Zonal Flow and Jupiter's Dynamo

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Motivation: how do the zonal flows match onto the magnetic interior?

Jupiter and Saturn have strong zonal flows $\sim 100m/sec$ at the surface.

Strong eastward jet at the equator, alternating westward/eastward bands at higher latitudes.

Are (some of) the jets confined to the stably stratified upper atmosphere or do they penetrate deep into the interior?

If they are deep, what happens when they interact with the dynamo region in the interior?

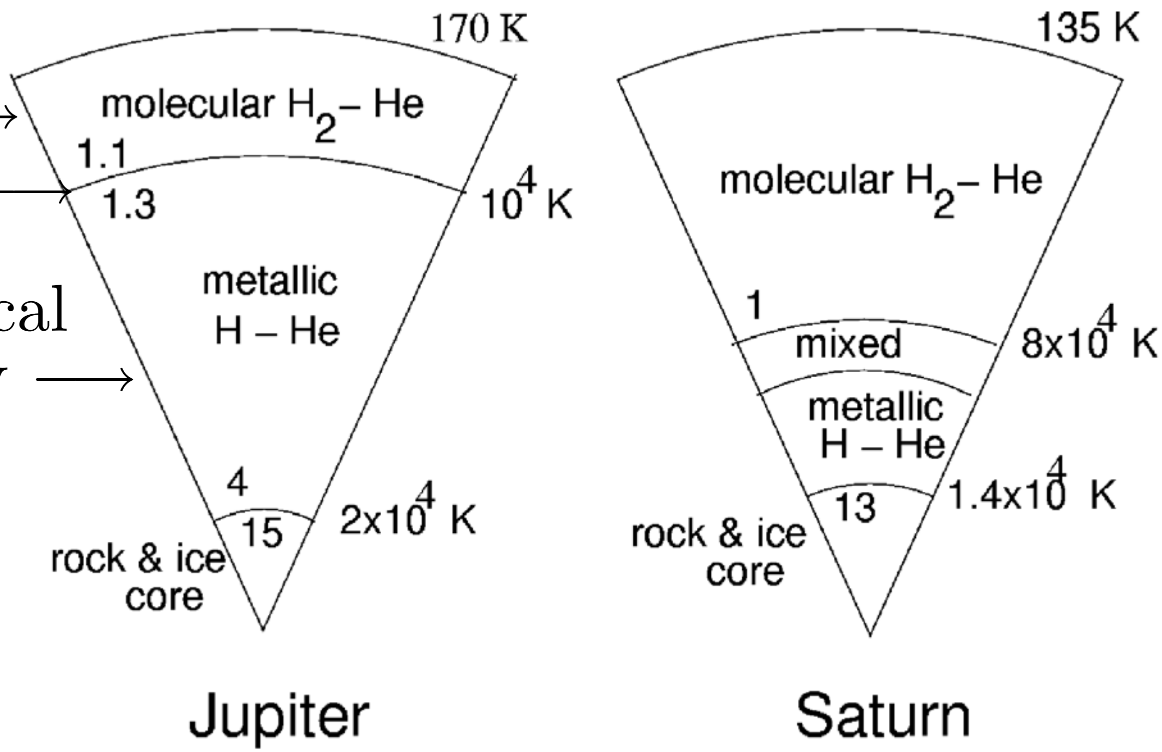
Internal structure of Giants

Magnetic field locks metallic core: typical velocity there 10^{-3} metres/sec compared with 100 metre/sec surface zonal flows. Low core speed from uniformity of magnetic field rotation period, and dynamo estimates.

