On the Fate of $\boxed{\text{PV Homogenization}}$ with Magnetic Linkage and its Implication for Jet Formation in M.H.D.

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K.I.T.P. Dynamo Conference
7/14/08

Outline

i.) Motivation:

PV Homogenization → what? / → why?

ii.) Some Theory (with B-linkage)

→ exact results – simple

→ mean eddy + fluctuations – not so simple

iii.) Implications (furthur study)

→ 2D MHD vs 2D Hydro ∞ finite

* → Jet formation and momentum transport

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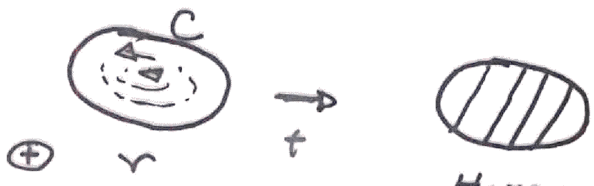
Prandtl, Batchelor '56
Rhines + Young '82

i.) Motivation

→ **WHAT** is PV Homogenization?



Homogeneous field ω within C

- → closed streamline bound
- → nested iso-contours
- → viscosity (diffusion)

⇒ expulsion of $\nabla\omega$ to boundary layer

→ time scales:

$t \to \infty$, ν finite

a) nested iso-contours:

$$\tau_1 \cong \left(\frac{L}{V}\right) Re^{1/3}$$

b.) homogenization:

$$\tau_2 \sim (L/V)\, Re$$

4.

→ Why?

- ubiquitous in 2D advection, $\nabla\cdot\underline{v}=0$

$q = \nabla^2\phi$	2D-hydro
$q = \nabla^2\phi + \beta y$	GFD - β plane
$q = A$	Flux Expulsion
$q = \begin{cases}\phi - \rho_s^2\nabla^2\phi \\ n - \rho_s^2\nabla^2\phi\end{cases}$	Drift Waves

all → $\partial_t q + \underline{v}\cdot\nabla q = \nu\nabla^2 q$, $\nabla\cdot\underline{v} = 0$

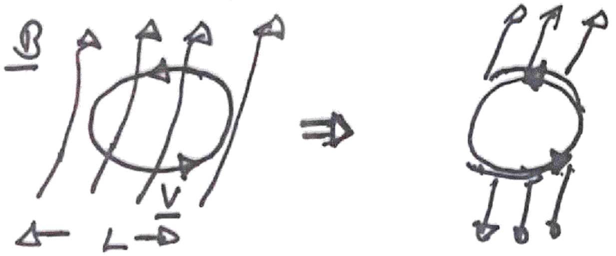
- "Relaxation" Principle ↔ Expulsion
  - → $t\to\infty$ evolution of gyre ↔ general circulation

*→ jets / zonal flows (c.f. Rhines '94)
- extended vortex with $\beta + \langle\omega\rangle' \to 0$
- no local fluctn ↔ zonal avg. enstrophy exchange

→ selective decay ↔ Minimum Enstrophy

5.

- so natural to ask: re: MHD ..... (~~tacho~~cline)



- fate with magnetic linkage ?

- $\delta\Sigma/\delta(\phi) \neq 0$

$= F(\underline{v}, b, \nu, \eta \ldots)$

dependencies ?

- $L^2\,\delta\Sigma/\delta\phi \gtrless 1$ crossing

$\to$ 'homogenization criterion' ?

- Proceed via:
  - 'exact' streamline analysis $\to \nu, \eta$ a la' P-B.
  - mean + fluctuations $\to \nu_T, \eta_T$ ....

Aside:

5a.

Proving the P-B Theorem:

$\left\{ \begin{array}{l} t \to \infty \text{ prior } \nu \to 0 \\ \text{nested streamlines established} \end{array} \right.$

$$(\partial_t + \underline{v}\cdot\underline{\nabla})\,q = \nu \nabla^2 q$$

$$\int_{C_n} dA_n\, \nabla\cdot(\underline{v}\, q) = \int_{C_n} dA_n\, \underline{\nabla}\cdot(\nu \underline{\nabla} q)$$

$$0 = \nu \oint ds\, \hat{n}\cdot\nabla q(\phi_n)$$

$$= \nu \oint ds\, \frac{\delta q(\phi_n)}{\delta\phi_n}\, \hat{n}\cdot\nabla\phi_n$$

$$= \nu\, \Gamma_n \frac{\delta q(\phi_n)}{\delta\phi_n}$$

$$\Rightarrow \boxed{\delta q(\phi_n)/\delta\phi_n = 0, \quad \text{all } C_n}$$

6.

