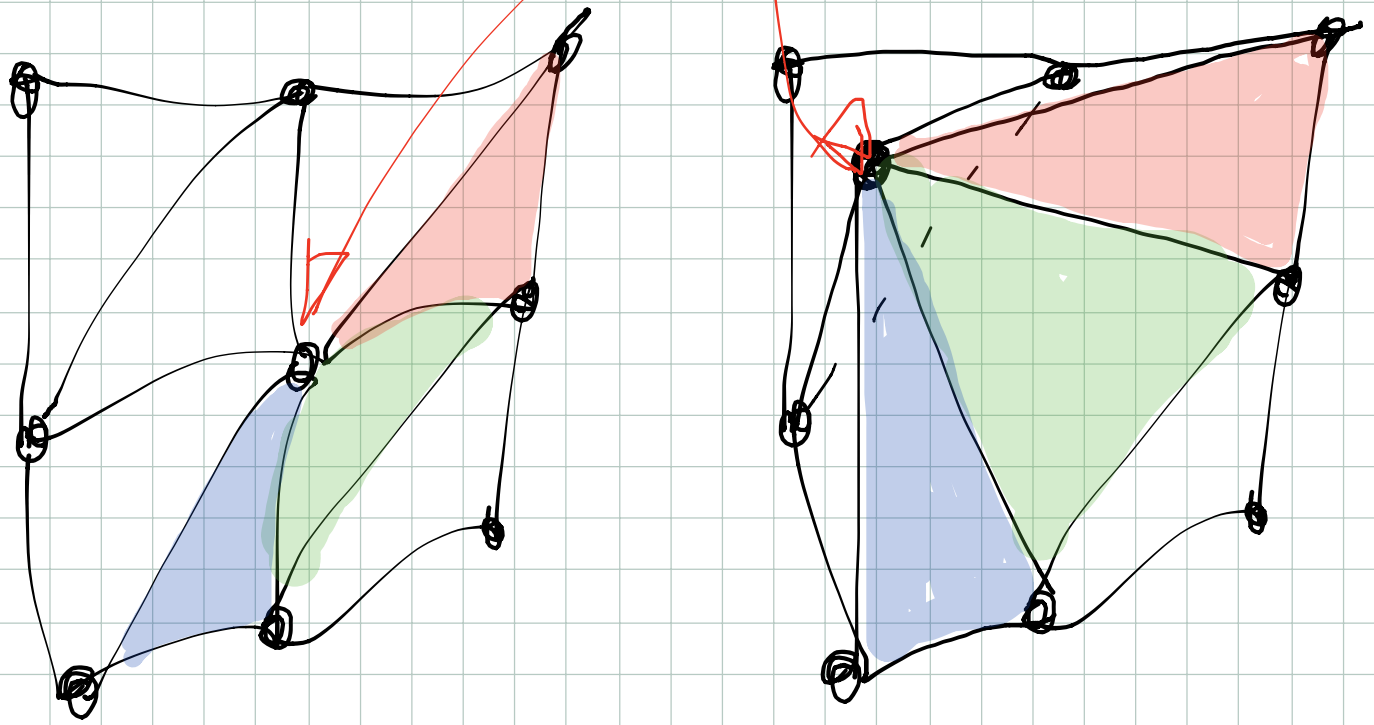
Think about mass being enclosed between particles.

Wow! That implies that $\delta = \frac{\Delta m}{\Delta V} \overset{1D}{=} \frac{\Delta m}{\Delta x}$ is defined everywhere in the interval.


total densities
= sum of "stream"
densities
1
2
3
# of streams
$\bar{V} = \frac{\sum_{i}^{\#s} s_i v_i}{\sum_{i}^{\#s} s_i}$
streams
average velocity
stream
density
neighbors
7 point stencil
v_x [arbitrary]
ρ [arbitrary]
x
1
0.5
0
−0.5
−1
100
10
1
0
0.2
0.4
0.6
0.8

A 2-D example

moved this point.

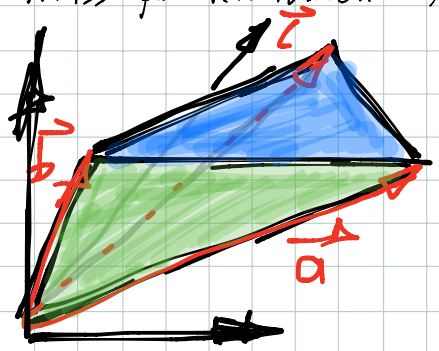


1D → LINE ; 2D → triangle ; 3D → tetrahedron

- just "painting" triangles with a weight $\frac{1}{V} = \frac{2}{\vec{a} \times \vec{b}}$

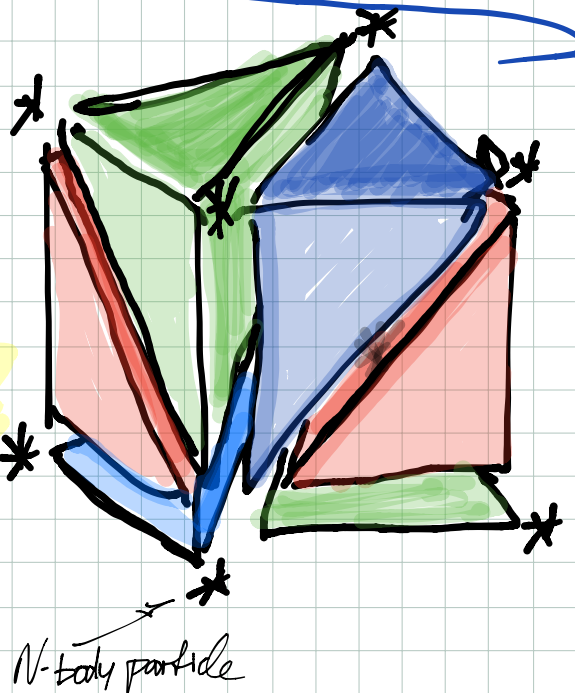
3 dimensional manifold in 6D Phase Space

- Natural tessellation tiles unit cube & splits it into six equal size tetrahedra.
- mass per tetrahedron = 1/6 of DM particle mass.

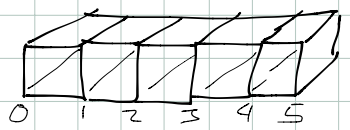


$$V = \frac{|\vec{a}\cdot(\vec{b}\times\vec{c})|}{6}$$

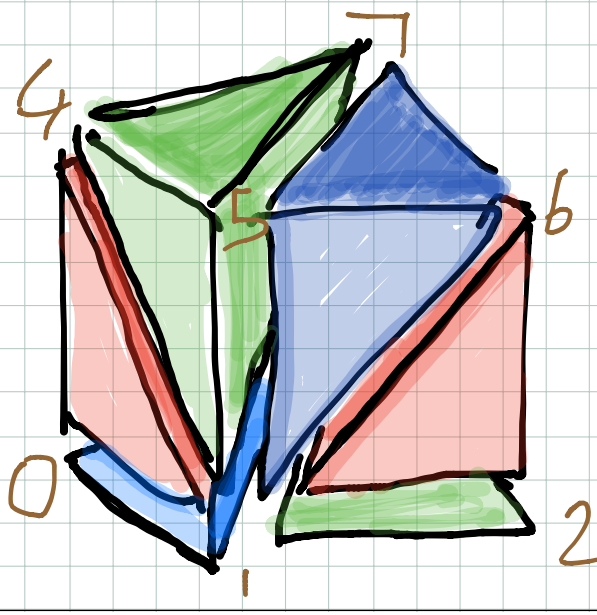
$$\Rightarrow \rho = \frac{m_p}{6V} = \frac{m_p}{|\vec{a}\cdot(\vec{b}\times\vec{c})|}$$