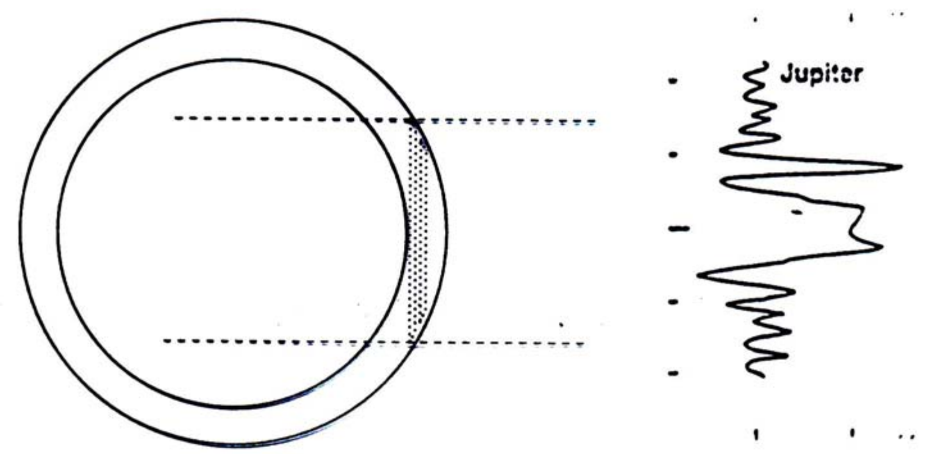
Low electrical conductivity $\longrightarrow$

Transition zone $\longrightarrow$

High electrical conductivity $\longrightarrow$

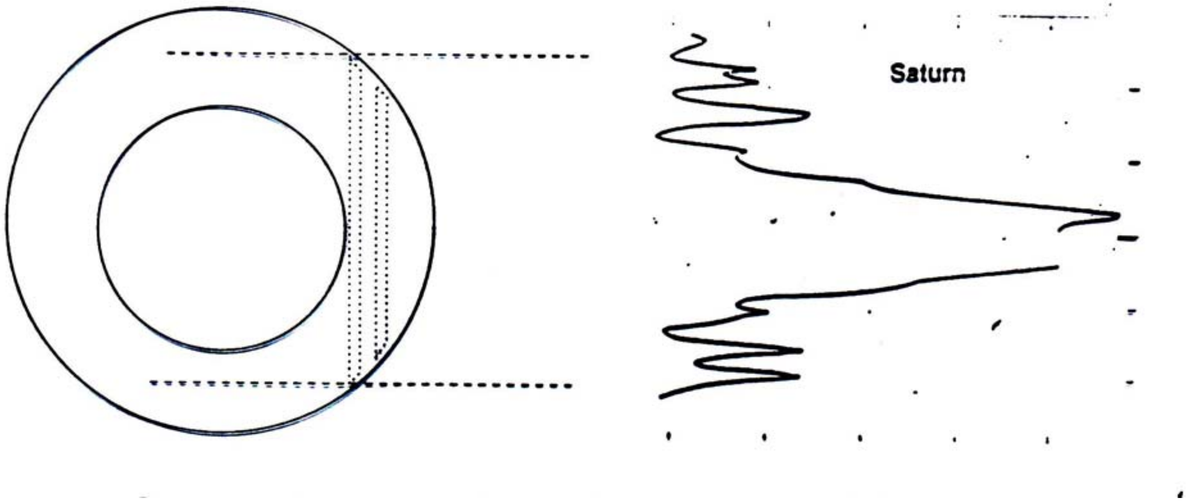


Zonal flows in Giant Planets: banded east-west jets



Are zonal flows deep, 15,000 km, driven by convection in molecular H/He layer, or shallow, confined to stably stratified surface layers?