II.) Some Theory

a) Exact Streamlines

$\nabla\cdot\underline{v} = 0$

$\nabla\cdot\underline{b} = 0$

$$(\partial_t + \underline{v}\cdot\nabla) q = \frac{\underline{b}\cdot\nabla j}{4\pi\rho} + \nu \nabla^2 q$$

$$b = \nabla A \times \hat{z}, \quad j = -\nabla^2 A$$

$$(\partial_t + \underline{v}\cdot\nabla) A = \eta \nabla^2 A$$

re-scale; $(\partial_t \to 0)$

$$\boxed{\begin{aligned} \nabla\cdot(\tilde{\underline{v}}\tilde{q}) &= \frac{1}{M^2}\nabla\cdot(\tilde{\underline{b}}\tilde{j}) + \frac{1}{Re}\nabla^2\tilde{q} \\ \nabla\cdot(\tilde{\underline{v}}\tilde{A}) &= \frac{1}{Rm}\nabla^2\tilde{A} \end{aligned}}$$

$Re = \frac{u\ell}{\nu}$, $Rm = \frac{u\ell}{\eta}$

$M \equiv u/v_A(B) \rightarrow$ Mach #

$\ell \rightarrow$ cell size

7.

integration $\Rightarrow$

$$\frac{1}{Re}\oint_c ds\, \hat{n}\cdot\nabla\tilde{q} = \oint_c ds\, \hat{n}\cdot\left(\tilde{\underline{v}}\tilde{q} - \frac{\tilde{\underline{b}}\tilde{j}}{M^2}\right)$$

$\Rightarrow$ normalized PV gradient:

$$\boxed{\frac{\delta\tilde{q}}{\delta\tilde{\phi}} = -\frac{Re}{M^2}\oint_c ds\, \hat{n}\cdot\tilde{\underline{b}}\tilde{j}}$$

useful to define geometrical factor

$$\boxed{g_c = \oint_c ds\, \hat{n}\cdot\tilde{\underline{b}}\tilde{j}}$$

so

$$\left|\frac{\delta\tilde{q}}{\delta\tilde{\phi}}\right| \simeq \underbrace{\frac{B^2\ell}{u\nu}}_{\text{magnitude}} |g_c| \rightarrow \left\{\begin{array}{l}\text{field,} \\ \text{streamline}\end{array}\right. \text{orientation}$$

8.

if employ 'Zeldovich relation':

$$B^2 \simeq R_m B_0^2$$ (relate $B \leftrightarrow$ mean B specified)

$\langle \rangle$

$$\Rightarrow \boxed{\left| \frac{d\tilde{q}}{d\phi} \right| \approx \frac{B_0^2 L^2}{\eta r} = Ha^2}$$

→ cell Hartmann # (predictable!) quantifies cross-field gradient (not a surprise...)

→ magnetic stresses can inhibit viscous PV mixing for sufficient B_0

→ $Ha \sim 1$ is homogenization boundary

9.

- What of g_c? - non-trivial bit...

$$\boxed{g_c = -\oint_c ds\, \hat{n} \cdot \tilde{b}\, \tilde{j}}$$

notable that $g_c \to 0$ for:

1) $\hat{n} \cdot \tilde{b} = 0$ i.e. $\underline{v}$ aligned $\underline{b}$

→ cell magnetically disconnected from surroundings...

$\delta q/\delta\phi \neq 0 \Leftrightarrow$ non-aligned field, flow

→ bending required tension

2) $\tilde{j}$ uniform on C

$$\Rightarrow g_c = -j \oint ds\, \hat{n} \cdot \hat{b} \to 0$$

→ no net magnetic stress on bounding streamline....

concrete (albeit simple) example:

$$\left.\begin{aligned}\phi &= x^2+y^2\\ A &= mx+y\,(x^2+y^2)\end{aligned}\right\}\quad g_\phi = 8\pi m\phi$$

⇒ non-trivial, inhomogeneous PV state

⇒ Comment:

→ simple, clean yet $\boxed{\text{contrived}}$

— exact streamlines,

micro ν, η

vs. more useful

— coarse grained streamlines,

mean field ν_T, η_T

→ can formulate Prandtl–Batchelor in terms either....

b.) Mean-Field Formulation (not so simple..)