- Number the edges of the cube
- think of lattice
- Looping over

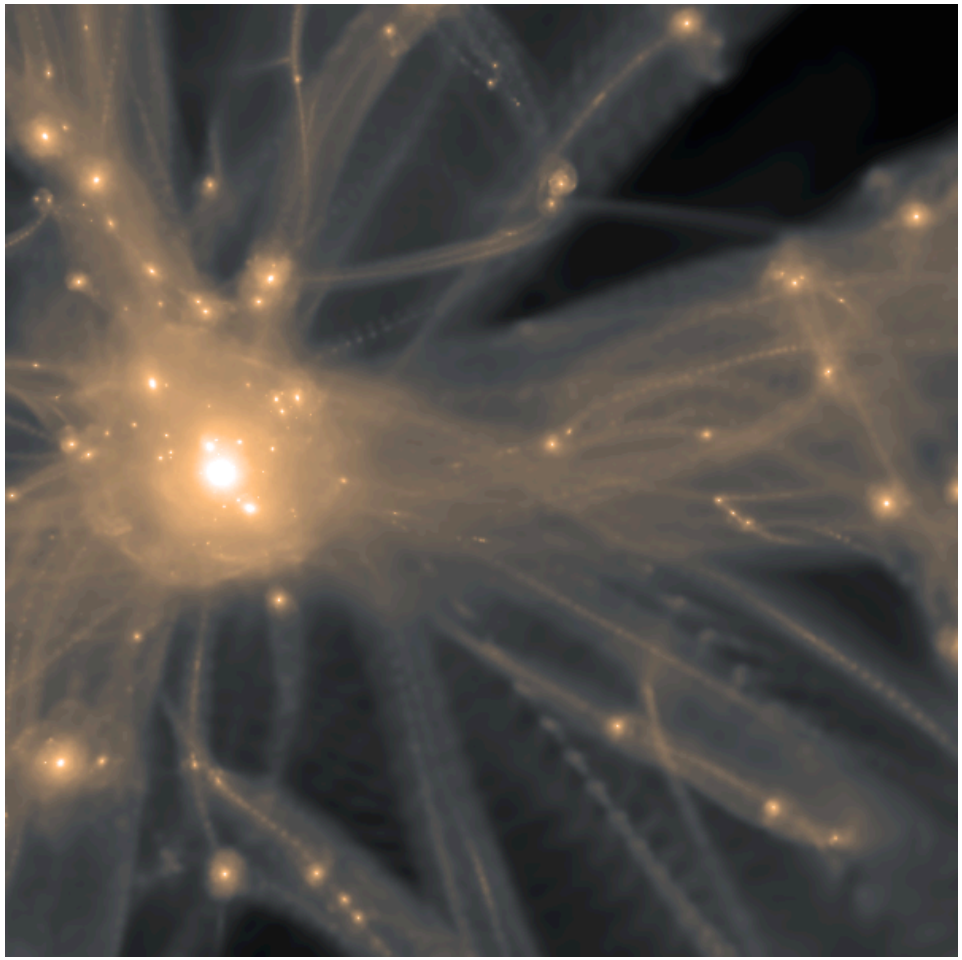


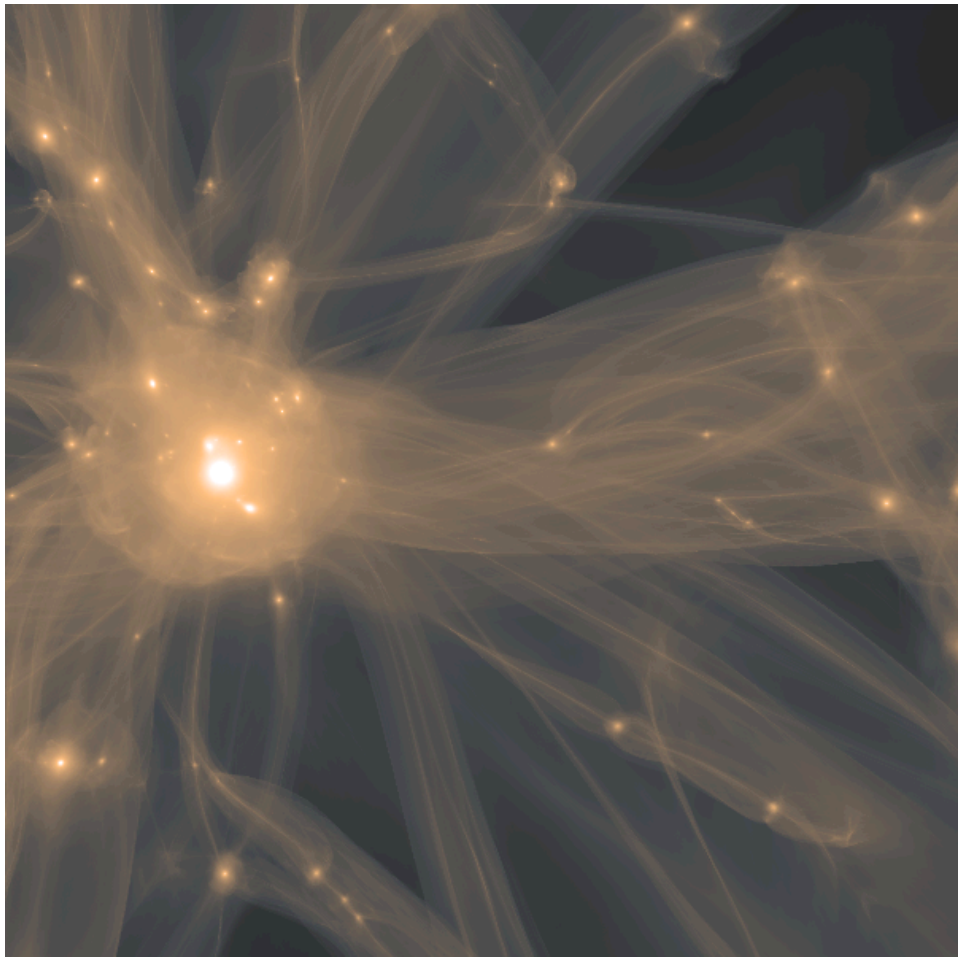
the initial cartesian (LAGRANGIAN) lattice generates the 6N tetrahedra.



Use DM sheet to get distribution function

Renderings of same warm DM simulation data

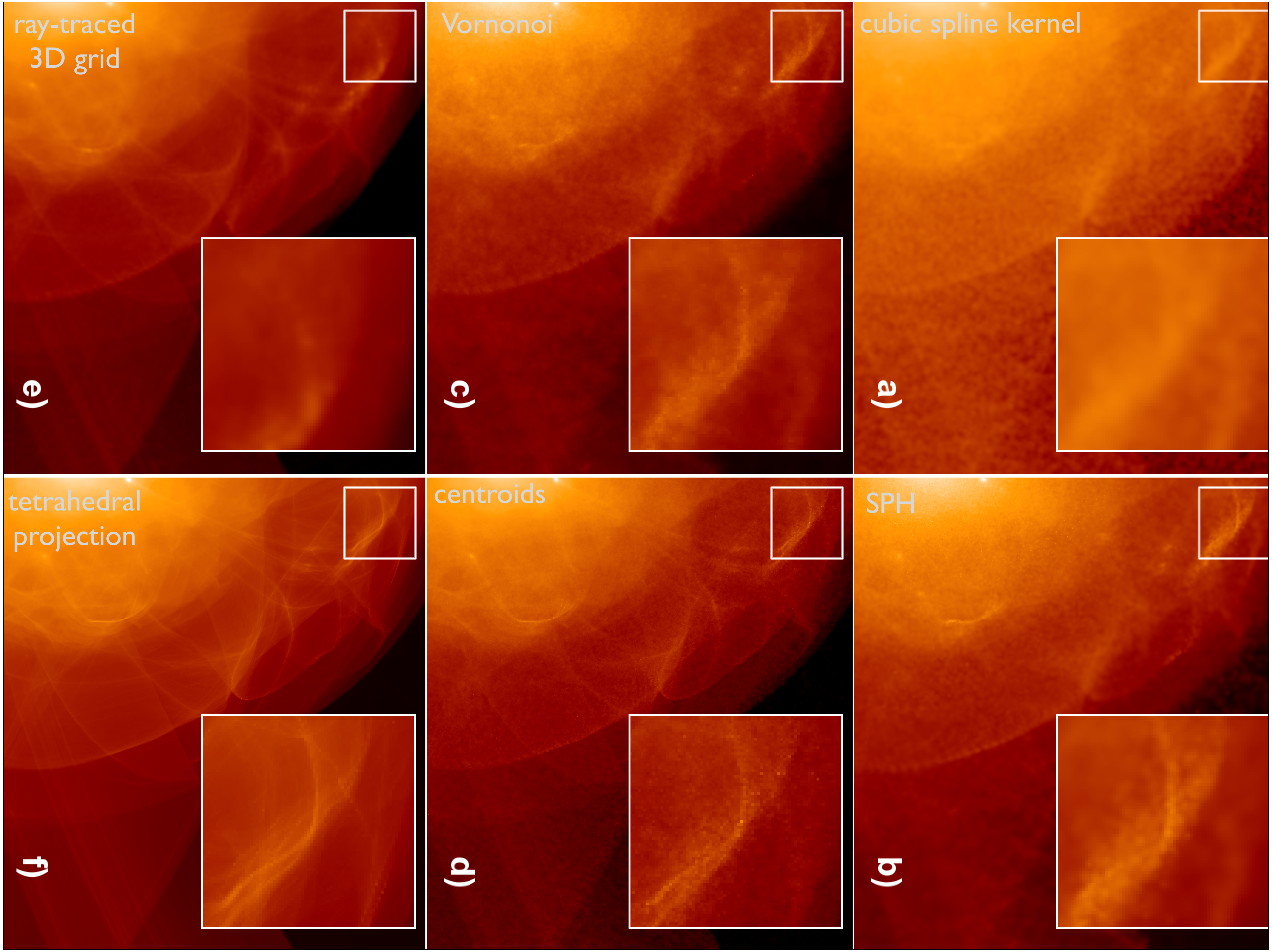




Mass is spread out ⇒ fragmentation reduced

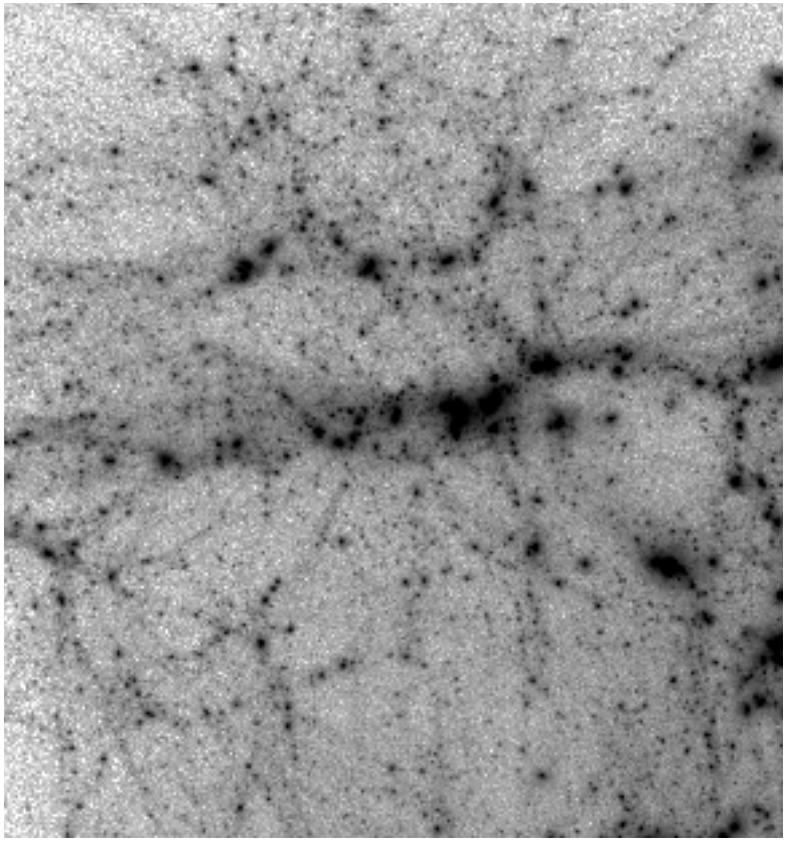
Adaptive kernel filtered

Kaehler et al. 2012 full tet rendering

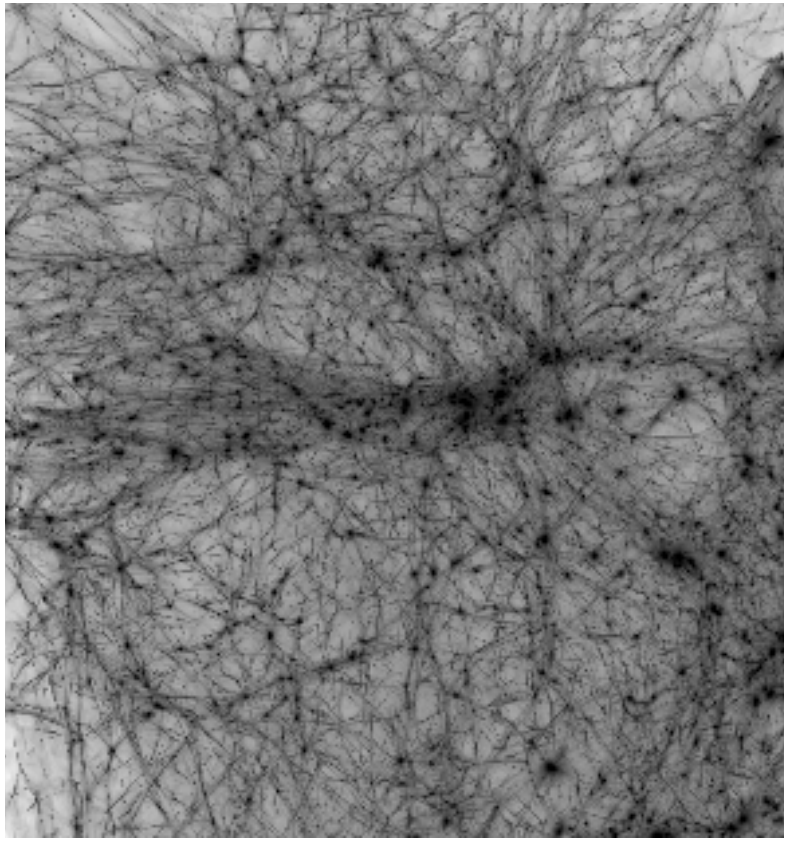


ray-traced
3D grid
e)
Vornonoi
c)
cubic spline kernel
a)
tetrahedral
projection
f)
centroids
d)
SPH
b)

Much more intricate web structure...