Jupiter: Large radius ratio, narrowly confined bands



Saturn: Smaller radius ratio, less confined bands

Galileo Probe

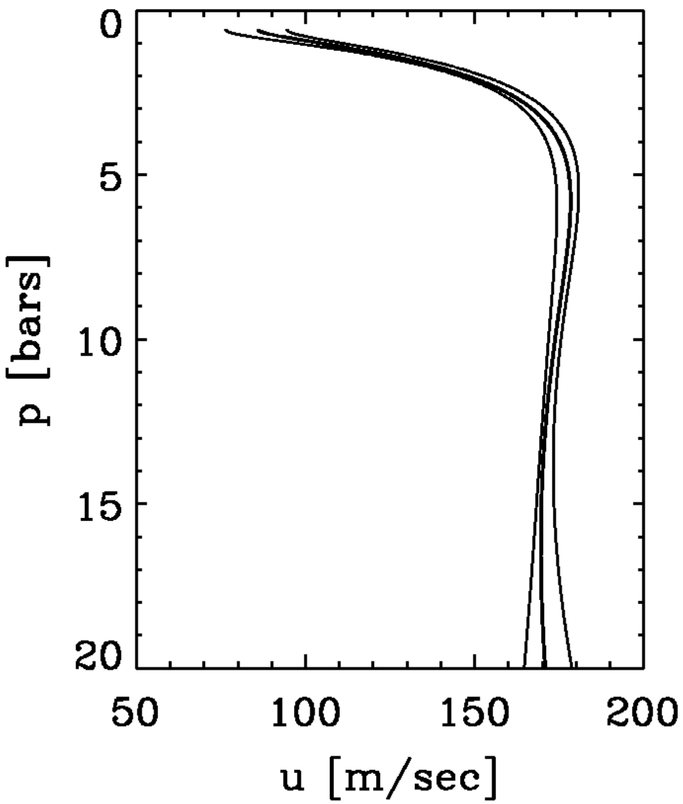


Figure 4. Jupiter's zonal winds at 7.4°N latitude obtained by Doppler tracking of the Galileo Probe signal (Atkinson *et al* 1997). The thick curve is the nominal wind profile and the thin curves bound the uncertainty envelope.

Probe entered 7° N, in eastward equatorial jet. Found velocity increases inward, supporting deep convection model.

Boussinesq spherical convection models

Boussinesq convection models solving the rotating convection equations do a remarkably good job of reproducing Giant Planet zonal flows.

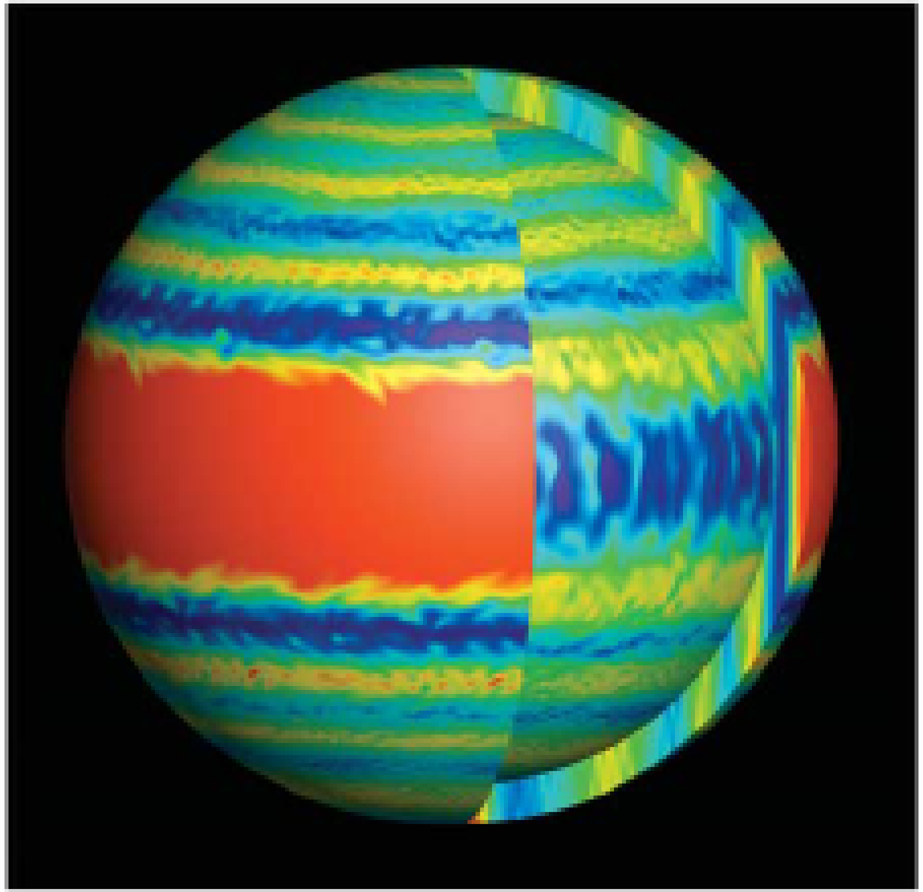
Anelastic compressible models also produce long-lived banded zonal flows, though there is more time-dependent activity near the surface.

However, all these models use a stress-free boundary condition at the bottom of the layer, and no magnetic field, so issue of how zonal flow gets from 100 metres/sec down to 10^{-3} metressec not addressed.