→ ~~seek~~ examine homogenization of mean Q, A $(t\to\infty)$

i.e.

$$\begin{cases}\dfrac{\partial Q}{\partial t} + \underline{\nabla}\cdot(\underline{v}\,Q + \overline{\tilde{v}\tilde{q}}) = \underline{\nabla}\cdot(\underline{B}\,j + \overline{\tilde{b}\tilde{j}}) + \nu\nabla^2 Q\\[2ex] \dfrac{\partial A}{\partial t} + \underline{\nabla}\cdot(\underline{v}A + \overline{\tilde{v}\tilde{a}}) = \eta\nabla^2 A\end{cases}$$

take $\overline{\tilde{v}\tilde{q}} = -\nu_T\underline{\nabla}Q$, $\overline{\tilde{u}\tilde{a}} = -\eta_T\underline{\nabla}A$

⇒ de-dim:

$$\begin{cases}\underline{\nabla}\cdot(\underline{v}Q) = \dfrac{1}{M^2}\underline{\nabla}\cdot\left(\underline{B}\,\underline{J} + \dfrac{B_p^2}{\lambda}\overline{\underline{b}j}\right) + \underline{\nabla}\cdot\left(\dfrac{\underline{\nabla}Q}{\tilde{R}_e}\right)\\[2ex] \underline{\nabla}\cdot(\underline{v}A) = \underline{\nabla}\cdot(\nabla A/\tilde{R}_m)\end{cases}$$

small scale str. $(\sim b/B)$

$\lambda = \ell/L$ (flctn / cell) ; $1/\tilde{R}_e = \dfrac{\nu+\nu_T}{VL}$ $\quad M = u/v_A(B)$

$1/\tilde{R}_m = (\eta+\eta_T)/VL$

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as before:

$$\boxed{\frac{\delta\Omega}{\delta\phi} = -\frac{\tilde{R}e}{M^2}\oint ds\,\hat{n}\cdot\{\underbrace{\underline{B}\,j}_{\text{mean field torque}} + \underbrace{\frac{B_p^2}{\lambda}\overline{\tilde{b}\tilde{j}}}_{\text{flctns. (mean square)}}\}}$$

but $\rightarrow$ $\hat{n}\cdot\underline{B}\,j = \hat{n}\cdot[\hat{z}\times\underline{\nabla}A]\,\nabla^2 A$

$$= -\left(\frac{\partial A}{\partial s}\right)\nabla^2 A$$

$\hookrightarrow$ along streamline gradient

but $\rightarrow$ $\underline{V}\cdot\underline{\nabla}A = \frac{1}{\tilde{R}_m}\nabla^2 A$ {from flux advection

$$\Rightarrow V^*\frac{\partial A}{\partial s} = \frac{\nabla^2 A}{\tilde{R}_m} = \frac{J}{\tilde{R}_m}$$

and $\frac{\partial A}{\partial s} = \frac{1}{\tilde{R}_m}\,J/|\nabla\phi|$

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$*$ d-less

skipping further crank $\Rightarrow$

$$\boxed{\left|\frac{\delta\Omega}{\delta\phi}\right| = \frac{1}{M^2}\frac{\tilde{R}e}{\tilde{R}_m}\langle J_*^2\rangle_\phi + \frac{B_p^2}{\lambda M^2}\frac{\tilde{R}e}{R_m}\langle \overline{\tilde{J}_*^2}\rangle_\phi}$$

$\hookrightarrow$ no η_T dep!

$\rightarrow$ a more complex story ....

$\rightarrow$ M^2, B_p^2, λ, P_m, P_{m_T} all enter

$\leftrightarrow$ coupling to OH dissipation...

$\rightarrow$ a "simple problem" is no longer "simple" .....

$\rightarrow$ future numerical studies.....

14.

iii.) Implications → Further Study

(a.) → 2D Hydro vs 2D MHD

- well known "weak" B_0 → large $\langle \tilde{B}^2 \rangle$ → "magnetize" 2D turbulence
- quenching η_T, enstrophy cascade ⇒ 2D MHD cascade ....
- how weak is "weak"?