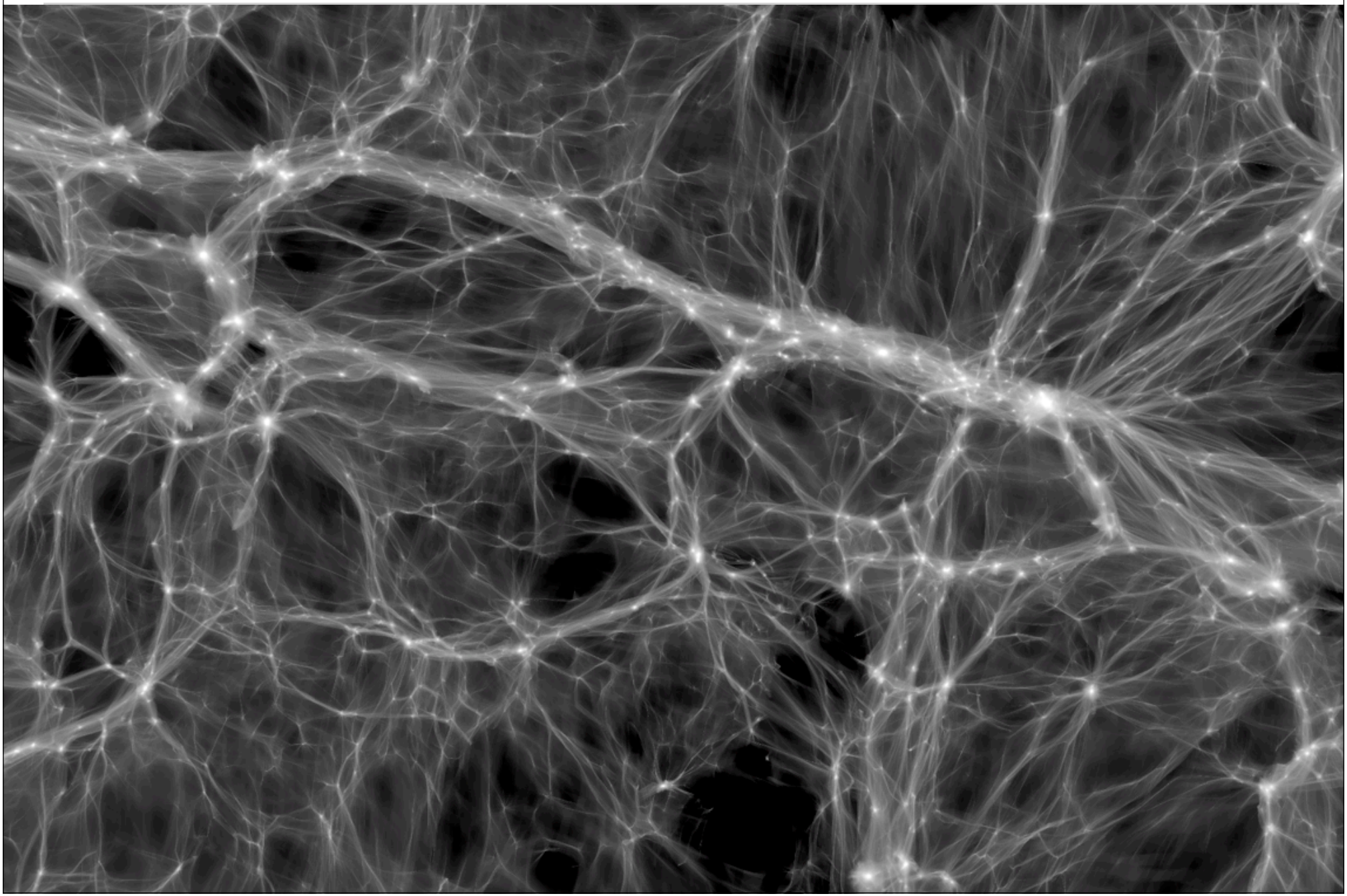
rendering points for particles.

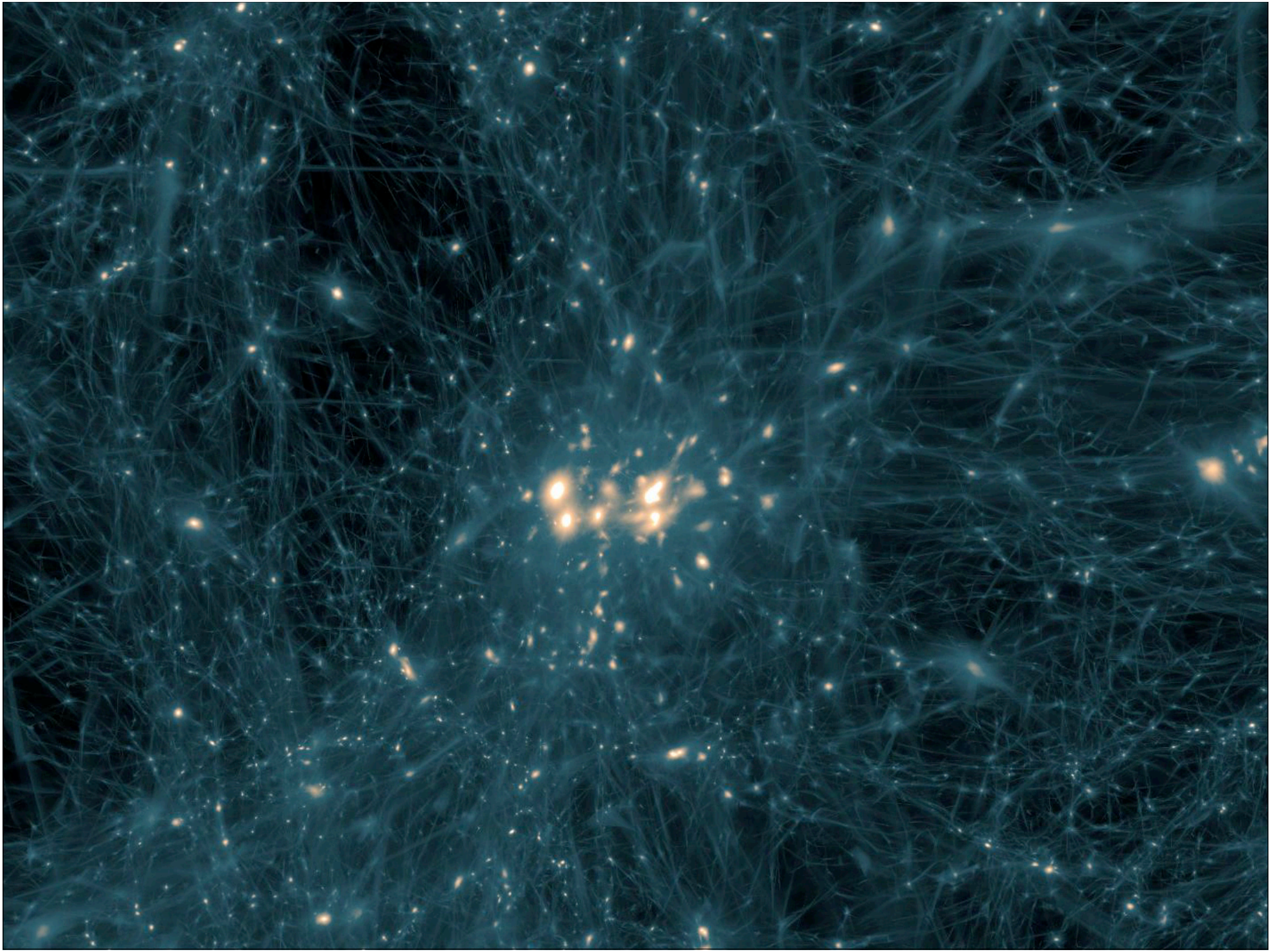


rendering tetrahedral phase space cells.

Same simulation data!