Boussinesq spherical convection model

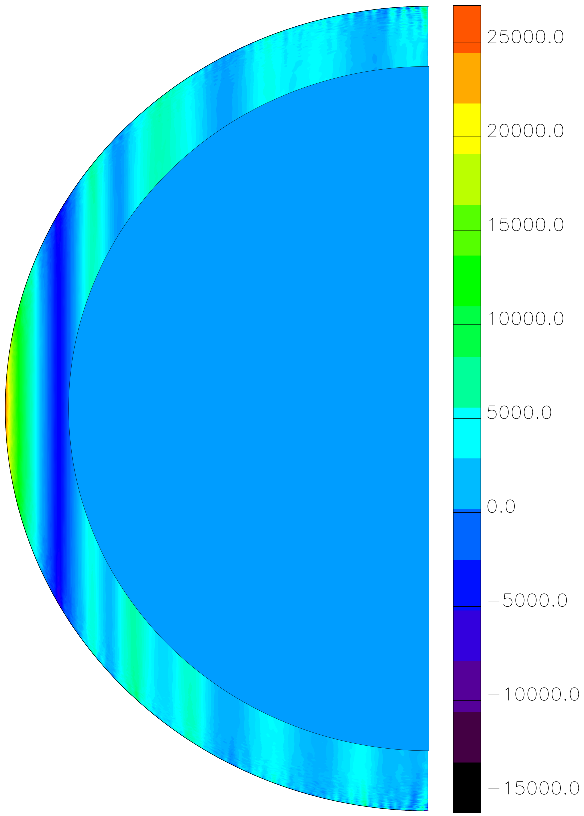
u_ϕ: $E = 3 \times 10^{-6}$, $Pr = 0.1$, $\eta = 0.9$, $Ra \sim 100 Ra_{crit}$

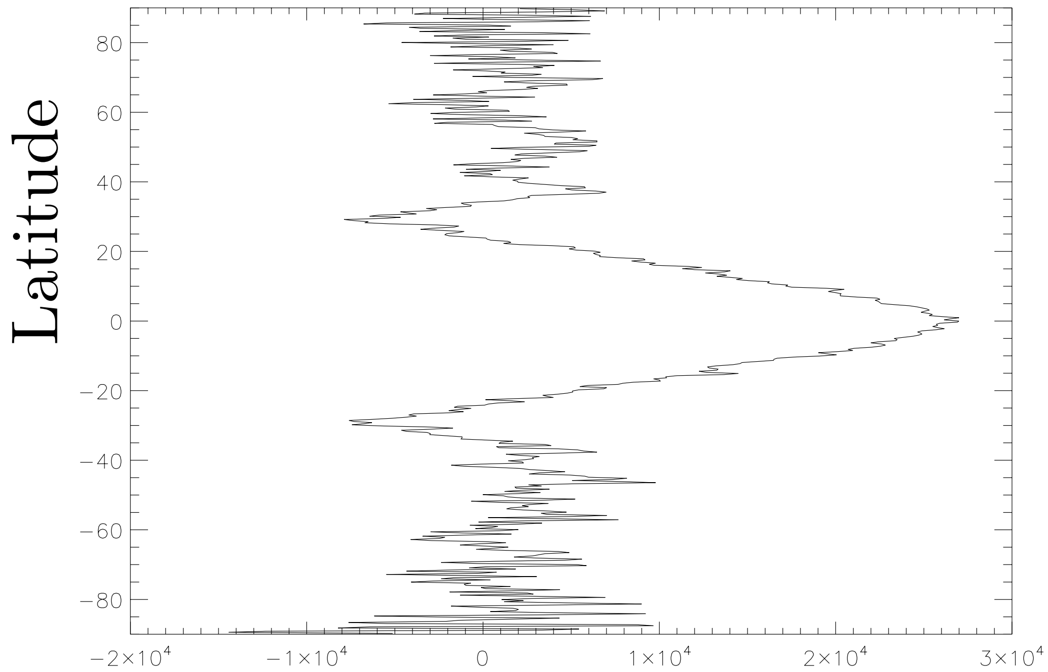
From Heimpel, Aurnou and Wicht, 2005



Zonal Flow: Compressible Case

$E = 3 \times 10^{-6}$, $Pr = 0.1$, $\eta = 0.85$, $N_\rho = 5.0$, $n = 2$





Meridional section   Surface zonal flow

Averaged over ϕ

Dissipation constraints

Liu et al. 2008 pointed out that large zonal flows generate ohmic dissipation when they reach the electrically conducting region if the magnetic field cuts the shearing zonal fow transversely.