⇒ for cellular flow (ℓ) →

$Ha \gtrsim 1$ ⇒ $B_0^2 > \nu M / \ell^2$?!

ν dependence not yet investigated.

(b.) Jets and Angular Momentum Transport

→ tachocline : arena for GFD → GMHD → PV homogenization

15.

→ recent studies { P.D., et. al. '07 (C.U.P. Volume); Tobias, P.D., Hughes '07 Ap. J. }

⇒ weak fields "Alfvenize" Rossby wave turbulence

⇒ magnetic stresses inhibit jet formation for $B_0^2/n > (B_0^2/n)_{crit}$

↔ Many questions : (see Tobias, tomorrow PM)

- breakdown of PV homogenization in jet vortex? → field, scale dependence criterion
- relation to $Ha \sim 1$ cross-over scaling?
- behavior of partially Alfvenized cases?

↔ ideal application of mean field calculation ...

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Note: Conservation energy between ZF and DW

RPA equations

DW $$\frac{\partial}{\partial t}\left|\tilde{V}_{\mathrm{DW}}\right|^2 + \sum_k \left(\gamma_{\mathrm{L},k} + C_k(N)\right)\left|\tilde{V}_{\mathrm{DW},k}\right|^2 = \frac{2}{B^2}\sum_{q_x}\int d^2k \frac{q_x^2 k_\theta^2 k_x \left|V_{\mathrm{ZF},q}\right|^2}{\left(1+k_\perp^2\rho_s^2\right)^2} R(k,q_x)\frac{\partial\langle N\rangle}{\partial k_x}$$

ZF $$\left(\frac{\partial}{\partial t} + \gamma_{\mathrm{damp}}\right)\left|V_{\mathrm{ZF}}\right|^2 = -\frac{2}{B^2}\sum_{q_x}\int d^2k \frac{q_x^2 k_\theta^2 k_x \left|V_{\mathrm{ZF},q}\right|^2}{\left(1+k_\perp^2\rho_s^2\right)^2} R(k,q_x)\frac{\partial\langle N\rangle}{\partial k_x}$$

Coherent equations — Pump (S)

DW $$\frac{dP^2}{d\tau} = P^2 - 2\,P\,Z\,S\cos(\Psi)$$ (S: beat wave)

ZF $$\frac{dZ^2}{d\tau} = -\frac{\gamma_{\mathrm{damp}}}{\gamma_L} Z^2 + 2\,P\,Z\,S\cos(\Psi)$$

$$\boxed{\frac{\partial}{\partial t} W_{\mathrm{d}}\Big|_{\text{by ZF}} = -\frac{\partial}{\partial t} W_{\mathrm{ZF}}\Big|_{\text{by DW}}}$$

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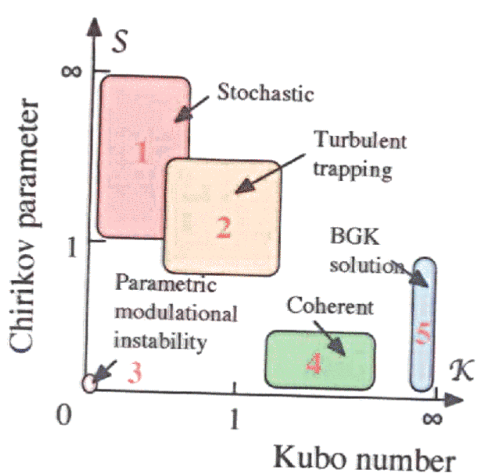

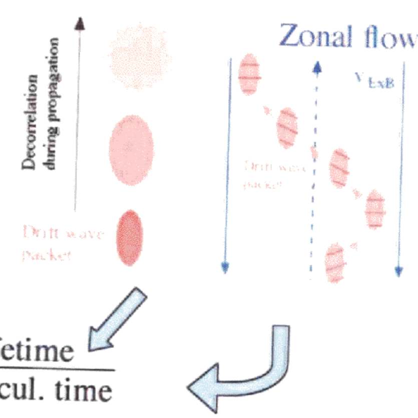

$$\frac{\text{DW lifetime}}{\text{Packet-circul. time}}$$

Regime	Keywords
1	k_r Diffusion
2	Turbulent trapping
3	Single wave modulation
4	Reductive perturbation
5	DW trapping in ZF