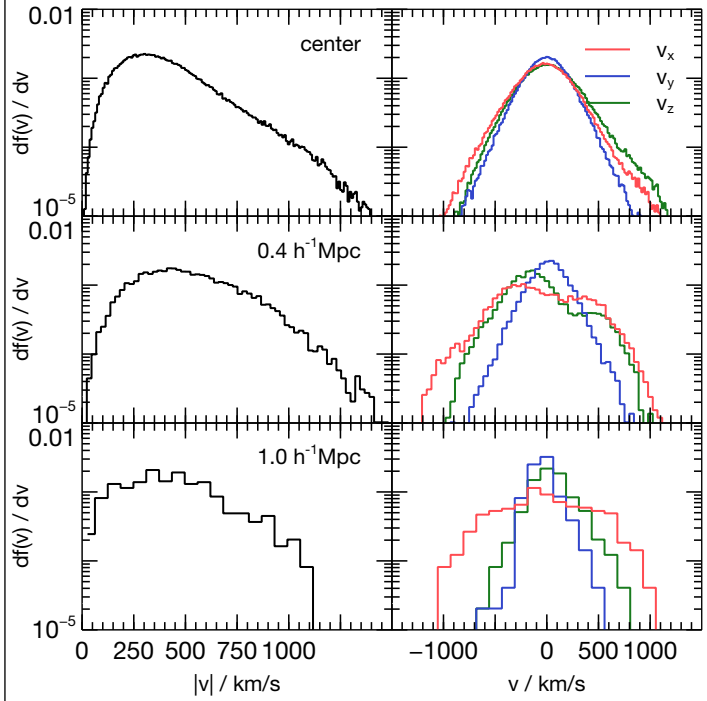
Density information everywhere in space

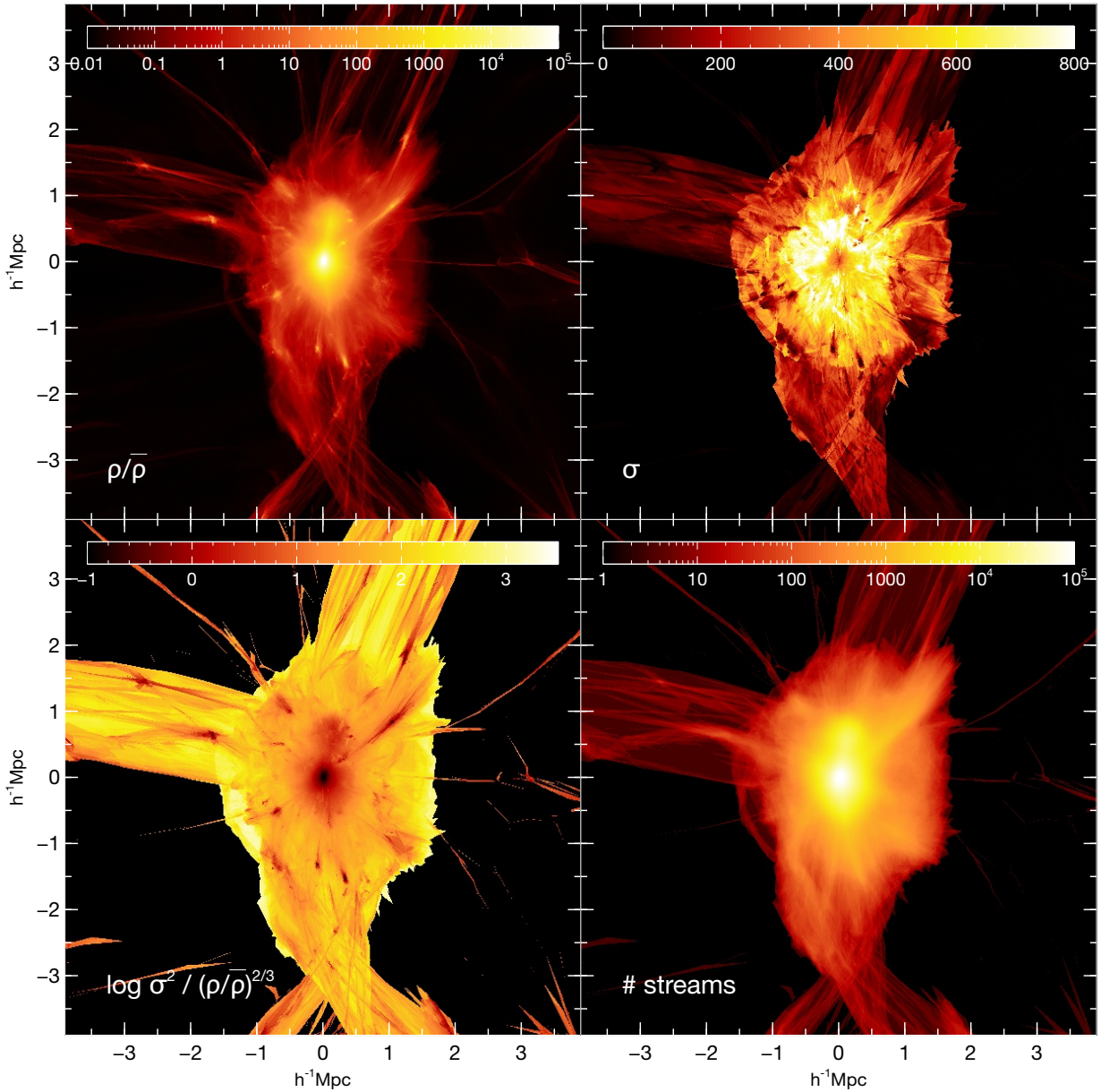




A first glimpse: analyzing phase space

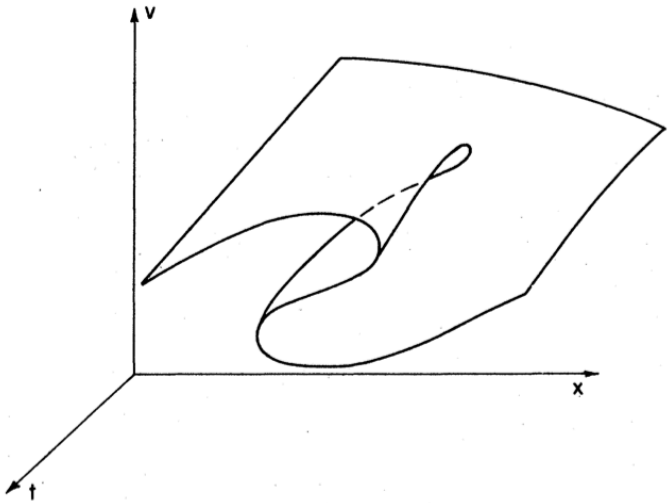
can probe fine-grained phase space structure.



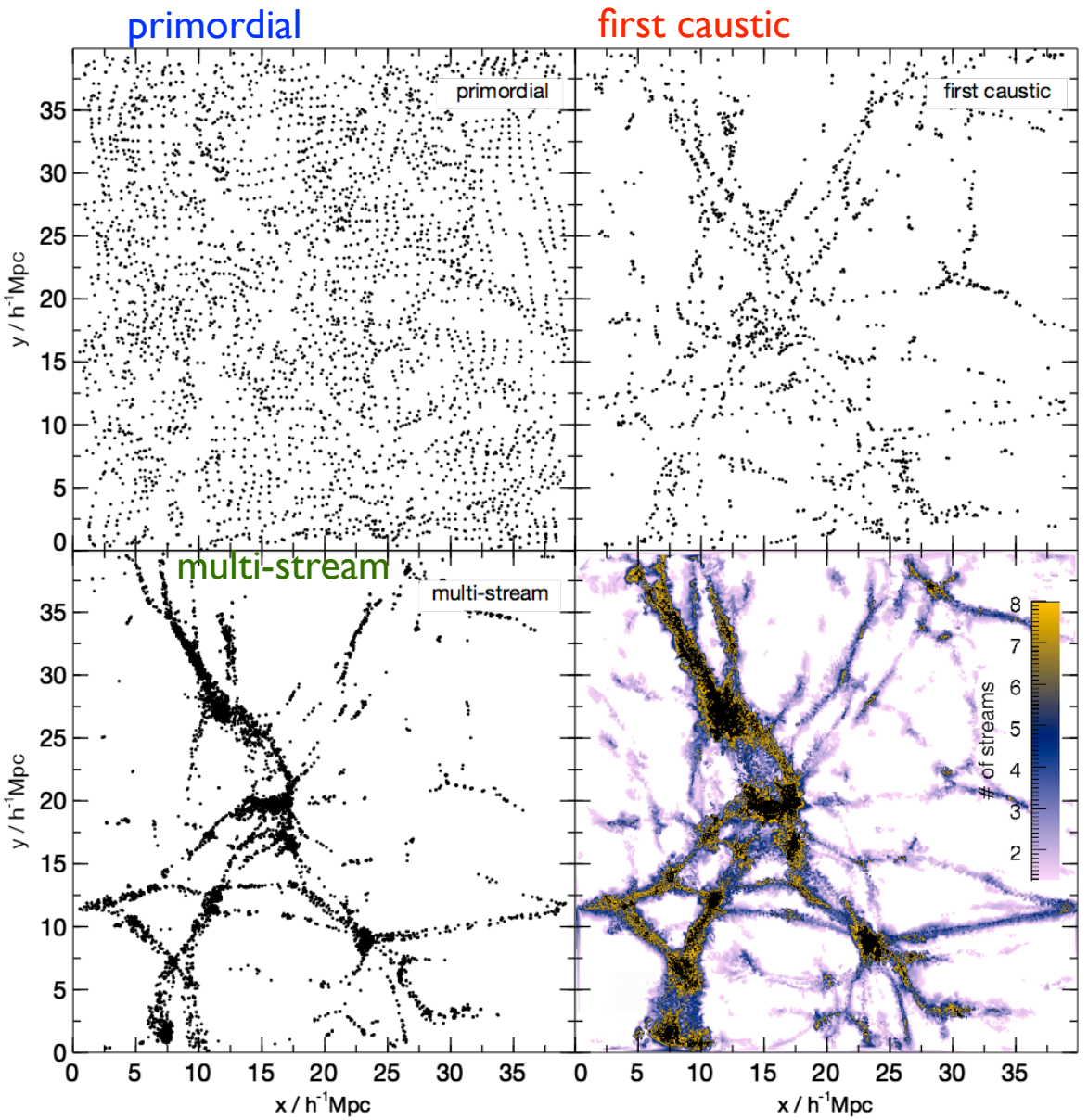


From caustics to multistream...

Use the local number of foldings



works remarkably well to understand dynamics of LSS



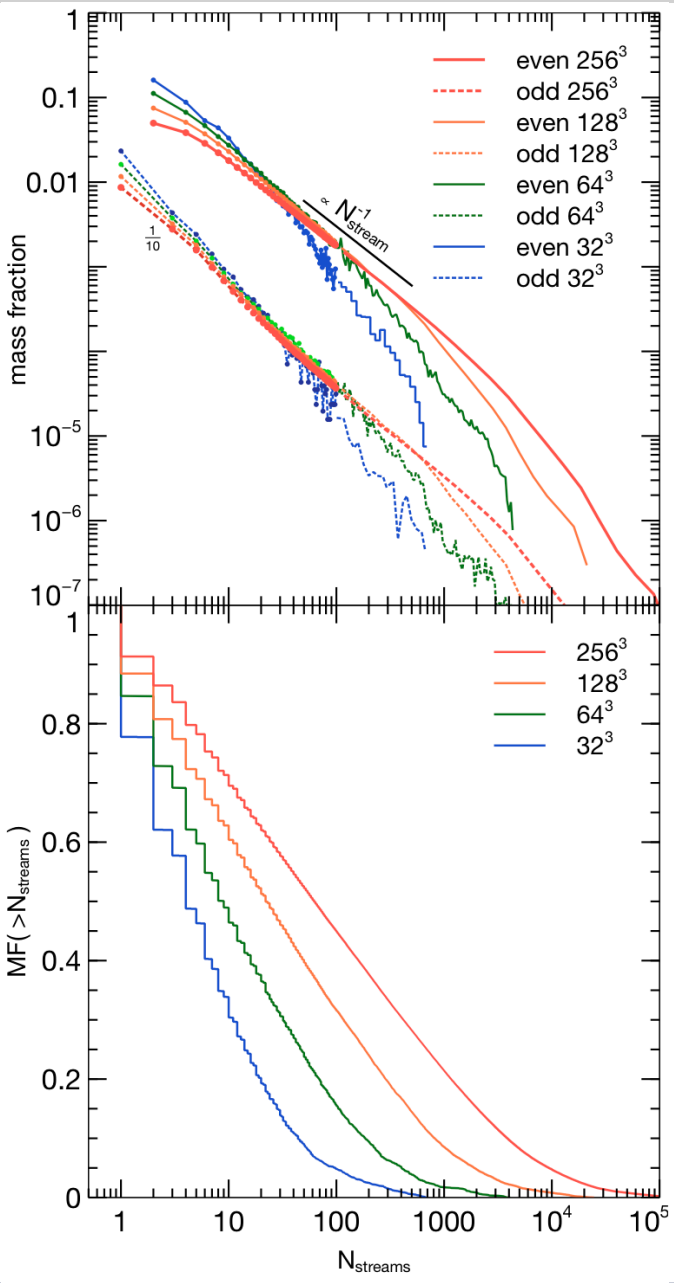
What mass fraction is multi-stream?

or, how much mass is collapsed?

almost everything,
a few% in caustics

approaches power-law?
(cf. also Shandarin et al. 2011)

Numbers increase
with resolution
just as you expect for CDM



So, what volume fraction is multi-stream?

or, how much volume is LSS?