Aligned fields can have much less dissipation.

What happens if the ohmic dissipation is the same order as the total heat flux coming out of the planet?

Perhaps surprisingly, it is possible to have the ohmic (or viscous) dissipation much larger than the heat flux through the planet. Work done by buoyancy comes out of the energy flux, and is returned through dissipation. Energy balance gives no constraint, but Entropy balance does provide a constraint.

Entropy balance

In a steady, the entropy balance in the molecular H/He layer is essentially

$$\frac{F_{out}}{T_{top}} = \frac{F_{in}}{T_{bottom}} + \int \frac{Q_{diss}}{T}\, dv + S$$

Here S is the entropy generated in the layer by thermal conduction, and simulations show this is small compared to the other terms if the Nusselt number is large, as expected in giant planet convection.

Q_{diss} is the local rate of viscous and ohmic dissipation. If the total is $\mathcal{Q}_{diss}$, we can define T_{diss} as the average temperature where the dissipation occurs.

Neglecting S and assuming no heat sources in the molecular H/He layer, so $F_{out} = F_{in}$,

$$\frac{F_{out}}{T_{top}} = \frac{F_{out}}{T_{bottom}} + \frac{\mathcal{Q}_{diss}}{T_{diss}}$$

Omitted terms are positive, and $T_{diss} < T_{bottom}$ so

$$\mathcal{Q}_{diss} < F_{out}\left(\frac{T_{bottom}}{T_{top}} - 1\right)$$

(Entropy constraint).

If ohmic dissipation is dominant, expect T_{diss} close to T_{bottom} and given S is small, expect near equality.

$$\left(\frac{T_{bottom}}{T_{top}} - 1\right) \sim 40$$

for Jupiter.

No-slip boundary

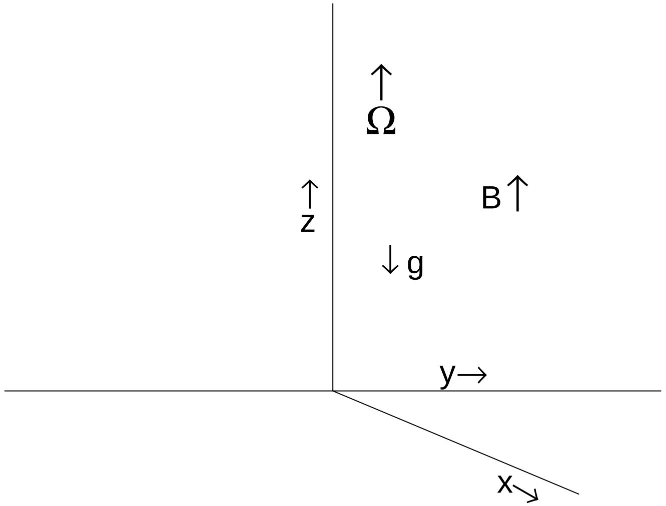
If magnetic field locks interior together, simple model is to apply no-slip boundary at the base of the layer.

In 3D spherical simulations, this destroys the high latitude persistent zonal bands, in both Boussinesq and compressible cases.

Ekman boundary layer pumps fluid close to boundary, where friction destroys zonal flow.

However, this is an $O(E^{1/2})$ effect, not so small in simulations, very small in Jupiter, so this is inconclusive.

Low R_m variable conductivity layer



x eastward, y latitudinal, z radial.

Axisymmetric, independent of x.

$\mathbf{B} = B_0\hat{z}$, High latitude, assume rotating about z-axis. Electrical conductivity $\sigma(z)$, goes to zero as $z \to \infty$

$\mathbf{u} = (u_x, \frac{\partial\psi}{\partial z}, -\frac{\partial\psi}{\partial y})$, $u_x = u_0 \sin ky$ as $z \to \infty$.