References

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Balescu, Itoh
Sagdeev, Hasegawa, Chen, Zonca
Taniuti, Weiland, Champeaux
Kaw, Smolyakov, PD

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Self-regulating System Dynamics

Simplified Predator-Prey model

DW $\frac{\partial}{\partial t}\langle N\rangle = \gamma_L \langle N\rangle - \gamma_2 \langle N\rangle^2 - \alpha \langle U^2\rangle\langle N\rangle$

ZF $\frac{\partial}{\partial t}\langle U^2\rangle = -\gamma_{damp}\langle U^2\rangle + \alpha\langle U^2\rangle\langle N\rangle$

Cyclic bursts

$\gamma_2 \to 0$

(No self damping)

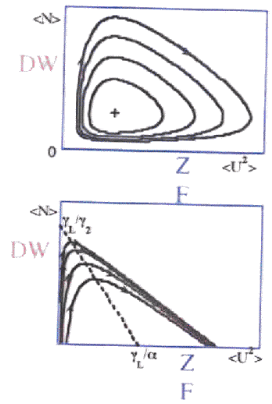
Single burst

(Dimits shift)

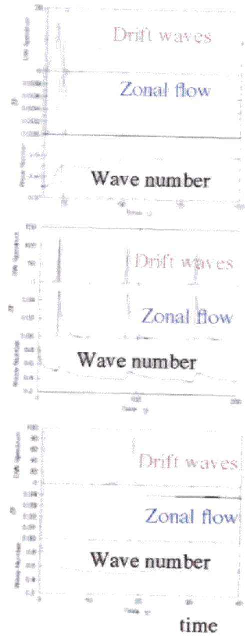
$\gamma_{damp} \to 0$

(No ZF friction)

transport quenched



Stable fixed point



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Self-regulation: Co-existence of ZF and DW

$$\begin{cases} \frac{\partial}{\partial t} W_d = \gamma\left[\nabla n_0, \cdots\right] W_d - \alpha W_d W_{ZF} \\ \frac{\partial}{\partial t} W_{ZF} = \gamma_{damp}\left[\cdots\right] W_{ZF} + \alpha W_d W_{ZF} \end{cases}$$

W_d : drift wave energy

W_{ZF} : zonal flow energy

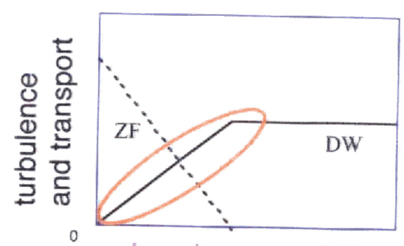
(v_{ii}, q, ε geometry) (+ rf, etc.)

$$\boxed{W_d \sim \frac{\gamma_{damp}}{\alpha}}$$

Transport coefficient

$$\boxed{\chi_i \sim \frac{\gamma_{damp}}{\omega_{eff}} \chi_{gB} \Rightarrow \chi_i = \mathcal{R}\chi_{gB}}$$

"$\mathcal{R}$ – Factor"



e.g. Carreras Lin

Confinement enhancement

Includes other reduction effects

(i.e., cross phase)

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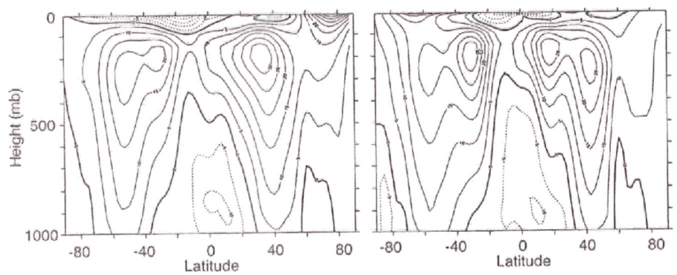
II.) Atmospheric Jets (c.f. G. Vallis '06)

→ persistent feature in atmospheric wind pattern

⇐ westward mid-latitude jet

⇒ eastward sub-tropical jet



subtropical:
– $\nabla T)_{\text{merid.}}$ driven

mid-latitude:
– less structure, shear

(*)
→ How does dipole shear form ?
– symmetry breaking ?
– structure ?