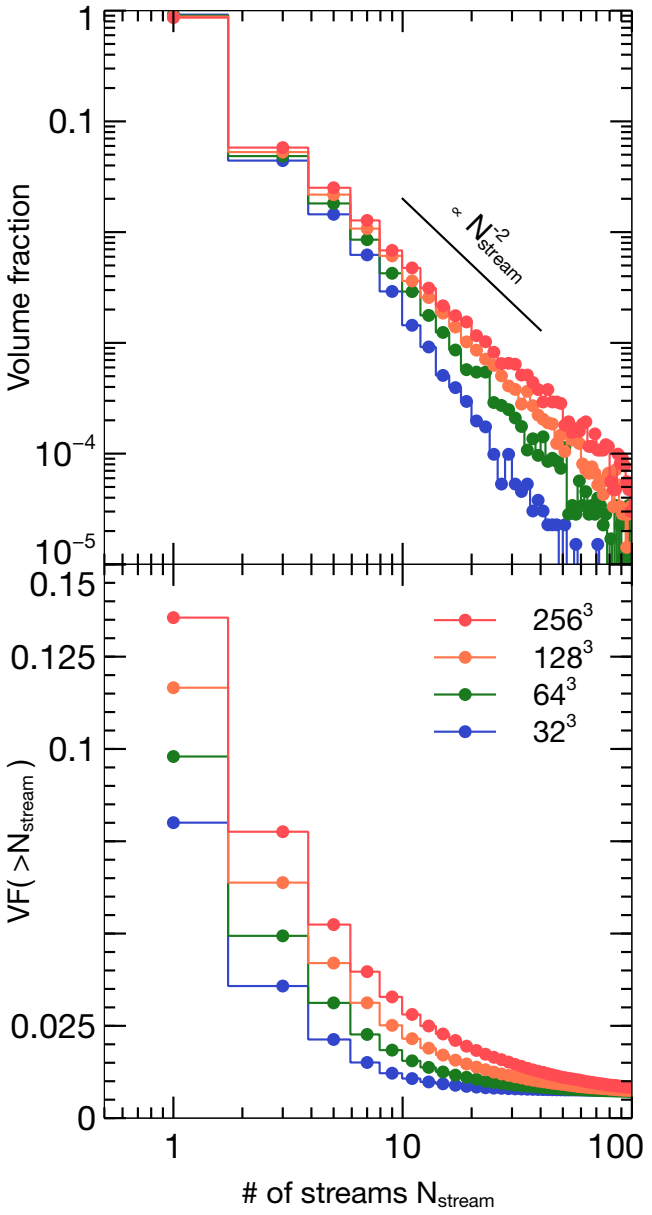
again...,
approaches power-law

AGAIN,
Continues to change with resolution

In particular:
The volume fraction of voids cannot even be determined.

This is CDM : clumps on all scales, maybe down to earth masses.

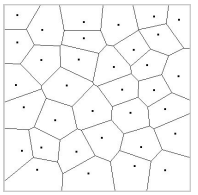
Voids, Sheets, Filaments can be sensibly defined only for a given spatial scale.

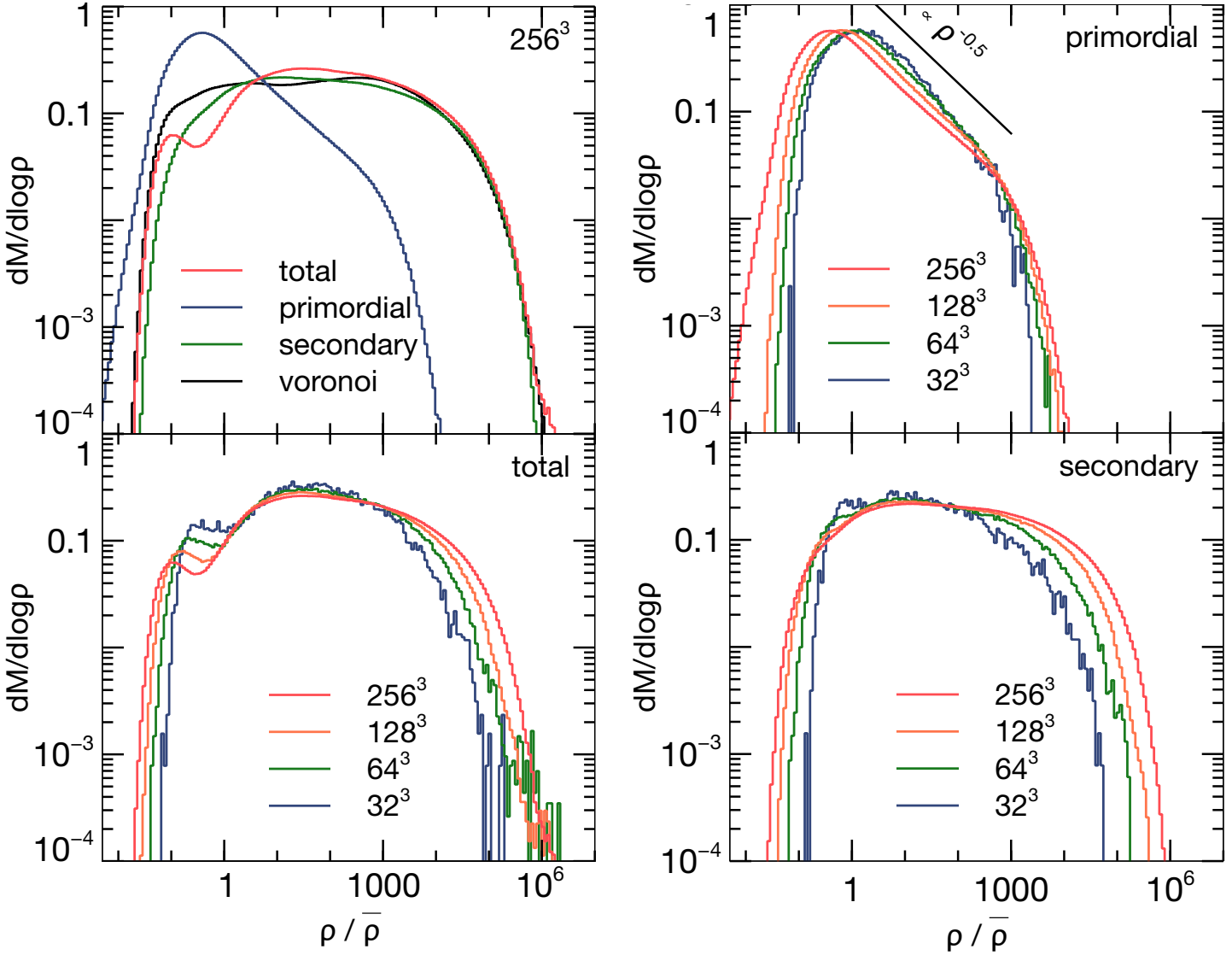


The density distributions

(mass weighted)

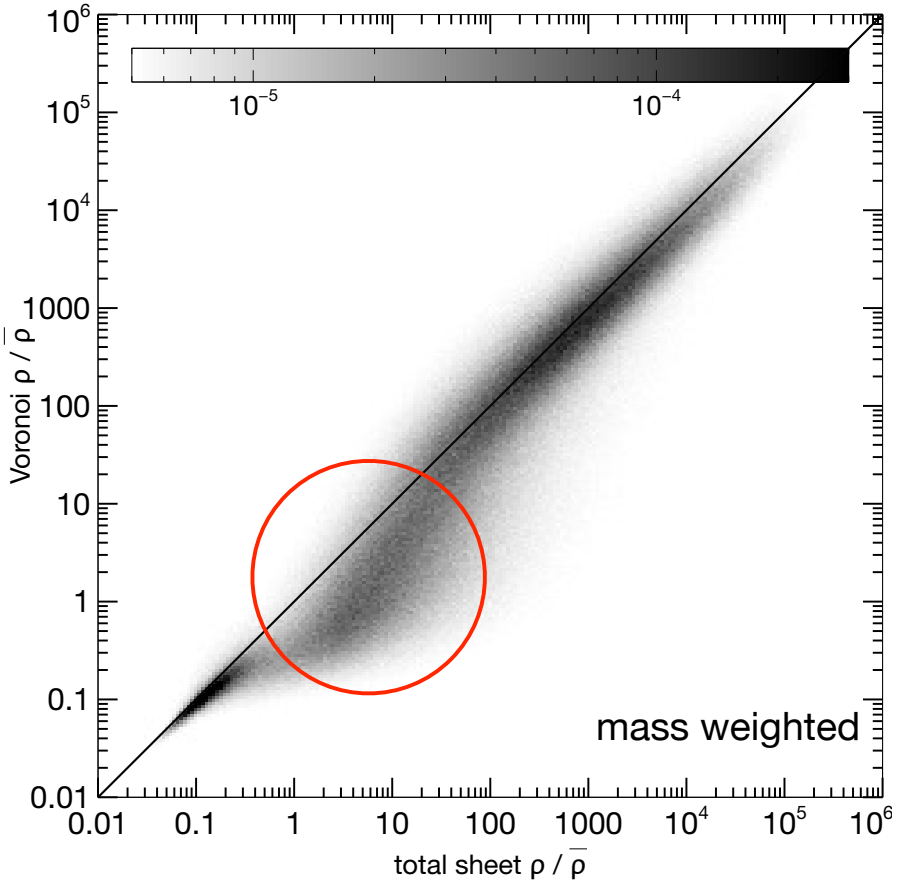
Voronoi:



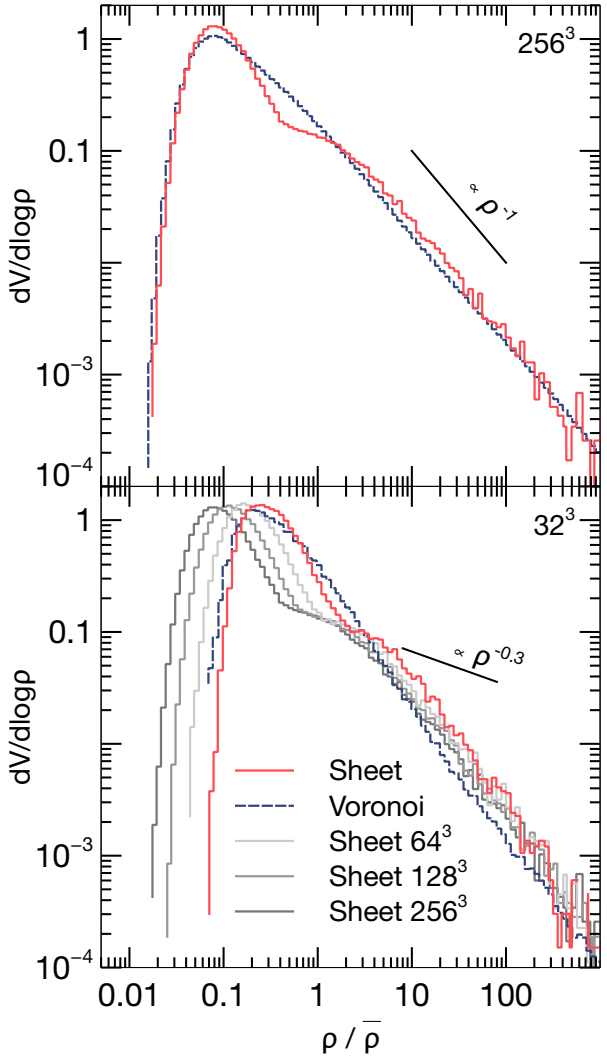


Comparison with Voronoi densities

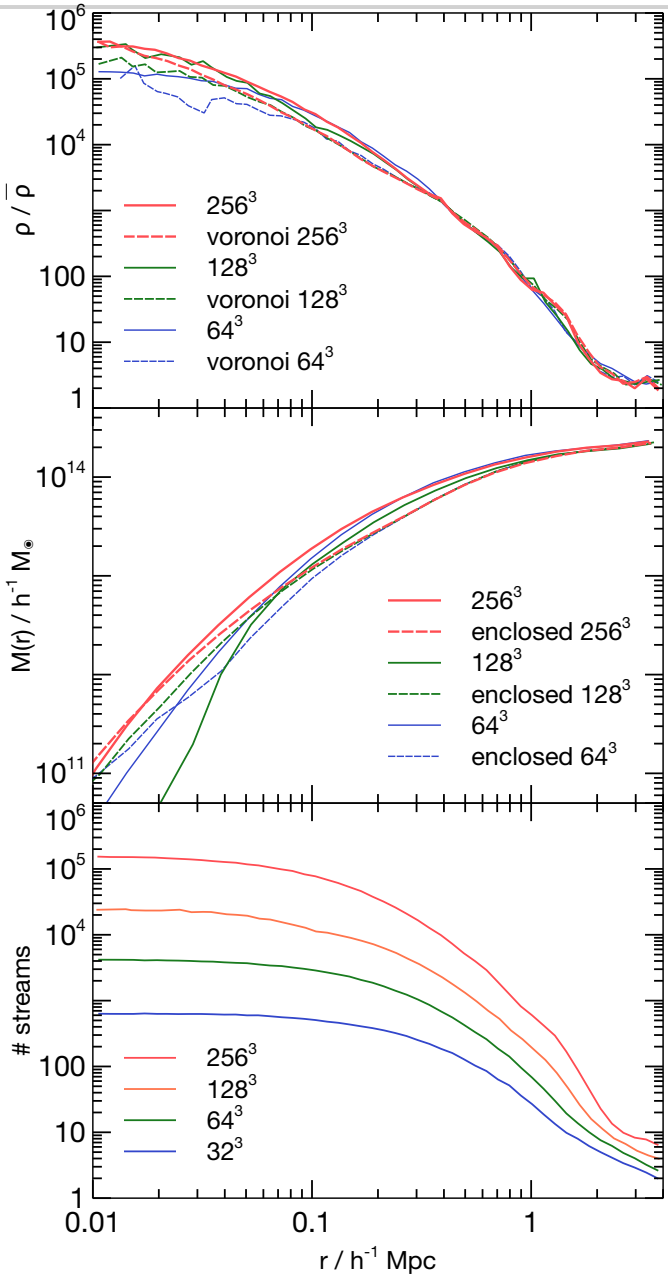
Much of the difference is at modest overdensities!

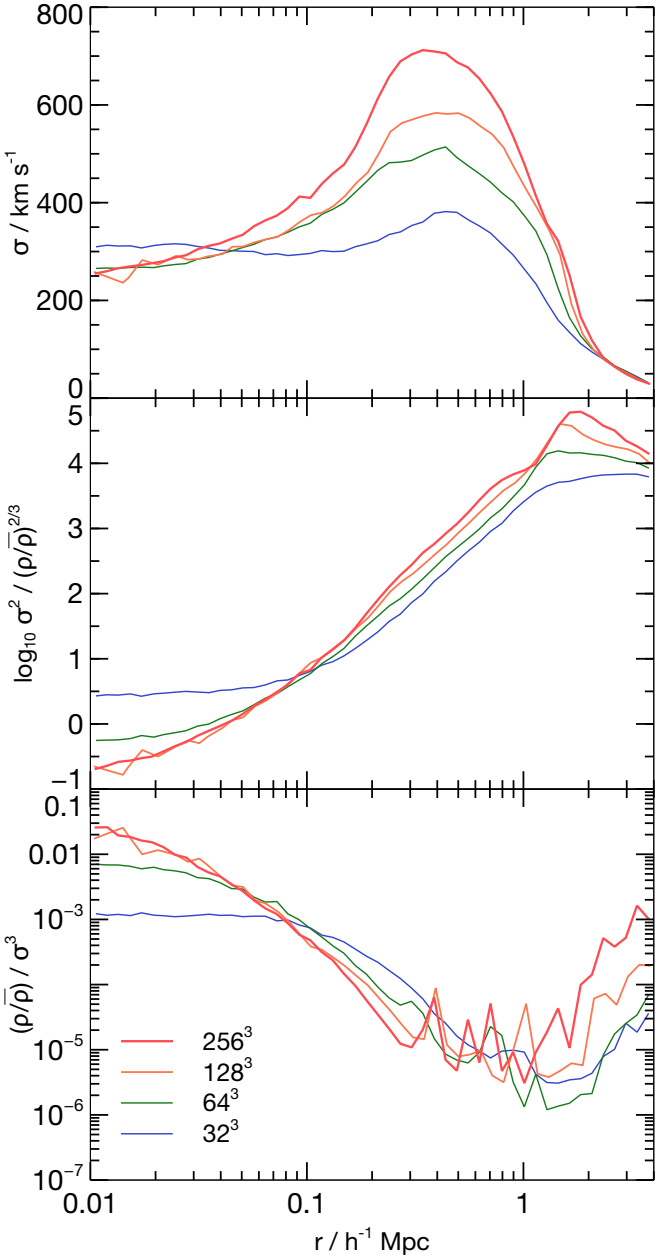


But they occupy a lot of volume.



Radial profiles





New numerical methods

Problems of the N-body method

Vlasov-Poisson system

$$\frac{\partial f}{\partial t} = -\frac{\mathbf{p}}{m} \cdot \boldsymbol{\nabla}_x f - \boldsymbol{\nabla}_x \phi \cdot \boldsymbol{\nabla}_p f$$

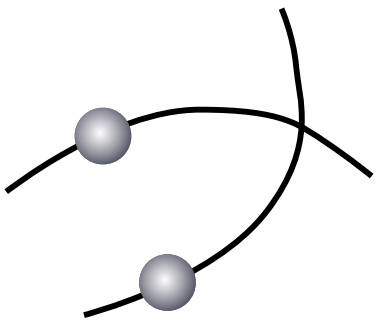
Distribution function

$$f(\mathbf{x}, \mathbf{p}, t) = \sum_{i=1}^{N} \delta_D(\mathbf{x} - \mathbf{x}_i(t))\, \delta_D(\mathbf{p} - \mathbf{p}_i(t))$$

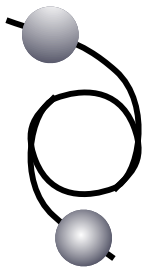
⇒ eq. of motion for N massive particles, not a continuum

Main Problem: two-body effects, can be reduced by force softening

Scattering

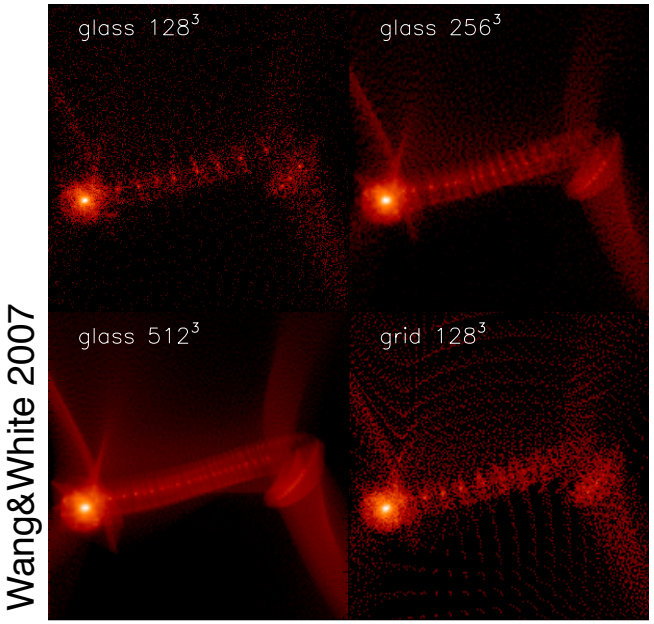


Clumping/
Fragmentation



Most problematic for non-CDM simulations!