$$x - \text{momentum} : -2\Omega\rho\frac{\partial\psi}{\partial z} = \frac{B_0}{\mu}\frac{\partial b_x}{\partial z}$$

$$\text{magnetic wind equation} : -2\Omega\rho\frac{\partial u_x}{\partial z} = \frac{B_0}{\mu}\frac{\partial j_x}{\partial z}$$

together with induction equation gives

$$\frac{d}{dz}\Lambda\frac{d}{dz}\left(\frac{j_x}{\Lambda^2}\right)+\frac{d}{dz}\Lambda\frac{dj_x}{dz}-\frac{k^2}{\Lambda}j_x=0$$

where local Elsasser number

$$\Lambda=\frac{\sigma(z)B_0^2}{2\rho\Omega}$$

Boundary conditions (i) $j_x \to 0$ as $z \to \infty$, and from the Integral of the magnetic wind equation

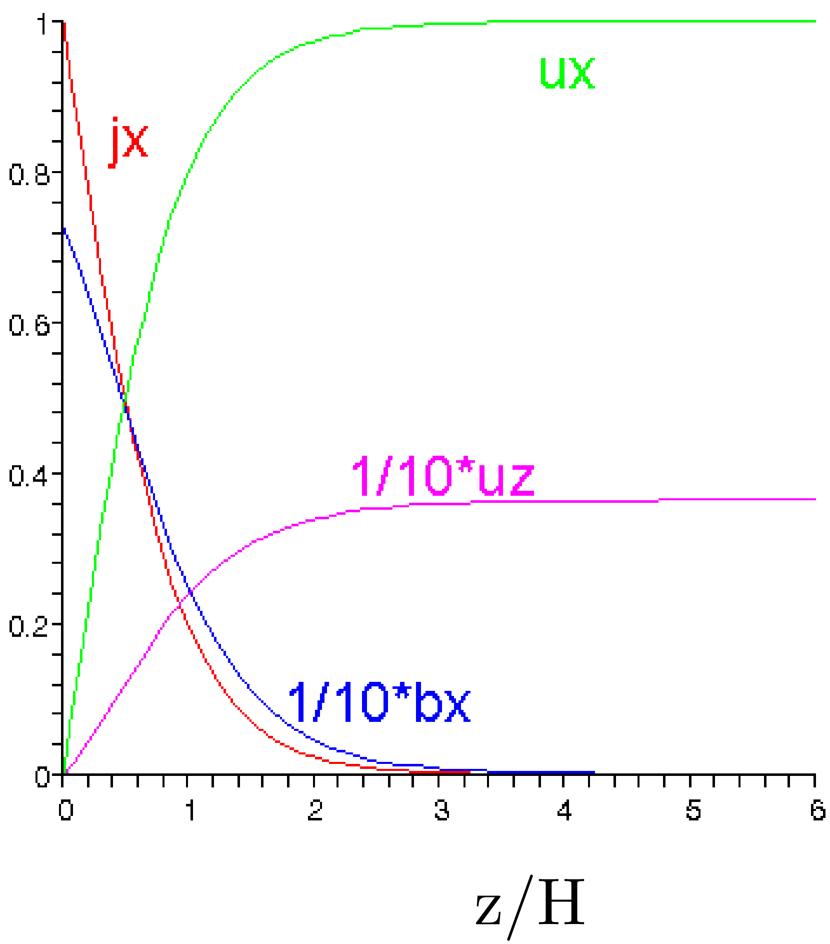
$$j_x=\frac{2\rho\Omega}{B_0}(u_0-u_x)$$

so if zonal flow $u_x=0$ on some level $z=0$,

$$\text{(ii)} \qquad j_x=\frac{2\rho\Omega}{B_0}u_0$$

As in Ekman layers, there is a suction, i.e. u_z is finite as $z \to \infty$. To get this, need to assume $\psi=0$ on $z=0$.

Results from Model



Current and b_x decrease rapidly as conductivity gets small. Change in zonal flow occurs where the current exists.

$\Lambda = \Lambda_0 \exp(-z/H)$
$kH = 0.5$, $(H \sim 300$ km$)$,
$\Lambda_0 = 1$

Issues raised by the Model

Ohmic dissipation is large, because u_x is large. Strong constraint to get dissipation below the entropy limit.

Helps if H is small (conductivity falls off very rapidly in z) and $\Lambda_0 = \sigma_0 B_0^2 / 2\Omega\rho$ large.

Another problem is the suction, which provides scale independent damping of the jets: much larger than Ekman suction, because variable conductivity layer is thicker than an Ekman layer.