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→ Minimalist Understanding

– subtropical excitation ⇒ { wave radiation

– outgoing waves ⇒ $\phi \sim e^{ik_y y} \to 0$, $y \to \infty$

$\omega \to \omega - i\nu$

$\delta k_y = i\nu / V_{gr,y}$ $V_{gr,y} = \dfrac{2\beta k_x k_y}{(k^2)^2} > 0$

$$\boxed{k_x k_y > 0}$$

– but momentum flux

$$\langle \tilde{v}_y \tilde{v}_x \rangle = -\sum_k k_y k_x |\phi_k|^2 < 0$$

Key Point: Energy Radiation
↔ Momentum Convergence

– Rossby waves "backward"

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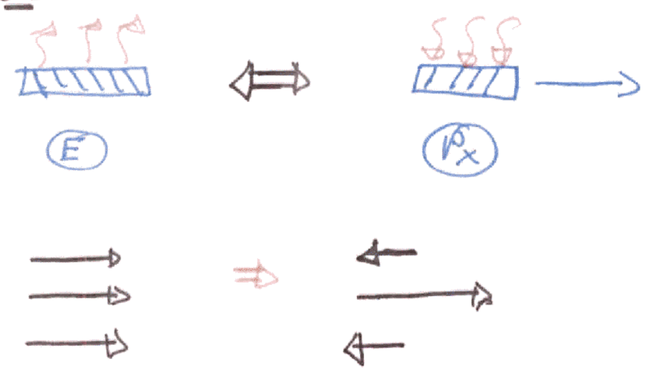
→ cont'd

$\boxed{\text{Energy Radiation} \leftrightarrow \text{Momentum Convergence}}$

- i.e.



- form dipole via:
  - momentum influx locally boosts eastward flow ⇒ subtropical jet
  - resulting momentum deficit generates westward mid-latitude jet

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→ Collisionless Saturation of Zonal Flows

- if all collisional drag, diffusion → 0

$$\partial_t \{ \langle V_\theta \rangle - \rho_\theta \, \rho_s \} = \langle \tilde{v}_r \tilde{n} \rangle - \frac{1}{\langle u \rangle'} \partial_r \langle \tilde{v}_r \tilde{u}^2 \rangle$$

⇒ stationarity:

$$\underbrace{\langle \tilde{v}_r \tilde{u}^2 \rangle}_{\text{potential enstrophy flux}} \sim \int dr \, \underbrace{\langle \tilde{v}_r \tilde{n} \rangle \langle u \rangle'}_{\text{production by } \Gamma_0,\ \langle u \rangle'}$$

- calculating $\langle \tilde{v}_r \tilde{u}^2 \rangle$ non-trivial (c.f. Gurcan, P.D., Hahm '06)
  → $\langle \tilde{u}^2 \rangle$ not even close to passive tracer
  → $\langle \tilde{v}_r \tilde{n}^2 \rangle \neq \langle \tilde{v}_r \tilde{\omega}^2 \rangle$

- What of ZF KH Instability?
  $\langle u \rangle' \to 0$ is signature ....

(17)

→ Potential Enstrophy Flux

– novel feature: spreading of specific quantity

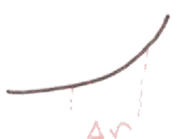
– origin Z.F. $\left\langle \begin{array}{c} \langle \tilde{q}^2 \rangle \\ \updownarrow \\ d\langle u \rangle / dr \end{array} \right.$ $\Rightarrow \left\{ \begin{array}{l} \text{Reynolds} \\ \text{Force} \end{array} \right.$

∴ ⇒ transport $\langle \tilde{q}^2 \rangle$ must alter flow (akin J_h in dynamo)

⇒ for levels a la' MLT (with L_ε), NOT SMALL

– jumps in $\langle \tilde{v}_r \tilde{q}^2 \rangle \Rightarrow \left\{ \begin{array}{l} \text{Shear} \\ \text{Layers} \end{array} \right.$

ε



$$\Delta r \frac{d\langle u \rangle}{dr} \left(\partial_t \langle V_\theta \rangle \right) \approx \langle \tilde{v}_r \tilde{q}^2 \rangle \Big|_{x_-}^{x_+}$$

– feedback loop

a.) seed shear → $\Delta \langle \tilde{v}_r \tilde{q}^2 \rangle \rightarrow \partial_t \langle V_\theta \rangle \neq 0$

b.) $\langle V_\theta \rangle' \rightarrow$ enhanced $\Delta \langle \tilde{v}_r \tilde{q}^2 \rangle$

– especially relevant to edge

(15.)