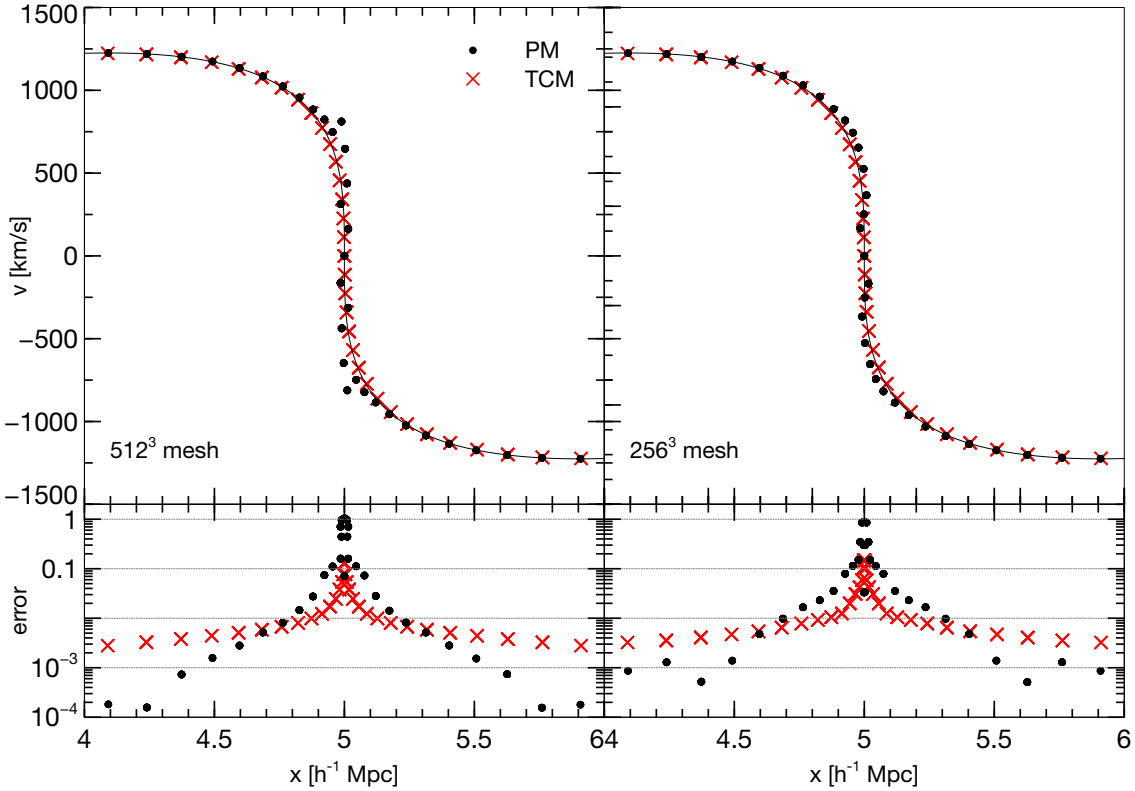
(cf. Wang&White 2007, Mellott



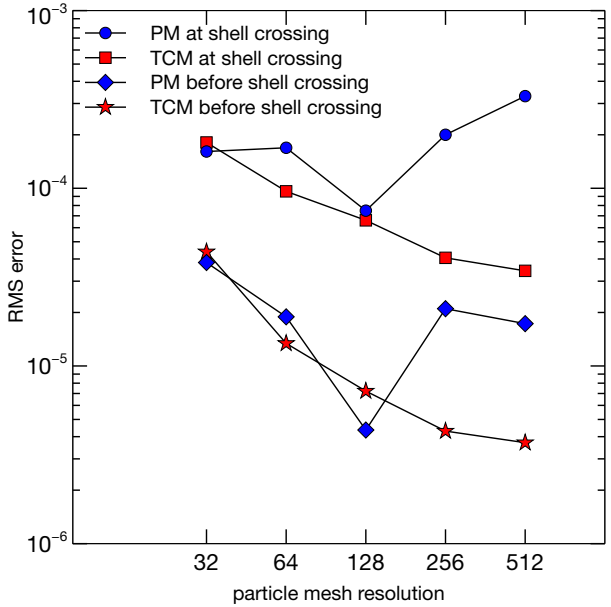
Wang&White 2007

Plane wave collapse I

Plane wave at shell-crossing



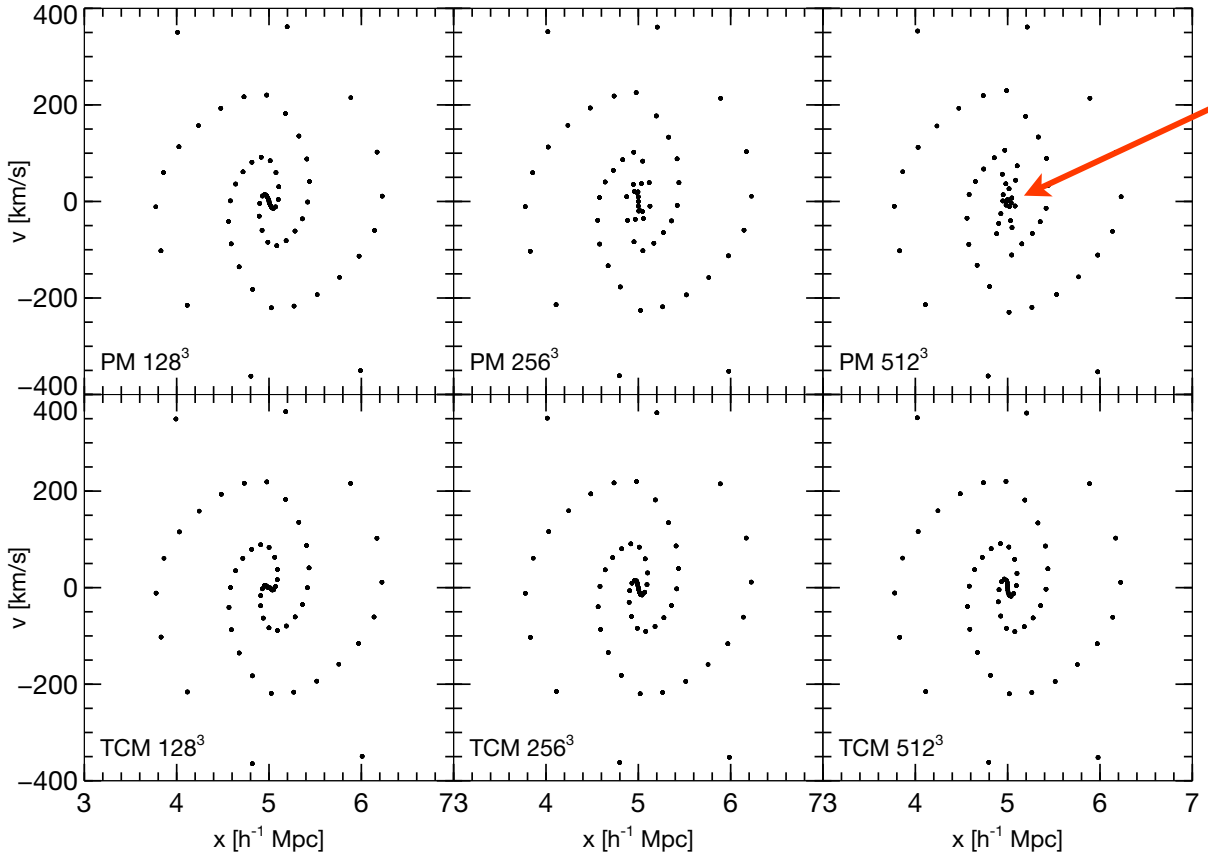
Convergence?



Hahn, Kaehler, Abel 2012, in prep.