→ Zonal Flow Structure

– stationary, standard regime

$$\boxed{\langle V_\theta \rangle = \frac{1}{\nu} \left\{ \Gamma_0 - \frac{1}{\partial \langle u \rangle / \partial r} \left(D_0 \langle (\nabla \tilde{q})^2 \rangle + \partial_r \langle \tilde{v}_r \tilde{q}^2 \rangle \right) \right\}}$$

– exact result; in terms macroscopics

– flow structure $\left\langle \begin{array}{l} \rightarrow \text{dissipation profile} \\ \rightarrow \text{enstrophy spreading} \end{array} \right.$

– Zonal Flow Shear:

$$\langle V_\theta \rangle' \cong -\frac{\nu'}{\nu^2} \Gamma_0 - \frac{1}{\nu \langle u \rangle'} \left\{ D_0 \langle (\nabla \tilde{q})^2 \rangle + \partial_r \langle \tilde{v}_r \tilde{q}^2 \rangle \right\}'$$

– shear ↔ ν', $D_0 \langle (\nabla \tilde{q})^2 \rangle'$, spreading

– $\langle V_\theta \rangle'$ up → $\langle \tilde{u}_r \tilde{n} \rangle$ drops → fixed Γ_0 demands $D_0\, \partial \langle n \rangle / \partial r$

∴ collisional $\left\{ \begin{array}{l} \text{particle} \\ \text{heat} \end{array} \right.$ transport critical for flow dynamics near marginal.

(14.)

→ "No-Slip" Momentum Theorem (H-W)

$$\partial_t \left\{ \langle V_\theta \rangle \underset{P_{rel}}{-} \left(- \frac{\langle \tilde{u}^2 \rangle}{d\langle u \rangle / dr} \right) \right\} + \underset{drag}{\nu \langle V_\theta \rangle}$$

$$= \underbrace{\langle \tilde{v}_r \tilde{n} \rangle}_{\text{driving flux}} - \left(\frac{d\langle u \rangle}{dr} \right)^{-1} \left\{ \underbrace{D_0 \langle (\nabla \tilde{u})^2 \rangle}_{\text{diffusion}} + \partial_r \underbrace{\langle \tilde{v}_r \tilde{u}^2 \rangle}_{\text{transport of Pot. Enstr.}} \right\}$$

- no Reynolds modelling ....
- Similar QG, but:

$$\langle \tilde{v}_r \tilde{n} \rangle = \underbrace{\Gamma_0}_{\text{fixed net flux}} + \underbrace{D_0 \, \partial \langle n \rangle / \partial r}_{\text{collisional flux}}$$

is driver.

collisional flux → negligible but for ITB, Dimits

- Pseudomomentum $\sim \langle \tilde{u}^2 \rangle$ independent $k_\parallel^2 D_\parallel / \omega_k$, etc.

no restrictions ....

(13.)

→ Momentum Theorem

- observe: $\left\{ \begin{array}{l} \text{3D system} \\ \text{Conserved PV} \leftrightarrow \text{2D dynamics} \end{array} \right.$ but:

$$\left\{ \begin{array}{l} \boxed{u = \nabla^2 \phi - n} \\ \dfrac{du}{dt} = D_0 \nabla^2 u \end{array} \right. \qquad \frac{d\langle u \rangle}{dr} = \frac{d}{dr} \langle \nabla^2 \phi \rangle - \frac{d\langle n \rangle}{dr}$$

2D evolution eqn.

- Potential Enstrophy Balance:

PV flux

$$\partial_t \langle \tilde{u}^2 \rangle + \frac{\partial}{\partial r} \langle \tilde{v}_r \tilde{u}^2 \rangle = -\langle \tilde{v}_r \tilde{u} \rangle \, d\langle u \rangle / dr - D_0 \langle (\nabla \tilde{u})^2 \rangle$$

(as before)

but:

$$\partial_t \langle V_\theta \rangle = \langle \tilde{v}_r \nabla^2 \tilde{\phi} \rangle - \nu \langle V_\theta \rangle$$

only vorticity flux evolves zonal flow

∴

- now momentum conservation coupled to transport ......

(12.)

II.) Zonal Flow Momentum - DWT

→ H-W system ($\perp$ $A = 1$)

$$\frac{dn}{dt} = -D_\parallel \nabla_\parallel^2(\phi - n) + D_0 \nabla^2 n$$

$$\frac{d}{dt}\nabla^2\phi = -D_\parallel \nabla_\parallel^2(\phi - n) + D_0 \nabla^2 \nabla^2 \phi$$

→ minimal relevant system

- 3D, parallel dissipation
- drift wave instabilities
- finite $\langle \tilde{v}_r \tilde{n} \rangle$ → transport

→ simple but detailed,

i.e. $k_\parallel^2 D_\parallel/\omega_* \gtrless 1$, $\langle n\phi \rangle \neq 0$, damped modes .....

→ Zonal Flow structure in H-W?

(11.)

→ Meaning?

- Fluid: 2 coupled component — zonal flow / quasi-particle flow
- $\partial_t P_{rel} = \ldots$ ⇒ "No Slip"

  except by preferential damping or excitation of one component
- absent $\tilde{F}$, 0 ⇒ can't accelerate $\langle v_x \rangle$ with stationary turbulence

→ Stationarity ⇒ Dipole

$$\boxed{\langle v_x \rangle = \frac{1}{\nu\beta^*}\left\{ \langle \tilde{F}\tilde{\omega} \rangle - \mu\langle(\nabla\tilde{\omega})^2\rangle - \partial_y \langle \tilde{v}_y \tilde{\omega}^2 \rangle \right\}}$$

forcing region $\sim \langle \tilde{F}\tilde{\omega} \rangle / \nu\beta^*$ → Eastward jet

viscous damping region ("beach") $\sim -\dfrac{\mu\langle(\nabla\tilde{\omega})^2\rangle}{\nu\beta^*}$ → Westward jet

10.

→ **Extended Charney-Drazin Theorem**

(Charney + Drazin '61 ; Rhines + Holland '79)

$$\partial_t \left\{ \overset{\text{flow}}{\langle V_x \rangle} - \left(- \frac{\overset{\text{pseudomomentum}}{\langle \tilde{\omega}^2 \rangle}}{\beta^*} \right) \right\} + \nu \langle V_x \rangle$$

$$= \underbrace{\frac{\langle \tilde{f} \tilde{\omega} \rangle}{\beta^*}}_{\text{forcing}} - \underbrace{\frac{\mu \langle (\nabla \tilde{\omega})^2 \rangle}{\beta^*}}_{\text{viscous damping}} - \underbrace{\frac{1}{\beta^*} \partial_y \langle \tilde{v}_y \tilde{\omega}^2 \rangle}_{\text{enstrophy spreading}}$$

→ Pseudomomentum ~ Wave Momentum Density (WMD)

- enstrophy → intensity
- β → orientation

- not tied to weak nonlinearity

→ β effect ⇒ zonal acceleration w/o net momentum input

9.

→ Some Theory

- zonal mean flow

$$\partial_t \langle V_x \rangle = - \partial_y \langle \tilde{v}_y \tilde{v}_x \rangle - \nu \langle V_x \rangle$$

$$= \langle \tilde{v}_y \tilde{\omega} \rangle - \nu \langle V_x \rangle$$

Reynolds Force ⟺ Vorticity Flux (Taylor, '15)

- Vorticity Flux ⟺ $\boxed{\text{Enstrophy Balance}}$

$$\partial_t \langle \tilde{\omega}^2 \rangle + \partial_y \langle \tilde{v}_y \tilde{\omega}^2 \rangle + \beta \langle \tilde{v}_y \tilde{\omega} \rangle$$

$$= \langle \tilde{f} \tilde{\omega} \rangle - \mu \langle (\nabla \tilde{\omega})^2 \rangle$$

i.e. Reynolds Force → Production via { Vorticity flux, $\nabla \langle \omega \rangle$ }

∴

- Zonal Momentum Linked to Enstrophy Balance

Configuration Space Approach?

Coming Attractions:

S. Tobias on

"Jet Formation in MHD"

→ Tachocline discussion, next week.