Plane wave collapse II

Plane wave long after first shell-crossing

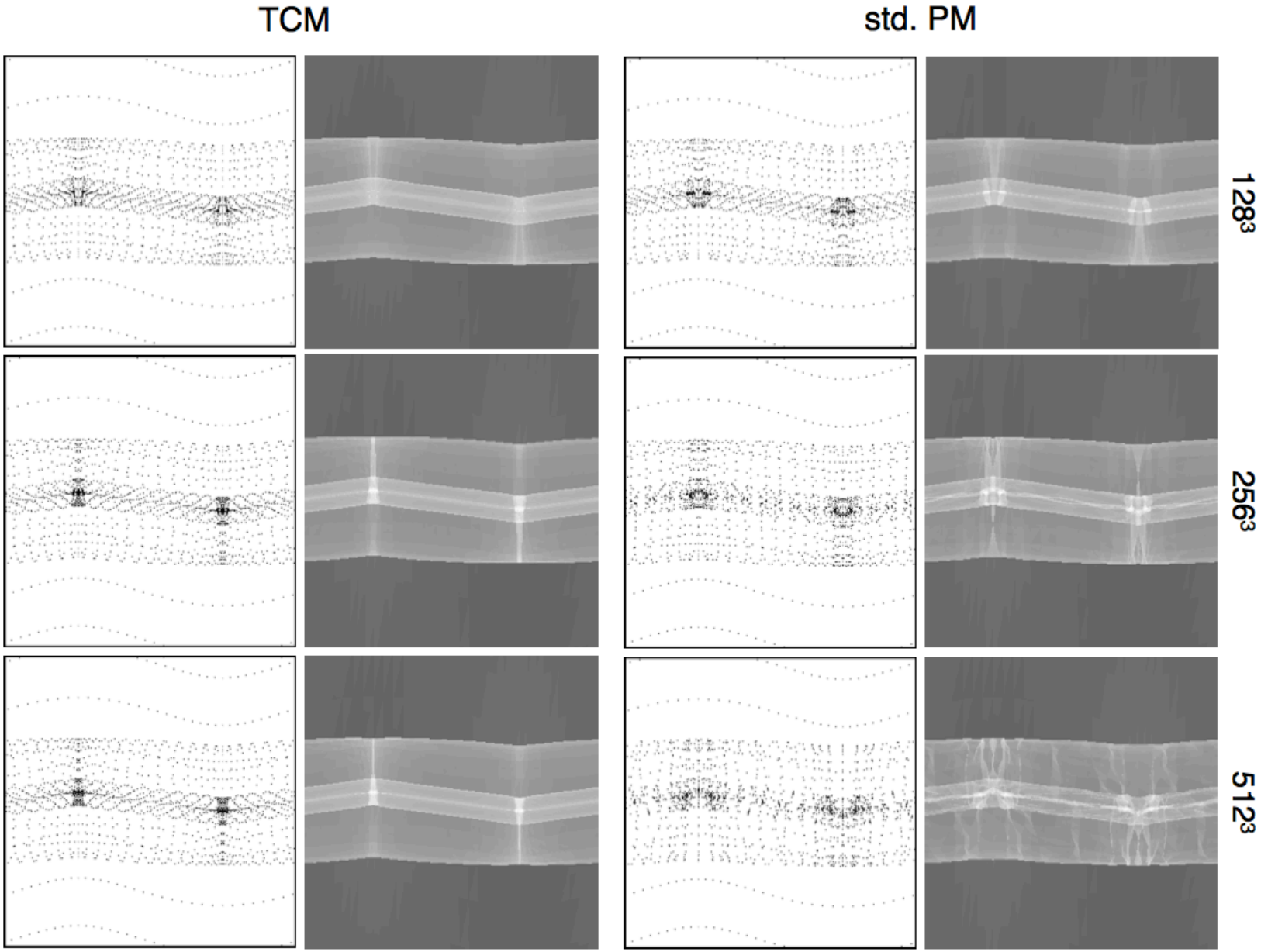


two-body effects

force res ≈ 8x mass res

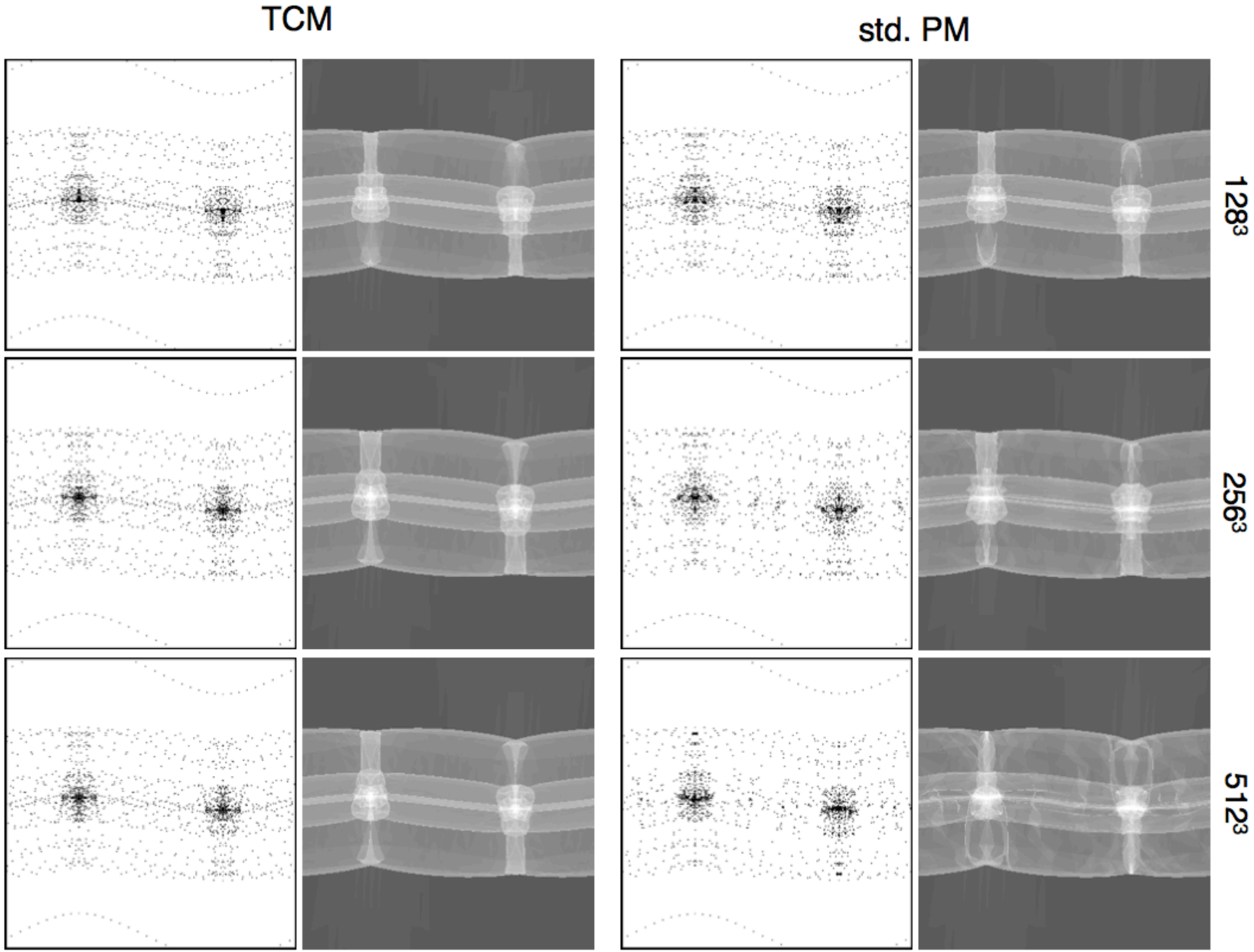
Antisymmetrically perturbed pancake I

Valinia et al. 1997 **early stage, shortly after shell-crossing in 2nd dim.**

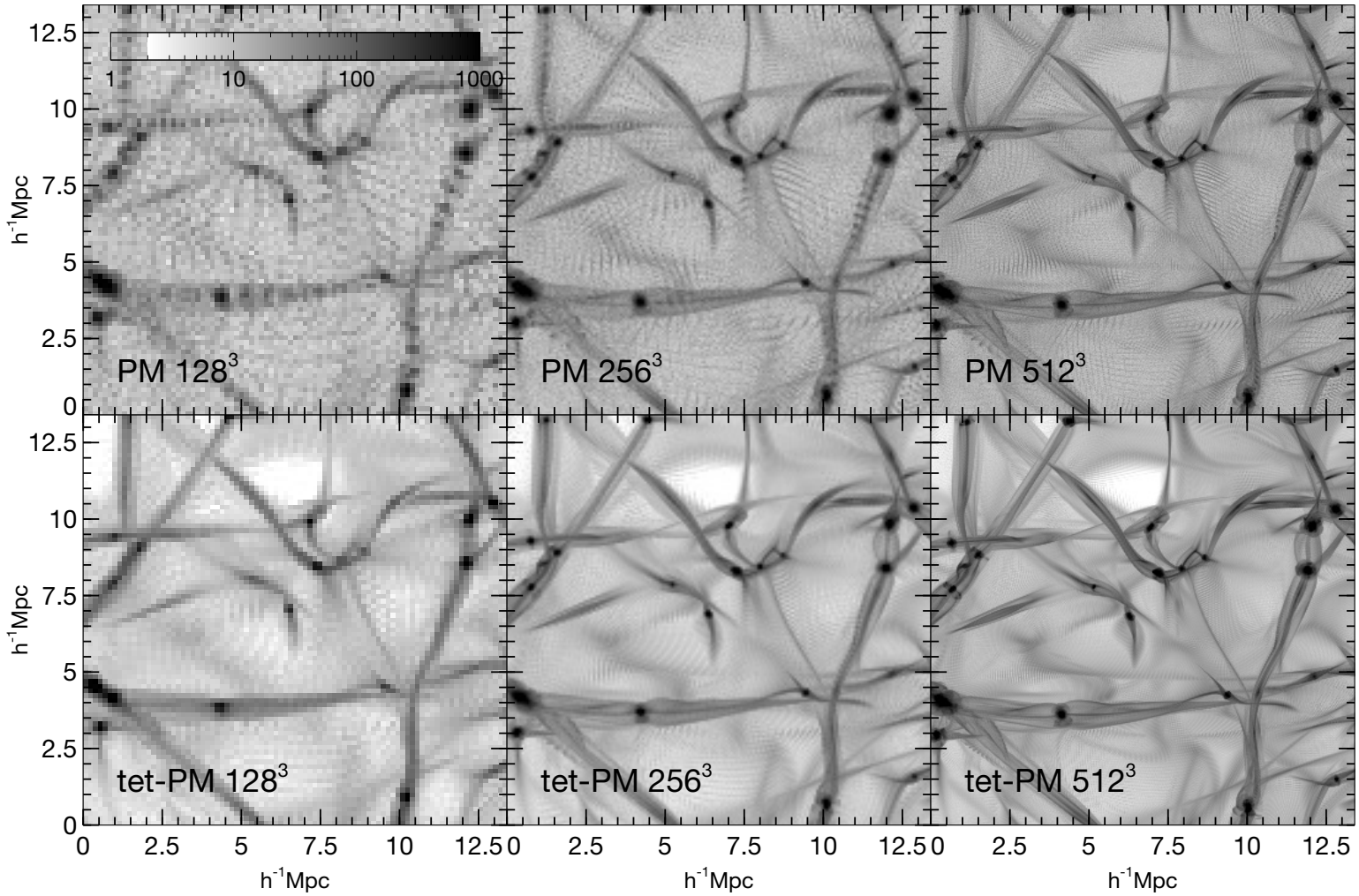


Antisymmetrically perturbed pancake II

Valinia et al. 1997 **late stage**



55eV warm/hot cosmological run

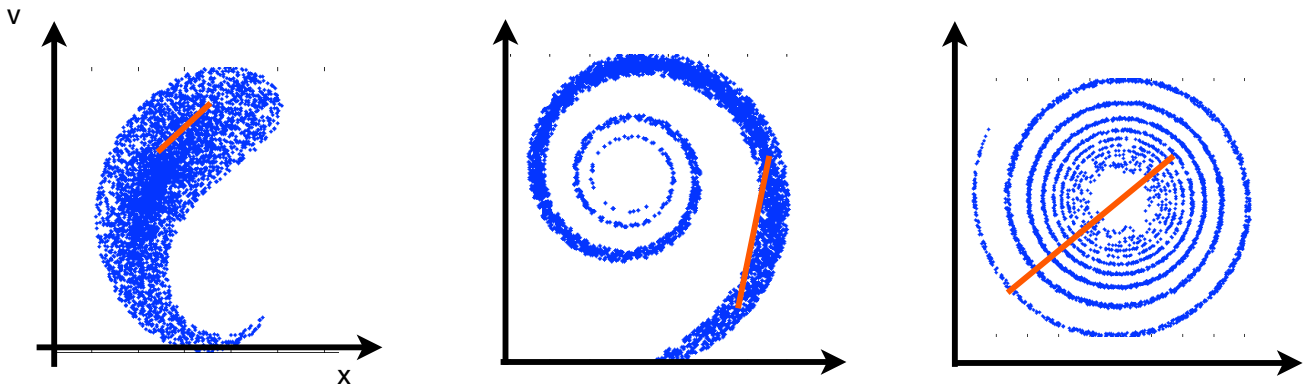


no fragmentation visible

mass functions problematic,
halo finding needs some work ...

So, it's totally awesome or what?

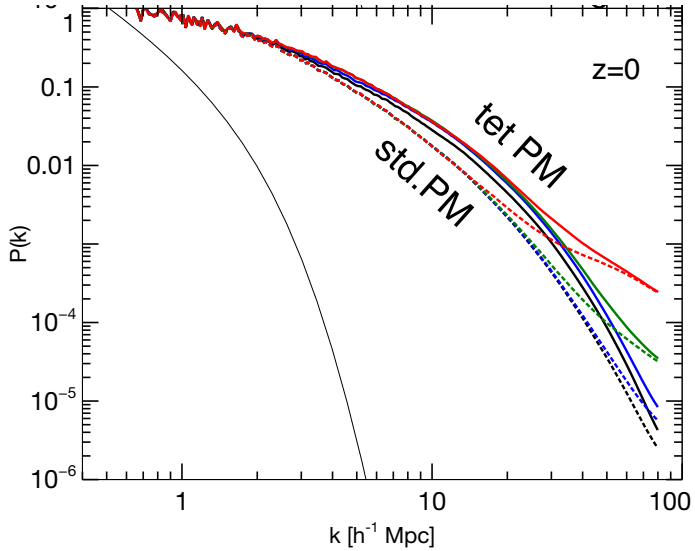
Mixing
need increasingly larger number of elements to trace the surface



otherwise bias density towards center

power spectra too high and convergence problems due to halo profiles

will need refinement



#particles

64^3
128^3
256^3
512^3

Summary

- Dark matter occupies 3D sheet in 6D phase space
- Can reconstruct this from existing sims!
- New ways to analyze LSS:
  - collapsed mass and volume not converged
  - direct access to fine grained phase space structure
  - implications for direct&indirect DM detection -> stay tuned
- Visualization helps to find errors and analyzing in detail the information provided by cosmological N-body simulations
- Can use as density field for new N-body codes
  - Less two-body effects, but convergence issues still... stay tuned!